

Empirical Essays on Inferring Information from Options and Other Financial Derivatives



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Abstract

This thesis consists of three essays on inferring information from option contracts and other financial derivatives in the U.S. market as well as in the international markets.

The **first** essay examines corporate bankruptcy probabilities inferred from option prices and credit default swaps (CDS) spreads around the 2008 financial crisis in the U.S. market. Option pricing framework is used where the risk-neutral density of the underlying asset is assumed to be a mixture of two lognormals augmented with a probability of default, to calibrate to the market option prices. The CDS model assumes a constant default probability which is solved from the non-linear equation that equates the present value of expected premium payments with the present value of expected payoffs. The essay documents that both sources provide ex-ante bankruptcy probabilities, but there is no significant evidence suggesting one predicts the other.

The **second** essay constructs volatility indices for 15 markets around the world and examines implied volatility spillover between these markets. Volatility indices are constructed using option prices based on the new VIX methodology with modification to address its limitations. Spillover effects are then examined using vector autoregressive analysis, impulse response functions and forecast error variance decomposition. Empirical results show that the U.S. is unambiguously the dominant source of uncertainty in the world. Correlation between markets largely depends on geographical proximity. The findings support the notion of informationally efficient international stock markets, in that information transmitted

from one market to another is processed within one or two days.

The **third** essay further investigates spillover effects in variance risk premiums, which has been interpreted as the difference between the realised variance under the physical measure and the risk-neutral measure. Realized variance under the physical measure is constructed for each market using the HAR-RV model, which is able to capture long-memory characteristic of volatility. Risk-neutral expectation of future variance is approximated by a portfolio of option contracts, as calculated in the **second** essay. Steps are taken to address serial correlation and dependence, and variance risk premium spillovers are examined using vector autoregressive analysis, impulse response functions, and Granger Causality tests. The findings are consistent with those found in implied volatility spillovers. The U.S. market is the distributor of uncertainty in the global market. Information transmitted from one market to another is quickly digested, but it may take longer in crisis period due to greater uncertainty.

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Declaration

I hereby declare that this PhD thesis is being submitted for the degree of Doctor of Philosophy at the University of Lancaster, United Kingdom. It has not been submitted before for any degree or examination at any other university or other institute of higher learning.

I certify that the thesis is my own, unaided work. It does not contain any material previously published or written by another person except where due acknowledgement is made in the text.

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Contents

- 1 Introduction** **1**
 - 1.1 Derivatives 1
 - 1.2 Structure of the thesis 3

- 2 Bankruptcy Probabilities Inferred from Option Prices and CDS Spreads** **5**
 - 2.1 Introduction 5
 - 2.2 Literature review 9
 - 2.2.1 Introduction 9
 - 2.2.2 Accounting-based models 11
 - 2.2.3 Market-based models 16
 - 2.2.4 Econometric models using option prices 21
 - 2.2.5 Methods to estimate risk-neutral densities from option prices 24
 - 2.3 Methodology 29
 - 2.3.1 Framework using option prices 29
 - 2.3.2 Framework using CDS spreads 33
 - 2.4 Data 36
 - 2.5 Empirical results 40
 - 2.5.1 Comparison of bankruptcy probabilities across firms 56

| | | |
|----------|--|-----------|
| 2.5.2 | Comparison between option implied bankruptcy probabilities and CDS bankruptcy probabilities | 59 |
| 2.6 | Conclusions | 61 |
| 2.7 | Appendix | 64 |
| 3 | Model Free Expectation of Implied Volatility and International Spillovers | 94 |
| 3.1 | Introduction | 94 |
| 3.2 | Literature review | 96 |
| 3.2.1 | International stock market spillovers under the historical measure | 98 |
| 3.2.2 | International stock market spillovers under the forward looking measure | 107 |
| 3.3 | Construction of model free implied volatility indices | 111 |
| 3.3.1 | CBOE procedure for constructing VIX | 113 |
| 3.3.2 | Problems associated with the CBOE procedure and solutions | 116 |
| 3.3.3 | Construct implied variance – the BKM methodology | 120 |
| 3.3.4 | Data | 121 |
| 3.3.5 | Empirical results | 123 |
| 3.4 | Implied volatility spillover effect | 126 |
| 3.4.1 | Data | 127 |
| 3.4.2 | Preliminary discussion | 128 |
| 3.4.3 | VAR model specification | 130 |
| 3.4.4 | Impulse response analysis | 133 |
| 3.4.5 | Forecast error variance decomposition | 138 |
| 3.4.6 | Empirical results for groups European area, Asia, and North America . . . | 140 |
| 3.5 | Conclusions | 141 |
| 3.6 | Appendix | 144 |

| | |
|---|------------|
| 4 Variance Risk Premiums and International Spillover Effects | 161 |
| 4.1 Introduction | 161 |
| 4.2 Literature review | 163 |
| 4.3 Construction of Variance Risk Premiums | 174 |
| 4.4 Data | 178 |
| 4.5 Empirical results | 181 |
| 4.5.1 Variance risk premium dynamics | 181 |
| 4.5.2 Spillover analysis | 183 |
| 4.6 Conclusions | 196 |
| 4.7 Appendix | 199 |
| 5 Concluding remarks | 219 |

List of Tables

- 2.1 Sample data description 64
- 2.2 Descriptives of cross-sectional option data used to estimate the MLNbk model 65
- 2.3 Granger Causality tests for bankruptcy probabilities inferred from option prices
and 1 year CDS spreads 66
- 3.1 Equity indices underlying option data 144
- 3.2 Group description 144
- 3.3 ADF unit root tests 145
- 3.4 Lag order selection for VAR(p) model 145
- 3.5 R^2 of VAR regressions 145
- 3.6 Publicly calculated and disseminated volatility indices 146
- 3.7 Comparison of volatility indices calculated using BKM procedure and those
publicly calculated and disseminated 147
- 3.8 Descriptive statistics for calculated volatility indices and Bloomberg source . . 148
- 3.9 Option information on calculating BKM volatility indices 149
- 3.10 Correlation coefficients for differenced implied volatility series 150
- 4.1 HAR-RV regressions 199
- 4.2 Descriptive statistics of realised variance and volatility, implied variance and
volatility, and variance risk premium 200

| | | |
|------|---|-----|
| 4.3 | Pairwise correlations of (return) variance risk premiums | 201 |
| 4.4 | Ljung-Box test on [return] variance risk premiums | 201 |
| 4.5 | GARCH fitting for subsample variance risk premiums | 202 |
| 4.6 | Vector Autoregression Estimates for post-crisis subsample | 203 |
| 4.7 | VAR Granger Causality test for post-crisis subsample | 204 |
| 4.8 | Validity check for post-crisis VAR estimation | 205 |
| 4.9 | Vector Autoregression Estimates for crisis subsample | 206 |
| 4.10 | VAR Granger Causality test for crisis subsample | 206 |
| 4.11 | Validity check for crisis VAR estimation | 207 |
| 4.12 | Vector Autoregression Estimates for pre-crisis subsample | 208 |
| 4.13 | VAR Granger Causality test for pre-crisis subsample | 208 |
| 4.14 | Validity check for pre-crisis VAR estimation | 209 |

List of Figures

- 2.1 Bankruptcy probabilities inferred from option prices and CDS spreads for Bear Stearns 67
- 2.1 Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Bear Stearns 68
- 2.2 Bankruptcy probabilities inferred from option prices and CDS spreads for Lehman Brothers 69
- 2.2 Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Lehman Brothers 70
- 2.3 Bankruptcy probabilities inferred from option prices and CDS spreads for Merrill Lynch 71
- 2.3 Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Merrill Lynch 72
- 2.4 Bankruptcy probabilities inferred from option prices and CDS spreads for Washington Mutual 73
- 2.4 Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Washington Mutual 74
- 2.5 Bankruptcy probabilities inferred from option prices and CDS spreads for Wachovia 75

| | | |
|------|--|----|
| 2.5 | Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Wachovia | 76 |
| 2.6 | Bankruptcy probabilities inferred from option prices and CDS spreads for AIG | 77 |
| 2.6 | Continued Bankruptcy probabilities inferred from option prices and CDS spreads for AIG | 78 |
| 2.7 | Bankruptcy probabilities inferred from option prices and CDS spreads for J.P. Morgan Chase | 79 |
| 2.7 | Continued Bankruptcy probabilities inferred from option prices and CDS spreads for J.P. Morgan Chase | 80 |
| 2.8 | Bankruptcy probabilities inferred from option prices and CDS spreads for Bank of America | 81 |
| 2.8 | Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Bank of America | 82 |
| 2.9 | Bankruptcy probabilities inferred from option prices and CDS spreads for Citigroup | 83 |
| 2.9 | Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Citigroup | 84 |
| 2.10 | Bankruptcy probabilities inferred from option prices and CDS spreads for Goldman Sachs | 85 |
| 2.10 | Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Goldman Sachs | 86 |
| 2.11 | Bankruptcy probabilities inferred from option prices and CDS spreads for Morgan Stanley | 87 |
| 2.11 | Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Morgan Stanley | 88 |

| | |
|---|-----|
| 2.12 Bankruptcy probabilities inferred from option prices and CDS spreads for Wells Fargo | 89 |
| 2.12 Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Wells Fargo | 90 |
| 2.13 Bankruptcy probabilities inferred from option prices across healthy firms and distressed firms | 91 |
| 2.14 Bankruptcy probabilities inferred from option prices across healthy firms and distressed firms | 92 |
| 2.15 Times series plots of stock prices for all firms | 93 |
| 3.1 Option trading hours for foreign markets on the same calendar day (in Tokyo time) | 151 |
| 3.2 IV_S&P/TSX 60 vs. VIXC | 152 |
| 3.3 IV_S&P 500 vs. VIX | 153 |
| 3.4 System autocorrelations with 2 Std.Err. bounds, for the Global group | 154 |
| 3.5 Impulse responses to one S.D. innovations with 2 Std.Err. bounds, for the Global group. Variable ordering: $US_t, Eurozone_t, UK_t, JP_t$ | 155 |
| 3.6 Impulse responses to one S.D. innovations with 2 Std.Err. bounds, for the Global group. Variable ordering: $US_t, Eurozone_{t+1}, UK_{t+1}, JP_{t+1}$ | 156 |
| 3.7 Impulse responses to one S.D. innovations with 2 Std.Err. bounds, for the Global group. Variable ordering: $Eurozone_t, UK_t, US_t, JP_{t+1}$ | 157 |
| 3.8 Forecast error variance decomposition for the Global group | 158 |
| 3.9 Generalised impulse responses for group European area | 159 |
| 3.10 Numerical intergration method used in the CBOE procedure | 160 |
| 4.1 Time series plots of HAR-RV annualised expected realised variance | 210 |
| 4.2 Autocorrelation plots of HAR-RV realised variance | 211 |

| | | |
|-----|--|-----|
| 4.3 | Variance risk premiums | 212 |
| 4.4 | Autocorrelation plots of variance risk premiums | 213 |
| 4.5 | Standardized residuals plots of subsamples | 214 |
| 4.5 | Continued standardized residuals plots of subsamples | 215 |
| 4.6 | Impulse response analysis for post-crisis subsample | 216 |
| 4.7 | Impulse response analysis for crisis subsample | 217 |
| 4.8 | Impulse response analysis for pre-crisis subsample | 218 |

Chapter 1

Introduction

1.1 Derivatives

A *derivative* can be defined as a financial contract whose value is derived from an underlying asset. Over the past 40 years, derivatives have experienced explosive growth and have become increasingly important in Finance. Many different types of derivative contracts have been developed and used by financial institutions, fund managers, and corporate treasurers. Derivatives can be used for hedging, speculation or arbitrage, and they play a key role in risk measurement and risk transfer. Prices of these derivatives contain information beyond that conveyed by the basic underlying asset. Extracting information from these derivatives may provide useful views on risk management and asset allocation.

Options are one of the most popular derivatives traded in both exchanges and over-the-counter markets. Option prices are contingent upon all possible future payoffs, hence they contain a rich source of information about the underlying asset's distribution at option maturity. The risk-neutral distribution identified in the option pricing framework provides an appealing way to extract forward looking information.

Credit default swaps (CDS) are the most popular and liquidly traded credit derivatives. A

CDS contract provides insurance against the risk of default by a particular company, known as the reference entity. During the 2007 subprime mortgage crisis and the subsequent 2008 financial crisis, CDS contracts continued to trade actively, though with dramatically increased protection cost, while many other credit derivatives, such as securitized assets, ceased to trade due to lack of transparency. The nature of a CDS contract and its popularity and liquidity in trading provide a natural channel in extracting the reference entity's default risk over the life of the CDS contract.

In this thesis, I extract information from option prices and CDS spreads to examine bankruptcy probabilities estimated for a number of financial companies during the 2008 financial crisis. In contrast with early empirical studies on bankruptcy predictions which use accounting- and equity market-based information, derivatives markets provide extra useful information ex-ante.

Volatility plays a key role in asset pricing and risk management. Implied volatility is embedded in option prices and can be obtained by equating the market price of an option with the price indicated by an option pricing model. Option-implied volatilities reveal important information on the dynamics of volatility evolution, and they are used to monitor the market's opinion on the volatility of a particular asset.

A *Volatility index* is an index of implied volatilities over a specific horizon calculated from a basket of call and put options on a representative equity index. The Chicago Board Options Exchange (CBOE) was the first to introduce a volatility index, called the VIX, designed to measure the market's expectation of 30-day volatility of the S&P 500 index, which then quickly became the benchmark risk measure of the U.S. equity market. Many exchanges across the world have developed their own respective volatility index following the steps of the CBOE. Volatility indices have become core instruments for risk management and they underly a number of volatility derivatives such as VIX futures, options and variance swaps

following their introduction.

A *variance swap* is an over-the-counter derivative contract that allows one to speculate or hedge the movement of the volatility of an underlying asset. It is an agreement to exchange the realised variance between times t and T for a prespecified fixed variance. The variance risk premium is quantified as the difference between the realised variance and the risk-neutral expectation of future variance. The fixed variance can be synthesized using a portfolio of call and put options, which approximates the risk-neutral expected variance. The introduction of volatility indices has made quantifying the variance risk premium readily convenient.

