Inequality Under Frictional Labour Markets

A thesis submitted for the degree of Doctor of Philosophy

by

Emil Kostadinov

Department of Economics

University of Essex

September 2016

Dedicated to my parents for their endless support

Acknowledgements

I am in debt to my supervisors, Carlos Carrillo-Tudela and Melvyn Coles. During my first year as a graduate student at Essex, Carlos taught a course in search theory; his knowledge, enthusiasm and ability to demonstrate theory's relevance is what sparked my own interest in the subject. I am grateful for his guidance and support throughout my research and his availability as an advisor. The third chapter of this thesis is joint work with Melvyn. His approach to economics has been a constant source of inspiration; the discussions with him and his informal, direct, and in many other ways unique, style of supervision have deeply influenced my understanding of what is interesting.

I am grateful to my committee members Helen Weeds and Ben Etheridge for their feedback and suggestions. On a number of occasions I have discussed aspects of this thesis with Eric Smith, Joao Santos-Silva, Tom Crossley and Giovanni Mastrobuoni, and their comments are reflected throughout these chapters.

I would like to acknowledge and appreciate the financial support of the Economic and Social Research Council and extend my gratitude to the Economics Department at the University of Essex for providing an open and stimulating environment. I thank my fellow PhD students and my friends. Finally, I am grateful for the continuous support of my parents, to whom this thesis is dedicated.

Abstract

This thesis studies the emergence of, and the interaction between, inequality in earnings and inequality in wealth, when labour markets are frictional.

Chapter 1 investigates the implications of search frictions and human-capital accumulation for the equilibrium distribution of wages when firms invest optimally in match-specific productivity. Optimal investment choice is incorporated in a framework along the lines of Burdett et al. (2011) and equilibrium is characterised. The effect of the rate of human capital accumulation on equilibrium dispersion of firm productivities and wages is analysed in a numerically solved version of the model.

Chapter 2 studies the empirical relationship between wealth and two labour market outcomes - re-employment wages and unemployment durations. The analysis complements a closely related literature by exploiting new data from the Survey of Income and Program Participation. As in prior studies, negative relationship between net worth and hazard rates to employment is documented. In disagreement with prior studies, the relationship between re-employment wages and net worth is found to be non-monotonic and it is argued that prior findings likely result from misspecification. The implications of the relationship serve as a motivation for the third chapter.

Chapter 3 (joint with Melvyn Coles) presents a model of the consumption-leisure tradeoff for risk-averse workers when labour markets are frictional. Optimal behaviour is that of a life-cycle consumer - work when young and save for retirement (non-participation) later - planning retirement efficiently. The analysis has highly tractable implications for wealth dynamics which emphasise life-cycle motives, labour-market decisions, persistent differentials in ability and heterogeneity in initial wealth. The model's empirical relevance is assessed; it is demonstrated

that it provides an empirically convincing explanation for much of the between-households inequality in wealth.

Contents

List of Tables

List of Figures

Chapter 1

On-the-Job Search, Human Capital Accumulation and Endogenous Firm Productivity

1.1 Introduction

When workers' productivity increases with experience, ex-ante identical firms in a frictional labour market characterised by churning have differential pay policies, not only because higher pay retains workers for longer but also because it implies that firms attract more experienced work-force in equilibrium (Burdett et al., 2011). It has been long recognized that the relationship between pay policies and turnover provides incentives for firms to also differentiate themselves in terms of productivity (Mortensen, 2000; Acemoglu and Shimer, 2000; Quercioli, 2005; Mortensen, 2005). Intuitively, a firm whose workers are less likely to quit gains by investing in the productivity of the match (or in the workers' human capital) implying positive relationship between firm productivity and wage offers. The possibility of endogenous determination of productive differentials across firms is interesting because empirically these differentials are large and persistent (Bartelsman and Doms, 2000; Lentz and Mortensen, 2008) and essential for improving the fit of standard models of wage dispersion (Postel-Vinay and Robin, 2002).

This paper studies firms' pay policies and decisions to invest in match-specific productivity in a labour market where workers accumulate human capital through experience and search randomly. Since high-paying firms see their workers quit less often and also attract more experienced workforce they have incentive to make larger productivity investments; given that their productivity is higher (both match productivity and average productivity of workers) they tend to pay higher wages. Such feedback links the rate of human capital accumulation and the properties of the technology transforming investment into productivity gains to the equilibrium distribution of pay policies, firm productivities, and wages.

To address the question I present a steady-state model of frictional labour market featuring on-the-job search, human capital accumulation, matching and job-creation, and a firm-level decision to invest in productive capacity. The environment is closely related to Burdett et al. (2011) and, in particular, the supply-side of the labour market is identical to the more recent generalization by Burdett et al. (2016). Workers enter the market with different ex-ante abilities, accumulate human capital exponentially while employed, search randomly for employment when unemployed and for better-paying employment when employed. As in Burdett et al. (2016) meeting rates of employed and unemployed workers differ by a constant fraction, but in addition depend on equilibrium market tightness. Firms create "job sites" which at any point in time are either vacant or filled[1]. Upon entry a vacancy commits to a time-invariant job-specific investment, which determines its productivity once matched and flow cost while vacant, and piece-rate[2] paid to any employee following a match. I characterize the steady-state equilibrium and, in particular, show that higher-paying firms invest more in match-specific productivity. I then parameterize the model consistently with

[1] As I abstract from issues of firm size, the term "firm" refers to one of these job sites.

[2] As human capital accumulation implies that the productivity of a match changes over time, modelling explicitly how the total product is divided between worker and firm is challenging. The assumption of piece-rate offers (also used in other studies such as Bagger et al. (2014) and Fu (2011)) circumvents the difficulty.

Burdett et al. (2016) and investigate numerically how the rate of human capital accumulation and the parameters related to investment affect equilibrium dispersion of productivity and piece-rate offers. The results imply that high rates of accumulation and high returns to investment imply more disperse and positively skewed offer distributions, and have quantitatively large effect.

The labour market environment in this paper is most closely related to Burdett et al. (2016) and Burdett et al. (2011) but differs insofar as I focus on endogenous productivity dispersion and job-creation. Firms' decision to invest in productivity at the point of posting a vacancy closely resembles the setup in Mortensen (2000) but there workers do not accumulate human capital. By merging these two ideas, the analysis here contributes to the literature by emphasising the interaction between human-capital accumulation, pay policies, and productivity investment in the determination of equilibrium distribution of earnings and firm productivity. To the extent that growth rates of human capital (i.e. the steepness of the learning curve) differ across labour-market segments[3], wage and productivity dispersion will also differ. Furthermore, if the rates of human capital accumulation or the cost/return to investment in productivity for firms can be affected by policy, the analysis here presents a framework for evaluating its distributional consequences.

On the other hand, my emphasis on the interaction between human capital accumulation, pay policies and investment in productivity, is similar to Fu (2011) but the analysis here differs in two important respects. First, I model human capital accumulation as universal (learning-by-doing) while Fu (2011) emphasises its emergence as a consequence of costly firm-specific training decision. Second, I model investment in productivity as match-specific, while in her model a firm investing in training increases its workers' general human capital. While the main question of interest is very similar, the underlying assumptions are, in a sense, diametrically opposed.

The paper proceeds as follows. Section 1.2 formulates the model. Sections 1.3

[3] For example, Bagger et al. (2014) show that it differs substantially by education, with most educated workers observing largest growth rates.

and 1.4 state the optimisation problem of workers and firms respectively. Section 1.5 defines steady-state equilibrium and the latter is characterised in Section 1.6. Section 1.7 presents the results from the numerical analysis. Section 1.8 concludes.

1.2 Environment

Time is continuous. A continuum of heterogeneous workers, normalized to unit mass, and a continuum of ex ante identical firms interact in a frictional labour market. All agents are risk-neutral and discount the future at rate, r. The system is in steady state.

Workers enter the labour market unemployed, with no experience, and with different initial productive abilities, summarised by the random variable, ϵ. At a constant Poisson rate, ϕ, a worker leaves the labour market for good. New workers enter the labour force at the same rate, implying that the distribution of ϵ among workers, $A(\epsilon)$, is time-invariant.

A worker's productivity is summarised by an individual-specific variable, y, determined by ability and experience. For a worker of ability ϵ who has been employed for x years

$$y(\epsilon, x) = \epsilon e^{\rho x}$$

Human capital is general, accumulates at a constant rate, ρ, during employment and does not depreciate. To prevent infinite accumulation, assume $\phi > \rho$.

Both employed and unemployed workers search. An unemployed worker meets a random vacancy at Poisson rate $\lambda_u(\theta)$, while an employed worker meets a random vacancy at Poisson rate $\lambda_e(\theta)$. The number of meetings in the economy is determined by a matching technology and market tightness, θ, is the ratio of vacancies to effectively searching workers[4]. I assume $\lambda_u > \lambda_e$ and $\lambda \equiv \lambda_e/\lambda_u$, the relative search intensity of employed as compared to unemployed workers, to be

[4]During most of Sections 1.3 and 1.4 explicit notational reference to market tightness is suppressed for brevity.

constant.

Firms create "job sites" (Mortensen, 2000) which at any point in time are either vacant or filled. As I abstract from issues of firm size, the term "firm" refers to one of these job sites. A firm enters the labour market by posting a vacancy, simultaneously chooses the productivity of its job opening and commits to paying a constant fraction $\tau \in (0,1)$ of the future match product to any prospective employee. To set ideas, imagine that the firm opens a vacancy by investing in capital, k, and the investment maps into a unique time-invariant firm-specific productivity, $p(k)$. To capture the idea that investment is costly, assume that while still vacant, a more productive firm incurs a larger flow cost, $c(k)$, perhaps because of expenses necessary to preserve productive capacity when a job is still idle[5]. Assume $p'(.) > 0$, $p''(.) < 0$, $\lim_{k\to\infty} p'(k) = 0$, $c'(.) > 0$, and $c''(.) \geq 0$ (see below). A vacancy meets random workers at rate $\eta(\theta)$ and becomes a "job" upon meeting a worker who accepts the match. Jobs are destroyed exogenously at a Poisson rate, δ, or when the matched worker finds better-paid employment. Upon destruction the job becomes a vacancy (with the same amount of capital) while the worker becomes unemployed (upon exogenous destruction) or transits to higher-paid employment. In equilibrium free entry drives the value of posting a vacancy to zero.

A firm of productivity $p(k)$ matched with a worker of productivity y produce flow of output $p(k)y$. The match product is shared according to the pre-specified piece rate, τ - at each point of time the worker receives flow wage $\tau p(k)y$ and the firm receives flow profit $(1-\tau)p(k)y$. For notational purposes, let $z \equiv \tau p(k)$. Assume that the flow of benefits to an unemployed worker is proportional to her productivity, $z_b = by$, where b is a policy-set parameter[6].

The flow of meetings between workers and vacancies is described by a matching

[5]Alternatively, one can think of $c(k)$ as the opportunity cost of not filling a vacancy.

[6]This implies that unemployment benefits are proportional to a worker's productivity, rather than their most recent wage. The specification is preferred for analytical convenience - unemployment income proportional to past wages will imply heterogeneity of reservation pay rates among the pool of unemployed workers, an aspect from which the analysis here abstracts. For a discussion see Burdett et al. (2011)

technology, $m(u, 1-u, v)$, where u and v are the unemployment rate and the measure of vacancies. Employed and unemployed workers are assumed 1-to-λ perfect substitutes in the matching technology, that is $m(u, 1-u, v) = m(u + \lambda(1-u), v)$. Accordingly let $\theta = v/(u+\lambda(1-u))$ denote labour market tightness. In what follows, I assume directly a Cobb-Douglas matching function of the form

$$m(u + \lambda(1-u), v) = \beta(v)^{\alpha}[u + \lambda(1-u)]^{1-\alpha}$$

In order for the number of meetings prescribed by the matching function to equal the number of meetings accruing to workers, the meeting rates are related to market tightness and the matching parameters in the following way[7]

$$\begin{aligned} \eta(\theta) &= m(1/\theta, 1) = \beta\theta^{\alpha-1} \\ \lambda_u(\theta) &= m(1, \theta) = \beta\theta^{\alpha} \\ \lambda_e(\theta) &= \lambda\lambda_u(\theta) = \lambda\beta\theta^{\alpha} \end{aligned}$$

1.3 Workers' Behaviour

This section formulates the dynamic problem faced by workers and characterises their optimal behaviour given any profile of firms' behaviours (summarised by the distribution of z across vacancies, $F(z)$) and labour market tightness. It is important to notice that given z workers have no preference over individual firms' combinations of payout policies (τ) and firm-specific productivities ($p(k)$). This property is convenient because in turn will imply that given z a firm's equilibrium turnover is independent of k which yields a tractable investment problem. For the same reason the workers' problem is identical to the one in Burdett et al. (2016). For completeness, I state the workers' problem, its solution, and discuss the main intuition but do not state the proofs explicitly as they can be found in Burdett

[7]This is a standard specification of matching function in the context of on-the-job search; for example see Dolado et al. (2009)

et al. (2016).

Let $W^U(y|F(.),\theta)$ be the lifetime utility of an unemployed worker with productivity y given offer distribution $F(z)$ and tightness, θ; let $W^E(y,z|F(.),\theta)$ be the lifetime utility of a worker with productivity y employed at z and facing offer distribution $F(z)$. Recall that the offer distribution and tightness are determined in equilibrium.

An unemployed worker with productivity y faces random death risk, discounts the future, receives flow income by and meets firms at rate λ_u. Upon meeting a vacancy she decides whether to match based on a comparison of her expected lifetime utilities under unemployment and under employment at that firm. The Bellman equation for the value of unemployment is then

$$(r+\phi)W^U(y|.) = by + \lambda_u \int_{\underline{z}}^{\bar{z}} \max\left[W^E(y,z'|.) - W^U(y|.), 0\right] \partial F(z') \tag{1.1}$$

A worker with productivity y employed at a firm paying z faces random death and job destruction risks, discounts the future, receives flow income zy, accumulates human capital and meets new vacancies at rate λ_e. Upon meeting a new vacancy the worker compares her expected lifetime utilities under her current employer and under the one just met and chooses optimally (assuming that a worker never optimally quits into unemployment - which is true in equilibrium). Since working at higher piece rate is always better for the worker, the value of employment increases with z. Therefore, it is immediate that a worker quits a firm to match with another firm if and only if the new firm offers higher z. We also adopt the convention that if a worker is offered exactly her current z she remains with the incumbent firm. The Bellman equation for the value of employment is then

$$\begin{aligned}(r+\phi \quad + \quad & \delta)W^E(y,z|.) = zy + \delta W^U(y|.) + \frac{\partial W^E(y,z|.)}{\partial y}\rho y + \\ + \quad & \lambda_e \int_{z}^{\bar{z}} \max\left[W^E(y,z'|.) - W^E(y,z|.), 0\right] \partial F(z') \end{aligned} \tag{1.2}$$

Denote $q(z) \equiv \phi + \delta + \lambda_e(1 - F(z))$, the rate at which a worker employed at z leaves her employer. The optimal behaviour of a worker as implied by (1.1) and (1.2) is then completely characterised by the following

Claim 1. *Optimal workers' behaviour is fully described by a set of reservation values of z for employed and unemployed workers.*

1. *The reservation value of an employed worker is her current z.*

2. *The reservation value for an unemployed worker, z^R, satisfies*

$$\begin{aligned} (r+\phi)z^R(F(z),\theta) &= (r+\phi-\rho)b + \\ &+ [\lambda_u(r+\phi-\rho) - \lambda_e(r+\phi)] \int_{z^R}^{\bar{z}} \frac{1-F(z')}{q(z')+r-\rho} \partial z' \end{aligned} \tag{1.3}$$

3. *Sufficient condition for existence and uniqueness of a solution is $\bar{z} > b(r + \phi - \rho)/(r+\phi)$*

Proof. See Proposition 1 in Burdett et al. (2016). □

The main arguments behind the proof are the following. First, $W^U(y)$ is independent of z and $W^E(y,z)$ is increasing in z. Therefore, worker behaviour is indeed described by cutoff, or reservation values of z, at which she optimally changes states. Second, since all payoffs are proportional to y, then so are the value functions. That is, there exist real valued α^U and $\alpha^E(z)$ such that

$$W^U(y) = \alpha^U y, \text{ and } W^E(y,z) = \alpha^E(z)y$$

Proposition 1 can be then proved by substituting the latter back into the value functions, evaluating at z^R and solving the resulting system of equations. The proportionality of the value functions to y implies that the reservation value of an unemployed worker does not depend on her productivity. The latter is due to the fact that workers are paid in piece rates and that human capital accumulates at constant rate.

When there is no learning by doing, $\rho = 0$, and unemployed and employed workers search at the same intensity, $\lambda_e = \lambda_u$, (1.3) implies that $z^R = b$. When $\rho = 0$ and $\lambda_e < \lambda_u$, (1.3) implies that $z^R > b$, because by entering into employment a worker forgoes the opportunity to search at a higher intensity, hence making unemployment relatively more valuable. Positive rate of human capital accumulation, $\rho > 0$, makes employment relatively more valuable and, therefore, decreases the reservation value of z. At a sufficiently high ρ, the implied z^R may even become negative - workers might be willing to pay in order to be able to stay in employment and accumulate experience.

1.4 Firms' Behaviour

I now turn attention to the optimal behaviour of firms, taking the reservation value of workers, z^R, and market tightness, θ, as given. Let $J(z, k, \epsilon, x)$ be the value to a firm with capital, k, paying z from being matched with a worker with initial ability ϵ and experience x. Let $V(z, k)$ be the value to a firm from posting a vacancy with capital k committed to paying z.

A firm with capital k, paying according to z, matched with a worker of productivity y produces an instantaneous flow $p(k)y$ and receives flow profit $(p(k)-z)y = (p(k)-z)\epsilon e^{\rho x}$. It discounts the future, gets more productive over time due to human capital accumulation, and becomes vacant if exogenously destroyed or its worker quitted. Its value is therefore implicitly defined by the ordinary differential equation

$$(r + q(z))J(z, k, \epsilon, x) = (p(k) - z)\epsilon e^{\rho x} + \frac{\partial J(z, k, \epsilon, x)}{\partial x} + (\delta + \lambda_e(1 - F(z)))V(z, k) \quad (1.4)$$

Solving (1.4) (integrating by parts with respect to x over the interval $[x, \infty)$, using an integration factor $e^{-(r+q)x}$ and assuming $\lim_{x'\to\infty} J(.,x')e^{-(r+q(.))x} = 0$) yields

$$J(z,k,\epsilon,x) = \frac{(p(k)-z)\epsilon e^{\rho x}}{q(z)+r-\rho} + \frac{\delta + \lambda_e(1-F(z))}{q(z)+r} V(z,k) \tag{1.5}$$

Direct examination then confirms that indeed $\lim_{x'\to\infty} J(.,x')e^{-(r+q(.))x} = 0$.

A vacancy that invested k and committed to pay according to z incurs a flow cost $c(k)$ and meets a random worker at Poisson rate $\eta(\theta)$. The value of a vacancy offering $\{z,k\}$ is then given by

$$(r+\eta(\theta))V(z,k) = -c(k) + \eta(\theta)M(z,k)$$

where $M(z,k)$, the expected value from a match, is an appropriately weighted average of the job value (see (1.12)). The firm chooses its $\{z,k\}$ upon entering the market so that its vacancy value is maximised. The Bellman equation for a vacancy is then

$$(r+\eta(\theta))V = \max_{z,k}\{-c(k) + \eta(\theta)M(z,k)\} \tag{1.6}$$

Given free entry, V is driven down to 0 in equilibrium.

1.5 Equilibrium

Let U^ϵ, N^ϵ(x), and $H^\epsilon(x,z)$ be the unemployment rate, the distribution of experience among unemployed workers, and the joint distribution of experience and z among employed workers of ability ϵ, respectively.

Definition A steady-state equilibrium is a set of pay and investment policy pairs, $Z \times K$, an offer distribution, $F(z)$, over Z, associated distribution of productivities, a reservation offer rate, z^R, unemployment rate, U^ϵ, steady state distributions, $N^\epsilon(x)$ and $H^\epsilon(x,z)$, and market tightness, θ, such that

- Given $F(z)$ and θ, the reservation offer rate, z^R, is given by (1.3); worker's behaviour is optimal.

- Given z^R, $F(z)$, and θ,

$$\{z,k\} = \arg\max_{z,k} V(z,k), \forall \{z,k\} \in Z \times K$$

  The behaviour of each firm is individually optimal.

- The distribution of offers, $F(z)$, is consistent with individual firms' optimal offer policies.

- U^ϵ, $N^\epsilon(x)$, and $H^\epsilon(x,z)$ are consistent with equilibrium turnover given optimal behaviour and market tightness.

- The market tightness, θ, is such that the free entry condition holds

$$0 = V \equiv V(z,k), \forall \{z,k\} \in Z \times K$$

1.6 Characterisation

1.6.1 Turnover and distributions

To characterise equilibrium, I first derive expressions for U^ϵ, $N^\epsilon(x)$, and $H^\epsilon(x,z)$. Notice that conditional on z, turnover is independent of firms' investment decisions.

Consider U^ϵ. The inflow of workers to this pool over a time interval of length dt is $(\phi+\delta(1-U^\epsilon))dt$ (new labour-market entrants and previously employed workers whose jobs were destroyed) with associated outflow $(\lambda_u+\phi)dtU^\epsilon$ (previously unemployed workers who became employed or left the labour market). Equating the flows yields a steady-state unemployment rate

$$U \equiv U^\epsilon = \frac{\phi+\delta}{\phi+\delta+\lambda_u} \tag{1.7}$$

which is independent of ability, ϵ.

Consider $N^{\epsilon}(x)$, the pool of unemployed workers of type ϵ with experience below x. The inflow of workers to this pool is $(\phi + \delta(1-U)H^{\epsilon}(x,\bar{z}))dt$ (new labour-market entrants or previously employed workers with experience below x whose jobs were destroyed) and the outflow is $(\phi + \lambda_u)dtUN_{\epsilon}(x)$ (this pool is only left through transitions to employment or non-participation, as experience is constant during an unemployment spell), yielding

$$N_{\epsilon}(x) = \frac{\phi(\phi + \delta + \lambda_u) + \delta\lambda_u H_{\epsilon}(x,\bar{z})}{(\phi + \lambda_u)(\phi + \delta)} \tag{1.8}$$

Consider $H_{\epsilon}(x,z)$, the pool of workers of type ϵ with experience no more than x earning at rate no more than z. The inflow to this pool is $UN_{\epsilon}(x)\lambda_u dt$ (previously unemployed workers of experience below x who found a job) and outflow

$$(1-U)H_{\epsilon}(x,z)q(z)dt + (1-U)(H_{\epsilon}(x,z) - H_{\epsilon}(x-dt,z)) + O(dt^2)$$

(the first term describes workers who transited to unemployment, non-participation or employment at rate above z; the second term describes those who remained employed at rate below z but accumulated experience above x; $O(dt^2)$ accounts for the fact that some workers both accumulated experience above x and left employment at pay rate below z, but this term is of order dt^2). Equating inflow to outflow and rearranging yields the first-order ordinary differential equation in $H_{\epsilon}(x,z)$

$$(\phi + \delta)N_{\epsilon}(x)F(z) = q(z)H_{\epsilon}(x,z) + \frac{\partial H_{\epsilon}(x,z)}{\partial x} \tag{1.9}$$

which can be solved by directly integration using factor $e^{q(z)x}$. By solving simultaneously (1.8) and (1.9) closed-form expressions for the steady-state distributions are obtained. Given θ and $F(.)$ all flows are determined by the worker's problem and therefore the steady-state distributions are identical to the ones in Burdett et al. (2016) where further elements of the derivation are discussed in detail. In particular, U_ϵ, $N_\epsilon(x)$, and $H_\epsilon(x,z)$ are independent of the worker's type, ϵ, and

$$H(x,z) \equiv H^\epsilon(x,z) = \frac{(\phi+\delta)F(z)}{q(z)}\left[1-e^{-q(z)x}\right] - \tag{1.10}$$

$$- \frac{\delta\lambda_u F(z)}{\delta\lambda_u + \lambda_e(1-F(z))(\phi+\lambda_u)}\left[e^{-\frac{\phi(\phi+\delta+\lambda_u)}{\phi+\lambda_u}x} - e^{-q(z)x}\right]$$

$$N(x) \equiv N^\epsilon(x) = 1 - \frac{\delta\lambda_u}{(\phi+\lambda_u)(\phi+\delta)}e^{-\frac{\phi(\phi+\delta+\lambda_u)}{\phi+\lambda_u}x} \tag{1.11}$$

1.6.2 Value of a vacancy

Upon entering the market a vacancy chooses k (which determines its flow cost during recruitment and affects the product of any future match) and posts z (which affects the probability that a future match will be left by a worker, but also the distribution of experience among workers among whom the firm recruits). Since search is random, a vacancy meets a random worker. If it offers $z < z^R$ no worker accepts; if it offers $z \geq z^R$, all unemployed workers, as well as all workers employed at $z' < z$ accept. Notice that firm's expected turnover is independent of k conditional on z. Then the expected job value for a vacancy offering $\{z,k\}$ is

$$M(z,k) = \int_{\epsilon'} \frac{\lambda_u U^{\epsilon'}}{\lambda_u U^{\epsilon'} + \lambda_e(1-U^{\epsilon'})}\left(\int_{x'=0}^{\infty} J(.,\epsilon',x'|.)dN^{\epsilon'}(x')\right)dA(\epsilon') + \tag{1.12}$$

$$+\int_{\epsilon'} \frac{\lambda\lambda_u U^{\epsilon'}}{\lambda_u U^{\epsilon'} + \lambda_e(1-U^{\epsilon'})}\left(\int_{x'=0}^{\infty}\int_{z'\in[z^R,z)} J(.,\epsilon',x'|.)dH^{\epsilon'}(x',z')\right)dA(\epsilon')$$

whenever $z \geq z^R$ and

$$M(z,k) = 0$$

for $z < z^R$.

Given equations (1.7), (1.11), (1.10) and (1.5) a closed form expression for the expected value of a job for a vacancy posting given $\{z, k\}$ can be obtained. To derive it I conduct the integration in (1.12) directly (full workings are demonstrated in Appendix 1.A) and after imposing the free entry condition in (1.6), it follows that for all $z \in [\underline{z}, \bar{z}]$

$$0 = -c(k) + \frac{\eta(\theta)\tilde{\epsilon}(\phi+\delta+\lambda_u)}{(\phi+\delta+\lambda_e)} \left[\frac{p(k)-z}{q(z)+r-\rho}\right] \left[a_0 + \lambda a_1 \frac{F(z)}{q(z)-\rho}\right] \tag{1.13}$$

where

$$\begin{aligned} a_0 &= \frac{\phi\delta\lambda_u}{(\phi+\lambda_u)(\phi(\phi+\delta+\lambda_u)-\rho(\phi+\lambda_u))} \\ a_1 &= \frac{\phi(\phi+\delta-\rho)\lambda_u}{(\phi(\phi+\delta+\lambda_u)-\rho(\phi+\lambda_u))} \\ \tilde{\epsilon} &= \int_\epsilon \epsilon' dA(\epsilon') \end{aligned}$$

A firm choosing z faces a tradeoff: a lower z means higher profit flow whoever the firm is matched with; a higher z means lower probability of the worker leaving the firm and also higher expected productivity of the match. For future references, let $a(z) \equiv a_0 + \lambda a_1 F(z)/(q(z)-\rho)$.

1.6.3 Investment choice

I now turn to the optimal choice of investment, k. Suppose that the pair $\{z^*, k^*\}$ is an optimal policy. Since z^* is optimal, the envelope theorem implies that the optimal choice of k requires (differentiating (1.13) with respect to k at the optimum)

$$c_k(k^*) = \frac{\eta\tilde{\epsilon}(\phi+\delta+\lambda_u)}{(q(z^*)+r-\rho)(\phi+\delta+\lambda_u)} \left[a_0 + \lambda a_1 \frac{F(z^*)}{q(z^*)-\rho}\right] p_k(k^*) \tag{1.14}$$

It is easy to see that the restrictions on $p(.)$ and $c(.)$ guarantee that the latter identifies a maximum and the solution is unique. Furthermore, combining (1.13) and (1.14) yields

$$z^* = p(k^*) - p_k(k^*)\frac{c(k^*)}{c_k(k^*)} \tag{1.15}$$

The right-hand side is strictly increasing in k and therefore (1.15) describes a one-to-one mapping between z and k irrespective of the functional form of $F(.)$. By offering higher pay rates a vacancy expects to match with more productive workers and also to keep them for longer. Since their productivity augments its own, it finds investment in capital more profitable - firms that offer high pay rates invest more in capital. Henceforth, let $k(z)$ denote the optimal choice of investment for a firm offering z, and let $p(z) \equiv p(k(z))$ and $c(z) \equiv c(k(z))$.

1.6.4 Offer distribution

I now characterise the equilibrium distribution of z. First, notice that all vacancies yield zero value in expectation and in particular the value from offering z^R is the same as from offering the highest equilibrium rate $\bar{z}$. Equating $V(z^R, k(z^R))$ to $V(\bar{z}, k(\bar{z}))$ (assuming non-degenerate distribution) yields an expression for the highest equilibrium offer, $\bar{z}$ in terms of z^R, which can be conveniently expressed as

$$\left(\frac{p(\bar{z}) - \bar{z}}{c(\bar{z})}\right) = \left(\frac{p(z_R) - z_R}{c(z_R)}\right) \frac{\delta(\phi + \delta + r - \rho)}{(\delta + \lambda(\phi + \lambda_u))(\phi + \delta + \lambda_e + r - \rho)} \tag{1.16}$$

Notice that given (1.15), the restrictions on $p(.)$ and $c(.)$ guarantee that $(p(z) - z)/c(z)$ is invertible, and for each level of tightness and z_R, (1.16) determines a unique $\bar{z}$.

Next, equating $V(z, k(z))$ to $V(z^R, k(z^R))$ yields

$$\left(\frac{a_0}{q(z)+r-\rho} + a_1 \frac{\lambda F(z)}{(q(z)+r-\rho)(q(z)-\rho)}\right) =$$

$$= \frac{p(z_R)-z_R}{c(z_R)} \frac{c(z)}{p(z)-z} \frac{a_0}{\phi+\delta+\lambda_e+r-\rho}$$

Denoting the right-hand side by $b_0(z)$ the latter can be restated as the quadratic equation in F

$$b_0\lambda_e^2 F^2 - [b_0(2(\phi+\delta+\lambda_e-\rho)+r)\lambda_e + (\lambda a_1 - \lambda_e a_0)]F +$$

$$(\phi+\delta+\lambda_e-\rho)[b_0(\phi+\delta+\lambda_e+r-\rho)-a_0] = 0$$

with discriminant

$$D = (b_0\lambda_e r + (\lambda a_1 - \lambda_e a_0))^2 + 4b_0\lambda_e(\phi+\delta+\lambda_e-\rho)a_1 > 0$$

and roots

$$F_{1,2} = 1 + \frac{r+2(\phi+\delta-\rho)}{2\lambda_e} + \frac{\phi}{2b_0\lambda_e(\phi+\lambda_u)} \pm \frac{\sqrt{D}}{2b_0\lambda_e\lambda_u} \tag{1.17}$$

The root consistent with F being distribution is the smaller of the two (the other one exceeds one). The equilibrium offer distribution is therefore determined uniquely given z_R and θ.

1.6.5 Tightness

Finally, equilibrium tightness is determined by the free entry condition. Evaluating (1.13) at $z = z^R$, expressing the matching rates in terms of tightness, and discarding the no-trade solution (dividing both sides of the equation by θ^{α}) yields

$$\theta^{1-2\alpha} = \frac{\phi\delta\beta^2\tilde{\epsilon}[(\phi+\delta+\beta\theta^{\alpha})/(\phi+\delta+\lambda\beta\theta^{\alpha})][p(z^R)-z^R]/c(z^R)}{(\phi+\delta+\lambda\beta\theta^{\alpha}+r-\rho)(\phi+\beta\theta^{\alpha})(\phi(\phi+\delta+\beta\theta^{\alpha})-\rho(\phi+\beta\theta^{\alpha}))} \quad (1.18)$$

Notice that the right-hand side is positive, continuously decreasing in θ and approaches zero as θ limits to infinity. The behaviour of the left-hand side expression, however, depends on the sign of $(1-2\alpha)$. It is immediate that $\alpha < 0.5$ is sufficient (but not necessary) for existence of a unique $\theta(z^R) > 0$ and given this the relationship between θ and z_R is negative[8]. Further, notice that as z^R limits to infinity, the solution to (1.18) limits to zero[9].

1.6.6 Existence

Equilibrium is now fully characterised. To summarize, given z^R market tightness is determined by (1.18). Given θ and z_R, (1.17) determines the offer distribution. Given $F(.)$ and θ, (1.3) determines z_R. Further, for each $z \in [z_R, \bar{z}]$ the associated optimal capital investment is determined by (1.15) and the steady-state distributions are given by the results in Section 1.6.1.

Given this, an equilibrium is a zero of the function $T(z^R)$ defined as

$$T(z^R) \equiv (r+\phi)[z^R - z^R(F(z|z^R),\theta(z^R))] = \quad (1.19)$$

$$= (r+\phi)z^R - (r+\phi-\rho)b -$$

$$-[\lambda_u(\theta|z^R)(r+\phi-\rho) - \lambda_e(\theta|z^R)(r+\phi)] \int\limits_{z^R}^{\bar{z}(z^R)} \frac{1-F(z'|z^R)}{q(z'|z^R)+r-\rho} dz'$$

where $z^R(F(.),\theta)$ is the solution to (1.3), $\theta(z^R)$ is the solution to (1.18) and

[8] Establishing a weaker sufficient condition analytically is hindered by the complexity of the expression and is not pursued. In the numerical exercises below I find that a unique solution obtains under every parameterisation used.

[9] As $(p(z^R)-z^R)/c(z^R) = p'(z^R)/c'(z^R)$ limits to zero as long as $\lim_{z\to\infty} p'(z) = 0$.

