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NUMERICAL INVESTIGATION OF AN INCOMPRESSIBLE FLOW OVER A BACKWARD FACING STEP USING A UNIFIED FRACTIONAL-STEP, ARTIFICIAL COMPRESSIBILITY AND PRESSURE-PROJECTION (FSAC-PP) METHOD

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ABSTRACT

This study focuses on an incompressible and laminar flow problem behind a backward facing step by employing a recently developed Fractional-Step, Artificial Compressibility and Pressure-Projection (FSAC-PP) method. The FSAC-PP approach unifies Chorin's fully-explicit Artificial Compressibility (AC) and semi-implicit Fractional-Step Pressure-Projection (FS-PP) methods within the framework of characteristic-based (CB) Godunov-type schemes for solving the incompressible Navier-Stokes equations. The FSAC-PP approach has been originally introduced for low and moderate Reynolds number flows in conjunction with microfluidic and wide range of multiphysics applications. In this work, we demonstrate the applicability of the novel FSAC-PP method to macro-scale separated flows at a moderate Reynolds number. The computational results obtained with the FSAC-PP approach have been compared to the AC method and experimental data to highlight its favorable accuracy and convergence properties for separated flows.

1. INTRODUCTION

The numerical solution of the Navier-Stokes equations has fallen into the two categories of compressible and incompressible solvers within the field of Computational Fluid Dynamics (CFD). The absence of density changes at low Mach numbers prohibits the direct application of the Navier-Stokes equations to incompressible flows and hence researchers proposed algorithms over the past decades to circumvent this shortcoming. Chorin [1] introduced the Artificial Compressibility (AC) method where a pseudo-time derivative of the pressure is added to the continuity equation. In addition to this, Chorin [2] devised the Fractional-Step Pressure-Projection (FS-PP) method which is relying on the Helmholtz-Hodge decomposition to enforce the divergence-free (incompressibility) constraint at each time level. Patankar and Spalding [3] introduced another class of incompressible Navier-Stokes solvers which is called pressure correction algorithm. In their Semi-Implicit Method for Pressure-Linked Equations (SIMPLE), an initial pressure field is imposed and corrected via the continuity equation. Recently, a novel numerical procedure has been proposed by Könözsy [4], and Könözsy and

Drikakis [5] labeled as the FSAC-PP method to unify the advantageous features of Chorin's fully-explicit AC and semi-implicit FS-PP methods within the framework of characteristic-based (CB) Godunov-type schemes. This approach was originally developed for low and moderate Reynolds number flows in conjunction with a wide range of microfluidic and multiphysics applications. The FSAC-PP method was validated for multi-species variable density flows in a Y-junction channel [6]. The pseudo-compressibility type schemes can also be successfully applied to predict trapping and positioning of cryogenic propellants through acoustic liquid manipulation in microgravity space environment [7]. A comparative study was published by Tsoutsanis et al. [8] to assess the performance of structured, unstructured, incompressible and compressible solvers where the FSAC-PP method was capable of predicting and resolving vortical flow structures accurately even with lower-order interpolation schemes exhibiting low numerical dissipative behaviour. In the present work, we employ the FSAC-PP method to a backward facing step problem to further validate and investigate its characteristics and numerical features for a moderate Reynolds number flow with separation.

2. COMPUTATIONAL METHOD

The FSAC-PP method employs a modified set of governing equations relying on the unification of the AC and FS-PP methods of Chorin [1,2] for solving the incompressible Navier-Stokes equations. The perturbed continuity equation with a pseudo-pressure derivative term can be written in a semi-discrete form as

$$\frac{1}{\beta}\frac{p^{(n+1)}-p^{(n)}}{\Delta\tau}+\nabla\cdot\mathbf{u}^{(n)}=0, \tag{1}$$

where β is the artificial compressibility parameter which is responsible for ensuring convergence of the numerical solution as a convergence parameter. The pseudo-time τ derivative of the pseudo-pressure p is inherited from the AC formulation of Chorin [1] to predict an initial guess for the real pressure field when the pressure-projection step will be carried out by solving a pressure-Poisson equation subsequently. The momentum equation can be written based on the FS-PP method of Chorin [2] as

$$\frac{\mathbf{u}^{*}-\mathbf{u}^{(n)}}{\Delta\tau}+\left[(\tilde{\mathbf{u}}\cdot\nabla)\tilde{\mathbf{u}}\right]^{(n)}=\upsilon\nabla^{2}\mathbf{u}^{(n)}, \tag{2}$$

