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Stiffness Tailoring of Elliptical Composite Cylinders for Axial Buckling Performance

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Abstract

Automated fiber placement (AFP) machines have made it possible to tailor the material stiffness properties in a laminated composite structure by fiber steering. The so-called variable stiffness (VS) laminate can be designed to improve the structural performance of a composite component. Herein, using a metamodeling based design optimization (MBDO) method, elliptical composite cylinders with VS laminate are designed and optimized for maximum axial buckling capacity. The effect of cross-sectional aspect ratio of the elliptical cylinders on the potential improvement of the buckling capacity is also investigated. As the baseline for comparison, for each cross-sectional aspect ratio, the buckling capacity of elliptical cylinders with quasi-isotropic (QI) and optimum constant stiffness (CS) laminates are also calculated. It is found that the buckling capacity of an elliptical composite cylinder can be improved by fiber steering up to 118% compared with its best CS counterpart.

Keywords: Stiffness tailoring, Variable stiffness, Elliptical composite cylinder, Buckling, Design optimization

1. Introduction

Thin-walled laminated composite shells and panels are widely used in aerospace applications. In aerospace structures, any weight savings through reduction of skin thickness must be balanced against the requirement for structural stability; therefore, it is of a great importance to study the buckling performance of composite structures for aerospace applications. In this regard, composite cylinders with circular cross-sections have mostly been studied in the literature due to their axisymmetric geometry that makes their analysis and design simpler. However, aerodynamic or other geometric considerations could dictate use of non-circular cross sections for aerospace applications such as in blended wing body aircraft designs. As a consequence, the non-uniformity of the curvature makes flatter portions with large radius of curvature more susceptible to buckling than other parts that have small

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radius of curvature, resulting in a considerably reduced buckling capacity [1]. From another point of view, this reduced buckling performance can be attributed to the inefficient use of material in such structures. Therefore, efficient tailoring of the material properties (e.g. stiffness) in different parts seem to be able to compensate the structural inefficiency resulting in more involvement of the structure in carrying the load and consequently improvement in the structural performance (e.g. buckling capacity).

For metallic materials, material tailoring to improve the structural performance is possible by several ways including varying the wall thickness [2], varying the lattice rib angle of isogrid structures [3], and using curvilinear stiffeners [4]. Although they are practical methods to be used in aerospace structures, they do not benefit from many advantages of composite materials with the directional properties. Combination of all these methods may however manifest in promising highly-tailored solutions not available today.

For fiber reinforced composite materials, stiffness tailoring has become possible by Automated Fiber Placement (AFP) machines that are capable of steering the fiber/tows on the tool surface to continuously change the fiber orientation angles resulting in a laminate with spatially varying stiffness. The resulting so-called variable stiffness (VS) composite laminate offers the designers more scope to use the directional properties of the material more efficiently to produce VS composite cylinders [5, 6, 7, 8, 9] with better structural performance compared with their traditional Constant Stiffness (CS) counterparts. However, this improvement is at the expense of adding more complexity to the design optimization process because of the increased number of design variables. Therefore, developing a computationally efficient design optimization tool to study the potential improvement in the structural performance of VS composite structures is of a great importance. Besides, there are a number of manufacturing limitations and process-induced defects that need to be addressed in successful using of AFP [10] and more specifically fiber steering [11] in VS structures. The defects induced by the in-plane bending deformation such as local wrinkling, fibre misalignment, and irregular thickness distribution due to steering are unavoidable. Therefore, there is a minimum curvature of the tow path beyond that these local defects are not manageable. Gaps and overlaps are other inevitable defects that exist in fiber steered laminates. Several methods including staggering, tow overlap and tow drop techniques have been developed to reduce this defect. However, there still remain some some thickness nonuniformity because of overlaps and resin rich areas that are not covered due to the gaps. They certainly affect the structural performance of VS laminates [12] and should be addressed to fully harness the improvements predicted by design optimization of such structures.

Tatting [13] was among the first researchers who studied the potential buckling improvement of VS composite cylinders. In his study, the axial variation of stiffness in a circular cylinder resulted in a slight improvement of the axial buckling load compared with its CS counterpart. More significant improvements were found for cases that involve section forces that vary circumferentially, i.e. in bending. Therefore, extensive research works have been performed subsequently to improve the bending-induced buckling capacity of circular VS composite cylinders [5, 7, 8, 9]. The inhomogeneity of the section load in the skin arising from overall bending gives designers suitable scope to vary the orientation angle of fibers from the compressive side to the tension side so that the section loads are redistributed more efficiently. As a result, the VS design provides a more effective load transfer path between the loading points and the supports so that the directional properties of the fibers are fully exploited for maximum buckling capacity.

For an objective different from the buckling performance, White and Weaver [14] have studied the potential of stiffness tailoring to achieve bend-free states in elliptical cylinders and ellipsoid of revolutions under internal pressure. Their results showed that for elliptical cylinders with considerably large aspect ratio, it is possible to tailor the stiffness to have completely bend-free states. In case of the buckling performance of non-circular cylinders, a few studies have shown considerable potential improvement for elliptical VS composite cylinders made by fiber steering [1, 6, 15] compared with their CS counterparts. Sun and Hyer [1] considered an elliptical cylinder with a cross-sectional aspect ratio of 0.7 and a quasi-isotropic laminate as the baseline case for which they could improve the buckling capacity by stiffness tailoring up to 30% for what they considered as small cylinders and 35% for the larger cylinders. In a following work, Lo and Hyer [16] studied the fundamental frequencies of VS cylinders and their results revealed a negligible sensitivity of the fundamental frequencies of VS elliptical cylinders compared with their QI counterparts. Khani et al. [6], however, considered the best CS design as the baseline case. As a result, the buckling load improvement for a VS design over its best CS counterpart was found to be 17.9% for linear and 21.3% for nonlinear analysis in their study. Khani et al. [15] also improved the buckling capacity of a longitudinally stiffened elliptical composite cylinder subjected to bending moment by fiber steering. In all cases, the results showed that the reduction of the buckling capacity of the elliptical cylinders compared with a circular one can be compensated to a great extent by using the continuously varying fiber orientation.

To the best of the authors' knowledge, the effect of the cross-sectional aspect ratio on the potential improvement in the buckling capacity of elliptical VS composite cylinders has not been studied yet. In this paper, for elliptical composite cylinders with different cross-sectional aspect ratios, the optimum constant orientation angles of the laminate for maximum axial buckling is determined first. Afterwards, for VS design, the optimum fiber orientation angle distribution in the circumferential direction is calculated. For both cases, a multi-step surrogate based design optimization method [7, 8, 9] is used. Finally, the potential improvement of the buckling capacity for the VS design compared with its counterparts with constant stiffness (CS) and quasi-isotropic (QI) laminates is studied for different cross-sectional aspect ratios.

