



**Balancing of mixed-model parallel U-shaped assembly lines considering model sequences**

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## Detailed Response to Reviewers

### Dear Reviewers,

Thanks for reviewing the paper and providing your useful comments once again. All comments have been handled individually. Please find below the detailed responses to your comments.

### Reviewer #1:

#### **Comment 1-1:**

My decision is accepting.

#### **Response 1-1:**

Thank you for your comment. We are happy to see that you found the changes made in the manuscript in the previous round satisfactory.

### Reviewer #2

#### **Comment 2-1:**

In response 2-1 provided by the authors, they state that " this paper gives an insight about the mixed-model U-line idea which has potential application opportunities to produce large-sized products such as buses, trucks and cars." This response shows that the concept is so far from practice and industrial realities. Because, as originated from Toyota, U-shaped assembly lines are designed for small products or parts and these lines have small number of tasks. Therefore, the paper is not suitable for publication in IJPR.

#### **Response 2-1:**

Thank you for your comment. The following image (see Fig. 1) shows the practical application of a hybrid U-shaped line at a Japanese leading car manufacturer's assembly plant<sup>1</sup>.



Fig. 1. U-line configuration at Honda's car assembly plant<sup>1</sup>

If the U-shaped line is aimed to be utilised for producing small products (as you said), then it can have a layout which limits working in between the two branches of the line. In other words, no space is available between the two (front and back) branches of the line to locate operators/workstations. Instead, the workstations can be located to the outer side of the line which limits access to the inner side. It is clear that such a configuration is not practical for producing large items, e.g. cars. However, the U-line configuration given in Fig. 1 allows performing tasks on both sides of the product being assembled on the line. Therefore, there is no doubt that such a configuration can easily be applied for producing large-sized products.

<sup>1</sup> <http://s3.caradvice.com.au/thumb/480/240/wp-content/uploads/2016/04/honda-cell-production-plant-ARC-factory-01.jpg>

**Reviewer #3****Comment 3-1:**

The paper can be accepted in its current form.

**Response 3-1:**

Thank you for your comment and encouraging thoughts in the previous round.

**Reviewer #4****Comment 4-Overall:**

The paper deals with the balancing problem of mixed model U-lines in a parallel U-shaped line system as the authors called. It seems that the paper is a modified version of their previous study which deals with the problem of balancing parallel single model U-lines (Kucukkoc and Zhang, 2015a) by considering the lines are mixed model.

When the advantages of parallel straight line balancing concept are taken into account the idea of the paper is good enough but I have two major concerns about the paper.

**Response 4-Overall:**

Your comment is acknowledged. Please find below the responses to your concerns.

**Comment 4-1:**

The authors say in Section 4.3 that: "At the beginning of the assignment procedure (when the final assignment configuration is not known), the most challenging issue is the lack of knowledge on which model will appear in workstations located at the back of the line since the total number of workstations is not known. The algorithm overcomes this problem by considering the maximum processing times of the tasks common in different models when assigning a task from the back of the line (or precedence relationships diagram)."

Considering the maximum processing time of a task is not an appropriate approach to overcome the problem of model mix combinations in the workstations. For example if we look at the Figure 2 (The changing model combinations through different production cycles) we see that model mix combinations of (E,E) and (D,D) will occur in the first crossover workstation of Line II. The remaining model mix combinations of (D,E) or (E,D) will never occur in such an assignment. Let's say that these workstation-task assignments shown in Figure 2 are the best assignments for the problem.

Now suppose that the first task in the precedence diagram of Line II requires 5 minutes for model D and 3 minutes for model E. Similarly the last task of the precedence diagram requires 5 minutes for model D and 6 minutes for model E. Then consider the cycle time is 10 minutes. In such a situation, we cannot able to assign the first and the last tasks of Line II to a crossover workstation because in the case of model D appears in the front side of the line we take into account the actual processing time of the first task for model D (5 minutes) and the maximum processing time of the last task (6 minutes). The total processing time of these two tasks will be 11 minutes and will exceed the cycle time. Thus we miss the opportunity of achieving the assignment shown in Figure 2. Actually, the assignment shown in Figure 2 is a feasible assignment because as I explained above only two model mix combinations (E,E) and (D,D) will occur in related workstation and total processing time of the first and the last tasks does not exceed the cycle time for both of these combinations. But it will be missed by the reason of considering maximum of the processing times of a task common in different models.

The assignment results which are missed in this manner are able to be better results compared to obtained ones. This is the most important weakness of the solution approach presented in the paper.

**Response 4-1:**

Your comment is appreciated. The main advantage of a U-line system is the opportunity to start assigning tasks from both front and back of the precedence relationships diagram, simultaneously. This works perfectly in single-model U-lines. However, in mixed-model U-lines, the processing time of a task may show difference from one model to another, as you mentioned. Therefore, to satisfy the capacity constraint when assigning a task, it must be known which model to appear in each workstation located at the back of the line. However, this is challenging as the number of stations is not known until all tasks have been assigned.

When assigning a task from the back of the line, the algorithm proposed here considers the maximum processing time of a task common between product models. As you exemplified, it is obvious that some assignment solutions to be missed in this manner are able to be better results compared to obtained ones. That is why the algorithm proposed here handles not only the line balancing problem but also the model sequencing problem, as explained in the same section (Section 4.3). Thus, the combinations of models are changed by the algorithm during the assignment procedure to be able to obtain better assignment solutions.

Furthermore, the paper contributes to knowledge by not only proposing a new and comprehensive algorithm but also introducing an innovative assembly line concept, MPUL. The authors believe in that a possible drawback that may be caused by the proposed algorithm due to the sophistication of the problem should not shade the success of the line concept proposed.

#### Comment 4-2:

I see that walking times operators in crossover and common workstation are ignored in the paper. The line layout which is called as parallel U-shaped lines by the authors has been addressed as "embedded U-lines" in the study of Miltenburg. (Miltenburg, J., 2001, U-shaped production lines: A review of theory and practice, International Journal of Production Economics, 70 (3), 201-214.) (Figure 2C). Miltenburg says in the mentioned study that "The embedded U-line arrangement in Fig. 2C has a large U-line encircling a small U-line, all manned by two operators. This is the least common complex U-line, because it may take up a lot of space and so require a lot of walking." In the light of this information I think ignoring the walking times of operators in common and crossover workstations is not an appropriate approach for practice in such a line layout.

#### Response 4-2:

Thank you for your comment and addressing to this valuable research, Miltenburg (2001). There are three major differences between our concept and that of Miltenburg (2001). To make a direct comparison, the "embedded U-line arrangement" presented in Miltenburg (2001) is given in Fig. 2 and the mixed-model parallel U-line concept proposed in our study is given in Fig. 3.

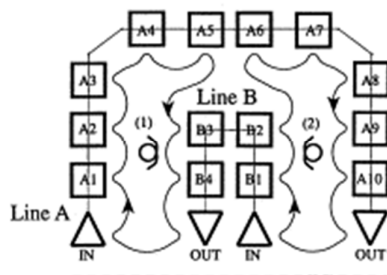


Fig. 2 (Miltenburg (2001)).

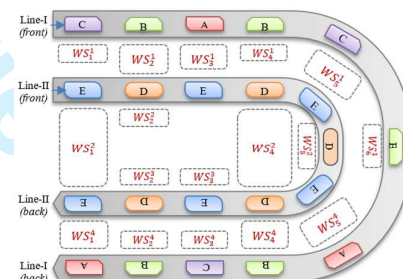


Fig. 3 (current work).

- First of all, as clearly seen from Fig. 2 and understood from the statement of "The embedded U-line arrangement in Fig. 2 has a large U-line encircling a small U-line, all manned by two operators..." given in Miltenburg (2001), the whole system is operated by only two operators. This requires operators walk quite a long distance across the line from one workstation to another. However, in our concept, each operator is only responsible with one workstation utilised in only one line – called regular stations, or between the opposite areas of two lines – called multi-line stations (or branches – called crossover workstations). You will notice 17 workstations (which equivalent to 17 operators) in Fig. 3.
- Secondly, the inner line (small U-line as called in the study of Miltenburg (2001)) shown in Fig. 2 does not allow operators work between its two (front and back) branches. This requires other operators (who work between Line-A and Line-B) constantly travel back and forth between the two lines to complete all tasks.
- Finally, as mentioned in Miltenburg (2001), "A bigger disadvantage (of the embedded U-line arrangement) is that the same operator is usually not able to operate the entrance and exit operations of a line". This is caused by the different movement direction of lines in

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embedded U-line arrangement (see Fig. 2). Nevertheless, in our model, the lines move in the same direction. Therefore, the same operator operates on either *the entrance of both lines or the exit of both lines*.

Consequently, the walking distance in our system is much smaller than of that required in embedded U-line arrangement. Of course walking times are worth to investigating in crossover and common workstations to reflect real-life conditions and make the concept even more practical. However, including such a constraint to an already complex problem at this stage makes it even harder to solve. This has also been explained in the last paragraph of Section 3.

