CoA Note No. 104

THE COLLEGE OF AERONAUTICS

CRANFIELD

SUPERSONIC FLUTTER OF CYLINDRICAL SHELLS

by

D. J. JOHNS

Note No. 104
July, 1960.

THE COLLEGE OF AERONAUTICS

CRANFIELD

Supersonic Flutter of Cylindrical Shells

- by -

D. J. Johns, M.Sc.(Eng.), M.I.A.S.

SUMMARY

The general theories of thin elastic cylindrical shells as derived by Love and Novozhilov are compared and then used in a simple binary flutter analysis which permits the existence of both axial and circumferential waves of deformation. Linear piston theory has been used and the results obtained indicate that the axi-symmetric mode of deformation is the most critical. Comparisons are then made with other published results and apparent inconsistencies in those papers are found to arise from certain assumptions made in the deformation equations used. In a further axisymmetric mode analysis the use of a travelling wave form of radial deflection is shown to give similar results as standing wave forms when applied to a shell of finite length.

LIST OF CONTENTS

NOTATION

a	Radius of cylinder
A, B, C	Modal coefficients in expressions for u, v, w
c	Speed of sound in air
D	$= Eh^3/12(1 - \nu^2)$ Flexural rigidity/unit width of shell
E	Young's Modulus
F	Function defined after equation 3.6
h	Shell thickness
i	$\sqrt{-1}$
L	Shell length between supports
m	Number of axial half waves
M	Mach No.
n	Number of circumferential waves
p	Wave number of axial waves in infinitely long cylinder
q	Dynamic pressure
r, s	Number of axial half waves in assumed modes
u, v, w	Axial, tangential and radial displacements of the middle surface of the shell (Sign Convention as in Ref. 13)
U	Velocity of Supersonic Flow
$\bar{V}$	Flutter speed parameter from Ref. 6 $= \psi\alpha/\pi^4(1 - \nu^2)$
x	Axial co-ordinate measured in a streamwise direction
Z	Lateral external pressure on the shell

Notation (Continued)

α	Shell bending parameter = $h^2/12a^2$
β	$(M^2 - 1)^{\frac{1}{2}}$
λ_m	$\frac{m\pi a}{L}$ other suffices used are r and s
ν	Poissons ratio
ρ	Density of supersonic flow
σ	Shell mass per unit surface area
θ_m	Parameter defined in equation 6.6
ϕ	Circumferential shell co-ordinate
ψ_n	Flutter speed parameter for shell with n circumferential waves $\left(= \frac{2qL^3}{MD}\right)$
Ω	Flutter frequency of oscillation
Ω_m	Vibration frequency for mode m = m, n = 0

1. Introduction

Investigations into the panel flutter of cylindrical shells with the wind direction along the cylinder axis fall into two distinct categories; relating to unstiffened, infinitely long, cylinders and cylinders stiffened by rings and longerons.

The former problem has been examined in Refs. 1 - 3 and, although these various approaches are quite dissimilar, the conclusion was reached that infinite cylinders are extremely susceptible to panel flutter. Miles (Ref. 1) showed that a type of travelling wave with a wavelength which is small compared with the radius of the cylinder, and without a node around the circumference, is the most critical type of instability. Stepanov (Ref. 3) showed a similar result using piston theory for the aerodynamic forces, but Miles in another paper (Ref. 4) questions the validity of Stepanov's work.

The question as to whether travelling wave instability based on linearised aerodynamic theory is significant for panels of finite length depends essentially on the wavelengths at which instability is predicted, and on whether an unstable travelling wave would experience sufficient growth before reaching the downstream support where it presumably would be reflected to form a standing wave. A method of dealing with the ring-stiffened cylinder starting from a travelling wave analysis is presented in Ref. 2 but no numerical results are given. Other analyses which deal specifically with standing waves on a ring-stiffened cylinder of finite length are presented in Refs. 5 - 6. These papers have been examined in considerable detail by Fung in Ref. 7 which contains a most valuable review of the entire panel flutter problem. It is stated in Ref. 7 that a major point of controversy concerns the choice of the appropriately simplified equations governing the deformations of the thin walled elastic shell, and related to this is the choice of flutter mode and the question as to how the critical flutter speed depends upon the number of circumferential and axial waves in that mode.

To avoid computational difficulties in using the general theory for cylindrical shells, which reduce to three simultaneous equations in terms of the axial, tangential and radial deformation components (u, v, w) recourse may be had to the well known Flugge's equations for thin cylinders (Ref. 8) which give way successively to the Donnells equations (Ref. 9), the shallow-shell equations (Ref. 10) and the Goldenveiser equations for cylinders of medium length (Ref. 11).