It should be noted that the buckling behavior is sensitive to any manufacturing subtleties or imperfections which is as yet quantitatively unknown for elliptical cylinders. Therefore, the linear analysis may overpredict the buckling capacity of elliptical cylinders. However, as a first step in analysing VS elliptical cylinders a design optimization based on linear analysis has much utility and could lead to further studies into the postbuckling regime. It is worth noting that elliptical cylinders are expected to be less imperfection sensitive than circular cylinders because of the fact that the inherent axisymmetry -a known contributing factor for imperfection sensitivity- is broken itself. For further understanding and discussion on postbuckling behavior of VS composite shells see [17, 18, 19].

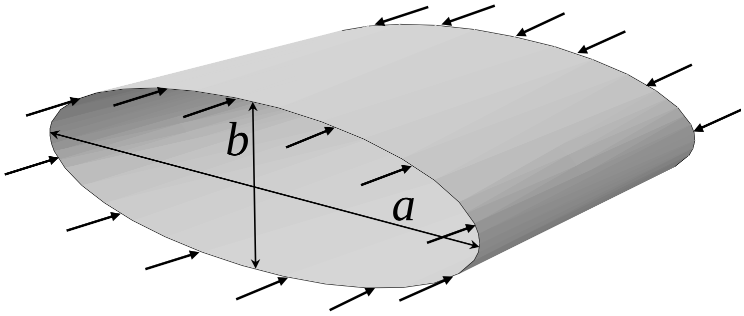


Figure 1: The elliptical composite cylinder with axial load.

2. Modeling of elliptical VS composite cylinders

Figure 1 illustrates the problem being considered in this study, in which a composite cylinder with an elliptical cross section is subjected to a compressive axial load. The simply supported boundary conditions were considered for the buckling analysis. To this end, the multipoint constraint (MPC) option built in ABAQUS [20] was used for both ends of the cylinders that can also guarantee the uniform axial displacement at the boundaries. The elliptical composite cylinders with different cross-sectional aspect ratios (b/a) were modeled in the commercial finite element (FE) software ABAQUS of which S8R5 shell elements were used for the FE analysis. The size of the finite elements for each elliptical cylinder was determined by a convergence study. As illustrated in Fig. 2, the axial narrow bands with equal widths were generated on the surface of the cylinders that represent the regions in which the fiber orientation angles are assumed to be constant, but different from their adjacent ones in case of VS laminate. This way, a piece-wise constant model is used to approximate the continuous variation of the fiber orientation angles in the circumferential direction on the surface of the cylinders. Two stacking sequences were considered for the composite cylinders in this work; (Layup 1) A balanced symmetric 8-ply laminate with $[\pm\theta_1/\pm\theta_2]_s$, and (Layup 2) A 16-layer, antisymmetric, specially orthotropic laminate with $[\theta_1, -\theta_1, -\theta_1, \theta_1, \theta_2, -\theta_2, -\theta_2, \theta_2, -\theta_2, \theta_2, \theta_2, -\theta_2, -\theta_1, \theta_1, \theta_1, -\theta_1]$ stacking sequence. In both cases, the composite plies were made of AS4D/9310 carbon/epoxy materials for which the mechanical properties are given in Table 1. The lengths of the major and minor axes are chosen so that the circumference of the ellipse is equal to that of a circle with a diameter of $D = 381$ mm. Therefore, in all cases the same amount of materials is used. Table 2 lists the geometrical properties of the elliptical cylinders considered for modeling, analysis, and design optimization in this study. The length of the cylinders in all cases is considered to be $L = 381$ mm.

As stated above, for VS composite cylinders the orientation angle in each ply (θ_j) is allowed to circumferentially vary on the surface of the cylinders. Considering the orientation angle in each narrow band as a design variable, results in a very large number of design variables that substantially increases the computational complexity of the design optimization problem. Therefore, to reduce the number of the design variables, the orientation angles of certain narrow bands in each ply (T_i's) are considered to be the design variables (Fig. 3). A

Figure 2: (a) The continuous circumferential variation of orientation angles in a VS ply, and (b) the piecewise constant model to approximate the orientation angle variation. The orientation angle in a VS ply is assumed to be constant in each narrow band, but different from its neighboring regions.

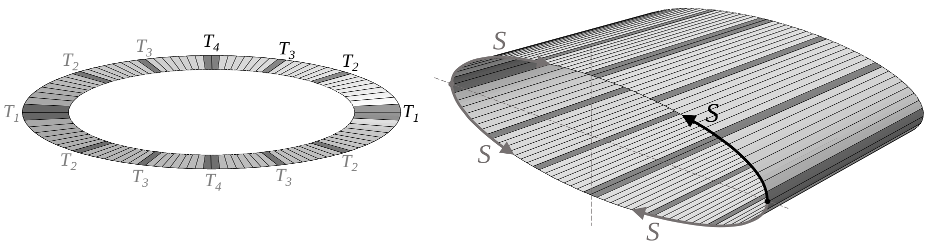
Table 1: Mechanical properties of each unidirectional composite ply made of carbon/epoxy materials.

Property	AS4D/9310
E_1 (GPa)	134
$E_2 = E_3$ (GPa)	7.71
$G_{12} = G_{13}$ (GPa)	4.31
G_{23} (GPa)	2.76
$\nu_{12} = \nu_{13}$	0.301
ν_{23}	0.396
Thickness (t) (mm)	0.127

Table 2: Geometrical properties of the elliptical composite cylinders.

Aspect ratio b/a	Major axis a (mm)	Minor axis b (mm)	Eccentricity $e = \sqrt{1-(b/a)^2}$	Length L (mm)
0.01	600.35	5.99	1.000	381
0.1	589.56	58.95	0.995	381
0.2	569.82	113.97	0.980	381
0.3	545.85	163.75	0.954	381
0.4	520.12	208.05	0.917	381
0.5	494.18	247.09	0.866	381
0.6	468.88	281.33	0.800	381
0.7	444.75	311.33	0.714	381
0.8	422.02	337.62	0.600	381
0.9	400.76	360.70	0.436	381
1.0*	381.0	381.0	0.000	381

*Circular



(a) (b)

Figure 3: The piece-wise constant model to approximate the continuous variation of the orientation angle in each ply of a VS elliptical cylinder (a) front view and (b) side view.

linear interpolation between the values of the orientation angles in these points is used to calculate the orientation angles in other narrow bands as formulated in Eq. 1:

$$\phi_k = T_i + \frac{S_k - S_i}{S_{i+1} - S_i}(T_{i+1} - T_i); \quad k = 1, \ldots, 7 \text{ and } i = 1, \ldots, 3 \tag{1}$$

where ϕ_k is the orientation angle of the k^{th} narrow band located between the i^{th} and $(i+1)^{th}$ design variables of which the orientation angles are T_i and T_{i+1}, respectively. S also indicates the arc length each narrow band has from a reference point (major axis ends). As shown in Fig. 3 and formulated in Eq. 1, the symmetry of the orientation angle distribution with respect to the major and minor axes of the elliptical cross-section was maintained. Therefore, the orientation angles in four equidistant narrow bands on a quarter of the surface of the elliptical cylinder were considered as design variables in each θ_j-ply of Layup 1 or Layup 2 stacking sequences resulting in 8 design variables to be determined for maximum buckling capacity of each cylinder.