where the pressure gradient term is cancelled out in the first fractional-step to predict an intermediate velocity field $\mathbf{u}^{*}$ without taking into account the real pressure field, and $\tilde{\mathbf{u}}$ represents a velocity field through the characteristics-based (CB) Godunov-type discretization of the convective flux term. In other words, the first step of the FSAC-PP formulation based on Eq. (1) is consistent with the AC method [1] and the second step according to Eq. (2) is consistent with the Helmholtz-Hodge decomposition based FS-PP [2] approach of Chorin. The reason for unifying the AC [1] and FS-PP [2] formulations is to make the CB Godunov-type convective flux discretization scheme compatible with the elliptical-type pressure-projection (PP) method along the characteristics. From a theoretical point-of-view, it is important to highlight that the Godunov-type convective flux term can be derived from a hyperbolic system of governing equations consistently with the AC method [1], and the PP step is compatible with the elliptical-type FS-PP [2] approach. In this way, we can retain the excellent accuracy and convergence properties of the CB Godunov-type scheme making compatible the hyperbolic-type convective flux term discretization with the elliptical-type FS-PP [2] method. The proposed novel FSAC-PP formulation [4,5] is a unified solution method to the incompressible Navier-Stokes equations, because the pseudo time derivative of pressure does not change the original characteristic of the system of governing equations and hence the hyperbolic nature is retained which allows the convective fluxes to be treated with a CB Godunov-type scheme. For solving incompressible flow problems, the CB Godunov-type scheme was first introduced by Drikakis et al. [9] within the formulation of the hyperbolic-type AC method [1] of Chorin. With higher-order interpolation schemes with the solution of the local Riemann problem, the unified FSAC-PP method can be classified as a Godunov-type method. The contribution of the viscous effect is treated numerically on the right hand side of the momentum equation (2) where υ is the kinematic viscosity of the fluid. After predicting an initial pressure field by solving the perturbed continuity equation (1) and computing the convective fluxes through a CB Godunov-type scheme, the second fractional-step is performed to recover the real pressure field gradient as

$$\frac{\mathbf{u}^{(n+1)}-\mathbf{u}^{*}}{\Delta\tau}=-\nabla p^{(n+1)}. \tag{3}$$

The Helmholtz-Hodge decomposition requires the incompressibility (divergence-free) constraint to be exactly satisfied at time level *(n+1)*, therefore by taking the divergence of Eq. (3), a pressure-Poisson equation can be constructed by

$$\nabla^{2} p^{(n+1)}=\frac{1}{\Delta\tau}\nabla\cdot\mathbf{u}^{*}. \tag{4}$$

After solving the pressure-Poisson equation (4) by a point S.O.R (Successive Over-Relaxation) method with relaxation factor ω = 1.7 performing an approximate solution through a few sub-iterations, the velocity field can be updated as

$$\mathbf{u}^{(n+1)}=\mathbf{u}^{*}-\Delta\tau\nabla p^{(n+1)}, \tag{5}$$

which is consistent with the FS-PP method [2] of Chorin. It is important to note that the solution of the Poisson equation requires high computational demand in general, however, since an initial pressure field is already predicted through the perturbed continuity equation (1), the Poisson equation does not need to be satisfied exactly when the unified FSAC-PP approach is employed. Due to the elliptic nature of the pressure-Poisson equation, its numerical solution stabilizes the pressure field and accelerates the convergence of the perturbed continuity equation (1). The iterations from Eq. (1) to (5) have to be repeated until the continuity equation of incompressible flows $\nabla \cdot \mathbf{u}^{(n+1)} = 0$ is satisfied. Therefore, the convergence criterion can be prescribed within a small threshold value [4,5] as

$$\max\left(\beta\left|\nabla \cdot \mathbf{u}^{(n+1)} - \nabla \cdot \mathbf{u}^{(n)}\right|\right) < \varepsilon, \tag{6}$$

where ε is the convergence tolerance and set to be equal to 10^{-6} in the present study. The pseudo-temporal accuracy is advanced by using a fourth-order explicit Runge-Kutta time-integration scheme and the spatial interpolation/reconstruction has been approximated by a first-order scheme and a third-order polynomial [4].

