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CRANFIELD



LINEAR BUCKLING OF AN AXIALLY REINFORCED
PRESSURISED CYLINDER

by

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SUMMARY

An analysis is presented using small deflection theory for the buckling of a pressurised, axially reinforced cylinder, which is subjected to axial compression.

Various approximations to the analysis are discussed and some results are presented which show the effects of internal pressure and various structural parameters on both panel buckling and overall buckling.

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NOTATION

a	cross sectional area of stringer
b	stringer pitch
D	flexural rigidity of skin
D'	effective flexural rigidity of skin-stringer combination
E	Young's modulus of skin
E _s	Young's modulus of stringer
I	moment of inertia of a stringer about median plane of skin
ℓ	length of cylinder
m	number of half waves in an axial direction
M _(x, y, xy)	resultant stress couples in cylinder
n	number of waves in a circumferential direction
N _(x, y, xy)	stress resultants in median plane of cylinder
N' _y	$N_y - pR$
p	internal pressure
R	radius of cylinder
t	thickness of skin
t _s	equivalent thickness of stringer sheet ($= \frac{a}{b}$)
u, v, w,	displacements in x, θ and z directions
α	cylinder parameter = $\left[\frac{t^2}{12R^2} \right]$
β	ratio of cylinder flexural rigidities = $\left[\frac{D'}{D} \right]$
ε ₁ , ε ₂ , γ	median plane strains
ζ, η, ξ	arbitrary constants in expressions for displacements
θ	cylinder co-ordinate
λ	$\frac{km\pi}{\ell}$
μ	axial reinforcement parameter = $\left[1 + \frac{E_s t_s}{Et} (1 - \nu^2) \right]$
ν	Poisson's ratio
σ	stress
φ ₁	pressure parameter = $\left[\frac{pR(1 - \nu^2)}{Et} \right]$
φ ₂	axial force parameter = $\left[\frac{-N_x(1 - \nu^2)}{Et} \right]$
x (x, y, xy)	changes in curvature

1. Introduction

In the original investigations into the buckling of circular cylinders carried out by Timoshenko, Southwell, Flügge and others, a small deflection theory was used to establish the elastic deflection equations. Experiments however, show that, particularly for the case of a cylinder under an axial load, the classical deflection theory considerably over-estimates the critical stress.

In 1934, Donnell first proposed the use of a non-linear theory to explain these discrepancies and this theory has been developed and used extensively in recent years for predicting the elastic buckling of cylindrical shells. Much of the recent work in this field is summarised by Nash¹ and Thielemann².

In the last report use is made of the non-linear theory to analyse the effect of both internal pressure and orthotropic properties on the post buckled behaviour of axially loaded cylindrical shells. The experimental investigations which are also carried out indicate good agreement with the theoretical results. However, the solution of the large deflection equations suggested by Thielemann, involves considerable computational difficulty, and it would seem reasonable in investigating the influence of axial stiffening and internal pressure, to initially use small deflection theory in order to show more readily the effect of the shell parameters on the critical stress.

An investigation has recently been undertaken by McKenzie³, who used the small deflection theory in solving the problem of buckling of an axially reinforced cylinder subjected to axial end load and internal pressure. McKenzie's solution is, in fact, restricted to the problem of a cylinder having a large number of axial stiffeners. Furthermore, he is only concerned with the overall buckling of the stiffened shell after panel buckling has developed.

In this paper both panel buckling and overall buckling of the shell are considered, and the influence of the axial reinforcement is discussed.