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## Balancing of mixed-model parallel U-shaped assembly lines considering model sequences

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### Abstract

As a consequence of increasing interests in customised products, mixed-model lines have become the most significant components of today's manufacturing systems to meet surging consumer demand. Also, U-shaped assembly lines have been shown as the intelligent way of producing homogeneous products in large quantities by reducing the workforce need thanks to the crossover workstations. As an innovative idea, we address the mixed-model parallel U-shaped assembly line design which combines the flexibility of mixed-model lines with the efficiency of U-shaped lines and parallel lines. The multi-line stations utilised in between two adjacent lines provide extra efficiency with the opportunity of assigning tasks into workstations in different combinations. The new line configuration is defined and characterised in details and its advantages are explained. A heuristic solution approach is proposed for solving the problem. The proposed approach considers the model sequences on the lines and seeks efficient balancing solutions for their different combinations. An explanatory example is also provided to show the sophisticated structure of the studied problem and explain the running mechanism of the proposed approach. The results of the experimental tests and their statistical analysis indicated that the proposed line design requires fewer number of workstations in comparison with independently balanced mixed-model U-lines.

**Keywords:** Assembly line balancing; design of production systems; production planning; U-shaped assembly lines; mixed-model assembly lines.

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## Balancing of mixed-model parallel U-shaped assembly lines considering model sequences

### Abstract

As a consequence of increasing interests in customised products, mixed-model lines have become the most significant components of today's manufacturing systems to meet surging consumer demand. Also, U-shaped assembly lines have been shown as the intelligent way of producing homogeneous products in large quantities by reducing the workforce need thanks to the crossover workstations. As an innovative idea, we address the mixed-model parallel U-shaped assembly line design which combines the flexibility of mixed-model lines with the efficiency of U-shaped lines and parallel lines. The multi-line stations utilised in between two adjacent lines provide extra efficiency with the opportunity of assigning tasks into workstations in different combinations. The new line configuration is defined and characterised in details and its advantages are explained. A heuristic solution approach is proposed for solving the problem. The proposed approach considers the model sequences on the lines and seeks efficient balancing solutions for their different combinations. An explanatory example is also provided to show the sophisticated structure of the studied problem and explain the running mechanism of the proposed approach. The results of the experimental tests and their statistical analysis indicated that the proposed line design requires fewer number of workstations in comparison with independently balanced mixed-model U-lines.

**Keywords:** Assembly line balancing; design of production systems; production planning; U-shaped assembly lines; mixed-model assembly lines.

### 1. Introduction

The manufacturing industry has experienced crucial changes with the industrial revolution emerged in 18<sup>th</sup> century in England. Mass production techniques have been put into practice by companies to increase capacity and so productivity. Following these developments, which built a base for today's high-performance manufacturing systems, the first moving-belt was constructed by Henry Ford and his colleagues in early 20<sup>th</sup> century at Highland Park assembly plant. This was the pioneering attempt to establish an assembly line, which builds the major and the most significant parts of modern production systems in several industries, *e.g.*, automotive, electronics, home appliances *etc.* (Kucukkoc *et al.* 2015).



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3 An assembly line is a sequence of workstations linked to each other by a conveyor or moving  
4 belt, on which homogeneous products are consecutively assembled in an efficient way. The  
5 problem of determining which task will be assembled in which workstation with the aim of  
6 minimising the total number of workstations and/or cycle time is called the *assembly line*  
7 *balancing problem*. Some constraints must be satisfied to obtain a feasible line balance, such as  
8 capacity constraint, precedence relationships constraint, task assignment constraint, *etc.*  
9 (Kucukkoc and Zhang 2015e). The assembly line balancing problem was first studied by  
10 Salveson (1955) in its simplest version (where a simple straight line was considered with a  
11 single commodity of product on the line) and has gained continuing interest from academics as  
12 well as practitioners since then.

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18 In its traditional version, assembly lines have a straight structure on which a series of  
19 workstations are located sequentially, to produce only one type of product in large quantities.  
20 The problem of balancing such lines is called *simple assembly line balancing problem*, which is  
21 an NP-hard combinatorial optimisation problem, as shown by Wee and Magazine (1982).  
22 However, as shown by Thomopoulos (1967) and several studies following this, mixed-model  
23 lines (where more than one model of a base product are assembled on the same line) carry  
24 several practical advantages over single-model lines.

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30 This paper contributes to knowledge by presenting the first and original research results on  
31 *mixed-model parallel U-shaped assembly line (MPUL shortly)* concept, which was recently  
32 introduced by Kucukkoc and Zhang (2015e). So that, as an innovative approach, the model  
33 variation flexibility is introduced to the parallel U-shaped assembly line system with the aim of  
34 obtaining well-balanced line configurations by reducing idle times. The addressed line system  
35 combines the advantages of its sub-configurations, *i.e.*, mixed-model lines, parallel lines and U-  
36 shaped lines as will be explained in the following sections. Thanks to the proposed design, the  
37 multi-line stations and crossover stations are of the key factors which provide advantages. As  
38 the leading car manufacturers set new goals for advanced flexible manufacturing systems (see  
39 for example Ford Motor Company (2015)), which can easily adapt to varying customer demand,  
40 the proposed MPUL system can replace the conventional line configurations. Thus, the MPUL  
41 system can play crucial roles in “*giving customers the features, fuel efficiency and technology*  
42 *they want anywhere in the world*” as stated by Ford Motor Company (2015). To solve the  
43 balancing problem on MPULs and handle its complexity, which will increase to a great extent  
44 due to the contained line configurations which are already complex alone, a heuristic solution  
45 approach is also proposed and explained, as another contribution of this research.

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The remainder of this paper is organised as follows. Section 2 reports the review of literature  
while Section 3 introduces and defines the proposed MPUL system in details. Section 4  
proposes a possible heuristic solution approach for the MPUL balancing problem and explains

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3 its running principles. Section 5 presents an explanatory example and exhibits the running  
4 principle of the heuristic algorithm. The sophisticated structure of the problem and challenging  
5 issues are also discussed in the same section. The results of the experimental tests are reported  
6 in Section 6 and the conclusions and possible industrial implications of the work are given in  
7 Section 7 followed by the future research directions.  
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## 10 11 12 **2. Related work**

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14 During the last six decades, various types of line configurations along with several objectives  
15 and constraints have been studied by academics and practitioners. Thomopoulos (1967)  
16 proposed a mixed-model assembly line system, where a variety of product models having  
17 similar product characteristics are assembled. Mixed-model lines provide advantages over  
18 single-model lines as a variety of similar products can be assembled on the same line with no  
19 need of setup times between model changes (Kucukkoc, Karaoglan, and Yaman 2013).  
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23 Assembly lines can be divided into two main groups in terms of the line shape: *straight lines*  
24 and *U-shaped lines* (Miltenburg and Wijngaard 1994). In U-shaped lines (or U-lines, shortly),  
25 the entrance and the exit of the line system are very close to each other. Operators may handle  
26 work-pieces both on the front and back of the line thanks to the formed U-shaped line  
27 configuration. Operators located in crossover workstations can perform tasks from both front  
28 and back of the line. Thus, idle times are reduced and resource utilisation is increased thanks to  
29 the crossover stations located in between front and back of the U-line. Miltenburg and  
30 Wijngaard (1994) introduced the U-line balancing problem and a series of researchers followed  
31 them; *e.g.*, see Urban (1998), Scholl and Klein (1999), and Urban and Chiang (2006) for exact  
32 solution approaches; Erel *et al.* (2001), Gökçen *et al.* (2005), Hwang *et al.* (2008) and  
33 Sabuncuoglu *et al.* (2009) for heuristic/meta-heuristic solution approaches on simple U-line  
34 configurations. Miltenburg (2001) also introduced the embedded U-line arrangement, which  
35 considers “a large U-line encircling a small U-line, all manned by two operators”. The  
36 difference between the *embedded U-line arrangement* (Miltenburg 2001) and the MPUL  
37 concept described in the current work will be provided in Section 3.  
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41 Almost decade ago, Gökçen *et al.* (2006) introduced the line parallelisation idea to minimise  
42 idle times by maximising the use of shared resources. Parallel line configuration provides the  
43 opportunity of building multi-line stations in which operators can perform jobs from both of the  
44 adjacent lines. This also helps obtain well-balanced line configurations as there is more chance  
45 to assign tasks in different combinations. The line parallelisation idea has been applied to  
46 mixed-model lines, where more than one model of a base product is assembled on the same line,  
47 by Ozcan *et al.* (2010). Ozcan *et al.* (2010) introduced the parallel mixed-model line system and  
48 demonstrated the requirement of considering balancing and sequencing problems  
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3 simultaneously on those lines through experimental tests. In another study, Ozcan *et al.* (2010)  
4 located more than one two-sided line in parallel to each other and introduced the parallel two-  
5 sided assembly line system. A tabu search approach was also developed to find efficient  
6 solutions for the parallel two-sided assembly line balancing problem. Kucukkoc and Zhang  
7 (2015c, 2015d) developed a genetic algorithm approach and an ant colony optimisation based  
8 approach for solving the parallel two-sided assembly line balancing problem with different  
9 objectives. Kucukkoc and Zhang (2014b, 2014a) improved the parallel two-sided assembly line  
10 system in a mixed-model production environment and proposed agent-based ant colony  
11 algorithms for solving the problem efficiently. In their latter study, Kucukkoc and Zhang  
12 (2015b) enhanced the agent-based ant colony optimisation algorithm by integrating a genetic  
13 algorithm based model sequencing mechanism. As the common consequence of these  
14 researches, it was shown that locating two lines in parallel to each other helps minimise the total  
15 number of workstations.  
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23 As the pioneering research in parallelisation of the U-shaped lines, the study by Kucukkoc and  
24 Zhang (2015a) introduced the parallel U-shaped assembly line system. However, model  
25 variations across the lines have not been considered in their study. Instead, a single product  
26 model is produced on each of the U-shaped lines located in parallel to each other. Mixed-model  
27 production have been extensively studied on individual U-shaped lines. Sparling and  
28 Miltenburg (1998) introduced the mixed-model U-line (MMUL) balancing problem and  
29 presented an approximate solution approach with a numerical example. The model sequencing  
30 problem was not considered in their research. Kim *et al.* (2006), developed a new genetic  
31 approach, called endosymbiotic evolutionary algorithm, to deal with both balancing and  
32 sequencing problems in MMULs. Kara *et al.* (2007), presented a multi-objective approach  
33 (enhanced with a neighbourhood generation method) for solving the same problem with the aim  
34 of minimizing the absolute deviations of workloads, part usage rate and cost of setups. Özcan *et al.*  
35 (2011), Kazemi *et al.* (2011) and Hamzadayi and Yildiz (2012) developed genetic algorithm  
36 based approaches for balancing and sequencing MMULs. While Özcan *et al.* (2011) considered  
37 the stochastic task processing times, Kazemi *et al.* (2011) allowed the assignment of common  
38 tasks into different stations. Parallel workstations and zoning constraints were considered in  
39 Hamzadayi and Yildiz (2012). Hamzadayi and Yildiz (2013) and Kara (2008) proposed  
40 simulated annealing algorithms for balancing and sequencing MMULs. In a latter study, Kara  
41 and Tekin (2009) presented a mixed integer programming formulation which minimises the  
42 number of workstations for a given model sequence. Lian *et al.* (2012) developed a modified  
43 colonial competitive algorithm for solving the line balancing and model sequencing problems in  
44 MMULs simultaneously. The objective was to minimise the absolute deviations of workloads.  
45 The performance of the algorithm was compared with that of existing algorithms through test  
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3 problems. Li *et al.* (2012) studied the problem of sequencing minimum product sets in MMUL.  
4 While the line balancing problem was not considered in their research, a branch and bound  
5 algorithm was proposed to minimise the work overload. Rabbani *et al.* (2012) addressed the  
6 mixed-model two-sided assembly lines in a multiple U-shaped layout environment. A mixed-  
7 integer programming formulation and a genetic algorithm based heuristic were developed to  
8 simultaneously minimize the cycle time and the number of workstations. In another study,  
9 Rabbani *et al.* (2012) considered only the line balancing problem on MMULs and presented a  
10 genetic algorithm approach with the aim of minimising crossover workstations considering  
11 operator travel times. Manavizadeh *et al.* (2013) developed a simulated annealing approach  
12 which assigns operators with different skill levels into workstations upon the line balance is  
13 obtained. Another simulated annealing approach was developed by Dong *et al.* (2014) to  
14 minimise the expectation of work overload time when balancing and sequencing MMULs. For  
15 the same problem, Manavizadeh *et al.* (2015) developed a multi-objective heuristic algorithm  
16 which incorporates the minimization of the cycle time, the wastages in each station and the  
17 work overload. Kucukkoc and Zhang (2015e) introduced the MPUL concept, where the variants  
18 of a base product can be assembled on each of the parallel U-lines in different model mixes.  
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21 As seen from this comprehensive survey, only the study by Kucukkoc and Zhang (2015e)  
22 incorporated model variations on parallel U-shaped lines. Kucukkoc and Zhang (2015e) brought  
23 the MPUL idea to the attention of academia. However, no solution method was proposed in  
24 their study. Also, they have not presented research results, which prove the advantages of  
25 MPULs over conventional (or independently balanced) MMULs. As a continuing research built  
26 over Kucukkoc and Zhang (2015e), this paper fills in this gap through presenting new and  
27 original research results obtained from a new solution method developed for MPULs. The  
28 individual model sequences of the lines have been considered to prevent infeasible solutions and  
29 violation of capacity limits. Furthermore, a paired-sample t-test was conducted to statistically  
30 prove the advantage of MPUL over independently balanced MMULs.  
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### 45 **3. Problem description**