3. GEOMETRICAL AND SIMULATION SETUP

The geometry employed for the present study is shown in Figure 1. We use non-dimensional units and have $h = S = 1$, $L_1 = 4$ and $L_2 = 10$ with 41566 grid points. The Reynolds number is $Re = 100$ relying on the experiment of Armaly et al. [10], thus we consider the fluid flow as laminar and steady-state. The small and large channels are shorter with respect to the reference data [10] to reduce computational time. A fully-developed analytical laminar velocity profile has been imposed at the inlet of the smaller channel to match the profile at the transition from the smaller to the larger channel. After a length L_2, the velocity profile does not change which was observed based on the experiments, and thus we can use a smaller channel length.

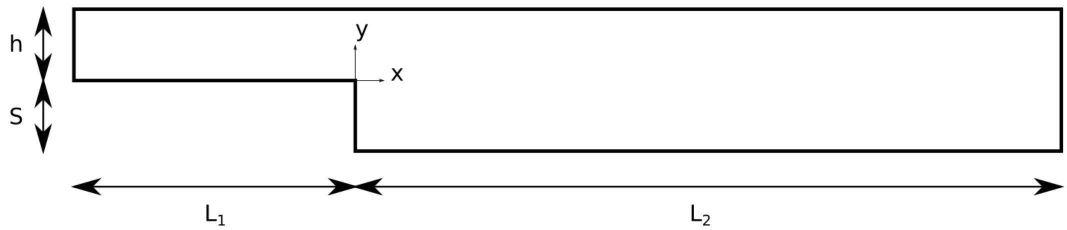

Figure 1: Geometrical setup for the backward facing step problem.

4. RESULTS AND DISCUSSION

The velocity profiles for the AC and FSAC-PP method are plotted in Figures 2 and 3 compared to the reference data [10]. First of all, the velocity profile at the outlet of the smaller channel (*x/S* = 0) exhibits a difference between both simulation methods and reference data [10]. We note that the experimental data is slightly off center while both FSAC-PP and AC methods provide a more centered velocity profile. The reason for the difference is due to the shape of the recirculation area downstream which, by means of pressure propagation, is influencing the upstream velocity profile. This imbalance is further seen by the difference in the velocity profile downstream of the larger channel. The downward momentum is kept by the experimental data which shows a momentum deficit in the upper half of the channel and a momentum excess at the bottom half, compared to the FSAC-PP and AC methods. Near the outlet boundary, the velocity profiles of all three sources approach similar shape. However, the differences overall are minute and both of the FSAC-PP and AC methods are closely matched. Due to the forced separation over the convex corner at $x = y = 0$, the reattachment length is a parameter allowing us to further judge the applicability of the FSAC-PP method which has not been investigated before to separated flows. The dimensionless reattachment length for this case was stated by Armaly et al. [10] to be x/S = 3.05 and we have given the reattachment lengths obtained with the FSAC-PP and AC methods in Table 1. It has been emphasized by Könözsy and Drikakis [5] that the FSAC-PP approach shows high accuracy for internal flows even with lower-order spatial interpolation schemes. Therefore, we have also performed a simulation with a first-order scheme to further validate this type behaviour of the FSAC-PP method for separated flows. These values have been included in Table1. We can see that this behaviour is also valid for the investigated separated flow in this study. Compared to the AC method, the FSAC-PP approach predicts the reattachment length four and seven times more accurately by employing first- and third-order schemes, respectively.

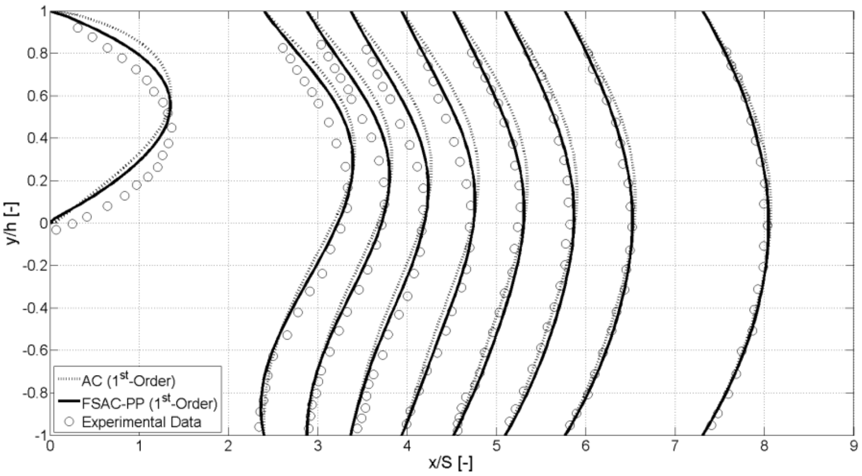

Figure 2: Velocity profiles at different stream-wise locations with a first-order interpolation scheme [4] compared to the experimental data of Armaly et al. [10].