2. The Basic Equations

The equations of equilibrium of an element of a stiffened cylinder can be obtained by suitably modifying the equations for an unstiffened shell as presented by Timoshenko⁴ which are

$$\begin{aligned}
 R \frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial \theta} - pR \left(\frac{\partial^2 v}{\partial x \partial \theta} - \frac{\partial w}{\partial x} \right) &= 0, \\
 \frac{\partial N_y}{\partial \theta} + R \frac{\partial N_{xy}}{\partial x} - \frac{\partial M_y}{R \partial \theta} + \frac{\partial M_{xy}}{\partial x} + RN_x \frac{\partial^2 v}{dx^2} &= 0, \\
 RN_x \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 M_{yx}}{\partial x \partial \theta} + R \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{R \partial \theta^2} - \frac{\partial^2 M_{xy}}{\partial x \partial \theta} + N'_y + & \\
 + p \left(w + \frac{\partial^2 w}{\partial \theta^2} \right) &= 0. \tag{2.01}
 \end{aligned}$$

The sign convention is the same as that given in Ref. 4, Fig. 231.

For a long shell which is stiffened in an axial direction the stress resultants and couples can be modified to become

$$\begin{aligned}
 N_x &= E_s t_s \epsilon_1 + \frac{Et}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2), \quad N_y = \frac{Et}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1) \\
 N_{yx} = N_{xy} &= \frac{\gamma t E}{2(1+\nu)}, \quad N'_y = N_y - pR, \tag{2.02}
 \end{aligned}$$

and

$$\begin{aligned}
 M_x &= -D (\beta x_x + \nu x_y), \quad M_y = -D (x_y + \nu x_x), \\
 -M_{yx} = M_{xy} &= D(1-\nu) x_{xy}, \tag{2.03}
 \end{aligned}$$

where

$$\begin{aligned}
 \epsilon_1 &= \frac{\partial u}{\partial x}, \quad \epsilon_2 = \frac{\partial v}{R \partial \theta} - \frac{w}{R}, \quad \gamma = \frac{\partial u}{R \partial \theta} + \frac{\partial v}{\partial x}, \\
 x_x &= \frac{\partial^2 w}{\partial x^2}, \quad x_y = \frac{1}{R^2} \left(\frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2} \right), \quad x_{xy} = \frac{1}{R} \left(\frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x \partial \theta} \right)
 \end{aligned}$$

These equations are derived on the assumption that the axial reinforcement is symmetrically disposed about the skin centre line. If this assumption is not fulfilled, additional terms may be necessary in the expressions given in equations (2.02) and (2.03).

Substitution of equations (2.02) and (2.03) into (2.01) yields

$$\begin{aligned} \mu R^2 \frac{\partial^2 u}{\partial x^2} + R \frac{(1+\nu)}{2} \frac{\partial^2 v}{\partial x \partial \theta} - \nu R \frac{\partial w}{\partial x} - R \phi_1 \left(\frac{\partial^2 v}{\partial x \partial \theta} - \frac{\partial w}{\partial x} \right) + \left(\frac{1-\nu}{2} \right) \frac{\partial^2 u}{\partial \theta^2} = 0, \\ R \frac{(1+\nu)}{2} \frac{\partial^2 u}{\partial x \partial \theta} + \left(\frac{1-\nu}{2} \right) R^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial \theta^2} - \frac{\partial w}{\partial \theta} + \alpha \left[\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^3 w}{\partial \theta^3} + R \frac{2 \partial^3 w}{\partial x^2 \partial \theta} \right. \\ \left. + R^2 (1-\nu) \frac{\partial^2 v}{\partial x^2} \right] - R^2 \phi_2 \frac{\partial^2 v}{\partial x^2} = 0, \quad (2.04) \\ R \nu \frac{\partial u}{\partial x} + \frac{\partial v}{\partial \theta} - w - \alpha \left[\frac{\partial^3 v}{\partial \theta^3} + 2(1-\nu) R^2 \frac{\partial^3 v}{\partial x^2 \partial \theta} + \nu R^2 \frac{\partial^3 v}{\partial x^2 \partial \theta} + R^4 \beta \frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial \theta^4} \right. \\ \left. 2R^2 \frac{\partial^4 w}{\partial x^2 \partial \theta^2} \right] + \phi_1 \left(w + \frac{\partial^2 w}{\partial \theta^2} \right) - \phi_2 R^2 \frac{\partial^2 w}{\partial x^2} = 0. \end{aligned}$$