$F(z|z^R)$ is the solution to (1.17). Notice that

$$T((r+\phi-\rho)b/(r+\phi)) < 0$$

(as search has option value) and

$$\lim_{z^R \to \infty} T(z^R) = +\infty$$

as (1.18) implies that θ approaches zero in the limit. As $T(.)$ is continuous, a disperse equilibrium with $z^R > (r+\phi-\rho)b/(r+\phi)$ exists. The latter, however, need not be unique for any values of the parameters. Differentiating (1.19) with respect to z^R yields

$$\begin{aligned} T_{z^R} &= (r+\phi) - \frac{\partial\theta}{\partial z^R}\beta\alpha\theta^{\alpha-1}[(r+\phi-\rho)-\lambda(r+\phi)]\int_{z^R}^{\bar{z}(z^R)} \frac{1-F(z'|z^R)}{q(z'|z^R)+r-\rho}dz' \\ &+ \frac{[(r+\phi-\rho)-\lambda(r+\phi)]\beta\theta^{\alpha}}{\phi+\delta+\lambda_e+r-\rho} \\ &- [(r+\phi-\rho)-\lambda(r+\phi)]\beta\theta^{\alpha}\int_{z^R}^{\bar{z}(z^R)} \frac{\partial}{\partial z^R}\left[\frac{1-F(z'|z^R)}{q(z'|z^R)+r-\rho}\right]dz' \end{aligned}$$

While the first three terms of the sum are positive, the last may be negative. This being said, in the numerical exercise that follows unique disperse equilibrium obtains under the parameterisation used.

1.7 Numerical analysis

This section presents the results from numerically solving the model and, in particular, discusses how equilibrium dispersion depends on the rate of human capital accumulation and the parameters governing investment choice. The discussion centres on the implications for offer and productivity distributions rather than the distribution of piece-rates or wages across employed workers. The equilibrium distribution of offers has the immediate interpretation of the exact counterpart of frictional wage dispersion - workers of the same ability and experience who become employed simultaneously, draw initial wages from this distribution. The overall wage distribution (among all employed or only the newly-employed) generated by this class of models is more complicated as it depends on the distributions of experience among employed (which is endogenous) and on the distribution of abilities in the population (which is primitive). As discussed in Burdett et al. (2011) and Burdett et al. (2016) the wage density generated by the model inherits the shape of the ability distribution, but some features, in particular related to the right tail of the distribution depend crucially on the underlying dispersion.

1.7.1 Parameterization

I start by assuming that $\beta = 1$ and $\alpha = 0.5$; that is the matching function is given by

$$m(v, U + \lambda(1-U)) = v^{1/2}(U + \lambda(1-U))^{1/2}$$

and use this functional form for the calibration of the other parameters[10]. For the search cost and productivity functions I choose the following functional forms[11]

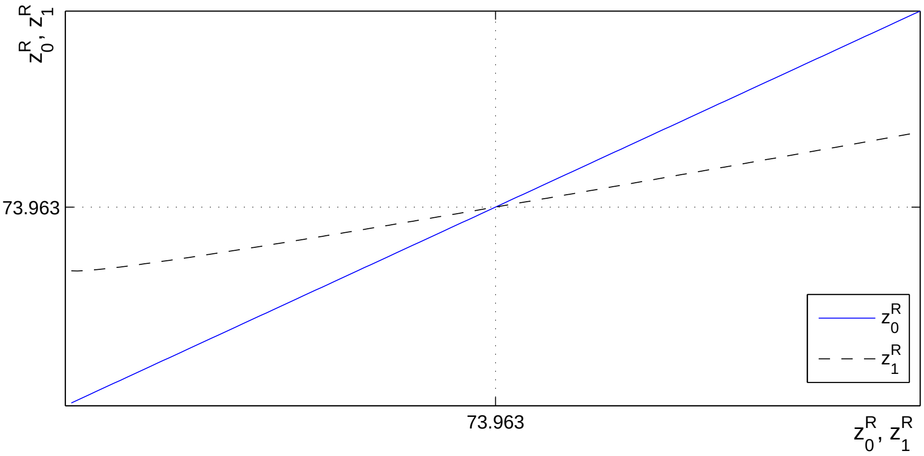
$$\begin{aligned} c(k) &= ck \\ p(k) &= pk^{\gamma} \end{aligned}$$

Setting the time period equal to one month, I choose parameters following Burdett et al. (2016). Accordingly I set $r = 0.0041$, implying annual interest rate of 5 per cent; $\phi = 0.0021$, consistent with average labour-market life of 40 years;

[10] It turns out that all following results are robust qualitatively to specifying different values of the matching elasticity as long as α is not very close to 1.

[11] In this formulation, the choice of p matters in the determination of the other parameters only as a scale factor. All the results that follow are reported using $p = 1000$.

Figure 1.7.1: Fixed point for z^R



$\delta = 0.012$.

In a benchmark specification (necessary to calibrate the other parameters) I set $\lambda = \lambda_e/\lambda_u = 0.038/0.141$, as estimated by Burdett et al. (2016) for a sample of medium-skilled workers in the BHPS (this identifies λ and sets η for the benchmark); $\rho = 0.0020$ as fitted there to match the observed mean-min ratio with the one obtained in the model for the same sample; and $\gamma = 0.4$. What remains is to set b and $c/\tilde{\epsilon}$ (c and $\tilde{\epsilon}$ are not separately identified but the ratio is sufficient for parameterising the model). As discussed in Burdett et al. (2016) the ratio b/z^R identifies uniquely the mean-min ratio generated by the model. Further, notice that given tightness, $c/\tilde{\epsilon}$ is uniquely determined as a function of z^R by the free entry condition (1.18). Using these observations, I identify the two parameters as follows. Given any guess for z^R, I set $c/\tilde{\epsilon}$ consistently (given tightness), solve the firms' problem, set b consistently with the mean-min ratio observed by Burdett et al. (2016) and check that the value from the guess is consistent with the reservation z from the workers' problem (1.3). I iterate until $T(z^R) = 0$[12]. This procedure identifies b and $c/\tilde{\epsilon}$.

Table 1.7.1: Benchmark specification

Parameter/variable	Value
Parameterized	
β	1.0000
α	0.5000
λ	0.2695
λ_u	0.1410
λ_e	0.0380
η	7.0920
ρ	0.0019
γ	0.5000
p	1000.0000
Calibrated	
b	448.6623
$c/\tilde{\epsilon}$	6094.0014
Endogenous variables	
$\bar{z}$	1033.0102
z_R	735.9138
$k(\bar{z})$	426.8667
$k(z_R)$	2.1663

1.7.2 Results

Given the discussed parameterisation, I now present the results from solving the model under different values of ρ and γ. As discussed above uniqueness of equilibrium is not guaranteed in general. Under the proposed parameters, however, equilibria are always unique. In particular, consider the following procedure. Starting with a grid for z^R, let z_0^R denote an initial guess. Under the guess the firms' problem can be solved using (1.18) (identifying θ), (1.16) (identifying $\bar{z}$ and (1.17) (identifying $F(.|z_0^R)$). Given these (1.3) identifies a unique optimal reservation value for workers given firm behaviour. Let z_1^R denote the associated solution. Then z_0^R is an equilibrium if and only if $z_0^R = z_1^R$. Under the benchmark specification ($\rho = 0.0020$ and $\gamma = 0.4$), Figure 1.7.1 plots z_0^R and z_1^R for different values of z_0^R. The figure illustrates that a unique fixed point exists and further shows that over the closed interval of z_0^R plotted $T(z^R)$ behaves as a contraction.

[12] As discussed further this is feasible as for the chosen parameters $T(.)$ behaves like a contraction. See below for further discussion.

It turns out that the same applies for different values of ρ and γ. This suggests the feasibility of numerically solving the differently parameterised versions of the model by guessing z^R, updating through (1.3) and iterating until convergence. This is the method I follow.

To illustrate the relationship between dispersion and the rate of human capital accumulation I set $\gamma = 0.4$ and solve the model for a number of different values of ρ. The left panels of Figure 1.7.2 plot the resulting cumulative and density distributions of offers (and productivity) over a normalized support. The top panel of Figure 1.7.3 plots the limits of the support against ρ. The results suggest that the distribution gets wider and more right-skewed as the rate of human-capital accumulation increases. Markets characterised by high accumulation are therefore likely to be described by more dispersion both in terms of productivity and wages.

Similarly the right panels of Figures 1.7.2 and the bottom panel of 1.7.3 illustrate the relationship between γ and the properties of the distribution. When investment yields larger productivity increases, the resulting steady-state distributions are more unequal. Both the shape and the support of the distributions are highly sensitive and relatively large values of γ can generate extremely unequal distributions.

Two important points should be discussed. First, in all cases the offer densities are continuously decreasing. However, this does not imply that wage densities are. While offer distributions are defined over z wages are further determined by individual abilities and histories of experience. For example, if the distribution of abilities is unimodal, then so will be the distribution of wages generated by the model. Second, since each z corresponds to a unique $p(k(z))$, with increasing relationship, the shape of the dispersion of z is always identical to the shape of the distribution of productivities. This is a typical feature of models featuring exogenous productivity dispersion (although here the direction of causality it is not that more productive firms offer higher wages but firms that choose to offer

Figure 1.7.2: Offers distributions, $F(z)$, and $f(z)$

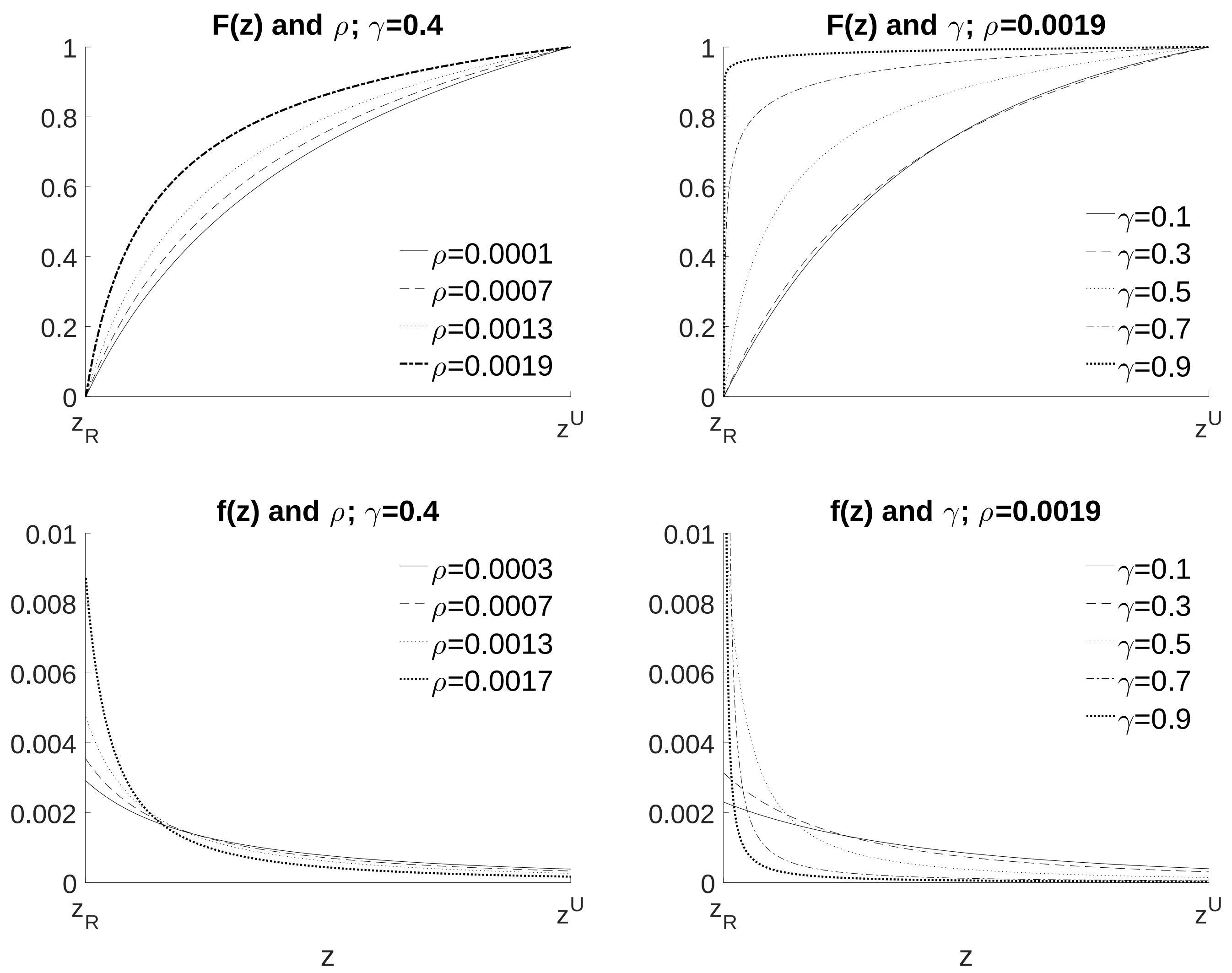
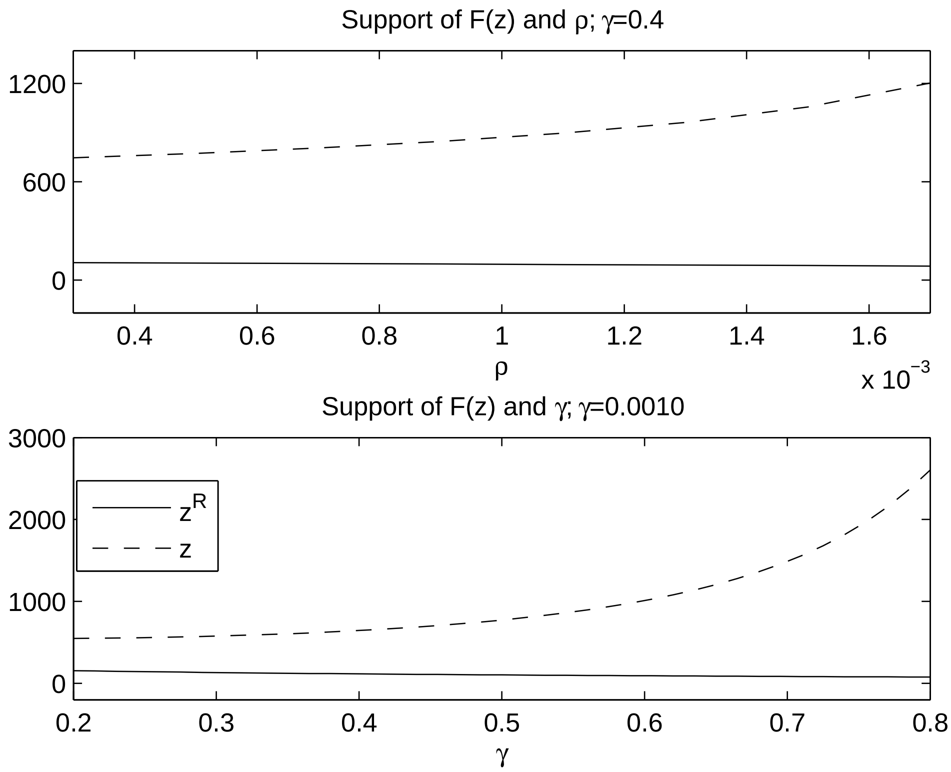


Figure 1.7.3: Support of $F(z)$



high wages also optimally choose to invest more in productivity). It should again be qualified that the distributions are defined over z rather than wages and the latter also depend on ability and experience. In particular, since more experienced workers sort with more productive firms, the overall wage density (for workers of the same ability) is more unequal than the distribution of firm productivities.

1.8 Conclusion

This paper presents a model of frictional labour markets where workers accumulate experience while working, search both when unemployed and employed and ex-ante identical firms create vacancies by making costly capital investments in productivity while simultaneously committing to piece rates. In equilibrium, firms with more generous pay policies also invest more and highly experienced (and productive) workers sort with more productive firms. Numerical solutions to the model demonstrate that equilibrium dispersion increases with the rate of human capital accumulation and with the elasticity of productivity with respect to capital investment. By varying the related parameters the model is able to generate different equilibrium distributions including ones that are extremely unequal.

Appendix

1.A Derivation of expected value of a job

This appendix derives the closed form expression for the expected value of a job given $\{z, k\}$, that is

$$M(z,k) = \int_{\epsilon'} \frac{\lambda_u U^{\epsilon'}}{\lambda_u U^{\epsilon'} + \lambda_e(1-U^{\epsilon'})} \left(\int_{x'=0}^{\infty} J(.,\epsilon',x'|.) dN^{\epsilon'}(x') \right) dA(\epsilon') + \quad (1.20)$$

$$+ \int_{\epsilon'} \frac{\lambda\lambda_u U^{\epsilon'}}{\lambda_u U^{\epsilon'} + \lambda_e(1-U^{\epsilon'})} \left(\int_{x'=0}^{\infty} \int_{z' \in [z^R, z)} J(.,\epsilon',x'|.) dH^{\epsilon'}(x',z') \right) dA(\epsilon')$$

First, notice that by the results from Section 1.6.1 the unemployment rate and steady state distributions are independent of ϵ. Plugging in (1.5)

$$
\begin{aligned}
M(z,k) \quad = \quad & \frac{\lambda_u}{\lambda_u U + \lambda_e(1-U)} \left[\int_{\epsilon'} U \left(\int_{x'=0}^{\infty} \frac{(p(k)-z)\epsilon'}{q(z)+r-\rho} e^{\rho x'} dN(x') \right) dA(\epsilon') + \right. \\
& \left. \lambda(1-U) \int_{\epsilon'} \left(\int_{x'=0}^{\infty} \int_{z' \in [\underline{z}, z)} \frac{(p(k)-z)\epsilon'}{q(z)+r-\rho} e^{\rho x'} dH(x',z') \right) dA(\epsilon') \right] =
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\phi+\delta+\lambda_u}{\phi+\delta+\lambda_e} \left[U \frac{(p(k)-z)}{q(z)+r-\rho} \int_{\epsilon'} \epsilon' \left(\int_{x'=0}^{\infty} e^{\rho x'} dN(x') \right) dA(\epsilon') + \right. \\
& \left. \lambda(1-U) \frac{(p(k)-z)}{q(z)+r-\rho} \int_{\epsilon'} \epsilon' \left(\int_{x'=0}^{\infty} e^{\rho x'} \int_{z' \in [\underline{z}, z)} dH(x',z') \right) dA(\epsilon') \right] =
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\phi+\delta+\lambda_u}{\phi+\delta+\lambda_e} \frac{(p(k)-z)}{q(z)+r-\rho} \left[U \int_{\epsilon'} \epsilon' dA(\epsilon') \left(\int_{x'=0}^{\infty} e^{\rho x'} dN(x') \right) + \right. \\
& \left. \lambda(1-U) \int_{\epsilon'} \epsilon' dA(\epsilon') \left(\int_{x'=0}^{\infty} e^{\rho x'} H_x(x',z) dx' \right) \right]
\end{aligned}
$$

The first equality follows as $\frac{\lambda_u}{\lambda_u U+\lambda_e(1-U)} = \frac{\phi+\delta+\lambda_u}{\phi+\delta+\lambda_e}$, $\frac{(p(k)-z)}{q(z)+r-\rho}$ is constant conditional on $\{z,k\}$, and given x', $e^{\rho x'}$ does not vary with z. The second equality follows because turnover is independent of ϵ, and because

$$
\int_{z' \in [\underline{z}, z)} H_{xz}(x',z') dz' = H_x(x',z) - H_x(x',\underline{z}) = H_x(x',z)
$$

Next, differentiating (1.11) and (1.10) with respect to x yields the densities

$$
\begin{aligned}
\frac{\partial N(x)}{\partial x} \quad &= \quad \frac{\phi(\phi+\delta+\lambda_u)\delta\lambda_u}{(\phi+\lambda_u)^2(\phi+\delta)} e^{-\frac{\phi(\phi+\delta+\lambda_u)}{\phi+\lambda_u}x} \\
\frac{\partial H(x,z)}{\partial x} \quad &= \quad \frac{F(z)\phi(\phi+\delta+\lambda_u)}{\delta\lambda_u + \lambda_e(\phi+\lambda_u)(1-F(z))} \times \\
& \quad \left[\frac{\delta\lambda_u}{\phi+\lambda_u} e^{-\frac{\phi(\phi+\delta+\lambda_u)}{\phi+\lambda_u}} + \lambda_e(1-F(z)) e^{-q(z)x} \right]
\end{aligned}
$$

Using these above, the expressions for the integrals can be stated as

$$\int\limits_{x'=0}^{\infty} e^{\rho x'} N_x(x')dx' = \frac{\phi\delta\lambda_u(\phi+\delta+\lambda_u)}{(\phi+\lambda_u)^2(\phi+\delta)} \int_{x=0}^{\infty} e^{\left[\rho - \frac{\phi(\phi+\delta+\lambda_u)}{\phi+\lambda_u}\right]x'} dx'$$

and

$$\int\limits_{x'=0}^{\infty} e^{\rho x'} H_x(x',z)dx' = \frac{F(z)\phi(\phi+\delta+\lambda_u)}{\delta\lambda_u + (\phi+\lambda_u)\lambda_e(1-F(z))} \times$$
$$\left[\frac{\delta\lambda_u}{\phi+\lambda_u} \int\limits_{x'=0}^{\infty} e^{\left(\rho - \frac{\phi(\phi+\delta+\lambda_u)}{\phi+\lambda_u}\right)x'} dx' + \lambda_e(1-F(z)) \int\limits_{x'=0}^{\infty} e^{(p-q(z))x'} dx'\right]$$

and direct integration yields

$$\begin{aligned}
\int\limits_{x'=0}^{\infty} e^{\left(\rho - \frac{\phi(\phi+\delta+\lambda_u)}{\phi+\lambda_u}\right)x'} dx' &= \frac{\phi+\lambda_u}{\phi(\phi+\delta+\lambda_u) - \rho(\phi+\lambda_u)} \\
\int\limits_{x'=0}^{\infty} e^{(p-q(z))x'} dx' &= \frac{1}{q(z)-\rho}
\end{aligned}$$

Then after some basic manipulations

$$U \int\limits_{x'=0}^{\infty} e^{\rho x'} dN(x') = a_0$$

and

$$\lambda(1-U) \int\limits_{x'=0}^{\infty} e^{\rho x'} H_x(x',z)dx' = \lambda a_1 \frac{F(z)}{q(z)-\rho}$$

as in (1.13).

Chapter 2

Wealth and Labor Market Outcomes: Evidence from the SIPP 1996-2000

2.1 Introduction

Under incomplete insurance, accumulated wealth can serve as a cushion for smoothing consumption when income is temporarily low, in particular, during spells of unemployment. In search models this implies a relationship between wealth and the behaviour of the unemployed. Intuitively, a wealthy worker can sustain consumption over a long non-employment spell and can afford to be more selective when presented with a job opportunity. On the other hand, a wealth-poor worker needs to escape unemployment quickly even at the cost of accepting less appealing job offers. This intuition was first formalized by Danforth (1979) in an environment along the lines of McCall (1970) where workers are risk-averse. He demonstrated that under DARA preferences reservation wages increase with wealth, with the implication that wealthy workers experience longer unemployment spells and end up working at better-paying jobs. More recent work, including Lentz and Tranaes (2005), Rendon (2006), Lise (2013), and Eeckhout and Sepahsalari

(2013), has investigated the relationship between wealth and search behaviour under more general environments. A universal prediction from the above is that unemployment durations increase with wealth. This is due to either a positive effect of wealth on reservation wages or a negative effect on search effort, or a combination of both[1].

A number of studies have investigated empirically the reduced form predictions of this hypothesis. Bloemen and Stancanelli (2001) use data from the Dutch Socio-Economic Panel from 1988-1989 with information on self-reported reservation wages and assets. They document that unemployed high-net-worth workers report higher reservation wages and experience longer non-employment spells on average. Alexopoulos and Gladden (2006), using US SIPP data from 1984-1987, and Lammers (2014), using the Dutch DNB Household Survey data from 1993-2008, document the same relationship. Algan et al. (2003) use French data from the European Panel (collected by Eurostat) over 1993-1996 and document negative relationship between net worth and unemployment durations. These results have been widely interpreted as direct evidence in favour of the theoretical models above (Browning et al., 2007; Lentz, 2009).

This paper complements the above literature by investigating the empirical relationship between workers' household net worth, re-employment wages and hazard rates out of non-employment in the 1996 panel of the Survey of Income and Program Participation. The analysis here is very similar to the above, yet differs in one important aspect. Rather than using self-reported reservation wages (which are unavailable in the data) I use observed re-employment wages. There are no theoretical reasons why reservation wages are superior to re-employment wages

[1] In Lentz and Tranaes (2005) wealth has no effect on reservation wages by construction but has a negative effect on search effort in equilibrium. Lise (2013) develops a model along the lines of Burdett and Mortensen (1998) featuring risk-averse workers and endogenous search effort. While reservation wages are allowed to depend on wealth, he shows that in equilibrium they don't and behaviour is affected only through an effect on search effort. Eeckhout and Sepahsalari (2013) consider a directed search model and show that under DARA preferences, wealthy workers direct their search to more productive (and better paying) jobs where they also face longer queues. In Rendon (2006) transition rates are exogenous (there is no search effort) but reservation wages depend positively on wealth.

as an object of study when interest lies in the relationship between wealth and the decision to accept a job offer. A worker's re-employment wage is by definition higher than her reservation wage but if re-employment wages are systematically higher for some group of the population it must be that so is the reservation wage. Further, observed re-employment wages are objective, unlike self-reported reservation wages, and are available for all workers who experience a transition from non-employment into employment, while in all of the above studies information on reservation wages is collected only from unemployed persons in few of the survey waves. As a result I have significantly larger sample, which ultimately allows for further flexibility in modelling the main relationships of interest.

In documenting the relationship between re-employment wages and net worth I start with a specification identical to the one in Bloemen and Stancanelli (2001) and obtain remarkably similar results - net worth enters the conditional mean of log-wage through a quadratic polynomial and the estimates imply an increasing concave relationship. However, non-parametric estimation of the underlying relationship identifies a non-monotonic conditional mean - re-employment wages decrease with net worth while the latter is negative (as for about 12 percent of the households surveyed and 20 percent of the relevant estimation sample) and then increase when positive. Re-employment wages are lowest not for the most asset-poor workers but for those with close to zero net worth. I then demonstrate that the pattern survives even after controlling for a broad range of observables but disappears when accounting for past wages. I postpone my interpretation of these results until later but at this stage assert that they are inconsistent with the findings from the closely related literature above.

Next I estimate a proportional hazards model of the hazard rates out of non-employment with respect to net worth. I find a negative relationship with the caveat that workers with close to zero net worth experience on average longer durations, of the same magnitude as those for wealthiest workers, conditional on observables. Subject to this, the relationship between wealth and hazard rates is broadly consistent with the view that wealth affects search behaviour as predicted by theory.

The exposition is structured as follows. Section 2.2 describes the data used. Section 2.3 discusses the relationship between re-employment wages and net worth. Section 2.4 presents the analysis of hazard rates. Section 2.5 concludes.

2.2 Data

I use data from the 1996 panel of the Survey of Income and Program Participation. SIPP is a nationally representative, longitudinal, multi-stage stratified sample of civilian, non-institutionalized US households. The survey has been conducted since 1984 but comes in panels of length three or four years where different panels sample different households. A household can be followed for four years at most.

A household is interviewed every four months (a *wave*) and data is collected for each of the preceding four months (the *reference period*) for each household member. During each interview a set of *core* questions on demographics, income (from various sources), employment, program participation and others are asked. In particular, within the core questionnaire respondents provide a weekly calendar of employment status, as well as monthly earnings, hours worked, and further job characteristics from up to two jobs. In addition, a set of different *topical* questions are asked each wave.

The 1996 panel samples 36,730 households (95,300 individuals) over the period 1996-2000[2]. A household is interviewed twelve times and during the third, sixth, ninth, and twelfth wave detailed data on assets and liabilities are collected at both household and individual level. Asset categories covered in the survey include wealth in bank accounts, stocks/funds, private pension accounts (but not claims to future Social Security benefits), vehicles, home and business equity, and other wealth. Liabilities covered include credit card debt, loans and "other debt". A household's net worth is identified as the difference between all recorded assets

[2]These are the numbers from the first wave.

and liabilities. Wealth measures used in the analysis are based on these records. Appendix 2.B gives detailed information of the available asset and liabilities data.

During the first wave all respondents who were not employed at the beginning of the reference period are asked when was the last time they worked. I use this to identify the duration of non-employment for workers whose spell started before the beginning of the panel. As a result the spell data used in Section 2.4 does not suffer from problems of left censoring. At the first interview respondents are also asked to report the total number of years they have worked for more than six months. I use this as a basis measuring labour-market experience. In what follows a worker's labour market experience is therefore defined as the number of years she has worked for at least six months.

In the SIPP hourly wages are only reported by workers compensated by the hour, while monthly earnings (EPM), hours worked per week (HPW) and weeks in each month (WPM) are available for all observations[3]. Hence, for comparability, I define a worker's wage at month t as $EPM_{it}/(WPM_t HPW_{it})$ (effective hourly earnings). I define a worker's re-employment wage as the average wage received over the first two full months ($t+1$ and $t+2$) of an employment spell. The hourly starting wage (W_t) is then

$$\begin{aligned} W_{it} &= \frac{WPM_{t+1}}{WPM_{t+1}+WPM_{t+2}} \frac{EPM_{it+1}}{WPM_{t+1}HPW_{it+1}} \\ &+ \frac{WPM_{t+2}}{WPM_{t+1}+WPM_{t+2}} \frac{EPM_{it+2}}{WPM_{t+2}HPW_{it+2}} \end{aligned}$$

The reason to focus only on initial wages (rather than, for example, average wage over the observed duration of a spell) is to abstract from tenure-related wage growth when employment spell durations vary across observations. Under this convention transitions resulting in job spells shorter than two full months are

[3]The analysis here models wages rather than earnings for comparability with prior studies. The empirical regularities documented, however, also hold if total monthly earnings are used as regressand.

excluded from the sample[4].

All nominal variables (wealth, earnings, wages, and income from other sources) are deflated by the CPI with base December, 1996[5]. At each point in time the assets of an individual are identified with the latest observed level of the assets from the particular category.

Finally, as all surveys, data quality could be a concern in the SIPP. Particularly relevant issues relate to the quality of wealth and employment status data. Appendix 2.A discusses some of the surveys better known deficiencies and the implications for my analysis.

2.3 Wealth and re-employment wages

This section turns attention to the relationship between workers' re-employment wages and their households' net worth. To summarise the latter consider the regression specification for an individual i who experienced a non-employment-to-employment transition in month t

$$\log W_{it} = \alpha + g(NW_{it\_}) + \beta_2 D_{it} + \beta_3 P_{it} + \beta_4 O_{it} + \epsilon_{it} \tag{2.1}$$

where W_{it} is the starting wage of individual i defined above; $NW_{it\_}$ is her household's latest observed (prior to the associated non-employment-to-employment transition) net worth; $g(.)$ is a pre-specified functional form for the relationship of interest; D_{it}, P_{it}, and O_{it} are vectors of demographic, productivity (such as education), and outside-option related (such as the income of other household members) characteristics. Recall that since assets are only observed annually, $NW_{it\_}$ may have been measured at any point between one week and twelve months prior to the transition. Aside from W_{it} being a re-employment rather than reservation

[4]Some workers report flat earnings profiles during employment spells, while others report sequences of higher and lower earnings depending on the number of weeks there are in a month. I take two-month weighted average to correct for biases due to this discrepancy.

[5]Asset and liabilities are only available at yearly frequency. I deflate these by the CPI at the time when they were observed. As a result their deflated levels only change at annual frequency.

Table 2.3.1: Descriptive statistics: wage equations

	Heads	Spouses		Heads	Spouses
Wage	10.65	10.15	**Education**		
Net worth	92075	95052	*Elementary*	0.04	0.04
Household size	3.46	3.59	*< High school*	0.15	0.13
Age	39.70	39.73	*High school*	0.32	0.33
Female	0.55	0.73	*Some college*	0.31	0.30
Race			*Undergraduate*	0.12	0.15
White	0.80	0.87	*Master's +*	0.04	0.04
Black	0.15	0.08	*Professional*	0.01	0.01
Other	0.05	0.06	**Other**		
Marital			Experience	18.09	16.96
Married	0.66	1.00	NE spell (wks)	6.34	7.16
Single	0.13	0.00	HH income	1758	2865
Divorced	0.21	0.00	Hours per week	35.76	33.39
Observations	4589	3355		4589	3355

"Wage" is re-employment wage; "HH income" is the income of all other household members; "NE spell" is the duration of the intervening spell of non-employment.

wage, (2.1) is (up to differences in $g(.)$ and specification of the control set) the relationship estimated in the papers discussed in the introduction (Bloemen and Stancanelli, 2001; Alexopoulos and Gladden, 2006; Lammers, 2014). Notice that since $NW_{it\_}$ is pre-determined there could be no question of causal influence running directly from W_{it} to $NW_{it\_}$ but as long as factors that jointly determine W_{it} and $NW_{it\_}$ are absent from the control set, simultaneity is present.

The sample for estimation is restricted to individuals within family households (household net worth may be uninformative in non-family households) and versions of (2.1) are estimated separately for household heads and spouses[6]. Further, individuals who report to be retired or to suffer from work-preventing disabilities at any point during the survey, are excluded. As wealth data is first recorded in the third wave of the survey all observations from the first twelve months are excluded. Subject to this the sample includes all workers that experienced a non-employment-to-employment transition[7] resulting in an employment spell of length

[6]Household head is defined as the individual who owns the household's home or in whose name rent is paid. This is different to the definition in Bloemen and Stancanelli (2001) where the household head is the husband. Every household has a household head but not necessarily a spouse.

