Modelling Wind Turbine Wakes at Middelgrunden Wind Farm

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Abstract

As part of the development of an offshore wind farm layout optimisation tool, this paper explores the accuracy and computational time of wake models applied to Middelgrunden Wind Farm outside of Copenhagen, Denmark. In this study, four years of data from 2001 to 2004 are used to test the applicability, accuracy, and computational time of the Jensen, Larsen, Ishihara, and a simplified version of the Ainslie Eddy-Viscosity wake models. This study has shown that the size of the directional sector used in the comparison and if that directional sector is applied to all turbines' incoming wind velocities or just the northernmost greatly affects the results. From this it is found that the Larsen wake model provides the best balance between accuracy and computational time. It also shows that even a simplified version of a field model takes significantly longer to compute than an analytic model. This study has also shown that using directional sectors of ±15° these models perform similarly to previous studies at Nysted and Horns Rev indicating that the close spacing (2.4D) at Middelgrunden is not too close for the use of these models.

Keywords: Middelgrunden wind farm, wake modelling, layout optimisation

1 Introduction

With continuing growth in the size of offshore wind farms, it has become increasingly more important to optimise the layout of wind farms in order to ensure that the wind farm extracts energy effectively. To this end, it is important to model and understand the turbine interactions offshore. In the development of a layout optimisation tool to be used to aid in the decision making process for future offshore wind farm projects, a comparative study of wind turbine wake models has been completed.

As a layout optimisation tool would be required to evaluate several different layouts, it is important for the wake model implemented as part of this tool to have both high accuracy and low computational time. In order to classify the existing wake models it was decided to use data available for Middelgrunden Wind Farm in Denmark to compare four existing wake models. The analytic models of Jensen, Larsen, and Ishihara were compared in terms of accuracy and computational time to one another and to a simplified representation of the Ainslie Eddy-Viscosity field model. The Middelgrunden site poses a unique opportunity as the turbines are spaced at only 2.4D. Though this close spacing is in a non-dominant wind direction, looking specifically at the time periods when the wind is in this direction allows us to establish how these wake models compare for closely spaced turbines.

This paper will first outline the approach taken in this analysis in terms of how data was selected, and the impact that the data selection criteria had on the results, as well as the formulation used for each of the wake models. Following this, the results of the study are presented before the conclusions and scope for further work is outlined.

2 Approach

The advantage of the Middelgrunden site over other wind farms is that 10-minute averaged data for four years (2001-2004) is available courtesy of the Virtual Wakes Laboratory and Middelgrunden Windfarm Cooperative. Using this data and subsets of this data, it was possible to apply the wake models and compare the results. The site is, however, not the best suited for a wake study given that the dominant wind direction is perpendicular to the single line of turbines. Therefore the reduction in annual energy production (AEP) due to the wake effect is minimal.

The wake modelling done as part of this study can therefore be further subdivided into two major steps: data selection/filtering and the application of the wake models to the selected data periods.

2.1 Data Selection

Given previous studies of the wakes and modelling the turbulence intensity of the flow at Middelgrunden [1, 2] it was decided to use a similar methodology for the selection of data. The Middelgrunden wind farm is comprised of twenty Bonus B-76/2000 turbines placed along a single arc in a roughly North-South orientation. Wakes are therefore only expected when the wind direction is parallel to the dominant direction of this arc (357°). As wakes are the focus of this study, it was important to filter out the data periods during which the wind was perpendicular to the arc of the turbines resulting in little or no wake effect. Though winds from the South would be expected to result in measurable wake effects it is not considered in this study as due to the proximity to shore and as a result of the shorter fetch a more significant speed-up is observed [1].

It was also important to use time periods where data was available for all twenty turbines, all twenty were grid connected, and all were generating power. In order to do this, the data-set was filtered based on the mean active power for each interval to ensure that they were generating, and based on the generator RPM in order to ensure that they were grid-connected. Any time intervals where any one wind turbine was not operating or was in an error-state was immediately filtered out.