#### 46 **3.1. Main characteristics and advantages**

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48 The parallel U-shaped assembly line system is a combination of two or more U-shaped  
49 assembly lines, represented by  $L_h$  ( $h = 1, \dots, H$ ), located in parallel to each other. Two or more  
50 different product models, where each model on  $L_h$  is represented by  $m_{hj}$  ( $j = 1, \dots, M_h$ ), are  
51 produced on each of the U-shaped assembly lines. Each product model produced on each of the  
52 lines has its own set of tasks, where a task is represented by  $t_{hi}$  ( $i = 1, \dots, T_h$ ), performed  
53 according to predefined precedence relationships.  $P_{hi}$  represents the set of predecessors of task  
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3  $t_{hi}$  for model  $m_h$ . Each task ( $t_{hi}$ ) for model  $m_{hj}$  on line  $L_h$  requires a certain amount of  
4 processing time, symbolised with  $pt_{hji}$ , to be performed. In such a parallel U-shaped assembly  
5 line system, operators located in between two adjacent U-lines will have the opportunity of  
6 performing their jobs on both of the lines. Meanwhile, operators located in the centre of the  
7 inner U-shaped line will have the flexibility of performing tasks on the front and back of the  
8 line. The proposed parallel U-shaped assembly line system is illustrated in Figure 1.  
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14 Figure 1. The proposed parallel U-shaped assembly line system.  
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18 As it can be seen from Figure 1, two U-shaped lines are located in parallel to each other.  
19 Operators located in multi-line stations are allowed to perform their jobs belonging to the  
20 models assembled on both Line-I and Line-II. This brings the opportunity of assigning tasks to  
21 the workstations in different combinations. By this way, the idle times of workstations are  
22 reduced (therefore the efficiency of the whole line system is increased) as well as the  
23 communication between workers is increased. It is also possible to utilise regular workstations  
24 instead of multi-line stations depending on the overall efficiency of the line configuration  
25 obtained.  
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29 The proposed system also carries the advantages of U-shaped lines as crossover stations are  
30 allowed to be utilised in the centre of the inner line. The operators located in these workstations  
31 can travel between the front and back of the line to help perform jobs on both branches of the U-  
32 line. Also, there are regular stations in which tasks from only one branch of the line are  
33 accomplished.  
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37 As an advantage of the proposed assembly line system, each of the parallel lines may have a  
38 different cycle time regardless of the other one. On one hand, this increases the flexibility of the  
39 system because it is possible to produce products in different throughput rates. On the other  
40 hand, the complexity of the problem of balancing this line system increases as it is needed to  
41 determine common time slots between the two lines to be able to assign tasks in multi-line  
42 stations. Gökçen *et al.* (2006) used least common multiple (LCM) based approach to make  
43 modelling easier when different cycle times are subject to consideration in such a parallel line  
44 system configuration (Zhang and Kucukkoc 2013; Kucukkoc and Zhang 2014c). This procedure  
45 will not be repeated here due to page limit.  
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53 The MPUL concept differs from the embedded U-line arrangement addressed in Miltenburg  
54 (2001) in three ways. First of all, in embedded U-line arrangement, the whole system is  
55 operated by only two operators. This requires operators walk quite a long distance across the  
56 line. However, in our concept, each operator is only responsible with one workstation utilised in  
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3 only one line – called regular stations, or between the opposite areas of two lines – called multi-  
4 line stations (or branches – called crossover workstations). Secondly, the inner line (small U-  
5 line as called in the study of Miltenburg (2001)) does not allow operators work between its two  
6 (front and back) branches. This requires other operators constantly travel back and forth  
7 between the two lines to complete all tasks. Finally, as mentioned in Miltenburg (2001), “A  
8 *bigger disadvantage (of the embedded U-line arrangement) is that the same operator is usually*  
9 *not able to operate the entrance and exit operations of a line*”. This is caused by the different  
10 movement direction of lines in embedded U-line arrangement. Nevertheless, in our model, the  
11 lines move in the same direction. Therefore, the same worker operates on either the entrance of  
12 both lines or the exit of both lines.  
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### 19 **3.2. Challenging issues**

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21 On two-sided assembly lines, where both sides of the line (called left and right sides) are used,  
22 some tasks can be performed on the left side of the line while some tasks belonging to the same  
23 model can be performed on the opposite (right) side of the line. There may be precedence  
24 relationships between those tasks performed on the opposite sides of the lines. This  
25 phenomenon is called interference and extra attention is needed to avoid violation of precedence  
26 relationships as this may cause infeasible solutions. In the line system proposed in this research,  
27 the situation is similar to the two-sided lines because it is allowed to perform jobs on both sides  
28 of the Line-II (see Figure 1). When a model belonging to Line-II is being assembled, two  
29 different operators – one in the multi-line station between two adjacent lines while the other one  
30 is in crossover station or regular station in the centre of Line-II – can work on the same work-  
31 piece. The challenge in this situation is to ensure precedence relationships among tasks caused  
32 by technological or organisational requirements.  
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40 Capacity constraint is another must have requirement that needs to be satisfied in assembly line  
41 balancing problems. Each workstation has a limited time, called cycle time, in which they need  
42 to complete their tasks. In a paced (synchronous) assembly line, all workstations are linked to  
43 each other via a conveyor or any other transportation system. The synchronisation is achieved  
44 by transferring semi-finished product models between stations at a pre-determined and fixed  
45 time interval. In the proposed line system, each workstation utilised on the same line has the  
46 same capacity, irrespectively whether or not it is a multi-line station. The capacity of a multi-  
47 line station is determined by the cycle time of the line on which the multi-line station is  
48 constructed. Crossover stations and regular stations adhere to the cycle time of Line-II.  
49 Obviously, when an LCM based approach is applied, both lines will be balanced using the same  
50 cycle time (but modified task times) and this makes modelling and solving the problem easier.  
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In some cases, there can be other constraints caused by the safety rules or allocations of the workstations, such as positive-negative zoning constraints and positional constraints. The positive zoning constraint requires the two tasks be assigned in the same workstation while in the negative zoning constraint two tasks must be assigned in different workstations. Also, in some situations, a certain task may need to be operated at a certain workstation. This is assured by positional constraints.