[7]I define transitions with intervening non-employment of less than two weeks to be

more than two months. This leaves 4589 observations for household heads and 3355 for spouses. Table 2.3.1 presents some descriptive statistics. The average reemployment wage is higher for household heads while the average income of other household members is higher for the spouses. These suggest that typically the individual recorded as household head is the main earner in the observed household. The average ages for the two groups are very close but household heads have worked on average about a year more than the spouses. The observed differences in education are small. Net worth is on average higher in households where the spouse experienced a transition (likely because all these households consist of married couples).

In what follows I report the results from estimating different versions of (2.1). In all cases the equation is fitted by OLS and identifies the parameters of the conditional mean. It should be noted, however, that the main results obtain when the equation is estimated by median regression, weighted least squares[8], or by PPML when the wage, rather than its log, is modelled as an exponential function of the linear index (see Santos-Silva and Tenreyro (2006)). For brevity, the results from alternative estimation procedures are not reported in the text. All following tables report the p-values obtained through heteroskedasticity-consistent estimation of the standard errors. As discussed further, I also use two different measures of wealth, depending on how home equity is handled, with little change in results.

First, Table 2.3.2 reports the coefficients from a log-wage OLS regression using a specification identical (up to variable definitions) to the one in Bloemen and Stancanelli (2001). The columns labelled B&S (2001) present Bloemen and Stancanelli (2001)'s estimates while columns labelled SIPP present mine. The

employment-to-employment; those involving a longer intervening spells are defined as non-employment-to-employment. This is in the tradition of previous work (e.g. Nagypál (2008)).

[8]The SIPP samples disproportionately from areas with high poverty and, in this sense, its sample is not representative of the US population. Since sampling weights are available, WLS estimation of (2.1) is feasible but it turns out that the results are very close to the ones obtained by OLS, in terms of both coefficients and standard errors. A possible reason is that the covariates (in particular, net worth) are highly correlated with the selection criteria.

Table 2.3.2: Comparison to Bloemen and Stancanelli (2001)

	Heads		**Spouses**	
	B&S	SIPP	B&S	SIPP
Any children	0.084*	−0.0423	−0.10	−0.0057
Female	−1.14**	−0.435		−0.632
ln(Age)	4.95**	5.432***	7.75*	3.326***
$\ln(\text{Age})^2$	−0.67**	−0.757***	−1.09*	−0.469***
Unemployment income	0.044*	0.207***	0.09	0.172***
Other income	−0.0027	0.0141**	−0.0025	0.0127**
ln(Hours)	−0.15**	−0.0515	0.094	−0.0805
Educ_2	0.018	0.111***	0.093	0.139***
Educ_3	0.14**	0.219***	0.12	0.213***
Educ_4	0.20**	0.566***	0.36**	0.541***
Female × ln(Hours)	0.31**	0.0277		0.0739
Net worth	0.029**	0.0145**	0.052**	0.0234**
Net worth ^{2}	−0.0012**	−0.0015***	−0.0019**	−0.0003**
House	0.10	0.0972***	−0.060	0.124***
Observations	284	4659	284	3396

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

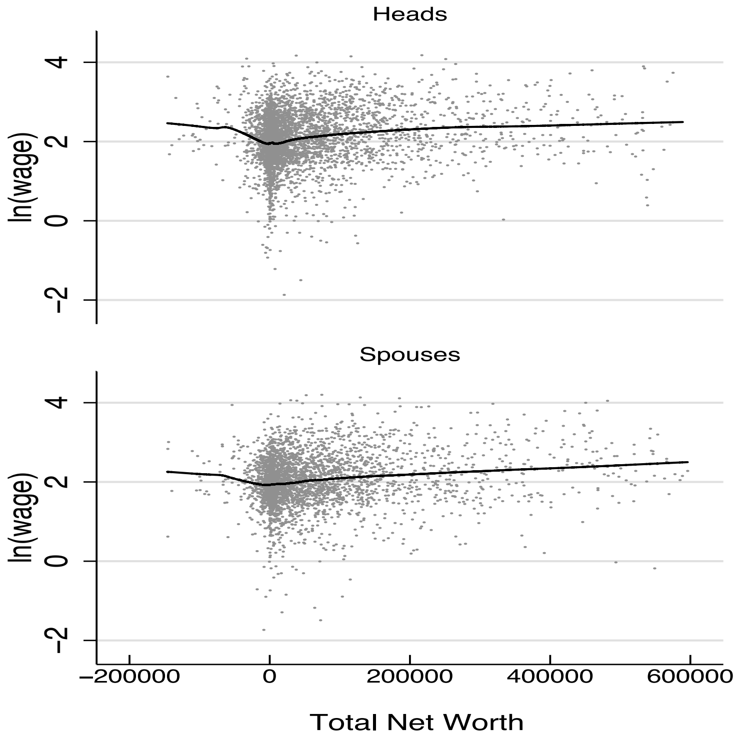
- The income (wealth) variables in Bloemen and Stancanelli (2001) are measured in 1000s (10000s) of 1984 Dutch guiders. The corresponding variables here are measured in 1000s (10000s) of 1996 US dollars.

results are remarkably similar (notice that all nominal quantities are measured in different currencies and the equation is estimated in two different countries) and imply a positive and convex relationship between wages and net worth. Perhaps the most striking difference relates to the number of observations which differ by factor of twenty.

Next, to assess the adequacy of quadratic relationship with respect to net worth, Figure 2.3.1 presents scatterplots of the log-wage against net worth and fits a locally weighted scatterplot fitting (LOWESS) curve through the data [9]. The observed relationship between log-wages and net worth is non-monotonic. Starting from the left tail of the net worth distribution, mean wages decrease as net worth increases and attain a minimum near zero net worth. Then as net worth increases, log-wages increase again, at a decreasing rate. The pattern is observed both for household heads and for spouses, but is somewhat more pronounced

[9] The graph shows the middle 98 % of the observations for net worth. The bandwidth is set to 1000 dollars (deflated) - a reasonably small value given the support of the net worth distribution.

Figure 2.3.1: Wages and net worth



for household heads. This suggests that quadratic polynomial in net worth is inadequate specification for $g(.)$ in (2.1). Not accounting appropriately for the shape of the relationship could lead to misleading results - notice that about 19 percent of household heads and 20 percent of spouses in the estimation sample have negative net worth (see Appendix 2.B).

Since non-parametric estimation with a large set of controls is computationally infeasible I recode the continuous net worth variable into a categorical ordering of households by net worth and use category indicators as regressors. In addition to inducing a semi-parametric flavour to the estimation this approach has the advantage of reducing the influence of extreme net worth observations, and improving the robustness of results to measurement issues associated with the continuous wealth variables. I assign households into twelve net worth categories in ascending order, allocating those with net worth between -100 and 100 dollars into the third category. Appendix 2.B describes how the net worth categories are created from the pooled asset data. Importantly, the net worth categories are not

Table 2.3.3: Wage regression, household heads

Net worth	(1)	(2)	(3)	(4)
NW1	0.420***	0.319***	0.166**	0.128*
	(0.000)	(0.000)	(0.006)	(0.041)
NW2	0.225***	0.166***	0.0683	0.0386
	(0.000)	(0.001)	(0.200)	(0.477)
NW4	0.144**	0.103**	0.0576	0.0316
	(0.002)	(0.006)	(0.157)	(0.449)
NW5	0.266***	0.212***	0.129**	0.0978*
	(0.000)	(0.000)	(0.004)	(0.038)
NW6	0.333***	0.274***	0.159**	0.125*
	(0.000)	(0.000)	(0.001)	(0.016)
NW7	0.376***	0.316***	0.172***	0.135**
	(0.000)	(0.000)	(0.000)	(0.002)
NW8	0.445***	0.374***	0.230***	0.201***
	(0.000)	(0.000)	(0.000)	(0.000)
NW9	0.498***	0.418***	0.218**	0.188*
	(0.000)	(0.000)	(0.009)	(0.025)
NW10	0.666***	0.564***	0.359***	0.321***
	(0.000)	(0.000)	(0.000)	(0.000)
NW11	0.579***	0.515***	0.281***	0.239**
	(0.000)	(0.000)	(0.000)	(0.001)
NW12	0.764***	0.712***	0.426***	0.385***
	(0.000)	(0.000)	(0.000)	(0.000)
Observations	4589	4589	4437	4437

p-values in parentheses

equally sized (Table 2.B.1).

Table 2.3.3 presents the coefficients on net worth category indicators (households close to zero net worth serving as reference category) from estimation of four regression specifications nested by (2.1) for household heads (Tables 2.D.1 and 2.D.2 in the Appendix show the full specifications, including controls, for household heads and spouses). The first specification (column (1)) only includes a constant and the net worth indicators; column (2) includes demographic controls; (3) includes productivity controls; (4) includes controls related to the worker's outside option. Household heads in the top (bottom) net worth category have on average 76 (42) percent higher re-employment wages than workers close to zero net worth. Similar results obtain for spouses. Observables explain about two-thirds of the re-employment wage gap between top (bottom) and zero-net worth workers but the qualitative pattern from Figure 2.3.1 is preserved after each round of extra controls.

One concern with measuring wealth by total household net worth is that it includes illiquid assets (in particular, home equity) that are conceivably unsuitable for smoothing consumption[10]. To address this issue I consider an alternative definition of net worth which subtracts home equity from total net worth. It should be noted that even this measure could be subject to the same criticism - for example, it includes wealth in pension accounts which carry penalty upon early withdrawal. Tables 2.D.5 and 2.D.6 (in the Appendix) report the coefficients on the net worth categories for the same regression specifications as above but using this net worth definition[11]. The results are very similar to before. The pattern identified above is, therefore, not the result of treating home equity inappropriately.

A significant part of the relationship between wages and net worth is explained by observable characteristics. To explore this more closely Table 2.3.4 reports the mean values of a subset of the observables for households heads from different net worth categories (Table 2.D.4 in the Appendix presents the same information for spouses). Workers around zero net worth are on average the youngest; least experienced, educated or likely to be married; most likely part of low-income households. Towards the tails of the net worth distribution the incidence of characteristics associated with high earnings increases steadily. Further, workers in the bottom net worth categories have education profiles consistent with high earning potential. The most striking difference between them and workers towards the right end of the distribution is that the former are younger and less experienced. To state this simply, the most asset-poor workers don't have low re-employment wages plainly because they are fundamentally high earners.

This is at odds with the findings of Bloemen and Stancanelli (2001), Alexopoulos and Gladden (2006) and Lammers (2014). It is unlikely that the discrepancy is due to the conceptual difference between reservation and re-employment wages

[10]While this criticism has some merit, it is probably extreme to think that a household's property value is fully irrelevant as a source of financing in face of unexpected events.

[11]The coefficients of the control variables are not reported to keep the tables concise but are available upon request. Individuals are now grouped into only eleven categories but the third group again contains individuals with zero net worth - for details see Appendix 2.B

Table 2.3.4: Characteristics by net worth, household heads

Net worth	1	2	3	4	5	6	7	8	9	10	11	12
Age	35.29	34.66	33.20	34.51	35.83	38.75	40.63	44.41	44.50	48.18	49.46	52.77
Experience	15.12	13.83	8.40	12.72	15.00	17.99	19.36	22.72	23.55	25.33	26.49	28.51
Female	0.50	0.61	0.65	0.63	0.59	0.50	0.53	0.48	0.46	0.41	0.46	0.42
Spell (wks)	4.91	5.25	11.81	6.83	5.90	5.60	6.19	6.26	6.29	6.43	6.10	6.41
HH (net) income	1639	1200	403	849	1141	1563	1866	2017	2488	2738	3511	4250
Married	0.71	0.55	0.35	0.47	0.58	0.72	0.74	0.77	0.83	0.86	0.85	0.89
Education												
Elementary	0.03	0.04	0.09	0.09	0.05	0.05	0.03	0.01	0.02	0.02	0.00	0.00
< *High school*	0.09	0.15	0.34	0.27	0.21	0.15	0.14	0.13	0.10	0.06	0.05	0.03
High school	0.27	0.35	0.31	0.35	0.39	0.33	0.31	0.34	0.31	0.28	0.27	0.17
< *Degree*	0.36	0.37	0.25	0.24	0.27	0.35	0.35	0.32	0.29	0.30	0.30	0.28
Undergraduate	0.18	0.06	0.00	0.05	0.08	0.08	0.14	0.16	0.20	0.21	0.25	0.29
Master's	0.03	0.02	0.00	0.01	0.01	0.03	0.04	0.03	0.06	0.10	0.12	0.17
PhD	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.01	0.02
Professional	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.00	0.04
Observations	390	452	318	350	691	566	391	350	343	299	250	189

Note: Columns correspond to net worth categories

- if most asset-poor individuals report low reservation wages but somehow end up working at high-paying jobs, their subjective assessment of reservation wages is flawed. Another possibility is that the different results are due to differences in sample selection. In particular, while I include unmarried household heads in the estimation sample, some of the above studies do not. Table 2.D.3 (in the Appendix) estimates the equations only on the sample of married heads and shows that this is not the case. Further, the results are obtained for different countries (of the above only Alexopoulos and Gladden (2006) uses US data) and at country level the underlying relationship could differ. A more likely explanation, however, is that by imposing quadratic form for the underlying relationship, the long right tail of net worth becomes extremely influential and drives the results. It is also difficult to reconcile the results with the view that (2.1) identifies a causal effect of wealth on re-employment (or reservation) wages. To make this claim one has to assume that all factors that jointly determine wages and net worth (for example, a workers permanent earnings and ability) are appropriately accounted for in the control set. Direct inspection of the full list of controls in Table 2.D.1 reveals that virtually all variables related to ability are discrete while the dependant variable is continuous. One possibility for a continuous noisy measure of ability that could serve as a proxy in (2.1) is past wages. For this two requirements should be satisfied. First, past wages should be ignorable. From theoretical point of view, this requirement is likely met[12]. The second requirement, that the correlation between omitted ability and net worth is zero once past wage is partialed out, is more difficult to defend - for example, positive transitory component in past wages may imply higher wealth at the time of the transition. While recognising this issue, column (2) of Table 2.D.7 (in the Appendix) reports, proactively, the results from a simple regression of the re-employment wage on net-worth-category indicators, education, age, marital status and the average wage earned at a worker's latest

[12] In the class of models discussed in the introduction past wages matter for an unemployed worker's reservation wage only insofar as they contain information of the worker's earning potential. However, it is easy to think of alternative environments where past wages have causal effect on re-employment wages.

employment spell prior to transition[13]. Past wages explain most of the variation otherwise attributed to wealth, except for the three top net worth categories. If one believes that the proxy is suitable (in particular, that transitory variations in wages over the observed window can't explain systematic movements between net worth categories) the results suggest that wealth has essentially no effect on re-employment wages over most of the net worth distribution. While suggestive, this result's causal interpretation should be taken with care in view of the discussion above.

Irrespective of the exact nature of causal forces involved, the results demonstrate unambiguously that the most asset-poor workers are young and highly educated high-earners. To investigate whether their liabilities could be attributed to particular sorts of expenditure, Table 2.3.5 reports the average financial position of household heads from the twelve net worth categories separately by asset group. Households in the third net worth category hold less wealth and own less debt, in almost every asset class, than other households. From there, both assets and liabilities increase in value towards the tails of the net worth distribution. Households in the bottom category are the only ones with negative, on average, home equity position but their largest component of debt is loans (which is also disproportionately large in comparison to its relative share of total liabilities across net worth categories). "Loans" corresponds to survey questions about "student loans, home-improvement loans, lines of credit besides credit card". Given their age and educational profile, it is sensible to conjecture that student loans dominate their balance sheet. Unfortunately, no finer decomposition is available in the data and at this stage this can't be established unambiguously.

[13] As a preceding employment spell (of duration more than two months) is not observed for many workers, the number of observations drops to 2771. Column (1) of Table 2.D.7 reports the estimates over the same sub-sample from an identical regression excluding the past wage and identifies the same pattern as Table 2.3.3.

Table 2.3.5: Assets by net worth, heads

	1	2	3	4	5	6	7	8	9	10	11	12
Assets												
Held in bank	1297	384	16	278	621	1661	2966	4752	7706	12728	20308	59250
Stocks/funds	654	22	0	54	248	466	1213	2476	5991	11461	34969	869039
IRA/KEOGH	455	97	0	29	69	531	683	2182	4617	8162	19110	60689
Vehicles (net)	2018	837	98	1119	4175	5960	6843	7663	8160	10489	11265	14954
Business equity	-2894	68	0	17	50	261	276	1020	1401	3805	8312	53674
Other	398	189	10	109	351	1327	1845	3935	7070	13578	25060	109148
Home equity	-2456	777	84	374	2077	10525	25845	43684	63187	87550	127914	166508
Total wealth	-529	2374	209	1981	7590	20731	39671	65712	98131	147773	246939	1333262
Liabilities												
Credit card debt	9477	2605	44	621	1415	2504	2302	3102	3349	3063	2941	3152
Loans	11892	1424	148	465	521	871	1404	984	1185	1258	1121	458
Other debt	6230	1136	15	106	497	648	936	809	582	539	331	485
Total unsecured debt	27599	5165	207	1193	2432	4022	4642	4895	5116	4860	4392	4095
Net worth												
Total	-28128	-2790	2	788	5158	16709	35028	60817	93015	142914	242546	1329167
Net of home equity	-25672	-3567	-82	314	2081	6184	9183	17133	29828	55364	114632	1162659

Note: Columns correspond to categories based on total net worth

2.4 Wealth and the duration of non-employment

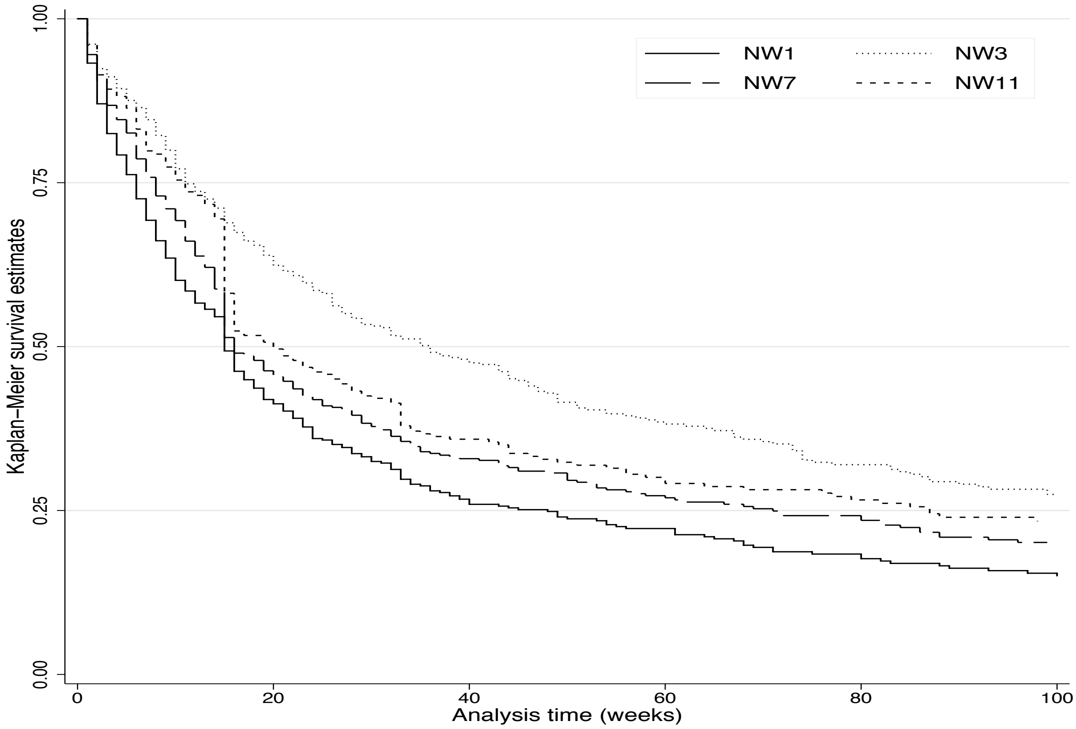
I now turn attention to the relationship between net worth and hazard rates to employment. Starting from the pooled data, I exclude all individuals of age less than 16 years, the retired and those reporting being unable to work because of chronic health condition or disability. I transform the original data into a weekly panel dataset where each individual's employment status is observed weekly, core-module variables are observed monthly, and assets are observed annually. The SIPP reports five distinct employment states. I recode these so that they are consistent with only two employment states (employment and non-employment). As before, the amount of assets held by an individual at any week is identified with the most recently observed amount (See Appendix 2.B).

Identification of spell durations for individuals observed continuously who were employed in the first week of the first wave is straightforward. For those observed continuously but not employed at the beginning of the first reference period, I use data from the first-wave topical module where they report the last time they worked. In about five percent of the relevant cases these records are missing and I exclude the associated spells. Spells of individuals who left the sample for some time and returned as non-employed are also excluded. As before, I exclude non-employment spells shorter than two weeks (which I interpret as job-to-job transitions). This leaves a final sample of 7313 non-employment spells (210443 weeks at risk) for household heads, with 4779 ending in employment, and 5942 spells (227918 weeks at risk) for spouses, 3603 ending in employment[14]. Median exit times are 13 and 15 weeks respectively.

Figure 2.4.1 shows the Kaplan-Meier estimates of the survivor function for household heads from four different net worth categories (the others are omitted for readability). While survival rates typically decrease with wealth, as before a discontinuity occurs for households close to zero net worth who are least likely to

[14]Notice that the time-at-risk does not correspond to the number of weekly observations reported later due to the spells whose duration was constructed using the first-wave topical module.

Figure 2.4.1: Survivor function by net worth, Kaplan-Meier



transit into employment. Hence, in the regression specifications that follow I keep on using net-worth-category indicators[15].

To summarise the relationship between hazard rates out of non-employment and net worth I use a discrete time approximation to the continuous time proportional hazards model. Let $\theta(t, X_{it}, \epsilon_t)$ be the continuous time hazard rate for an individual i with observed characteristics X_{it} and unobserved ϵ_t who has been non-employed for amount of time t. Under the proportional hazards assumption this can be represented as

$$\theta(t, X_{it}, \epsilon_{it}) = \theta_0(t) exp(\beta' X_{it} + \epsilon_{it}) \equiv \theta_0(t)\lambda_{it} \tag{2.2}$$

implying that all individuals share the same underlying pattern of time dependence, summarized by $\theta_0(t)$, the baseline hazard rate, and observed characteristics

[15]The estimated survival probabilities seem to discontinuously decrease at 18 weeks, and exhibit less visible discontinuities every other 18 weeks. This pattern likely results from seam bias in the data - the SIPP reference period is 4 months. The regression specifications that follow account for this by attributing all discontinuities at the seams to seam bias.

affect hazard rates by scaling λ_{it}. Both t and X_{it} are only observed at discrete weekly intervals[16]. The probability that a worker is still not employed after j weeks (the survivor probability) is

$$S(j, X_{ij}^h) = exp\left(-\int_0^1 \theta_0(u)\lambda_{i1}du - ... - \int_{j-1}^j \theta_0(u)\lambda_{ij}du\right) \quad (2.3)$$

where

$$X_{ij}^h \equiv \{X_{ij}, X_{i,j-1}, ..., Xi1\}$$

is the history of X_{it} for $t \in \{1, 2, ..., j\}$. An individual is still not employed after j weeks if they did not experience a transition in the first, or second, ..., or $j-1$'st week.

The probability of a spell ending during week j (the discrete hazard rate) is

$$h(j, X_{ij}^h) = \frac{S(j-1, X_{i,j-1}^h) - S(j, X_{ij}^h)}{S(j-1, X_{i,j-1}^h)}$$

A spell ends at week j if an individual is not employed in week $j-1$ but employed in week j. Using (2.3) to substitute the survivor functions the discrete time hazard can be expressed independently of history:

$$h(j, X_{ij}) = 1 - exp(-exp(\beta' X_{ij} + \gamma_j + \epsilon_{ij})) \quad (2.4)$$

where

$$\gamma_j \equiv \int_{j-1}^j \theta_0(u)du$$

is the discrete time counterpart of the baseline hazard rate. Therefore, subject to appropriately accounting for γ_j the parameters in (2.2) are identified by a

[16]In fact, observed covariates only change at monthly (or for assets annual) frequency. However, this is unlikely to result in significant time aggregation bias as long as one treats the time varying covariates as monthly averages.

Table 2.4.1: C-log-log regression, household heads

Net worth	(1)	(2)	(3)	(4)
NW1	0.632***	0.479***	0.608***	0.510***
	(0.000)	(0.000)	(0.000)	(0.000)
NW2	0.407***	0.276**	0.428***	0.315***
	(0.000)	(0.002)	(0.000)	(0.001)
NW3	0.139	-0.0000103	0.223*	0.0861
	(0.114)	(1.000)	(0.028)	(0.411)
NW4	0.527***	0.369***	0.552***	0.431***
	(0.000)	(0.000)	(0.000)	(0.000)
NW5	0.391***	0.242**	0.389***	0.278**
	(0.000)	(0.004)	(0.000)	(0.002)
NW6	0.439***	0.287***	0.390***	0.306***
	(0.000)	(0.001)	(0.000)	(0.001)
NW7	0.432***	0.333***	0.452***	0.371***
	(0.000)	(0.000)	(0.000)	(0.000)
NW8	0.289***	0.192*	0.282**	0.204*
	(0.001)	(0.031)	(0.002)	(0.032)
NW9	0.255**	0.182*	0.258**	0.201*
	(0.005)	(0.046)	(0.006)	(0.036)
NW10	0.209*	0.160	0.223*	0.179
	(0.025)	(0.090)	(0.021)	(0.069)
NW11	0.200*	0.166	0.181	0.145
	(0.030)	(0.072)	(0.055)	(0.130)
Observations	152147	152147	148712	148712

Standard errors clustered by individual. NW12 is reference category.

complementary log-log regression of the event that a transition occurs in week j on the covariates (Jenkins, 2005).

To account for duration dependence I follow a semi-parametric piecewise-constant approach - 26 indicator variables, corresponding to discrete intervals of cumulative spell duration, are included as regressors. These are chosen so that they are narrower towards the beginning of the spell, where the survivor function has higher curvature. Appendix 2.C describes how the intervals are generated. To insure that this way of modelling duration dependence is not inadequate I also estimate the relationships through a Cox model, which imposes no specific form of the duration dependence, with no significant change of results[17]. All results that follow include these indicators but, for brevity, their coefficients are not reported.

[17]While valuable as a robustness check, the Cox model is best suited to continuous time duration data.

A well-known data-quality problem of the SIPP (and all other major surveys) is seam bias (see Appendix 2.A) - unusually large proportion of labour market transitions appear to occur at the seam between subsequent waves. To address the issue I include as a regressor a variable which indicates whether a particular week is the first week of a reference period. As a result, any significant difference between the probability of a transition taking plays at the seams rather than at other point of the reference period is attributed to bias.

Table 2.4.1 presents the net-worth coefficients from ML estimation of (2.4) for household heads (Tables 2.D.8 and 2.D.9 in the Appendix report the full specification for heads and spouses; Tables 2.D.10 and 2.D.11 report the results when the alternative net-worth definition is used). All tables report regression coefficients and, unlike the wage-regressions above, the top net-worth category serves as reference. For household heads the hazard rate into employment declines with net worth[18], except for a sharp discontinuity around zero net worth where hazard rates are very low and close to those of the wealthiest individuals. To get a feeling of the magnitude of the differences, notice that in a c-log-log regression, coefficients' exponentiated values identify hazard ratios. For example, all else equal, household heads from the bottom net-worth category have $(e^{0.510}-1) \approx 67\%$ higher hazard rate than their wealthiest counterparts. For household heads the results are robust with respect to net-worth concept.

The pattern for spouses is different. First, they tend to have overall longer non-employment durations. Next, above the fifth net-worth category (and the second when home equity is netted out) hazard rates do not differ significantly across groups. Further, no discontinuity around zero net worth occurs - in fact, under the benchmark net-worth definition hazard rates there are the highest, although insignificantly different from those in the left tail. It should be noted that both for household heads and for spouses, accounting for observable characteristics does not affect the underlying relationship as significantly as in the case of re-

[18]While coefficients do not monotonically decline in value, they do not differ in statistical significance sense between any two adjacent net-worth groups, except around the third category.

employment wages.

Except for the discontinuity around zero net worth, the results indicate that hazard rates to employment decline with wealth. In view of the theoretical literature discussed in the introduction, this could be rationalized by a positive effect of wealth on reservation wages and/or a negative effect on search effort. As in the previous section, it is unlikely that the net worth coefficients identify causal effect as simultaneity is probably present. However, there is an important difference. It is reasonable to think that wealth accumulation is, if anything, lower during non-employment. If some unobserved factor causes individuals to stay out of employment for longer, it will likely imply that they have lower net worth. Omitting relevant variables will then induce a positive bias in the relationship between net worth and hazard rates, while the estimated relation is, in fact, negative. Therefore even if the coefficients do not identify the causal effect, the latter is most likely negative.

2.5 Conclusion

This paper explores the reduced-form relationship between wealth, re-employment wages and hazard rates into employment, in the 1996 panel of the Survey of Income and Program Participation. It complements the literature by studying a new dataset. As in related previous studies I find that the relationship between net worth and hazard rates is negative. This evidence is consistent with the theoretical prediction of search models featuring risk-averse workers. In disagreement with prior studies I show that net worth and wages are not monotonically related - in fact, re-employment wages decline with net worth while the latter is negative (as for about 20 percent of the sample) and then increase when positive, attaining a minimum for workers around zero net worth. I argue that prior estimates are based on inappropriate specification of the main relationship of interest. The pattern is robust even after controlling for a broad range of observables but disappears when accounting for past wages. This finding is inconsistent either with theoretical predictions or with the view that causal effects are identified by the approach used here and in previous studies.

Causality aside, the fact than the most wealth-poor individuals are high-earners raises interesting question on its own. Inspection of the data reveals that the major difference between those at the left tail of the net worth distribution and the asset-rich is that the former are younger. Understanding the pattern motivates a fusion of life-cycle and labour-market dimensions.

Appendix

2.A Data quality

This section discusses some well-understood issues regarding the quality of income, labour force status, and assets data in the SIPP. The discussion is largely based on Czajka and Denmead (2008) and Czajka et al. (2003).

It is well-documented that major surveys underestimate aggregate earned income of US households in comparison to data based on administrative records and estimates based on the SIPP are lower than other surveys. This discrepancy, however, can be largely attributed to sample selection (the SIPP surveys disproportionately in regions with high concentration of poverty) and top-coding of income (the SIPP estimates of average income at the top/bottom of the distribution are lowest/highest among major surveys). While this raises concerns about income-data reliability, the differences among surveys are, in fact, small (Czajka and Denmead, 2008).

With respect to labour-market status data a well-known problem in the SIPP is seam bias. The survey reports a disproportionately large amount of transitions occurring "at the seam" between waves. It should be noted, however, that seam bias is a problem in all surveys and given its relatively short reference period the SIPP has a comparative advantage in this respect. In estimating hazard rates, I include controls for the two weeks around the seam and attribute any significant differences in comparison to the rest of the reference period to such bias.

In comparison to surveys that collect asset data (in particular SCF and PSID), the SIPP estimates substantially lower average and somewhat lower median household net worth. As compared to SCF both assets liabilities are underestimated. [19]. As with income, however, most of the differences seem to be attributable to sample selection and top-coding practices. The main empirical results in this paper are based not on continuous measures of net worth but on categorical ordering of households by net worth, and as long as the categories are sufficiently well identified results should not be sensitive to measurement error.

2.B Definition of wealth and debt variables

This section describes the wealth and debt variables used in the analysis and how they were created from the variables available in the 1996 SIPP panel, waves 3,6,9, and 12.

The SIPP collects asset and liabilities data for a number of categories at individual level. In addition, it provides recode variables at household level that sum the individual level assets and liabilities across all household members except for those related to the value of property and vehicles. The latter are identical for each household member and enter the household-level recodes only once through the values reported by the household reference person. All the variables I use in my analysis are based on these household-level recodes.

Household liquid wealth is defined as the sum of the interest rate earning assets held in banks and other institutions, and equity in stocks and mutual funds (*thhintbk*, *thhintot*, and *rhhstk* in the SIPP recodes). Pension wealth is defined as the sum of equity in IRA and KEOGH accounts for all household members (*thhira* in the SIPP recodes). Total business equity is based on the variable *thhbeq*; net equity in vehicles is based on *thhvehcl*; other household wealth is defined as the sum of net equity in real estate other than the household's home (*thhore*), and total other assets (*thhotast*).

[19] Czajka et al. (2003) report that the SIPP estimate of median assets is 83 percent of the SCF estimate while the estimate of median liabilities is 97 percent of the SCF estimate.

Net home equity is based on the home equity recode (*thhtheq*) which is identically equal to the sum of the current value of home property (*tpropval*), reported value of mobile home (*tmhval*), net of the total debt owned on the former (*thhmortg*).

Total household wealth (*thhtwlth*) is the sum of all aforementioned asset categories. It should be noted that it already includes net rather than gross asset positions with respect to value of the household's home and vehicles.

Credit card debt is the sum of credit card debts owed in own name by all individual household members (*ealidab*) plus the sum of all credit card debts owed jointly by subsets of household members (*ealjdab*). Similarly household debt on loans sums across household members their individual loans (*ealidal*) and jointly owed loans (*ealjdal*). Finally, other household debt includes the sum of all individual "other" debts across household members (*ealidao* and *ealjdao*). The loan debt includes the amount of money "owed for loans obtained through a bank or credit union, other than car loans or home equity loans". "Other" debt includes money owed "for any other debt not yet mentioned (include medical bills not covered by insurance, money owed to private individuals, and any other debt not covered; exclude mortgages, home equity loans and car loans)". The sum of household credit card debt, loan debt and other debt is identically equal to the total household unsecured debt (*rhhuscbt*).