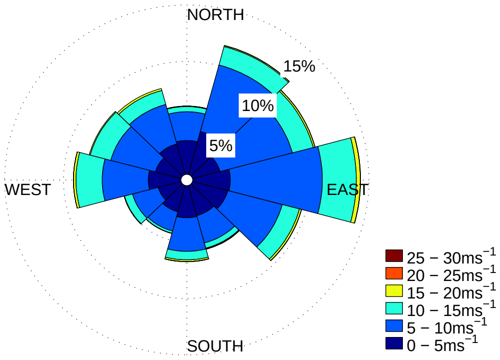


Figure 1: Characteristic wind rose for Middelgrunden Wind Farm based on time-series data from 2001-2004. Data used courtesy of The Middelgrunden Windfarm Cooperative.

Based on these filtering techniques, a number of different sector sizes were considered to observe how this affected the accuracy of the wake models. For each case, the same 357° azimuth was considered. It was also later decided to relax the direction criteria such that turbine 1, the northernmost turbine, was only checked against the incoming wind direction rather than all of the turbines. This is similar to the methodology used in similar studies at Horns Rev [3, 4].

2.2 Wake Models

As this study was completed as part of the development of a layout optimisation tool, it was decided to consider analytic wake models as these would be sufficiently fast to implement as part

Table 1: Data Selection Scenarios

Sector Size	Turbines Checked	Time Intervals
60°	All	1646
30°	All	25
60°	Turbine 1	4701
30°	Turbine 1	2299
20°	Turbine 1	1609
10°	Turbine 1	930
2°	Turbine 1	248

of the optimisation tools. For comparision purposes, a simplification of a field model, the Simplified Ainslie Eddy-Viscosity Model was also implemented. All four of the models under consideration are generally not recommended for use below 4D, though accurate results have been seen for as low as 1.7D. Middelgrunden therefore offers an interesting site to consider as the turbines are spaced at 2.4D [2].

Wake models in general require the thrust curve of the turbine to compute the velocity deficit through conservation of momentum. Some models also take into account the mixing of the air and therefore require a value for the ambient turbulence intensity. For this study, the thrust and power curves for the Bonus B76/2000 were provided in the literature [1]. Previous studies have also identified the ambient turbulence intensity to be approximately 13% which was used in this study [2].

2.2.1 Jensen Model

The simplest of the analytic wake models is the Jensen model which was originally devised in the 1980's. This wake model is based on momentum balance through the rotor plane of a single turbine and assumes that the wake expands linearly behind the rotor [3–6].

As the wake is assumed to expand linearly downstream of the turbine, the wake diameter d_w is given by:

$$d_w = d_r \times (1 + 2ks) \tag{1}$$

where d_r is the rotor diameter, k is the wake decay factor, and s is the non-dimensional distance downwind of the turbine ($s = \frac{x}{d_r}$, where x is the perpendicular distance downwind of the turbine) [6–8].

The wake decay factor, k, describes the relative persistence of the wake downstream of the turbine and can be related to the ambient turbulence intensity (I_a) [8, 9].

$$k = \frac{1}{2} I_a \tag{2}$$

According to this model, the wind velocity deficit experienced by a downstream turbine scales proportionally to the ratio of the rotor area that lies within a wake and is given by:

$$D_{i,j} = \frac{1 - \sqrt{(1 - C_{T_j})}}{(1 + 2ks)^2} \cdot \frac{A_{ij}}{A_i} \tag{3}$$

where C_{T_j} is the thrust coefficient of the upwind turbine j, A_{ij} is the area of intersection between the downstream turbine's rotor plane and the wake of the upstream turbine, and A_i is the rotor swept area of the downwind turbine i [8]. It is important to note that this model assumes that the thrust coefficient C_T does not exceed 1.

The above formulation accounts only for the wake behind a single turbine. However, further development of this model by Katic et al. [10] led to a means of superposing multiple single wakes to compute the total velocity deficit experienced by a turbine due to the combined effect of multiple upwind turbines using a root-sum-square formulation.

Using this updated formulation, the total velocity deficit factor D is given by:

$$D_i = \sqrt{\sum_j (D_{ij})^2} \tag{4}$$

The velocity experienced by the downwind turbine is therefore:

$$u_i = u_\infty \cdot (1 - D_i) \tag{5}$$

where u_∞ is the free stream wind speed.