### 3.3. Model changes

Introducing the product model diversity to such a complex production environment makes modelling and solving the problem harder to a great extent. The most important factor that contributes to this complexity is the change in product models on the parallel lines from one production cycle to another. A production cycle is defined as a phase of the line system where there is a different mix of products in the workstations. When the cycle times of the parallel lines are different, model changes on the lines will take place at different times and this yields a quite complex situation to manage in multi-line stations. The total number of different production cycles that can appear in a parallel assembly line system depends on the product mix assembled on the lines. Formulations on how to calculate production cycles will be provided in Section 4.

To give an example, let us assume a parallel U-shaped assembly line system composed of two lines, Line-I and Line-II, where a mix of different product models are assembled on each of the U-lines, *i.e.*, models A, B, and C on Line-I and models D and E on Line-II. The cycle times of Line-I and Line-II are 10 time-units and 20 time-units, respectively. Therefore, the moving speed of Line-I is as double of that of the Line-II. If the model sequences on Line-I and Line-II are considered as  $MS_1 = ABCB$  and  $MS_2 = ED$ , possible production cycles will be as in Figure 2.

Figure 2. The changing model combinations through different production cycles.

To identify workstations (symbolised with  $WS_q^z$ ), the working space is divided into different zones ( $z = 1, 2, \dots, 4$ ) and queues ( $q = 1, 2, \dots, Q$ ; where  $Q = 6$  in this example) as seen from Figure 2. The workstations located in between two adjacent lines and assigned tasks from both of the lines are called *multi-line stations* (e.g., see  $WS_2^1, WS_3^1, WS_5^1$ , etc.) and operators located in these workstations can perform their jobs on both of the lines. Workstations located in between the front and back branches of Line-II and assigned tasks from both branches are called *crossover stations* (e.g.,  $WS_1^2$  and  $WS_4^2$ ) and operators located in these stations can perform

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3 their jobs on either branch. As in traditional configurations, *regular stations* (e.g.,  $WS_2^2$  and  
4  $WS_2^3$ ), in which operators perform their jobs for only one specific line and/or branch are also  
5 allowed.  
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#### 9 **4. Mixed-model parallel U-line heuristic (MPUH)**

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11 This section proposes a heuristic algorithm to find balancing solutions for the MPUL system  
12 introduced in the previous section.  
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##### 14 **4.1. Pseudo-code of MPUH**

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16 The heuristic procedure proposed by Kucukkoc and Zhang (2015a) for parallel U-line systems  
17 has been improved and adapted for the MPULs. Thus, the algorithm proposed in this paper,  
18 called *mixed-model parallel U-line heuristic (or MPUH shortly)*, integrates the modifications of  
19 two well-known heuristics, the *ranked positional weight method* (Helgeson and Birnie 1961)  
20 and the *maximum number of successors* (modified from Tonge (1960)), in an MPUL production  
21 environment. The main aim is to minimise the number of workstations as a primary goal and to  
22 minimise the line length (or the number of queues) as an additional goal. Therefore, if two  
23 solutions, which has the same number of workstations but different lengths are obtained, the one  
24 with the lower length is preferred. As the MPUL system is a much more complex problem in  
25 comparison with its single-model version, it requires an even more sophisticated solution  
26 procedure to obtain efficient balancing solutions. Therefore, the MPUH procedure, which uses a  
27 newly introduced task priority index ( $\omega_{hi}^{PI}$ ) value for selecting tasks, comprises several  
28 improvements as the details will be explained in this section. The superscript “PI” is used here  
29 as an acronym for priority index. The task priority index ( $\omega_{hi}^{PI}$ ) of a task is the multiplication of  
30 its positional weight index ( $\omega_{hi}^{PW}$ ) and the number of successors index ( $\omega_{hi}^{NS}$ ); i.e.,  $\omega_{hi}^{PI} =$   
31  $\omega_{hi}^{PW} \times \omega_{hi}^{NS}$ . In this equation, the positional weight index ( $\omega_{hi}^{PW}$ ) of a task is determined by its  
32 positional weight ( $PW_{hi}$ ) which is calculated as follows;  $PW_{hi} = wpt_{hji} + \sum_{j \in S_{hi}} wpt_{hji}$ ,  
33 where  $S_{hi}$  represents the set of successors of task  $t_{hi}$  on line  $L_h$ . The term  $wpt_{hji}$  denotes the  
34 weighted processing time of task  $t_{hji}$  and is calculated as  $wpt_{hji} = \left( \sum_{j=1}^{M_h} d_{hj} pt_{hji} \right) / \sum_{j=1}^{M_h} d_{hj}$   
35 where  $h = 1, \dots, H$ ;  $i = 1, \dots, T_h$  (to recapitulate,  $pt_{hji}$  is the processing time of task  $t_{hji}$ ).  
36 Afterwards, on each of the lines, the tasks are sequenced in descending order based on their  
37  $PW_{hi}$  values and the positional weight index ( $\omega_{hi}^{PW}$ ) of the task which has the lowest  $PW_{hi}$  value  
38 is set to ‘1’. The  $\omega_{hi}^{PW}$  value of the task which has the second lowest  $PW_{hi}$  value is set to 2 and  
39 this is repeated until every task is assigned a  $\omega_{hi}^{PW}$  value. If there are two or more tasks which  
40 have the same positional weights, the lowest numbered task gets the higher positional weight  
41 index.  
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To calculate the number of successors index ( $\omega_{hi}^{NS}$ ) of a task, the total number of successors ( $NS_{hi}$ ) of this task is calculated as  $NS_{hi} = \text{card}\{S_{hi}\}$ , where  $\text{card}\{S_{hi}\}$  corresponds to the length of the set of successors of task  $t_{hi}$  on line  $L_h$ . On each of the lines, tasks are sequenced in descending order based on their  $NS_{hi}$  values and the  $\omega_{hi}^{NS}$  value of the task which has the lowest  $NS_{hi}$  value is set to '1'. The  $\omega_{hi}^{NS}$  value of the task which has the second lowest  $NS_{hi}$  value is assigned 2, and so on. These calculations will be exemplified in Section 5.

The pseudo-code of the proposed MPUH procedure is given in Figure 3.

Figure 3. The pseudo-code of MPUH.

#### 4.2. Diversification

A stochastic assignment procedure is applied in MPUH to alternate assignment positions during the balancing process and have more diversified as well as efficient balancing solutions. For this aim, depending on the value of a newly introduced random boolean variable ( $isFrontArea$ ), the value of  $z$  index is randomly determined after a new task is assigned. Thus, for example, if the assignment procedure is being performed on the front area of the line system ( $isFrontArea = true$ ),  $z$  is randomly assigned 1 or 2 ( $z = \text{randomIntegerBetween}[1,2]$ ). When there is no available task for the current assignment area (front or back), the value of  $isFrontArea$  is alternated ( $isFrontArea = false$ ), and  $z$  is assigned a random value depending on the value of  $isFrontArea$  as exemplified in the pseudo-code.

#### 4.3. MPS principle

At the beginning of the assignment procedure (when the final assignment configuration is not known), the most challenging issue is the lack of knowledge on which model will appear in workstations located at the back of the line since the total number of workstations is not known. The algorithm overcomes this problem by considering the maximum processing times of the tasks common in different models when assigning a task from the back of the line (or precedence relationships diagram). For this aim, a newly introduced term, namely deserved task time ( $DT_{hi}$ ), is used as will be explained below.

When assigning tasks to the front areas of the lines, the task processing times belonging to the actual models are used by the algorithm, as different from the back of the line. The algorithm generates possible model sequences and the *minimum part set* (MPS) principle (Bard, Darel, and Shtub 1992) is used to determine model mixes on the lines. The MPS approach was used by Ozcan *et al.* (2010) for parallel mixed-model lines and Kucukkoc and Zhang (2014a) for mixed-

model parallel two-sided lines. The  $MPS_h$  ( $h = 1, \dots, H$ ) is a vector which represents the smallest set having the same proportions of different product models as the demands. It represents the mix of product models on line  $L_h$ , such that  $MPS_h = (d_{h1}, \dots, d_{hM_h})$ , where  $d_{hj} = D_{hj}/gcd_h$  (where  $j = 1, \dots, M_h$  and  $h = 1, \dots, H$ ). The  $D_{hj}$  denotes the demand of model  $m_{hj}$  ( $j = 1, \dots, M_h$ ) on  $L_h$  ( $h = 1, \dots, H$ ). The  $gcd_h$  ( $h = 1, \dots, H$ ) is the greatest common divisor of the demands of the product models assembled on the same line ( $L_h$ , where  $h = 1, \dots, H$ ). Obviously, the total demand is met by  $gcd_h$  times repetition of producing the  $MPS_h$ . The model sequence of line  $L_h$  is represented with  $MS_h$  and the length of  $MS_h$  for one  $MPS_h$ , which means the total number of products on line  $L_h$  for one  $MPS_h$ , is calculated as follows;  $S_h = \sum_{j=1}^{M_h} d_{hj}$ . Thus, the maximum number of different model combinations for a determined model sequence pattern on two lines is calculated as  $MS_{max} = (S_1 \times S_2)$ . This also regulates how many different production cycles ( $\phi = 1, \dots, \phi$ ) the system should be split into ( $\phi = MS_{max}$ ) (Ozcan *et al.* 2010).