In this thesis, I construct volatility indices for 15 markets around the world. Following the notion of a variance swap, I construct the variance risk premium using the square of the volatility index as the proxy for the risk-neutral expected variance. I then investigate implied volatility and variance risk premium spillover effects across the world's equity markets.

This thesis aims to provide information inferred from the derivatives markets from different perspectives. The main contributions include: first, estimating ex-ante bankruptcy probabilities from derivative instruments and providing a direct comparison between option-implied probabilities and CDS-implied probabilities; second, constructing volatility indices for a wider range of markets using a revised methodology, which addresses issues associated with the CBOE methodology; third, providing empirical spillover analysis of variance risk premiums across the world.

1.2 Structure of the thesis

The thesis is organised as follows. Chapters 2 to 4 are three individual essays with respective literature reviews contained within each chapter.

Chapter 2 examines bankruptcy probabilities inferred from option prices and CDS spreads

for 12 U.S. financial institutions around the 2008 financial crisis period. This chapter begins with a literature review on empirical bankruptcy prediction studies, as well as literature on estimating risk-neutral densities from option prices. I specify a mixture of two lognormal densities augmented with a probability of bankruptcy, and calibrate this density to option prices to infer the bankruptcy probability parameter. CDS bankruptcy probabilities are solved from a non-linear equation that equates the present value of the expected premium payments with the present value the expected payoff. Bankruptcy probabilities inferred from the two sources are then compared for each of the firms and groups of firms.

Chapter 3, "Model free expectation of implied volatility and spillover effects", first constructs volatility indices for 15 equity markets around the world. The methodology to construct the volatility index is based on the CBOE procedure in constructing the VIX, but with modified smoothing steps to address the shortcomings of the CBOE procedure. Volatility indices are then grouped based on geographical location to study the spillover effects. Vector autoregressive methodology is employed for spillover analysis.

Chapter 4, "Variance risk premiums and international spillover effects", quantifies variance risk premium building upon some of the results presented in chapter 3. The risk-neutral expected future variance is taken as the square of the implied volatility calculated in chapter 3. The realised variance under the physical measure is constructed using the HAR-RV model. The dynamics of variance risk premiums for the sample examined are presented, and spillover effects for subsamples are investigated. Some issues concerning heterogeneity and autocorrelation in variance risk premiums are addressed in employing vector autoregressive analysis.

Chapter 5 summarizes and concludes the thesis, and points out directions for future research.

Chapter 2

Bankruptcy Probabilities Inferred from Option Prices and CDS Spreads

2.1 Introduction

In the field of default probabilities and bankruptcy estimation, various kinds of historical data and estimation models have been considered. Mainstream examples are accounting based models and market based models. An accounting based model, as its name suggests, predicts bankruptcy based on accounting information – a set of accounting ratios as predictor variables. The model evolves from early stage linear discriminant analysis (Altman (1968) and related literature in section 2.2.2) to the later advanced parametric (Shumway (2001)) and non-parametric models (Peat (2008)) in the aim to overcome the limiting statistical assumptions of multivariate discriminant analysis. The main conclusion of this body of research is that financial ratios provide a significant indication of the likelihood of financial distress.

Market based models, in particular, the Merton type structural model, are another type of default prediction model. As opposed to accounting based models, market models make use of market information, such as equity price, which reflects all information contained in

accounting data and expectations about the firm's future performance. It relates the credit quality of a firm to the firm's economic and financial conditions. It is thus more appropriate to use market information in a default prediction setting. Examples are Black and Scholes (1973), Merton (1974), and related literature in section 2.2.3. Market models have developed as a competing rival of accounting based models. A number of studies have examined empirically the relevance of accounting variables versus market variables in explaining bankruptcy, but conclusions are difficult to draw whether one is superior to the other. While some studies find Merton type structural models outperform accounting based models, others find that structural models have little forecasting power.

This raises the intriguing question as to whether information about bankruptcy can be inferred from the derivative markets. It was not until recently that econometric models emerged which make use of derivative market instruments, i.e. options and credit default swaps. Option prices are considered forward-looking; they reflect the aggregate market expectation about the underlying asset price until the expiration of the option contract. Option data contain high information efficiency. Empirical studies have found that option-implied volatilities and densities have strong forecasting ability, especially when forecast horizon is one month or three months (Liu et al. (2007), Shackleton et al. (2010)). Various papers which studied index options have found evidence that option prices contain incremental information on volatility forecasting (Blair et al. (2001), Poon and Granger (2003), Jiang and Tian (2005), Liu et al. (2007) and Taylor (2011)). Chen and Fong (2012) finds supporting evidence that risk-neutral densities are useful forecasting tool in extreme market conditions.

However, the use of option prices to infer bankruptcy probabilities has been limited. One example is Câmara et al. (2012) who assume a single lognormal density augmented with a probability of bankruptcy for the underlying asset. Taylor et al. (2014) generalise the density to be a mixture of two lognormals augmented with a probability of bankruptcy to avoid the

rigid shape of the lognormal density, and they find the density reproduces options prices better than the single lognormal density. The double lognormal density does not necessarily avoid the rigid shape of each lognormal component, but it produces a kurtosis higher than that of the single lognormal density that creates fatter tails. Information contained in option prices can be extracted if the appropriate risk-neutral density for the underlying asset can be identified. There is a huge literature on estimating risk-neutral densities including parametric fitting, non-parametric method, extensions of price dynamics, volatility smile, etc., and the choice depends on researchers' purposes and preferences. In this study, I follow Taylor et al. (2014) who assume a mixture of two lognormal densities.

An alternative source of information which provides forward looking measure of bankruptcy probabilities is CDS spreads. CDS contracts, which protect against default of a reference entity, play a crucial role in risk management by providing a measure of probability of default. Bharath and Shumway (2008) construct a model using CDS information to directly extract a measure of default probabilities. In this study, I follow the general idea of their method with revision on certain parameter inputs.

This study examines empirically bankruptcy probabilities inferred from option prices and CDS spreads, and compares the information conveyed in both markets. To the best of my knowledge, this is the first study that makes a direct comparison of the bankruptcy probabilities between the option market and the CDS market. In this study, I include 12 American financial institutions to investigate bankruptcy risk around the 2008 global financial crisis, as this crisis period is a natural setting to test the models. I find that option prices and CDS spreads provide risk-neutral bankruptcy probabilities ex-ante. Consistent with the results found in Taylor et al. (2014), distressed firms have higher chances of bankruptcy than relative healthy firms. Option market and CDS market provide complementary information in assessing bankruptcy risk, but one does not lead the other. I emphasize that all probabilities

obtained from option prices and CDS spreads are risk-neutral. Real-world probabilities embed investors' risk preferences. In theory, any change in the market's risk tolerance should be sufficient to explain a risk-neutral change.

The context of bankruptcy is worth a clarification in order to understand what the probabilities inferred from option prices or CDS spreads actually measure. In the option pricing model, bankruptcy is assumed to be a particular state in the risk-neutral density for the future underlying stock price when the future stock price goes to zero. The theoretical value zero assigned to bankruptcy is an extreme way to fatten the left tail of the risk-neutral density, but it does not necessarily mean that the bankruptcy can only happen when the stock price is exactly zero. In fact, Lehman Brothers and Washington Mutual share prices were still worth a few dollars (\$3.65 and \$1.69) one day prior to their filings of bankruptcy.

The probability of default in the CDS pricing model measures the probability of a credit event that triggers the settlement of the CDS contract. The credit event, defined in the ISDA Agreement, refers to either of the following: 1) bankruptcy (the reference entity has filed for relief under bankruptcy law); 2) failure to pay (the reference entity fails to make interest or principal payments when due); 3) debt restructuring (the configuration of the debt obligations is changed); and 4) obligation default, obligation acceleration, and repudiation/moratorium (these are very rare)¹. Mergers and acquisitions, however, are not defined as credit events that will trigger CDS payout.

Clearly, the probability of default defined in the two pricing models are not exactly measuring the same underlying, except for the case when companies file for bankruptcy that the two measures are loosely equivalent. However, the emphasis about the default probability in this study is on the overall state of distress of each firm, which may lead to filing for bankruptcy, or triggering a credit event, or maybe nothing at all if firms manage to get out of distress. In this sense, option-implied probabilities and CDS implied-probabilities both

¹See Markit Credit Indices Primer.

provide information on to what extent the company is in financial distress. Qualifying any of the following criteria can be thought of a signal for financial distress: 1. The firm has gone bankrupt or acquired by other firms during the crisis. 2. The share price fell below \$5 from a previous high level and remained low for a long period of time.

The study proceeds as follows. The next section provides a comprehensive literature review on bankruptcy estimation models and econometric methods associated with density estimation. Section 2.3 details models used to extract bankruptcy probabilities using option prices and CDS spreads. Data is discussed in section 2.4 and section 2.5 describes empirical results. Section 2.6 concludes. A list of Figures and Tables are in Appendix in section 2.7.

2.2 Literature review

2.2.1 Introduction

The literature on bankruptcy prediction dates back to the 1930s when initial studies began to concern the use of ratio analysis to predict future bankruptcy. These studies focused on individual ratios and compared those of financially distressed firms to those of successful firms. In 1935, the Bureau of Business Research at University of Illinois (Smith and Winakor (1935)) published some results of a ratio study of failing industrial firms. The study analysed 24 ratios to determine common characteristics of failing firms and compared them with the average ratio across 29 failing firms. A number of ratios were identified as good indicators of distress of a firm. These are accounting ratios typically measuring profitability, liquidity, and solvency.

Research up to mid-1960s focused on univariate analysis; the most widely recognized is that of Beaver (1966). The univariate studies laid the groundwork for multivariate bankruptcy prediction models using discriminant analysis led by Altman (1968). Multivariate discrim-

inant analysis is a statistical technique which considers a number of accounting ratios at the same time to determine the most significant ones, and then classifies an observation into a qualitative group, in this case bankruptcy or non-bankruptcy, depending on the observation's individual score calculated from those ratios. Discriminant analysis was a very popular method at the early stage of bankruptcy prediction models, but advancements in knowledge have made other methods available, including probit and logit analysis, and survival/duration analysis later in the 1980's to late 1990's.

Despite its simple structure and easy application, discriminant analysis lacks a theoretical background in determining bankruptcy risk. The Merton-type contingent claims valuation approach is a more appealing alternative. The model is based on Black and Scholes (1973) and Merton (1974) option pricing framework, where equity can be viewed as a European call option on the value of the firm's assets, with exercise price being the face value of the debt. When the option expires, shareholders either exercise the option and pay off debtholders if the option expires in-the-money, or do nothing – in this case, the firm goes bankrupt. The probability of the outcome happening either way is embedded in the Black-Scholes option pricing formula. This kind of model is referred to as Merton-type structural model or market-based model in that the observed variables are market measures.

It was not until very recently that the econometric technique emerged in the literature to infer bankruptcy probabilities from option prices. This is a significant move beyond accounting-based models and market-based models because, first, the econometric methods are more advanced method than prior statistical procedures and avoid strong assumptions of structural models; and second, option prices are forward-looking which contain rich information about the future and carry higher information efficiency than accounting numbers or equity market data. Prominent example is that of Taylor et al. (2014) which develops a parametric risk-neutral density model with a probability of default, and calibrate the model to market

option prices. Option implied densities have strong forecasting abilities (Liu et al. (2007), Shackleton et al. (2010)). The model is useful in providing an ex-ante measure of probability risk which can be applied to other risk management. The literature in this strand is relatively small, which opens up opportunities for researchers to further investigate derivatives.

The rest of this section details related literature covering four areas aforementioned – accounting-based models, market-based models, econometric models, and methods associated with estimating risk-neutral densities from option prices. The volume of existing literature is huge and it is very difficult to cover it all. As a result, this review tends to be selective.

2.2.2 Accounting-based models

Beaver (1966) pioneered the study of bankruptcy prediction models using univariate analysis. Following many earlier accounting ratio studies, Beaver compared mean values of 30 ratios of 79 matched pairs of failed and non-failed publicly owned corporations in 38 industries, but unlike most studies, Beaver took a step further to test individual ratio's predictive ability in classifying bankrupt and non-bankrupt firms. Six ratios were chosen on the basis of the lowest percentage error when comparing the dichotomous outcome as bankrupt or non-bankrupt from an optimal cut-off point to the actual failure status. He finds that cash flow to total debt has the strongest predictive ability, followed by net income to total assets and total debt to total assets, with three liquid-asset ratios, namely working capital to total assets, current ratio, and no-credit interval, performing least well. In his suggestions for future research, Beaver indicated that a multi-ratio model may predict better than single ratios, and so began the evolution of bankruptcy prediction models.

Altman (1968) was the first study to employ multivariate discriminant analysis (MDA). He developed a five-factor (out of 22) model to classify a firm as bankrupt or non-bankrupt by referring to a Z-score which falls below or above a certain range. Model predictability is mea-

sured by the percentage of firms correctly classified. Altman finds that the model has a high predictive ability at 95% accuracy for the initial sample one year before failure and 83% accuracy for two years before failure. However, the model performs considerably poorer when predicting bankruptcy up to three, four, and five years prior to failure, where the predictive accuracy drops below 50%. The model reliability is tested using a hold-out sample where the test results are impressive. Altman concluded that bankruptcy can be accurately predicted up to two years prior to the failure, and called for future research on smaller-sized and unincorporated entities where business failure is more likely to occur. Altman et al. (1977) subsequently updated the original Z-score model to a ZETA-analysis adjusting for changes in reporting standards since 1960s. They find that the ZETA discriminant model is extremely accurate for up to five years before failure.