3. Design optimization

The design optimization problem for this study is one of finding the optimum orientation angles on a limited number of specified narrow bands in each ply (T_i's) shown in Fig. 3 so that the buckling load is maximum, as given in Eq. 2:

$$\begin{aligned} \text{Max.} \quad & F_{cr} = f(T_i\text{'}s) \\ s.t. \quad & 0 \leq T_i \leq 90; \quad i = 1, \ldots, 8 \end{aligned} \tag{2}$$

where F_{cr} is the buckling capacity in terms of the design variables. In general, using any optimization method to find the optimum solution for Eq. 2 would require numerous function evaluations with the proportional computational cost associated with them. That is because of the iterative procedures performed in any computational optimization process, no matter if it is a gradient based or a non-gradient based method. The computational cost will be even more challenging in case of using evolutionary optimization approaches that are population based methods and a large number of function calls (high fidelity FEA in this study) are required. To reduce the computational expense, metamodel-based design optimization (MBDO) methods are used in which the costly high fidelity functions are replaced by low-cost analytical approximation functions so-called *surrogate models* or *metamodels* [21]. Besides the substantially improved computational efficiency, MBDO methods are also capable of alleviating the noisy behavior of the high fidelity models by replacing them with smooth analytical surrogate models [21, 22]. In its simplest form, a surrogate model is constructed by performing high fidelity modeling/simulations (FEA in this study) for a limited number of well-distributed sample or training points referred to as the design of experiments (DOE), followed by a curve fitting procedure to the responses. The resulting curve, or hyper-surface in case of higher dimensions, is a low cost approximation for the costly high fidelity model that is used for function calls in MBDO process.

Among several metamodeling techniques developed for engineering design optimization purposes [21, 22], the radial basis functions (RBF) offer accurate and reliable predictions at a relatively low computational cost for buckling response of VS composite structures [7, 8]. Therefore, RBF-based metamodels were used in this study to construct surrogate models relating the buckling load (F_{cr}) to the design variables (T_i's). Since metamodels are approximation of the high fidelity models, there are some errors associated with the responses predicted by them compared to the responses computed by the high fidelity models. These errors are calculated by comparing the responses computed from metamodels and those from high fidelity models in a number of so-called test points. In this study, the number of test points is considered to be one tenth of the DOE training points (one tenth of the sample size). To reduce the errors associated with metamodeling, a multi-step design optimization method [23] was used in which the design domain is narrowed down in multiple steps of the optimization process so that a converged optimum design emerges with negligible error when the result is compared with the high fidelity FE model.

The same MBDO method was also used to find the optimum orientation angles for the CS designs. However, it should be noted that in the CS cases the design variables were considered to be θ_1 and θ_2 as they are not varying on the surface of the elliptical cylinders. Therefore, the computational time needed to find the best CS laminate is considerably lower than the time needed to find the VS one because of the reduced number of design variables. The buckling capacity of all elliptical cylinders was also calculated for specific QI laminates with $[\pm 45/0/90]_s$ (for Layup 1) and $[\pm 45/0/90]_{2s}$ (for Layup 2) stacking sequences since using quasi-isotropic (QI) laminates is fairly common in composite structures.

4. Results and discussion

4.1. Buckling design for Layup 1 (spiral mode shape)

Using the material system with Layup 1 and properties given in Table 1, the buckling loads of elliptical cylinders with the cross-sectional aspect ratios listed in Table 2 are shown in Fig. 4. The buckling load for a cylinder with QI layup was calculated for each aspect ratio using FEA. Using the MBDO method, the CS laminate that results in the maximum buckling capacity for that cylinder was calculated in the second part. Finally, in the third part, MBDO was also used to find the circumferential distribution of the orientation angles that gives the maximum buckling capacity for VS design. These three procedures were separately performed for each elliptical cylinder. The results shown in Fig. 4 for the best CS designs reveal that the axial buckling capacity of the elliptical cylinders decreases almost linearly when the cross-sectional aspect ratio decreases from its maximum (b/a=1.0, F_{cr}=160 kN) to its minimum (b/a=0.01, F_{cr}=2.6 kN) value. Comparing the buckling capacities of the CS designs with their QI counterparts, it is concluded that the structural inefficiency, caused by the eccentricity of the elliptical cross-section, can be slightly compensated by substituting the QI with the optimum CS laminate whereas the VS design can greatly compensate this structural inefficiency. For instance, the 51.2% reduction of the buckling load for a CS elliptical cylinder with b/a=0.5 (F_{cr} = 77.4 kN) compared with a circular cylinder (F_{cr} = 158.7 kN) can be reduced to 14.6% by fiber steering (F_{cr} = 135.5 kN) as shown in Fig. 4. The reason for such improvement, as also noted by [1, 6, 15], is the more efficient distribution of the compressive load in the VS design. For further investigation, the distribution of the axial section force (SF_1) along the circumference of the elliptical cylinders were calculated from the FEA results:

$$SF_1 = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{11} dz \tag{3}$$

Figure 5 shows the axial section force (SF_1) distribution over one half of the circumference (from one end of the major axis to the other end) of an elliptical cylinder (b/a=0.7) for QI, CS, and VS cases. It shows how the VS design, by varying the orientation angles, efficiently exploits the directional properties of the composite layers for optimum distribution of the axial section forces resulting in improved buckling capacity. It is observed that the axial compressive section force is reduced in the flatter portions of the cross-section and is increased in the curved portions. In other words, the reduced section force in the flatter portions, which are structurally more susceptible to buckling, is balanced by increasing the section force in the curved portions which are structurally more stable in an elliptical cylinder. The optimum usage of the directional properties of the composite laminate via fiber steering in the VS design can also be observed in Fig. 6c in which almost the whole surface of the elliptical cylinder is involved in carrying the buckling load.

As stated above, the efficient load path in a VS laminate is provided by optimum distribution of stiffness via fiber steering. Figure 7 shows the fiber orientation angle distribution on a quarter circumference of three elliptical cylinders with different cross-sectional aspect ratios (b/a=0.2, b/a=0.5, b/a=0.8). It reveals a general trend of the optimum orientation angles (θ_1 and θ_2) to start from small values in the curved portions of the circumference (laminate is stiff in axial direction) to large values in the flatter portions (laminate is soft in axial direction). This way, a considerable portion of the compressive load is shifted from the flatter parts of the elliptical cylinder that are structurally less stable to the curved parts that are structurally more stable.