Throughout the paper I use two alternative definitions of net worth. The benchmark definition uses the SIPP recode for total household net worth (*thhtnw*). The latter is identically equal to the "total household wealth" (*thhtwlth*) net of total unsecured debt (*rhhuscbt*). Netting out only the unsecured debt is necessitated by the fact that *thhtwlth* already subtracts the value of secured debt. Net worth based on the first definition therefore includes all home related assets and liabilities. In addition, I use a definition of net worth excluding households' asset positions in the housing market. The net worth net of the housing position is identically equal to th total household net worth recode (*thhtnw*) minus the home equity (*thhtheq*). The latter is equal to the value of the household's property (*tpropval*) minus the value of the mortgage (*thhmortg*).

Table 2.B.1: Net worth categories, group sizes

	Net worth	SIPP Heads	SIPP Spouses	Wages Heads	Wages Spouses	Hazards Heads	Hazards Spouses
NW1	≤ -7769	0.06	0.06	0.08	0.09	0.09	0.09
NW2	$(-7769, -100]$	0.06	0.05	0.10	0.08	0.10	0.08
NW3	$(-100, 100]$	0.04	0.01	0.07	0.02	0.08	0.01
NW4	$(100, 1651]$	0.04	0.03	0.08	0.05	0.08	0.05
NW5	$(1651, 9887]$	0.10	0.08	0.15	0.13	0.14	0.12
NW6	$(9887, 25737]$	0.10	0.10	0.12	0.12	0.10	0.12
NW7	$(25737, 47415]$	0.10	0.10	0.09	0.11	0.09	0.09
NW8	$(47415, 74961]$	0.10	0.11	0.08	0.09	0.08	0.09
NW9	$(74961, 114442]$	0.10	0.11	0.07	0.09	0.07	0.09
NW10	$(114442, 180752]$	0.10	0.11	0.07	0.09	0.06	0.09
NW11	$(180752, 336453]$	0.10	0.12	0.05	0.07	0.07	0.08
NW12	≥ 336453	0.10	0.13	0.04	0.06	0.06	0.10
Observations		84519	63529	4589	3355	7313	5942

Note: The final columns refer to the net worth at the time of transition/right-censoring.

For comparability between the different estimation exercises I define the net worth categories based on the distribution of net worth for the households in the whole survey. Under both definitions households with net worth between -100 and 100 constitute the third category.

2.B.1 Net worth including home equity

In the pooled asset data, households with net worth between -100 and 100 dollars (deflated) are located between the 12th and 15.8th percentiles of the net worth distribution. I assign these households to the third net worth category. The first two categories are the two equally large groups of households with lower net worth. The fourth category complements the third up to the 20th percentile. All the next categories correspond to a decile of the net worth distribution. Even in the overall data from the survey the twelve categories are not of equal size. Further when constructing the samples for each of the empirical exercises, the sizes of the groups change. Table 2.B.1 reports the sizes of these groups for all of the constructed samples.

2.B.2 Net worth without home equity

In the pooled asset data, households with net worth between -100 and 100 dollars (deflated) are located between the 19.5th and 24.6th percentiles of the net worth distribution. With home equity subtracted from the total net worth the individuals with zero wealth are shifted towards the right in the distribution. I choose to assign these to the third net worth category and as a result there are eleven rather than twelve categories. Table 2.B.2 reports the sizes of these groups for all constructed samples.

Table 2.B.2: Net worth categories, no home equity, group sizes

		SIPP		**Wages**		**Hazards**	
Net worth		Heads	Spouses	Heads	Spouses	Heads	Spouses
NW1	≤ -6040	0.10	0.10	0.12	0.14	0.12	0.13
NW2	$(-6040, -100]$	0.10	0.08	0.14	0.12	0.13	0.12
NW3	$(-100, 100]$	0.05	0.02	0.08	0.02	0.09	0.02
NW4	$(100, 1222]$	0.05	0.04	0.09	0.05	0.09	0.05
NW5	$(1222, 4827]$	0.10	0.08	0.13	0.12	0.13	0.10
NW6	$(4827, 9706]$	0.10	0.09	0.11	0.11	0.09	0.10
NW7	$(9706, 17891]$	0.10	0.11	0.10	0.10	0.09	0.10
NW8	$(17891, 36771]$	0.10	0.11	0.07	0.10	0.07	0.09
NW9	$(36771, 82401]$	0.10	0.12	0.07	0.09	0.06	0.09
NW10	$(82401, 209829]$	0.10	0.12	0.06	0.08	0.07	0.09
NW11	≥ 209829	0.10	0.13	0.04	0.07	0.06	0.10
Observations		84519	63529	4589	3355	7313	5942

Note: The final columns refer to the net worth at the time of transition/right-censoring.

2.C Modelling duration dependence

To account for duration dependence in (2.4) I generate 26 indicator variables taking a value of 1 if the current length of a spell for a particular observation falls within some interval and 0 otherwise. Since the aggregate hazard rates declines at a decreasing rate I give higher weight to the first weeks of a spell. Table 2.C.1 presents the indicators created and the corresponding intervals of elapsed time in weeks.

Table 2.C.1: Duration dependence

Indicator	Duration (weeks)
1-15	1-15
16	$(15, 17]$
17	$(17, 19]$
18	$(19, 21]$
19	$(21, 23]$
20	$(23, 25]$
21	$(25, 29]$
22	$(29, 33]$
23	$(33, 37]$
24	$(37, 45]$
25	$(45, 55]$
26	$(55, \infty]$

2.D Tables and figures

Table 2.D.1: Wage regression, household heads

	(1)	(2)	(3)	(4)
NW1	0.420***	0.319***	0.166**	0.128*
	(0.000)	(0.000)	(0.006)	(0.041)
NW2	0.225***	0.166***	0.0683	0.0386
	(0.000)	(0.001)	(0.200)	(0.477)
NW3	0	0	0	0
	(.)	(.)	(.)	(.)
NW4	0.144**	0.103**	0.0576	0.0316
	(0.002)	(0.006)	(0.157)	(0.449)
NW5	0.266***	0.212***	0.129**	0.0978*
	(0.000)	(0.000)	(0.004)	(0.038)
NW6	0.333***	0.274***	0.159**	0.125*
	(0.000)	(0.000)	(0.001)	(0.016)
NW7	0.376***	0.316***	0.172***	0.135**
	(0.000)	(0.000)	(0.000)	(0.002)
NW8	0.445***	0.374***	0.230***	0.201***
	(0.000)	(0.000)	(0.000)	(0.000)
NW9	0.498***	0.418***	0.218**	0.188*
	(0.000)	(0.000)	(0.009)	(0.025)
NW10	0.666***	0.564***	0.359***	0.321***
	(0.000)	(0.000)	(0.000)	(0.000)
NW11	0.579***	0.515***	0.281***	0.239**
	(0.000)	(0.000)	(0.000)	(0.001)
NW12	0.764***	0.712***	0.426***	0.385***
	(0.000)	(0.000)	(0.000)	(0.000)
State	X	✓	✓	✓
Metro area		0.144***	0.101**	0.105**
		(0.000)	(0.002)	(0.002)
Num. persons		-0.0119	0.00306	0.00381
		(0.277)	(0.732)	(0.708)
Any children		-0.0369	-0.0202	-0.0186
		(0.131)	(0.424)	(0.468)
Age		0.0458***	0.0338***	0.0329***
		(0.000)	(0.000)	(0.000)
Age^2		-0.0006***	-0.0005***	-0.0005***
		(0.000)	(0.000)	(0.000)
Female		-0.335***	-0.310***	-0.281***
		(0.000)	(0.000)	(0.000)
Married		0.1269***	0.0850***	0.0821***
		(0.000)	(0.007)	(0.009)
Observations	4589	4589	4437	4437

p-values in parentheses

continued on next page

continued from previous page

	(1)	(2)	(3)	(4)
Race				
Black		0.0497	0.0405	0.0501
		(0.102)	(0.211)	(0.143)
Native American		0.0374	0.0716	0.0754
		(0.669)	(0.413)	(0.389)
Asian		0.0208	0.00460	0.0140
		(0.550)	(0.915)	(0.740)
Education				
< High school			0.126**	0.133**
			(0.001)	(0.001)
High school			0.178***	0.187***
			(0.000)	(0.000)
< Degree			0.257***	0.266***
			(0.000)	(0.000)
Undergraduate			0.536***	0.542***
			(0.000)	(0.000)
Master's			0.746***	0.749***
			(0.000)	(0.000)
PhD			0.113	0.125
			(0.795)	(0.773)
Professional			0.863***	0.862***
			(0.000)	(0.000)
Enrolled			-0.0291	-0.0283
			(0.531)	(0.524)
Experience, years			0.0101*	0.00624
			(0.016)	(0.149)
Experience^2			-0.0000546	-0.0000121
			(0.553)	(0.898)
Spell, wks				-0.00867*
				(0.015)
Spell^2				0.0000738
				(0.344)
UI income				0.000143***
				(0.000)
Business income				0.0000933
				(0.345)
Other income				0.0000120
				(0.476)
HH income (net)				0.00000384
				(0.461)
Work hrs				0.00101
				(0.291)
Constant	1.707***	0.957***	0.973***	1.034***
	(0.000)	(0.000)	(0.000)	(0.000)
Observations	4589	4589	4437	4437

p-values in parentheses

Table 2.D.2: Wage regression, spouses

	(1)	(2)	(3)	(4)
NW1	0.278**	0.326***	0.187*	0.175*
	(0.002)	(0.000)	(0.025)	(0.032)
NW2	0.220*	0.243**	0.179*	0.169*
	(0.011)	(0.003)	(0.026)	(0.032)
NW3	0	0	0	0
	(.)	(.)	(.)	(.)
NW4	0.0730	0.103	0.0252	0.0214
	(0.454)	(0.264)	(0.793)	(0.822)
NW5	0.219**	0.276***	0.177*	0.172*
	(0.008)	(0.000)	(0.025)	(0.026)
NW6	0.282***	0.363***	0.222**	0.204*
	(0.001)	(0.000)	(0.006)	(0.011)
NW7	0.359***	0.458***	0.296***	0.284***
	(0.000)	(0.000)	(0.000)	(0.000)
NW8	0.368***	0.498***	0.326***	0.305***
	(0.000)	(0.000)	(0.000)	(0.000)
NW9	0.381***	0.524***	0.317***	0.292***
	(0.000)	(0.000)	(0.000)	(0.001)
NW10	0.523***	0.650***	0.458***	0.434***
	(0.000)	(0.000)	(0.000)	(0.000)
NW11	0.476***	0.624***	0.397***	0.363***
	(0.000)	(0.000)	(0.000)	(0.000)
NW12	0.667***	0.765***	0.482***	0.454***
	(0.000)	(0.000)	(0.000)	(0.000)
State	X	✓	✓	✓
Metro area		0.120***	0.0769*	0.0765*
		(0.005)	(0.047)	(0.048)
Num. persons		-0.0546***	-0.0299*	-0.0311*
		(0.000)	(0.007)	(0.008)
Any children		0.0607	0.0435	0.0573
		(0.085)	(0.171)	(0.084)
Age		0.0256***	0.000490	-0.000441
		(0.000)	(0.953)	(0.958)
Age^2		-0.0004***	-0.0001	-0.0001
		(0.000)	(0.306)	(0.405)
Female		-0.369***	-0.347***	-0.336***
		(0.000)	(0.000)	(0.000)
Race				
Black		0.0416	0.0578	0.0582
		(0.377)	(0.208)	(0.190)
Native American		0.135	0.134	0.134
		(0.175)	(0.095)	(0.077)
Asian		-0.0149	-0.00919	-0.00468
		(0.842)	(0.892)	(0.947)
Observations	3355	3355	3045	3045

p-values in parentheses

continued on next page

continued from previous page

	(1)	(2)	(3)	(4)
Education				
< High school			-0.0487	-0.0582
			(0.300)	(0.191)
High school			0.0679	0.0584
			(0.155)	(0.183)
< Degree			0.107**	0.0937**
			(0.004)	(0.006)
Undergraduate			0.374***	0.351***
			(0.000)	(0.000)
Master's			0.447***	0.434***
			(0.000)	(0.000)
PhD			0.649***	0.649***
			(0.000)	(0.000)
Professional			0.880***	0.841***
			(0.000)	(0.000)
Enrolled			-0.00802	-0.0113
			(0.924)	(0.889)
Experience, years			0.0160***	0.0128**
			(0.001)	(0.006)
Experience^2			-0.000239*	-0.000188
			(0.014)	(0.053)
Spell, wks				-0.00556*
				(0.041)
Spell^2				0.0000297
				(0.655)
UI income				0.000129*
				(0.027)
Business income				0.0000136***
				(0.000)
Other income				-0.000000
				(0.988)
HH income (net)				0.0000103*
				(0.044)
Work hrs				0.000238
				(0.878)
Constant	1.695***	1.590***	1.894***	1.960***
	(0.000)	(0.000)	(0.000)	(0.000)
Observations	3355	3355	3045	3045

p-values in parentheses

Table 2.D.3: Wage regression, married heads

Net worth	(1)	(2)	(3)	(4)
NW1	0.426***	0.379***	0.208**	0.168*
	(0.000)	(0.000)	(0.003)	(0.021)
NW2	0.258***	0.248***	0.112	0.0927
	(0.000)	(0.001)	(0.123)	(0.206)
NW3	0	0	0	0
	(.)	(.)	(.)	(.)
NW4	0.160*	0.149	0.0695	0.0481
	(0.032)	(0.058)	(0.363)	(0.535)
NW5	0.286***	0.280***	0.173*	0.146*
	(0.000)	(0.000)	(0.010)	(0.033)
NW6	0.327***	0.336***	0.203**	0.173*
	(0.000)	(0.000)	(0.004)	(0.016)
NW7	0.349***	0.374***	0.206**	0.176*
	(0.000)	(0.000)	(0.004)	(0.017)
NW8	0.468***	0.461***	0.279***	0.256***
	(0.000)	(0.000)	(0.000)	(0.001)
NW9	0.526***	0.540***	0.316***	0.290***
	(0.000)	(0.000)	(0.000)	(0.000)
NW10	0.663***	0.669***	0.435***	0.404***
	(0.000)	(0.000)	(0.000)	(0.000)
NW11	0.542***	0.602***	0.334***	0.302**
	(0.000)	(0.000)	(0.000)	(0.001)
NW12	0.763***	0.835***	0.516***	0.477***
	(0.000)	(0.000)	(0.000)	(0.000)
Observations	3045	3045	2952	2952

p-values in parentheses

Table 2.D.4: Characteristics by net worth, spouses

	1	2	3	4	5	6	7	8	9	10	11	12
Age	33.86	34.55	33.75	34.08	35.04	37.19	39.55	42.22	44.14	45.67	48.10	47.71
Experience	12.37	13.28	9.85	12.26	14.03	15.33	17.04	18.28	20.43	21.32	21.43	22.83
Female	0.72	0.64	0.57	0.62	0.71	0.73	0.74	0.77	0.80	0.74	0.80	0.75
Spell (wks)	7.75	6.09	9.25	7.47	7.32	6.36	7.65	7.46	7.35	7.22	6.53	7.05
HH (net) income	2577	1912	1345	1695	1957	2342	2566	3137	3666	3652	4310	4851
Married	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Education												
Elementary	0.02	0.06	0.08	0.11	0.06	0.05	0.04	0.02	0.03	0.03	0.01	0.00
< *High school*	0.11	0.19	0.33	0.25	0.20	0.14	0.13	0.08	0.09	0.07	0.05	0.02
High school	0.26	0.36	0.43	0.37	0.39	0.33	0.35	0.39	0.34	0.28	0.27	0.21
< *Degree*	0.35	0.27	0.14	0.21	0.26	0.30	0.29	0.31	0.30	0.33	0.36	0.28
Undergraduate	0.18	0.10	0.02	0.06	0.09	0.14	0.12	0.16	0.18	0.19	0.22	0.27
Master's	0.04	0.02	0.00	0.01	0.01	0.03	0.04	0.03	0.05	0.08	0.06	0.14
PhD	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.05
Professional	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.02	0.02
Observations	314	258	51	167	434	402	364	318	310	290	236	211

Note: Columns correspond to net worth categories

Table 2.D.5: Wage regression, no home equity, heads

	(1)	(2)	(3)	(4)
NW1	0.429***	0.316***	0.173***	0.136**
	(0.000)	(0.000)	(0.000)	(0.002)
NW2	0.274***	0.195***	0.103*	0.0716
	(0.000)	(0.000)	(0.016)	(0.091)
NW3	0	0	0	0
	(.)	(.)	(.)	(.)
NW4	0.139**	0.0948*	0.0558	0.0348
	(0.002)	(0.029)	(0.199)	(0.424)
NW5	0.230***	0.170***	0.0870*	0.0594
	(0.000)	(0.000)	(0.034)	(0.149)
NW6	0.358***	0.267***	0.160***	0.130**
	(0.000)	(0.000)	(0.000)	(0.002)
NW7	0.379***	0.280***	0.156***	0.126**
	(0.000)	(0.000)	(0.001)	(0.006)
NW8	0.498***	0.396***	0.211***	0.178**
	(0.000)	(0.000)	(0.000)	(0.001)
NW9	0.615***	0.490***	0.268***	0.244***
	(0.000)	(0.000)	(0.000)	(0.000)
NW10	0.642***	0.538***	0.305***	0.256***
	(0.000)	(0.000)	(0.000)	(0.000)
NW11	0.710***	0.622***	0.357***	0.319***
	(0.000)	(0.000)	(0.000)	(0.000)
Observations	4589	4589	4437	4437

p-values in parentheses

Table 2.D.6: Wage regression, no home equity, spouses

	(1)	(2)	(3)	(4)
NW1	0.269***	0.295***	0.139	0.119
	(0.001)	(0.000)	(0.077)	(0.125)
NW2	0.106	0.147	0.0757	0.0616
	(0.173)	(0.052)	(0.323)	(0.420)
NW3	0	0	0	0
	(.)	(.)	(.)	(.)
NW4	-0.0200	0.00581	-0.0657	-0.0668
	(0.825)	(0.947)	(0.473)	(0.465)
NW5	0.189*	0.228**	0.124	0.112
	(0.018)	(0.002)	(0.112)	(0.151)
NW6	0.205**	0.263***	0.130	0.113
	(0.008)	(0.000)	(0.089)	(0.135)
NW7	0.231**	0.274***	0.140	0.121
	(0.004)	(0.000)	(0.088)	(0.140)
NW8	0.358***	0.441***	0.241**	0.211**
	(0.000)	(0.000)	(0.003)	(0.009)
NW9	0.368***	0.432***	0.216*	0.190*
	(0.000)	(0.000)	(0.013)	(0.029)
NW10	0.406***	0.492***	0.262**	0.231**
	(0.000)	(0.000)	(0.002)	(0.007)
NW11	0.521***	0.583***	0.312***	0.273**
	(0.000)	(0.000)	(0.001)	(0.003)
Observations	3355	3355	3045	3045

p-values in parentheses

Table 2.D.7: Wage regression, previous wage, household heads

	(1)	(2)
NW1	0.150**	0.0654
	(0.009)	(0.242)
NW2	0.0615	-0.00523
	(0.267)	(0.924)
NW4	0.00814	-0.0269
	(0.885)	(0.621)
NW5	0.113*	0.0456
	(0.021)	(0.342)
NW6	0.107*	0.0239
	(0.037)	(0.633)
NW7	0.144*	0.0556
	(0.012)	(0.324)
NW8	0.250***	0.048
	(0.000)	(0.110)
NW9	0.196**	0.0677
	(0.004)	(0.309)
NW10	0.388***	0.257***
	(0.000)	(0.000)
NW11	0.304***	0.171*
	(0.000)	(0.015)
NW12	0.420***	0.215*
	(0.000)	(0.014)
$log(W_{t\_})$		0.299***
		(0.000)
Observations	2771	2771

Regressions include constant and controls for age, sex, education and marital status

Table 2.D.8: C-log-log regression, household heads

	(1)	(2)	(3)	(4)
NW1	0.632***	0.479***	0.608***	0.510***
	(0.000)	(0.000)	(0.000)	(0.000)
NW2	0.407***	0.276**	0.428***	0.315***
	(0.000)	(0.002)	(0.000)	(0.001)
NW3	0.139	-0.0000	0.223*	0.0861
	(0.114)	(1.000)	(0.028)	(0.411)
NW4	0.527***	0.369***	0.552***	0.431***
	(0.000)	(0.000)	(0.000)	(0.000)
NW5	0.391***	0.242**	0.389***	0.278**
	(0.000)	(0.004)	(0.000)	(0.002)
NW6	0.439***	0.287***	0.390***	0.306***
	(0.000)	(0.001)	(0.000)	(0.001)
NW7	0.432***	0.333***	0.452***	0.371***
	(0.000)	(0.000)	(0.000)	(0.000)
NW8	0.289***	0.192*	0.282**	0.204*
	(0.001)	(0.031)	(0.002)	(0.032)
NW9	0.255**	0.182*	0.258**	0.201*
	(0.005)	(0.046)	(0.006)	(0.036)
NW10	0.209*	0.160	0.223*	0.179
	(0.025)	(0.090)	(0.021)	(0.069)
NW11	0.200*	0.166	0.181	0.145
	(0.030)	(0.072)	(0.055)	(0.130)
NW12	0	0	0	0
	(.)	(.)	(.)	(.)
Metro		-0.0108	-0.0312	-0.0104
		(0.765)	(0.396)	(0.778)
Number of persons		-0.0601***	-0.0396**	-0.0324**
		(0.000)	(0.001)	(0.009)
Age		0.0130	-0.0410***	-0.0360**
		(0.071)	(0.000)	(0.001)
Age^2		-0.0003***	0.0003	0.0002
		(0.000)	(0.065)	(0.137)
Female		-0.666***	-0.631***	-0.620***
		(0.000)	(0.000)	(0.000)
Married		-0.142**	-0.193***	-0.165***
		(0.003)	(0.000)	(0.001)
Enrolled		0.185*	0.133	0.145
		(0.015)	(0.085)	(0.061)
Race				
Black		-0.0116	-0.0246	-0.0493
		(0.790)	(0.581)	(0.270)
Native American		-0.132	-0.0550	-0.0634
		(0.220)	(0.613)	(0.560)
Asian		-0.0333	-0.0345	-0.0514
		(0.705)	(0.698)	(0.565)
Observations	152147	152147	148712	148712

p-values in parentheses

continued on next page

continued from previous page

	(1)	(2)	(3)	(4)
Education				
$<$ *High school*			-0.119	-0.108
			(0.163)	(0.210)
High school			-0.0156	0.0070
			(0.849)	(0.932)
$<$ *Degree*			0.122	0.154
			(0.140)	(0.062)
Undergraduate			0.200*	0.260**
			(0.029)	(0.005)
Masters			0.247*	0.330**
			(0.031)	(0.004)
PhD			0.0122	0.0923
			(0.958)	(0.692)
Professional			0.279	0.320
			(0.111)	(0.069)
Experience, years			0.0358***	0.0382***
			(0.000)	(0.000)
Experience^2			-0.0005***	-0.0006***
			(0.000)	(0.000)
UI income				-0.0004***
				(0.000)
Business income				0.0002***
				(0.000)
Other income				-0.0001*
				(0.039)
HH income (net)				-0.0001***
				(0.000)
Constant	-3.276***	-2.681***	-2.160***	-2.144***
	(0.000)	(0.000)	(0.000)	(0.000)
Observations	152147	152147	148712	148712

p-values in parentheses

Table 2.D.9: C-log-log regression, spouses

	(1)	(2)	(3)	(4)
NW1	0.411***	0.276***	0.344***	0.265**
	(0.000)	(0.001)	(0.000)	(0.002)
NW2	0.480***	0.304***	0.427***	0.336***
	(0.000)	(0.000)	(0.000)	(0.000)
NW3	0.503***	0.231	0.509**	0.411**
	(0.000)	(0.118)	(0.001)	(0.010)
NW4	0.334***	0.141	0.231*	0.135
	(0.000)	(0.146)	(0.030)	(0.209)
NW5	0.290***	0.166*	0.263**	0.176*
	(0.000)	(0.036)	(0.002)	(0.045)
NW6	0.191*	0.0829	0.150	0.0699
	(0.013)	(0.296)	(0.078)	(0.417)
NW7	0.163*	0.0755	0.147	0.0741
	(0.042)	(0.355)	(0.093)	(0.401)
NW8	0.172*	0.0827	0.126	0.0660
	(0.033)	(0.313)	(0.144)	(0.447)
NW9	0.155	0.109	0.139	0.0901
	(0.056)	(0.184)	(0.103)	(0.297)
NW10	0.0294	-0.0038	0.0499	-0.0011
	(0.727)	(0.964)	(0.564)	(0.990)
NW11	0.0680	0.0392	0.0359	0.0067
	(0.427)	(0.647)	(0.685)	(0.940)
NW12	0	0	0	0
	(.)	(.)	(.)	(.)
Metro		-0.0517	-0.0807	-0.0650
		(0.200)	(0.057)	(0.126)
Num. persons		-0.0570***	-0.0342*	-0.0311*
		(0.000)	(0.017)	(0.030)
Age		0.0112	-0.0321**	-0.0268*
		(0.201)	(0.007)	(0.024)
Age^2		-0.0002*	0.0002	0.0002
		(0.017)	(0.083)	(0.180)
Female		-0.751***	-0.711***	-0.690***
		(0.000)	(0.000)	(0.000)
Enrolled		0.205*	0.346***	0.372***
		(0.036)	(0.001)	(0.000)
Race				
Black		-0.0756	-0.0579	-0.0629
		(0.286)	(0.438)	(0.398)
Native American		0.166	0.185	0.170
		(0.193)	(0.155)	(0.190)
Asian		-0.0287	-0.0244	-0.0442
		(0.756)	(0.803)	(0.653)
Observations	172477	172477	162502	162502

p-values in parentheses

continued on next page

continued from previous page

	(1)	(2)	(3)	(4)
Education				
< *High school*			-0.143	-0.123
			(0.169)	(0.237)
High school			-0.108	-0.0866
			(0.267)	(0.371)
< *Degree*			-0.0750	-0.0397
			(0.443)	(0.685)
Undergraduate			-0.0861	-0.0277
			(0.410)	(0.792)
Masters			0.121	0.173
			(0.345)	(0.178)
PhD			0.0678	0.169
			(0.776)	(0.483)
Professional			-0.0612	-0.0346
			(0.794)	(0.886)
Experience, years			0.0389^{***}	0.0396^{***}
			(0.000)	(0.000)
Experience^2			-0.0007^{***}	-0.0007^{***}
			(0.000)	(0.000)
UI income				-0.0002^{*}
				(0.016)
Business income				0.0002^{***}
				(0.000)
Other income				-0.0000
				(0.112)
HH income (net)				-0.0001^{***}
				(0.000)
Constant	-3.482^{***}	-2.657^{***}	-2.258^{***}	-2.282^{***}
	(0.000)	(0.000)	(0.000)	(0.000)
Observations	172477	172477	162502	162502

p-values in parentheses

Table 2.D.10: C-log-log, no home equity, heads

	(1)	(2)	(3)	(4)
NW1	0.620***	0.465***	0.590***	0.496***
	(0.000)	(0.000)	(0.000)	(0.000)
NW2	0.391***	0.268**	0.412***	0.298**
	(0.000)	(0.001)	(0.000)	(0.001)
NW3	0.165	0.00317	0.229*	0.0925
	(0.055)	(0.973)	(0.020)	(0.365)
NW4	0.477***	0.311***	0.508***	0.389***
	(0.000)	(0.000)	(0.000)	(0.000)
NW5	0.444***	0.300***	0.441***	0.328***
	(0.000)	(0.000)	(0.000)	(0.000)
NW6	0.416***	0.296***	0.417***	0.324***
	(0.000)	(0.001)	(0.000)	(0.001)
NW7	0.367***	0.246**	0.354***	0.268**
	(0.000)	(0.005)	(0.000)	(0.004)
NW8	0.391***	0.311***	0.397***	0.333***
	(0.000)	(0.001)	(0.000)	(0.001)
NW9	0.228*	0.153	0.199*	0.154
	(0.012)	(0.095)	(0.034)	(0.106)
NW10	0.303***	0.277**	0.304**	0.276**
	(0.001)	(0.002)	(0.001)	(0.004)
NW11	0	0	0	0
	(.)	(.)	(.)	(.)
Observations	152147	152147	148712	148712

p-values in parentheses

Table 2.D.11: C-log-log, no home equity, spouses

	(1)	(2)	(3)	(4)
NW1	0.314***	0.148*	0.167*	0.0828
	(0.000)	(0.048)	(0.037)	(0.305)
NW2	0.285***	0.123	0.184*	0.0931
	(0.000)	(0.112)	(0.028)	(0.272)
NW3	0.226	-0.0275	0.126	0.0141
	(0.077)	(0.834)	(0.372)	(0.921)
NW4	0.155	-0.0359	-0.00314	-0.112
	(0.090)	(0.702)	(0.976)	(0.283)
NW5	0.158*	0.0126	0.0632	-0.0368
	(0.038)	(0.873)	(0.460)	(0.672)
NW6	0.125	-0.00795	0.0492	-0.0362
	(0.109)	(0.922)	(0.566)	(0.676)
NW7	0.0754	-0.0311	0.0296	-0.0465
	(0.342)	(0.700)	(0.728)	(0.589)
NW8	0.0732	-0.00162	0.0147	-0.0520
	(0.359)	(0.984)	(0.862)	(0.542)
NW9	0.0782	-0.00961	0.0186	-0.0281
	(0.327)	(0.905)	(0.824)	(0.738)
NW10	-0.0413	-0.0718	-0.0637	-0.0960
	(0.619)	(0.387)	(0.458)	(0.266)
NW11	0	0	0	0
	(.)	(.)	(.)	(.)
Observations	172477	172477	162502	162502

p-values in parentheses

Chapter 3

Labour market frictions, endogenous retirement, and wealth

3.1 Introduction

Respondents to household surveys identify financing consumption after retirement and insurance against unexpected events as the two most important reasons for saving[1]. While theoretical models emphasising precautionary motives have proved successful in replicating broad trends in empirical wealth distributions, direct evidence shows that precautionary wealth is too small a proportion of total wealth[2]. Long-term life-cycle motives should instead be the dominant force behind central tendencies of the wealth distribution.

Canonical versions of the life-cycle model[3] take a household's earning process

[1]For example, see Cagetti (2003) and Schunk (2009) for descriptive statistics on self-reported saving motives.

[2]Fulford (2015) shows that the median household in the SCF reports to need a little more than a month's income in savings for "emergencies and other unexpected things that may come up". Hurst et al. (2010) finds that precautionary savings account for less than 10 percent of total household wealth in the PSID, and attribute previous higher estimates (e.g. Carroll and Samwick (1997)) to pooling together business owners and other household - two groups that otherwise hold the same low ratio of precautionary to total wealth. This is in line with earlier studies such as Lusardi (1998) and Guiso et al. (1992).

[3]For example, Hubbard et al. (1994a) and Gourinchas and Parker (2002).

and retirement age as exogenous. In fact, earnings are determined by individual labour market experiences and a large literature studying the interaction between labour market decisions and life-cycle phenomena has emerged. Moreover, the assumption that workers retire at the same age conceals considerable heterogeneity. In the US, labour-force participation declines steadily after the age of 55 (with discontinuous falls around the Social Security early and full retirement ages) but about 20 percent of 70-75 year-olds (12 percent of 75-80 year-olds) are still participants[4]. Retirement is a decision rather than an exogenous feature of the environment. Burbidge and Robb (1980) proposed a modification of the pure life-cycle model where households choose efficiently when to retire in order to maximise lifetime utility from consumption and leisure. Versions of this model have been used extensively to study optimal retirement age[5]. It should be noted that in this tradition labour markets are not modelled explicitly and agents are assumed to follow a sequence of employment and non-participation (retirement) for exogenous reasons.

It has been documented that certain groups of the population (e.g. the poor) save significantly less than implied by plain life-cycle models [6]. Various explanations have been proposed, including ones rooted in behavioural economics[7]. But when retirement is a decision, a rational household's optimal saving policy should be consistent with its plan about when to retire - a logic somewhat explicit in

[4]These are the figures from the 2012 CPS as reported by Toossi (2013).

[5]Recent studies include Bloom et al. (2007, 2014), D'Albis et al. (2012) and Kuhn et al. (2015) who investigate optimal retirement in the context of demographic changes, increases in longevity or expenditure on healthcare.

[6]For example, in an influential study, Dynan et al. (2004), using alternative strategies for isolating permanent earnings, documented that saving rates are increasing with measures of permanent earnings consistently across specifications in three different US surveys (PSID, SCF and CEX). Hubbard et al. (1994b) use PSID to show that a significant fraction of households with low lifetime earnings have pre-retirement wealth accumulation too small to be consistent with the perfect-market version of the life-cycle model. They further show that asset-poor households have inconsistently low saving rates, to the extent that low wealth is an "absorbing state over lengthy periods of time".

[7]Examples of rational explanations include persistent differences in time preference rates or subsistence parameters (Dynan et al., 2004); differences in Social Security replacement rates across high and low earning households (Huggett and Ventura, 2000); consumers deriving utility directly from wealth (Carroll, 2000), among others. For examples of behavioural explanations see (Laibson et al., 1998), Bernheim et al. (2001) and Benartzi and Thaler (2013), among others.

defined contribution plans where workers choose saving rates. If expected time of retirement varies with expected long-term earnings[8], then so will saving behaviour. For example, the poor might save little because they intend to work until older ages. If leisure is a normal good then its pursuit at older ages might be prohibitively costly for some.

This paper presents a model of a frictional labour market where risk-averse workers enjoy leisure when non-participants and earn a wage when employed but transit from non-participation into employment only through a spell of frictional unemployment. Labour is indivisible. Workers are characterised by time-invariant productivity which maps into a wage rate and differ by initial wealth endowments. When sufficiently asset-poor, workers of any productivity plan to work indefinitely. Efficient retirement plans along the lines of Burbidge and Robb (1980) arise endogenously from more asset-rich employed worker's decisions and optimal labour-market and consumption/saving policies are consistent with the retirement plan. Sufficiently asset-rich workers never work, enjoy leisure and consume out of interest income.