2.2.2 Larsen Model

A subsequent analytic model that was developed was the Larsen Model which was included as part of the European Wind Turbine Standards II (EWTS-II) [11]. This model is also an analytic wake model, however, unlike the Jensen model it does not assume a linear expansion, nor does it assume that the deficit is equal in the radial direction [3, 6, 12]. The model is based on a closed-form solution to the Reynolds-Averaged Navier-Stokes (RANS) equations based on Prandtl mixing theory.

Below are the key equations of the Larsen method:

$$u_\infty - u_i = -\frac{u_\infty}{9}\left[C_{T_j} A_i (x + x_0)^{-2}\right]^{\frac{1}{3}} \left[r^{\frac{3}{2}} \left(3c_1^2 C_{T_j} A_i (x + x_0)\right)^{-\frac{1}{2}} - \left(\frac{35}{2\pi}\right)^{\frac{3}{10}} \left(3c_1^2\right)^{-\frac{1}{5}}\right]^2 \tag{6}$$

$$R_w = \left(\frac{35}{2\pi}\right)^{\frac{1}{5}} \left(3c_1^2\right)^{\frac{1}{5}} \left(C_t A x\right)^{\frac{1}{3}} \tag{7}$$

The parameters x_0 and c_1 are given by:

$$x_0 = \frac{9.5d}{\left(\frac{2R_{9.5}}{d_{\text{eff}}}\right)^3 - 1} \tag{8}$$

$$c_1 = \left(\frac{d_{\text{eff}}}{2}\right)^{\frac{5}{2}} \left(\frac{105}{2\pi}\right)^{-\frac{1}{2}} \left(C_{T_j} A_i x_0\right)^{-\frac{5}{6}} \tag{9}$$

where d_{eff} is the effective rotor diameter, and $R_{9.5}$ is the wake radius at a distance of 9.5 rotor diameters downstream of the turbine. This term includes a correction to include the ground effect.

$$d_{\text{eff}} = d\sqrt{\frac{1 + \sqrt{1 - C_{T_j}}}{2\sqrt{1 - C_{T_j}}}} \tag{10}$$

$$R_{9.5} = 0.5(R_{nb} + min(H, R_{nb}) \tag{11}$$

where H is the hub height, and R_{nb} is an empirically found relationship related to the ambient turbulence:

$$R_{nb} = max\left[1.08d, 1.08d + 21.7d(I_a - 0.05)\right] \tag{12}$$

No agreed upon method exists for superposing the single wakes modelled by the Larsen wake model, however, either linear superposition or root-sum-square superposition tend to be used. For this study, a similar root-sum-square superposition as was used in the Jensen model is used similar to eq. (4).

2.2.3 Ishihara Model

The Ishihara model is one of the lesser known analytic wake models which is rarely used in practice. Uniquely this model accounts for not only the ambient turbulence, however, includes a term for the mechanically generated turbulence in the wake recovery zone. This model was originally developed based on wind tunnel experiments, and therefore includes a number of empirical constants. Little work has been done in validation or calibration of this model and it is likely necessary for the empirical constants to be adjusted to better represent real wind farms [4, 13, 14]. Like the other models described, this is a single wake model for which a root-sum-square method has been implemented to account for the superposition of single wakes.

In this model, the wake diameter is given by:

$$d_w = \frac{k_1 C_{T_j}^{\frac{1}{4}}}{0.833} d^{1-\frac{p}{2}} x^{\frac{p}{2}} + d \tag{13}$$

where p is a function of the ambient turbulence I_a and the mechanically generated turbulence I_w.

$$p = k_2 \left(I_a + I_w\right) \tag{14}$$

The mechanically generated turbine turbulence is given by:

$$I_w = \frac{k_3 C_T}{max\left(I_a, 0.03\right)} \left(1 - exp\left[-4\left(\frac{x}{10d}\right)^2\right]\right) \tag{15}$$

For a single wake, the velocity experienced by a downstream turbine is given by:

$$u_i = \frac{\sqrt{C_T} u_\infty}{32} \left(\frac{1.666}{k_1}\right)^2 \left(\frac{x}{d}\right)^{-p} exp\left(-\frac{r^2}{d_w^2}\right) \tag{16}$$