#### 4.4. Complex task selection procedure

The list of *available tasks* on  $L_h$  (where  $h = 1, \dots, H$ ) is symbolised with  $ATL_h$ . When determining available tasks, the tasks in the unassigned task lists for Line-I and Line-II ( $UTL_1$  and  $UTL_2$ ) are checked one-by-one. If a task being checked (called a candidate task) is from the front of the precedence relationships graph, either the task should have no predecessors or all of its predecessors (if any) must have been assigned and completed. However, if the task is from the back of the precedence relationships graph, either the task should have no successors or all of its successors (if any) must have been assigned and completed. Also, the remaining capacity in the current workstation should be large enough to perform the task. To determine whether there is enough capacity, there are three important components which need to be considered: (i) the workload of the current workstation, (ii) the earliest starting time of the candidate task, and (iii) the deserved task time ( $DT_{hi}$ ) of the candidate task. The  $DT_{hi}$  is a newly introduced term for MPUL system, because, at the beginning of the balancing process (when the final balance and so the total number of workstations are uncertain), it is not known which model will appear at the back of the line (*i.e.*, Line-I back and Line-II back). Due to this uncertainty, the maximum processing time of the candidate task among different models being produced on the same line will be considered when assigning a task from the back of the line,  $DT_{hi} \leftarrow \max_{m_{hj} \in M_h} \{pt_{hji}\}$ . However, as it is known which model will appear on the front of the lines, the task processing time of the relevant model will be used when assigning a task from the front of the precedence relationships graph,  $DT_{hi} \leftarrow pt_{hji}$ . Therefore, the capacity constraint will be satisfied by task  $t_{hji}$  if the following condition is fulfilled:  $C \geq DT_{hi} + \max\{WT_q^{z\phi}, EST_{hi}^{\phi}\}$  for all production

cycles. The term  $EST_{hi}^{\varphi}$  corresponds to the earliest starting time of task  $t_{hi}$  in production cycle  $\varphi$  and is determined by the latest completion time of its predecessors (if the task is from the front of the line) or successors (if the task is from the back of the line).

When selecting and assigning tasks to workstations, the task which has the best priority index ( $\omega_{hi}^{PI}$ ) value is preferred. However, the best  $\omega_{hi}^{PI}$  depends on the position of the available task. If the task is from the front of the precedence relationships graph (regardless of Line-I or Line-II), then the best priority index corresponds to the maximum  $\omega_{hi}^{PI}$ . On the other hand, if the task is from the back of the precedence relationships graph (regardless of Line-I or Line-II), the best priority index corresponds to the minimum  $\omega_{hi}^{PI}$ .

To increase the diversity of the solutions obtained and scan the search space more effectively, randomness is allowed when selecting tasks. Randomness is a commonly applied technique in assembly line balancing approaches, especially in priority rule-based methods, and as mentioned by Otto and Otto (2014), the quality of the solutions obtained by such methods can be improved by “*applying several passes of this priority rule with some kind of random influence*”. For this aim, a random number,  $r_1 \in (0,1)$ , is determined by the algorithm. If  $r_1 \leq RI$ , where  $RI \in [0,1]$  is the randomness index and determined by the user at the beginning, a random task is selected among the available ones. If  $r_1 > RI$ , then the task which has the best  $\omega_{hi}^{PI}$  value among the available tasks is selected. Another randomly determined number,  $r_2 \in (0,1)$ , is used to decide on the list from which the task will be picked, *i.e.*,  $ATL_1$  or  $ATL_2$ . If  $r_2 \leq 0.5$ , the task is tried to be selected from  $ATL_1$ . If  $r_2 > 0.5$ , the algorithm tries to assign a task from  $ATL_2$ . If any of the lists is empty, then the task is picked from the other list. When a task ( $t_{hi}$ ) is assigned to a workstation ( $WS_q^z$ ) on line ( $L_h$ ), its workload time is increased by  $DT_{hi} + \max\{WT_q^{z\varphi}, EST_{hi}^{\varphi}\}$  for all production cycles,  $WT_q^{z\varphi} \leftarrow DT_{hi} + \max\{WT_q^{z\varphi}, EST_{hi}^{\varphi}\}$  where  $\varphi = 1, \dots, \phi$ .

$DT_{hi}$  is also used when updating  $EST_{hi}^{\varphi}$  values of tasks. If the task is assigned from the front of the precedence relationships graph, then  $EST_{hi}^{\varphi}$  values of its successors will be set to  $DT_{hi} + \max\{WT_q^{z\varphi}, EST_{hi}^{\varphi}\}$ . On the contrary, if the task is assigned from the back of the precedence relationships graph, then  $EST_{hi}^{\varphi}$  values of its predecessors will be set to  $DT_{hi} + \max\{WT_q^{z\varphi}, EST_{hi}^{\varphi}\}$ .

## 5. An explanatory example

A small-scale numerical example is given here to show the sophisticated structure of the MPUL balancing problem and the running principle of MPUH, explicitly.

### 5.1. Problem data

Let us consider two U-shaped lines, Line-I and Line-II, located in parallel to each other, as in the example given in Section 3. Two different sets of models are produced on each of the parallel lines, *i.e.*, models A and B on Line-I, and models C and D on Line-II. A common precedence relationships diagram is built among tasks belonging to the models produced on each of the parallel lines. The processing times of tasks and the precedence relationships are given in Table 1. The *IM* column presents the immediate successor(s) of the corresponding task.

The model demands are assumed  $D_{1A} = 24$ ,  $D_{1B} = 72$ ,  $D_{2C} = 48$  and  $D_{2D} = 48$  for a planning horizon of 960 time-units. Thus, the cycle times of the lines are simply calculated as  $C_1 = C_2 = 10$  time-units. When the MPS principle explained in Section 4 is applied, the model mixes on Line-I and Line-II are obtained as  $d_1 = (1,3)$  and  $d_2 = (1,1)$ , respectively, which means  $S_1 = 4$  and  $S_2 = 2$ . This leads to a maximum of  $\phi = MS_{max} = 8$  different production cycles on the lines.

Table 1. Input data for the numerical example (task times in time units).

### 5.2. Steps of MPUH

Table 2 shows the task selection principle of MPUH through an example solution for the problem given. Note that only the first 14 steps are provided due to page limit. The task selected from the list of available tasks at each step, and the completion time of the selected task for each production cycle can be seen from the table. In the *Completion Time* column, the models that will appear in workstations located at the front of the line are also given in brackets. As the specific models that will appear in workstations located at the back of the line are unknown, they are marked with '[M]'. In this situation, the maximum task processing time among all models on the same line is considered (as explained in Section 4). Model sequences are assumed  $MS_1 = \{BBAB\}$  and  $MS_2 = \{CD\}$ .

Table 2. The task selection procedure.

As seen from Table 2, the assignment procedure starts from the front area of Line-II ( $z = 2$ ,  $q = 1$ ). Amongst the available tasks from Line-I and Line-II, the algorithm selects task  $2 \in ATL_2$ , which has the maximum  $\omega_{hi}^{PI}$  value among the tasks on Line-II, to assign in workstation  $WS_1^2$ . The completion times in this workstation are updated as 2, 5, 2 and 5 for production

cycles 1, 2, 3 and 4, respectively. The earliest starting times of the successors of task 2 on Line-II are also updated for each production cycle. In the next step (step 2),  $z$  is assigned 1 ( $z = 1$ ) and task  $1 \in ATL_1$  is assigned to workstation  $WS_1^1$ . The completion times and earliest starting times are updated and this cycle continues until there is no available task for this queue. The algorithm proceeds to the next queue in step 12. In step 14, task  $5 \in ATL_2$  is assigned to the multi-line station utilised on Line-I ( $WS_2^1$ ). The operator allocated in  $WS_2^1$  operates on both lines, *i.e.*, he/she performs operations for models A and B being assembled on Line-I and models C and D being assembled on Line-II. As seen from the table, no crossover station is utilised in the first 14 steps provided.

The algorithm consecutively determines which product model will appear on the front of the line as the new queue is utilised based on the model sequence given ( $MS_1 = \{BBAB\}$  and  $MS_2 = \{CD\}$ ). However, the final configuration of product models, including the back of the lines, is achieved once all the tasks have been assigned and so the total number of workstations has been determined. In such an environment where it is not known which model will exist in the back of the line, it is not easy to assign tasks to the workstations by ensuring that capacity constraint is satisfied while pursuing to obtain an efficient line balance.

### 5.3. Final solution

Figure 4 gives the best balancing solution obtained after MPUH algorithm was run for 50 iterations ( $maxBalancingItNumber = 50$ ). Note that many different solutions may be obtained during these iterations thanks to the stochastic task assignment procedure of MPUH. The procedure (and the sample task assignments) given in the previous subsection can be followed when building a sample balancing solution in any iteration. However, the best solution obtained eventually (given in Figure 4) is completely different from this sample balancing solution, as expected. As seen from the figure, 10 workstations are constructed; of which four are multi-line stations (*i.e.*,  $WS_1^1$ ,  $WS_2^2$ ,  $WS_3^3$  and  $WS_3^3$ ), one is crossover station (*i.e.*,  $WS_1^2$ ) and five are regular stations (*i.e.*,  $WS_1^3$ ,  $WS_1^4$ ,  $WS_2^1$ ,  $WS_3^1$  and  $WS_3^2$ ). The multi-line stations and crossover stations will be influenced from the model changes during production cycles. This is exhibited in Table 3, in which the final workload times of workstations are reported across four production cycles. In accordance with the number of workstations obtained, possible production cycles are also provided in Table 3. As can be seen from the table, the model type that will exist in the workstations located in the back of the line tightly depends on the total number of workstations utilised across the lines.