Ever since Altman's study, numerous papers applying, improving and extending discriminant analysis to predict bankruptcy have appeared in the literature. These studies include models for medium and large manufacturing firms, among others, by Deakin (1972), Blum (1974), and Moyer (1977); small business by Edmister (1972); and models for specific industries such as Sinkey (1975), Santomero and Vinso (1977), and Martin (1977) on commercial banks; Altman (1973) on railroads; Altman and Loris (1976) on broker/dealers, and Altman (1977) on savings and loan associations. At the same time, considerable efforts have been made to replicate and extend these models on an international level. Altman (1984) provides an extensive survey on the works by both academics and practitioners in ten countries: Japan, Germany, Switzerland, Australia, United Kingdom, Ireland, Canada, The Netherlands, Brazil and France.

The multivariate discriminant analysis has been the primary method used for bankruptcy prediction models for the decade between late 1960s and 1970s, however, the conclusion are difficult to assess because the models play loose with certain statistical assumptions. Dis-

criminant analysis requires equal variance-covariance matrices of predictors for both failed and non-failed groups, and that the predictors are normally distributed. Violation of these conditions renders the conclusion unreliable. Ohlson (1980) developed a conditional logit model to avoid problems with respect to MDA. The analysis uses maximum likelihood to estimate parameters of some probability function, which are then used to compute the probability a firm fails, which is referred to as O-Score. The fundamental estimation problem becomes: given that a firm belongs to some pre-specified population, what is the probability that the firm goes bankrupt within a pre-specified time period. This way, no assumptions have to be made about the distribution of predictors. Ohlson also pointed out an issue with data collection that realistic evaluation of the model requires that the predictors are available prior to the event of bankrupt, however, previous studies have not explicitly mentioned this point since most of them have used Moody's Manual² to derive financial ratios, and the Manual does not indicate at which point in time the data was made available. Implicitly presuming a report is available at the fiscal year-end date may be inadequate for a pure forecasting purpose as it may lead to 'back-casting' if a firm files for bankruptcy after a fiscal year but prior to the release of financial statements. The findings of the study are that it is possible to identify four statistically significant predictors in assessing probability of failure, but the error-rate of the sample is larger than other studies, even after accounting for the data timing factor.

Zavgren (1985) argues that the dichotomous partition resulting from discriminant analysis is much less useful for an investor than a cardinal evaluation of financial risk, and the logit model provides significantly better probability estimates than discriminant analysis of the same data. He criticizes Ohlson's choice of independent variables lack of theoretical determination; instead, he employs a factor analysis to determine the attribute vector so that

²Moody's Manuals are a series of annual manuals published by the Moody's Corporation containing brief reports on companies. Volumes in each year are divided into manuals named for the types of the companies they contain, e.g. Industrial, Bank & Finance, International, etc..

no significant attributes should be omitted. Zavgren also traced the significance of the coefficients for each of the variable for five years and found the pattern is congruent with a priori expectation. He concluded that the model proved highly significant in detecting up to five years prior to the failure. Lau (1987) further extended dichotomous classification into a five financial state model and estimated the probability that a firm enters each of the states, which provides a measure of the firm's financial position on a continuous scale.

Zmijewski (1984) pointed out that researchers typically estimate bankruptcy prediction models on non-random samples which result in two estimation biases: choice-based sample bias and sample selection bias. The choice-based sample bias results when the sample is selected based on knowledge of some variables, so the probability of a firm entering the sample depends on the variable's attributes. Sample selection bias results when only firms with complete data are used to estimate the model and incomplete data observations occur non-randomly. Zmijewski used a weighted exogenous sample maximum likelihood probit model and a bivariate probit model respectively to assess the two biases and found clear evidence of their existence. However, he claims that the biases, in general, do not affect statistical inferences or overall classification rates.

Greene (2008) points out that the differences between MDA models and standard form logit and probit models are not as significant as once believed, because, MDA model can be constructed as nothing more than a linear probability model. Thus the difference between MDA and logit, probit model should not be that great. Two major problems arising from these standard or 'simple' models are: one, restrictive IID assumptions; two, inability to capture firm-specific heterogeneity in model estimation. There have been a number of attempts to overcome these limiting statistical assumptions, either by selecting a parametric method with fewer distributional assumptions, or by moving to a non-parametric approach.

The mixed logit model is an example of an advanced form logit model (parametric model)

that can accommodate firm-specific heterogeneity across firms. The essence of the method is to decompose the stochastic error component into two parts: one is IID that does not depend on underlying parameters, and the other is heteroskedastic representing the unobserved heterogeneity across firms, whose distribution depends on underlying parameters and observed data. The major advantage of the mixed logit model is that it allows for complete relaxation of the IID assumption by allowing all unobserved variances and covariances to be different up to identification, making the model highly flexible. However, a relative weakness of the mixed logit model is the complexity of estimation as well as the lack of a single globally efficient set of parameter estimates.

Another class of parametric model is survival or duration models which have become increasingly popular in financial distress research. Survival models are concerned with examining the length of time interval between transition dates where the transition is marked by the occurrence of an event, such as corporate failure. The duration can be modelled by a non-negative random variable T until transition occurs; or can be modelled under different probability distributions, such as, a cumulative distribution, a survivor function, a probability density function, or a hazard function. The primary benefits of survival analysis are in the area of censoring and time-varying covariates. Shumway (2001) argues that models like traditional MDA and standard logit are static models which are inappropriate for the use of bankruptcy prediction because of the nature of bankruptcy data. He pointed out that static models fail to account for each firm's period at risk because the model only uses one set of explanatory variables of which observations are usually made one period before failure, but firms' characteristics may change from year to year, and thus introduce selection bias. Shumway develops a discrete-time hazard model, which can be thought of as a binary logit model, to explicitly control for time and incorporate time-varying covariates. This way, many more observations are utilized and the model produces more efficient out-of-sample

forecasts. Shumway found that half of the accounting ratios used in previous studies are not statistically significant bankruptcy predictors, and the model which produces best out-of-sample forecast uses a combination of accounting ratios and market-driven variables.

Two main types of non-parametric approach that have been used in the literature are neural networks and recursive partitioning. The distinguishing feature of non-parametric methods is that there is no a priori knowledge about the form of the function which is being estimated. Peat (2008) presented these two models for credit risk analysis, each with a numerical example. He showed that the empirical application of both models has demonstrated their potential in the credit risk analysis context, with the best model outperforming a standard MDA model.

Despite the frequent use and improvement of accounting ratio based models, there are reasons to question the effectiveness of probability measures that are based on accounting data. The most important concern about accounting data is that financial statements are designed to reflect firms' past performance and may not be informative about future status, while measure of bankruptcy probability is in nature about the likelihood of future events. In addition, as discussed in Hillegeist et al. (2004), the conservatism principle required in accounting reporting systems causes asset values to be understated, of which the biased asset valuation will limit the performance of any accounting-based probability measure.

2.2.3 Market-based models

As early as the study of Beaver (1968), the stock market has been recognized as a potential alternative to provide a superior source of information regarding bankruptcy probability prediction. Shumway (2001) includes three market-driven variables: market size, past stock returns, and idiosyncratic standard deviation of stock returns in his estimation model and finds that the inclusion of market information provides the best out-of-sample forecast.

Option-pricing models based on Black and Scholes (1973) and Merton (1974) (BSM) provide a natural starting point in extracting bankruptcy probability related information from market prices. Equity can be viewed as a call option on the value of the firm's future assets. Equity holders are residual claimants of firm's assets and determine when the firm declares bankrupt. Under BSM framework, the strike price of the call option is equal to the face value of the firm's debt and option expires when debt matures. At maturity time, equity holders will exercise the call option and pay off the debt holders if the firm's assets value is greater than the face value of debt, or let the option expire and declare bankruptcy if the value of the firm's assets is below the face value of debt. If the firm files for bankruptcy, the ownership is transferred costlessly to debt holders. The payoff for equity holders is either the difference between the value of firm's assets and the face value of firm's debt or zero otherwise. The probability of each outcome is embedded in the BSM model. Denote V_E is the value of equity, V_A is the value of firm's assets, X is the face value of firm's debt maturing in t periods, r is the continuously compounded risk-free discount rate. Then the BSM European call option formula, stated in terms of equity on a firm's assets is

$$V_E = V_A N(d_1) - X e^{-rt} N(d_2), \quad (2.1)$$

where

$$d_1 = \frac{\ln \frac{V_A}{X} + (r + 0.5\sigma_A^2)t}{\sigma_A \sqrt{t}}, \quad (2.2)$$

and

$$d_2 = \frac{\ln \frac{V_A}{X} + (r - 0.5\sigma_A^2)t}{\sigma_A \sqrt{t}} = d_1 - \sigma_A \sqrt{t}, \quad (2.3)$$

$N(d_1)$ and $N(d_2)$ are standard cumulative normal probabilities of d_1 and d_2 , and σ_A is the volatility of the value of assets. The probability that the option expires in-the-money at maturity is represented by $N(d_2)$, and $1 - N(d_2) = N(-d_2)$ is the probability that the option expires

out-of-the-money, that is, the firm goes bankrupt. Equation (2.3) shows that the risk-neutral probability of bankruptcy is a function of the distance between firm's assets V_A and the face value of the debt X , relative to the volatility of firm's assets σ_A , which is referred to as the distance-to-default.

Merton's model is considered the first structural model³ in assessing credit risk. Several commercial vendors provide default probabilities based on option pricing models, KMV⁴ being the most famous. While the basic approach is similar to standard BSM model, the implementation differs in several ways. KMV implemented a model developed by Vasicek-Kealhofer (see Kealhofer (2003)) known as KV model. The model is a generalisation of BSM framework and allows for various classes and maturities of debt. It assumes that the firm's equity is a perpetual option with the default point acting as the absorbing barrier for the firm's assets value. Bankruptcy occurs when the value of assets hits the default point. Instead of using the cumulative normal distribution to convert distance-to-default into default probabilities, KMV uses an empirical distribution of actual defaults based on its large, proprietary database. KMV also make proprietary adjustments to accounting information that they use to calculate the value of debt. Other option-related studies include Cheung (1991), Kealhofer et al. (1998), and Core and Schrand (1999).

The Merton model, however, is fairly parsimonious due to its rather restrictive assumptions, one being that default can only occur at the maturity of the zero-coupon bond. Subsequent studies have explored more appropriate default boundaries within structural framework. Black and Cox (1976) introduced another approach that takes into account early de-

³In credit risk literature there are two main types of models that describe default processes: structural models and reduced-form models. Structural models use the evolution of firm's structural variables to determine time of default which is endogenously generated within the model; whereas in reduced-form models, default is exogenously determined. Reduced-form models do not consider the relation between default and firm's economic and financial conditions. For the relevance of the purpose of this study, structural models are described in detail; for a review of reduced form models, see Elizalde (2006).

⁴KMV was acquired by Moody's in April 2002. Before the merger, Moody's model to assess default probability is a hybrid one that combines BSM structural model and a statistical model determined on the basis of historical data. Details of the model can be found in Sobehart and Stein (2000).

fault possibilities. In their model, they assume an exogenously determined threshold level of asset value, below which default occurs. In contrast to the Merton model, default can occur at any time. The same idea was employed by Longstaff and Schwartz (1995). Alternatively, default thresholds can be determined endogenously, which allows the stockholders to decide when to default so that they maximize firm's value. Examples are Leland (1994), Leland and Toft (1996), and Anderson et al. (1996). The difference between exogenous and endogenous default barrier models are in the assumption underlying the default decision. While these studies focus on addressing early default issues, other studies try to relax one of the Merton model assumptions by considering stochastic interest rates. Examples are Kim et al. (1993), Nielsen et al. (1993), and Longstaff and Schwartz (1995).

Such market-based structural models provide an appealing alternative because it counters most of the criticisms about accounting ratio based models. It provides guidance about theoretical determinants of bankruptcy risk and structure to extract information from market prices. Market data should reflect all information contained in accounting data and also contain information not in accounting statements, and it reflects investors' expectations about a firm's future performance and hence should be more appropriate in prediction context. Market prices are less influenced by management than are accounting statements. Over the past decade, both practitioners and researchers have examined the contribution of the Merton model. The very first authors are practitioners employed by either KMV or Moody's before they merged. Falkenstein and Boral (2001) find that the Merton model is a powerful measure of default risk, and Kealhofer and Kurbat (2002) show that the model outperforms alternatives and it captures all information contained in accounting and agency ratings. Other papers, including Sobehart and Keenan (1999), Stein (2000), Sobehart and Stein (2000) argue that Merton-type models are not sufficient to predict bankruptcy probability and calls for the need for a hybrid model which combines equity market information

with accounting ratios and agency ratings, an example is Moody's hybrid model in Sobehart and Stein (2000).

A number of studies have addressed empirically the relevance of accounting-based models and market-based models in explaining bankruptcy. Shumway (2001) develops a simple hazard model using three market-driven variables to determine firm's bankruptcy risk, and finds adding market variables on top of previously identified accounting variables helps improve forecasting accuracy. Hillegeist et al. (2004) extend Shumway's method by using Merton's option model in a discrete hazard framework to examine the predictive ability of accounting-based variables. They take into account dividend rate and replace the continuous compounded risk free rate by continuous compounded expected return on assets, μ , to adjust for actual risk. In order to estimate BSM bankruptcy probability, V_A , σ_A , and μ must be estimated since these values are not directly observable. Hillegeist et al. (2004) estimate V_A and σ_A by simultaneously solving equation (2.1) and the optimal hedge equation, and the estimates of V_A are then used to estimate μ . They find that traditional accounting-based variables do not add incremental information beyond standard option variables. Another study by Vassalou and Xing (2004) uses a similar approach that adopts an iterative procedure to estimate V_A and σ_A . However Vassalou and Xing (2004) do not adjust for dividends, and as pointed out by Hillegeist et al. (2004), their method for calculating μ often results in negative numbers which is inconsistent with asset pricing theory. Studies by Du and Suo (2007) and Bharath and Shumway (2008) examine the model's predictive power in a similar way. While Hillegeist et al. (2004) find BSM model provides significantly more information than Altman's Z-Score and Ohlson's O-Score, Du and Suo (2007) and Bharath and Shumway (2008) have negative conclusions about the accuracy of the model forecast. Campbell et al. (2008) estimate hazard models incorporating the BSM bankruptcy probability measure but find it adds little forecasting power after conditioning on other variables.