The model combines the main abstraction of the life-cycle hypothesis with endogenous retirement choice and emphasises how the latter emerges as a result of optimal behaviour when labour markets are frictional. It, hence, presents a framework for analysing the relationship between labour-market behaviour, asset accumulation and retirement strategies. A closely related paper by Rogerson and Wallenius (2013) discusses the role of non-convexities in the worker's problem, and labour indivisibility in particular, for generating abrupt transitions from employment to non-participation[9]. Our analysis also relies on a non-convexity due to

[8]Two empirical observations seem robust across countries and specifications. First, household wealth is positively related to the probability of retirement at any age. For example, Imbens et al. (2001) show that large lottery gains lead to significant reduction of labour supply, particularly for those around retirement; Brown et al. (2010) finds that receipt of inheritance increases probability of retirement especially when inheritance is unexpected. Second, descriptive evidence (Kallestrup-Lamb et al., 2016; Bender et al., 2014) suggests that controlling for wealth, individual earnings are inversely related to the probability of retirement - an observation suggestive of opposing income and substitution effects of earned income on consumption of leisure over the life cycle.

[9]In addition to labour indivisibility, they suggest two different sources of non-convexity -

labour indivisibility, but as the latter results from frictions it further implies that workers specialise in work early in life and in leisure later. When labour markets are frictional quits to non-participation are suboptimal if an individual intends to return to employment later. Our theoretical environment is also similar to the one in Krusell et al. (2008) but we focus on the implications of labour indivisibility for wealth accumulation while they study the flows between labour market states.

The environment abstracts from a number of features customarily present in life-cycle models, including income and mortality risk, health and expenditure shocks, and intergenerational transfers. As a result, the analysis abstracts from precautionary and bequest motives, while permanent income and consumption-leisure choice are emphasised as main motives for saving behaviour. This is in line with the earlier discussion of the empirical magnitude of precautionary wealth. One implication for the analysis is that the implied consumption profiles are flat and consumption inequality is age independent (see Storesletten et al. (2005)). As a result the model delivers highly tractable empirical predictions about the evolution of wealth distributions in the presence of persistent differentials in earning ability. We employ the 1996 and 2001 panels of the Survey of Income and Program Participation to explore some empirical aspects of the latter. First, we estimate non-parametrically the observed age profiles of the conditional on earnings net worth distributions. We document that wealth gets increasingly dispersed with earnings. The dispersion increases with age and is driven by pronounced lengthening of the right tail of wealth while the bottom quantiles vary little with age. The tenth percentile of wealth increases with age only for households with high observed earnings. We argue that these observations are readily interpretable through the prism of the model. Next we turn to individual households' observed saving outcomes. In the absence of uncertainty the model emphasises the role of persistent differences in earning ability on optimal saving policies. To frame empirical results more closely to the theoretical context we construct permanent

non-linearity of wages with respect to hours worked (to which they attribute the retirement decision in French (2005)) and fixed time and consumption costs earnings proxies and investigate how permanent earnings and wealth map empirically into median household saving outcomes. In order to limit the influence of extreme observations we follow a double stage least absolute deviations estimation (as proposed by Amemiya (1982)).The evidence is suggestive of the model's empirical adequacy in aspects where its implications differ from traditional life-cycle models.

As documented in Section 3.5.3[10], while households with low earnings have low net worth on average, households at the left tail of the net worth distribution have high earnings, high levels of education, and are younger than their counterparts in the right tail. In an application of the model we demonstrate how such pattern could emerge as a result of education choice early in life. In particular, we ask how initial endowments of wealth and abilities (productivities prior to obtaining education) relate to the optimality of investing in costly education, given that subsequent behaviour is as in the model. The analysis implies that, given wealth, investment is only optimal for workers of sufficiently high ability; given ability, it is only optimal for workers with sufficiently low wealth. These imply that wealth dynamics induced by costly education early in life provides an explanation for the observed pattern.

The rest of this paper is organised as follows. Section 3.2 presents the model and formulates the optimal consumption/saving and labour-market strategies for workers of given earning ability. Section 3.3 analyses how optimal policies differs across workers with different productivity. Section 3.4 extends the framework by introducing an education choice at the beginning of life and tracks the implications for early-age wealth dynamics. Section 3.5 presents some empirical results. Section 3.6 concludes.

[10]See Chapter 2 of this thesis for a similar result obtained using a different dataset.

3.2 Model

3.2.1 Environment

Time is continuous. An infinitely-lived[11] risk-averse worker has productivity w and wealth A. Labour is indivisible. The worker borrows and lends at a constant risk-free rate, r, and derives utility from consumption and leisure. Preferences are additively separable with flow utility from consumption, $u(c)$, such that $u'(.) > 0$, $u''(.) < 0$, $\lim_{c \to 0} u'(c) = \infty$ and flow utility, u_b, from leisure.

At any point in time the worker is in one of three labour-market states - non-participant, job-searcher or employed. The labour market is frictional as she becomes employed only upon accepting a job offer. When employed she earns w and decides whether to remain employed or quit into non-participation or unemployment. When non-participant she earns no income, enjoys u_b, and decides whether to remain non-participant or transit to unemployment. When searching she earns unemployment income, $b < w$, enjoys no leisure, samples job offers at Poisson rate λ, and decides to remain unemployed or transit to non-participation. For simplicity, assume she faces no layoff risk while employed.

An important assumption is that earned wage, w, is time-invariant[12]. While the analysis generalises to i.i.d. income uncertainty the focus on long-term earning ability is consistent with an emphasis on life-cycle rather than precautionary saving motives. Finally, given flat earning profiles, assume that the rate of time preference equals the real interest rate, implying a taste for flat consumption profiles as well.

Given this environment we now turn attention to workers' optimisation problem.

[11]The environment generalises to exogenous Poisson death process but this brings no extra insight.

[12]A constant growth rate in w generates convex regions in the value function for employment, inducing agents to pursue strategies that convexify their payoffs (for example, lotteries) - behaviour from which we abstract.

3.2.2 Optimisation problem

Let $V^{np}(A,w)$, $V^{js}(A,w)$ and $V^{e}(A,w)$ be the lifetime utilities from non-participation, job search and employment, and $V_A^j(.) \equiv \partial V^j(.)/\partial A$ in state j. Further, let $V^n(A,w) \equiv \max\{V^{np}(A,w), V^{js}(A,w)\}$ denote lifetime utility from non-employment. The values from working or not given $\{A,w\}$ are described by the system of Bellman equations (see Appendix 3.A.1)

$$rV^n(A,w) = \max \left\{ \begin{array}{r} \max_{c\geq 0}[u(c) + u_b + V_A^n(A,w)(rA - c)] \\ \\ \max_{c\geq 0}[u(c) + V_A^n(A,w)(rA + b - c) + \\ \lambda \max(V^e(A,w) - V^n(A,w), 0)] \end{array} \right\} \tag{3.1}$$

$$rV^e(A,w) = \max \left\{ \begin{array}{l} \max_{c\geq 0}[u(c) + V_A^e(A,w)(rA + w - c)] \\ \\ rV^n(A,w) \end{array} \right\} \tag{3.2}$$

These summarise the discussion of labour-market states and optimal behaviour as stated in Section 3.2.1. It is immediate that in any state optimal consumption requires that the marginal utility of consumption equals the marginal value of the asset[13]

$$u'(c^i(A,w)) = V_A^i(A,w), \forall i \in \{np, js, e\} \tag{3.3}$$

Consider an employed worker. Using the (3.3) and (3.2), her discounted lifetime utility is

$$rV^e(A,w) = u(c^e(A,w)) + u'(c^e(A,w))(rA + w - c^e(A,w)) \tag{3.4}$$

[13]Without ad-hoc constraints on borrowing a worker will not face liquidity constraints as long as $rA+b>0$. The equality counterpart to the latter identifies the natural borrowing limit.

Assuming differentiability, total differentiation of (3.4) with respect to time implies

$$u''(c^e(A,w))c^e_A(A,w)(rA+w-c^e(A,w))^2=0 \tag{3.5}$$

Following the same argument for non-participating workers, optimal consumption and wealth dynamics requires

$$u''(c^i(A,w))c^i_A(A,w)\dot{A}^2=0, i\in\{np,e\} \tag{3.6}$$

where $\dot{x}\equiv\partial x/\partial t$. Workers attain perfect consumption smoothing within each spell of employment or non-participation. There are two types of strategies consistent with (3.6). One possibility is that a worker consumes all her income at every instant ($c^e=rA+w$ or $c^{np}=rA$). If ever optimal, this is optimal forever - wealth remains constant and the worker solves the same problem in every future state. Alternatively, a worker could follow a flat consumption path over the duration of a spell and accumulate/decumulate assets until changing employment state. No other strategies could be optimal according to (3.6).

Inspection of (3.1) reveals that perfect consumption smoothing is not optimal for unemployed workers. We postpone the discussion of their optimal strategies until Section 3.2.4 and first characterise the behaviour of the employed.

3.2.3 Optimal behaviour of employed

Given (3.6) one potentially optimal strategy for an employed worker is to work forever and consume all income in perpetuity. Similarly a potentially optimal strategy for a non-participant is to never seek employment but enjoy leisure and consume asset income forever - that is retire permanently. Notice that in both cases a worker consumes her permanent income.

Other potentially optimal strategies involve asset accumulation/decumulation and switches between employment, non-participation and unemployment[14]. Quits into non-participation allow the worker to enjoy leisure immediately; if planning to become employed again, however, she will experience a spell of frictional unemployment. The tension is resolved by a third possible strategy - she works and saves in order to accumulate sufficient wealth to retire in future. Formally, consider three possibly optimal labor-market strategies for an employed worker given $\{A, w\}$:

Strategy 1: (Work forever) Work and consume permanent income, $c(A, w) = rA+w$. If ever optimal, this is optimal forever. The associated lifetime payoff is $\Pi^E(A, w) = u(rA + w)/r$.

Strategy 2: (Permanently retire) Never participate and consume permanent income, $c(A, w) = rA$. If ever optimal, this is optimal forever. The associated lifetime payoff is $\Pi^R(A, w) = (u(rA) + u_b)/r$.

Strategy 3: (Optimal retirement plan) Work, consume permanent income[15] $c(A, w) < rA + w$ and save. Once a threshold amount of wealth, $A^R(w)$, is accumulated, retire permanently and consume $c(A, w) = rA^R(w)$ forever after.

The rest of this section demonstrates that given any initial $\{A, w\}$ one and only one of these strategies is optimal. As a starting point, consider the optimal behaviour of a worker pursuing Strategy 3.

3.2.3.1 Characterisation of the optimal retirement plan

By construction strategies 1 and 2 are consistent with optimal consumption dynamics described by (3.6). Consider a worker pursuing strategy 3. While employed

[14] It is easy to see that an employed worker never quits into unemployment. Suppose she does. Then unemployment is preferred to both employment and non-participation. Then (3.1) implies that the value from unemployment is identical to the value of employment at wage $b < w$. Standard arguments imply that the value of employment is increasing in w which is a contradiction.

[15] The discussion of what permanent income is when a worker chooses the span of working life is postponed until Section 3.2.3.1.

she saves. Then (3.6) requires that consumption is constant for the duration of the employment spell. Let $c^*(w)$ denote its optimal level. Once the worker accumulates a stock of wealth $A^R(w)$ she retires and consumes $rA^R(w)$ forever. Optimal consumption smoothing and separability between consumption and leisure imply $c^*(w) = rA^R(w)$.

Let $\tau(A, A^R)$ be the optimal time to retirement given current wealth and wealth at retirement[16]. Solving the wealth accumulation equation $\dot{A} = rA + w - rA^R$ forward from 0 to $\tau(A, A^R)$ yields

$$\begin{aligned}
\int_0^\tau \frac{\dot{A}}{rA + w - rA^R} dt &= \int_0^\tau dt \\
\frac{1}{r} \ln(rA + w - rA^R)\Big|_0^\tau &= \tau \\
\tau(A, A^R) &= \frac{1}{r} \ln\left(\frac{w}{rA + w - rA^R}\right) \qquad (3.7)
\end{aligned}$$

Time to retirement decreases with wealth. In the limit as A approaches A^R, τ approaches zero. As rA approaches $w - rA^R$ time to retirement approaches infinity.

A worker earns labour income only in annuity until retirement. Given A^R and (3.7) the discounted value of future labour income is

$$\begin{aligned}
w \int_0^{\tau(A,A^R)} e^{-rt} dt &= \frac{w}{r}\left(1 - e^{-r\tau(A,A^R)}\right) \\
&= \frac{c^*(w) - rA}{r}
\end{aligned}$$

and after rearranging

$$\frac{c^*(w)}{r} = A + w \int_0^\tau e^{-rt} dt \qquad (3.8)$$

[16]Time to retirement depends on w but the argument is suppressed for brevity here as well as in the expressions for lifetime payoffs derived below.

(3.8) reveals that when planning for retirement workers consume permanent income. Given τ a worker behaves identically to a pure life-cycle consumer who faces the same time to retirement exogenously and is subject to an exogenous stream of earnings of the same present value.

The worker consumes c^* in perpetuity and after time $\tau(A, A^R)$ enjoys leisure in perpetuity. Therefore her lifetime payoff is

$$u(c^*) \int_0^\infty e^{-rt} dt + u_b \int_{\tau(A,A^R)}^\infty e^{-rt} dt$$

Integrating and substituting (3.7), the payoff from pursuing strategy 3 in terms of A^R is

$$\Pi(A, A^R) = \frac{u(rA^R)}{r} + \frac{rA + w - rA^R}{rw} u_b \tag{3.9}$$

Maximizing (3.9) with respect to A^R yields the familiar first-order condition for optimal consumption-leisure choice:

$$u'(c^*) = \frac{u_b}{w} \tag{3.10}$$

The worker chooses A^R (and, equivalently c^*) in order to equalise the marginal rate of substitution of consumption for leisure to her earned income. At the margin by delaying retirement for an instant she loses the opportunity to enjoy an immediate flow of leisure, u_b, but is able to gain an extra flow of income w which increases her utility from consumption by $wu'(c^*)$.

Consumption during saving for retirement is increasing in w/u_b. All else equal, workers with strong preference for leisure consume less and accumulate wealth faster so they are able to enjoy leisure sooner in the future. Similarly high-wage workers consume more during employment and the subsequent spell of retirement.

Using (3.9) the payoff from strategy 3 in terms of the model parameters is

$$\Pi^P(A, w) = \frac{u(c^*(w))}{r} + \frac{rA + w - c^*(w)}{rw} u_b \tag{3.11}$$

Let $\underline{A}^E(w) \equiv (c^* - w)/r$. If $A < \underline{A}^E(w)$ strategy 3 is not feasible as $\dot{A}_{|A<\underline{A}^E} = rA + w - c^* < 0$, i.e. workers decumulate wealth if consuming c^*. Let $\bar{A}^E(w) \equiv A^R(w)$.

3.2.3.2 Optimal consumption during employment

While easy to see that strategies 1-3 are consistent with (3.6), they need not be either optimal or the only optimal strategies. The next result demonstrates their optimality among a set of solutions characterised by the following property (later verified to be consistent with the prescribed optimal behaviour):

Property 1. *Whenever it is optimal for an employed worker to quit into non-employment, she retires permanently.*

Assuming Property 1 allows us to temporarily disregard strategies involving cycles between the three labour market states and implies the following result:

Proposition 1. *Optimal consumption of employed workers and retirement*

Conditional on Property 1

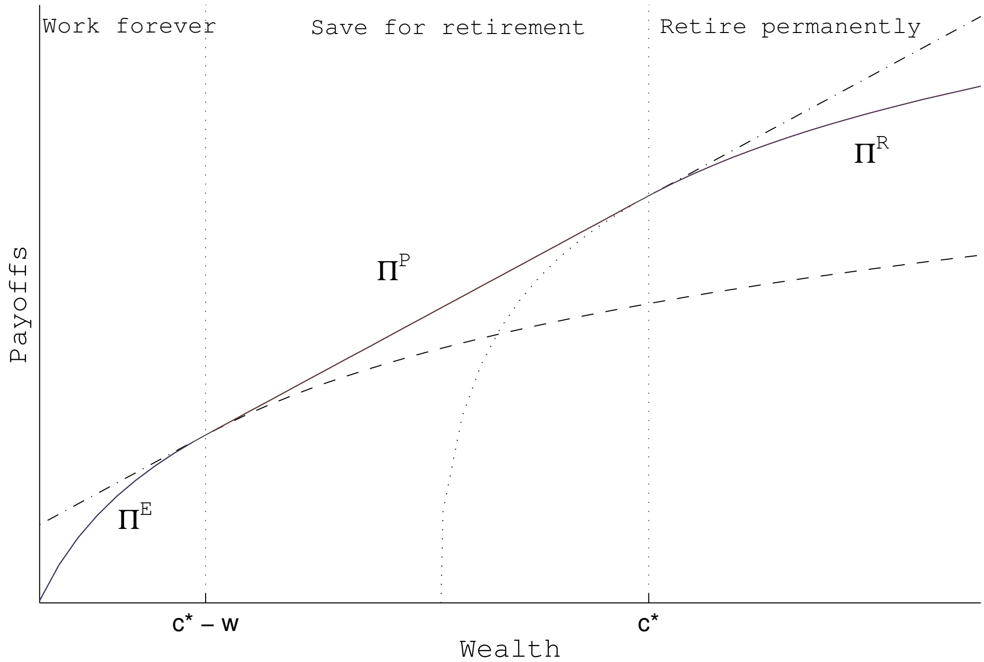
(i) If $A \le \underline{A}^E(w)$, an employed worker's optimal strategy is to work forever and consume permanent income. Her lifetime utility is $V(A, w) = \Pi^E(A, w)$.

(ii) If $A \in (\underline{A}^E(w), \bar{A}^E(w))$, an employed worker's optimal strategy is to save for retirement and consume permanent income. Her lifetime utility is $V(A, w) = \Pi^P(A, w)$.

(iii) If $A \ge \bar{A}^E(w)$, the worker's optimal strategy is to retire permanently and consume permanent income. Her lifetime utility is $V(A, w) = \Pi^R(A, w)$.

Proof. The result is proved directly. Suppose that $V(A, w)$ as stated above solves the workers' Bellman equations. Under this conjecture optimal consumption is as

Figure 3.2.1: Wealth and labor market strategies



stated. Given this optimal consumption the choice of $V(A,w)$ is consistent with the Bellman equations. Workings are demonstrated in Appendix 3.A.2. □

Sufficiently asset-poor workers have high value from earned income and do not plan to retire. The asset-rich receive large interest income which can finance consumption while they also enjoy leisure. Between the extreme cases workers save for retirement. The payoff from optimally saving for retirement is linear in wealth and identifies the convex envelope of the payoffs from working forever and retiring (Figure 3.2.1). When labour is supplied indivisibly at the cost of foregone leisure, saving for retirement convexifies the payoffs from the "pure" actions of work and retirement, allowing workers to achieve an optimal combination of leisure and consumption over the life cycle.

3.2.4 Optimal behaviour of non-employed and solution

The validity of Proposition 1 relies on Property 1 which is a conjecture about the behaviour of non-employed workers. To complete the solution we characterise the latter and show that Property 1 is indeed a feature of the model. To keep a clear focus on wealth accumulation, the analysis of non-employed workers' behaviour is delegated to Appendix 3.A.3, and only the main results and intuitions are listed here. Let $S(A, w) \equiv bu'(c^n(A, w)) + \lambda(V^e(A, w) - V^n(A, w))$. Since non-employed workers can freely change state between unemployment and non-participation, non-participation is preferred if and only if $S(A, w) < u_b$, and as demonstrated later $S(.)$ is strictly decreasing in A. Optimal behaviour of non-employed workers is then fully characterised by the following

Proposition 2. ***Optimal consumption of non-employed workers***
Conditional on Property 1 two wealth levels $\underline{A}^U < \bar{A}^U$ exist such that

(i) A non-employed worker with $A \in [-b/r, \underline{A}^U]$ seeks employment and dissaves. Optimal consumption and savings dynamics is described by the system

$$
\begin{aligned}
V_A^n(A) &= u'(c^n(A)) \\
c_A^n(A) &= \frac{\lambda(u'(c^n(A)) - u'(c^e(A))}{u''(c^n(A))(rA + b - c^n(A))}
\end{aligned}
$$

with terminal conditions $c^n(-b/r) = 0$ and $V^n(-b/r) = \frac{u(0)+\lambda V^e(-b/r)}{r+\lambda}$. This regime ends at $\underline{A}^U$ where $S(\underline{A}^U) = u_b$.

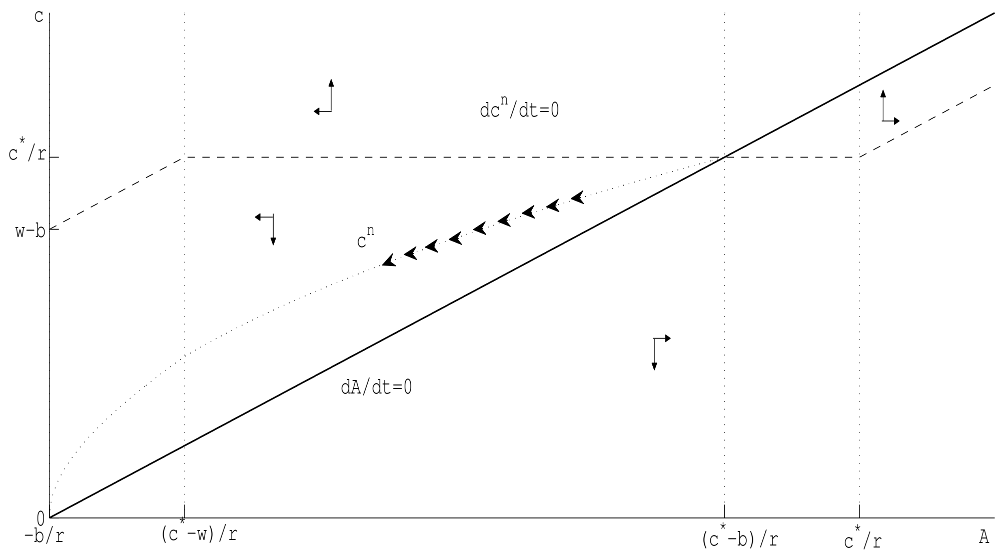
(ii) A non-employed worker with $A \in (\underline{A}^U, \bar{A}^U)$ is non-participant, consumes $c^n(\underline{A}^U)$, and dissaves. This regime ends at $\bar{A}^U$ where $\bar{A}^U = c^n(\underline{A}^U)/r$.

(iii) A non-employed worker with $A \geq \bar{A}^U$ retires permanently, consumes rA, and saves 0.

Proof. The validity of 2 follows by construction from the arguments in Appendix 3.A.3. □

The main arguments behind the result are as follows. Non-employed workers can freely change state between unemployment and non-participation. A worker sufficiently close to the natural borrowing limit relies on unemployment income and future employment prospects to prevent unsustainable debt accumulation,

Figure 3.2.2: Phase diagram



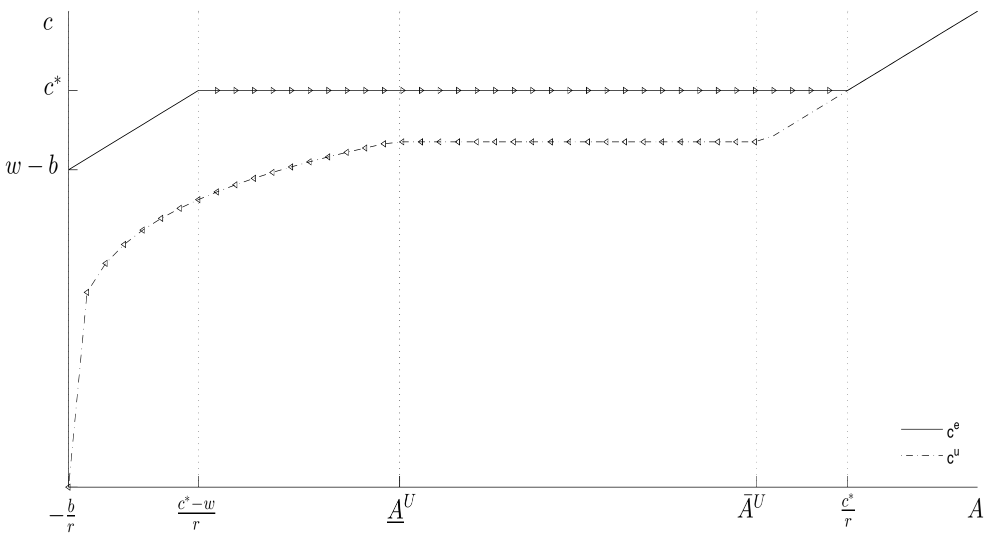
The phase portrait only applies for $A \in [-b/r, \underline{A}^U)$.

hence optimally avoids leisure. For wealth levels between the borrowing limit and $\underline{A}^U$, unemployment dominates non-participation. The tension between desire for smooth consumption and constraint on borrowing implies a unique saddle path solution characterised by a steady decline of consumption and dissaving (Figure 3.2.2). At wealth levels between $\underline{A}^U$ and $\bar{A}^U$ the worker postpones job-search, pursues leisure and dissaves until unemployment dominates again. At sufficiently high wealth, the perspective of an employment opportunity far into the future is discounted sufficiently so that the worker specialises in enjoying leisure and retires.

Figure 3.2.3 illustrates the dynamics of consumption and wealth for employed and non-employed workers over the wealth distribution's support. Unlike employed and non-participating workers, the unemployed do not attain perfect consumption smoothing since they cannot insure against the risk of not finding a job. As they approach the borrowing limit the probability of not finding employment before accumulating unsustainable debt increases. As a result they limit the pace of dissaving due to precautionary motives.

The next result completes the solution by affirming that the behaviour described by Propositions 1 and 2 implies Property 1.

Figure 3.2.3: Consumption



Theorem 1. *Bellman equations* (3.1) *and* (3.2) *imply:*

(i) The value function for employed workers, $V^e(A, w)$, is as stated in Proposition 1.

(ii) The value function for non-employed workers, $V^n(A, w)$, is as stated in Proposition 2.

(iii) Property 1 holds.

Proof. Appendix 3.A.4. □

The model implies optimal labour-market behaviour with natural life-cycle interpretation and endogenous retirement decision. As rigidities prevent workers from continuously adjusting their labour supply, they achieve optimal consumption-leisure tradeoffs over the life cycle. They choose to work early in life so that they have sufficient assets to retire later. While valuing leisure, they avoid temporary quits into non-participation as finding employment involves costly search. When choosing consumption they simultaneously plan retirement and retire where the marginal value of extra earned income just equals the value of leisure. Consumption/saving decisions and retirement plans depend on wealth endowment and long-term earning ability. In the limit, the most asset-poor workers of given earning ability do not save.

The analysis demonstrates that, qualitatively, endogenous retirement provides an explanation for the well documented empirical fact of low wealth being an "absorbing state" (Hubbard et al., 1994b), while not deviating fundamentally from the main life-cycle abstractions. The next section turns attention to the model's implications for the relationship between long-term earnings and saving behaviour.

3.3 Earnings heterogeneity

Within this framework, earnings have two opposing effects on retirement plans. The income effect of higher earnings, as a wealth transfer, encourages pursuit of leisure (earlier retirement). However, high earnings imply high opportunity cost of leisure (by retiring early a worker forgoes more income and consumption) and the substitution effect results in higher consumption and postponement of retirement. To see how the two effects interact it is more convenient to work with the total effect on consumption. Implicitly differentiating $c^*(w)$ in (3.10) with respect to w and rearranging

$$\frac{\partial c^*(w)}{\partial w}\frac{w}{c^*(w)} = -\frac{u'(c^*)}{u''(c^*)c^*} \tag{3.12}$$

which links the elasticity of consumption with respect to earnings depends on the elasticity of intertemporal substitution/risk aversion properties of preferences. For future references, let $R(.)$ denote the coefficient of relative risk aversion. Recall that $\bar{A}^E(w) = c^*(w)/r$ identifies the upper bound of the wealth distribution for employed workers of ability w, or equivalently their wealth at retirement. It is strictly increasing in w with the rate of increase depending on the risk-aversion properties of the utility function. $\underline{A}^E(w) = (c^*(w) - w)/r$ identifies the lower bound below which workers do not save. The relationship between $\underline{A}^E$ and w could be increasing, decreasing or non-monotonic, depending on preference parameters. The range of wealth levels where workers save, $\bar{A}^E - \underline{A}^E = w/r$, is proportional to earnings.

Consider a set of workers facing the same time to retirement, $\bar{\tau}$. By (3.7)

$$e^{-r\bar{\tau}} = \frac{rA - c^*(w) + w}{w}$$

and after rearranging

$$A|_{\tau=\bar{\tau}} = (1 - e^{-r\bar{\tau}})\underline{A}^E + e^{-r\bar{\tau}}\bar{A}^E$$

The locus of $\{A, w\}$ pairs where workers face the same retirement horizon is a weighted mean of the $\underline{A}^E$ and $\bar{A}^E$ loci. The weight equals the discount factor for time $\bar{\tau}$ into the future. Figure 3.3.1 illustrates the latter by mapping the plane into times to retirement for different values of the risk aversion coefficient, all assuming isoelastic utility, $u(c) = c^{1-\rho}/(1-\rho)$[17]. The two effects cancel out exactly when preferences for consumption are described by log-utility so that all workers with zero wealth face the same time to retirement. As a result scaling up earnings scales permanent income proportionally and homothetic saving policies obtain. If instead $R(c) > 1$, time to retirement relates to earnings non-monotonically but starts decreasing after a threshold. Above the threshold, higher earnings translate to less than proportionate increases in permanent income as workers plan to retire sooner. The opposite occurs when relative risk aversion is below unity.

More precisely, recall the accumulation equation for employed savers

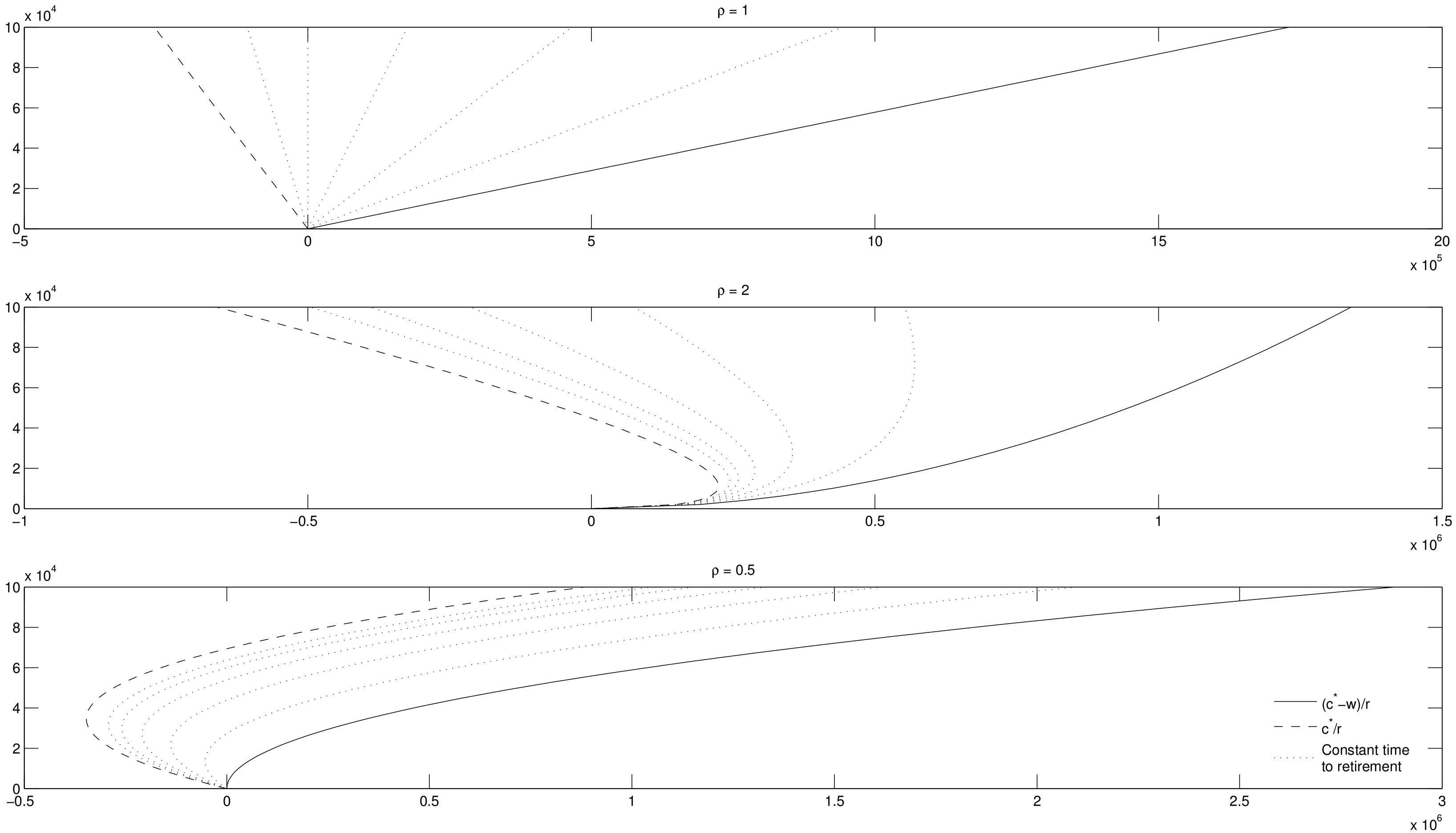
$$\dot{A} = rA + w - c^*(w) \tag{3.13}$$

Consider a set of workers of different w optimally choosing to save the same amount $\bar{\dot{A}}$. Rearranging (3.13), these workers have stock of wealth

$$A|_{\dot{A}=\bar{\dot{A}}} = \frac{\bar{\dot{A}}}{r} + \underline{A}^E(w)$$

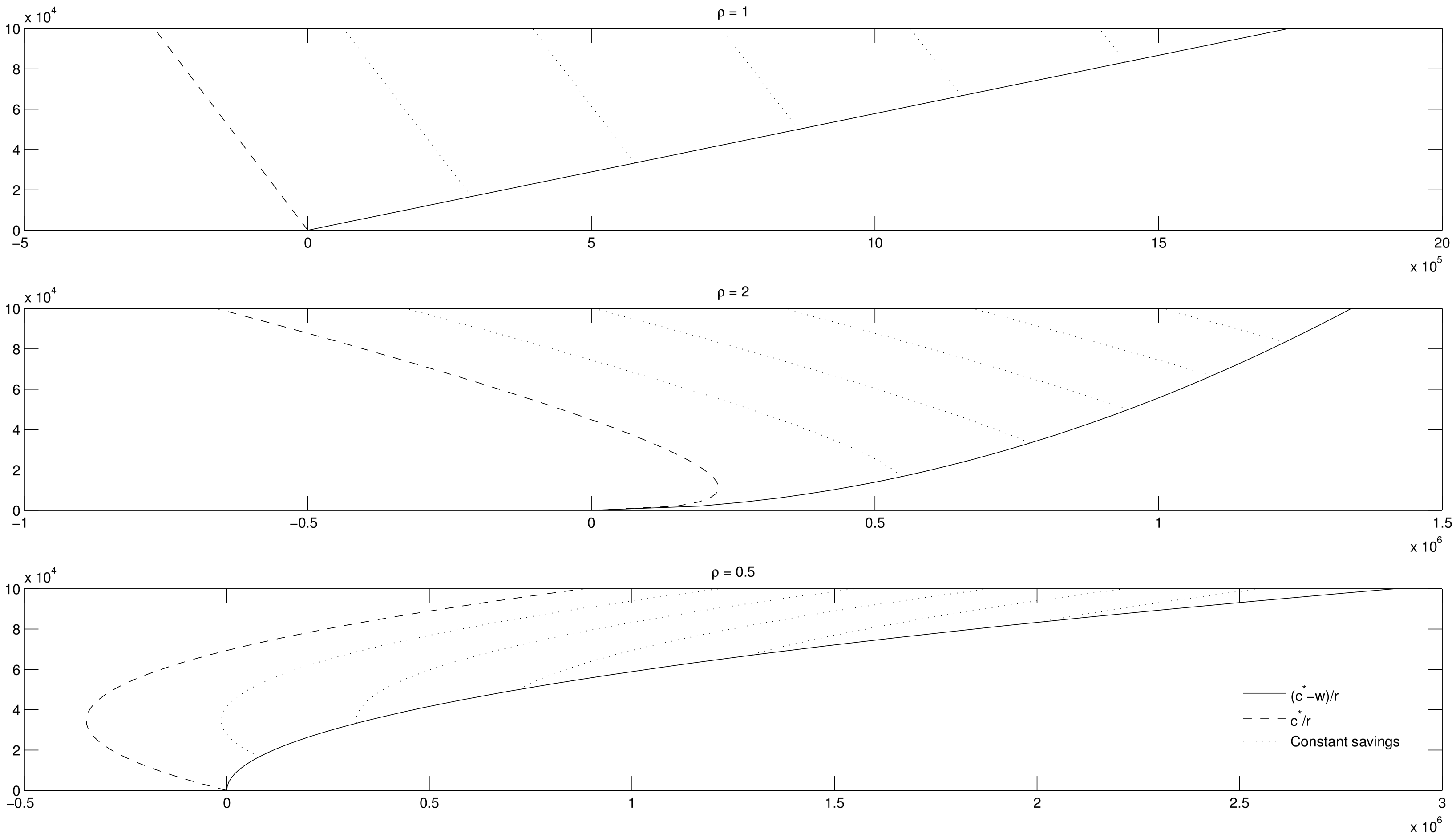
[17] Figures 3.3.2 and 3.3.1 are plotted for $r = 0.05$ and u_b set to imply (somewhat arbitrarily) that a worker with zero wealth earning 60000 per year retires in exactly 40 years. ρ stands for the coefficient of relative risk aversion.

