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Pricing Holder-Extendable Options in a Stochastic Volatility Model with an Ornstein-Uhlenbeck Process

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ABSTRACT

Holder-extendable options are characterized by two maturity dates, which means the option can be exercised at either the expiration date or the extended maturity date. This paper develops a pricing framework for holder-extendable options which deals with the extended version of a stochastic volatility model with an Ornstein-Uhlenbeck (OU) process. The extended model allows correlation between volatility and asset returns. The method uses Fourier inversion techniques that does not require an initial guess of the characteristic functions. A closed-form pricing formula for holder-extendable options is derived for logarithmic asset price dynamics.

Keywords: Holder-extendable option, Ornstein-Uhlenbeck process, Fourier inversion, stochastic volatility.

1. Introduction

Options are derivatives which give the holder the right to trade certain underlying assets at a fixed price at specific period of time. Options are cheaper than stocks, and buying options may limit risks and has the potential for higher profits. Standard European and American options have one maturity date, where the former can only be exercised at maturity, and the latter can be exercised at any time right up until its maturity. However, to cater the needs of investors, it brings existence to non-standard options or known as exotic options with custom-made payoff structures. One of many extensions to the standard option payoffs is to having a dual-maturity as discussed in Buchen (2004), and extendable option is one of the options that is characterized by two maturity dates.

To our best knowledge, extendable options were first discussed by Longstaff (1990) in which numerous applications of extendable contracts are provided, for instance extendable warrants and extendable bonds. Dias and Rocha (2000) also used the extendable options framework to oil prices in a jump-diffusion model with mean-reversion, whereas within the same framework, Abinzano and Navas (2008) priced the equity of a firm. Recently, Koussis et al. (2013) studied the features of extendable contracts for product development.

Under the Black-Scholes model (Black and Scholes, 1973), Longstaff (1990) and Ibrahim et al. (2014) discuss extendable options when the contract is extendable once. Chung and Johnson (2011) extend the work by presenting a general pricing formula for extendable options, where the contract can be extended more than once. On the other hand, Gukhal (2004) presented a closed-form solution for extendable options under the Merton jump-diffusion model (Merton, 1973), and Peng and Peng (2012) present the price for extendable options when the dynamics of the underlying asset price follows a fractional process with jumps.

Other than modeling jumps in option pricing model, it is also well-known as a stochastic volatility model. Prominent models that capture asset returns variability are the Heston model (Heston, 1993), the Stein-Stein model (Stein and Stein, 1991), and the Schöbel-Zhu model (Schobel and Zhu, 1999). The Schöbel-Zhu model is stochastic volatility model with a mean-reverting Ornstein-Uhlenbeck process. Moreover, the Schöbel-Zhu model extends the Stein-Stein model by allowing correlation between asset returns and volatilities.

This study develops a theoretical pricing framework of European call holder-extendible options in the Schöbel-Zhu model. The paper is structured as follows: In Section 2, we examine the construction of the option price as distribution functions. Section 3 presents the characteristic functions of holder extendable options, and the closed-form pricing formula. Section 4 concludes our work.

2. The Schöbel-Zhu Model

The Schöbel-Zhu model assumes that the logarithmic asset price and the volatility $v(t)$ follows the following dynamics:

$$dx(t) = \left[r - \frac{1}{2}v^2(t)\right] dt + v(t)\, dw^S(t),$$
$$dv(t) = \kappa[\theta - v(t)]\, dt + \sigma\, dw^v(t),$$

where $x(t) = \ln S(t)$ and $< dw^S\, dw^v >= \rho\, dt$.

Let C be a European call option C with asset price S, strike K, and maturity date T_0, and $I_1,\ I_2$ are critical prices[1]. The holder of an extendable option has three choices with different outcomes: if at time $T_0,\ \ S(T_0) < I_1$, the holder of the call option may let the option to expire; or if $S(T_0) > I_2$, the holder may exercise the call option; or if $I_1 \leq S(T_0) \leq I_2$, the holder may pay a fee P to extend the maturity of the call option to a future date T_1 with a new strike X. On that account, the payoff for a European call holder-extendable option can be represented as:

$$HC = \max\left[0,\ S(T_0) - K,\ C(S(T_0),\ X,\ T_1 - T_0) - P\right]. \tag{1}$$