To solve the above equations the following modes are assumed which are appropriate for a simply supported shell

$$\begin{aligned} u &= \xi \cos n\theta \cos \frac{\lambda x}{R}, \\ v &= \eta \sin n\theta \sin \frac{\lambda x}{R}, \\ \text{and } w &= \zeta \cos n\theta \sin \frac{\lambda x}{R}, \end{aligned} \quad (2.05)$$

In these equations ξ , η and ζ are arbitrary constants, $\lambda = \frac{Rm\pi}{l}$ and m and $2n$ refer to the number of axial and circumferential half waves.

These modes are identical with those assumed by Timoshenko⁴ and McKenzie³ and include the possibility of axi-symmetric ring shaped buckles and asymmetric chess board shaped buckles. However, in their present form equations 2.05 are inadequate to include the diamond-shaped buckling mode which is indicated in the experiments conducted by Thielemann². Substitution of equations 2.05 into the equilibrium equations 2.04 yields the stability determinantal equation as follows

$$\left| \begin{array}{l} \mu\lambda^2 + \left(\frac{1-\nu}{2}\right)n^2 \\ \left(\frac{1+\nu}{2}\right)n\lambda \\ \nu\lambda \end{array} \right. , \left. \begin{array}{l} \lambda n \left[\frac{(1+\nu)}{2} - \phi_1 \right], \quad (\nu - \phi_1)\lambda \\ \lambda^2 \left(\frac{1-\nu}{2}\right) (1 + 2\alpha) + n^2(1 + \alpha) - \phi_2 \lambda^2, \quad n + \alpha n(n^2 + \lambda^2) \\ n + \alpha n^3 + 2\alpha(1 - \nu)n\lambda^2 + \nu\alpha n\lambda^2, \quad 1 + \alpha\beta\lambda^4 + \alpha n^4 + \\ \quad 2\alpha n^2\lambda^2 - \phi_1(1 - n^2) - \phi_2\lambda^2 \end{array} \right| = 0 \quad \dots\dots (2.06)$$

The solution of this equation leads to the critical value of the axial force parameter ϕ_2 for various values of the pressure parameter ϕ_1 , and of the terms describing the effects of the axial stiffeners μ and β for any assumed values of n and λ . There is of course a minimum value of ϕ_2 , and it is the determination of the values of n and λ corresponding to this minimum value which constitutes one major problem.

3. The axisymmetric buckling solution ($n = 0$)

For this case the deformation modes become simply

$$\begin{aligned} u &= \xi \cos \frac{\lambda x}{R} , \\ \text{and } w &= \zeta \sin \frac{\lambda x}{R} , \end{aligned} \quad (3.01)$$

and these modes correspond to the case of ring shaped buckles.

In the equation 2.04, it is observed that the $\frac{\partial}{\partial \theta}$ terms vanish, and that $\nu = 0$, in which case the equation becomes

$$\mu R \frac{d^2 u}{dx^2} = \frac{dw}{dx} \left[\nu - \phi_1 \right] , \quad (3.02)$$

and

$$R\nu \frac{du}{dx} - w - \alpha R^4 \beta \frac{d^4 w}{dx^4} + \phi_1 w - \phi_2 R^2 \frac{d^2 w}{dx^2} = 0 ,$$

These can be combined to give

$$\alpha R^4 \beta \frac{d^4 w}{dx^4} + \phi_2 R^2 \frac{d^2 w}{dx^2} + w \left[\left(1 - \frac{\nu^2}{\mu}\right) - \phi_1 \left(1 - \frac{\nu}{\mu}\right) \right] = 0. \quad (3.03)$$

For the case of an unstiffened shell (i.e. $\mu = \beta = 1$) and no internal pressure (i.e. $\phi_1 = 0$), equation 3.03 reduces to

$$D \frac{d^4 w}{dx^4} - N_x \frac{d^2 w}{dx^2} + Et \frac{w}{R^2} = 0. \quad (3.04)$$

This result corresponds to the classical equation for an unstiffened shell.