For this model, the k parameters were empirically found based on the wind tunnel studies to be:

$$k_1 = 0.27 \tag{17a}$$
$$k_2 = 6.00 \tag{17b}$$
$$k_3 = 0.004 \tag{17c}$$

2.2.4 Simplified Ainslie Eddy-Viscosity Model

The final of the wake models used is a simplified version of the Ainslie Eddy-Viscosity field model. The Ainslie Eddy-Viscosity model solves the RANS equations using an eddy-viscosity closure term [15, 16]. This model is widely used in commercial wind resource assessment packages such as WindFarmer, OpenWind, and WindPRO.

The simplified version, developed by Mike Anderson of RES [17] allows the Ainslie Eddy-Viscosity model to be simplified, requiring far less computational time without significantly affecting the result.

Based on the full solution of the eddy-viscosity model it was found that the initial Gaussian shape profile is preserved downstream. Therefore the only parameters of the wake are the centerline velocity profile behind the rotor and the wake width. These assumptions, supported by the full solution to the Navier-Stokes equations, simplify the governing equations to a single ordinary differential equation with the same wake initialization parameters at a distance of two rotor diameters behind the turbine as the original Ainslie Eddy-Viscosity model. The simplified ODE for the center line velocity, u_c, can therefore be given to be:

$$\frac{du_c}{dx} = \frac{16\varepsilon\left(u_c^3 - u_c^2 - u_c + 1\right)}{u_c C_T} \tag{18}$$

As this is a first-order differential equation, a numerical integration scheme using a 4th order Runge-Kutta method is implemented to quickly solve the for the wake effect. It should be noted that in this methodology, all parameters including u_c, u_∞, b, x, and r are non-dimensionalised using the free-stream wind velocity u_∞ and the rotor diameter d as appropriate.

This center line velocity can then be substituted into Ainslie's equation assuming a Gaussian shape profile:

$$1 - \frac{u}{u_\infty} = (u_\infty - u_c)\, exp\left(-3.56\left(\frac{r}{b}\right)^2\right) \tag{19}$$

where the wake width, b, is given by:

$$b = \sqrt{\frac{3.56C_T}{8D_m(1 - 0.5D_m)}} \tag{20}$$

the center line velocity deficit, D_m is given by:

$$D_m = 1 - \frac{u_c}{u_i} \tag{21}$$

The model is initialised two rotor diameters behind the turbine where the initial center line velocity deficit, D_{mi}, is taken to be:

$$D_{mi} = C_T - 0.05 - (16C_T - 0.5)\frac{I_a}{10} \tag{22}$$

This approach has been validated to show that it gives very similar results to the full eddy-viscosity approaches solved using a numerical integration scheme such as Crank-Nicholson [17, 18].

3 Results

For the seven cases outlined in table 1 each of the four wake models described in section 2.2 was run. The total normalised production value for each of the twenty turbines was then computed across the entire data-set while the computational time was measured. The analysis was also repeated for individual wind speed bins to observe the model accuracy at specific wind speed ranges. All wake models were formulated in Matlab 2013a and executed on a Dell PowerEdge R415 with Operton 427HR Processor (2.5 GHz) and 66 GB RAM.

3.1 Computational Time

As would be expected, the computational time for each of the wake models was roughly linear with the number of time intervals for which the wakes needed to be computed.

As can be seen from fig. 2, for each case the Larsen and Ishihara models were consistently the quickest with very little difference between them, while the Simplified Ainslie Eddy-Viscosity model was consistently the slowest.

3.2 Direction Constraint Applied to All Turbines

Following the approach given in section 2, the directional criteria were first imposed on all the turbines. Applying the direction constraint in this

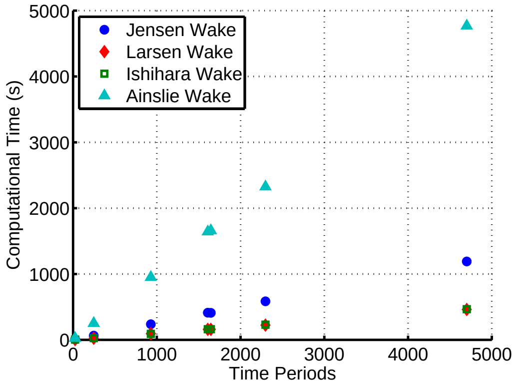


Figure 2: Computational Time

manner lead to fewer valid time periods as is indicated in table 1. In fact, reducing the sector size to 20° led to no valid time periods in the dataset. Therefore the application of the direction constraint to all the turbines is limited to only considering 60° and 30° sectors.