Hence, under risk-neutral measure $\mathbf{Q}$, the price is the discounted payoff at risk-free rate r :

$$HC = e^{-r(T_0-t)}\mathbb{E}^{\mathbf{Q}}\left\{\max\left[0,\ S(T_0) - K,\ C(S(T_0),\ X,\ T_1 - T_0) - P\right]\right\}, \tag{2}$$

or:

$$\begin{aligned} HC &= e^{-r(T_0-t)}\left[\mathbb{E}^{\mathbf{Q}}\left\{(S(T_0) - K)[\mathbf{1}_{\{S(T_0)>I_2\}}]\right.\right. \\ &\quad + \left.\left.[C(S(T_0),\ X,\ T_1 - T_0) - P][\mathbf{1}_{\{I_1\leq S(T_0)\leq I_2\}}]\right\}\right] \\ &= e^{-r(T_0-t)}\left[\mathbb{E}^{\mathbf{Q}}\left\{(S(T_0) - K)[\mathbf{1}_{\{S(T_0)>I_2\}}]\right.\right. \\ &\quad - \left.\left.P[\mathbf{1}_{\{I_1\leq S(T_1)\leq I_2\}}]\right\}\right] \\ &\quad + e^{-r(T_1-t)}\mathbb{E}^{\mathbf{Q}}\left\{(S(T_1) - X)[\mathbf{1}_{\{I_1\leq S(T_0)\leq I_2, S(T_1)>X\}}]\right\}. \end{aligned} \tag{3}$$

Let $\mathbf{Q}^S$ and $\mathbf{Q}^T$ denote probability measure and $T-$forward measure, respectively. By the Radon-Nikodym derivative, we have:

$$\left.\frac{d\mathbf{Q}^S}{d\mathbf{Q}}\right|_{\mathcal{F}_T} = e^{-r(T-t)-x(t)+x(T)}, \tag{4}$$

$$\left.\frac{d\mathbf{Q}^T}{d\mathbf{Q}}\right|_{\mathcal{F}_T} = 1. \tag{5}$$

where $x(T) = \ln S(T)$. Therefore, in view of Equations (4) and (5), Equation

[1] The critical prices are obtainable using a root-search algorithm to $I_2 - K = C(I_2,\ X,\ T_1 - T_0) - P$ and $C(I_1,\ X,\ T_1 - T_0) - P = 0$, where $I_1 < I_2$ and $K > I_1$.

(3) can be expressed as such:

$$\begin{aligned} HC &= S(t)\,\mathbb{E}^{\mathbf{Q}^S}[\mathbf{1}_{\{x(T_0)>\ln I_2\}}] - K\,e^{-r(T_0-t)}\,\mathbb{E}^{\mathbf{Q}^T}[\mathbf{1}_{\{x(T_0)>\ln I_2\}}] \\ &\quad + S(t)\,\left(\mathbb{E}^{\mathbf{Q}^S}\left[\mathbf{1}_{\{x(T_1)>\ln X,\,x(T_0)\le \ln I_2\}}\right] - \mathbb{E}^{\mathbf{Q}^S}\left[\mathbf{1}_{\{x(T_1)>\ln X,\,x(T_0)\le \ln I_1\}}\right]\right) \\ &\quad - X\,e^{-r(T_1-t)}\,\left(\mathbb{E}^{\mathbf{Q}^T}\left[\mathbf{1}_{\{x(T_1)>\ln X,\,x(T_0)\le \ln I_2\}}\right] - \mathbb{E}^{\mathbf{Q}^T}\left[\mathbf{1}_{\{x(T_1)>\ln X,\,x(T_0)\le \ln I_1\}}\right]\right) \\ &\quad - P\,e^{-r(T_0-t)}\,\left(\mathbb{E}^{\mathbf{Q}^T}\left[\mathbf{1}_{\{x(T_0)\le \ln I_2\}}\right] - \mathbb{E}^{\mathbf{Q}^T}\left[\mathbf{1}_{\{x(T_0)\le \ln I_1\}}\right]\right). \end{aligned} \tag{6}$$