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 Figure 4. The best balancing solution obtained after 50 iterations.  
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 Table 3. The workload times of workstations across production cycles.  
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To give an example regarding the workload variations in multi-line stations, let us consider workstation  $WS_1^1$  given in Figure 4. In the first production cycle, model B and model C will be under operation on Line-I and Line-II, respectively. The operator working in this workstation will complete tasks 2 and 6 on model B on Line-I and he/she will then perform task 3 on model C belonging to Line-II. Thus, the workload of this workstation will be 9 time units in production cycle 1. In the next production cycle, the workload of  $WS_1^1$  will remain the same although model D will appear on Line-II. This is because both models, C and D, require the same amount of time for task 3. However, this situation will change in the next production cycle with the launch of models A and C. The workload time of  $WS_1^1$  will increase to 10 time units in production cycle 3. In another multi-line station,  $WS_2^2$ , the model combinations on Line-I and Line-II and so workload time of the workstation will change four times; *i.e.*, workload time will be 9, 9, 9 and 10 time units in production cycles 1, 2, 3 and 4, respectively. Similarly, there will be changes in the model combinations in the crossover station,  $WS_1^2$ . In production cycles 1 and 3, models C and D will exist on the front and back of Line-II, respectively, by requiring 8 time units of workload time. In production cycles 2 and 4, models D and C will appear and fill up the capacity of  $WS_1^2$  with 10 time units.

If these two lines were balanced independently, a total of 11 workstations (*i.e.*, 6 workstations for Line-I and 5 workstations for Line-II) would be needed to perform tasks for all models. Therefore, it is clear that the proposed MPUL design helps save one workstation for this particular case.

## 6. Experimental tests

As there is no suitable comparable result reported in the literature, to show the practical benefits of the proposed MPUL system, standard test problems have been derived from the literature and solved using the proposed MPUH algorithm under two different conditions: *independent balancing (IB)* and *MPUL*. The MPUH algorithm has been coded in Java SE 7u4 environment and run on a PC with 3.1GHz Intel Core™ i5-2400 CPU. A total of 24 test cases have been formed using the test problems, *i.e.*, one test problem on Line-I and another test problem on Line-II. In IB, Line-I and Line-II have been considered as two separate MMUL systems and the two lines were balanced separately such that no multi-line stations were allowed. In *MPUL*

condition, a MPUL system has been established where the two lines were located in parallel to each other, as proposed. Thus, it was aimed to measure the benefit of the proposed MPUL system against existing MMUL system. Based on some preliminary tests, the randomness index parameter is assumed  $RI = 0.5$  to obtain more diversified solutions, as contextualised in Section 4. The algorithm was run 100, 200 and 300 iterations for test cases 1-6, 7-15 and 16-24, respectively, and the solution which gives the minimum number of workstations ( $NS$ ) value was taken for each test case.

Table 4 presents both the design and the results of the experimental tests. The problems considered on each line are given in Line-I and Line-II columns for each test case. The cycle times of the lines and the minimum part sets (see  $MPS$  column) belonging to the models produced on particular lines are presented in the table. The table comparatively reports the results of test cases solved as well. The  $IB$  column reports the sum of the  $NS$  values for Line-I and Line-II when the lines were balanced independently. In the  $MPUL$  column, the number of queues ( $NQ$ ) and the  $NS$  values of the obtained solutions are reported for each test case. The  $Diff$  column denotes the relative difference in the  $NS$  values between the two solution strategies and is calculated as  $Diff = (IB_{NS} - MPUL_{NS})/IB_{NS}$ .

Table 4. The design and results of experimental tests.

As can be seen from the results, the proposed MPUL design helps reduce the number of workstations in 11 out of 24 test cases. The largest difference (0.33 or 33%) was observed for test case 5 for which the  $IB$  and  $MPUL$  solutions were obtained 6 and 4, respectively. Thus,  $MPUL$  system helped save two workstations for this particular case. The second (0.25) and the third (0.17) best improvements have been recorded for test cases 4 and 1, respectively. For test case 4, the proposed  $MPUL$  design requires 6 workstations while 8 workstations would be needed when the lines were balanced independently. Hence, a 25% improvement has been achieved. In test case 1, the  $MPUL$  design helped save one workstation with a 17 percent improvement over the  $IB$  condition. In test cases 13 and 21, two workstations were saved while one workstation was saved in test cases 9, 10, 14, 16, 17 and 22. For the remaining 13 test cases,  $MPUL$  heuristic requires the same number of workstations with  $IB$  solution.

A paired sample t-test was conducted to statistically analyse the experimental test results and determine whether the proposed  $MPUL$  system makes a significant improvement over  $IB$ . For this aim, the  $NS$  values obtained by the two solution strategies were used as input data. The hypotheses stated at the 95% confidence interval ( $\alpha = 0.05$  significance level) are as follows:

$H_0$ : There is no statistically significant mean difference in NS values obtained by IB and MPUL ( $\mu_{MPUL} = \mu_{IB}$ ).

$H_1$ : The mean difference in NS values obtained by IB and MPUL are statistically significant ( $\mu_{MPUL} \neq \mu_{IB}$ ).

The data was analysed using Minitab 16 statistical software package. The test results presented in Table 5 indicated that there was a significant difference in the mean NS values of MPUL ( $13.96 \pm 7.60$ ) and IB ( $14.58 \pm 7.59$ );  $t(23) = -3.98, p < 0.001$ . Specifically, these results confirmed that the proposed MPUL system helps minimise the total number of workstations in comparison with independently balanced MMULs. This has clearly demonstrated that the parallelisation of MMULs provides promising advantages which need further investigation in future studies.

Table 5. The paired sample t-test results.

## 7. Discussion and conclusions

The main aim of this study was to introduce a flexible as well as efficient manufacturing system design which may replace traditional U-shaped lines, in future. For this aim, two MMULs were located in parallel to each other and a unique line configuration was obtained. So that, tasks belonging to the models being produced in an inter-mixed sequence on both of the lines can be performed at multi-line stations utilised in between two adjacent lines. Also, as emphasised in details in this paper, the proposed line system combines the advantages of its individual sub-configurations (*i.e.*, mixed-model lines, parallel lines and U-shaped lines) which have already been discussed widely in the literature. The MPUL balancing problem was defined and discussed by taking model sequences into consideration. It was illustratively shown that the feasibility and so the quality of a balancing solution obtained is affected by the sequences of models being assembled on the lines. The dynamism of model combinations in multi-line stations and crossover stations and their effects on the workload times of these workstations have also been exhibited clearly.

In addition to the benefits of the proposed system, such a complex system design brings some difficulties which require sophisticated solution procedures for building efficient balancing techniques. As an advantage of the U-shaped lines, the balancing process starts from both front and back areas of the lines and this provides advantages over straight lines. However, as the number of workstations that need to be utilised is uncertain at the beginning of the balancing process, it is not known which models will appear in the workstations located at the back of the



1  
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3 line. In this environment, a heuristic algorithm was proposed to generate feasible model  
4 sequences considering minimum part sets and build balancing solutions considering these model  
5 sequences.  
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8 An explanatory example has been provided to show the running principle of MPUH and the  
9 implementation of the MPUL idea. Experimental tests have also been conducted to examine the  
10 superiority of the MPUL design against independently balanced mixed-model lines. The  
11 experimental test results and their statistical analysis have confirmed the advantages of the  
12 proposed design in minimising the total number of workstations. As a possible industrial  
13 application of the research, the techniques implemented in this study can easily be adopted by  
14 practitioners, *e.g.*, line managers in car manufacturing plants, as explained in the paper. By this  
15 way, companies will be able to produce customised products designed based on their customers'  
16 requests while reducing the need for workforce.  
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## 22 23 **8. Future research directions**

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25 The outcome of this paper made it clear that the MPUL design has a promising potential to yield  
26 more efficient line systems. Therefore, the implementation of the MPUL idea and the MPUH  
27 algorithm can be improved in future studies. Developing an improved procedure to detect which  
28 model will be produced on the back of the line at a specific workstation should be considered.  
29 Also, other heuristics/metaheuristics can be developed to solve the MPUL balancing problem  
30 and their performances can be compared with that of the MPUH. In doing so, different model  
31 sequencing procedures may also be integrated into the model so that the model sequencing and  
32 the line balancing problems can be solved simultaneously.  
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38 As the efficiency of a production system is strongly related to its ergonomic conditions,  
39 considering ergonomic issues when designing assembly lines is gaining even more importance  
40 in recent studies. For this reason, the current work can also be extended considering the physical  
41 strains, psychological strains, working postures and skill levels of the operators. Furthermore,  
42 some slackness may also be allowed for operators during their working period. This will have  
43 significant effects on the performance of the operators because zero idle times may result in  
44 more mistakes which yield reduced efficiency. Finally, standing on hard surfaces for a  
45 prolonged time can produce circulatory problems in the legs and feet of operators. Therefore,  
46 workplaces should allow enough room for operator movements.  
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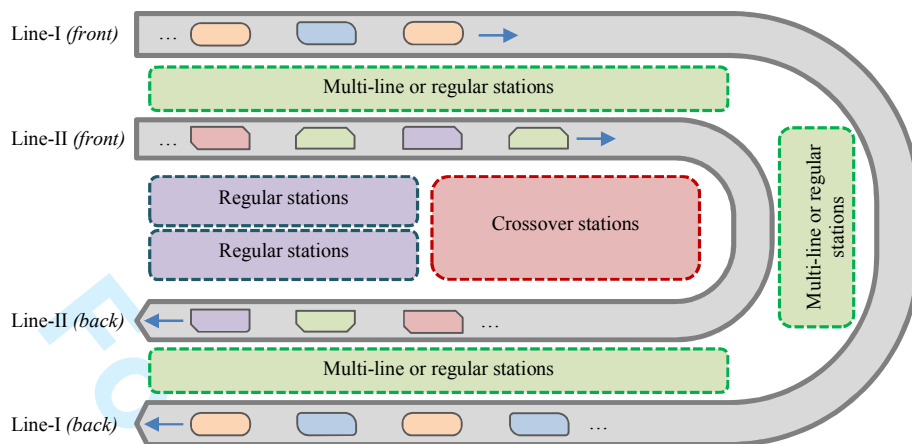


Figure 1. The proposed parallel U-shaped assembly line system.