Figure 3.3.1: Time to retirement



The dotted curves track pairs of wealth and earnings implying constant time to retirement. The leftmost line identifies 50 years to retirement, while the rightmost identifies 10 years to retirement.

Figure 3.3.2: Savings



The dotted curves track pairs of wealth and earnings implying constant savings.

This identifies an "iso-saving" set for $\bar{\dot{A}}$. It is geometrically represented in the (A, w)-plane as a horizontal translation of the $\underline{A}^E$ locus by distance $\bar{\dot{A}}/r$. Figure 3.3.2 plots some iso-saving curves in the case of isoelastic utility for three benchmark values of the risk aversion coefficient. Consider "active" saving rates out of earned income, $(w - c^*(w))/w$. Differentiating with respect to w and rearranging

$$\frac{\partial[(w - c^*)/w]}{\partial w} = \frac{c^*(w)}{w^2}\left[1 - \frac{1}{R(c^*)}\right]$$

Active saving rates increase with earnings when $R > 1$ and are independent when $R = 1$. Similarly for "total" saving rates out of earned income, $\dot{A}/w$,

$$\frac{\partial(\dot{A}/w)}{\partial w} = \frac{r}{w^2}\left[\left(1 - \frac{1}{R(c^*)}\right)A^R - A\right]$$

Total saving rates depend on risk aversion and on the distance between a worker's current wealth and their target retirement wealth.

To summarize the discussion, under endogenous retirement optimal saving behaviour depends on retirement plans. The relationship between saving rates and earnings depends on how workers substitute consumption for leisure. If the income effect on leisure dominates, high earners retire earlier hence save more than proportionately in comparison to low earners; the opposite occurs if the substitution effect dominates. The model presents a highly tractable description of wealth dynamics in the presence of heterogeneity in permanent earnings and wealth, implying the following. Asset-poor workers accumulate wealth slowly and remain in the left tail of the distribution for long time; the asset-rich accumulate wealth quickly until retirement; the relationship between total saving and wealth is linear. High earners retire with more assets and are found over a broader support of wealth; they save more or less than proportionately in comparison to low earners depending on the interaction between income and substitution effects of earnings. The empirical relevance of these predictions is discussed in Section 3.5, while Section 3.4 turns attention to the optimal choice of education and its implications for the distribution of wealth and earnings.

3.4 Education choice

Investment in education allows individuals to expand their long-term earning prospects early in life but involves significant costs that could largely affect subsequent life-cycle outcomes. This section employs the above framework to explore the optimality of educational investment when individuals also plan retirement efficiently.

More concretely, suppose that at the beginning of time a worker with ability w_0 and wealth A_0 can choose to invest in education or not[18]. For simplicity the choice is binary (interpreted as pursuit of a higher degree) and acquisition is instantaneous. Education costs a fixed fee, k, and increases productivity. In particular, suppose that a worker with ability w_0 earns w_0 without education and $w_0(1+e)$ with education. For tractability assume that following the decision the worker becomes employed immediately, hence precautionary motives do not affect the choice[19].

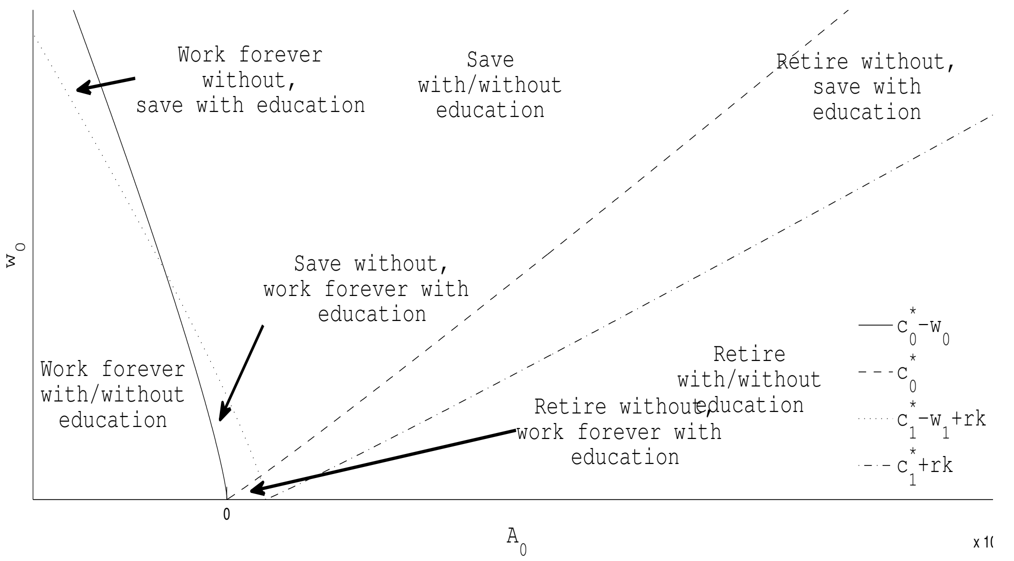
Let $V_0(A_0, w_0) \equiv V^e(A_0, w_0)$, $V_1(A_0, w_0) \equiv V^e(A_0 - k, w_0(1+e))$, $c_0(A_0, w_0) \equiv c^e(A_0, w_0)$, $c_1(A_0, w_0) \equiv c^e(A_0 - k, w_0(1+e))$, $\tau_0(A_0, w_0)$ and $\tau_1(A_0, w_0)$ denote the lifetime payoffs, optimal consumptions and times to retirement to a worker with $\{w_0, A_0\}$ if investing (state 1) or not (state 0) in education. Given Proposition 1 and (3.7)

$$\begin{aligned} rV_0(A_0, w_0) &= u(c_0(A_0, w_0)) + e^{-r\tau_0(A_0,w_0)}u_b \\ rV_1(A_0, w_0) &= u(c_1(A_0, w_0)) + e^{-r\tau_1(A_0,w_0)}u_b \end{aligned}$$

[18] As demonstrated later, postponing educational choice has no option value in this setting. Even if the choice was available later in life, workers would still invest when young.

[19] This is a strong assumption but simplifies the analysis considerably, as it allows us to abstract from the behaviour of unemployed workers near the natural borrowing limit and its inductive implications over the whole support of wealth. Thus the analysis emphasises life-cycle motives in isolation.

Figure 3.4.1: Optimal labor market strategies conditional on education



As the worker becomes employed immediately a degree is pursued if and only if $V_1(A_0, w_0) \geq V_0(A_0, w_0)$. The decision maps ability and wealth endowment into a productivity level and initial wealth for the consumption problem. Notice that the endowment $\{w_0, A_0\}$ pins down the optimal labour-market strategies conditional on investing in education or not (see Figure 3.4.1).

A worker is indifferent to education if and only if

$$V^e(A_0, w_0) = V^e(A_0 - k, w_0(1 + e)) \tag{3.14}$$

or equivalently

$$u(c_1(A_0, w_0)) - u(c_0(A_0, w_0)) = (e^{\tau_0(A_0,w_0)} - e^{\tau_1(A_0,w_0)})e^{-r}u_b$$

that is if the gain in consumption (in utility terms) just equals the loss of utility due to possibly delaying retirement. The set of abilities and wealth endowments where indifference obtains is henceforth referred to as the "education efficiency frontier". Appendix 3.B constructs the frontier by analysing the decision of a worker based on her ex-ante (without education) and ex-post (with education) labour market strategies but the main intuition and results are discussed here.

At sufficiently low wealth, individuals work forever irrespective of educational attainment. They only trade consumption possibilities and are always just willing to substitute a unit of wealth for r extra units earnings. The benefit from education is a perpetual stream of extra earnings. At the margin this is compared in present value terms to the one off cost of education incurred at the time of the decision. Workers of sufficiently high ability enjoy higher extra earnings because education complements ability. This implies a threshold wealth-independent ability where education becomes optimal and gives rise to a flat region of the education efficiency frontier.

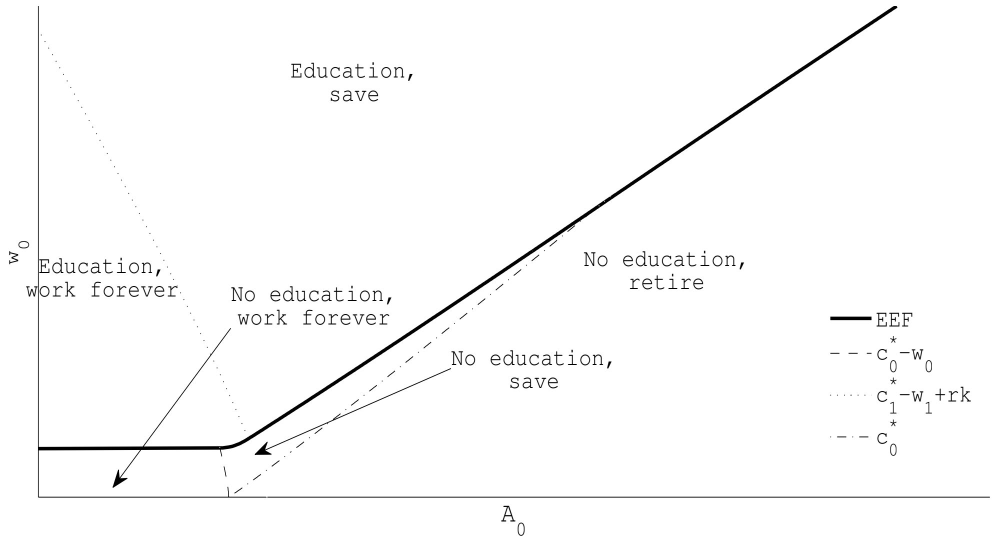
Workers with larger wealth endowment save for retirement. They enjoy boosted earnings only in annuity. The higher their wealth endowment, the sooner they retire - higher earnings translate to lower increases in permanent income and consumption. They willingly substitute a unit of wealth for $r \times (w_0)/(rA^R - rA) > r$ extra earnings. The threshold ability where indifference obtains therefore increases with the wealth endowment. In the limit, individuals sufficiently wealthy to retire immediately both with and without education, have no value from obtaining a degree - no one obtains education to retire immediately.

These arguments imply[20] that the education efficiency frontier is a continuous, (weakly) upward-sloping schedule in the (A_0, w_0)-plane (Figure 3.4.2). Given any wealth endowment, investment is optimal only for workers of sufficiently high ability; given ability investment is optimal only for sufficiently wealth-poor workers. This further demonstrates that given the stationary nature of the decision problem the assumption of educational investment being available only early in life is without loss of generality - as they accumulate wealth during employment workers of any ability lose value by postponing investment. Education is pursued early in life as when wealth is low, retirement is far in the future, and higher earnings result in larger increases in permanent income.

As a result high-earning workers start employment with less assets than low

[20]See Appendix 3.B for a more thorough treatment.

Figure 3.4.2: Education efficiency frontier and optimal strategy



earners with identical pre-education wealth endowment, who find educational investment unprofitable. To the extent that education is sufficiently expensive, it is a reason for significant wealth decumulation early in life, for those who invest. This implies that the left tail of the marginal wealth distribution is populated not by the income poor, but by young educated high earners, as documented empirically in Section 3.5.3.

3.5 Empirical analysis of wealth dynamics

While highly stylized the model combines the main abstraction of life-cycle theory with the view of endogenous retirement, and suggests a tractable description of wealth dynamics emphasising life-cycle motives and persistent differences in earnings. This section presents some descriptive evidence for the evolution of wealth of households with different earnings from the 1996 and 2001 panels of the Survey of Income and Program Participation. The analysis focuses on the household, rather than the individual as unit for analysis and selects a sample of middle-aged households that are unlikely to be in transitory stages of their life. We start by describing the estimation sample, and proceed to investigate how wealth and long-term earning ability map empirically into median saving outcomes. Finally, we document the age profile of the conditional on observed earnings net worth distribution, and demonstrate that a calibrated version of the model is able to account for the documented facts. We conclude that, while stylized, the model presents a data-consistent description of households' life-cyclical wealth dynamics.

3.5.1 Data and summary statistics

The SIPP is a US household-based survey running since 1984. It comes in a series of 3-to-4-year panels, each featuring a different nationally representative sample of households. A household is interviewed once every four months and upon the interview data on demographic, income and employment-related (among others) outcomes are collected for each household member retrospectively at monthly level over the latest four months. Comprehensive data on the stocks of assets and liabilities at household level is collected once a year during the interviews taking place in the third, sixth, ninth and twelfth waves[21](The 1996 panel consists of 12 waves, while the 2001 panel only consists of 9.). The first round of interviews for the 1996 (2001) panel took place between August and November 1995 (February 2001 and June 2001). Wealth data was first collected between March and June 1996 (October 2001 and January 2002). At this stage 27120 (22099) households were interviewed. Attrition resulted in only 22438 (20026) of these being followed up until the last round of interviews.

Given the focus on long-term wealth dynamics, using the full panel sample for the analysis is inappropriate. First, about 30% of the households are non-family and some family households contain no spouse. Further, the composition of some family households changes across the waves. Associating reported net worth with the stock of accumulated life-cycle resources relevant for decisions in these cases is flawed. Second, availability of wealth data at only annual frequency

[21]See Czajka et al. (2003) for a comprehensive comparison of the wealth information in the SIPP and the other two major household surveys - PSID and SCF. While the SCF is widely considered to contain the most comprehensive and reliable data on wealth, it is a cross sectional survey. On the other hand, while the PSID has very long panel, in the few releases where wealth data is collected it excludes assets in pension accounts which we perceive as fundamental for life-cycle considerations.

Table 3.5.1: Descriptive statistics, SIPP and sample for estimation

	1996		**2001**	
	SIPP	Sample	SIPP	Sample
Demographics				
Age, head	49.817	43.239	50.340	43.914
	(16.947)	(7.317)	(16.850)	(7.451)
White, head	.843	.895	.833	.881
	(.364)	(.306)	(.372)	(.323)
Retired, any	.292	0	.287	0
	(.455)	(0)	(.453)	(0)
Enrolled, any	.053	0	.047	0
	(.224)	(0)	(.211)	(0)
Household				
Family	.699	1	.693	1
	(.458)	(0)	(.461)	(0)
Married	.548	1	.544	1
	(.498)	(0)	(.498)	(0)
Female, head	.459	.297	.476	.346
	(.498)	(.457)	(.499)	(.476)
Education, head				
< *High school*	.199	.105	.167	.089
	(.399)	(.307)	(.373)	(.285)
High school	.292	.286	.291	.265
	(.455)	(.452)	(.454)	(.441)
< *Degree*	.280	.293	.289	.300
	(.449)	(.455)	(.453)	(.458)
Degree +	.229	.317	.254	.346
	(.420)	(.465)	(.435)	(.476)
Wealth/Earnings				
Earnings	33676.94	59682.43	36217.97	64201.19
	(39559.54)	(43952.64)	(40952.39)	(46521.17)
Income	41571.06	65805.93	44097.34	67532.04
	(38521.82)	(47156.46)	(39742.95)	(48086.64)
Net worth	117452.7	122067.2	144091.5	160955.0
	(507787.7)	(290673.4)	(560610.7)	(284907.2)
Observations	27120	4946	22099	4780

suggests that the analysis should also take place at annual frequency. Constructing consistent annual earnings from monthly observations requires that households are interviewed continuously during the length of the panel. Third, households where the head or spouse is retired or in full-time education are likely to be in transitory phases of their life with respect to saving decisions. In view of these, the sample is restricted to married family households, with the same household head and spouse over the panel duration, interviewed in every wave of the survey, both the household head and the spouse are between 30 and 59 years old and neither is enrolled in full time education or retired[22]. The subsample used in what follows thus contains 4946 households from the 1996 panel (4780 households from the 2001 panel), observed annually for four (three) consecutive years. Table 3.5.1 presents some descriptive statistics for the full SIPP sample (from the cross-section of observations at the time of the third wave interview, when asset data was first collected) and the sample for estimation[23]. Unsurprisingly (e.g. see Table 1 in Alan et al. (2015)), the two samples represent distinct populations. On average, households in the estimation sample are younger, more highly educated, wealthier, and earn more than those in the full SIPP sample. Earnings represent the overwhelming share of their total income.

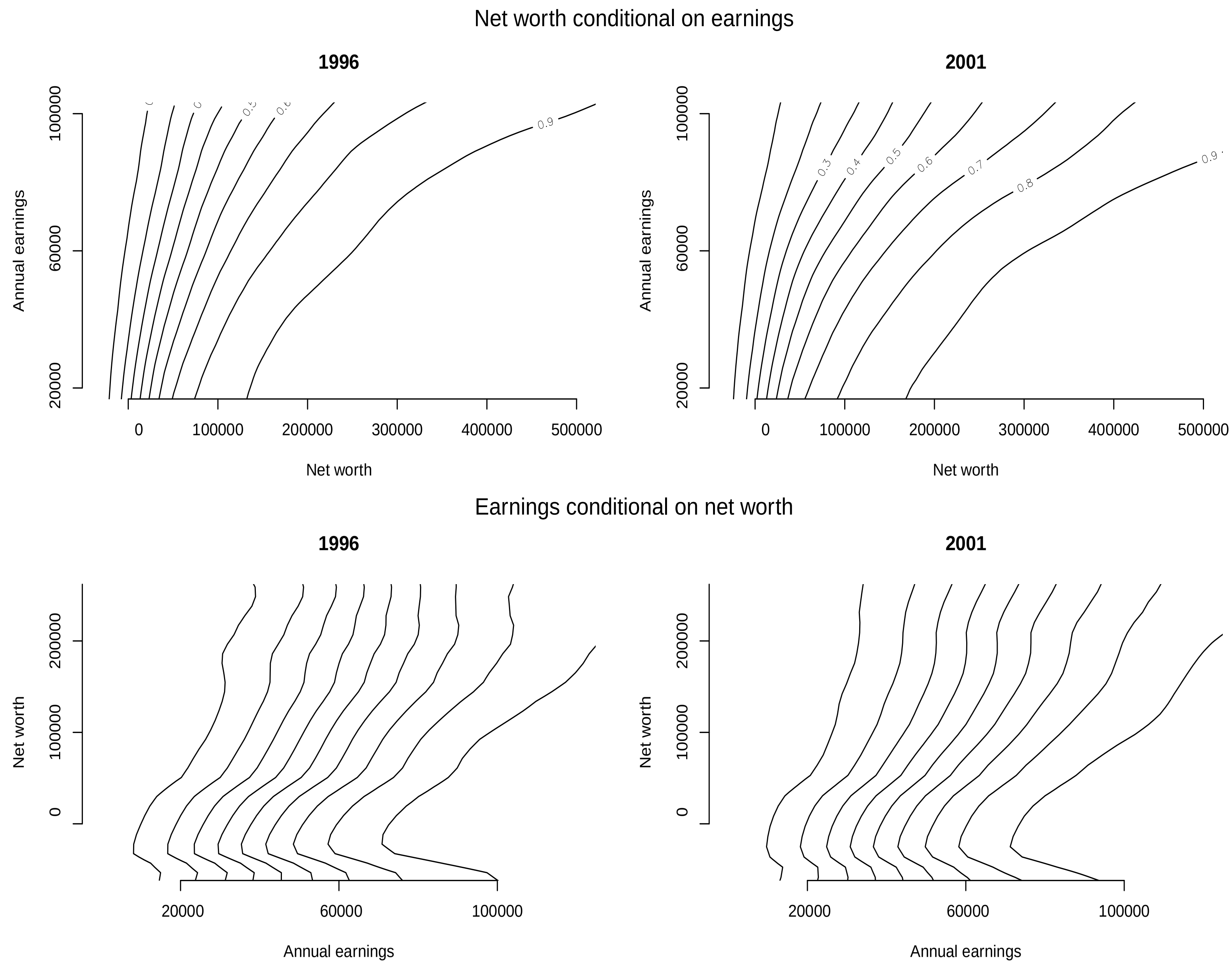
Next, turn attention to the cross sectional distribution of net worth and earnings. To summarize two aspects of these, we estimate non-parametrically the distribution of net worth reported in the first wave of the survey conditional on average annual earnings over the sample period, as well as the distribution of annual earnings conditional on net worth[24]. Figure 3.5.1 presents contour plots for the estimated conditional CDFs with the curves on the graph representing the quantiles of the conditional distribution functions.

[22]This sampling decision is consistent with prior studies (e.g. see Dynan et al. (2004) and Alan et al. (2015)).

[23]Henceforth, all income and wealth related quantities are reported in real terms after being deflated using CPI.

[24]The estimation uses the method of Li and Racine (2008), and employs Epanechnikov kernels and data-driven bandwidth selection. The procedure favours bandwidths close to 10000 real dollars for both variables but the results are robust under moderate deviations from this. The reasons for using such specification are briefly discussed below.

Figure 3.5.1: Distributions of net worth and earnings



The top panels of the figure plot the quantiles of net worth conditional on earnings. Net worth is highly disperse and its dispersion increases with earnings. Up to the middle 80 percent, the distribution for high-earners dominates the one for low-earners. Top quantiles expand significantly while the 10th percentile is nearly constant over the distribution of earnings.

The bottom panels plot the quantiles of earnings conditional on net worth. Households around zero net worth earn least on average, in particular, less than households in net debt[25]. To explore this more closely, Table 3.5.2 reports the average earnings, age, and educational attainment of household heads and spouses for households grouped in seven categories increasing in net worth. The results for earnings are consistent with the pattern identified by the bottom panel of Figure 3.5.1. Households within the bottom 25 percent of the sample are similar in terms of age, and average age increases across net worth quartiles. Average educational attainment tracks closely the pattern in earnings. Households in the left tail of the distribution are not only high earners relative to those at zero net worth, but also more highly educated. The 5 percent of households in the left tail have educational attainment comparable to the second quartile, yet they are somewhat younger and earn less. Households around zero net worth have the worst educational record and are predictably the lowest earners. The pattern implies that high earning households incur costly expenditures early in life unlike low-earners of similar wealth. As demonstrated in Section 3.4, costly education provides an explanation for this observation.

3.5.2 Dynamics of wealth over the distribution of earnings

This section describes the empirical relationship between observed household saving outcomes on one hand, and permanent earnings, net worth and age, on the

[25] It is well known that kernel-based methods suffer from boundary bias. As observations get more disperse near the limits of the marginal distributions' support distant observations become disproportionately influential and bias the estimator towards the mean. An Epanechnikov kernel does not eliminate the bias but has compact support and hence uses no information from distant observations. Hence, given the shape of the quantiles, the non-monotonicity in the relationship is, if anything, more strongly pronounced than suggested by the graph.

Table 3.5.2: Summary statistics by net worth

Net worth	< -12561	$[12561, -529)$	$[-529, 716)$	$[716, 17233)$	$[17233, 76850)$	$[76850, 198050)$	≥ 198050
Net worth quantiles	$[0, 0.05)$	$[0.05, 0.1)$	$[0.1, 0.12)$	$[0.12, 0.25)$	$[0.25, 0.5)$	$[0.5, 0.75)$	$[0.75, 1]$
Earnings	47248	40128	28017	39540	51907	67833	96769
Age	41.19	40.88	41.20	41.33	42.36	44.45	46.31
Education, head							
< *High school*	0.0962	0.175	0.271	0.217	0.104	0.0394	0.0234
High school	0.347	0.308	0.375	0.337	0.322	0.266	0.136
< *Degree*	0.318	0.321	0.260	0.282	0.346	0.334	0.226
Degree +	0.238	0.196	0.0938	0.165	0.228	0.361	0.615
Education, spouse							
< *High school*	0.130	0.142	0.281	0.243	0.120	0.0511	0.0209
High school	0.377	0.354	0.427	0.346	0.342	0.308	0.168
< *Degree*	0.243	0.346	0.104	0.270	0.305	0.319	0.257
Degree +	0.251	0.158	0.188	0.141	0.233	0.321	0.554

Households are grouped into 7 net worth categories. Two equally large groups span the bottom 10 percent of the sample. The third group spans 2 percent of the sample close to zero net worth. The fourth group complements the sample to the 25th percentile. The top three groups are equally large and each corresponds to 25 percent of the sample. Age is average age of household head and spouse.

other. The purpose of the exercise is twofold. First, it seeks to assess the adequacy of the model's implications for wealth dynamics, summarized in Figure 3.3.2, in reduced form, when permanent earnings are appropriately accounted for. To the extent that the model is seen as a description of data-generating process, reduced-form estimation of (3.13) is informative of the models broader adequacy. Further, wealth dynamics in a wide class of life-cycle models is driven by a mapping from permanent earnings, net worth, and age into saving outcomes. Reduced-form estimation of such mapping's empirical counterpart is informative about the aspects in which different frameworks are able to account for the data. We exploit the panel dimension of the data to construct proxies for permanent earnings and explicit measures of household saving based on observed changes of net worth across periods.

Given the panel's short duration, explicit total saving of individual households is observed only over a period of three or four years. It is plausible that households with given permanent earnings will have a much more variable distribution of annual as opposed to long-term saving outcomes. Given this and the likelihood of extreme observations we choose to model the median of the conditional distribution. Formally, we are interested in estimating

$$Median(A_{i,t+1} - A_{i,t}) = F(A_{i,t}, \hat{w}_i, age_{i,t}) \tag{3.15}$$

where $\hat{w}$ is appropriately defined measure of permanent earnings[26]. We confine attention to linear in parameters form for $F(.)$ but allow for the form of the linear dependence to differ across specifications (see below). Two key issues are apparent immediately. First, observed net worth at time t enters on both sides. As measurement error is likely in net worth records, direct estimation of (3.15)

[26]Recall that in the model saving is age-independent conditional on wealth. It should be noted that this is, first, inconsistent with standard versions of the life-cycle model and, second, obtains as retirement plans rather than age determine expected lifetime income. Including age[27] as a regressor in (3.15) allows for testing the relevance of this implication; furthermore, if age effects are actually important, accounting for them is desirable in modelling the relationship of interest.

will identify severely downwards biased relationship (in addition to the standard measurement error problems for quantile regression). Second, permanent earnings are unobserved and have to be predicted at a first stage in a way consistent so that the second stage is estimated by a median regression.

To tackle measurement error in net worth we use lagged, rather than contemporaneous observations of net worth as regressors. In particular, the specifications for the 1996 panel use $A_{i,t-2}$ instead of $A_{i,t}$ as regressors, and the results from the 2001 panel use $A_{i,t-1}$ instead[28]. It should be noted that lagged net worth is not treated as in instrument for current net worth. More precisely, consider two lags of the discrete time counterpart of (3.13)

$$\begin{aligned} A_{i,t+1} - A_{i,t} &= \frac{e^r - 1}{r}(rA_{i,t} + w_i - c(w_i)) \\ A_{i,t} - A_{i,t-1} &= \frac{e^r - 1}{r}(rA_{i,t-1} + w_i - c(w_i)) \end{aligned}$$

and substitute the solution for $A_{i,t}$ from the second equation into the first

$$A_{i,t+1} - A_{i,t} = \frac{e^r(e^r - 1)}{r}(rA_{i,t-1} + w_i - c(w_i))$$

The latter suggests that the relationship between $A_{i,t+1} - A_{i,t}$ and $rA_{i,t-1}$ conditional on permanent earnings is just scaling up the coefficients of the original relationship by proportion $e^r \approx (1 + r)$. As long as the measurement error in wealth is of the form $(\mathrm{me}_i + \mathrm{me}_{i,t})$ and $\mathrm{me}_{i,t}$ is not serially correlated, this eliminates the bias due to $A_{i,t}$ entering both sides of the equation.

In order to construct proxies for permanent earnings we follow the literature and instrument current earnings in a first stage including the instruments as well as the other second-stage regressors. We consider as instruments lagged labour income and education. Education is likely a suitable instrument as it does not vary considerably in the population of middle aged households and is strongly

[28]Recall, that the data contains four net worth observations for the 1996 panel and three for the 2001 panel.

related to permanent labour income. In particular, we use the interaction of the education levels (grouped into four categories) of the household head and spouse. Lagged earnings are likely correlated with the permanent component of earnings, however, they also include lagged transitory shocks. It is intuitive that the longer the lag the more convincing the instrument is. Given the length of the SIPP panel the longest lag we can use is three years in the 1996 data and two years in the 2001 data (that is instrument earnings at $t+1$ with earnings at $t-2$ in the 1996 panel and $t-1$ in the 2001 panel) but this is likely sufficient for household-level data[29]. As an empirical justification for the suitability of the instruments, note that Dynan et al. (2004) find permanent income proxies based on measures of education, lagged earnings or consumption to imply very similar conclusions about the relationship between savings and permanent income in three different US surveys. Once the first-stage equation is estimated the permanent earnings proxies are constructed as the fitted-values at constant age.

To limit the influence of extreme earnings observations, the first-stage is also fitted by LAD. The whole two-stage procedure, originally proposed by Amemiya (1982), gives rise to the double-stage least absolute deviations (DSLAD) estimator. Following his suggestion, we redefine the dependent variable for the second stage as $p\Delta A_t + (1-p)\widehat{\Delta A_t}$, where $\Delta A_t \equiv A_{t+1} - A_t$, $\widehat{\Delta A_t}$ is the fitted value from a median regression of ΔA_t on all explanatory variables from the first stage, and $p \in [0,1]$ is a value chosen by the econometrician[30]. Consistently with Amemiya (1982) second-stage estimates are reported for two different values of p - 0.2 and 0.5 - but we also implement robustness checks using other values, including $p = 1$. In all results that follow we report bootstrapped standard errors based on 1000 pairwize replications.

[29]For example, Blundell et al. (2008) find no evidence of a moving average component in excess of MA(1) in income growth data from the PSID.

[30]This reformulation of the dependent variable was suggested by Amemiya (1982) as a generalisation of a 2SLS property. While no formal procedure for choosing p was proposed, it was suggested that p in the range of 0.2 to 0.5 leads to an estimator that significantly outperforms $p = 1$ in terms of efficiency. Kim and Muller (2004) later demonstrated that the choice of p is inconsequential in median regressions except in very small samples.