Flugge's equations were used in Ref. 2 as were also Donnell's equations, but no numerical results were given for the flutter of ring-stiffened cylinders. Donnell's equations were used in Ref. 6 and the existence of a minimum flutter speed was demonstrated for a flutter mode containing both axi al and circumferential waves. This result is rather surprising since it can be inferred from Refs. 3 and 5, which both used Goldenveiser's equation, that the most critical flutter mode is axi-symmetric. The accuracy of the Goldenveiser equation is suspect however since it neglects axial bending stiffness of the shell, whereas Donnell's equation is inaccurate when the condition $n^2 >> 1$ is violated, i.e. n must be greater than 3.

Thus it can be seen that there is a need to derive more general flutter analyses for ring-stiffened cylinders of finite length, without the limiting assumptions of previous papers, and it is the purpose of this present paper to start from the general theories of cylindrical shells developed by Love and Novozhilov and to determine the critical flutter speed as a function of the numbers of axial and circumferential waves. An attempt will also be made to apply a travelling wave analysis to a cylinder of finite length in the manner outlined in Ref. 2. Linear piston theory will be used in the analyses and only loadings normal to the shell surface will be considered. Although the analyses will only strictly apply to thin shells with no axial stiffening i.e. no longerons, the effects of such stiffeneing will be discussed briefly.

2. General Theory of Cylindrical Shells

To establish the differential equations for the displacements, u, v and w, which define the deformation of a shell the following procedure is followed. The equations of equilibrium of forces and moments (six in all) are derived for an element of the thin shell in terms of the deformations of the element. These general equations were obtained by Love (Ref. 12) and are presented by Timoshenko (Ref. 13). If the ssumption is then made that the membrane forces in the shell are much smaller than the critical values required to produce lateral buckling of the shell the following three equations can eventually be obtained (Ref. 13 eq. 303).

$$\frac{\partial^2 u}{\partial x^2} + \frac{1-\nu}{2a^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{1+\nu}{2a}\frac{\partial^2 v}{\partial x\,\partial \phi} - \frac{\nu}{a}\frac{\partial w}{\partial x} = 0 \qquad (2.1)$$

$$\frac{1+\nu}{2a}\frac{\partial^2 u}{\partial x\,\partial \phi} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1}{a^2}\frac{\partial^2 v}{\partial \phi^2} + \alpha\left[(1-\nu)\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{a^2\partial \phi^2}\right]$$

$$- \frac{1}{a^2}\frac{\partial w}{\partial \phi} + \alpha\left[\frac{\partial^3 w}{\partial x^2\partial \phi} + \frac{\partial^3 w}{a^2\,\partial \phi^3}\right] = 0 \qquad (2.2)$$

$$-\frac{\nu}{a}\frac{\partial u}{\partial x} - \frac{1}{a^2}\frac{\partial v}{\partial \phi} + \alpha\left[(2-\nu)\frac{\partial^3 v}{\partial x^2\partial \phi} + \frac{\partial^3 v}{a^2\partial \phi^3}\right] + \frac{w}{a^2} +$$

$$\alpha\left(a^2\frac{\partial^4 w}{\partial x^4} + \frac{2\,\partial^4 w}{\partial x^2\partial \phi^2} + \frac{\partial^4 w}{a^2\partial \phi^2}\right) = \frac{Z(1-\nu^2)}{Eh} \qquad (2.3)$$

where $\alpha = h^2/12a^2$.

It should be emphasised that, according to Novozhilov (Ref. 14) the development given by Love is not free from inadequacies with regard to certain small terms, some of which are retained, and others, which are of the same order of magnitude, are rejected. This point is suggested also by the lack of symmetry of the terms in equations 2.1 - 2.3. If the method formulated by Novozhilov is followed the equations obtained are,

$$\frac{\partial^2 u}{\partial x^2} + \frac{1-\nu}{2a^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{1+\nu}{2a}\frac{\partial^2 v}{\partial x\,\partial \phi} - \frac{\nu}{a}\frac{\partial w}{\partial x} = 0 \qquad (2.4)$$

$$\frac{1+\nu}{2a}\frac{\partial^2 u}{\partial x\,\partial \phi} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1}{a^2}\frac{\partial^2 v}{\partial \phi^2} + \alpha\left[2(1-\nu)\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{a^2\partial \phi^2}\right]$$

$$- \frac{1}{a^2}\frac{\partial w}{\partial \phi} + \alpha\left[(2-\nu)\frac{\partial^3 w}{\partial x^2\partial \phi} + \frac{\partial^3 w}{a^2\partial \phi^3}\right] = 0 \qquad (2.5)$$