Though appealing as the structural model is over accounting-based models, it suffers from a number of strict assumptions which are not true in reality. For example, it assumes normality of stock returns, does not distinguish between types of debt, and assumes that the firm only has a zero coupon debt and the default happens only at maturity. It is not surprising that the empirical evidence on the performance of market-based models is mixed.

2.2.4 Econometric models using option prices

Options market provides a rich source of information as option prices are considered forward-looking. Option prices reflect aggregate risk-neutral market expectations about the underlying stock price until the maturity of the option contract. These data contain high information efficiency. Particularly during crisis periods, traditional bankruptcy prediction models fail to provide reasonable signals in advance of the crisis, while option data provides an appealing way to extract information due to its forward-looking nature.

Using option prices to directly extract bankruptcy probabilities is relatively new in the literature. Capuano (2008) developed a framework by applying the minimum cross-entropy method to infer bankruptcy probabilities from an entire set of option prices. The framework is based upon the Merton model where the default probability is defined as the probability when the underlying assets value fall below a threshold, and is expressed as the cumulative distribution function of the value of the assets up to the default barrier. In order to calculate this quantity, option prices are used and the minimum cross-entropy method is used for the optimising problem. The cross-entropy approach can be interpreted as a measure of the discrepancy between a prior probability density function reflecting prior knowledge and a posterior density. The Lagrangian multiplier is formed incorporating several moment constraints given by the theory of risk-neutral pricing to minimise the discrepancy. Capuano (2008) applied the framework to ten largest U.S. banking groups namely Bank of America,

Citigroup, J.P. Morgan, Wachovia, Wells Fargo, Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, and Morgan Stanley, around the 2008 financial crisis to see if the model is able to capture market sentiments. The term structure of probabilities of default reported in the author's Appendix 1 suggests a less than 1% default probability of all financial institutions examined when the financial crisis broke out in September 2008.

Vilsmeier (2011) argues that despite offering an appealing tool to extract bankruptcy information, the framework of Capuano (2008) suffers serious problems regarding its numerical stability and also the accuracy of the estimates. He suggests some technical modifications to the approach and hence considerably improves the general application of the approach. The first modification concerns the estimation procedure for the minimum cross-entropy. The Lagrange multipliers are solved numerically via Newton-Raphson algorithm in Capuano (2008), however, Vilsmeier (2011) argues that the search for the roots of the system is infeasible in many applications and converges only for a small number of constraints when the initial guess of multipliers are near the final solution. The modification is done by defining a lower bound on the value of the cross-entropy function. The second modification concerns the determination of the optimal probability of default. Vilsmeier (2011) suggests an ad hoc procedure which is based on the evolution of the Lagrange multipliers when estimating for different default barriers, and the numerical evaluation show clearly its accuracy. He reported a default probability range between 0.0078% and 20% when applied to a set of user-specified data. He also uses option data to test and find convincing results that the framework is able to anticipate elevated risk of Bank of America relative to J.P. Morgan months before its downgrade by Moody's. Zer (2014) also employs the methodology to estimate default probabilities but his paper is focused on the relationship between firms' disclosure decisions and the market expected value of default probabilities.

A paper by Câmara et al. (2012) also examines the default rates for financial companies

for a period of time including the subprime mortgage crisis using options data. They assume that the stock price follows a delta-geometric random walk, for which in each period the stock price can go to zero with a certain probability δ or it follows a geometric random walk with probability $1 - \delta$. Then the probability density function of the stock price is effectively a delta-lognormal distribution with a probability of default. Given Black-Scholes option pricing framework, closed-form solutions can be derived for this model. The probability of default is obtained by minimising the sum of squared errors between observed option prices and prices calculated from the model. Câmara et al. (2012) investigates a sample of 144 financial firms with traded options in the U.S. for which the data is available through Option Metrics. The sample period span from December 1996 to October 2008. They found that the default probabilities for global financial service firms increase steadily during the subprime crisis period. They also compare the model performance versus Moody's KMV model and credit ratings, and found the model surpassed credit ratings and matched or exceeded KMV in anticipating the magnitude of the crisis.

Taylor et al. (2014) consider a similar approach but adopts a mixture of two lognormal densities augmented with a positive probability of bankruptcy. They argue that a mixture of lognormal densities characterize market information better than a single lognormal distribution. They calibrate the model to daily stock and option prices of six financial institutions (J.P. Morgan, Bank of America, Bear Stearns, Merrill Lynch, Lehman Brothers, and AIG) during 2008 and 2009 on major events. American option contracts with one month and three months maturity are used. Their results show that the acquiring institutions e.g. J.P. Morgan and Bank of America have lower average probability of bankruptcy of 0.7% and 0.9%, whereas those of the acquired institutions e.g. Bear Stearns and Merrill Lynch are high at 3.1% and 2.3%, respectively. Lehman Brothers has the highest bankruptcy probability which shot up to 54% on September 15, 2008 when it declared bankruptcy. The probabil-

ity of bankruptcy of Bear Stearns was 27% when J.P. Morgan offered to acquire at a low price. The results are generally consistent with other studies that bankruptcy probabilities increase during crisis period and that financially distressed institutions have higher probability of bankruptcy.

2.2.5 Methods to estimate risk-neutral densities from option prices

Option prices provide a rich source of information for estimating asset price distributions when options expire. This is because option prices are contingent upon all possible future payoffs and hence the distribution at maturity captured under risk-neutral measure. Following Breeden and Litzenberger (1978), it is well known that the underlying risk-neutral distributions extracted from option prices can be estimated from the second derivative of the European option prices with respect to the corresponding exercise prices. As a result, there exists a direct link between European option pricing formulae and associated functional forms of risk-neutral distributions.

The literature on extracting option implied risk-neutral densities is huge and still growing. Methods to estimate risk-neutral densities from option prices are generally grouped into four categories, (1) methods associated with extensions in price dynamics; (2) parametric density function fitting methods; (3) nonparametric methods; and (4) volatility smile approaches.

Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Wiggins (1987), and Stein and Stein (1991) are among the first in the first category to address inadequate price dynamics assumptions of Black-Scholes option pricing formula by incorporating stochastic volatility. Volatility is assumed to follow a stochastic process of its own and uncorrelated with changes in the underlying prices. Numerical methods are used to derive solutions to European call option prices, however, in most cases these are not closed-form. Heston (1993)

allows volatility to follow a mean-reverting square-root diffusion process, and allows arbitrary correlation between volatility and spot returns to explain return skewness and term structure effects. Based on characteristic functions techniques, a closed-form solution for a European call option price can be derived.

Bates (1991) investigates whether there were negative expectations prior to the financial market crash in October, 1987. He assumes a jump-diffusion process for the underlying asset price and fits the model to S&P 100 index option prices. He finds that the model characterises the stylized facts better than the Black-Scholes formula, which is based on the assumption that the asset price follows a geometric Brownian motion. Risk-neutral distributions associated with these pricing models can be numerically obtained by twice differentiating the European option prices with respect to exercise prices.

Researchers have enriched the literature by considering individually or together with other extensions stochastic interest rates, pure jump or jump diffusion models. Influential examples are stochastic interest rate models of Merton (1974) and Amin and Jarrow (1992), pure jump models of Madan and Chang (1996), binomial method of Rubinstein (1994) and Jackwerth and Rubinstein (1996), stochastic volatility and jump models of Bates (1996) and Scott (1997), Duffie et al. (2000), and stochastic volatility and stochastic interest rate models of Bailey and Stulz (1989), Amin and Ng (1993), Bakshi and Chen (1997), and Scott (1997).

The literature in the second category assumes a functional form of the underlying price distribution instead of the price dynamics. The method is flexible, general, and direct, allowing for a variety of possible shapes of terminal distributions, and better able to capture the characteristics of price distribution when options expire.

The simplest way to relax Black-Scholes assumptions is to assume a mixture of two log-normal distributions instead of a single lognormal distribution, as in Ritchey (1990), Melick and Thomas (1997), Bahra (1997), Söderlind and Svensson (1997), and Gemmill and Saflekos

(2000). Melick and Thomas (1997) apply the mixture of three lognormal distributions to the crude oil market during the Persian Gulf crisis. Three densities each represent a state driven by conditions in the oil market, namely, a pre-crisis situation, a severe disruption to the oil market, and a continuation of an unsettled situation in the Gulf region. The probability attached to each density is the likelihood of such state happening. They apply the model to American option prices and find the estimated distribution is significantly different from that recovered using single lognormal technique. The mixture of lognormal distribution is sensible when asset prices depend on one or two future states that can be learned before options expire, but as Taylor (2011) pointed out, the technique can be used generally as risk-neutral densities even when there is no obvious motivation for a set of future states. Liu et al. (2007) is another example which applies a mixture of lognormal densities.

A second distribution, lognormal polynomial density function, developed by Madan and Milne (1994) is an elegant theory of contingent claims valuation that assumes the density of standardized returns is the standard normal density multiplied by a linear combination of Hermite polynomials, which serve as an orthogonal basis. The method involves more mathematics than others, and has strong theoretical foundations, but negative estimated densities are sometimes inferred.

Similar but more complicated functions of lognormal and polynomial terms are given by the Edgeworth expansion method. Jarrow and Rudd (1982) use the Edgeworth expansion to adjust the Black-Scholes pricing formula. The true price distribution and the approximating probability distribution are linked by the differences between the moments. In this way, they derive a functional form of the true density by augmenting the lognormal density with its four moments, which in fact is based on Black-Scholes formula with adjusted variance, skewness, and kurtosis.

A more flexible distribution in the literature is the generalised beta distribution first ex-

plored by Bookstaber and McDonald (1987). They propose and apply the generalised distribution of the second kind (GB2) with four parameters, which are required to obtain four moments of future asset prices. The density is inherently flexible with many well-known distributions as limiting or special cases, such as lognormal distribution. Examples are Sherrick et al. (1996), Aparicio and Hodges (1998), and Liu et al. (2007).

Like parametric methods, nonparametric methods are employed to extract risk-neutral densities aiming at achieving flexibility from more general functions instead of assuming restricted shapes of distributions. Aït-Sahalia and Lo (1998) use kernel regression to estimate risk-neutral densities nonparametrically from option prices. They assume that the option pricing formula is an arbitrary non-linear function of a vector of explanatory variables including exercise price, forward price, time to maturity, interest rate, and dividend yield. By regressing kernel estimators across these five dimensions, they construct an option pricing formula, which is then twice differentiated with respect to exercise prices to obtain the risk-neutral density. The method makes the assumptions that parameter values remain constant over time, which does not always hold. In addition, the method also suffers from the intensity of data needed. Ait-Sahalia and Duarte (2003) describe an alternative nonparametric method to enforce the constraint that the density is nonnegative. Fewer data are then required.

Bondarenko (2003) employs a convolution approach which has similarities with both nonparametric smoothing methods and parametric mixture methods. It involves constructing a set of admissible densities consisting of functions which can be represented as the convolution of a fixed kernel and an arbitrary density function. The kernel determines the smoothness of the densities. The optimal density is the one which provides the best fit to option prices. The weights of the component normal densities are obtained by solving a quadratic programming problem.

Buchen and Kelly (1996) suggest estimating the risk-neutral density using a nonparametric maximum entropy method. They start with the maximum entropy distribution, one that does not assume any prior form and will evolve only as information arrives. This is particularly suitable for the situation where only a limited number of options and strike prices are available from the option market. In their study, information refers to strike prices and the corresponding option prices. Because no interpolations or extrapolations are needed for densities between strike prices and beyond available range of strike prices, the resulting distribution is considered least committed to unknown information and hence least biased.

Another kind of method is a curve-fitting method, which extracts risk-neutral densities from implied volatility functions. The strategy is to fit the volatility smile from observed market option prices with a parametric function of volatility, such as quadratic, as in Shimko (1993). The resulting approximated volatilities are translated back to Black-Scholes option price formula to acquire risk-neutral densities. However, plausible volatility functions do not guarantee nonnegative densities for all strike prices, especially in left tails. Malz (1997a,b) develop a strategy to modify the method and guarantees the tails are well behaved. He fits the implied volatility to the space defined by pairs of option deltas and implied volatilities instead of options prices and strike prices. It is argued that smoothing can be done easily in the option implied volatility and option delta space. Another strategy of this kind is to use more parameters by fitting a cubic spline to the observed implieds, either as a function of strike price (Campa et al. (1998) or as a function of delta (Bliss and Panigirtzoglou (2002, 2004)). The rationale is that the general cubics are constrained so that the functions and their first two derivatives are continuous. But there is trade-off between a perfect fit and smoothness of the fitted function.

2.3 Methodology

2.3.1 Framework using option prices

Recall that the theoretical price of a European option can often be written as the discounted expectation of final payoff. This is valid when an appropriate risk-neutral probability distribution of the price of the underlying asset can be found. Then, for call options,

$$C(X) = e^{-rT} E[(S_T - X)^+] = e^{-rT} \int_X^{\infty} (x - X) \psi(x) dx, \quad (2.4)$$

with X the strike price, T the time until expiry and $\psi(x)$ denoting the risk-neutral density (RND).

In Black-Scholes framework, where the stock price follows a Geometric Brownian Motion, the probability distribution of the future stock price used to price a European option is a lognormal risk-neutral density. Hence, assuming the asset pays dividends at a constant rate q , the lognormal RND is

$$f_Q(x|S, \sigma) = \frac{1}{x\sigma\sqrt{2\pi T}} \exp\left(-0.5 \left[\frac{\ln x - (\ln S + (r - q - 0.5\sigma^2)T)}{\sigma\sqrt{T}} \right]^2\right).$$

RNDs are defined for all $x \geq 0$, $\psi(x) \geq 0$ and $\int_0^{\infty} \psi(x) dx = 1$. The futures price of the underlying asset excluding arbitrage opportunity is

$$F = S e^{(r-q)T}.$$

For the futures price F , $\int_0^{\infty} x\psi(x) dx = F$.