Following Schobel and Zhu (1999), Equation (6) can be written in terms of probabilities as follows:

$$\begin{aligned} HC &= S(t)\,\mathbf{F}^{\mathbf{Q}^S}(S(T_0)>I_2) - K\,e^{-r(T_0-t)}\,\mathbf{F}^{\mathbf{Q}^T}(S(T_0)>I_2) \\ &\quad + S(t)\left[\underbrace{\mathbf{F}^{\mathbf{Q}^S}(S(T_1)>X,\,S(T_0)\le I_2)}_{I} - \underbrace{\mathbf{F}^{\mathbf{Q}^S}(S(T_1)>X,\,S(T_0)\le I_1)}_{II}\right] \\ &\quad - X\,e^{-r(T_1-t)}\left[\underbrace{\mathbf{F}^{\mathbf{Q}^T}(S(T_1)>X,\,S(T_0)\le I_2)}_{III} - \underbrace{\mathbf{F}^{\mathbf{Q}^T}(S(T_1)>X,\,S(T_0)\le I_1)}_{IV}\right] \\ &\quad - P\,e^{-r(T_0-t)}\left[\mathbf{F}^{\mathbf{Q}^T}(S(T_0)\le I_2) - \mathbf{F}^{\mathbf{Q}^T}(S(T_0)\le I_1)\right]. \end{aligned} \tag{7}$$

3. The Closed-Form Solution

In this section, we derive the characteristic functions using the approach presented in Scott (1997) to obtain analytical solutions for the probabilities in Section 2. Here, we apply stochastic calculus to compute the characteristic functions directly. Let us define the characteristic functions as follows:

$$\begin{aligned} f_S(\phi_1) &= \mathbb{E}^{\mathbf{Q}}\left[e^{-r(T_0-t)-x(t)+(1+i\phi_1)x(T_0)}\right], \\ f_T(\phi_1) &= \mathbb{E}^{\mathbf{Q}}\left[e^{i\phi_1 x(T_0)}\right] \end{aligned}$$

$$\begin{aligned} f_S(\phi_1,\,\phi_2) &= \mathbb{E}^{\mathbf{Q}}\left[e^{-r(T_1-t)-x(t)+i\phi_1 x(T_0)+(1+i\phi_2)x(T_1)}\right], \\ f_T(\phi_1,\,\phi_2) &= \mathbb{E}^{\mathbf{Q}}[e^{i\phi_1 x(T_0)+i\phi_2 x(T_1)}]. \end{aligned}$$

The characteristic functions for $f_S(\phi_1)$ and $f_T(\phi_1)$ are as provided in Schobel and Zhu (1999). These are given in the following lemma.

Lemma 3.1. *(Schobel and Zhu, 1999) The characteristic functions with respect to the asset price measure and the $T-$forward measure for an underlying asset* *in the Schöbel-Zhu model are given by:*

$$\begin{aligned} f_S(\phi_1) &= \exp\left\{ i\phi_1[r(T_0-t)+x(t)] - \frac{i(\phi_1-i)\rho}{2}\left[\frac{v^2(t)}{\sigma}+\sigma(T_0-t)\right]\right. \\ &\quad + \frac{1}{2}D(t,T_0;z_1,z_3)v^2(t) + B(t,T_0;z_1,z_2,z_3)v(t) \\ &\quad \left. + C(t,T_0;z_1,z_2,z_3)\right\}, \qquad (8) \\ f_T(\phi_1) &= \exp\left\{ i\phi_1[r(T_0-t)+x(t)] - \frac{i\phi_1\rho}{2}\left[\frac{v^2(t)}{\sigma}+\sigma(T_0-t)\right]\right. \\ &\quad + \frac{1}{2}D(t,T_0;\hat{z}_1,\hat{z}_3)v^2(t) + B(t,T_0;\hat{z}_1,\hat{z}_2,\hat{z}_3)v(t) \\ &\quad \left. + C(t,T_0;\hat{z}_1,\hat{z}_2,\hat{z}_3)\right\}, \qquad (9) \end{aligned}$$

where

$$\begin{aligned} z_1(\phi_1) &= -\frac{1}{2}\left[(i(\phi_1-i))^2(1-\rho^2) - i(\phi_1-i)\left(1-\frac{2\kappa\rho}{\sigma}\right)\right], \\ z_2(\phi_1) &= i(\phi_1-i)\frac{\kappa\theta\rho}{\sigma}, \\ z_3(\phi_1) &= \frac{1}{2}i(\phi_1-i)\frac{\rho}{\sigma}, \\ \hat{z}_1(\phi_1) &= -\frac{1}{2}\left[(i\phi_1)^2(1-\rho^2) - i\phi_1\left(1-\frac{2\kappa\rho}{\sigma}\right)\right], \\ \hat{z}_2(\phi_1) &= i\phi_1\frac{\kappa\theta\rho}{\sigma}, \\ \hat{z}_3(\phi_1) &= \frac{1}{2}i\phi_1\frac{\rho}{\sigma}, \end{aligned}$$

and functions $B(t,T_0)$, $C(t,T_0)$ and $D(t,T_0)$ are defined as such:

$$\begin{aligned} B(t,T_0) &= \frac{1}{\sigma^2\Gamma_1}\left\{\frac{[\kappa\theta\Gamma_1-\Gamma_2\Gamma_3]+\Gamma_3[\sinh[\Gamma_1(T_0-t)]+\Gamma_2\cosh[\Gamma_1(T_0-t)]]}{\cosh[\Gamma_1(T_0-t)]+\Gamma_2\sinh[\Gamma_1(T_0-t)]}\right. \\ &\quad \left. -\kappa\theta\Gamma_1\right\}, \\ C(t,T_0) &= -\frac{1}{2}\left\{\ln[\cosh[\Gamma_1(T_0-t)]+\Gamma_2\sinh[\Gamma_1(T_0-t)]] - \kappa(T_0-t)\right\} \\ &\quad + \frac{\kappa^2\theta^2\Gamma_1^2-\Gamma_3^2}{2\sigma^2\Gamma_1^3}\left\{\frac{\sinh[\Gamma_1(T_0-t)]}{\cosh[\Gamma_1(T_0-t)]+\Gamma_2\sinh[\Gamma_1(T_0-t)]} - \Gamma_1(T_0-t)\right\} \\ &\quad + \frac{(\kappa\theta\Gamma_1-\Gamma_2\Gamma_3)\Gamma_3}{\sigma^2\Gamma_1^3}\left\{\frac{\cosh[\Gamma_1(T_0-t)]-1}{\cosh[\Gamma_1(T_0-t)]+\Gamma_2\sinh[\Gamma_1(T_0-t)]}\right\}, \\ D(t,T_0) &= \frac{1}{\sigma^2}\left\{\kappa - \Gamma_1\frac{\sinh[\Gamma_1(T_0-t)]+\Gamma_2\cosh[\Gamma_1(T_0-t)]}{\cosh[\Gamma_1(T_0-t)]+\Gamma_2\sinh[\Gamma_1(T_0-t)]}\right\}, \end{aligned}$$

where

$$\begin{aligned} \Gamma_1 &= \sqrt{2\sigma^2 z_1+\kappa^2}, \\ \Gamma_2 &= \frac{\kappa-2\sigma^2 z_3}{\Gamma_1}, \\ \Gamma_3 &= \kappa^2\theta - z_2\sigma^2. \end{aligned}$$

From Equation (7), there are two-dimensional probabilities labeled as I, II, III and IV. In order to obtain the analytical solutions for the two-dimensional probabilities, we derive their corresponding characteristic functions, as given in the following lemma.

Lemma 3.2. *The characteristic functions with respect to the asset price measure and the $T-$forward measure for an underlying asset in the Schöbel-Zhu model are given as follows:*

$$\begin{aligned} f_S(\phi_1, \phi_2) \;=\; & \exp\{r[i\phi_1(T_0-t)+i\phi_2(T_1-t)]+i(\phi_1+\phi_2)x(t) \\ & -\frac{i\phi_1\rho}{2\sigma}[v^2(t)-\sigma^2(T_0-t)]-\frac{i(\phi_2-i)\rho}{2\sigma}[v^2(t)-\sigma^2(T_1-t)] \\ & +\frac{1}{2}D(t,T_0;\hat{z}_1,\hat{z}_3)v^2(t)+B(t,T_0;\hat{z}_1,\hat{z}_2,\hat{z}_3)v(t)+C(t,T_0;\hat{z}_1,\hat{z}_2,\hat{z}_3), \\ & +\frac{1}{2}D(t,T_1;w_1,w_3)v^2(t)+B(t,T_1;w_1,w_2,w_3)v(t) \\ & +C(t,T_1;w_1,w_2,w_3)\}, \end{aligned} \quad (10)$$

$$\begin{aligned} f_T(\phi_1, \phi_2) \;=\; & \exp\{r[i\phi_1(T_0-t)+i\phi_2(T_1-t)]+i(\phi_1+\phi_2)x(t) \\ & -\frac{i\phi_1\rho}{2\sigma}[v^2(t)-\sigma^2(T_0-t)]-\frac{i(\phi_2-i)\rho}{2\sigma}[v^2(t)-\sigma^2(T_1-t)], \\ & +\frac{1}{2}D(t,T_0;\hat{z}_1,\hat{z}_3)v^2(t)+B(t,T_0;\hat{z}_1,\hat{z}_2,\hat{z}_3)v(t)+C(t,T_0;\hat{z}_1,\hat{z}_2,\hat{z}_3), \\ & +\frac{1}{2}D(t,T_1;\hat{w}_1,\hat{w}_3)v^2(t)+B(t,T_1;\hat{w}_1,\hat{w}_2,\hat{w}_3)v(t) \\ & +C(t,T_1;\hat{w}_1,\hat{w}_2,\hat{w}_3)\}, \end{aligned} \quad (11)$$