$$-\frac{\nu}{a}\frac{\partial u}{\partial x} - \frac{1}{a^2}\frac{\partial v}{\partial \phi} + \alpha\left[(2-\nu)\frac{\partial^3 v}{\partial x^2\partial \phi} + \frac{\partial^3 v}{a^2\,\partial \phi^3}\right] + \frac{w}{a^2}$$

$$+ \alpha\left(a^2\frac{\partial^4 w}{\partial x^4} + \frac{2\,\partial^4 w}{\partial x^2\partial \phi^2} + \frac{\partial^4 w}{a^2\,\partial \phi^4}\right) = \frac{Z(1-\nu^2)}{Eh} \qquad (2.6)$$

The only changes in these equations compared with equations 2.1 - 2.3 occur in the square bracketed α terms in equation 2.2. By inspection of the terms in equations 2.2 and 2.5 it can be shown that the square bracketed α terms in v are negligible, and Vlasov (Ref. 15) has suggested that all the square bracketed α terms in equations 2.2 - 2.6 are small quantities of the same order as those which were disregarded in the derivation of equations 2.2 - 2.6. An order of magnitude comparison of all the terms in equations 2.2 - 2.6 does not support this suggestion but if the appropriate terms are neglected the following set of equations are obtained from the Love and Novozhilov developments,

$$\frac{\partial^2 u}{\partial x^2} + \frac{1-\nu}{2a^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{1+\nu}{2a}\frac{\partial^2 v}{\partial x \partial \phi} - \frac{\nu}{a}\frac{\partial w}{\partial x} = 0 \qquad (2.7)$$

$$\frac{1+\nu}{2a}\frac{\partial^2 u}{\partial x \partial \phi} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1}{a^2}\frac{\partial^2 v}{\partial \phi^2} - \frac{1}{a^2}\frac{\partial w}{\partial \phi} = 0 \qquad (2.8)$$

$$-\frac{\nu}{a}\frac{\partial u}{\partial x} - \frac{1}{a^2}\frac{\partial v}{\partial \phi} + \frac{w}{a^2} + \alpha\left(a^2\frac{\partial^4 w}{\partial x^4} + \frac{2\,\partial^4 w}{\partial x^2 \partial \phi^2} + \frac{\partial^4 w}{a^2 \partial \phi^4}\right) = \frac{Z(1-\nu^2)}{Eh} \qquad (2.9)$$

It can easily be shown that for the case of axi-symmetric deformations the above equations reduce to the simple result

$$D\frac{d^4 w}{dx^4} + \frac{Ehw}{a^2} = Z \qquad (2.10)$$

3. Standing Wave Flutter Analysis I

This first analysis is concerned with flutter of a cylindrical shell of length L, in which the simplified equations 2.7 - 2.9 will be used. The flutter mode assumed corresponds to that of a simply-supported shell and is defined by the expressions

$$u = \overline{U}_{(x,\phi)} \; e^{i\Omega t},$$

$$v = \overline{V}_{(x,\phi)} \; e^{i\Omega t},$$

and $$w = \overline{W}_{(x,\phi)} \; e^{i\Omega t},$$

where $$\overline{U} = \sum\sum A_{mn} \cos n\phi \cos \frac{\lambda_m x}{a} , \qquad (3.1)$$

$$\overline{V} = \sum\sum B_{mn} \sin n\phi \sin \frac{\lambda_m x}{a} , \qquad (3.2)$$

and $$\overline{W} = \sum\sum C_{mn} \cos n\phi \sin \frac{\lambda_m x}{a} , \qquad (3.3.)$$

When these expressions are substituted into equations 2.7 and 2.8 one obtains, after considerable manipulation,

$$A_{mn} = C_{mn} \; \lambda_m \left[(1-\nu)n^4 + (2-\nu+\nu^2)\lambda_m^2 n^2 - 2\nu \; \lambda_m^4 \right] \Big/ 2\,(\lambda_m^2 + n^2)^2 (\lambda_m^2 + \frac{1-\nu}{2} n^2) ,$$

and

$$B_{mn} = C_{mn} \; n \; \left[n^2 + (2+\nu)\lambda_m^2 \right] \Big/ (\lambda_m^2 + n^2)^2 ,$$

and inclusion of these expressions into equation 2.9 results in

$$\sum \sum C_{mn} \cos n\phi \sin \frac{\lambda_m x}{a} \left[\alpha (\lambda_m^2 + n^2)^4 + \lambda_m^4 (1-\nu^2) \right] \Big/ (\lambda_m^2 + n^2)^2 = \frac{Z \, a^2 (1-\nu^2) \, e^{-i\Omega t}}{Eh} \qquad (3.4)$$