If the axial stiffening and the internal pressure terms are included in the axisymmetric buckling equation 3.03, the condition for minimum ϕ_2 gives the result

$$2 \alpha \beta \lambda - 2 \left[1 - \frac{\nu^2}{\mu} - \phi_1 \left(1 - \frac{\nu}{\mu} \right) \right] \left(\frac{1}{\lambda} \right)^3 = 0, \quad (3.05)$$

where

$$\lambda^4 = \frac{1 - \frac{\nu^2}{\mu} - \phi_1 \left(1 - \frac{\nu}{\mu} \right)}{\alpha \beta}$$

These results could have been obtained directly from equation 2.06 with $n = 0$.

From equation 3.05, the critical axial stress becomes

$$N_{xc} = \frac{Et^2}{R \sqrt{3(1-\nu^2)}} \sqrt{\frac{\beta \left[1 - \frac{\nu^2}{\mu} - \phi_1 \left(1 - \frac{\nu}{\mu} \right) \right]}{1-\nu^2}} \quad (3.06)$$

For no axial stiffening equation 3.06 reduces to

$$\sigma_c = \frac{Et}{R \sqrt{3(1-\nu^2)}} \sqrt{1 - \frac{\phi_1}{1+\nu}}, \quad (3.07)$$

which can be compared with the classical buckling solution for an unpressurised cylinder under axial load given by Timoshenko as

$$\sigma_c = \frac{Et}{R \sqrt{3(1-\nu^2)}} \quad (3.08)$$

On examination it is found that the ϕ_1 term in equation 3.07 is very small compared with unity, so that it can be concluded that the influence of internal pressure on axisymmetrical ring shaped buckling is small, but is slightly destabilising. The fact that internal pressure has negligible effect on ring shaped buckling is observed by Thielemann.

4. A general buckling solution

The buckling determinant equation 2.06 can be expanded and expressed in the form

$$C_1 + C_2 \alpha = C_3 \phi_1 + C_4 \phi_2, \quad (4.01)$$

where

$$\begin{aligned} C_1 &= \lambda^4 (\mu - \nu^2), \\ C_2 &= (n^2 + \lambda^2)^4 - (2 + \nu)(3 - \nu) \lambda^4 n^2 + 2 \lambda^4 (1 - \nu^2) - \lambda^2 n^4 (7 + \nu) \\ &\quad + \lambda^2 n^2 (3 + \nu) - 2n^6 + n^4 \\ &\quad + (\mu - 1)(\beta - 1) \frac{2 \lambda^6}{1 - \nu} \left[\frac{1 - \nu}{2} \lambda^2 + n^2 \right] \\ &\quad + (\beta - 1) \lambda^4 (\lambda^2 + n^2)^2 \\ &\quad + (\mu - 1) \frac{2 \lambda^2}{1 - \nu} \left\{ (\lambda^2 + n^2)^2 \left[\left(\frac{1 - \nu}{2} \right) \lambda^2 + n^2 \right] \right. \\ &\quad \left. - 2n^4 + n^2 - (3 - \nu) n^2 \lambda^2 + (1 - \nu) \lambda^2 \right\}, \\ -C_3 &= (n^2 - 1)(\lambda^2 + n^2)^2 + \nu \lambda^4 + (\mu - 1) \lambda^2 (n^2 - 1) \left(\lambda^2 + \frac{2}{1 - \nu} n^2 \right), \end{aligned}$$

and

$$C_4 = \lambda^2 \left[\mu \lambda^4 + 2n^2 \lambda^2 \left(\frac{\mu - \nu}{1 - \nu} \right) + n^4 + 2 \lambda^2 \left(\frac{\mu - \nu^2}{1 - \nu} \right) + n^2 \right].$$

In these equations the small order terms α^2 , $\alpha \phi_1$, $\alpha \phi_2$, ϕ_1^2 , etc., have been neglected.