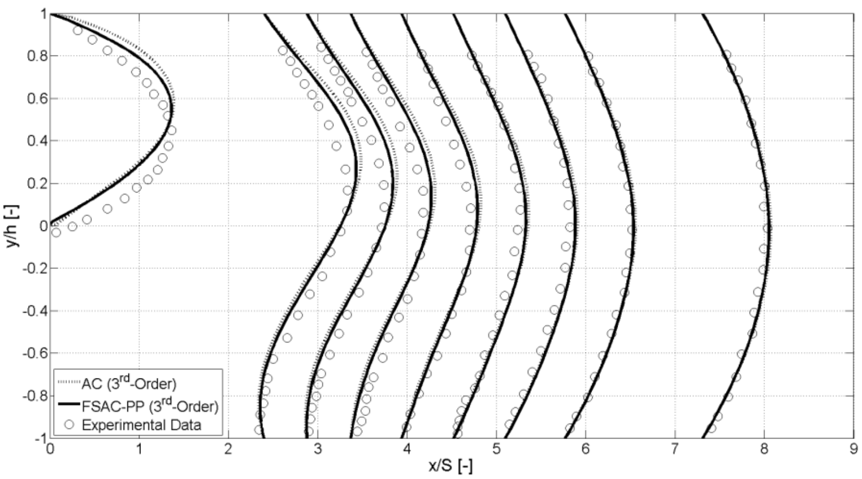

Figure 3: Velocity profiles at different stream-wise locations with a third-order interpolation scheme [4] compared to the experimental data of Armaly et al. [10].

Table 1: Reattachment length *L* and its deviation *D* to the experimental data of Armaly et al. [10] for first- and third-order spatial interpolation schemes.

	AC		FSAC-PP	
	1^{st}-order	3^{rd}-order	1^{st}-order	3^{rd}-order
L [-]	2.63	3.27	2.94	3.08
D [%]	-13.9	7.3	-3.6	1.0

Table 2: Computational time and convergence.

	AC		FSAC-PP	
	1^{st}-order	3^{rd}-order	1^{st}-order	3^{rd}-order
Iterations	30	37	24	26
CPU-time	2 min, 42 sec	3 min, 16 sec	3 min, 06 sec	3 min, 19 sec

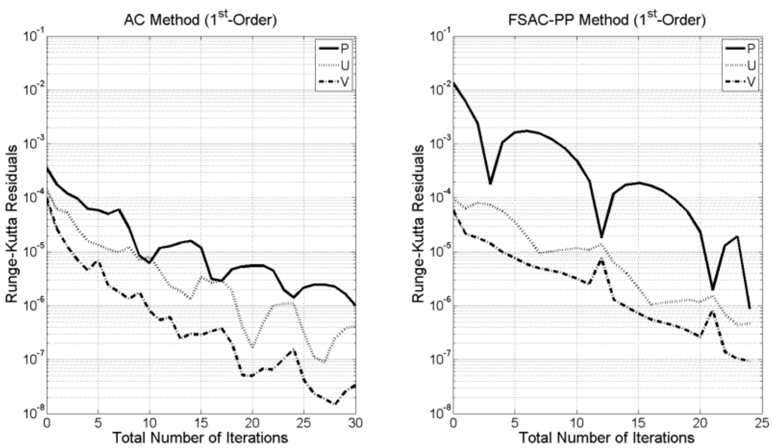

Figure 4: Convergence history by using a first-order interpolation scheme.

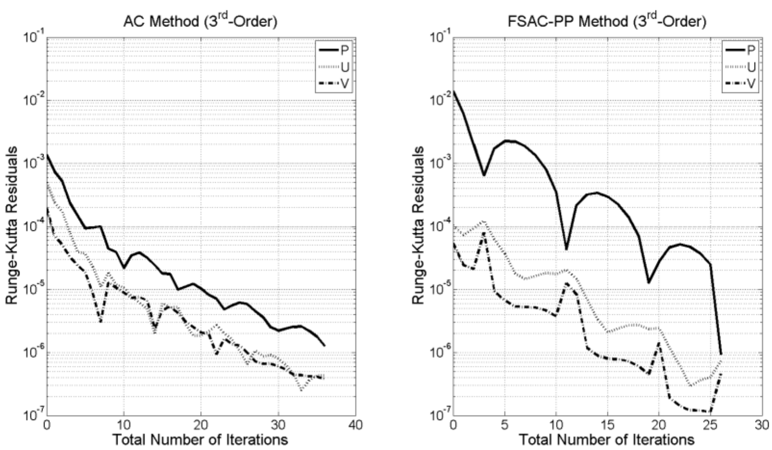

Figure 5: Convergence history by using a third-order interpolation scheme.