For the flutter problem the effective, external, lateral pressure Z contains aerodynamic terms and oscillatory inertia terms proportional to the shell mass. If linear piston theory is used to derive the aerodynamic forces, equation 3.4 becomes, for the mn th mode,

$$\frac{Eh}{a^2(1-\nu^2)}\left[\frac{\alpha(\lambda_m^2+n^2)^4+\lambda_m^4(1-\nu^2)}{(\lambda_m^2+n^2)^2}\right]\cos n\phi \sin\frac{\lambda_m x}{a}$$

$$= -\left[\frac{\rho U^2}{M}\frac{\lambda_m}{a}\cos n\phi\cos\frac{\lambda_m x}{a} + \left(\frac{\rho U i\Omega}{M} - \sigma\Omega^2\right)\cos n\phi\sin\frac{\lambda_m x}{a}\right] \qquad (3.5)$$

where U and M are the velocity and Mach Number of the supersonic flow passing in an axial direction along the external surface of the shell. If only two degrees of freedom are considered m = r and m = s with n fixed in each, then by applying the Galerkin process to equation 3.5 the following flutter determinant is obtained,

$$\begin{vmatrix} F_r + \frac{\rho U i\Omega}{M} - \sigma\Omega^2 , & \frac{\rho U^2}{LM}\cdot\frac{4rs}{s^2-r^2} \\ -\frac{\rho U^2}{LM}\,\frac{4rs}{s^2-r^2} , & F_s + \frac{\rho U i\Omega}{M} - \sigma\Omega^2 \end{vmatrix} = 0 , \qquad (3.6)$$

where $F_r = \frac{Eh}{a^2(1-\nu^2)}\left[\frac{\alpha(\lambda_r^2+n^2)^4+\lambda_r^4(1-\nu^2)}{(\lambda_r^2+n^2)}\right]$, with a similar expression for F_s, and the condition that (r + s) must be odd applies.

When the determinant is expanded, the imaginary and real terms can be separated giving expressions for the flutter frequency and speed respectively,

$$2\sigma\Omega^2 = F_r + F_s \qquad (3.7)$$

$$4\left(\frac{\rho U^2}{LM}\right)^2\left(\frac{4rs}{r^2-s^2}\right)^2 = (F_r - F_s)^2 + 4\left(\frac{\rho U\Omega}{M}\right)^2 . \qquad (3.8)$$

The term $\left(\frac{\rho U\Omega}{M}\right)$ represents the effects of aerodynamic damping which can probably be neglected as was justified in Ref. 16.

If this is done, equation 3.8 becomes after some manipulation, with $r > s$

$$\psi_n = \frac{2qL^3}{MD} = \left(\frac{L}{a}\right)^4 \left(\frac{r^2 - s^2}{8\ rs}\right) \Bigg[(\lambda_r^2 - \lambda_s^2)(2n^2 + \lambda_r^2 + \lambda_s^2) + \left(\frac{1 - \nu^2}{\alpha}\right) \left\{ \frac{\lambda_r^4}{\left(n^2 + \lambda_r^2\right)^2} - \frac{\lambda_s^4}{\left(n^2 + \lambda_s^2\right)^2} \right\} \Bigg] \qquad (3.9)$$

When $n = 0$ the result is obtained for axi-symmetric flutter, viz.

$$\psi_o = \frac{\pi^4}{8} \frac{(r^2 - s^2)^2 (r^2 + s^2)}{r\ s}, \qquad (3.10)$$

and equation 3.9 may be rewritten as

$$\psi_n = \psi_o + \left(\frac{L}{a}\right)^4 \left(\frac{r^2 - s^2}{8\ rs}\right) \Bigg[2n^2 (\lambda_r^2 - \lambda_s^2) + \left(\frac{1 - \nu^2}{\alpha}\right) \left\{ \frac{\lambda_r^4}{\left(n^2 + \lambda_r^2\right)^2} - \frac{\lambda_s^4}{\left(n^2 + \lambda_s^2\right)^2} \right\} \Bigg] \qquad (3.11)$$

It was pointed out in Ref. 16 that if aerodynamic damping can be neglected then "static" aerodynamic forces given by Ackeret are more accurate than those given by piston theory. The modification involves replacing M in ψ by $\beta = (M^2 - 1)^{\frac{1}{2}}$ and should lower the Mach No. range of applicability of equations 3.9 - 3.11.