It is found that for the case of no axial stiffening these equations do not reduce exactly to those given by Timoshenko⁴. The reason for this is that Timoshenko's solution is based on the equilibrium equations derived by Flügge which have small differences from the equilibrium equations 2.01. These differences are of little consequence and should not affect the solution for the critical axial stress. The evaluation of the critical axial stress, however, from equation 4.01, is difficult and further simplifying assumptions would appear desirable to obtain a closed form solution.

4.1. Panel buckling in a short cylinder (i. e. $\ell \ll R$) under axial load ($\beta = 1$)

For a reinforced cylinder with no internal pressure equation 4.01 becomes

$$C_1 + C_2 \alpha = C_4 \phi_2 \quad (4.02)$$

The problem is to find the values of λ and n which correspond to a minimum value of the axial loading parameter ϕ_2 . If it is assumed that $\beta = 1$ while $\mu > 1$, it is implied that even with heavy axial stiffening the value of n producing this critical ϕ_2 will correspond to, or be a multiple of, the number of stringers, where each acts as a nodal line and does not bend.

It is found that equation 4.02 reduces to the following form

$$\phi_2 = \frac{\lambda^4 (\mu - \nu^2) + \alpha (n^2 + \lambda^2)^2 F_{n\lambda}}{\lambda^2 F_{n\lambda}} \quad (4.03)$$

where

$$F_{n\lambda} = (\mu^2 + \lambda^2)^2 + (\mu - 1)\lambda^2 \left(\lambda^2 + \frac{2n^2}{1 - \nu} \right)$$

In equation 4.03 low order terms in λ^2 have been neglected which is justifiable for the case of the short cylinder, provided that $\ell \ll R$ and $\ell \gg \sqrt{Rt}$.

By differentiating ϕ_2 with respect to both λ and n , and equating to zero, one obtains the two equations,

$$F_{n\lambda}^2 = \frac{(\mu - \nu^2)}{\alpha(\lambda^4 - n^4)} \lambda^4 \left\{ 2\lambda^2 \left[n^2 + \lambda^2 + (\mu - 1)\left(\lambda^2 + \frac{n^2}{1 - \nu}\right) \right] - F_{n\lambda} \right\}, \quad (4.04)$$

and

$$F_{n\lambda}^2 = \frac{(\mu - \nu^2)\lambda^4}{\alpha(n^2 + \lambda^2)} \left[(n^2 + \lambda^2) + (\mu - 1) \frac{\lambda^2}{1 - \nu} \right] \quad (4.05)$$

When n is given by the particular stringer spacing, equation 4.04 yields the value of λ corresponding to the critical value of ϕ_2 . Solution of this equation is not attempted here. If however the stringer spacing is initially unknown, then the satisfaction of equations 4.04 and 4.05 simultaneously, results in a lower critical value of ϕ_2 .

From equations 4.04 and 4.05 the simple relationship is obtained between n and λ which is

$$n^2 = \nu \lambda^2. \quad (4.06)$$

It is interesting to note that this relationship is independent of the axial reinforcement parameter μ . Substitution of equation 4.06 into equation 4.05 gives

$$\lambda^2 = \sqrt{\frac{1 - \nu^2}{\alpha(1 + \nu)^4}}, \quad (4.07)$$

and

$$n^2 = \nu \sqrt{\frac{(1 - \nu^2)}{\alpha(1 + \nu)^4}}.$$

Again this result is independent of μ and corresponds with Timoshenko's result for an axially loaded short cylinder which suggests that a minimum ϕ_2 occurs when

$$\frac{(n^2 + \lambda^2)^2}{\lambda^2} = \sqrt{\frac{1 - \nu^2}{\alpha}}. \quad (4.08)$$

It is perhaps worth noting that by assuming axial stiffening, i. e. $\mu > 1$, one obtains explicit values for λ and n corresponding to a minimum value of ϕ_2 even though these values are independent of μ .