3.5.2.1 Estimation results

Given the above discussion we start by estimating two separate first-stage LAD regressions - one to construct our proxies and another to obtain $\widehat{\Delta A_t}$. We estimate them separately to insure against potential correlation between the error terms. The results are reported in Table 3.C.1. Unsurprisingly, the instruments are highly relevant and there is a significant (although small in magnitude) age effect. We construct the permanent earnings proxy as the prediction from the first equation at constant age (the mean age in each sample) and split households into four quartiles based on this measure. To allow for non-linear relationship we introduce permanent earnings in the second stage through a quartile indicator. Note that while the proxy likely suffers from measurement error, this has no consequences for the estimation as long as permanent-earnings quartiles are identified consistently.

As a baseline specification for the conditional median we estimate a second stage of observed wealth changes on permanent earnings quartile indicator, lagged net worth (included linearly) and age categories. Table 3.5.3 (3.5.4) reports the estimated coefficients for $p = 0.2$ ($p = 0.5$). Columns (1) to (5) ((6) to (10)) are estimated in the 1996 (2001) sample. The first specification (columns (1) and (6)) introduces permanent earnings through the proxy linearly, while the other columns allow for non-linearity by using quartile indicators. The second specification (columns (3) and (8)) allows for non-linear relationship with respect to net worth (including a square term) - the estimated coefficient is close to and insignificantly different from zero. Columns (4) and (9) further allow for an interaction of net worth and age, and yield estimates close to and insignificantly different from zero. The last specification (columns (5) and (10)) allows for interaction of net worth with earnings quartiles and yields coefficients insignificant from zero[31].

As a further illustration of the results Figure 3.5.2 plots the predicted median saving for the four earnings categories (obtained from columns (2) and (6) of Tables 3.5.3 3.5.4) against the median value of the proxy variable in each category,

[31]For all specifications we include all second-stage regressors in the first stage. The respective first-stage coefficients are also found insignificant. Results are not reported for brevity.

Table 3.5.3: Second stage, DSLAD, $p = 0.2$

	1996					2001				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(Intercept)	79.853	8083.005	6508.685	8528.913	7197.487	6458.820	4318.103	3998.990	5648.287	2197.275
	(5453.868)	(5193.896)	(5315.960)	(5585.134)	(5509.939)	(6041.422)	(6324.514)	(6176.372)	(6606.533)	(6165.757)
NW_{t-1}	0.022***	0.026***	0.029***	0.040	0.021***	0.026***	0.032***	0.029***	0.056*	0.020·
	(0.004)	(0.004)	(0.004)	(0.025)	(0.006)	(0.004)	(0.004)	(0.005)	(0.026)	(0.011)
PE	0.183***					0.238***				
	(0.012)					(0.011)				
(25,50]		2206.355***	2130.518***	2203.090***	2055.511***		4600.603***	4645.316***	4606.590***	4012.096***
		(327.077)	(318.851)	(333.625)	(403.831)		(391.492)	(405.155)	(408.546)	(501.473)
(50,75]		5728.661***	5631.567***	5709.911***	5102.821***		8923.701***	9132.997***	8965.278***	9356.404***
		(416.074)	(429.798)	(455.473)	(580.847)		(546.149)	(570.297)	(583.121)	(920.485)
(75,100]		12479.740***	12257.491***	12433.661***	12452.838***		16134.717***	16344.759***	16351.213***	15698.244***
		(676.300)	(660.578)	(682.962)	(1057.155)		(1059.636)	(1011.142)	(1097.004)	(1260.122)
Age	-162.090	-264.150	-182.073	-296.260	-211.316	-435.180	-12.516	-6.892	-104.298	84.788
	(262.788)	(250.247)	(255.938)	(275.777)	(265.924)	(285.298)	(303.428)	(294.855)	(326.001)	(296.119)
Age^2	1.555	2.956	1.882	3.444	2.272	3.850	-1.318	-1.249	0.049	-2.281
	(3.106)	(2.972)	(3.028)	(3.351)	(3.152)	(3.263)	(3.540)	(3.428)	(3.920)	(3.461)
NW_{t-1}^2			-0.000					0.000		
			(0.000)					(0.000)		
$NW_{t-1} \times Age$				-0.000					-0.001	
				(0.001)					(0.001)	
$NW_{t-1} \times$ (25,50]					0.006					0.021
					(0.009)					(0.015)
$NW_{t-1} \times$ (50,75]					0.012					0.004
					(0.009)					(0.014)
$NW_{t-1} \times$ (75,100]					0.004					0.012
					(0.010)					(0.013)
Num. obs.	4946	4946	4946	4946	4946	4780	4780	4780	4780	4780

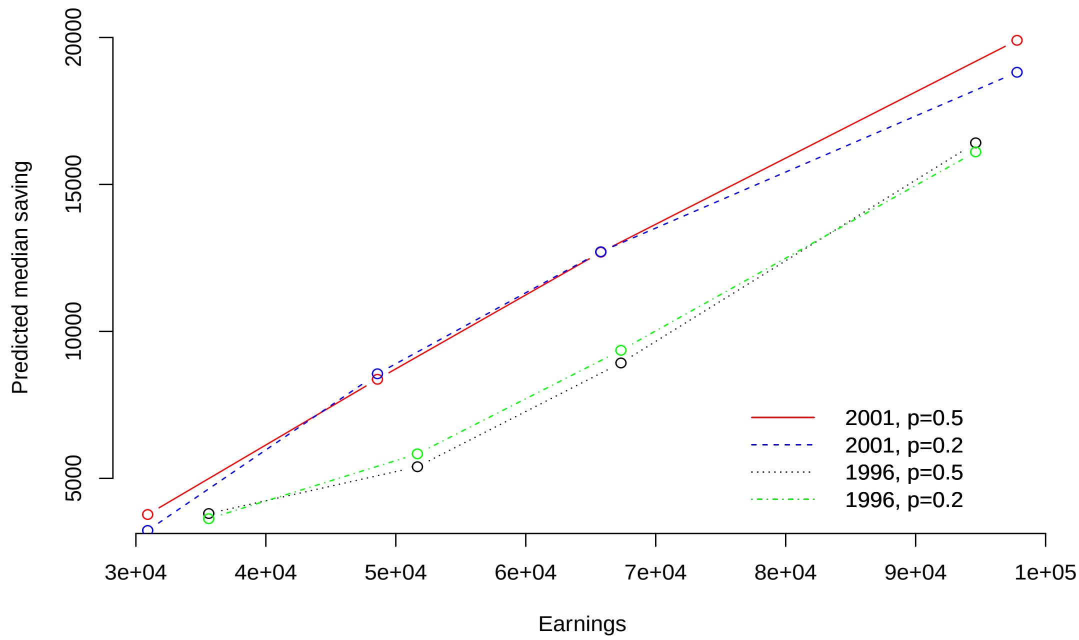
Bootstrap standard errors in parentheses. Controls omitted. NW: Net worth, PE: permanent earnings proxy.

Table 3.5.4: Second stage, DSLAD, $p = 0.5$

	1996					2001				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(Intercept)	-4316.045	3370.254	849.604	4436.982	-2800.177	-5425.998	338.977	-483.047	828.013	2913.194
	(9410.888)	(10583.272)	(10808.451)	(11022.162)	(11508.888)	(11012.092)	(11297.280)	(11145.651)	(11593.610)	(11022.863)
NW_{t-1}	0.021**	0.025**	0.035***	0.048	0.021·	0.027**	0.033**	0.028*	0.044	0.011
	(0.008)	(0.009)	(0.010)	(0.053)	(0.012)	(0.009)	(0.010)	(0.013)	(0.052)	(0.020)
PE	0.189***					0.236***				
	(0.023)					(0.029)				
(25,50]		1592.829*	1380.043·	1591.785*	1349.212		5330.609***	5288.238***	5195.955***	3226.520***
		(688.902)	(707.393)	(708.934)	(843.606)		(828.015)	(873.727)	(820.585)	(859.348)
(50,75]		5123.292***	4910.054***	5052.301***	4049.439**		9482.225***	9858.373***	9454.068***	10453.954***
		(846.812)	(862.645)	(902.518)	(1245.206)		(1294.843)	(1311.377)	(1265.005)	(1849.279)
(75,100]		12612.822***	12076.502***	12643.825***	13584.305***		15589.445***	16369.478***	15935.964***	15878.239***
		(1644.112)	(1531.269)	(1617.001)	(2261.109)		(2286.390)	(2154.795)	(2227.410)	(3036.421)
Age	32.235	-35.554	96.673	-111.280	272.692	79.476	97.273	141.641	61.679	4.577
	(452.162)	(506.304)	(516.757)	(543.025)	(555.280)	(529.083)	(532.290)	(525.117)	(567.303)	(524.174)
Age^2	-0.742	0.299	-1.517	1.454	-3.431	-1.684	-2.041	-2.544	-1.459	-1.110
	(5.419)	(5.940)	(6.076)	(6.575)	(6.560)	(6.195)	(6.135)	(6.050)	(6.848)	(6.107)
NW^2_{t-1}			-0.000					0.000		
			(0.000)					(0.000)		
$NW_{t-1} \times Age$				-0.000					-0.000	
				(0.001)					(0.001)	
$NW_{t-1} \times$(25,50]					0.010					0.052*
					(0.020)					(0.025)
$NW_{t-1} \times$(50,75]					0.023					0.010
					(0.022)					(0.028)
$NW_{t-1} \times$(75,100]					-0.001					0.020
					(0.020)					(0.024)
Num. obs.	4946	4946	4946	4946	4946	4780	4780	4780	4780	4780

Bootstrap standard errors in parentheses. Controls omitted. NW: Net worth, PE: permanent earnings proxy.

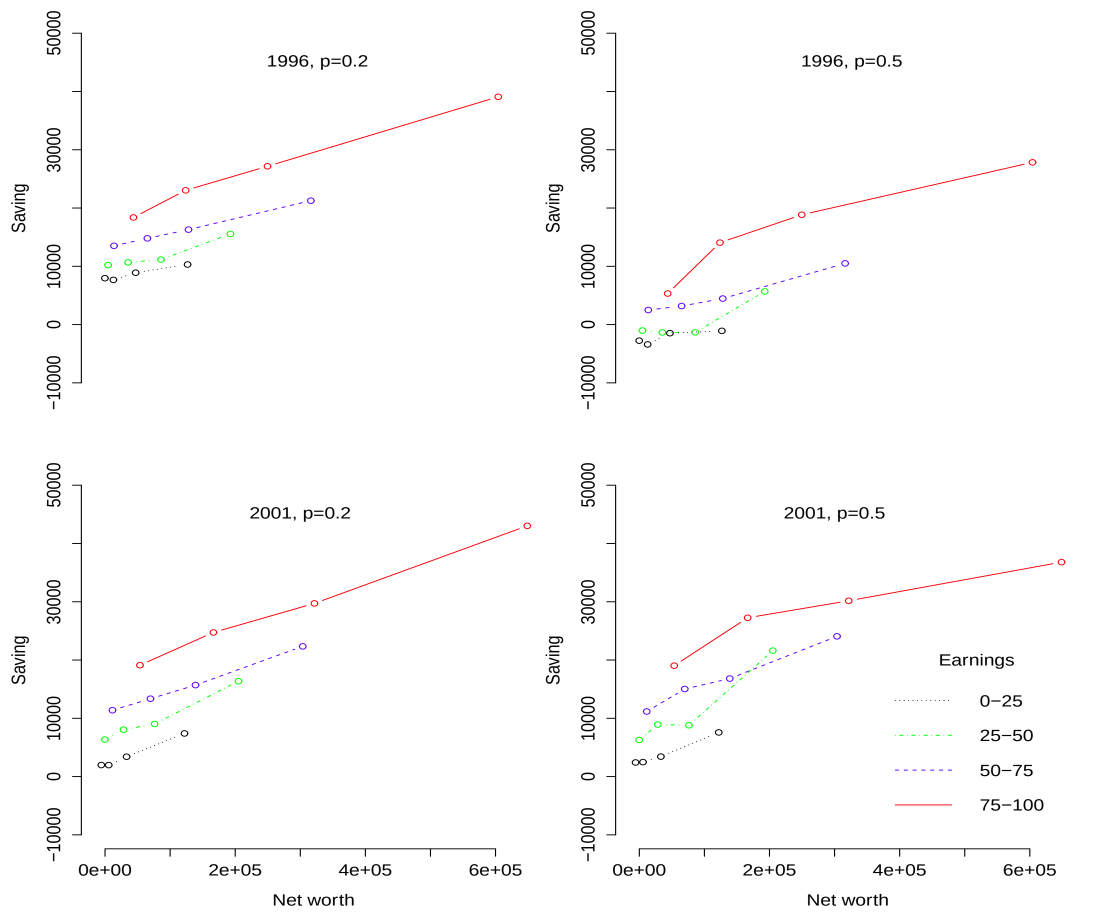
Figure 3.5.2: Earnings categories coefficients



holding age and net worth constant. The profiles are increasing and approximately linear. Interpreted directly through the model, such earnings profiles are consistent with approximately constant saving rates out of permanent earnings.

To allow for further flexibility of the functional form we split the households of each permanent earning category into four quartiles of observed lagged net worth and estimate a second-stage median regression of total savings on the interaction of the identified permanent earnings and net worth quartiles, as well as age indicators. This relaxes the assumptions of linear wealth effect and separability between wealth and earnings on the right-hand side, hence allowing for much more general functional forms. The estimated second-stage coefficients (which identify median savings for the respective group of households) are reported in Table 3.C.2 and plotted in Figure 3.5.3 against the median lagged net worth of the respective groups of households. Inspection of results suggests that the extra flexibility allowed brings little extra insight. The wealth profiles are similar across permanent-earnings quartiles and approximately linear, reasserting the previous results.

Figure 3.5.3: Median saving by permanent earnings/wealth category, conditional



In summary, median total saving increases substantially and significantly with permanent earnings. Saving is positively associated with net worth and the relationship is adequately approximated as linear; the coefficient of proportionality is within the magnitude of the real interest rate; and is approximately constant across age and permanent earnings groups. Households with high permanent earnings move quickly to the right of the net worth distribution and over time accumulate significantly larger stocks of wealth as compared to their less-earning counterparts. Conditional on permanent earnings and net worth, the differences in saving of households of different age are statistically insignificant. These results are strongly suggestive of the adequacy of (3.13) as describing household-level wealth dynamics. In particular, the model's implications for saving behaviour account for the observed saving outcomes in reduced form. Interestingly, the apparent age-invariance of saving and the increasing relationship with net worth (conditional on age and earnings) are not only supportive of the model, but also at odds with the implications of canonical life-cycle models[32]. While purely descriptive, the results are suggestive for the model's adequacy in explaining households' median saving outcomes and the evolution of wealth distributions in the presence of permanent earnings differentials and heterogeneity in initial wealth endowments.

3.5.3 The distribution of wealth and earnings

We next turn attention to the age profile of the conditional-on-earnings net worth distribution. We split the sample households into three age groups - $[30, 40)$, $[40, 50)$, and $[50, 60)$ - and estimate non-parametrically the net worth distribution conditional on earnings. Figure 3.5.4 presents the associated contour maps. The basic pattern identified in Figure 3.5.1 is still present and a few additional observations emerge. First, the distribution of net worth gets increasingly dispersed with age. Second, the higher dispersion is driven by lengthening of the right tail while the 10th percentile is relatively stable. Third, the increase in dispersion is more pronounced at high earnings. Fourth, the 10th conditional percentile is stable for low-earnings households and only increases with age at high earnings.

This pattern is qualitatively consistent with the model's predictions (Section 3.3) as long as preference parameters imply positive relationship between savings and earnings over the empirical support of the distribution. In particular, the "absorbing" property of low wealth is consistent with the data and the level of wealth where savings average zero is negatively related to earnings. To give some concrete substance to this claim we conduct the following exercise. First, we obtain the 10th, 50th and 90th percentile of wealth for households aged $[40, 50)$ and $[50, 60)$ in the four earnings quartiles. Then, using the observed earnings and net worth of households aged $[30, 40)$, we use the model to simulate their net worth

[32]In standard life-cycle models a wealthy household saves less than a wealth poor household of the same permanent income and age. Further, the mapping from net worth and permanent income onto saving is age-dependent.

Figure 3.5.4: Distribution of net worth conditional on earnings by age, data

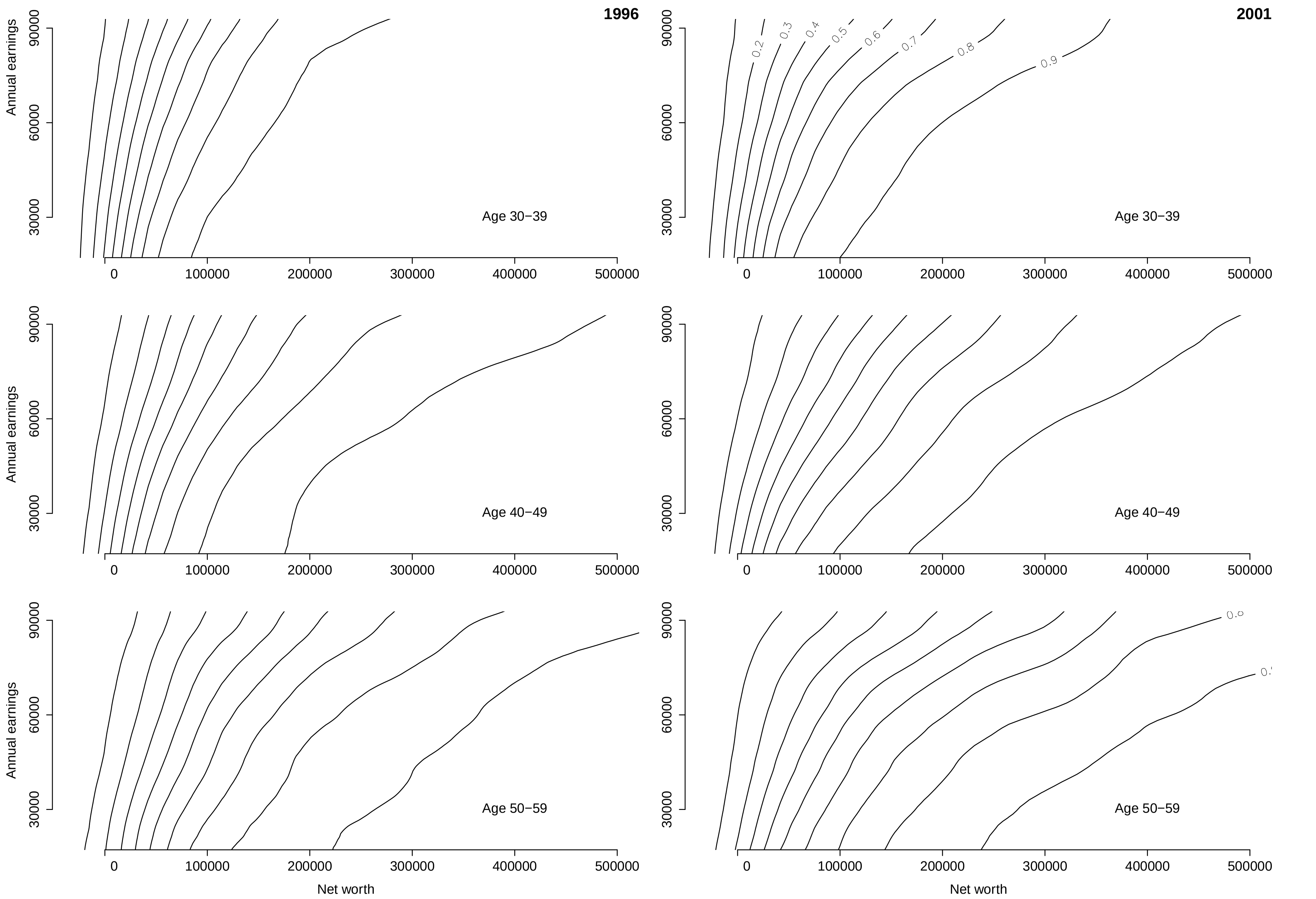


Figure 3.5.5: Distribution of net worth conditional on earnings by age, model, $\eta = 1$, $u_b = 1.029$

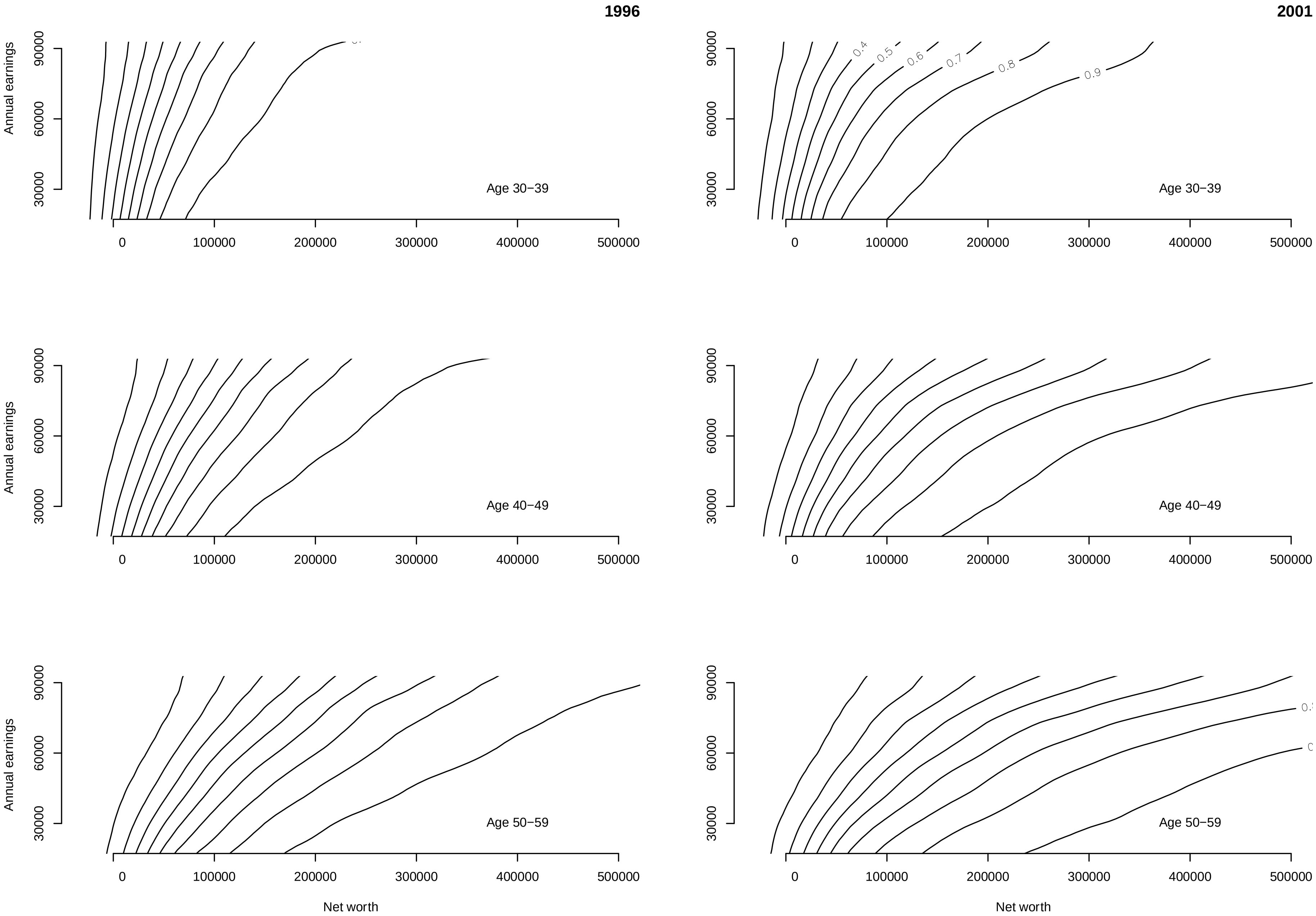


Table 3.5.5: Observed and fitted wealth quantiles

	Data E1	Data E2	Data E3	Data E4	Model E1	Model E2	Model E3	Model E4
2001								
Age 40-49								
Q10	-6841	2827	15247	49527	-7537	490	12817	41715
Q25	2167	20528	51319	126931	6075	19476	41980	104865
Q50	22477	67590	113063	232817	18979	51134	89908	223971
Q75	76266	158469	206203	410325	66971	117125	180800	434227
Q90	168246	260598	394196	703753	154617	224718	299780	686177
Age 50-59								
Q10	-4889	2625	10582	91449	-5422	15411	38913	88305
Q25	6078	33789	66748	178388	14313	43135	83978	187528
Q50	42096	91965	157163	330204	36894	90350	156483	371581
Q75	118628	202918	330131	596526	109036	190570	289049	685259
Q90	233016	372957	468283	979331	241446	350457	467381	1062598
1996								
Age 40-49								
Q10	-3182	912	5436	25902	-724	931	15424	30440
Q25	1947	15808	36059	62030	8815	23007	35035	69435
Q50	23211	46145	76251	139241	23302	47526	67208	130543
Q75	69531	97289	150125	275295	62076	96854	132254	223013
Q90	151776	174502	248018	512001	103201	164046	218501	448159
Age 50-59								
Q10	-1843	11732	25291	42452	7443	14096	42565	74693
Q25	11660	37942	58883	93251	21370	48390	70532	130653
Q50	49798	90335	128139	202956	42722	86969	120161	224257
Q75	110797	157556	238351	400601	101073	158677	217278	365676
Q90	231892	301952	398360	666746	160582	259395	345046	713299

10 and 20 years into the future for different values of u_b. The simulations, as in the model, assume that earnings remain constant over time and that preferences are given by a log-utility function. We choose the value ($u_b = 1.029$) that minimizes the sum of absolute deviations between simulated and observed median net-worth levels of the four earnings quartiles in the 2001 sample by equally weighting each of the resulting 8 targets. Table 3.5.5 reports the observed and simulated quantiles of net worth by earnings and age group, using the same parameterisation in the 1996 sample. Further, as before we estimate non-parametrically the conditional distribution of net worth on the simulated data and present the associated contour plots in Figure 3.5.5.

Before interpreting the results a discussion is in order. First, this procedure makes no difference between permanent and transitory earnings components - observed earnings are treated as a proxy for long term earnings. Second, it abstracts from possible heterogeneity in preference parameters in the population. Third, the model is extremely parsimonious - the above exercise amounts to fitting 8 targets by varying a single parameter. Fourth, as reflected in Table 3.5.5 the procedure systematically underestimates the wealth accumulation over the first ten years, with an offsetting error over the next ten-year period. Subject to this, the simulated wealth profiles fit their empirical counterparts closely and the simulated conditional quantiles in Figure 3.5.5 are very close to their observed counterparts in Figure 3.5.4. It should be noted that by considering quantiles rather than moments we make no attempt to describe extreme wealth observations - an issue that has attracted considerable attention in the literature and that dominates measures of inequality based on statistics such as the Gini index. There are, however, good reasons to think that extreme wealth accumulation is not best understood as a life-cyclical phenomenon[33]. The results suggest that wealth dynamics could be adequately attributed to pure life-cycle motives when retirement is efficient; wealth inequality reflects earnings inequality to a significant extent; while highly stylized, the theoretical description of wealth dynamics in Section 3.3 provides an adequate approximation to the data.

3.6 Conclusion

This paper studies the labour-market decisions of risk-averse workers in a world of frictional labour markets. The existence of labour-market rigidities imply that the tradeoff between consumption and leisure is resolved over the life cycle, with individuals working while young and saving in order to retire later. The model has highly tractable implications for wealth dynamics and educational investment which emphasise pure life-cycle motives, labour-market decisions (including optimal retirement), persistent earnings differentials and heterogeneity in initial

[33]See Carroll (2000) for a discussion.

wealth endowments. Using data from the Survey of Income and Program Participation we document how the household-level distribution of wealth and earnings evolves with age and how permanent earnings and wealth associate with median saving outcomes. The theoretical framework provides clear interpretation for the observed regularities. The evidence is suggestive that life-cycle motives and permanent-earnings heterogeneity are accountable for a significant part of the between-household differences in wealth.

Appendix

3.A Proofs and derivations

3.A.1 Derivation of Bellman equations

Let Δ be a discrete interval of time and $V(A,w) \equiv \max\{V_e(A,w), V^n(A,w)\}$.

Consider an employed worker with $\{A,w\}$. Her discrete-time Bellman equation is (suppressing the second argument for brevity)

$$V^e(A) = \max_c \{u(c)\Delta + \frac{1}{1+r\Delta} V(A')\}$$

where

$$A' = (1+r\Delta)A + w\Delta - c\Delta$$

Multiplying both sides by $(1+r\Delta)$ and subtracting $V(A,w)$

$$rV^e(A) = \max_c \{u(c) + \frac{V(A') - V(A)}{\Delta} + ru(c)\Delta\}$$

In the limit as Δ approaches 0 this implies (3.2).

Consider a worker with $\{A,w\}$ such that $V(A,w) = V^n(A,w)$. Her discrete-time Bellman equation is

$$V^n(A) = \max \left\{ \begin{array}{l} \max_{c\geq 0}[u(c)\Delta + u_b\Delta + \frac{1}{1+r\Delta}V^n(A'_{np})] \\ \\ \max_{c\geq 0}[u(c)\Delta + \frac{1}{1+r\Delta}(\lambda\Delta\max(V^n(A'_{js}), V^e(A'_{js}))+ \\ \\ \qquad\qquad\qquad\qquad +(1-\lambda\Delta)V^n(A'_{js}))] \end{array} \right\}$$

where the maximization is conditional on

$$\begin{aligned} A'_{np} &= (1+r\Delta)A - c\Delta \\ A'_{js} &= (1+r\Delta)A + b\Delta - c\Delta \end{aligned} \tag{3.16}$$

Multiplying both sides by $(1+r\Delta)$ and subtracting $V(A,w)$

$$rV^n(A) = \max \left\{ \begin{array}{l} \max_{c\geq 0}[u(c) + u_b + \frac{V^n(A'_{np})-V^n(A)}{\Delta}] \\ \\ \max_{c\geq 0}[u(c) + \frac{V^n(A'_{js})-V^n(A)}{\Delta} + \lambda\max(V^e(A'_{js}) - V^n(A'_{js}), 0)] \end{array} \right\}$$

In the limit as Δ approaches 0 this implies (3.1).

3.A.2 Proof of Proposition 1

Under property 1 the Bellman equation for an employed worker can be stated as

$$rV^e(A,w) = \max \left\{ \begin{array}{l} \max_{c\geq 0}[u(c) + \frac{\partial V^e(A,w)}{\partial A}(rA + w - c)] \\ u(rA) + u_b \end{array} \right.$$

Under the conjecture

$$rV(A,w) = \begin{cases} u(rA+w) & \text{if } A \leq \underline{A}^E \\ u(c^*) + (rA+w-c^*)u'(c^*) & \text{if } A \in (\underline{A}^E, \bar{A}^E) \\ u(rA) + u_b & \text{if } A \geq \bar{A}^E \end{cases}$$

Suppose $A \leq \underline{A}^E$. The Bellman equation of the worker is then

$$rV^e(A,w) = \max\{\max_c [u(c) + u'(rA+w)(rA+w-c)], u(rA)+u_b\}$$

Maximization implies that optimal consumption is $c^e = rA + w$. Substituting above

$$rV^e(A,w) = \max \left\{ \begin{array}{c} u(rA+w) \\ u(rA)+u_b \end{array} \right\}$$

The payoff from permanent retirement is not defined for negative wealth, hence working forever dominates if $\underline{A}^E \leq 0$ or if $A < 0 < \underline{A}^E$. Suppose $0 < A < \underline{A}^E$. Then both payoff functions increase in A and the payoff from retirement increases faster. At $A = \underline{A}^E$, $u(rA+w) = u(c^*)$ and $u(rA) + u_b = u(c^* - w) + u_b = u(c^* - w) + wu'(c^*)$. As $u(.)$ is convex, $u(c^*) - u(c^* - w) - wu'(c^*) > 0$. Therefore, working forever dominates and $rV^e(A,w) = u(rA+w)$ as conjectured.

Consider $A \in (\underline{A}^E, \bar{A}^E)$. The Bellman equation under the conjecture is

$$rV^e(A,w) = \max \left\{ \begin{array}{cc} \max_c & [u(c) + u'(c^*)(rA+w-c)] \\ u(rA)+u_b & \end{array} \right\}$$

Optimal consumption is $c = c^*$. Substituting above

$$rV^e(A,w) = \max \left\{ \begin{array}{c} u(c^*) + u'(c^*)(rA+w-c^*) \\ u(rA)+u_b \end{array} \right\}$$

At the upper bound of the subset $u(c^*) + u'(c^*)(r\bar{A}^E + w - c^*) = u(c^*) + u_b =$

$u((r\bar{A}^E)+u_b$, while the derivatives of the payoffs with respect to A satisfy $ru'(c^*) < ru'(rA)$ within the interior of the set. Hence saving for retirement dominates and $rV^e(A,w) = u(c^*) + u'(c^*)(rA + wic^*)$ as conjectured.

Finally, suppose $A \geq \bar{A}^E$. Under the conjecture the Bellman equation is

$$rV^e(A,w) = \max \left\{ \begin{array}{ll} \max_c & [u(c) + u'(rA)(rA + w - c)] \\ u(rA) + u_b & \end{array} \right\}$$

Optimal consumption is $c = rA$. Substituting above

$$rV(A,w) = \max \left\{ \begin{array}{c} u(rA) + u'(rA)w \\ u(rA) + u_b \end{array} \right\}$$

As $rA > c^*$ not working is strictly preferred and the conjecture is verified.

This completes the proof of Proposition 1.