3.1. Discussion of results

It is seen that the most critical value of ψ_n is ψ_o since the second term on the right hand side of equation 3.11 is always positive, and hence the most critical flutter mode is axi-symmetric. The most critical combination of modes r and s giving the lowest flutter speed was investigated in Ref. 17, which used equation 2.10 to define the elastic deformation of the shell, and found to be r = 2 and s = 1 whence $\psi_o = 274$.

It can also be seen that the second term on the right hand side of equation 3.11 has a minimum value at some value of n other than zero. By equating $\frac{d\psi_n}{dn}$ to zero and assuming that $n^2 \gg \lambda^2$ the following expression is obtained

$$n^6 = (r^2 + s^2) \frac{\pi^2 a^2}{L^2} \left(\frac{1 - \nu^2}{\alpha} \right) \qquad (3.12)$$

Shulman (Ref. 6) demonstrated the existence of a minimum flutter speed for $n \neq 0$ using Donnell's equations and linear piston theory in an analysis employing 8 modes. For a shell having the properties $\frac{h}{a} = \frac{1}{200}$ and $\frac{L}{a} = 6$ Shulman quoted a critical value of n = 8 and a minimum value of $\psi_n = 7.85 \times 10^4$. If the above shell parameters are substituted into equation 3.12 corresponding values of n = 9 and $\psi_n = 5 \times 10^4$ (when r = 2, s = 1) are obtained which are in reasonable agreement with Shulman's results. Table 1 gives numerical values of ψ_n for the above shell using equation 3.11 and also given are the corresponding values of $\bar{V}$ with Shulman's values for comparison. The agreement is good and the results indicate most strongly the fact that axi-symmetric flutter is the most critical mode. (Note $\bar{V} = \psi \alpha / \pi^4 (1 - \nu^2)$).

If the first term on the right hand side of equation 3.9 is neglected - which is equivalent to neglecting the α term or the bending stiffness of the shell - and if it is assumed that $n \gg \lambda^2$, the following result is obtained.

$$\psi_n = \psi_o \frac{12(1 - \nu^2)}{n^4} \left(\frac{a}{h} \right)^2 \qquad (3.13)$$

The above assumption, regarding the neglect of the bending stiffness, is the same as that made in the Goldenveiser's equation which was used in Ref. 5 and equation 3.13 corresponds to the result presented in Ref. 5. This suggests that ψ_n decreases as n increases and that $\psi_n < \psi_o$ for large n. This is obviously false since from equation 3.11 and Table 1 it is seen that $\psi_n >> \psi_o$ for large n.

4. Standing Wave Flutter Analysis II

In Section 3 the simplified elastic equations 2.7 - 2.9 were used and useful results have been obtained, but a further analysis is warranted to justify the omission of the various square bracketed α terms in equations 2.1 - 2.6. Since the neglect of the term in v in equations 2.2 and 2.5 can be easily justified the analysis of Section 3 has been repeated, including in equations 2.7 - 2.9 only the symmetric terms due to Novozhilov, i.e.

$$\alpha \left[(2-\nu) \frac{\partial^3 w}{\partial x^2 \partial \phi} + \frac{\partial^3 w}{a^2 \partial \phi^3} \right] \quad \text{in equation 2.8}$$

$$\text{and} \quad \alpha \left[(2-\nu) \frac{\partial^3 v}{\partial x^2 \partial \phi} + \frac{\partial^3 v}{a^2 \partial \phi^3} \right] \quad \text{in equation 2.9}$$

Following the procedure outlined in Section 3 the various corresponding results are,

$$A_{mn} = C_{mn} \ \lambda_m \left[\frac{(1-\nu)n^4 + (2-\nu+\nu^2)n^2\lambda_m^2 - 2\nu\lambda_m^4 + 2\alpha(1+\nu)}{2(\lambda_m^2+n^2)^2 \left(\lambda_m^2 + \frac{1-\nu}{2}n^2\right)} \right.$$

$$\left. \left[\left\{ (2-\nu)\lambda_m^2 + n^2 \right\} \left\{ \lambda_m^2 + \frac{1-\nu}{2} n^2 \right\} \frac{n^2}{(1-\nu)} \right] \right] ,$$

$$B_{mn} = C_{mn} \ n \left[n^2 + (2+\nu)\lambda_m^2 + 2\alpha \left\{ (2-\nu)\lambda_m^2 + n^2 \right\} \left\{ \lambda_m^2 + \frac{1-\nu}{2} n^2 \right\} \left(\frac{1}{1-\nu} \right) \right] \Big/ \left(\lambda_m^2 + n^2 \right)^2 ,$$

and F_r becomes

$$\frac{Eh}{a^2(1-\nu^2)} \left[\alpha \left\{ (\lambda_r^2 + n^2)^4 - 2n^2 \left[(2-\nu)\lambda_r^2 + n^2 \right] \left[(2+\nu)\lambda_r^2 + n^2 \right] \right\} + \lambda_r^4 (1-\nu^2) \right] \Big/ (\lambda_r^2 + n^2)$$

if terms in α^2 are neglected.