For the unstiffened shell however, the only result which is obtainable is in the form of equation 4.08.

Substitution of equations 4.07 into 4.03 gives the relationship

$$\phi_2 = 2 \sqrt{\alpha(1 - \nu^2)}.$$

This result agrees exactly with the classical solution for an unstiffened cylinder which shows that panel buckling in a reinforced shell occurs at the same value of resultant force as in an unreinforced shell, provided that equations 4.07 are satisfied.



4.2. Buckling in a long cylinder under axial load

For a long cylinder it is reasonable to assume that λ becomes small in the general buckling solution, equation 4.01, so that high order terms in λ may be neglected. Hence if β is small $\left(< \frac{1}{\lambda^4} \right)$

$$\lambda^2 n^2 (n^2 + 1) \phi_2 = \lambda^4 (\mu - \nu^2) + \alpha \left[n^8 + 4n^6 \lambda^2 - \lambda^2 n^4 (7 + \nu) - 2n^6 + n^4 + \lambda^2 n^2 (3 + \nu) \right] + \alpha (\beta - 1) n^4 \lambda^4 + \alpha (\mu - 1) \left[\frac{2 \lambda^2 n^2}{1 - \nu} (n^2 - 1)^2 \right] \quad (4.09)$$

For the case when $n = 1$ then

$$\phi_2 = \frac{\lambda^2}{2} (1 - \nu^2) \left[1 + \frac{E_s t_s}{Et} + \frac{\alpha (\beta - 1)}{(1 - \nu^2)} \right],$$

and the axial force/in. becomes

$$N_x = \frac{\lambda^2}{2} \left[Et + E_s t_s + \frac{Et \alpha (\beta - 1)}{(1 - \nu^2)} \right].$$

If $E_s = E$, the critical stress becomes

$$\sigma = \frac{\pi^2 ER^2}{2 \ell^2} \left[1 + \frac{\alpha t}{t + t_s} \frac{(\beta - 1)}{(1 - \nu^2)} \right].$$

This corresponds to Euler's formula for a stiffened shell buckling as a strut.

If $n > 1$, the value of λ which makes ϕ_2 a minimum in equation 4.09 is found from the relationship

$$\lambda^4 = \frac{\alpha n^4 (n^2 - 1)^2}{(\mu - \nu^2) + \alpha (\beta - 1) n^4} \quad (4.10)$$

When β and μ are both unity, this equation reduces to the form given by Timoshenko from a much simplified version of equation 4.09.

Substitution of 4.10 into 4.09 gives

$$\phi_2 = \frac{2(n^2 - 1)}{n^2 + 1} \alpha^{\frac{1}{2}} \left[(\mu - \nu^2) + \alpha (\beta - 1) n^4 \right]^{\frac{1}{2}} + \frac{\alpha}{n^2 + 1} \left[4n^4 - (7 + \nu) n^2 + (3 + \nu) \right]. \quad (4.11)$$

Timoshenko has justified the omission of the terms $4n^4 - (7 + \nu)n^2 + (3 + \nu)$ and by making a similar assumption, equation 4.11 reduces to that of Timoshenko for the unstiffened shell.

It is seen by inspection that ϕ_2 is a minimum when $n = 2$ and is

$$\phi_2 = \frac{6}{5} \left[\alpha (\mu - \nu^2) + \alpha^2 (\beta - 1)16 \right]^{\frac{1}{2}} \quad (4.12)$$

4.3. Buckling in a long cylinder under axial load and internal pressure

The assumptions and analysis will be similar to Section 4.2, and equation 4.01 reduces to become identical to equation 4.09, except that in this case an additional pressure dependent quantity appears on the right hand side. This quantity can be shown to be

$$\phi_1 (n^2 - 1)n^4 .$$

If $n = 1$, this term disappears and the obvious result is obtained, that the internal pressure has no effect on Euler buckling.