where

$$\begin{aligned} w_1(\phi_2) &= -\frac{1}{2}\left[(i(\phi_2-i))^2(1-\rho^2)-i(\phi_2-i)\left(1-\frac{2\kappa\rho}{\sigma}\right)\right], \\ w_2(\phi_2) &= i(\phi_2-i)\frac{\kappa\theta\rho}{\sigma}, \\ w_3(\phi_2) &= \frac{1}{2}i(\phi_2-i)\frac{\rho}{\sigma}, \\ \hat{w}_1(\phi_2) &= -\frac{1}{2}\left[(i\phi_2)^2(1-\rho^2)-i\phi_2\left(1-\frac{2\kappa\rho}{\sigma}\right)\right], \\ \hat{w}_2(\phi_2) &= i\phi_2\frac{\kappa\theta\rho}{\sigma}, \\ \hat{w}_3(\phi_2) &= \frac{1}{2}i\phi_2\frac{\rho}{\sigma}, \end{aligned}$$

and functions $B(t,T)$, $C(t,T)$, $D(t,T)$, z_1, z_2, z_3, $\hat{z}_1$, $\hat{z}_2$, and $\hat{z}_3$ are as defined in Lemma 3.1.

Hence, we obtain the pricing formula for a European call holder-extendable option as presented in the following proposition.

Proposition 3.1. *Under the Schöbel-Zhu model, the price of a European call holder-extendable option is given by:*

$$\begin{aligned} EC &= S(t)[1 - F_S(\ln I_2)] - Ke^{-r(T_0-t)}[1 - F_T(\ln I_2)] \\ &\quad + S(t)[F_S(\ln I_2, \ln X) - F_S(\ln I_1, \ln X)] \\ &\quad - Xe^{-r(T_1-t)}[F_T(\ln I_2, \ln X) - F_T(\ln I_1, \ln X)] \\ &\quad - Pe^{-r(T_0-t)}[F_T(\ln I_2) - F_T(\ln I_1)], \end{aligned} \tag{12}$$

where the probability distribution functions F are obtainable via the Fourier inversion formula (Shephard, 1991):

$$\begin{aligned} F_m(j) &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathbf{Re}\left[\frac{f_m(\phi_1)e^{-i\phi_1 j}}{i\phi_1}\right] d\phi_1 \\ F_m(j, \ln X) &= \frac{1}{4} + \frac{1}{2\pi}\int_0^\infty \mathbf{Re}\left[\frac{f_m(0,\phi_2)e^{-i\phi_1(\ln X)}}{i\phi_2}\right] d\phi_2 \\ &\quad - \frac{1}{2\pi}\int_0^\infty \mathbf{Re}\left[\frac{f_m(\phi_1,0)e^{-i\phi_1 j}}{i\phi_1}\right] d\phi_1 \\ &\quad + \frac{1}{2\pi^2}\int_0^\infty\int_0^\infty \mathbf{Re}\left[\frac{f_m(\phi_1,\phi_2)e^{-i\phi_1 j - i\phi_2(\ln X)}}{\phi_1\phi_2}\right] d\phi_1\, d\phi_2, \\ &\quad - \frac{1}{2\pi^2}\int_0^\infty\int_0^\infty \mathbf{Re}\left[\frac{f_m(\phi_1,-\phi_2)e^{-i\phi_1 j + i\phi_2(\ln X)}}{\phi_1\phi_2}\right] d\phi_1\, d\phi_2, \end{aligned}$$

with $m = S,\, T,$ and $j = \ln I_1,\, \ln I_2$.

This completes the pricing framework for European call holder-extendable options within the Schöbel-Zhu model, which is a stochastic volatility model with an Ornstein-Uhlenbeck process that allows correlation between asset returns and volatility.

4. Conclusion

Stochastic volatility model incorporates one of several important empirical characteristics of asset returns variability. In this study, we develop a theoretical pricing framework and derive a closed-form pricing solution for European call holder-extendable options under the Schöbel-Zhu model where the stochastic volatility has been specified by an Ornstein-Uhlenbeck process. Further investigation aims to incorporate jumps with stochastic volatility to capture other important empirical characteristics of asset return variability.

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