The corresponding result to equation 3.11 is

$$\psi_n = \psi_o + \left(\frac{L}{a}\right)^4 \left(\frac{r^2 - s^2}{8\ rs}\right) \left[2n^2 (\lambda_r^2 - \lambda_s^2) - 2n^2 \left\{ \frac{(4 - \nu^2)\lambda_r^2 + 4n^2\lambda_r^2 + n^4}{(n^2 + \lambda_r^2)^2} - \frac{(4 - \nu^2)\lambda_s^4 + 4n^2\lambda_s^2 + n^4}{(n^2 + \lambda_s^2)^2} \right\} + \left(\frac{1 - \nu^2}{\alpha}\right) \left\{ \frac{\lambda_r^4}{(n^2 + \lambda_r^2)^2} - \frac{\lambda_s^4}{(n^2 + \lambda_s^2)^2} \right\} \right] \qquad (4.1)$$

and it can be seen immediately that the additional terms do not affect the original conclusion that n = 0 gives the most critical mode. Also as $n \to \infty$ the additional terms vanish and the general statement may be made that the neglect of the square bracketed α terms in Section 3 was justified. To support this statement Shulman's shell parameters have been substituted into equation 4.1 and Table 1 shows the small changes in the values of ψ_n caused by the additional terms.

5. Travelling Wave Flutter Analysis: Infinite Length Cylinder

Miles (Ref. 1) showed that an axisymmetric flutter mode was the most critical for infinitely long shells, using a travelling wave analysis, and a similar result was shown in Sections 3 and 4 for finite length shells with a standing wave analysis. In this section therefore, only a simplified axi-symmetric analysis will be presented using equation 2.10 reproduced below,

$$D \frac{d^4 w}{d x^4} + \frac{Ehw}{a^2} = Z \qquad (2.10)$$

If it is assumed that the cylinder wall may deform into any number of sinusoidal waves of any wavelength and constant amplitude along its length, and that the motion is simple harmonic in time, then the radial deflection of the wall maybe written

$$w = \mathrm{Re}\left[C e^{-ip\left(x - \frac{\Omega t}{p}\right)} \right] \qquad (5.1.)$$

where C is the complex amplitude of the motion and p is the real wave number of the longitudinal waves.

Substiuting for Z and w into equation 2.10 yields the result,

$$Dp^4 + \frac{Eh}{a^2} = + \frac{\rho u^2}{M} ip - \frac{\rho u}{M} i\Omega + \sigma \Omega^2 \qquad (5.2)$$

By separating real and imaginery terms one obtains,

$$U = \frac{\Omega}{p} , \qquad (5.3)$$

and

$$Dp^4 + \frac{Eh}{a^2} = \sigma \Omega^2 , \qquad (5.4)$$

and substituting for Ω into equation 5.4 gives

$$U^2 = \left[Dp^2 + Eh/a^2 p^2 \right] / \sigma . \qquad (5.5)$$

The minimum value of U occurs when $p^4 = \frac{Eh}{a^2 D}$

and

$$U^2_{min} = Eh^2 / \sigma a \left[3(1 - \nu^2) \right]^{\frac{1}{2}} . \qquad (5.6)$$

This result agrees with that obtained by Stepanov (Ref. 3), who also used piston theory, from a rather different method of analysis. It should however be pointed out that the assumed deformation of equation 5.1 has the form of a wave travelling in the positive x direction with velocity $\frac{\Omega}{p}$, hence the flow of velocity U outside the vibrating cylinder is equivalent to a flow of velocity $U - \frac{\Omega}{p}$ outside the "stationary" cylinder. However it is seen that $\left(U - \frac{\Omega}{p}\right)$ is zero from equation 5.3 and we have an apparent inconsistency since for piston theory to be valid $U - \frac{\Omega}{p} >> c$, the speed of sound. This has been pointed out recently in Ref. 4 and is a consequence of limitations inherent in the use of piston theory when applied to travelling wave motions.