The absolute values of 3.6% and 1.0% show that the inclusion of the Godunov-type procedure in conjunction with a CB scheme to treat the convective flux terms with highly accurate results in close agreement with the reference data [10]. Furthermore, the interval in which the reattachment is calculated, i.e. 0.14 and 0.64 for FSAC-PP and AC in non-dimensional units, respectively, shows that the reattachment length prediction varies only marginally for the FSAC-PP method while the AC method shows a considerable dependence on the numerical scheme. The AC method performs less floating point operations per pseudo-time step, i.e. the pressure gradient is retained in the corresponding momentum equation which closes the system of equations, and no pressure-projection step needs to be carried out. Therefore, it may be argued that the AC method is faster than the FSAC-PP method. We have compared the convergence properties of both methods and summarized our findings in Table 2, and further show the convergence history of the first- and third-order spatial interpolation schemes in Figures 4 and 5. We can see that for both first- and third-order schemes, the FSAC-PP method is performing 20% and 30% iterations less, respectively. Since more computational demand is required for each pseudo-time step by using the FSAC-PP method, we have also measured the computational CPU-time. We can see that the AC method is faster for the first-order scheme and comparable for the third-order scheme as well. The inclusion of the pressure-Poisson equation in the FSAC-PP method requires more computational time during each pseudo-time step but has favorable stabilizing effects which in turn accelerate the pressure update in the perturbed continuity equation (1) at each pseudo-time level. In the present study, we perform ten Poisson sub-iterations which are based on computational experience. Fine tuning this parameter could speed up the convergence even further. The convergence history of first- and third-order schemes in Figures 4 and 5 shows that both AC and FSAC-PP method have a similar convergence behaviour for the u and v velocity component and reach the convergence criteria after approximately the same amount of iterations. However, the pressure takes longer to converge and we see that the speed up in convergence is due to the pressure stabilization of the pressure-Poisson solver. It was also reported by Könözsy and Drikakis [5] that the convergence properties of the FSAC-PP approach for microfluidic applications and internal flows at low Reynolds numbers exhibited a similar behaviour of numerical convergence. In addition to this, they also highlighted when the PP method was employed by itself, the PP method required higher number of iterations to converge due to the small time-step size. The Poisson equation itself may have advantageous properties, but due to its elliptical behaviour, Godunov-type methods with CB schemes cannot be employed straight away [4,5]. Overall, we can conclude that the inclusion of the PP step accelerates the numerical convergence of the FSAC-PP approach through the faster satisfaction of the perturbed continuity equation (1). The CB Godunov-type treatment of the convective flux terms is responsible for the high accuracy of the unified FSAC-PP approach even with lower-order spatial interpolation schemes, and these properties are also valid for separated flows as investigated in this work.

5. CONCLUSIONS

In this paper, we investigated the backward facing step problem by employing a recently developed unified Fractional-Step, Artificial Compressibility and Pressure-Projection (FSAC-PP) formulation for solving the incompressible Navier-Stokes equations. For modelling the aforementioned physical flow problem, a laminar, steady-state flow has been considered at a moderate Reynolds number (Re = 100). The numerical results have been compared to the reference data of Armaly et al. [10] and showed that the reattachment length is closely predicted by the FSAC-PP method even with a first-order spatial interpolation/reconstruction scheme. The spatial accuracy stems from the CB Godunov-type treatment of the convective term in conjunction with the solution of a local Riemann problem. An additional PP step has been carried out in addition to the solution of the perturbed continuity equation of the original AC method [1] of Chorin to stabilize and accelerate the numerical convergence of the unified FSAC-PP approach. For the backward facing step problem, the FSAC-PP method performed less iteration while computational CPU-times were comparable to the AC method. The reattachment lengths obtained with the AC method exhibited bigger differences compared to the FSAC-PP approach, and showed a considerable dependence on the order of the numerical scheme.

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