The variation of the orientation angles from the small value to the large one becomes less steep with increasing cross-sectional aspect ratio (b/a). It is an expected result since in larger cross-sectional aspect ratios the eccentricity decreases towards zero as in a circular state in which the axial symmetry of the cylinder leaves no possibility for VS design to

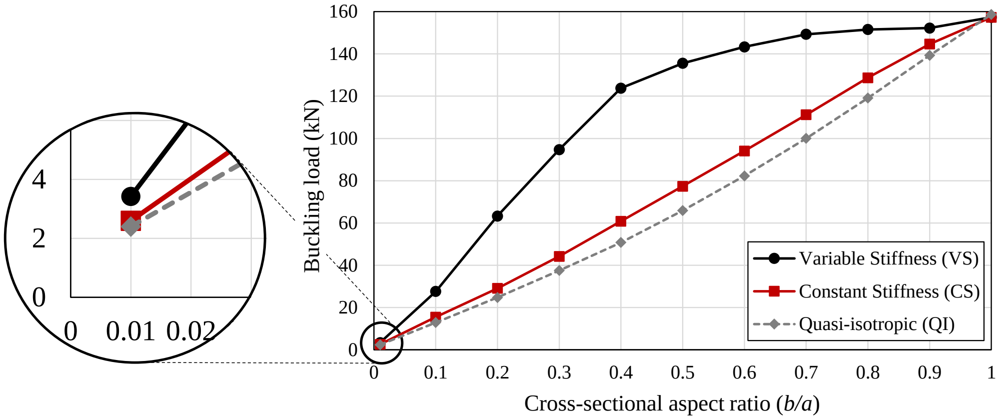


Figure 4: Buckling load of QI, CS, and VS elliptical cylinders with Layup 1 and different cross-sectional aspect ratios.

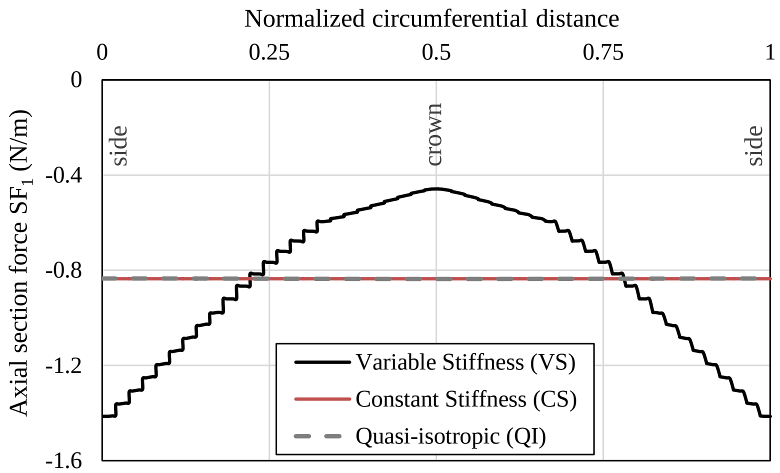


Figure 5: Axial section force (SF_1) distribution over the half circumference beginning from one end of the major axis in an elliptical cylinder with Layup 1 and b/a=0.7.

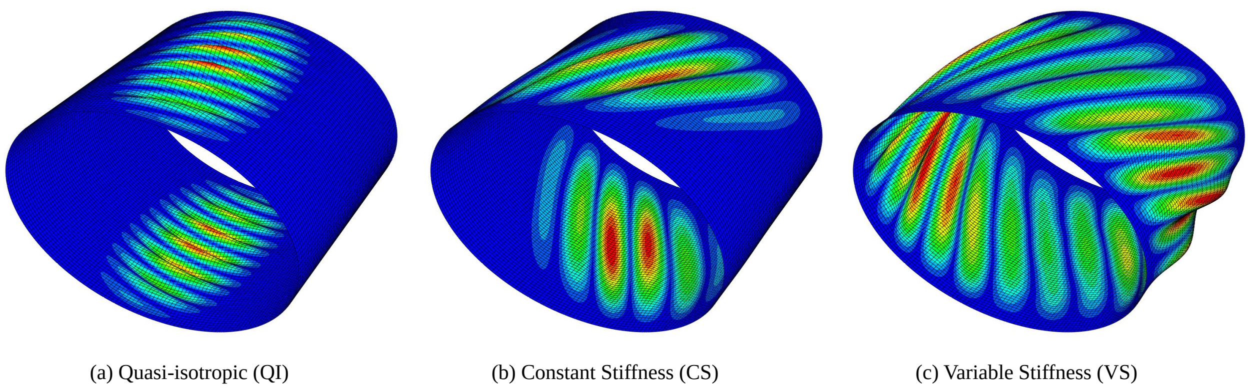


Figure 6: Buckling mode shapes of (a) QI, (b) CS, and (c) VS elliptical composite cylinders with Layup 1 and b/a=0.7.

take advantage of the non-uniformity of the structural curvature for improving the buckling capacity by fiber steering. As a result, the optimum VS design is actually the best constant stiffness design (CS) in circular cylinders.

The variation of the buckling mode shapes for CS and VS laminates shown in Fig. 8 also indicate that the potential for improvement in buckling capacity reduces with the increase in cross-sectional aspect ratio. In contrast, by reduction of the cross-sectional aspect ratio (b/a), the increased eccentricity of the elliptical cylinder provides more scope for fiber steering to take advantage of the structure's axial asymmetry and tailor the material stiffness to compensate the structural inefficiency in buckling performance. However, this increase of the improvement due to the increased eccentricity has a limit. Figure 9 shows the potential improvement available for the buckling capacity of VS elliptical cylinders of different cross-sectional aspect ratios compared with their best CS counterparts. The buckling capacity of the QI laminate are also included in Fig. 9 for comparison. It shows that the maximum improvement for the buckling capacity is about 118% for b/a=0.2, below that ($b/a < 0.2$; $e > 0.980$) the maximum improvement reduces. It is because of the fact that for very low cross-sectional aspect ratios the structure is almost two flat plates attached to each other at their ends. Therefore, there is a very limited area with a radius of curvature distinctively different from the other portions. In fact for increasing ellipticity, curvature becomes concentrated in the highly curved corners with essentially almost flat plates in between. In other words, Fig. 9 reveals that for $b/a < 0.2$ the highly curved regions do not buckle and the flatter regions in between buckle as isolated curved panels; i.e. the highly curved regions appear to act as panel breakers. This effect suggests that the solution could be lower bounded by flat plate buckling, presumably with clamped edges. In such cases, the eccentricity is not the driving factor for the optimum distribution of the orientation angles, and other factors are more important such as the distance from the edges.