Substituting F into the lognormal RND gives,

$$f_Q(x|F, \sigma) = \frac{1}{x\sigma\sqrt{2\pi T}} \exp\left(-0.5 \left[\frac{\ln x - (\ln F - 0.5\sigma^2 T)}{\sigma\sqrt{T}}\right]^2\right).$$

The underlying asset now has futures price F , and inserting the density into equation (2.4) leads to the Black-Scholes formula, which gives the price of a European option written on futures:

$$C_{BS}(X|S) = C_{BS}(X|F) = e^{-rt} \int_X^\infty (x - X) f_Q(x|F, \sigma) dx.$$

Many types of density functions provide reasonable fits to observed option prices, so there is plenty of scope for individual preferences. Taylor (2011) and Jackwerth (1999) have given an extensive review on the methods and types of density specifications. The lognormal density has only two parameters: F and σ . As the futures price F can be obtained for the stock price, there is only one free parameter σ , which restrains the density's flexibility to explain observed option prices. Thus, it is reasonable to look for flexible densities with more parameters.

One of the most popular parametric densities is a mixture of two lognormals (MLN), first proposed by Ritchey (1990). The density is weighted combination of two lognormal densities, where the weight p is between zero and one,

$$f_{Qmln}(x) = pf_Q(x|F_1, \sigma_1) + (1 - p)f_Q(x|F_2, \sigma_2),$$

where $F_1, F_2, \sigma_1, \sigma_2 > 0$.

The risk-neutrality constraint is given by

$$F = pF_1 + (1 - p)F_2.$$

There are five parameters $\theta = F_1, F_2, \sigma_1, \sigma_2, p$. The risk-neutrality constraint reduces the number of free parameters to four, which is sufficient to obtain flexible shapes. The option price given by a mixture of lognormal densities is then,

$$\begin{aligned} C_{BS}(X|\theta) &= e^{-rt} \int_X^\infty (x - X) f_{Qmln}(x|\theta) dx \\ &= e^{-rT} \int_X^\infty (x - X) [p f_Q(x|F_1, \sigma_1) + (1 - p) f_Q(x|F_2, \sigma_2)] dx \\ &= p C_{BS}(X|F_1, \sigma_1) + (1 - p) C_{BS}(X|F_2, \sigma_2). \end{aligned}$$

The mixture lognormal method guarantees a nonnegative estimated density, and it offers a possible interpretation that the density has two future states with p associated with the probability of a relevant future event. Since the mixture of lognormal density assumes the underlying asset price will always be positive, it is reasonable to go from this stage, to assume a value of zero for the underlying asset price to allow for a possible state of bankruptcy.

Following Taylor et al. (2014), I generalize the mixture of two lognormal densities to incorporate one more state in the event of a bankruptcy, where the underlying asset price is equal to zero. Weights p_1 and p_2 are allocated to two lognormal distributions and the remaining weight $1 - p_1 - p_2$ to the probability of bankruptcy. The probability density of the underlying asset is then,

$$f_{QlnB}(x|\theta) = p_1 f_Q(x|F_1, \sigma_1) + p_2 f_Q(x|F_2, \sigma_2), \quad x > 0,$$

with

$$f_Q(x|F_i, \sigma_i) = \frac{1}{x \sigma_i \sqrt{2\pi T}} \exp\left(-0.5 \left[\frac{\ln x - (\ln F_i - 0.5 \sigma_i^2 T)}{\sigma_i \sqrt{T}} \right]^2\right).$$

The risk-neutrality constraint is given by

$$F = p_1 F_1 + p_2 F_2.$$

Additional constraints are $F_1, F_2, \sigma_1, \sigma_2 > 0$, $p_1, p_2 \geq 0$, and $p_1 + p_2 \leq 1$.

I refer to this generalized distribution with a feature of bankruptcy as MLNbk. The MLNbk distribution has five free parameters. It has one extra free parameter than the MLN distribution. Taylor et al. (2014) find that the MLNbk describes option prices better than lognormal distribution with one parameter, lognormal distribution with bankruptcy feature with two free parameters as in Câmara et al. (2012), and the MLN distribution with four free parameters.

Inserting the MLNbk distribution into option pricing equation (2.4) gives the theoretical call option price,

$$C(X|\theta) = p_1 C_{BS}(X|F_1, \sigma_1) + p_2 C_{BS}(X|F_2, \sigma_2).$$

Call options are worthless if the firm goes bankrupt, but put options are worth X . So the theoretical put option price given by the MLNbk distribution is,

$$P(X|\theta) = p_1 P_{BS}(X|F_1, \sigma_1) + p_2 P_{BS}(X|F_2, \sigma_2) + (1 - p_1 - p_2)e^{-rT} X.$$

The parameter vector $\theta = \{F_1, F_2, \sigma_1, \sigma_2, p_1, p_2\}$ is estimated by minimizing the sum of the squared differences between the theoretical price and the observed market price denoted

C_M and P_M ⁵.

$$G(\theta) = \sum_{i=1}^N [(C(X_i|\theta) - C_M(X_i|\theta))^2 + (P(X_i|\theta) - P_M(X_i|\theta))^2] \quad (2.5)$$

2.3.2 Framework using CDS spreads

Credit default swaps are one example of credit derivatives, and credit derivative markets have experienced explosive growth in recent years, which have attracted attentions of dealers, investors, regulators, and lately, the general public. According to the International Swaps and Derivatives Association (ISDA), on a gross notional outstanding basis, the market has roughly doubled in size every year from 0.6 trillion dollars in 2001 to 62.2 trillion dollars in 2007⁶. After the breakout of the financial crisis in 2008, the level of CDS notional outstanding has been decreasing in every subsequent year since then, however, this decrease is attributable to portfolio compression, which is a process designed to terminate existing trades and replace them with a smaller number of trades with substantially smaller notionals that carry the same risk profile as the initial portfolio. In doing so, portfolio compression reduces the overall notional size and number of outstanding CDS contracts, and thereby improving derivatives risk management. ISDA reported 25.1 trillion dollars notional at year end 2012. Though notionals outstanding is declining, market risk transaction activity which measures trading volume during a period of time is thought to be a better way to look at the relative dynamics of the CDS market. A review of such activity shows that CDS transaction volumes as

⁵An alternative error minimisation function is, $\min \sum_{i=1}^{N_c} w_i (C(X_i|\theta) - C_M(X_i|\theta))^2 + \sum_{i=N_c+1}^{N_c+N_p} w_i (P(X_i|\theta) - P_M(X_i|\theta))^2$, the weighted squared error loss function introduced by Bliss and Panigirtzoglou (2002). This loss function is claimed particularly appropriate when the source of option price measurement error predominantly reflects the discreteness of option prices. Bliss and Panigirtzoglou (2002), Liu et al. (2007), and Taylor et al. (2014) applied equal weighting of squared errors to obtain parameters estimates for the mixture of log-normal risk-neutral density. This loss function may provide smaller G values and quicker convergence. The convergence of optimisation routine using equation 2.5 performs reasonably well in most estimates throughout our sample. Testing the robustness of the loss function is beyond the scope of this study, thus minimising the sum of the squared errors is retained in this study.

⁶See Markit report, The CDS Big Bang: Understanding the Changes to the Global CDS Contract and North American Conventions.

measured by notionals increased through 2011 to 2013, and the number of trades executed increase in 2013 after a slightly fall in 2012. Given the nature of a credit default swap, which is a contract bought by the protection buyer that protects against default of a certain reference entity, and the expanding size and engagement activity in the market, it is reasonable to assume that we can derive a measure of probability of default for a particular reference entity from CDS market data, and compare it with the estimate obtained from option data.

In a credit default swap, the protection buyer pays a series of periodic fixed payments known as premium to the protection seller to protect against default of a reference entity. In return, the protection seller does nothing unless the reference entity gets into financial difficulty. In that case, the protection seller will be obliged to buy back from the protection seller the defaulted bond at its face value. In a cash settlement case, the protection seller will pay the difference between the face value of the defaulted bond and the current value determined by an auction, to the protection buyer. The percentage of the value of the bond after default to its face value is known as the recovery rate.

Following the methodology adopted by Bharath and Shumway (2008), let S be the CDS spread, which is the amount paid per year as a percentage of the notional principal. Notional amount is assumed 1 dollar. Let T determine the life of the CDS contract. CDS contracts have maturities range from 6 month, 1,2,3...10 years, up to 20, 30 years. 5 year contract is the most liquid one, and I also examine 1 year contract in this study. Assume the premium payments are made once every three months, e.g. March, June, September, and December. Assume the probability of a reference entity defaulting between two consecutive payment days conditional on no prior default is p , and for simplicity, I assume the default only happens halfway through the three months. In the event of default, a final accrual payment is required by the protection buyer, which is equal to $0.5 * 0.25S$. The risk-free rate is r and recovery rate is δ .

The present value of the expected premium payments made by the protection buyer is then,

$$\sum_{t=0.25}^T \left[p(1-p)^{4t-1} e^{-r(t-\frac{1}{8})} 0.5 * 0.25S + (1-p)^{4t} e^{-rt} 0.25S \right],$$

where $t = 0.25, 0.5, 0.75, 1$ for 1 year contract, and $t = 0.25, 0.5, 0.75, 1, \dots, 5$ for 5 year contract.

The first term represents the present value of the expected accrual payments in the event of a default assuming it happens midway between two consecutive premium payments. The second term represents the present value of the expected periodic premium payments made at the end of each period provided that the reference entity survives until t .

Similarly, the present value of the expected payoff is given by

$$\sum_{t=0.25}^T p(1-p)^{4t-1} (1-\delta) e^{-r(t-\frac{1}{8})}.$$

An estimate of the recovery rate δ is needed in order to value the payoff. As of The CDS Big Bang in 2009, the new CDS contract has been standardized in a number of ways. The CDS contracts are grouped based on geographical regions, such as North America, Europe, Asia, etc., and classed into different types, such as standard corporate, index, sovereign, etc. Different classes of contracts have different estimates of recovery rates as input into models to price CDS spreads. As of my sample Standard North American Corporate Senior, I use the market consensus estimate 40% as recovery rate in the study.

Setting the present value of the expected premium payments equal to the present value of the expected payoff, so that the value of the contract is zero when both parties enter the contract, we can solve for the probability of default p from the resulting non-linear equation. Since the calculations assume no risk-premium associated with default, the resulting implied probability should be considered a risk-neutral measure. Note that p is assumed constant over the life of the CDS contract as a result of the modelling assumption. Of course

in reality the instantaneous jump to default rate may change at different point in time based on changes in the firm's fundamentals and market conditions. The default probability in this model represents the average default rate aggregated over the life of the CDS contract.

2.4 Data

The sample includes 12 American corporations in the financial industry⁷ – Lehman Brothers (LB), Washington Mutual (WM), Bear Stearns (BS), American International Group (AIG), Wachovia, Merrill Lynch (ML), Bank of America (BoA), Citigroup (Citi), Morgan Stanley (MS), Goldman Sachs (GS), J.P. Morgan Chase (JPM), and Wells Fargo (WF). The firms are selected based on data availability.

All of the firms in the sample experience financial distress to some extent throughout the crisis period. LB and WM filed for bankruptcy; BS, ML, and Wachovia were saved through acquisition by JPM, BoA, and WF, respectively. AIG was solely on the sell side of CDS contracts and almost drained its assets paying out CDS claims. It would not have survived without the government's four-time bailout. BoA was relatively safe in 2008 but ran into trouble in early January 2009 revealed by massive losses from the forced merger with ML. The government infused money to BoA as part of the agreement for BoA to acquire ML. Citi was showing less strains prior to September 2008, but the meltdown came in the week of November 17. Citi lost 60% of its market value that week and the government stepped in to rescue Citi in an effort to contain the financial crisis. GS, MS, JPM, and WF were relatively stable and secure compared to the rest the of sample.

Given the crisis period setting and ex-post knowledge of the survivorship of each firm,

⁷The original intention was to investigate a cross sectional investigation of corporate bankruptcy probabilities in all industries. However, CDS for the financial sector tend to be more actively traded than other sectors. This is reflected by larger net notional CDS positions for firms that provide credit guarantees and therefore represent counterparty risk to other market participants (Oehmke and Zawadowski (2017)). This is particularly manifest in financial services companies with exposure to the subprime mortgage market, and later in the global financial crisis period. Due to limited CDS trading activity in non-financial industries, the resulting sample includes firms with both CDS data and option data only in the financial industry.

one would naturally split the sample into two categories for comparison. However, one would be more interested in how things evolve over time, and in particular, how information contained in option prices and CDS spreads help to raise the flag when firms enter financial distress. Default probabilities inferred from option prices and CDS spreads, together with the risk-neutral densities derived from option prices provide ex-ante information in assessing the state of distress for each firm. Multimodal RNDs suggest investors have divergent views, and the emergence of bimodality in the RND, or parallel shift of RNDs can be interpreted as signals of potential crash in the stock price. I discuss such information for each of the firms in section 2.5.

Option data and CDS data are collected for each firm. Option data is obtained from OptionMetrics, which provides historical prices of options based on closing quotes at the Chicago Board of Options Exchange. Data sample starts in January 2007 and ends in October 2010 for healthy firms, and for dead firms option data ends when firms went bankrupt. All options traded on each of the firms are American. In order to avoid complexity of the early-exercise premium of American options, only out-of-the-money (OTM) calls and puts are used. Since the dividend yield is close to zero during crisis period, investors have little incentive to exercise OTM options before expiration, and thus it is reasonable to assume that a European option pricing model provides a good approximation to American option prices. To simplify equations and calculations, options are assumed to be written on futures contracts, with futures and options contracts having matching expiration time.

$$F = e^{rT}(S - pv(Div)).$$

For a strike price X , calls are defined to be OTM if $X > F$, and puts are OTM if $X < F$. The risk-free rate r corresponds to the zero-coupon rate that has the same maturity as the option's expiration, and is calculated by linear interpolation between the two closest zero-

coupon rates on the zero curve.