Figures 3a and 3b below show the normalised average power produced from each turbine under the two scenarios. From this it can be observed that all the wake models correctly predict a decrease in the power produced relative to the first turbine in the arc. For the two scenarios considered, the Larsen model was found to be the most accurate for the larger sector size(12.48% RMS error), while the Jensen model was the most accurate for the smaller sector (8.09% RMS error). The smaller sector size was found to have lower RMS errors for each of the models compared to the larger sector size indicating the models are generally more suitable for the smaller sector size. The Jensen and Ishihara models showed the greatest improvement with their RMS errors decreasing 10.62 percentage points and 8.15 percentage points respectively. The Larsen and Ainslie Eddy-Viscosity models, however, only showed a 1.28 percentage point and 3.04 percentage point decrease.

3.3 Direction Constraint Applied to Turbine 1 Only

Relaxation of the directional criteria as described insection 2.1 was similar to the methodology used by Gaumond et al. [3, 13] and Crasto & Castellani [4] in their analyses of wakes at Horns Rev. Relaxation of this directional criteria also allowed for smaller sector sizes to be investigated.

Table 2: RMS Error, Directional Criteria Applied to All Turbines

Sector	Jensen	Larsen	Ishihara	Ainslie
±15°	**8.09%**	11.19%	15.10%	10.08%
±30°	18.71%	**12.48%**	23.25%	13.13%

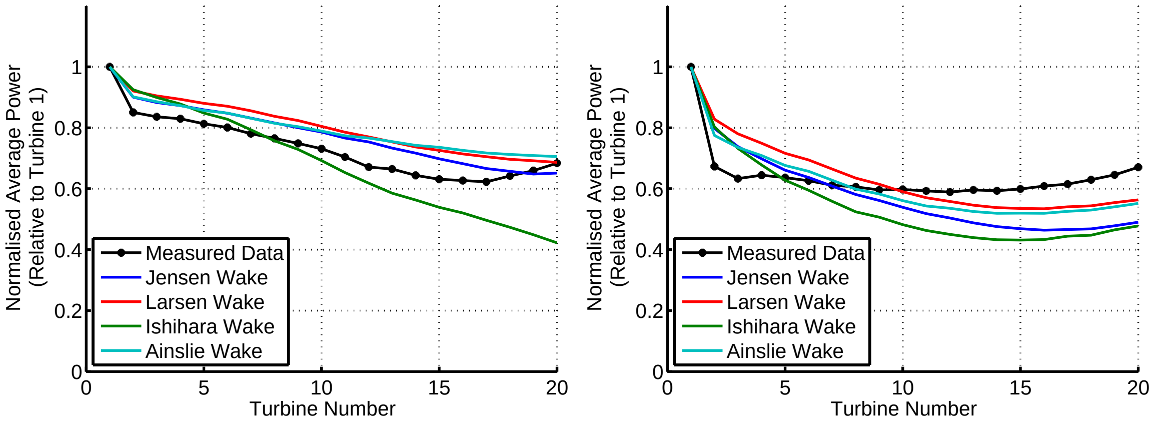
Figures 4a and 4b show the normalised power output from each of the turbines for the ±15° and ±30° sectors respectively. From these it can be observed that as in the previous scenarios a decrease in power output is observed down the line of turbines as would be expected. However, unlike the previous scenarios where the move from a ±30° sector to a ±15° sector resulted in improvements in the wake models, the application of the directional criteria to only the first turbine appears to increase in error as the directional sectors decrease in size (see table 3). Best performance was in fact observed for all the wake models when the largest sector size was considered. For this method of data selection, the Larsen model proved to be the most accurate for all but the smallest of the sector sizes when the Simplified Ainslie gave marginally better results.