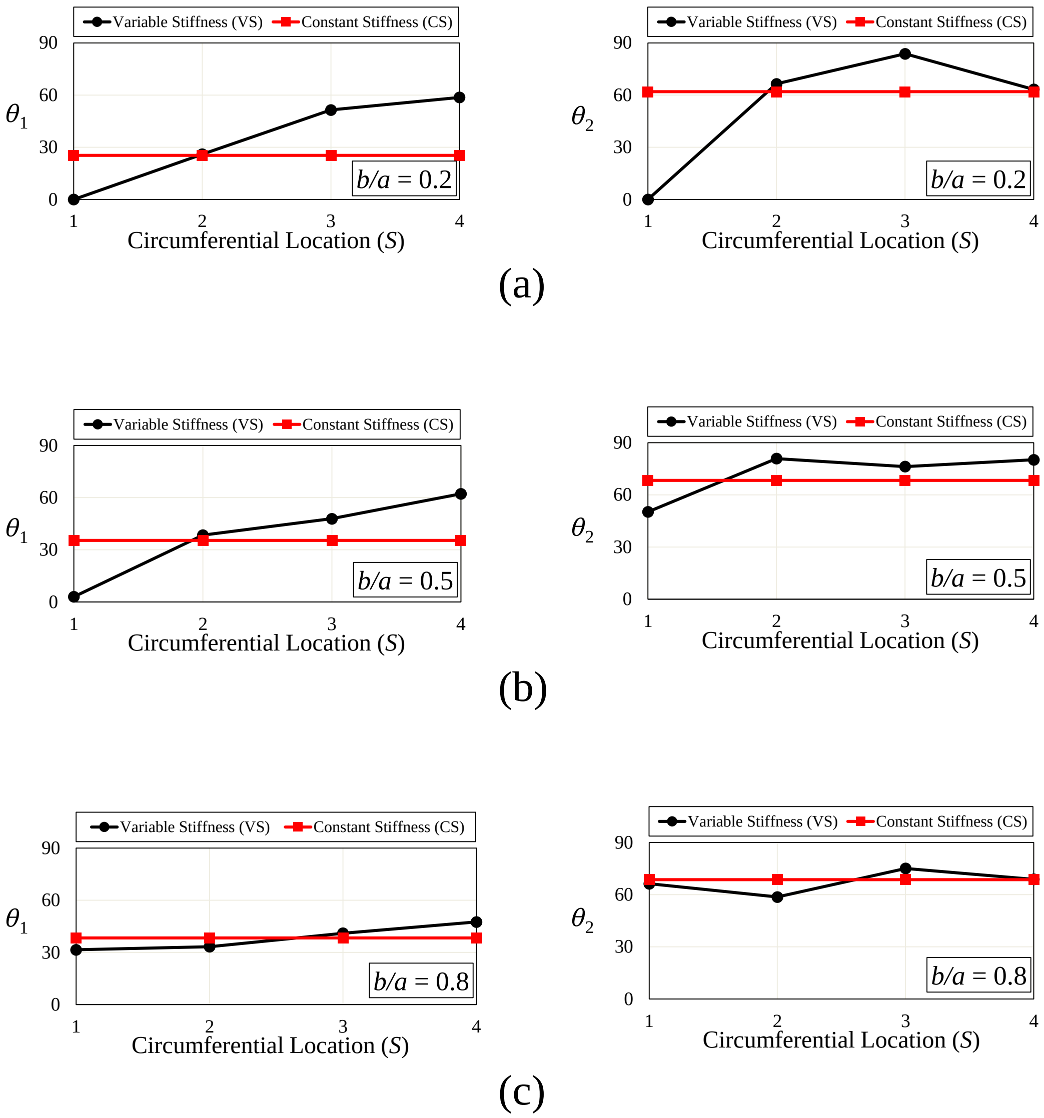
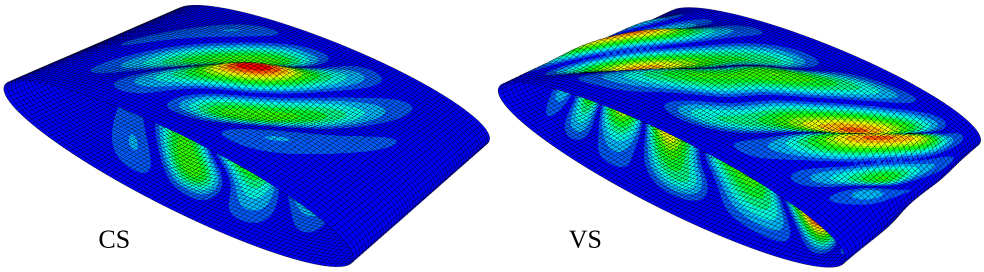
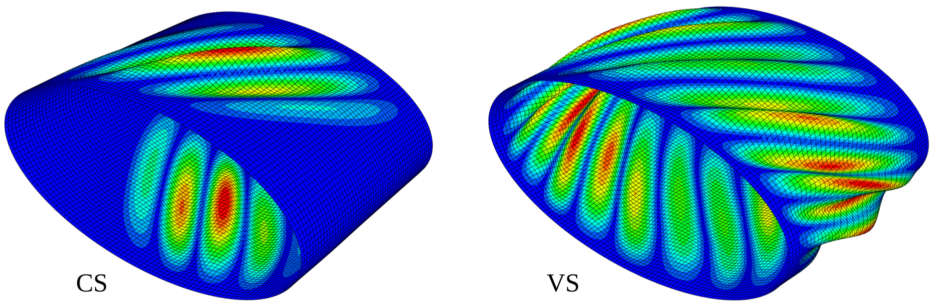


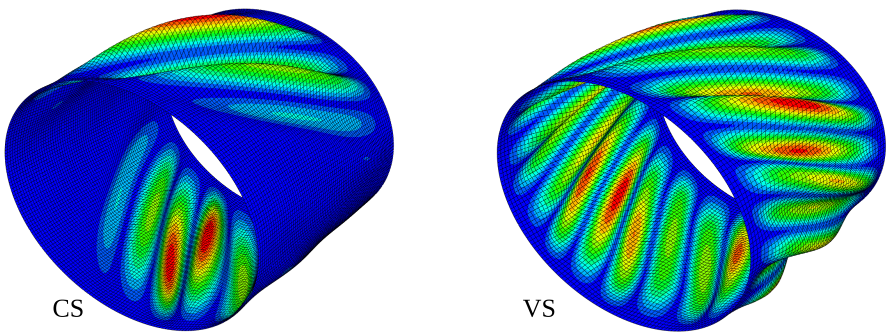
Figure 7: Optimum distribution of the orientation angles in elliptical cylinders with Layup 1 and (a) $b/a = 0.2$, (b) $b/a = 0.5$, and (c) $b/a = 0.8$.



(a) Aspect ratio *b/a* = 0.2



(b) Aspect ratio *b/a* = 0.5



(c) Aspect ratio *b/a* = 0.8

Figure 8: Buckling mode shapes of CS and VS elliptical cylinders with Layup 1 and (a) $b/a = 0.2$, (b) $b/a = 0.5$, and (c) $b/a = 0.8$.