Dividend information is obtained from the dividend distribution history in OptionMetrics. The distribution pattern for all firms is very regular — quarterly⁸, so it is highly predictable. Thus it can be assumed that the information about dividend information is made available to the public when futures contracts are priced. Since the amount of dividend distributed during the crisis period is very small, I neglect dividends in the calculations as it does not have a significant impact on the results⁹.

Option data for each firm is obtained once a month from 2007 to 2010. Option prices are then filtered. I restrict valid options to options with bid prices strictly positive, and exclude all options with bid price greater than ask price. The representative day on which the MLNbk density is sought is chosen to be the first trading day available for the month with the number of valid options greater than 5¹⁰, in other words, a minimum of 6 options including both calls and puts are required for model estimation. Options mature in, on average, one and a half months. When there are less than 6 available valid options during the first 10 days of the month, I drop the observation of this month. There are a small number of cases where I have dropped the monthly observation, either due to less than 6 available valid options or because the spot price is too low that the parameter estimates cannot be obtained. 8 out of 38 have been discarded for AIG due to extremely low spot price; 2 out of 30 for Citi, and 1 out of 44 for MS. The rest of the sample firms have satisfactory monthly observations. Only options with standard settlement are considered¹¹.

⁸All firms distribute dividends, and payments are made quarterly, e.g. January, April, July, and October as of Goldman Sachs, Morgan Stanley, J.P. Morgan Chase and Washington Mutual; or February, May, August, November as of Wachovia, Wells Fargo, Citigroup, Merrill Lynch, and Lehman Brothers; or March, June, September, December as of AIG and Bank of America.

⁹The amount of dividend payment per period ranges from \$0.1 to \$0.64 across firms pre-crisis, however it dropped significantly after the breakout of the financial crisis, especially for distressed firms, it is practically zero.

¹⁰I try not to choose the representative day with the most number of valid options trading, though liquidity may reflect better pricing on options. The idea is to maintain as consistent as possible the option life across the sample period.

¹¹Standard settlement specifies that 100 shares of underlying security are to be delivered at exercise; the strike price and premium multipliers are \$100 per tick. Non-standard settlement may deviate from the standard settlement specification. I restrict to options with standard settlement to avoid noise in estimation.

CDS data is provided by Credit Market Analytics (hereafter CMA) and is obtained through Datastream¹². CDS data is obtained from January 2007 to October 2010¹³. Daily CDS spreads are obtained for both the 1 year contract and the 5 year contract for each firm. The 5 year CDS contract is the most liquid contract in the market, and 1 year contract is also examined as I am more interested in default probabilities over a shorter period. Only CDS contracts on senior debt are selected, and the contracts are traded in US dollars.

1 year CDS prices provide expected bankruptcy probabilities over a one-year horizon, assuming default occurs between quarterly premium payment days conditional on no prior default; while option prices provide probabilities over on average 34 days, which is a relatively shorter time horizon. CDS probabilities are then scaled using the following equation to be comparable with option probabilities. The scaled probability is given by default within 34 days. Scaling is also done for 5 year CDS probabilities.

$$1 - P_{CDS_{34d}} = (1 - P_{CDS_{3m}})^{\frac{34}{66}},$$

with 66 trading days matching 3 month calendar days.

Table 2.1 gives a characteristic summary of firms in the sample and firm status, and time span for both option and CDS data for each of the firms. Columns 6-7 present option- and 1y CDS-implied (scaled) bankruptcy probabilities estimated on September 4, 2007, which is the earliest common start point for estimation across all firms. It can be seen at this point in time bankruptcy probabilities are less than 1% for all firms. It suggests that all firms are relatively secure and there is no concern that any firm is having financial difficulty, in other

¹²Both CMA data and Thomson Reuters data are available from Datastream, however, Thomson Reuters data is not available for some of the firms in the sample, and for those firms where data is available, the earliest history of data starts only from December 2007. CMA data is investigated in this study. CMA collects CDS data from around 40 members from the buy-side community, including investment banks, hedge funds, and asset managers who are active participants in the CDS market. Data represents aggregated average from all quoting parties on the same reference entity. CDS data for Lehman Brothers is obtained from Bloomberg.

¹³The reason that the data ends at October 2010 is because starting October 1 2010, clients are required to have a CMA license to access the data, but unfortunately the license is not available to us. In addition, I believe that the time span is sufficient for the purpose of contrast between pre and post crisis.

words, there appears to be no meaningful split between groups for which one are relative healthy and the other distressed. Thus, I put each firm on a continuum and discuss them individually in section 2.5. Table 2.2 provides descriptive statistics for option information used for parameter estimation and summary statistics for G values defined by equation (2.5). As can be seen from the quartile results in Panel B, the distributions of G -values are positively skewed.

2.5 Empirical results

This section first summarises estimated bankruptcy probabilities for each of the firms along with a closer look at certain stressful events. Then bankruptcy probabilities across firms are compared in section 2.5.1, and a comparison between probabilities inferred from option prices and CDS prices is described in section 2.5.2.

Bear Stearns (BS)

Bear Stearns was a New York based global investment bank and securities trading and brokerage firm that failed in March 2008, and was subsequently sold to J.P. Morgan Chase. Figure 2.1 shows that prior to August 2007, BS had a bankruptcy probability below 0.45% for option implied probabilities, and CDS probabilities were practically zero¹⁴. During the week of July 16, 2007, Bear Stearns disclosed that its two subprime hedge funds had lost nearly all of their value amid a rapid decline in the market for subprime mortgages. On August 1, 2007, investors in the two funds took action against Bear Stearns and its top board and risk management managers. This was the first legal action made against Bear Stearns. The Co-

¹⁴Figures 2.1 to 2.12 display bankruptcy probabilities estimated from option prices and CDS spreads (1 year and 5 year). Panel (a) display time series of estimated bankruptcy probabilities along with stock prices. Vertical axis on the left-hand-side (right-hand-side) corresponds to bankruptcy probabilities (stock prices). Panel (b) display scatter plots of option-implied probabilities against scaled 1y CDS-implied probabilities. Panel (c) show risk-neutral densities on certain event days.

President was then asked to resign as a result of the collapse of the two hedge funds. Option implied bankruptcy went up to 2.18% on August 1, and CDS probabilities went up as well.

On March 11, 2008, The Federal Reserve Board announced the creation of the Term Securities Lending Facility (TSLF), which will lend up to \$200 billion of Treasury securities for 28-day terms against certain type of securities including federal agency residential mortgage-backed securities. Investors interpret this announcement as a sign of difficulty for BS. Option implied probability rose up to 2.94% reflecting increased risk of bankruptcy. Liquidity dried up dramatically the next couple of days, the probability of bankruptcy was 3.6% on March 12 and 5.44% on March 13.

On March 14, The Federal Reserve Board approved the financing arrangement announced by J.P. Morgan Chase to provide Bear Stearns the liquidity for up to 28 days that the market refused to provide. The bankruptcy chance went up to 13.7% reflecting serious liquidity problem at BS. The RND shows obvious bimodality with the means of the two lognormal component being \$16 and \$42. The distinct bimodality of the RND reflects that investors hold different views on Bear Stearns' future stock price and that the price may fall sharply.

On March 17, J.P. Morgan offered to acquire BS at \$2 per share. This sale price represented a staggering loss as its stock had traded at \$93 a share as late as February 2008. The RND shows progressively spiked peak around \$5, revealing market consensus view that Bear Stearns is going to collapse. Option implied bankruptcy probability was 9.34%. A week later, the revised deal was struck that BS would be acquired by JPM at \$10 per share to quiet upset investors. After the acquisition was agreed, bankruptcy probabilities decreased to 0.72% and 0.42% in April and May.

It can be seen that the bankruptcy probabilities given by CDS spreads went up when BS gradually went into trouble, and was high at BS's crisis period, and settled down after the agreement of acquisition was made. We observe that the two sources of bankruptcy proba-

bilities agree with each other except that the magnitude of option implied probabilities are higher than those from CDS spreads during the crisis period. Both probabilities exhibit inverse relationship with the stock prices, in particular when stock price plummets, the two sources of bankruptcy probabilities shoot up in consensus.

Lehman Brothers (LB)

Figure 2.2 shows that LB's option implied probability was below 0.5% prior to mid-March 2008, and went up starting with the Bear Stearns crisis until it went bankrupt in September 2008. The RND on September 4, 2007 is well-behaved in that it is negatively skewed and does not have sharp spikes. At this point Lehman Brothers did not have known liquidity issues and therefore do not reveal any market concerns.

After Bear Stearns was taken over by JPM, market analysts suggest that LB would be the next major investment bank to fall due to its heavy exposure to credit derivatives linked to the sub-prime mortgages. During this time, LB relied on overnight Fed funding to survive BS crisis period. The RND is bimodal where one mode is as low as \$10 and the other above \$40, which evidenced market concern that Lehman Brothers may collapse. The bankruptcy probability was 6.59% on March 17.

In June, Lehman Brothers announced \$2.8 billion second-quarter loss as market volatility rendered many of its hedges ineffective. On June 3, the bankruptcy probability was 5.28% as analysts and investors anticipated the loss, reflecting deteriorated liquidity concern. During August, LB closed up 5% of its shares and held secret talks with state-controlled Korea Development Bank. As investors perceive that Korea Development bank was considering buying LB, this was considered positive news, and bankruptcy probabilities decreased from 2.86% in July to 2.34% in August.

However, gains quickly eroded as news came in that Korea Development Bank was strug-

gling to please the regulators and finding partners for the deal. The bankruptcy probability went up to 3.37% at the start of September, reflecting negative expectation about LB. LB's share price plunged to \$7.79 on September 9, after it was reported that the talks with Korea Development bank was put on hold. The RND shows more probability at the lower mode which indicates very low future price expectations. The bankruptcy probability went up to 17.56%. On September 12, the spike of the RND shows consensus view that Lehman Brothers is going to collapse. The bankruptcy probability was 25.63%, and three days later, LB declared bankruptcy on September 15.

CDS spreads for LB was obtained from Bloomberg, and gaps shown on CDS probabilities are due to missing values of CDS spreads. CDS probabilities were low before BS crisis, below 1%; they gradually went up as LB's situation deteriorates in July and August; and shoot up to 2.43% on September 12 for the last available data point before bankruptcy. LB's filing for bankruptcy triggered the CDS payout to buyers who sought for LB's credit default protection. The auction for LB's debt occurred on October 10, and the resulting price of LB's senior debt was only 8.625 cents on the dollar, which means that the sellers of LB's CDS contracts were obliged to pay the insured counterparties 91.375% of the face value of LB's senior debt. The Depository Trust and Clearing Corporation (DTCC) announced about \$5.2 billion in net funds transfer from net sellers to net buyers as a result of LB's CDS settlement.

Merrill Lynch (ML)

In November 2007, Merrill Lynch announced it would write-down \$8.4 billion in losses associated with the national housing crisis. As shown in Figure 2.3, option implied bankruptcy probability went up to 0.62% from 0.17% a month earlier, and stayed around 0.5% for the next couple of months prior to the BS crisis. On March 17, Merrill Lynch shows a similar perception of option-implied bankruptcy risk about 6.5% to that of Lehman Brothers, but the

RND does not reveal serious concern over future price drop.

In July 2008, ML announced \$4.9 billion fourth quarter losses for the company from defaults and bad investments in the ongoing mortgage crisis. Two weeks later on July 28, the company announced the sale of selected hedge funds and securities in an effort to reduce their exposure to mortgage related investments. The company's share price declined significantly. Bimodality emerges in the RND, and the bankruptcy probability was 2.35%.

On September 5, ML's stock was downgraded to 'conviction sell' by Goldman Sachs and warned of further losses. As Lehman Brothers came under severe liquidity issues, ML held talks with Bank of America and finally made a sale to BoA on September 15. The lower mode of the RND on September 15 became more peaked indicating investors' increased worry that ML is heading into trouble. The bankruptcy risk is 4.73%.

CDS probabilities are high at BS crisis and September 2008 when the break out of financial crisis occurred, with the latter higher than the former. But generally, bankruptcy probabilities given by both option prices and CDS spreads for ML are smaller than those for LB, reflecting the fact that ML was able to strike a deal with BoA whereas LB failed.

Washington Mutual (WM)

Washington Mutual Inc. was a savings bank holding company and the former owner of Washington Mutual bank, which was United States' largest savings and loan association bank until its collapse. At the start of the common date on September 4, 2007, the stock price is around \$40, and the RND is well-behaved, almost symmetric but with a small chance of future price falling below \$30.

On March 17, the RND shifted to a low level with mean around \$9. WM held a significant exposure to credit derivatives, and the bankruptcy probability was high at 11%. WM had sustained losses and difficulties as a result of the subprime mortgage crisis, and the situation

deteriorated quickly since July.

When Lehman Brothers declared bankruptcy and Merrill Lynch sold to Bank of America, the market hit a critical stage. The holding company of Washington Mutual bank received a credit rating agency downgrade, and option implied bankruptcy probability was high at 25.49% as of September 12, see Figure 2.4. Starting September 15, WM's customers started heading for the exits. Over the next 10 days, WM experienced a bank run of in total \$16.7 billion withdrawal in deposits, which eventually led to the seizure of the bank by Office of Thrift Supervision (OTS) and was placed into receivership with the Federal Deposit Insurance Corporation (FDIC). FDIC sold most of Washington Mutual bank's assets to J.P. Morgan Chase at a very low price, and WM filed for bankruptcy on September 26. The bankruptcy was the second largest in the U.S. history, with the largest being Lehman Brothers. Bankruptcy probability on the day before was 57.41%. The RNDs on both September 12 and 25 show significant spikes on the tail, indicating high volatility and panic over the survivorship of WM.