6. Travelling Wave Flutter Analysis: Finite Length Cylinder

In Ref. 2 a method of analysis was proposed for extending the travelling wave analysis to a cylinder of finite length. That method will be applied here to the problem of axi-symmetric flutter only, using equation 2.10. The configuration to be considered consists of an unstiffened cylinder with added rigid ring stiffeners which prevent radial deflections at the locations $x = \pm jL (j = 0, 1, 2, \ldots)$. As in sections 3 and 4 these stiffeners, whose positions define the length of the shell bays, are assumed not to interfere with the external flow of air, and the aerodynamic forces are given by linear piston theory.

The radial deflection of the wall may be written

$$w = Re \sum_{-\infty}^{\infty} C_m e^{-i\frac{m\pi x}{L}} e^{i\Omega t} \qquad (6.1)$$

provided that the coefficients C_m satisfy the constraining relations, which correspond to zero deflection at the ring locations

$$\sum_{m=-\infty \,(m \text{ odd})}^{\infty} C_m = 0 \qquad (6.2)$$

$$\sum_{m=-\infty \,(m \text{ even})}^{\infty} C_m = 0 \qquad (6.3)$$

It is assumed in equation 6.1 that w is periodic over two bay lengths in the x direction and, consequently equation 6.3 corresponds to motion which is identical in each bay, whereas equation 6.2 corresponds to motion having the same amplitude from bay to bay but with alternating direction.

In using equation 6.1 in equation 2.10 allowance must be made for the reaction forces exerted on the cylinder wall by the ring stiffeners at the end of each bay. If this is done and if equation 2.10, suitably modified, is multiplied by $e^{\frac{i m\pi x}{L}}$ and integrated over 2 bay lengths of the cylinder the resultant expressions obtained from the constraining relations of equations 6.2 and 6.3 are

$$\sum_{(m \text{ odd})\, m=-\infty}^{\infty} \frac{1}{\Theta_m} = 0 \qquad (6.4)$$

$$\sum_{m=-\infty \,(m \text{ even})}^{\infty} \frac{1}{\Theta_m} = 0 \qquad (6.5)$$

$$\text{where } \Theta_m = \frac{\sigma L^2 h}{\pi^2 \rho c^2 a} (\Omega_m^2 - \Omega^2) + \frac{iL}{\pi a} (mM - \frac{\Omega L}{\pi c}) \qquad (6.6.)$$

$$\text{and } \Omega_m^2 = (D \frac{m^4 \pi^4}{L^4} + \frac{Eh}{a^2}) \Big/ \sigma .$$

Attempts have been made with equations 6.4 and 6.5 to obtain closed form solutions for the flutter frequency and velocity using increasingly higher approximations in the summations but only the three term approximation to equation 6.5 was satisfactory in this respect. It is worth noting that very large values of m should not influence either equation 6.4 or 6.5 significantly, since then $\frac{1}{\Theta_m} \to 0$ as $m \to \infty$. The three-term approximation to equation 6.5 may be written as $\frac{1}{\Theta_o} + \frac{1}{\Theta_2} + \frac{1}{\Theta_{-2}} = 0$,

and substitution of the appropriate forms of equation 6.6 yields, eventually, the following expressions for flutter frequency and speed,

$$\Omega^2 = \frac{1}{3} (\Omega_o^2 + 2\,\Omega_2^2) \qquad (6.7)$$

$$\frac{2qL^3}{MD} = \frac{16\,\pi^3}{\sqrt{12}} = 144 \qquad (6.8)$$

The fact that this result is more conservative than the corresponding result ($\psi_o = 274$) of Section 3 is not surprising and it would be expected that as more terms are included in the approximation to either equation 6.4 or 6.5 a less conservative result would be obtained. This decrease in conservatism with a larger number of modes was noted in analyses of rectangular panels in References 16 and 18.

Thus starting from the travelling wave form of equation 6.1 it is seen that the inclusion of boundary conditions at the ring stiffeners, i.e. zero radial deflection at $x = \pm jL$, has led to a form of solution comparable with that obtained from the standing wave analysis of Section 3. Hence there is no intrinsic difference in the two types of solution and presumably the assumption of a downstream support for the shell has led to the travelling wave being reflected to form a standing wave.

7. Effect of Axial Stiffening

The previous analyses have been for thin cylindrical shells with no additional axial stiffening, and the inclusion of such effects would, in general, necessitate modifications to the elastic and inertia terms. Since the addition of heavy stringers would seem to preclude the possibility of axi-symmetric flutter - which would require the stringers to bend, it would appear that a more critical flutter mode might then exist for some value of n when the stringers lie along circumferential nodal lines and hence do not bend. It is suggested therefore that equation 3.9 be used in this case and the value of 2n assumed (i.e. number of nodes) should correspond to the number of stringers or a multiple of it whichever is the most critical.