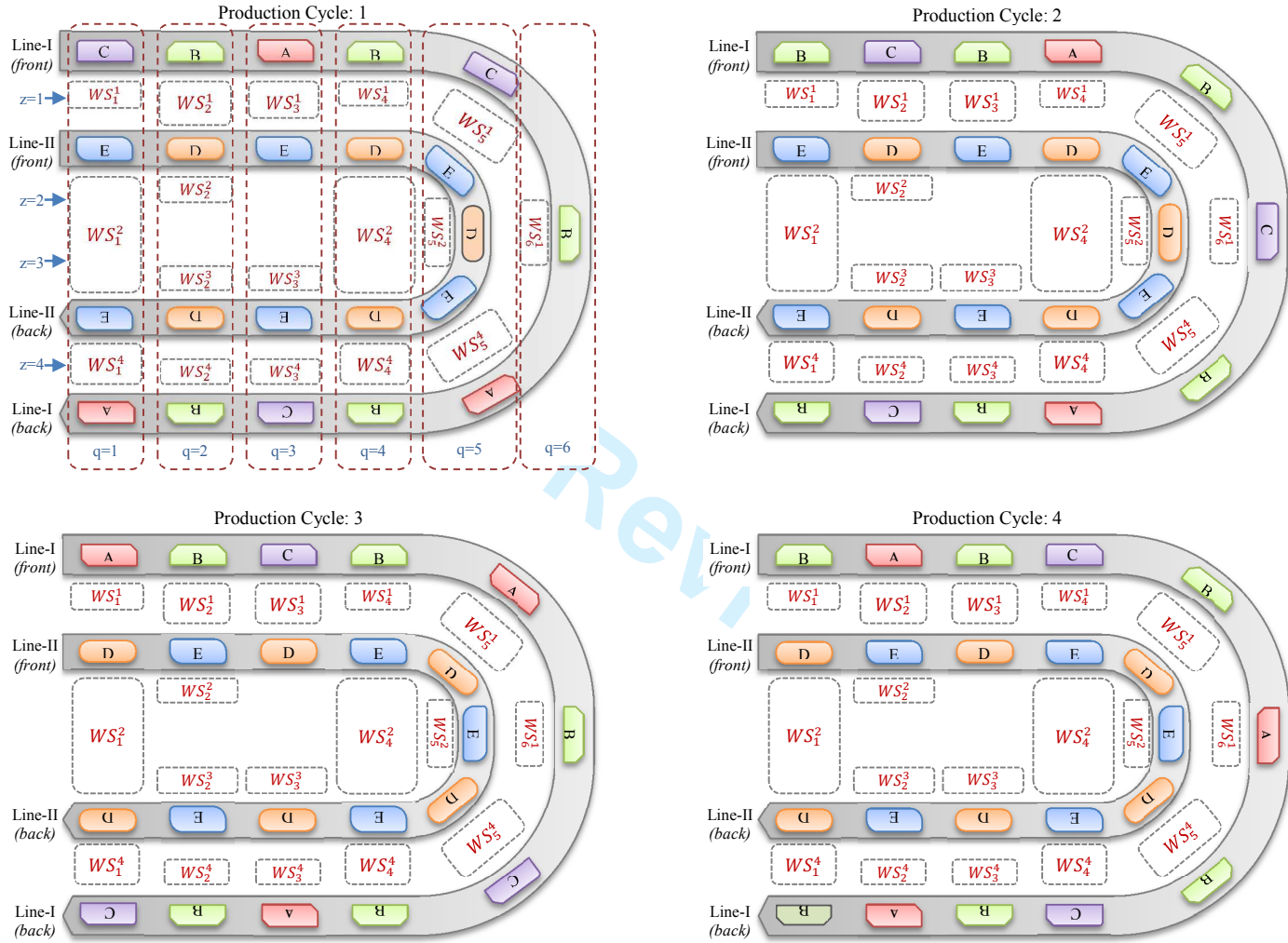


Figure 2. The changing model combinations through different production cycles.

**Start** by generating all possible model sequencing combinations for Line-I and Line-II.

If the lines have different cycle times, apply the LCM based approach to revise task times and determine a common cycle time (C), primarily.

Calculate task priority indexes ( $\omega_{hi}^P$ ) for all tasks on Line-I and Line-II.

Generate all possible model sequences for lines based on the minimum part sets and choose first model sequencing combination.

**While** there is untried model sequencing combination

Choose the next model sequencing combination.

**For** ( $balancingIt = 0$ ;  $balancingIt \leq maxBalancingItNumber$ ;  $balancingIt++$ )

Initialise unassigned tasks lists ( $UTL_1$  and  $UTL_2$  for Line-I and Line-II, respectively) and zone-queue indexes ( $z = randomIntegerBetween[1,2]$  and  $q = 1$ ). To remind,  $WS_q^z$  represents the workstation in zone  $z$  and queue  $q$ ; where  $z = 1, 2, \dots, 4$  and  $q = 1, 2, \dots, Q$ .

Initialise all workstation times for all production cycles ( $WT_q^{z\phi} \leftarrow 0$ ;  $z = 1, 2, \dots, 4$ ;  $q = 1, 2, \dots, Q$ ;  $\phi = 1, 2, \dots, \phi$ ) and earliest starting times of all tasks for all production cycles ( $EST_{hi}^\phi \leftarrow 0$ ;  $h = 1, 2$ ;  $i = 1, 2, \dots, T_h$ ;  $\phi = 1, 2, \dots, \phi$ ). Set the boolean variable  $isFrontArea = true$ .

**While**  $UTL_1 \neq \emptyset$  or  $UTL_2 \neq \emptyset$

Go to workstation  $WS_q^z$  and determine all available tasks from Line-I and Line-II, where  $ATL_1$  and  $ATL_2$  are the set of assignable tasks from Line-I and Line-II, respectively.

**While** there are available tasks from Line-I or Line-II ( $ATL_1 \neq \emptyset$  or  $ATL_2 \neq \emptyset$ )

Select and assign a task from the  $ATL_1$  or  $ATL_2$  in accordance with the task selection procedure.

Remove the assigned task from the relevant unassigned tasks list ( $UTL_1$  or  $UTL_2$ ).

Set the  $WT_q^{z\phi}$  to the finishing time of the assigned task for each production cycle. Finishing time of a task in production cycle  $\phi$  is the maximum of the following two options: *i*) the summation of the current workstation time ( $WT_q^{z\phi}$ ) and the deserved task time ( $DT_{hi}$ ) of the assigned task, or *ii*) the summation of the earliest starting time of the assigned task in production cycle ( $EST_{hi}^\phi$ ) and the deserved task time ( $DT_{hi}$ ) of the assigned task; to formulate,  $WT_q^{z\phi} \leftarrow DT_{hi} + \max\{WT_q^{z\phi}, EST_{hi}^\phi\}$ .

**If** (the assigned task is from the front of the precedence relationships graph)

Update  $EST_{hi}^\phi$  values of its immediate successors.

**else if** (the assigned task is from the back of the precedence relationships graph)

Update  $EST_{hi}^\phi$  values of its immediate predecessors.

**End if**

**If** ( $isFrontArea = true$ ),  $z = randomIntegerBetween[1,2]$ .

**else if** ( $isFrontArea = false$ ),  $z = randomIntegerBetween[3,4]$ .

**End if**

Update available tasks lists ( $ATL_1$  and  $ATL_2$ ) for the new situation.

**End while**

Reset earliest starting times of all tasks in  $UTL_1$  and  $UTL_2$  for all production cycles ( $EST_{hi}^\phi \leftarrow 0$ ;  $h = 1, 2$ ;  $i = 1, 2, \dots, T_h$ ;  $\phi = 1, 2, \dots, \phi$ ).

Alternate  $isFrontArea$  variable, i.e. if ( $isFrontArea = true$ ): then set  $isFrontArea = false$ ,

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1  
2  
3     or vice versa.  
4     If (isFrontArea = false), set  $z = \text{randomIntegerBetween}[1,2]$  and proceed to next queue  
5     ( $q \leftarrow q + 1$ ).  
6  
7     End while  
8  
9     End for  
10  
11 End while  
12  
13 Terminate the algorithm.
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Figure 3. The pseudo-code of MPUH.