CDS probabilities were small and flat before July 2008, and gradually increased in July and August. Starting early September, bankruptcy probabilities increased dramatically with significant jumps. It was 6.65% on September 8, and within a few days' time it went up to 21.2% on September 16, reflecting an accelerated expectation of default risk in the credit market. The magnitude of CDS bankruptcy probabilities for WM is much higher than those for BS, LB, and ML. The auction to determine the CDS settlement price was held on October 23, and the final result agreed by all parties was that the recovery rate on WM's senior debt was 57 cents on the dollar, and the sellers of CDS on WM would pay 43 cents on the dollar to CDS purchasers.

Wachovia

Wachovia was a diversified financial services company and was the fourth largest bank

holding company in the United States before Wells Fargo acquired it in 2008. On September 4, 2007, Wachovia's stock price was \$49. The RND appears almost unimodal, with the means of two modes very close to the future stock price, and the volatilities for the two log-normal component are small. Bankruptcy probability is less than 1%. On March 17, when Bear Stearns almost collapsed, the RND became bimodal with increased volatility for both lognormal components. The stock price fell to \$25.6 and the bankruptcy probability went up to 3.68%.

It can be seen in Figure 2.5 that prior to September 2008, Wachovia had bankruptcy probabilities below one half of 5%, while on September 15 when Lehman Brothers declared bankruptcy and Merrill Lynch sold to Bank of America, option implied bankruptcy probability went up to 14.2%. Due to the seizure of Washington Mutual by FDIC and the filing of bankruptcy the previous day, on September 26, Wachovia experienced a similar bank run of deposit withdrawals from businesses institutional investors, which amounts to 1% of the bank's total deposits. Its stock price plunged 27% and the option bankruptcy probability went up to 39.98%. The large outflow of deposits attracted regulators' concerns, and over the weekend FDIC has been considering what actions needed to be taken on Wachovia. As being deemed systematically important to the health of the economy, Wachovia was not allowed to fail, as did Washington Mutual. On September 29, FDIC announced to auction off Wachovia's banking assets to Citigroup and since then Citigroup has become the main source of liquidity provider to allow Wachovia to continue operating. The bankruptcy risk was high at 32.73%. The RND became a sharp spike at \$1.84 signalling the collapse of Wachovia.

On October 3, Wachovia and Wells Fargo announced a merger agreement of \$15 billion in stock. Wachovia preferred the deal with Wells Fargo despite Citigroup being its liquidity provider, because it would keep their business intact and there was far less overlap between the banks. Citigroup abandoned its attempt to purchase Wachovia's banking assets after

exploring its legal options against the discussion between Wachovia and Wells Fargo with no success. The Federal Reserve unanimously approved the merger with Wells Fargo on October 12, and the option implied bankruptcy probability decreased to 4.2% on the 13th, reflecting the market's relieved anxiety about the complete failure of Wachovia.

Compared to option implied probabilities, CDS probabilities are rather mild. It shows a slight increase during Bear Stearns crisis and early September turmoil. Though probabilities sharply increased after the collapse of Lehman Brothers and Washington Mutual, the highest bankruptcy probability was 3.96% on September 26. CDS probabilities show that the chances of bankruptcy generally decreased after the merger agreement between Wachovia and Wells Fargo was made; however, due to data limitation, I am unable to obtain option implied probabilities for after the merger.

American International Group (AIG)

AIG is the much troubled financial institution in the sample that survived the financial crisis through a series of government bailouts between September 2008 and March 2009. AIG was active in the credit derivative business, including CDSs and CDOs, and was severely affected by the liquidity dry-up during the financial crisis.

Figure 2.6 shows that prior to September 2008, the estimated chance of bankruptcy from option prices was below 2%, and increased after July 2008. On September 12, S&P put AIG on a negative credit watch, signalling a possible rate downgrade. The chance of bankruptcy was 13.89%. Monday September 15, AIG's credit rating was downgraded, forcing it to post additional collateral. AIG's financial crisis intensified and the chance of bankruptcy was 43.12%. Its share price dropped 60% to \$4.76 over the weekend. The fat tail of the RND reflects heightened fear about AIG's future uncertainty.

Seeing Lehman Brothers collapse without rescuing, the U.S. government decided that

AIG was too big to fail because an incredible amount of institutional investors – investment banks, hedge funds, mutual funds, and pension funds – invested in and also were insured by AIG. The Federal Reserve stepped in and announced an injection of \$85 billion on September 16 to allow it to deliver additional collaterals to prevent it from collapsing, and the government effectively seized control of AIG, making it the largest government bailout of a private company in U.S. history. The estimated chance of bankruptcy was nearly 50%. AIG received its second and third bailout from the government on Oct 8 and November 10, respectively, and its share price has been at only a few dollars throughout the first half of 2009.

In early March 2009, AIG received the fourth bailout and the bailout which was restructured to give the Fed preferred interests in life insurers. By then AIG had received more than \$170 billion taxpayers' money in bailouts, and two weeks after, AIG announced its plan to pay out \$165 million in executive bonuses. Word of bonuses stirred up outrage from the government as well as the general public. Under the pressure of the Treasury Secretary, the deal was cut to \$9.6 million going to the top 50 executives, as the firm was contractually obliged to pay them. At this stage, it was not clear where AIG stood in the recovery from the financial crisis.

CDS probabilities increased significantly to about 10% since the September 2008, and remained high throughout the first half of 2009. However, the CDS probabilities are as low as 2% from December 2008 to February 2009 when CDS spreads dropped significantly, possibly due to the intervention of the U.S. government in bailing out AIG, investors deem AIG unlikely to collapse.

J.P. Morgan Chase (JPM)

J.P. Morgan Chase, relative to all other financial institutions, has the lowest bankruptcy probabilities across the whole sample period. Option implied probabilities are below 2% and

CDS probabilities are below 1%. It was strong enough to stay profitable during the financial crisis and bail out two failing competitors, Bear Stearns and Washington Mutual.

Figure 2.7 shows that option implied bankruptcy probabilities were generally below 0.5% prior to the Bear Stearns crisis. JPM stepped in as a rescuer after intense negotiation between the two parties and the federal government. On March 17, JPM announced its plan to acquire BS in a stock swap valuing BS at \$2 per share. The estimated bankruptcy risk increased temporarily to 1.24%, and the RND has low probability of low future price. On March 24, JPM announced a revised deal at \$10 per share to pacify BS's angry shareholders, and the merger was complete in June. The bankruptcy risk dropped to 0.41% after consent with the new offer price. CDS probabilities show that the chances for bankruptcy increased during the Bear Stearns crisis period, but decreased as soon as the acquisition deal was approved.

On September 15 when Lehman Brothers filed for bankruptcy, the estimated chance of bankruptcy risk for JPM was just slightly above 1%. The relative stable RND reflects market perception about JPM's rather healthy financial conditions. On September 25, the Office of Thrift Supervision (OTS) seized Washington Mutual bank and placed it into receivership with the Federal Deposit Insurance Corporation (FDIC). JPM bought most of WM's banking operations from FDIC. The bankruptcy chance was 1.01% on September 26. The long left tail of the RND shows investors' sceptical concern about the benefit of the acquisition to JPM due to the significant write-downs and losses from WM's assets and debts. CDS probabilities give the same trend of estimates of bankruptcy risk. Probabilities jumped when Lehman Brothers collapsed, but were low prior to the critical day.

The Treasury Secretary Hank Paulson released a \$700 billion proposal to purchase troubled mortgage-related assets on September 20, called the Troubled Asset Relief Program (TARP), in order to stabilize the crashing market. However, the proposal was initially rejected by the House. Just a few days later, the Senate and the House passed the bill and the president

signed the \$700 billion Emergency Economic Stabilization Act into law. On October 14, The Treasury Department announces that it will invest up to \$250 billion in the nation's banks via the Capital Purchase Program (CPP), a subcomponent of the TARP. This allows the Treasury to purchase illiquid, difficult-to-value assets from banks and other financial institutions to improve the liquidity of these assets allowing participants to stabilize their balance sheets and avoid further losses. Nine large financial organizations announced their intention to subscribe to the facility through preferred stock investments in an aggregate amount of \$125 billion. JPM was one of them and received \$25 billion on October 28. The option implied bankruptcy risk was 1.32% and 1.65% respectively on October 14 and 28, relatively higher than pre-crisis period, reflecting market anxiety about collective collapse of major financial institutions, but the RND on October 28 has lower probability of future lower price.

CDS probabilities are also the lowest for JPM throughout the sample period relative to other companies. It fluctuates within 0.5% for most of the times with little volatility, with the highest record goes slightly above 0.5% in March 2009.

Bank of America (BoA)

BoA was relatively safe throughout the crisis from March to September 2008, but condition deteriorates since the start of 2009. During the Bear Stearns crisis, the option implied probability of bankruptcy for BoA, see Figure 2.8, was less than 1%, and on March 17 when JPM offered to buy Bear Stearns at \$2 per share, the estimated chance of bankruptcy for BoA was 0.52%, showing how slightly BoA was affected by the event. In September, before Lehman Brothers failed, BoA was the leading bidder to purchase Lehman Brothers. However, due to a lack of government backing for the takeover, BoA stopped bidding for Lehman Brothers, who filed for bankruptcy the next day. The same day, Merrill Lynch entered into agreement to be acquired by BoA. The bankruptcy probability for BoA on September 15

went up to 3.14%, reflecting some systemic credit risk stemming from Lehman Brother's bankruptcy.

BoA was one of the nine institutions signed on for the CPP program on October 14, and received \$25 billion on October 28. The estimated bankruptcy chances were 1.39% and 3.02%. The RNDs shown in Figure 2.8 up to October 28 do not show expectations of very low future price.

On January 16, 2009, in its earnings release, BoA revealed massive losses at Merrill Lynch in the fourth quarter, which necessitated an infusion of money which was previously negotiated with the government as part of the government-persuaded deal to acquire Merrill Lynch. The Treasury announced \$20 billion investment in BoA. The bank's stock price sank to \$7.18, the lowest in 17 years. The market capitalization of BoA was \$45 billion, less than what it offered to Merrill Lynch of \$50 billion. The bankruptcy risk on January 16 1.45%. The emergence of bimodality in the RND clearly shows concerns over BoA's financial stability which was not alleviated by the announcement of the government bailout.

CDS probabilities were a lot less volatile, with less than 0.5% for pre-crisis period and went up to as high as 1.5% between March and May 2009. BoA would not have gone in such difficulty had it not acquired Merrill Lynch through a forced deal by the U.S government.

Citigroup (Citi)

Citigroup was one of the largest bank holdings company in the U.S. as of 2008, but suffered huge losses during the global financial crisis and eventually entered into a rescue plan by the U.S. government. Prior to Bear Stearns running into trouble, bankruptcy risk for Citigroup was below 0.5%, see Figure 2.9. However, the bankruptcy chance went up to 2.76% on March 17 when JPM offered to buy Bear Stearns at \$2 per share. Citigroup had heavy exposure to troubled mortgages in the form of collateralized debt obligation (CDOs), com-

pounded by poor risk management; high bankruptcy risk reflects a possibility of hosing downturn at national level, however, Citigroup deemed it tiny and excluded them from their risk analysis. The RND has roughly equal weights for the two lognormal component, but with higher volatility for the lower mode.

On September 15, the estimated chance of bankruptcy was 4.35%, much higher than that of JPM and BoA, showing worsened conditions for Citigroup stemming from the subprime mortgage crisis. The RND does not show indication of serious deterioration of Citi's financial condition. The CPP program was introduced by the Treasury as a plan to help 'healthy' institutions to lend in tough market times rather than a bailout for the banks, but some participants turned out to be not so healthy and only needed further injections later. Citigroup was clearly one of them. Citigroup received their first injection of \$25 billion on October 28 and the bankruptcy risk was 2.54%.

November 17, Citigroup announced plans for about 52,000 new job cuts, on top of 23,000 cuts already made during 2008, resulting from four quarters of consecutive losses and reports that it was unlikely to be in profit again before 2010. By that time, Citigroup was insolvent despite its receipt of \$25 billion funds. The share price dropped and the chance of bankruptcy went up to 5.12%. The RND shifted to a lower mean around \$8, and became more peaked with increased volatility.

On November 23, government and regulators stepped in and approved a plan to stabilize the company and prevent it from failing. On the 24th, the government announced a massive stimulus package of \$20 billion of investment in Citigroup which gives the government a major say on its operations. Citigroup issued preferred shares to the Treasury and FDIC in exchange for protection against losses on a \$306 billion pool of commercial and residential securities held by it. The chance of bankruptcy was high at 6.36% on the 24th. The RND warns of a possible further fall in the already low share price, which indicates that investors

were not sure whether the government bailout would be sufficient in rescuing Citi.

The U.S. government was taking necessary actions to strengthen the financial system and protect taxpayers and the economy. In February 2009, Citigroup announced that the U.S. government would be taking 36% equity stake in the company by converting \$25 billion emergency aid into common shares. In June, it was announced that Citigroup was removed from Dow Jones Industrial Average due to a significant government ownership. Citi's share price has been at a very low level throughout 2009.

CDS probabilities are relatively low below 1% prior to 2009. Bankruptcy probabilities increased to about 2% throughout the first half of 2009 when Citigroup's stock price plummeted during this period.

Goldman Sachs (GS) and Morgan Stanley (MS)

Goldman Sachs and Morgan Stanley were the last two major investment banks in the Wall Street history who became traditional bank holding companies in September 2008. Figure 2.10 and Figure 2.11 show that the bankruptcy risk was below 1% for both companies prior to the failure of Bear Stearns. However, on March 17, the chances for bankruptcy went up to 2.24% for Goldman Sachs and 3.59% for Morgan Stanley. Similar to Citigroup, Goldman Sachs and Morgan Stanley suffered from the systemic credit risk stemming from the Bear Stearns crisis.

On September 15, bankruptcy chance for Goldman Sachs was 1.46% while 4.41% for Morgan Stanley. A few days later on September 21, both firms confirmed they would become traditional bank holding companies. In becoming holding companies, Goldman Sachs and Morgan Stanley would get access to the Federal Reserve's emergency lending facilities, and it will allow them to better organize their assets and place them in better position for mergers and acquisitions. The bankruptcy risk after the announcement was 1.84% and 6.86%

for Goldman Sachs and Morgan Stanley respectively. Upon completion of converting, both firms received \$10 billion funds from the government through CPP program on October 28. Bankruptcy risk was 4.76% and 9.36% for Goldman and Morgan. Clearly, the market was not confident whether both former investment banks would avoid following the footsteps of Lehman Brothers.