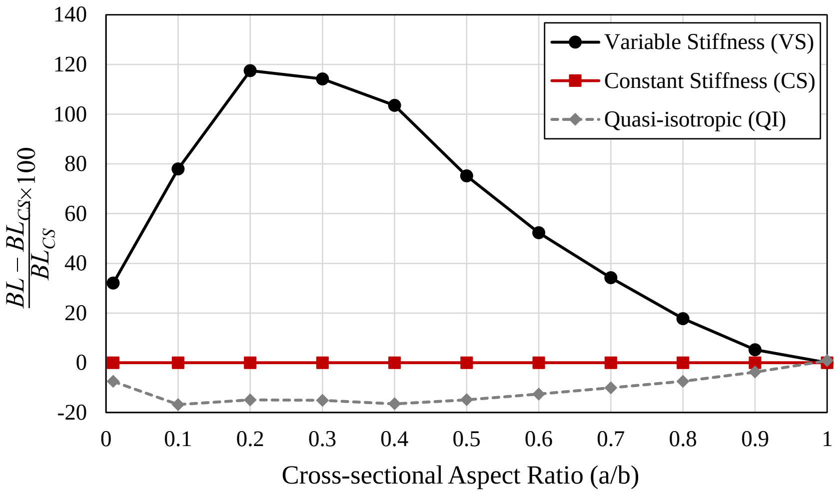


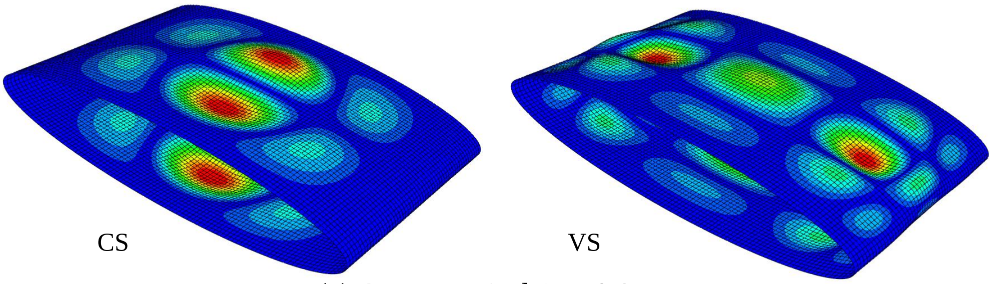
Figure 9: Buckling load improvement (VS) or decrease (QI) percentage compared with their best CS counterparts for different cross-sectional aspect ratios (b/a=0.01. . . 1) of the elliptical cylinders with Layup 1.

4.2. Buckling design for Layup 2 (doubly periodic mode shape)

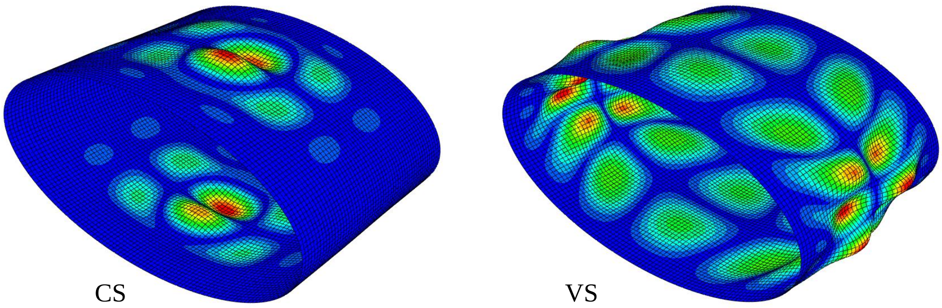
The spiral waves in the buckling mode shapes of Layup 1 (Figs. 6 and 8) arise from a degree of bend-twist coupling (i.e. D_{16} and D_{26}) [24] that is present at all points of the surface of the elliptical cylinders. This bend-twist coupling also results in a reduced buckling capacity. Therefore, the VS designs made by fiber steering for this type of laminate may seem to be oversold. To clear this doubt, another laminate with no bend-twist coupling was considered for fiber steering. Weaver [24] showed that the bend-twist coupling can be alleviated by increasing the number of plies and reducing the layer-wise orthotropy. To remove the bend-twist coupling, a 16-layer, antisymmetric, specially orthotropic laminate with $[\theta_1, -\theta_1, -\theta_1, \theta_1, \theta_2, -\theta_2, -\theta_2, \theta_2, -\theta_2, \theta_2, \theta_2, -\theta_2, -\theta_1, \theta_1, \theta_1, -\theta_1]$ stacking sequence (Layup 2) was used for buckling analysis of an elliptical cylinder with geometrical properties shown in Table 2. The θ-plies in this stacking sequence were also considered for fiber steering and the optimum VS design was obtained using MBDO method. Figure 10 shows the axial buckling modes of elliptical cylinders with (a) CS and (b) optimum VS laminates for three different cross-sectional aspect ratios (b/a=0.2, b/a=0.5, b/a=0.8). As expected, for such lamination that has no flexural/twist anisotropy (i.e. D_{16}=D_{26}=0) the buckling mode shape is conventionally doubly periodic without a spiral.

Figure 11 also shows the fiber orientation angle distribution on a quarter circumference of the three elliptical cylinders with Layup 2. Similar to Layup 1, it reveals the general trend of the optimum orientation angles' distribution that start from small values in the curved portions of the circumference and end with large values in the flatter portions for optimum redistribution of the compressive loads for maximum buckling capacity.

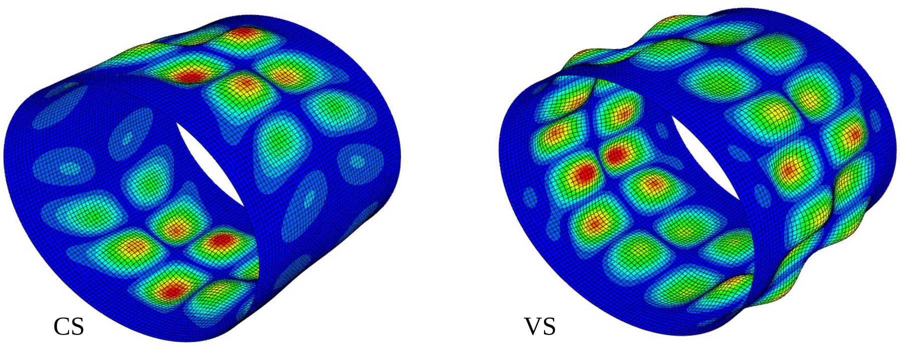
The potential buckling improvement of VS elliptical cylinders with Layup 2 compared to the CS counterparts for all cross-sectional aspect ratios is shown in Fig. 12. It shows that the maximum improvement (112%) of the buckling capacity for VS cylinder with Layup 2 occurs at b/a=0.3. The general trend, however, is similar to Layup 1 in which the buckling



(a) Aspect ratio *b/a* = 0.2



(b) Aspect ratio *b/a* = 0.5



(c) Aspect ratio *b/a* = 0.8

Figure 10: Buckling mode shapes of CS and VS elliptical cylinders with Layup 2 and (a) $b/a = 0.2$, (b) $b/a = 0.5$, and (c) $b/a = 0.8$.