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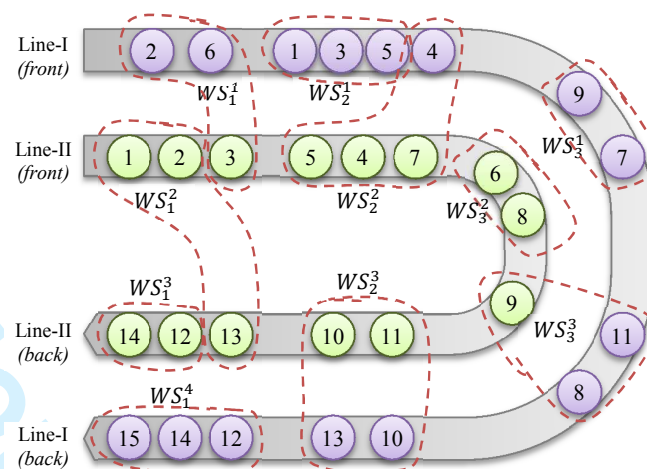


Figure 4. The best balancing solution obtained after 50 iterations.

Table 1. Input data for the numerical example (task times in time units).

Task No	Line-I								Line-II							
	IM	Time		Calculated Parameters					IM	Time		Calculated Parameters				
		A	B	$wpt_{hi}$	$PW_{hi}$	$\omega_{hi}^{PW}$	$\omega_{hi}^{NS}$	$\omega_{hi}^{PI}$		C	D	$wpt_{hi}$	$PW_{hi}$	$\omega_{hi}^{PW}$	$\omega_{hi}^{NS}$	$\omega_{hi}^{PI}$
1	3	5	5	5	46	14	15	210	3	4	3	3.5	45	13	13	169
2	5	4	4	4	41.5	13	13	169	4,5,6	2	5	3.5	74	14	14	196
3	4,7	1	2	1.75	36	11	14	154	7	2	2	2	38	11	12	132
4	7	2	1	1.25	34	10	12	120	7	3	3	3	40	12	11	132
5	8	3	2	2.25	33.5	9	11	99	9	3	3	3	36	10	10	100
6	9	4	3	3.25	47	15	10	150	8	5	4	4.5	27	7	8	56
7	10,12	3	4	3.75	31.5	8	9	72	9	2	2	2	34	9	9	81
8	10	4	5	4.75	29	7	8	56	10	5	5	5	18	5	6	30
9	11	7	6	6.25	40.5	12	7	84	11,12	1	1	1	30	8	7	56
10	13	2	0	0.5	21	5	6	30	13	1	1	1	8	3	5	15
11	13	4	4	4	28	6	5	30	13	3	3	3	12	4	4	16
12	14	3	3	3	18	3	4	12	14	8	8	8	19	6	3	18
13	14	4	4	4	20	4	3	12	14	2	2	2	6	2	2	4
14	15	5	5	5	12	2	2	4	-	0	2	1	2	1	1	1
15	-	1	2	1.75	2	1	1	1	-	-	-	-	-	-	-	-

Table 2. The task selection procedure.

Step	z	q	Available Tasks		Selected Task	Completion Time (in time-units)			
			$ATL_1$	$ATL_2$		$\varphi = 1$	$\varphi = 2$	$\varphi = 3$	$\varphi = 4$
1	2	1	1,2,6	1,2,14	$ATL_2\{2\}$	2 [C]	5 [D]	2 [C]	5 [D]
2	1	1	1,2,6	1,4,5,6	$ATL_1\{1\}$	5 [B]	5 [B]	5 [A]	5 [B]
3	2	1	2,3,6	1,4,5,6,14	$ATL_2\{1\}$	6 [C]	8 [D]	6 [C]	8 [D]
4	2	1	3	3,14	$ATL_2\{3\}$	8 [C]	10 [D]	8 [C]	10 [D]
5	1	1	2,3,6	4,5,6	$ATL_1\{3\}$	7 [B]	7 [B]	6 [A]	7 [B]
6	1	1	4,6	4,5	$ATL_1\{6\}$	10 [B]	10 [B]	10 [A]	10 [B]
7	4	1	15	14	$ATL_1\{15\}$	2 [M]	2 [M]	2 [M]	2 [M]
8	3	1	14	4,5,6,14	$ATL_2\{14\}$	2 [M]	2 [M]	2 [M]	2 [M]
9	4	1	14	12,13	$ATL_1\{14\}$	7 [M]	7 [M]	7 [M]	7 [M]
10	4	1	12	13	$ATL_1\{12\}$	10 [M]	10 [M]	10 [M]	10 [M]
11	3	1	-	4,5,6,12,13	$ATL_2\{12\}$	10 [M]	10 [M]	10 [M]	10 [M]
12	2	2	2,4,9	4,5,6,13	$ATL_2\{4\}$	3 [D]	3 [C]	3 [D]	3 [C]
13	1	2	2,4,9	5,6,7	$ATL_1\{2\}$	4 [B]	4 [B]	4 [B]	4 [A]
14	1	2	4,5	5,6,7	$ATL_2\{5\}$	8 [D]	8 [C]	8 [D]	8 [C]

Table 3. The workload times of workstations across production cycles.

#	Station	Production Cycle	$\varphi = 1$	$\varphi = 2$	$\varphi = 3$	$\varphi = 4$
1	$WS_1^1$	Assigned: Model[Tasks]	B[2,6] C[3]	B[2,6] D[3]	A[2,6] C[3]	B[2,6] D[3]
		Workload Time	9	9	10	9
2	$WS_2^2$	Assigned: Model[Tasks]	C[1,2] D[13]	D[1,2] C[13]	C[1,2] D[13]	D[1,2] C[13]
		Workload Time	8	10	8	10
3	$WS_1^3$	Assigned: Model[Tasks]	D[14,12]	C[14,12]	D[14,12]	C[14,12]
		Workload Time	10	8	10	8
4	$WS_1^4$	Assigned: Model[Tasks]	B[15,14,12]	B[15,14,12]	B[15,14,12]	A[15,14,12]
		Workload Time	10	10	10	9
5	$WS_2^1$	Assigned: Model[Tasks]	B[1,3,5]	B[1,3,5]	B[1,3,5]	A[1,3,5]
		Workload Time	9	9	9	9
6	$WS_2^2$	Assigned: Model[Tasks]	D[5,4,7] B[4]	C[5,4,7] B[4]	D[5,4,7] B[4]	C[5,4,7] A[4]
		Workload Time	9	9	9	10
7	$WS_2^3$	Assigned: Model[Tasks]	C[10,11]	D[10,11]	C[10,11]	D[10,11]
		Workload Time	B[13,10]	B[13,10]	A[13,10]	B[13,10]
		Workload Time	8	8	10	8
8	$WS_3^1$	Assigned: Model[Tasks]	A[9,7]	B[9,7]	B[9,7]	B[9,7]
		Workload Time	10	10	10	10
9	$WS_3^2$	Assigned: Model[Tasks]	C[6,8]	D[6,8]	C[6,8]	D[6,8]
		Workload Time	10	9	10	9
10	$WS_3^3$	Assigned: Model[Tasks]	D[9] B[11,8]	C[9] A[11,8]	D[9] B[11,8]	C[9] B[11,8]
		Workload Time	10	9	10	10

Table 4. The design and results of experimental tests.

Test Case #	Design				Results				Diff	
	Line-I		Line-II		Cycle Time	IB		MPUL		
	Problem	MPS (A-B)	Problem	MPS (C-D)		NS (Line I + Line II)	NQ	NS		
1	4-task	1-1	4-task	1-1	6	6	2	5	0.17	
2	4-task	1-2	4-task	2-1	7	4	1	4	0.00	
3	4-task	1-2	4-task	1-2	5	6	2	6	0.00	
4	5-task	1-1	5-task	1-1	4	8	2	6	0.25	
5	5-task	1-2	5-task	1-2	5	6	1	4	0.33	
6	5-task	1-2	5-task	2-1	6	4	1	4	0.00	
7	12-task	1-1	12-task	1-1	9	10	3	10	0.00	
8	12-task	1-2	12-task	1-2	11	8	2	8	0.00	
9	12-task	1-2	12-task	2-1	13	8	2	7	0.13	
10	15-task	1-2	15-task	2-1	9	14	4	13	0.07	
11	15-task	2-1	15-task	2-1	10	12	3	12	0.00	
12	15-task	1-1	15-task	1-1	12	10	3	10	0.00	
13	16-task	2-1	16-task	1-2	14	20	5	18	0.10	
14	16-task	1-1	16-task	1-1	16	18	5	17	0.06	
15	16-task	1-2	16-task	1-2	18	14	4	14	0.00	
16	25-task	1-1	25-task	1-1	11	28	7	27	0.04	
17	25-task	1-2	25-task	1-2	13	22	6	21	0.05	
18	25-task	2-1	25-task	1-2	14	20	5	20	0.00	
19	27-task	2-1	27-task	2-1	7	28	8	28	0.00	
20	27-task	1-1	27-task	1-1	8	24	6	24	0.00	
21	27-task	1-2	27-task	2-1	10	20	5	18	0.10	
22	30-task	1-1	30-task	1-1	10	20	5	19	0.05	
23	30-task	1-2	30-task	2-1	11	18	5	18	0.00	
24	30-task	1-2	30-task	1-2	9	22	6	22	0.00	

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Table 5. The paired sample t-test results.

	Sample Size	Mean	Standard Deviation	SE Mean
MPUL	24	13.96	7.60	1.55
IB	24	14.58	7.59	1.55
Difference	24	-0.625	0.770	0.157
95% CI for mean difference	: (-0.950; -0.300)			
t-value	: -3.98			
p-value	: 0.001			

For Peer Review Only

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