3.A.3 Characterisation of V^n and solution

Consider a non-employed worker with $\{A, w\}$. As she freely changes state between non-participation and job-search, (3.1) and property 1 suggest that the worker optimally seeks employment when $bV^n_A(A,w) + \lambda(V^e(A,w) - V^n(A,w)) \geq u_b$ and does not participate otherwise. Let

$$S(A,w) \equiv bu'(c^n(A,w)) + \lambda(V^e(A,w) - V^n(A,w)) \tag{3.17}$$

Consider a non-employed worker at the natural borrowing limit, $A = -b/r$. It is immediate that the only behaviour not violating the limit is to search for a job and consume nothing. The Inada condition implies that $S(-b/r, w)$ tends to infinity. Let $\underline{A}^U > -b/r$ be an amount of wealth such that $S(A,w) > u_b, \forall A \in [-b/r, \underline{A}^U)$, i.e. job search dominates for all amounts of wealth below $\underline{A}^U$. Then for any $A \in [-b/r, \underline{A}^U)$ the Bellman equation (3.1) reduces to

$$(r+\lambda)V^n(A) = u(c^n(A)) + u'(c^n(A))(rA + b - c^n(A)) + \lambda V^e(A) \tag{3.18}$$

Total differentiation of (3.18) with respect to time yields

$$[u''(c^n(A))\dot{c^n}(A) + \lambda(V^e_A(A) - V^n_A(A))]\dot{A} = 0 \tag{3.19}$$

One possibility for optimal wealth and consumption dynamics is to consume all income, implying $c^n(A, w) = rA + b$ and $\dot{A} = 0$. It is later verified that this is never optimal in the interior of $[-b/r, \underline{A}^U)$. Alternatively, consider the set of strategies implying $\dot{A} < 0$. Then by (3.19)

$$\dot{c^n}(A) = \frac{\lambda(u'(c^n(A)) - u'(c^e(A))}{u''(c^n(A))} \tag{3.20}$$

Over the duration of a job search spell consumption is declining over time.

Lemma 1. ***Optimality of job search***
There exists a unique $\underline{A}^U < \bar{A}^E$ such that job search is strictly preferred to non-participation for all $A \in [-b/r, \underline{A}^U)$ and $S(\underline{A}^U, .) = bu'(c^n(\underline{A}^U, .)) + \lambda(V^e(\underline{A}^U, .) - V^n(\underline{A}^U, .)) = u_b$.

Proof. Differentiation of (3.17) with respect to A implies

$$S_A(A) = bu''(c^n(A, w))c^n_A(A) + \lambda(u'(c^e(A)) - u'(c^n(A))) < 0$$

and as already established job search is strictly preferred to non-participation at the borrowing limit:

$$\lim_{A \to -b/r} S(A) > u_b$$

In the limit as A tends to $\bar{A}^E$ property 1 and proposition 1 imply that non-participation is strictly preferred to job search. Therefore there exists a unique

$\underline{A}^U$ such that $S(A) > u_b, \forall A \in [-b/r, \underline{A}^U)$ and $S(\underline{A}^U) = bu'(c^n(\underline{A}^U)) + \lambda(V^e(\underline{A}^U) - V^n(\underline{A}^U)) = u_b$. $\square$

The dynamics of consumption of unemployed workers over the interval $A \in [-b/r, \underline{A}^U)$ is illustrated in Figure 3.2.2. The set of $\{A, c^n\}$ pairs where wealth is stationary are identified by the locus $0 = rA + b - c^n(A)$, represented by a straight line with slope r in the (A, c)-plane, with $c^n(-b/r) = 0$. Points below the line are consistent with wealth accumulation and points above imply decumulation. On the other hand, equation (3.20), suggests that consumption is in steady state when $c^n(A, .) = c^e(A)$, increases above the locus, and decreases below. The stable saddle path consistent with the terminal condition $c^n(-b/r) = 0$ therefore lies between the $\dot{A} = 0$ and $\dot{c}^n = 0$ loci and prescribes decreasing consumption and wealth during job search. The identified saddle path illustrated, however, only applies within the interval $A \in [-b/r, \underline{A}^U)$.

As $0 < b < w$, there exists a unique level of A, henceforth denoted A^{SS}, where both consumption and wealth are at steady state, and furthermore $A^{SS} \in (\underline{A}_E, \bar{A}^E)$. It is straightforward to verify that $A^{SS} = (c^* - b)/r$, $\underline{A}^U < A^{SS}$ and indeed optimal saving behaviour during job-search involves wealth decumulation[34].

Lemma 1 suggests that to the right of $\underline{A}^U$ there exists a neighbourhood, $A^{NP \geq JS}$, where non-participation weakly dominates job search. Differentiation of (3.17) with respect to A and evaluation at any point in $A^{NP \geq JS}$ implies

$$\frac{\partial S(A | A \in A^{NP \geq JS})}{\partial A} = bu''(c^n(A)) \frac{\partial c^n(A)}{\partial A} + \lambda(u'(c^e(A) - u'(c^n(A))) < 0$$

under the assumptions (verified below) that consumption of non-employed is lower than that for employed and non-decreasing with wealth. Then non-participation is strictly preferred to job search for all $A > \underline{A}^U$.

Consider a non-employed worker with $A > \underline{A}^U$. As she optimally chooses

[34] Is job search still optimal at A^{SS}? Suppose so. Then $S(A^{SS}) = u'(c^*)(\frac{br+\lambda w}{r+\lambda}) = u_b(\frac{br+\lambda w}{wr+w\lambda}) < u_b$ which is a contradiction.

non-participation, her value function reduces to

$$rV^n(A) = u(c^n(A)) + u_b + u'(c^n(A))(rA - c^n(A))$$

Totally differentiating with respect to time

$$u''(c^n(A))\dot{c}^n(A)(rA - c^n(A)) = 0 \tag{3.21}$$

identifying two potentially optimal consumption/saving strategies. One possibility is to consume all income, $c^n(A) = rA$ (Strategy i). As this implies $\dot{A} = 0$, if ever optimal this is optimal forever. Substituting into the value function the discounted lifetime payoff is

$$rV^n(A|i) = u(rA) + u_b. \tag{3.22}$$

Alternatively, (3.21) holds if $c^n_A(A) = 0$ implying $c^n(A) = c^n(\underline{A}^U)$ and wealth decumulates[35]. The discounted lifetime payoff is

$$\begin{aligned} rV^n(A|ii) &= u(c^n(\underline{A}^U)) + u_b + u'(c^n(\underline{A}^U))(rA - c^n(\underline{A}^U)) \\ &= rV^n(\underline{A}^U) + r(A - \underline{A}^U)u'(c^n(\underline{A}^U)) \end{aligned} \tag{3.23}$$

Let $\bar{A}^U \equiv c^n(\underline{A}^U)/r$. Note that Strategy ii) is only feasible when $A \leq \bar{A}^U$ as consuming $c^n(\underline{A}^U)$ implies wealth accumulation for higher A.

Lemma 2. ***Optimality of non-participation***

A non-employed worker optimally chooses to not participate and decumulate wealth if and only if $A \in [\underline{A}^U, \bar{A}^U]$. Her value is function described by (3.23).

A non-employed worker optimally chooses to not participate and consume all income if and only if $A > \bar{A}^U$. Her value function is described by (3.22).

[35] Another possible strategy consistent with (3.21) is identified by $c^n_A(A) = 0$ and $\dot{A} > 0$. Such strategies are never optimal as worker never changes state yet does not maximize consumption but instead accumulates wealth indefinitely.

Proof. Let

$$T(A) \equiv rV^n(A|ii) - rV^n(A|i) = u(c^n(\underline{A}^U)) - u(rA) + u'(c^n(\underline{A}^U))(rA - c^n(\underline{A}^U))$$

Evaluating at $A = \underline{A}^U$ and rearranging

$$\frac{T(\underline{A}^U)}{c^n(\underline{A}^U) - r\underline{A}^U} = \frac{u(c^n(\underline{A}^U)) - u(r\underline{A}^U)}{c^n(\underline{A}^U) - r\underline{A}^U} - u'(c^n(\underline{A}^U)) > 0$$

where the inequality follows from the fact that $c^n(\underline{A}^U) > r\underline{A}^U$ (recall figure 3.2.2) and concavity of $u(.)$.

Differentiation of $T(.)$ with respect to A yields

$$\frac{\partial T(A)}{\partial A} = r(u'(c^n(\underline{A}^U) - u'(rA)) \leq 0, \forall A \in [\underline{A}^U, \bar{A}^U]$$

with equality at $\bar{A}^U$. The inequality follows by the definition of $\bar{A}^U$.

Evaluating $T(.)$ at $A = \bar{A}^U$ yields $T(\bar{A}^U) = 0$. Therefore $T(A) \geq 0, \forall A \in [\underline{A}^U, \bar{A}^U]$ and strategy ii is optimal within this interval. Furthermore, strategy ii is infeasible for $A > \bar{A}^U$ implying the optimality of strategy i. This completes the proof of Lemma 2.

□

The analysis in this section fully characterises the solution of a non-employed worker's problem conditional on Property 1 and implies Proposition 2.

3.A.4 Proof of Theorem 1

Conditional on Property 1 the strategies described by Propositions 1 and 2 solve the Bellman equations (3.1) and (3.2) by construction. Next, we verify that the conjectured solution implies Property 1 as well.

Sufficient condition for Property 1 is $V^e(A) > V^n(A), \forall A < \bar{A}^E$. Since $c^* > c^n(\underline{A}^U)$ (Figure 3.2.2), $\bar{A}^E > \bar{A}^U$. Further, $V^e(\bar{A}^E) = V^n(\bar{A}^E)$ by Proposition 1.

Since $c^n(A) < c^e(A), \forall A < \bar{A}^E$ (Figure 3.2.3), it follows that $u'(c^e(A)) < u'(c^n(A))$ and equivalently $V^e_A(A) < V^n_A(A), \forall A < \bar{A}^E$.

Hence both $V^e(A)$ and $V^n(A)$ are increasing with A and $V^n(A)$ is increasing at a higher rate over $A < \bar{A}^E$. Therefore, $V^e(A) > V^n(A), \forall A < \bar{A}^E$ and Property 1 holds.

3.B Education efficiency frontier

For brevity let "forever-workers", "savers", and "retirees" describe workers whose optimal strategy is to work forever, save for retirement and retire. The derivation of the education efficiency frontier proceeds by analysing the choice of education of workers based on their optimal employment strategies conditional on investing in education or not.

3.B.1 Forever-workers without education

Consider the set of workers whose optimal strategy would be to work forever if they chose to obtain no education. They are identified by the set of abilities and wealth endowments

$$FW_0 \equiv \{A_0, w_0 | rA_0 \leq c^*_0 - w_0\}$$

Should they invest in education instead their optimal strategy might be to either work forever or to save for retirement[36].

3.B.1.1 Forever-workers with education

The subset

$$FW_0^{FW} \equiv \{A_0, w_0 | rA_0 \leq \min\{c^*_0 - w_0, c^*_1 - w_0(1+e) + rk\}\}$$

[36] Investment in education involves a depletion of wealth and an increase in c^*, consumption during the wealth accumulation phase. Therefore a forever worker or saver without education is never an immediate retiree with education.

identifies forever-workers irrespective of education choice. They never enjoy leisure so their decision is driven solely by comparison of consumption possibilities with and without education. The latter depends both on wealth and earnings which are perfectly substitutable, and the benefit from extra earnings, $w_0 e$, is enjoyed in perpetuity. A worker optimally invests in education if and only if the perpetuity-discounted value of the extra flow of earnings exceeds the cost of education

$$\frac{w_0 e}{r} \geq k \tag{3.24}$$

Conditional on belonging to FW_0^{FW} optimality of education is independent of the wealth endowment. Workers with sufficiently high ability $w_0 > rk/e$ always pursue a degree. Less able workers, do not. The strict-equality counterpart of (3.24) identifies the education efficiency frontier over FW_0^{FW}.

3.B.1.2 Savers with education

The relative complement of FW_0^{FW} in FW_0

$$FW_0^S \equiv \{A_0, w_0 | rA_0 \in (c_1^* - w_0(1+e) + rk, c_0^* - w_0]\}$$

identifies forever-workers without education who save for retirement if they had education. The set is non-empty as long as $c_1^* - c_0^* < w_0 e - rk$ for some w_0[37]. Since $c^*(w)$ is increasing in w existence requires $w_0 e - rk > 0$, i.e. all members of FW_0^S lie above the education efficiency frontier for FW_0^{FW}. The value with education always exceeds the value without as

$$u(c_1^*) + \left(\frac{rA_1 + w_1 - c_1^*}{w_1}\right) u_b > u(c_1^*) > u(c_0^*) \geq u(rA_0 + w_0)$$

[37]The existence of this and some subsequently discussed sets depends on the form of preferences (see (3.10)). Figure 3.4.1 illustrates just one special case. For example, if risk aversion is low and the worker has weak preference for leisure, FW_0^S may be empty.

With education these workers achieve both higher consumption and the prospect of enjoying leisure some time in the future hence always invest.

In summary, workers who would work forever without education, optimally choose to pursue a degree if and only if $w_0 \geq rk/e$. Subject to this, their wealth endowment is irrelevant.

3.B.2 Savers without education

Consider the set of workers whose optimal strategy would be to save for retirement if they chose to obtain no education. They are identified by the set

$$S_0 \equiv \{A_0, w_0 | rA_0 \in (c_0^* - w_0, c_0^*)\}$$

Should they invest in education instead their optimal strategy might be to either work forever or save for retirement.

3.B.2.1 Forever-workers with education

The subset

$$S_0^{FW} \equiv \{A_0, w_0 | rA_0 \in (c_0^* - w_0, \min\{c_0^*, c_1^* - w_0(1+e) + rk\})\}$$

identifies the workers who would work forever were they to invest in education. To trade the perspective of future leisure they need sufficient increases in consumption.

S_0^{FW} is never empty. Consider a worker with $w_0 = rk/e$ and $rA_0 = c_0^*(rk/e) - rk/e$. This worker belongs to FW_0 and is just indifferent between working forever and saving for retirement, as well as between investing or not investing in education. In a neighbourhood to the right of $(rk/e, c_0^*(rk/e) - rk/e)$ (Figure 3.4.1), workers belong to S_0^{FW} (see discussion in section 3.B.1.2).

Workers are indifferent to education if and only if

$$u(rA_0 - rk + w_0(1+e)) - u(c_0^*) = \frac{(rA_0 + w_0 - c_0^*)}{w_0} u_b \tag{3.25}$$

that is if the gain in consumption just equals the discounted value of leisure they forego. Totally differentiating (3.25) with respect to A_0, the slope of the EEF is

$$\left.\frac{\partial w_0}{\partial A_0}\right|_{EF} = \frac{rw_0(u'(c_0^*) - u'(rA_0 - rk + w_0(1+e)))}{(rA_0 - c_0^*)u'(c_0^*) + w_0(1+e)u'(rA_0 - rk + w_0(1+e))} \geq 0$$

The inequality follows as both the numerator and the denominator are non-negative. As without education the workers save for retirement, (3.25) implies that $c_0^* \leq rA^0 - rk + w_0(1+e)$. Workers are willing to trade their prospect for future leisure only for a sufficiently large increase in consumption. As they work forever with education, $c_1^* \geq rA^0 - rk + w_0(1+e)$. Then for the denominator

$$\begin{aligned}
(rA_0 - c_0^*)u'(c_0^*) + w_0(1+e)u'(rA_0 - rk + w_0(1+e)) &\geq \\
(rA_0 - c_0^*)u'(c_0^*) + w_0(1+e)u'(c_1^*) &= \\
(rA_0 + w_0 - c_0^*)u'(c_0^*) &\geq 0
\end{aligned}$$

In the limit as $w_0 = rk/e$ and rA_0 approaches $c_0^*(rk/e) - rk/e$, (3.25) holds with equality and $\partial w_0/\partial A_0|_{EF}$ approaches zero. The education efficiency frontier for S_0^{FW} follows continuously from the frontier of FW_0 and describes an upward sloping curve in the $\{A_0, w_0\}$-plane.

Savers enjoy the benefit of boosted income only in annuity until retirement. Time to retirement declines with wealth and the wealthier a worker is, the larger is her present value of future leisure. A wealthier worker requires a larger increase in earnings to obtain education. As wealth increases workers indifferent to education have higher and higher ability.

3.B.2.2 Savers with education

The complement of S_0^{FW} in S_0

$$S_0^S \equiv \{A_0, w_0 | rA_0 \in (\max\{c_0^* - w_0, c_1 - w_0(1+e) + rk\}, \min\{c_0^*, c_1^* + rk\})\}$$

identifies workers whose optimal strategy is to save for retirement irrespective of education. The set is non-empty as long as $c^*(w_0) > c^*(w_0(1+e)) - w_0(1+e) + rk$ for some w_0.

Workers are indifferent to education if and only if

$$\left(\frac{(rA_0 + w_0 - c_0^*)}{w_0} - \frac{(rA_0 - rk + w_0(1+e) - c_1^*)}{(1+e)w_0}\right) u_b = u(c_1^*) - u(c_0^*) \quad (3.26)$$

that is, if the gain in consumption (in utility terms) just equals the loss of discounted value of leisure due to increase of time to retirement. Total differentiation with respect to A_0 implies

$$\left.\frac{\partial w_0}{\partial A_0}\right|_{EF} = \frac{rw_0(u'(c_0^*) - u'(c_1^*))}{u(c_1^*) - u(c_0^*)} > 0$$

The education efficiency frontier follows continuously from the one over S_0^{FW}.

3.B.3 Retirees without education

Consider the set of workers whose optimal strategy would be to retire immediately if they chose to obtain no education. They are identified by the set

$$R_0 \equiv \{A_0, w_0 | rA_0 \geq c_0^*\}$$

Should they invest in education instead their optimal strategy might be to work forever, save for retirement or still retire immediately.

The subset

$$R_0^R \equiv \{A_0, w_0 | rA_0 \geq c_1^* + rk\}$$

identifies the workers who retire immediately with or without education. As $u(rA_0) > u(rA_0 - rk), \forall A_0$ they never pursue a degree. If a worker retires immediately, the benefit of boosted earnings is not realised at all.

The subset

$$R_0^{FW} \equiv \{A_0, w_0 | rA_0 \in [c_0^*, c_1^* + rk - w_0(1+e))\}$$

identifies the workers who optimally work forever if they chose education. This set is non-empty.

Workers are indifferent to education if and only if

$$u(rA_0) + u_b = u(rA_0 - rk + w_0(1+e)) \tag{3.27}$$

Totally differentiating with respect to A_0

$$\left.\frac{\partial w_0}{\partial A_0}\right|_{EF} = \frac{r[u'(rA_0) - u'(rA_0 - rk + w_0(1+e))]}{(1+e)u'(rA_0 - rk + w_0(1+e))} > 0$$

The inequality follows because over (3.27) a forever-worker consumes more than a retiree for indifference to obtain.

The subset

$$R_0^S \equiv \{A_0, w_0 | rA_0 \in [\max\{c_0^*, c_1^* - w_0(1+e) + rk\}, c_1^* + rk\}$$

identifies workers who save for retirement if they chose education. This set is non-empty.

Workers are indifferent to education if and only if

$$u(rA_0) + u_b = u(c_1^*) + (rA_0 - rk + w_0(1+e) - c_1^*)u'(c_1^*) \tag{3.28}$$

Totally differentiating with respect to A_0 and using (3.28)

$$\left.\frac{\partial w_0}{\partial A_0}\right|_{EF} = \frac{rw_0[u'(rA_0) - u'(c_1^*)]}{u(c_1^*) - u'(rA_0)} > 0$$

The inequality follows as over (3.28) a worker will delay enjoyment of leisure only if they are able to attain higher consumption.

This completes the construction of the education efficiency frontier.

3.C Tables

Table 3.C.1: First stage

	1996		2001	
	Earnings_t	Saving_t	Earnings_t	Saving_t
$\textbf{Earnings}_{t-2}$			0.755^{***}	0.158^{**}
			(0.018)	(0.050)
$\textbf{Earnings}_{t-3}$	0.787^{***}	0.098^{*}		
	(0.026)	(0.043)		
Education				
HS,HS	1866.607	−3724.072	5330.778^{***}	−720.917
	(1175.814)	(2228.318)	(974.846)	(1520.350)
<D,HS	1753.077	−2423.443	6279.136^{***}	4571.777
	(1297.204)	(2265.825)	(1122.324)	(3051.242)
D+,HS	5094.001^{***}	-8191.107^{*}	10704.381^{***}	1116.796
	(1513.423)	(4175.289)	(1804.790)	(5171.671)
HS,<D	2434.049	−2777.476	7644.515^{***}	3352.449
	(1386.715)	(2797.656)	(1251.744)	(3016.435)
<D,<D	3700.294^{**}	−2735.786	8009.959^{***}	948.949
	(1295.635)	(2137.584)	(1096.206)	(2618.186)
D+,<D	8031.511^{***}	3276.548	10872.854^{***}	3780.029
	(1984.679)	(3281.118)	(1396.889)	(4381.975)
HS,D+	8414.996^{***}	3052.624	11169.124^{***}	3631.515
	(2133.960)	(5261.445)	(2941.858)	(6147.794)
<D,D+	6793.264^{***}	−1380.225	9241.357^{***}	−3193.662
	(1655.063)	(2856.892)	(1380.629)	(4078.870)
D+,D+	11651.517^{***}	5503.179	14592.591^{***}	8579.603^{*}
	(1519.232)	(3160.614)	(1288.134)	(3443.893)
Age				
Age	1598.386^{***}	−168.115	1724.869^{***}	42.241
	(475.047)	(1016.471)	(470.307)	(1081.641)
Age^2	-18.508^{***}	2.138	-19.290^{***}	−1.266
	(5.490)	(12.169)	(5.349)	(12.823)
$\textbf{Net worth}_{t-1}$			0.005	0.027
			(0.003)	(0.018)
$\textbf{Net worth}_{t-2}$	0.003	0.021		
	(0.003)	(0.017)		
Intercept	-22299.932^{*}	5553.979	-31880.985^{**}	−2442.103
	(9893.479)	(20919.004)	(9959.012)	(22287.167)
Num. obs.	4946	4946	4780	4780

Bootstrap standard errors. Education categories: <HS - no high school, HS - high school, <D - higher education but no degree, D+ - at least a degree. First term in education interaction is household head's education category. Base category is <HS,<HS. Education categories that are insignificant in both equations are omitted.

Table 3.C.2: Second stage, interactions, DSLAD

	1996, p=0.2				2001, p=0.2			
	NW1	NW2	NW3	NW4	NW1	NW2	NW3	NW4
Earnings								
[0 – 25]	7961	7662	8900	10308	1978	1967	3405	7401
	(5522)	(5508)	(5499)	(5617)	(6007)	(5965)	(6018)	(6135)
(25 – 50]	10202	10662	11159*	15559**	6356	8051	9032	16369**
	(5474)	(5519)	(5586)	(5764)	(6061)	(6099)	(6108)	(6221)
(50 – 75]	13514*	14796**	16294**	21258***	11387	13359*	15697*	22347***
	(5585)	(5515)	(5568)	(5397)	(6095)	(6050)	(6145)	(6089)
(75 – 100]	18384***	23037***	27166***	39089***	19112**	24732***	29718***	43027***
	(5479)	(5540)	(5550)	(6137)	(6210)	(6289)	(6432)	(6788)
Age	-242				87			
	(262)				(286)			
Age^2	2				-2			
	(3)				(3)			
Num. obs.	4946				4780			

Bootstrap standard errors in parentheses. Controls omitted.

Bibliography

Daron Acemoglu and Robert Shimer. Wage and technology dispersion. *Review of Economic Studies*, 67(4):585–607, 2000.

Sule Alan, Kadir Atalay, and Thomas F. Crossley. Do the Rich Save More? Evidence from Canada. *Review of Income and Wealth*, 61(4):739–758, 2015.

Michelle Alexopoulos and Tricia Gladden. Wealth, reservation wages, and labor market transitions in the US: evidence from the Survey of Income and Program Participation. *Working Paper, University of Toronto*, 2006.

Yann Algan, Arnaud Chéron, Jean-Olivier Hairault, and François Langot. Wealth effect on labor market transitions. *Review of Economic Dynamics*, 6(1):156–178, January 2003.

Takeshi Amemiya. Two Stage Least Absolute Deviations Estimators. *Econometrica*, 50(3):689–711, 1982.

Jesper Bagger, François Fontaine, Fabien Postel-Vinay, and Jean Marc Robin. Tenure, experience, human capital, and wages: A tractable equilibrium search model of wage dynamics. *American Economic Review*, 104(6):1551–1596, 2014.

Eric J Bartelsman and Mark Doms. Understanding Productivity: Lessons from Longitudinal Microdata. *Journal of Economic Literature*, 38(3):569–594, 2000.

Shlomo Benartzi and Richard H Thaler. Behavioral Economics and the Retirement Savings Crisis. *Science Magazine (New York, N.Y.)*, 339(March):1152–3, 2013.

Keith A. Bender, Kostas Mavromaras, Ioannis Theodossiou, and Zhang Wei. The Effect of Wealth and Earned Income on the Decision to Retire: A Dynamic Probit Examination of Retirement. *IZA Discussion Paper Series*, (7927), 2014.

B. Douglas Bernheim, Jonathan Skinner, and Steven Weinberg. What accounts for the variation in retirement wealth among U.S. households? *American Economic Review*, 91(4):832–857, 2001.

HG Bloemen and EGF Stancanelli. Individual wealth, reservation wages, and transitions into employment. *Journal of Labor Economics*, 19(2):400–439, 2001.

David E. Bloom, David Canning, Richard K. Mansfield, and Michael Moore. Demographic change, social security systems, and savings. *Journal of Monetary Economics*, 54(1):92–114, 2007.

David E. Bloom, David Canning, and Michael Moore. Optimal Retirement with Increasing Longevity. *Scandinavian Journal of Economics*, 116(3):838–858, 2014.

Richard Blundell, Luigi Pistaferri, and Ian Preston. Consumption inequality and partial insurance. *American Economic Review*, 98(5):1887–1891, 2008.

Jeffrey R Brown, Courtney C Coile, and Scott J Weisbenner. The Effect of Inheritance Receipt on Retirement. *Review of Economics and Statistics*, 92(2): 425–434, 2010.

Martin Browning, Thomas F. Crossley, and Eric Smith. Asset accumulation and short-term employment. *Review of Economic Dynamics*, 10(3):400–423, July 2007.

John B Burbidge and A Leslie Robb. Pensions and Retirement Behaviour. *Canadian Journal of Economics*, 13(3):421–437, 1980.

Ken Burdett, Carlos Carrillo-Tudela, and Melvyn Coles. Wage inequality: A structural decomposition. *Review of Economic Dynamics*, 19:20–37, 2016.

Kenneth Burdett and Dale Mortensen. Wage differentials, employer size, and unemployment. *International Economic Review*, pages 257–273, 1998.

Kenneth Burdett, Carlos Carrillo-Tudela, and Melvyn Coles. Human capital accumulation and labor market equilibrium. *International Economic Review*, 52 (3):657–677, 2011.

Marco Cagetti. Wealth Accumulation Over the Life Cycle and Precautionary Savings. *Journal of Business & Economic Statistics*, 21(3):339–353, July 2003.

Christopher D Carroll. Why Do the Rich Save So Much? In Joel B. Slemrod, editor, *Does Atlas Shrug? The Economic Consequences of Taxing the Rich.*, chapter 14, pages 465–84. Harvard University Press, 2000.

Christopher D Carroll and Andrew A Samwick. The Nature of Precautionary Wealth. *Journal of Monetary Economics*, 40(1):41–71, 1997.

John L Czajka and Gabrielle Denmead. Getting More from Survey Income Measures : Empirically-based Recommendations for Improving Accuracy and Efficiency. 2008.

John L Czajka, Jonathan E Jacobson, and Scott Cody. Survey Estimates of Wealth: A Comparative Analysis and Review of the Survey of Income and Program Participation. *Social Security Bulletin*, 65(1):63–69, 2003.

Hippolyte D'Albis, Sau Him Paul Lau, and Miguel Sánchez-Romero. Mortality transition and differential incentives for early retirement. *Journal of Economic Theory*, 147(1):261–283, 2012.

JP Danforth. On the Role of Consumption and Decreasing Absolute Risk Aversion in the Theory of Job Search. In *Studies in the Economics of Search*, pages 109–131. North-Holland, 1979.

JuanJ. Dolado, Marcel Jansen, and JuanF. Jimeno. On-the-job search in a matching model with heterogeneous jobs and workers. *Economic Journal*, 119(534): 200–228, 01 2009.

Karen E. Dynan, Jonathan Skinner, and Stephen P. Zeldes. Do the Rich Save More? *Journal of Political Economy*, 112(2):397–444, 2004.

Jan Eeckhout and Alireza Sepahsalari. Unemployment Risk and the Distribution of Assets. Technical report, 2013.

Eric French. The Effects of Health, Wealth, and Wages on Labour Supply and Retirement Behavior. *Review of Economic Studies*, 72(2):395–427, 2005.

Chao Fu. Training, search and wage dispersion. *Review of Economic Dynamics*, 14(4):650–666, aug 2011.

Scott L. Fulford. The surprisingly low importance of income uncertainty for precaution. *European Economic Review*, 79:151–171, 2015.

Pierre-Olivier Gourinchas and Jonathan A Parker. Consumption over the Life Cycle. *Econometrica*, 70(1):47–89, 2002.

Luigi Guiso, Tullio Jappelli, and Daniele Terlizzese. Earnings uncertainty and precautionary saving. *Journal of Monetary Economics*, 30(2):307–337, 1992.

R. Glenn Hubbard, Jonathan Skinner, and Stephen P. Zeldes. The importance of precautionary motives in explaining individual and aggregate saving. *Carnegie-Rochester Confer. Series on Public Policy*, 40(C):59–125, 1994a.

R. Glenn Hubbard, Jonathan Skinner, and Stephen P. Zeldes. Expanding the Life-Cycle Model - Precautionary Saving and Public Policy. *American Economic Review*, 84(2):174–179, 1994b.

Mark Huggett and Gustavo Ventura. Understanding why high income households save more than low income households. *Journal of Monetary Economics*, 45: 361–397, 2000.

Erik Hurst, Annamaria Lusardi, Arthur Kennickell, and Francisco Torralba. The Importance of Business Owners in Assessing the Size of Precautionary Savings. *Review of Economics and Statistics*, 92(1):61–69, 2010.

Guido W Imbens, Donald B Rubin, and Bruce I Sacerdote. Estimating the Effect of Unearned Income on Labor Earnings , Savings , and Consumption : Evidence from a Survey of Lottery Players. *American Economic Review*, 91(4):778–794, 2001.

Stephen P Jenkins. Survival analysis. *Unpublished manuscript, Institute for Social and Economic Research, University of Essex, Colchester, UK*, 2005.

Malene Kallestrup-Lamb, Anders Bredahl Kock, and Johannes Tang Kristensen. Lassoing the Determinants of Retirement. *Econometric Reviews*, 35(8-10):1522–1561, 2016.

Tae-Hwan Kim and Christiophe Christophe Muller. Two-stage quantile regression when the first stage is based on quantile regression. *Econometrics Journal*, 7: 218–231, 2004.

Per Krusell, Toshihiko Mukoyama, Richard Rogerson, and Ayşegül Şahin. Aggregate implications of indivisible labor, incomplete markets, and labor market frictions. *Journal of Monetary Economics*, 55(5):961–979, jul 2008.

Michael Kuhn, Stefan Wrzaczek, Alexia Prskawetz, and Gustav Feichtinger. Optimal choice of health and retirement in a life-cycle model. *Journal of Economic Theory*, 158:186–212, 2015.

David I Laibson, a Repetto, and J Tobacman. Self-control and saving for retirement. *Brookings Papers on Economic Activity*, 1(1):91–172, 1998.

Marloes Lammers. The effects of savings on reservation wages and search effort. *Labour Economics*, 2014.

Rasmus Lentz. Optimal unemployment insurance in an estimated job search model with savings. *Review of Economic Dynamics*, 12(1):37–57, January 2009.

Rasmus Lentz and Dale T. Mortensen. An Empirical Model of Growth through Product Innovation. *Econometrica*, 76(6):1317–1373, 2008.

Rasmus Lentz and T Tranaes. Job search and savings: Wealth effects and duration dependence. *Journal of Labor Economics*, 23(3):467–489, 2005.

Qi Li and Jeffrey S. Racine. Nonparametric Estimation of Conditional CDF and Quantile Functions With Mixed Categorical and Continuous Data. *Journal of Business & Economic Statistics*, 26(4):423–434, 2008.

Jeremy Lise. On-the-job search and precautionary savings. *Review of Economic Studies*, 80:1086–1113, 2013.

Annamaria Lusardi. On the Importance of the Precautionary Saving Motive. *American Economic Review*, 88(2):449–453, 1998.

John McCall. Economics of information and job search. *Quarterly Journal of Economics*, 84(1):113–126, 1970.

D. Mortensen. *Wage Dispersion: Why are Similar Workers Paid Differently?* Zeuthen lecture book series. MIT Press, 2005.

Dale T Mortensen. *Equilibrium unemployment with wage posting: Burdett-Mortensen meet Pissarides*, volume 243. Emerald Group Publishing Limited, 2000.

Éva Nagypál. Worker reallocation over the business cycle: The importance of employer-to-employer transitions. *Manuscript*, pages 1–55, 2008.

Fabien Postel-Vinay and Jean-Marc Robin. Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, 70(6):2295–2350, November 2002.

Elena Quercioli. Training, Turnover, and Search. *International Economic Review*, 46(1):133–143, 2005.

Silvio Rendon. Job search and asset accumulation under borrowing constraints. *International Economic Review*, 47(1):233–263, February 2006.

Richard Rogerson and Johanna Wallenius. Nonconvexities, retirement, and the elasticity of labor supply. *American Economic Review*, 103(4):1445–1462, 2013.

Joao M.C. Santos-Silva and Silvana Tenreyro. The log of gravity. *Review of Economics and Statistics*, 88(4):641–658, 2006.

Daniel Schunk. What determines household saving behavior? an examination of saving motives and saving decisions. *Journal of Economics and Statistics (Jahrbuecher fuer Nationaloekonomie und Statistik)*, 229(4):467–491, 2009.

K Storesletten, C I Telmer, and A Yaron. Consumption and risk sharing over the life cycle. *Journal of Monetary Economics*, 52(7):1331–1358, 2005.

Mitra Toossi. Labor force projections to 2022: the labor force participation rate continues to fall. *Monthly Labor Review*, (December):1–28, 2013.