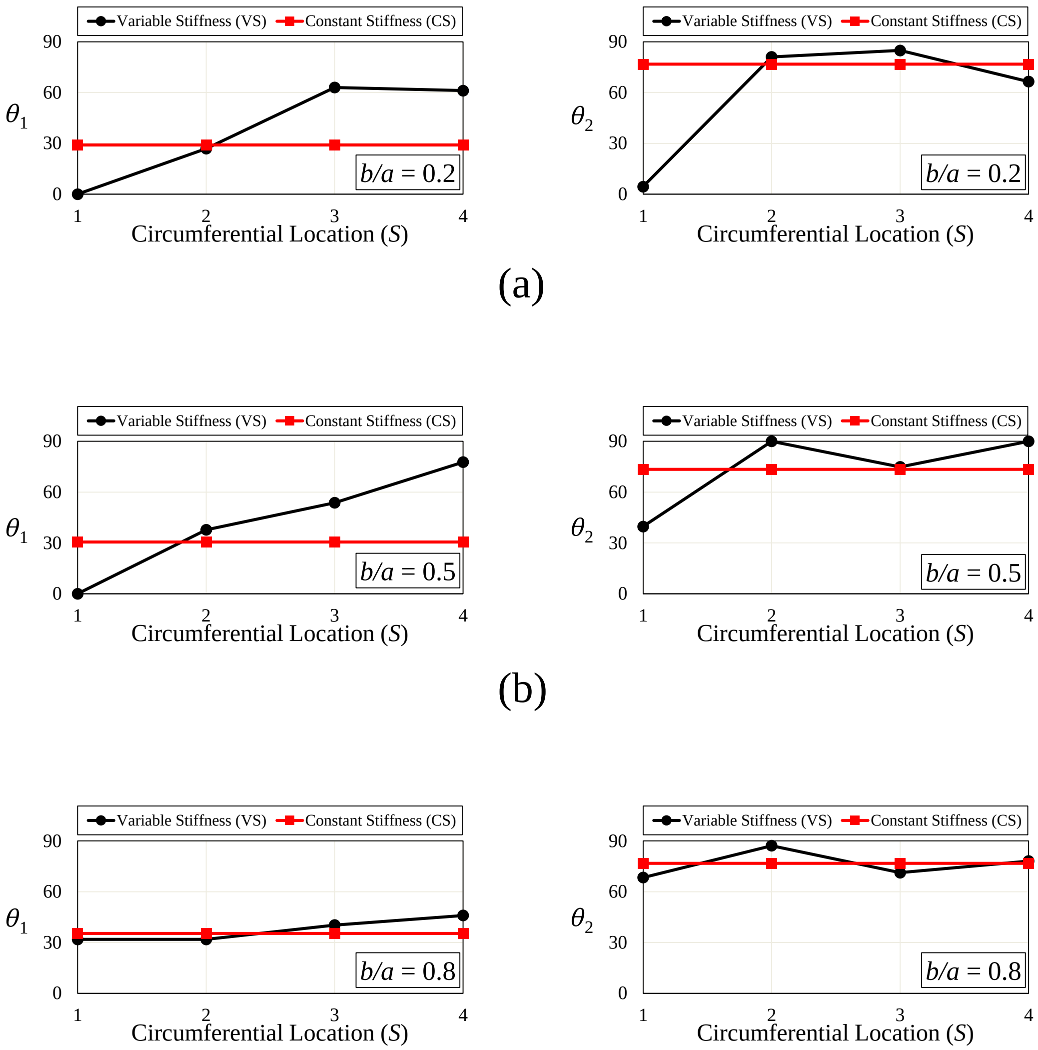


Figure 11: Optimum distribution of the orientation angles in elliptical cylinders with Layup 2 and (a) b/a = 0.2, (b) $b/a = 0.5$, and (c) $b/a = 0.8$.

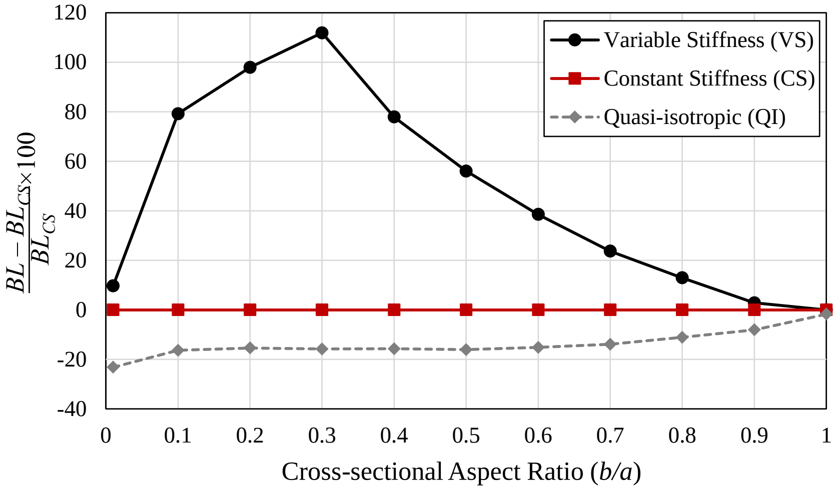


Figure 12: Buckling load improvement (VS) or decrease (QI) percentage compared with their best CS counterparts for different cross-sectional aspect ratios (b/a=0.01...1) of the elliptical cylinders with Layup 2.

capacity improvement increases by the decrease of the cross-sectional aspect ratio up to a certain limit and then decreases for aspect ratios below that.

5. Conclusions

A metamodeling based design optimization (MBDO) method was used to tailor the stiffness properties, by fiber steering, on the surface of laminated composite elliptical cylinders for maximum buckling capacity. The buckling capacity of the resulting variable stiffness (VS) composite cylinders was compared with their counterparts with quasi-isotropic (QI) and constant stiffness (CS) laminate. The results showed that the VS laminate, by spatially varying the fiber orientation angles on the circumference of the elliptical cylinders, efficiently uses the directional properties of the composite materials and provides an optimum distribution of the applied load on the elliptical cylinder and consequently improves the buckling capacity to a great extent. Stiffness tailoring was shown to significantly increase the involved portions of the elliptical cylinders in carrying the buckling load for VS designs compared with their QI and CS counterparts.