8. Conclusions

By starting from the general theory of deformation for thin cylindrical shells a simple result has been obtained (Equation 3.9) from a binary flutter analysis for a shell of finite length which indicates that the axi-symmetric flutter mode is the most critical. This equation reduces, after further simplifying assumptions, into forms comparable with earlier papers using Donnell's equation (Ref. 6) and Goldenveiser's equation (Ref. 5), and shows how certain apparent inconsistencies in these papers have been derived.

In further axi-symmetric flutter mode analyses the use of a travelling wave form of radial deflection has been shown to give similar results as standing wave forms for the finite length shell, and to give a very simple, but suspect, result for the infinitely long shell.

Although the analyses were for shells with no axial stiffening in the form of stringers, suggestions were made as to how such effects might be considered.

9. References

1. Miles, J.W. — Supersonic Panel Flutter of a Cylindrical Shell. Jour. Aero. Sci. Part I Vol. 24, 1957, pp 107-118; Part II Vol. 25, 1958, pp 312-316

2. Leonard, R.W., Hedgepeth, J.M. — On Panel Flutter and Divergence of Infinitely Long Unstiffened and Ring-Stiffened Thin Walled Circular Cylinders. NACA Report 1302, 1957.

3. Stepanov, R.D. — On the Flutter of Cylindrical Shells and Panels Moving in a Flow of Gas. (Translation) NACA TM. 1438, 1958.

4. Miles, J.W. — On Supersonic Flutter of Long Panels. Jour. Aero/Space Sci., Vol. 27, 1960, pp 476.

5. Strack, S.L., Holt, M. — Supersonic Panel Flutter of a Cylindrical Shell of Finite Length. IAS paper No. 60-22, 1960.

6. Shulman, Y. — Vibration and Flutter of Cylindrical and Conical Shells. O.S.R. TR. 59-776, M.I.T., 1958.

7. Fung, Y.C. — Panel Flutter. AGARD Manual on Aeroelasticity, 1959.

8. Flugge, W. — Statik und Dynamik der Schalen. Julius Springer, Berlin, 1939.

9. Donnell, L.H. — Stability of Thin-Walled Tubes Under Torsion. NACA Report 479, 1933.

10. Vlasov, V.S. — Basic Differential Equations in General Theory of Elastic Shells. NACA TM. 1241, 1951.

References (Continued)

11.	Goldenveiser, A.L.	Theory of Elastic Thin Shells. G.T.L., Moscow, 1953.
12.	Love, A.E.H.	Elasticity. 4th Edition Chap. 24, 1927 pp. 515
13.	Timoshenko, S.H. Woinowsky-Kriegar, S.	Theory of Plates and Shells. 2nd Edition. McGraw Hill Book Co., 1959.
14.	Novozhilov, V.V.	The theory of Thin Shells. Noordhoff Ltd., 1959.
15.	Vlasov, V.Z.	A General Theory of Shells. Moscow, 1949.
16.	Johns, D.J.	Some Panel Flutter Studies Using Piston Theory. Jour. Aero. Sci. Vol. 25, 1958, pp 679-684.
17.	Johns, D.J.	Supersonic Flutter of a Cylindrical Panel in an Axisymmetric mode. Jour. Roy. Aero. Soc. Vol. 64, 1960, pp 362-363.
18.	Hedgepeth, J.M.	Flutter of Rectangular Simply Supported Panels at High Supersonic Speeds. Jour. Aero. Sci. Vol. 24, 1957, pp 563-573.

TABLE 1

Critical Flutter Speed Parameters

$$\frac{h}{a} = \frac{1}{200}, \quad \frac{L}{a} = 6, \quad r = 2, \; s = 1 \; \psi_o = 274$$

n	$\psi_n \times 10^{-4}$	$\nabla \times 10^3$	$\nabla \times 10^3$	$\delta\psi_n \times 10^{-4}$
	Equation 3.11		Ref. 6	Correction Terms in Equation 4.1
1	2411	562	-	-0.040
2	450	105	-	-0.070
3	117	27.3	38.0	-0.070
4	41.5	9-67	13.1	-0.075
5	18.7	4.36	5.69	-0.080
6	10.1	2.36	3.15	-0.080
7	6.8	1.58	2.14	-0.080
8	5.42	1.26	1.83	-0.080
9	5.05	1.17	1.88	-0.080
10	5.20	1.21	2.22	-0.080
11	5.66	1.33	2.70	-
12	6.36	1.49	3.26	-