For $n > 1$ the value of λ which makes ϕ_1 a minimum is found from equations 4.10 with the following additional term in the numerator

$$\phi_1 n^4 (n^2 - 1) .$$

Hence it can be concluded that the effect of internal pressure is to reduce the axial wavelength of the buckle. A similar result has been shown by Thielemann.

5. The overall buckling of a heavily stiffened cylinder ($\beta \gg 1$)

Recently McKenzie³ has obtained a solution for the problem of the heavily reinforced cylinder subjected to an axial load and internal pressure. This solution was for the case of a cylinder having premature buckling of the panels so that some of the skin terms could be neglected in the equilibrium equations.

An order of magnitude analysis, using McKenzie's shell parameters, suggests that the full determinantal equation 2.06 can be reduced to the following

$$\begin{vmatrix} \mu\lambda^2 + \frac{1-\nu}{2}n^2 & , & \left(\frac{1+\nu}{2}\right)\lambda n & , & [v\lambda] \\ \left(\frac{1+\nu}{2}\right)\lambda n & , & (1-\frac{\nu}{2})\lambda^2 + n^2 & , & n \\ [v\lambda] & , & n & , & 1 + \alpha\beta\lambda^4 + \alpha n^4 + 2\alpha n^2\lambda^2 + \phi_1 n^2 - \phi_2 \lambda^2 \end{vmatrix} = 0 \quad \dots\dots (5.01)$$

The solution becomes

$$\phi_2 = \frac{\lambda^2 (\mu - \nu^2)}{\mu\lambda^4 + 2\lambda^2 n^2 \frac{(\mu - \nu)}{1 - \nu} + n^4} + \phi_1 \frac{n^2}{\lambda^2} + (\alpha\beta\lambda^4 + \alpha n^4 + 2\alpha n^2\lambda^2) / \lambda^2 \quad \dots\dots (5.02)$$

Equation 5.02 was solved graphically using the previously assumed shell parameters, from Ref. 3.

The minimum value of ϕ_2 is 1.37×10^{-2} , and corresponds to values of $\lambda = 4$ and $n = 7$.

The solution of the full determinantal equation 2.06 gave $\phi_2 = 1.33 \times 10^{-2}$ which compares favourably with the approximate solution above. The value for ϕ_2 obtained by McKenzie was

$$\begin{aligned} \phi_2 &= .607 \times 10^{-2} && \text{(Exact solution based on full determinant)} \\ \text{or } \phi_2 &= .702 \times 10^{-2} && \text{(Solution based on a simplified equation).} \end{aligned}$$

It would seem that the principal difference between this solution and that offered by McKenzie is in the definition of the axial force/in. N_x . The value used in this analysis, i. e. equation 2.02, was

$$N_x = E_s t_s \epsilon_1 + \frac{Et}{1 - \nu^2} (\epsilon_1 + \nu\epsilon_2) , \quad (5.03)$$

and the corresponding value used by McKenzie was

$$N_x = E_s t_s (\epsilon_1 + \bar{H} \frac{d^2 w}{dx^2}) , \quad (5.04)$$

where \bar{H} is the distance between the centroid of the stringer cross section and the median plane of the skin. It is seen that equation 5.04 neglects the skin terms in 5.03 but includes an additional curvature correction term. However, this term vanishes when the stringer material is symmetrically disposed as assumed in Section 2. This curvature term in McKenzie's analysis gives a large negative number replacing the square-bracketed terms in equation 5.01 and it can be shown that it is these terms which mainly cause the difference in results between equation 5.02 and the corresponding result of Ref. 3.

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