The effect of the cross-sectional aspect ratio (b/a) on the potential room for improvement of the axial buckling capacity of the elliptical cylinders was also studied and discussed. The results showed that the room for buckling capacity improvement percentage of VS designs over the CS ones increases by the decrease in cross-sectional aspect ratio from 0% (b/a=1: circular cylinder) up to more than 118% (b/a=0.2) for Layup 1 with $[\pm\theta_1/\pm\theta_2]_s$ stacking sequence and 112% (b/a=0.3) for Layup 2 with $[\theta_1, -\theta_1, -\theta_1, \theta_1, \theta_2, -\theta_2, -\theta_2, \theta_2, -\theta_2, \theta_2, \theta_2, -\theta_2, -\theta_1, \theta_1, \theta_1, -\theta_1]$ stacking sequence. For elliptical cylinders with higher eccentricity of the cross-section the buckling capacity improvement percentage decreases with the decrease in cross-sectional aspect ratio (b/a) down to 32% for Layup 1 and 10% for Layup 2 (b/a=0.01).

It is worth noting that a practical design, especially for aerospace application, must consider the manufacturing limitations such as minimum steering radius as well as the effects of gaps/overlaps generated by fiber steering. Therefore, all the improvements reported in this study are the potentials and the actual improvement may be lower when it is manufactured by the existing AFP facilities. However, with the emerging manufacturing technologies such as continuous tow shearing (CTS) [25], steering with small radii should be practically possible. Moreover, when it comes to large structures (i.e. aircraft fuselage), the orientation angle variation will be more feasible than in a small cylinder since the fibers/tows have larger space to steer from one orientation angle to another.

Future studies should address other loading conditions such as bending-induced or torsional buckling and combined loadings. Investigating the nonlinear post-buckling behavior of such VS composite structures is also worthy of future study. Failure constraints should also be considered in the design optimization of such structures to make sure they will not fail before buckling. More importantly, VS elliptical cylinders need to be manufactured and tested for experimental validation of the results which is a future plan for this study.

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References

[1] M. Sun, M. W. Hyer, Use of material tailoring to improve buckling capacity of elliptical composite cylinders, AIAA Journal 46 (3) (2008) 770–782.
[2] M. Paschero, M. W. Hyer, Improvement of axial load capacity of elliptical cylindrical shells, AIAA journal 47 (1) (2009) 142–156.
[3] M. Paschero, M. W. Hyer, Improvement of axial buckling capacity of elliptical lattice cylinders, AIAA journal 49 (2) (2011) 396–410.
[4] A. Y. Tamijani, R. K. Kapania, Buckling and static analysis of curvilinearly stiffened plates using mesh-free method, AIAA journal 48 (12) (2010) 2739–2751.
[5] A. W. Blom, P. B. Stickler, Z. Gürdal, Optimization of a composite cylinder under bending by tailoring stiffness properties in circumferential direction, Composite Part B: Engineering 41 (2) (2010) 157–165.
[6] A. Khani, M. M. Abdalla, Z. Gürdal, Circumferential stiffness tailoring of general cross section cylinders for maximum buckling load with strength constraints, Composite Structures 94 (9) (2012) 2851–2860.
[7] M. Rouhi, H. Ghayoor, S. V. Hoa, M. Hojjati, Effect of structural parameters on design of variable-stiffness composite cylinders made by fiber steering, Composite Structures 118 (2014) 472–481.
[8] M. Rouhi, H. Ghayoor, S. V. Hoa, M. Hojjati, Multi-objective design optimization of variable stiffness composite cylinders, Composites Part B: Engineering 69 (2015) 249–255.
[9] M. Rouhi, H. Ghayoor, S. V. Hoa, M. Hojjati, The effect of the percentage of steered plies on the bending-induced buckling performance of a variable stiffness composite cylinder, Science and Engineering of Composite Materials 22 (2) (2015) 149–156.
[10] D. H.-J. Lukaszewicz, C. Ward, K. D. Potter, The engineering aspects of automated prepreg layup: History, present and future, Composites Part B: Engineering 43 (3) (2012) 997–1009.

[11] B. C. Kim, H. Hazra, P. M. Weaver, K. Potter, Limitations of fibre placement techniques for variable angle tow composites and their process-induced defects, in: Proceedings of the 18th International Conference on Composites Materials, ICCM 2011, 2011.
[12] A. W. Blom, C. Lopes, P. Kromwijk, Z. Gürdal, P. Camanho, A theoretical model to study the influence of tow-drop areas on the stiffness and strength of variable-stiffness laminates, Journal of Composite Materials 43 (5) (2009) 403–425.
[13] B. F. Tatting, Analysis and design of variable stiffness composite cylinders, Ph.D. thesis, Virg. Tech, Blacksburg, Virginia (1998).
[14] S. White, P. M. Weaver, Bend-free shells under uniform pressure with variable-angle tow derived anisotropy, Composite Structures 94 (11) (2012) 3207 – 3214.
[15] A. Khani, M. M. Abdalla, Z. Gürdal, Optimum tailoring of fibre-steered longitudinally stiffened cylinders, Composite Structures 122 (2015) 343–351.
[16] H. C. Lo, M. W. Hyer, Fundamental frequencies of elliptical composite cylinders with circumferentially varying fiber orientation, AIAA Journal 50 (4) (2012) 928–937.
[17] Z. Wu, G. Raju, P. M. Weaver, Postbuckling analysis of variable angle tow composite plates, International Journal of Solids and Structures 50 (10) (2013) 1770–1780.
[18] S. C. White, G. Raju, P. M. Weaver, Initial post-buckling of variable-stiffness curved panels, Journal of the Mechanics and Physics of Solids 71 (2014) 132–155.
[19] S. C. White, P. M. Weaver, K. C. Wu, Post-buckling analyses of variable-stiffness composite cylinders in axial compression, Composite Structures 123 (2015) 190–203.
[20] ABAQUS Inc., Pawtucket, RI, USA, ABAQUS Version 6.11 User's Manual (2011).
[21] G. G. Wang, S. Shan, Review of metamodeling techniques in support of engineering design optimization, Journal of Mechanical Design, Transactions of the ASME 129 (4) (2007) 370–380.
[22] N. V. Queipo, R. T. Haftka, W. Shyy, T. Goel, R. Vaidyanathan, K. P. Tucker, Surrogate-based analysis and optimization, Progress in Aerospace Sciences 41 (1) (2005) 1–28.
[23] M. Rouhi, H. Ghayoor, S. V. Hoa, M. Hojjati, Multi-step design optimization of variable stiffness composite cylinders made by fiber steering, in: Proceedings of the American Society for Composites - 29th Technical Conference, La Jolla, CA, USA, Sep, 2014.
[24] P. M. Weaver, Anisotropy-induced spiral buckling in compression-loaded cylindrical shells, AIAA journal 40 (5) (2002) 1001–1007.
[25] B. C. Kim, K. Potter, P. M. Weaver, Continuous tow shearing for manufacturing variable angle tow composites, Composites Part A: Applied Science and Manufacturing 43 (8) (2012) 1347–1356.