Table 3: RMS Error, Directional Criteria Applied to Turbine 1

Sector	Jensen	Larsen	Ishihara	Ainslie
±1°	45.91%	41.76%	61.20%	**41.09%**
±5°	38.73%	**33.40%**	53.58%	34.19%
±10°	30.97%	**23.77%**	40.20%	26.15%
±15°	23.84%	**15.88%**	27.44%	18.67%
±30°	15.59%	**8.34%**	13.52%	11.23%

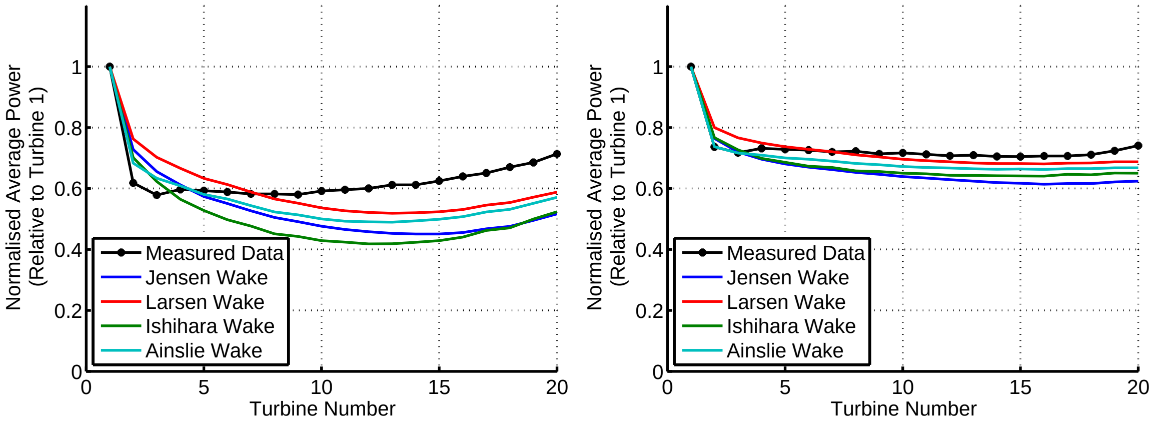
3.4 Model Sensitivity to Wind Speed

As would be expected, the behaviour of the wakes vary with the wind speed and the wake models are therefore more accurate when applied at certain wind speeds at this site. Figures 5a to 5c show the model behaviour at specific wind speeds. As can



(a) 357°±15° **(b)** 357°±30°

Figure 3: Wake Deficit - Direction Sector Applied to All Turbines



(a) 357°±15° **(b)** 357°±30°

Figure 4: Wake Deficit - Direction Sector Applied to Turbine 1

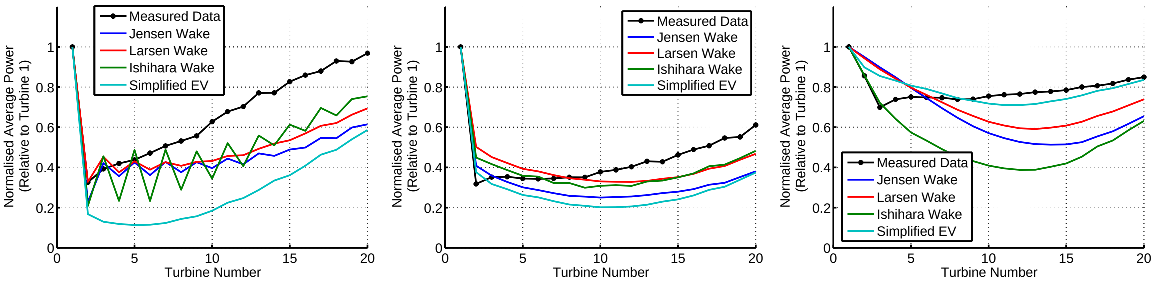
be seen in this series of figures, the wake models all perform best around $8\,\mathrm{m\,s^{-1}}$. High errors can be observed at both low and high wind speeds.

4 Discussion

The previous similar studies applied to Horns Rev found that the Larsen model best described the power deficit at Horns Rev [3, 4, 13]. These studies also found that decreasing the sector size beyond ±15° led to higher levels of error. Smaller sectors such as ±5° or ±1° therefore led to an over-estimation of the wake effect and the power deficits down a single line of turbines at Horns Rev. Similarly in the present study, smaller sectors such as and ±10° or ±5° lead to higher levels of RMS error. This result did, however, not hold for the analysis in which all turbines were compared against the direction criteria.

Checking all the turbines against the direction criteria lead to difficult results due in part to the amount of data constituting each data-set. The smaller sector size under consideration, ±15°, had only 25 valid time intervals thereby implying high levels of uncertainty. Though this scenario did result in lower RMS error than the case where the direction criteria was only applied to turbine 1, this needs to be further explored with larger data-sets.

In fact checking all the turbines against the di-



(a) 5 m s^{-1} Free Wind **(b)** 8 m s^{-1} Free Wind **(c)** 15 m s^{-1} Free Wind

Figure 5: Turbine Waked Wind Velocities

rection criteria resulted in lower levels of RMS error for similarly sized sectors. This is in fact as we would expect as comparing all turbines against the directional sector ensures that there is little variation in wind direction through the wind farm. It can be expected that the methodology which is similar to that of studies at Horns Rev, considering the direction only at one turbine, would be more applicable of the end-use in a layout optimisation tool.

Interestingly, the simplified field model was not significantly more accurate than a simpler analytic model and in fact only outperformed the analytic models on one occasion. The Simplified Ainslie Eddy-Viscosity model was, however, consistently the slowest as expected due to the iterative nature required in solving it. The Jensen model, though the simplest in principle requires a relatively complex computation to determine the ratio of the rotor plane area that is within a wake and therefore suffers as a result of this. The Larsen and Ishihara models likely have similar computational times as they are both relatively simple and require the same order of computations in order to compute the waked velocities.

It is important to note that none of the wake models implemented includes any kind of wake drift or wake meandering model. This omission does increase the uncertainty of these wake models, however, it is unclear to what degree [19–21].

The Bonus turbines in question are also known to have anemometers that give erroneously low readings [1]. Looking therefore at the non-normalised values, it can be observed that even at the first turbine the “modelled” power output is under-predicted. The use of these anemometer readings therefore introduces some uncertainty and it is worth exploring a similar study where better data might be available.

The average wind speed measured by the anemometer on turbine 1 over the data period is 6.6 m s^{-1} indicating that the optimal region of the models may in fact be very close to the average condition at the site leading to the low levels of RMS error observed. Had the site had an average condition further from the accurate region of the models we could expect larger levels of error.

5 Conclusion

This study explored modelling the wake effect at Middelgrunden wind farm. The study considered four different wake models, none of which are recommended for turbine spacing below 5D. This study has, however, shown that for turbines spaced at 2.4D all four models can give results on the order of 8-15% RMS error. Likely sources of this error are the error on the anemometer, the use of a global turbulence intensity, and the scarcity of data after the filtering process.

Though each wake model has different errors for each incoming wind velocity, the overall performance of the models was considered here. From this analysis it was found that the lowest RMS errors were on the order of 8% and achieved using either the Jensen or Larsen wake models depending on the data selection criteria. With the exception of one of the data selection scenarios, the Larsen wake model was consistently the most accurate. The Ainslie Eddy-Viscosity field model had RMS error values close to that of the Larsen model; however, they were consistently higher indicating that for the extra computational time there was no gain in accuracy. These preliminary results suggest that of the four models considered, the Larsen wake model constitutes the best compromise between accuracy and computational time regardless of the data selection criteria, and therefore would be best suited for implementation as part of a layout optimisation tool. Although the Ishihara model was often one of the quickest, it did consistently result in some of the highest errors, consistent with previous work at Horns Rev [3, 4, 13]. It is likely that the Jensen model required additional computational time compared to the other kinematic models due to the fact that it computes the fraction of the rotor plane that is within the wake of another turbine rather than including a radial term. It was also found that the computational time for each model could be approximated as a linear function of the number of 10-minute data points under consideration.

This work has, however, been unable to identify the most appropriate data selection criteria for these models. Further work should validate these models against additional wind farms and explore the data selection criteria at greater depth.

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