The RNDs of both companies display very similar characteristics. Prior to the BS crisis, the RNDs appear almost identical with unimodal and fatter left tail. On both dates when JPM offered to acquire BS and LB filed for bankruptcy, the RNDs show slightly concern over future share price drop. On October 28 when both companies received government funding through the CPP program, the RNDs has further increased mass in the lower mode of the lognormal component, reflecting concern over the systemic risk in the financial markets. However, there was no signal of serious deterioration of either firm's financial stability even during the most stressful market condition, reflecting two firms' relative healthy condition throughout the crisis period.

CDS bankruptcy probabilities for both firms exhibit similar trends, with low and less volatile shape for pre- and post- financial crisis period, and sharp spike after the break out of financial crisis in September 2008. The magnitude of bankruptcy risk is higher for Morgan Stanley than for Goldman Sachs, especially during the crisis period, with the highest being nearly 6% for Morgan and only 2% for Goldman.

Wells Fargo (WF)

Figure 2.12 shows that the bankruptcy probabilities for Wells Fargo are generally low across the sample period. Wells Fargo was only slightly impacted by the Bear Stearns crisis, with a bankruptcy risk of 0.88% on March 17 increased from 0.45% on March 3. It was the only bank in the United States to be rated AAA by Standard & Poor's in 2007 and became

one of the big four banks of the United States, along with Bank of America, Citigroup, and J.P. Morgan Chase, through acquisition of Wachovia.

Wells Fargo was less affected by the systemic credit risk after the collapse of Lehman Brothers and sale of Merrill Lynch. The bankruptcy risk on September 15 was 1.86%. Despite the low risk of bankruptcy, the RND warns a probable decrease in share price with higher probability in the lower mode lognormal component.

On October 3, 2008, Wachovia agreed to be bought by Wells Fargo for about \$14.8 billion in an all-stock transaction. This news came four days after the Federal Deposit Insurance Corporation (FDIC) made moves to have Citigroup buy Wachovia for \$2.1 billion. Citigroup protested Wachovia's agreement to sell itself to Wells Fargo and threatened legal action over the matter. However the deal with Wells Fargo overwhelmingly won shareholder approval since it valued Wachovia at about 7 times what Citigroup offered. To further ensure shareholders' approval, Wachovia issued Wells Fargo with preferred stock holding 39.9% of the voting power in the company. The bankruptcy risk for Wells Fargo on October 3 was 0.44%, decreased from early September, reflecting shareholders optimism about the acquisition. The shrinkage of the lower mode lognormal component suggests investors alleviated concern over WF's solvency stemming from the credit crunch.

October 14, the Treasury announced the CPP program and Wells Fargo signed on to participate in the program and on October 28, Wells Fargo received \$25 billion. The bankruptcy risk increased to 1.19% and 1.21% respectively on the two dates, reflecting systemic credit risk in the unstable financial system. The RND on October 14 shows concern about a small chance of mild decrease in future share price.

CDS bankruptcy probabilities are generally low for Wells Fargo. Probabilities increased slightly during the Bear Stearns crisis and the global financial crisis, with the highest being less than 1%.

2.5.1 Comparison of bankruptcy probabilities across firms

In this section, the sample is split into two groups for a comparison between estimated bankruptcy probabilities. Using ex-post knowledge of the survivorship of each firm, firms are grouped into relative healthy firms which survived the crisis – Bank of America, J.P. Morgan Chase, Goldman Sachs, Morgan Stanley, and Wells Fargo, and distressed firms – Bear Stearns, Lehman Brothers, Washington Mutual, Merrill Lynch, AIG, Wachovia, and Citigroup. The identification for distress is based on the ex-post knowledge of a firm either went bankrupt or acquired by other firms during the financial crisis, or the share price fell below \$5 from a previous high level and remained low for a long period of time.

Figures 2.13 shows option implied bankruptcy probabilities (including critical event days) for both relatively healthy firms and distressed firms. Panel (b) shows estimated bankruptcy probabilities for when data is available. After bankruptcy or acquisition the firm drops out of the sample. As can be seen bankruptcy risks for both groups exhibit a consensus trend — probabilities are low before March 2008 and after May 2009, and high during both the Bear Stearns crisis and the global financial crisis, with the only difference in the magnitude, especially during the crisis period. The probabilities around Bear Stearns crisis are generally smaller than the Lehman Brothers failure period starting September 2008.

Prior to Bear Stearns crisis in March 2008, the average probability was 0.20% and 0.31% respectively across healthy firms and distressed firms. During the Bear Stearns crisis, healthy firms were only slightly affected by the systemic risk while distressed firms started to reveal problems. Bankruptcy risk for healthy firms were less than 4% whereas that for distressed firms went above 5%, the highest being 20% for Bear Stearns just a couple of days before JPM offered its first deal. The probabilities decreased for both groups after Bear Stearns crisis. Starting September when Lehman Brothers collapsed until the first half of 2009, bankruptcy probabilities increased significantly for both groups. Healthy firms' bankruptcy probabilities

scattered between 1% and 7%, with a few estimates above 8% to about 14%; bankruptcy probabilities for distressed firms were dramatically higher which range from 4% to 50% on certain event days. These estimates are consistent with the market perception that distressed firms have higher chances of failure and are more fragile than healthy firms.

Figure 2.14 shows bankruptcy probabilities inferred from 1 year CDS spreads grouped the same way into healthy firms and distressed firms. Panel (b) shows estimated bankruptcy probabilities for when data is available. After bankruptcy or acquisition the firm drops out of the sample. Daily CDS spreads provide daily estimates of bankruptcy probabilities, which enable us to have a clearer view on how bankruptcy risks evolve on a daily basis. There is clearly a consistent trend among the two groups across the sample period, complemented with idiosyncratic variation with respect to individual firms. Bankruptcy chances increased during Bear Stearns crisis and are much higher in September 2008. The probabilities were less than 1% for healthy firms during Bear Stearns crisis, whereas probabilities for some distressed firms went over 2%.

In September, chances of bankruptcy jumped significantly for Washington Mutual and AIG, showing a spike in Figure 2.14(b). It went as high as 22.61% and 13.16% respectively on critical days. Washington Mutual filed for bankruptcy at the end of September and AIG only survived upon receiving a series of government bailouts. Wachovia and Merrill Lynch had a bankruptcy risk below 5% in September, reflecting the fact that both firms were able to strike a deal with another party and eventually to be acquired. The chance of bankruptcy for Lehman Brothers was only 2.24% on September 9, which was the last date data available. The relative low bankruptcy probability just a few days prior to its collapse may suggest that the market did not believe that Lehman would actually go bankrupt. On the other hand, bankruptcy risk for healthy firms was generally below 2%, except for Morgan Stanley which had a few probabilities between 2% and 6%.

Figure 2.15 plots share prices of all sample from January 2007 to September 2010 for when data is available. Observations disappear from the plots when firms went bankrupt or acquired by other firms. It shows clearly that share prices for all surviving firms dropped significantly since September 2008, and remained at a low level for several months before a slow recovery. The plummet in share prices coincides with an increase in estimated bankruptcy probability. The collective fall of share prices and increase of bankruptcy probabilities potentially reflect a systemic risk factor influencing the market.

In the CDS model, 1 year CDS probabilities provide the expected chance of bankruptcy in three months over a one year horizon while 5 year chances are over a 5 year horizon. Figures 2.1 to 2.12 show that in most cases 5 year CDS bankruptcy probabilities are higher than 1 year probabilities during calm periods whereas 1 year probabilities overtake 5 year probabilities in critical market conditions. This is consistent with market expectations that the 5 year CDS contract protects against longer horizon than the 1 year contract, in which there is higher chance that bad things can happen, thus under normal market conditions 5 year CDS bankruptcy probabilities are higher than 1 year probabilities. However, when there is systemic credit risk in the market and the financial system becomes unstable, the market worries about sudden jump to default risk in the near future. The protection seller would then increase CDS spreads, and thus render a heightened chance of bankruptcy embedded in shorter horizon CDS contract. 1 year CDS contract provides a view about default risk over a shorter horizon than does the 5 year contract, which is useful for analysts who want to assess credit risk in the near future. But the 5 year CDS contract is the most liquid contract in the market. It may contain more accurate pricing information than the 1 year CDS contract.

2.5.2 Comparison between option implied bankruptcy probabilities and CDS bankruptcy probabilities

Option implied bankruptcy probabilities are risk-neutral probabilities over the horizon defined by the options' time to maturity, in this case around 34 days. CDS bankruptcy probabilities are also risk-neutral probabilities of default within three months over the life of the CDS contract. CDS probabilities have been scaled to the same horizon as option implied probabilities for better comparison. The scatter plots in Figures 2.1 to 2.12 panel (b) plot option-implied probabilities against the scaled 1 year CDS-implied probabilities for each firm. The plots show that CDS-implied probabilities generally increase with option-implied probabilities. Visual comparison through Figures 2.1 to 2.12 suggests that the two sources agree about relative bankruptcy risk, but option prices give magnified estimates during crisis periods than CDS estimates. Clearly the two markets provide complementary information about default risk. The next logical question to ask is whether one market leads the other in providing relevant information. I attempt to answer this question by examining Granger Causality between the two sources.

Granger Causality test gives an idea of whether one time series is useful in forecasting the other. Option implied bankruptcy probabilities are said to Granger Cause CDS bankruptcy probabilities ($O \rightarrow C$) if CDS probabilities can be better predicted by histories of both option implied probabilities and CDS probabilities, and vice versa ($C \rightarrow O$). The null hypothesis is that the coefficients of lagged option implied probabilities are zero, in other words, option implied probabilities do not Granger Cause CDS probabilities, and vice versa. It is important to note that the test depends crucially on the number of lagged terms introduced in the model, which is usually chosen using an information criterion, such as the Akaike information criterion or the Schwarz information criterion. However, since we have a relatively small sample, it is very difficult to choose the optimal number of lags given the limited data history.

I include 2 lags in the test as it is intuitive that the value at each period is mostly influenced by its close previous values.

I perform a pairwise uni-direction Granger Causality test to see if information inferred from both option prices and CDS spreads predicts better than information from only one source. Tests are done between option implied bankruptcy probabilities and 1 year CDS bankruptcy probabilities. Time series of bankruptcy probabilities are monthly from 2007 to 2010. The tests are done for firms which are relative healthy and survived the crisis. As stationary assumption is required to perform the test, we could reasonably assume that the time series of probabilities are stationary as they keep at a low level when the market is peaceful, went up during crisis period, and eventually come down at a low level if the firm survives the crisis. I have done two sets of tests, one for the original time series that are at level, and the other for time series of first difference which are more applicable if the data are non-stationary.

Table 2.3 reports F statistics, p -values, and decisions for each uni-direction test. Results are consistent between tests on levels and tests on first differences, except for Citigroup and one direction for Wells Fargo. However, the causality conclusions are mixed. While results for Bank of America and Wells Fargo indicate option implied probabilities Granger Cause CDS probabilities for levels, those of Goldman Sachs indicate completely the opposite; results of J.P. Morgan and Morgan Stanley do not show causality between the two measures and those for Citigroup indicate both measures Granger Causes one another. Mixed conclusions can be caused by a non-optimal number of lags used in the test and the small sample. Daily time series of bankruptcy probabilities may provide better outcomes as daily measures may contain richer information about dependence on previous values. I consider the evidence about causal relations between the two series to be inconclusive.

2.6 Conclusions

This study makes use of option prices and CDS spreads to infer corporate bankruptcy probabilities during the 2008 global financial crisis, and makes comparisons of the information conveyed in both markets. An option pricing framework is used where the risk-neutral density is assumed to be a mixture of two lognormals augmented with a probability of default, to calibrate to the observed market option prices. The bankruptcy probability is thus obtained through least square estimation. The CDS model assumes a constant conditional default probability and equates the present value of expected premium payments with the present value of expected payoffs to solve for default probability. All probabilities inferred from option prices and CDS spreads are risk-neutral.

The sample includes 12 American firms in the financial industry. Risk-neutral densities extracted from option prices and the evolution of estimated bankruptcy probabilities are used to provide ex-ante information to assess the degree of financial distress for each firm. The emergence of bimodality in the risk-neutral densities can be interpreted as investors' divergent views on the future value of stock price, which reflects the market's concern of potential price crash. This information and estimated bankruptcy probabilities can help regulators and market participants monitor whether a firm is probably secure or default risky.

To avoid ex-post knowledge of splitting firms into healthy or distressed groups, firms are put on a continuum from September 4, 2007 and examined individually. The results show that the bankruptcy probabilities are low during calm market conditions and are high during crisis period. Firms are then grouped into distressed firms and relative healthy firms using ex-post knowledge of their survivorship, to compare the magnitude of estimated bankruptcy probabilities between groups. Results show that distressed firms have higher chances of bankruptcy than firms which are relatively healthy. Probabilities inferred from option prices and CDS spreads are then compared using Granger Causality tests. The conclusion is that

both markets provide complementary information, but it does not show significant evidence suggesting one predicts the other for the data examined. It is worth noting that Granger Causality tests depend heavily on the properties of the time series under investigation. Daily pairwise bankruptcy probabilities may provide more meaningful results on the causal relationship than monthly observations.

One observation from the analysis is that option-implied bankruptcy probabilities are higher than CDS-implied probabilities, especially on event days, the magnitude is a few multiples higher than CDS-implied probabilities. One possible explanation for the discrepancy could be that the assumptions of p in the two pricing models are not defined for exactly the same underlying. Option-implied p measures the probability of the stock price reaches zero, whereas CDS-implied p measures the probability that a well-defined credit event is triggered. Another explanation could come from the quoting of CDS spreads. The CDS spread quoted by one particular counterparty on one reference entity reflects the joint default risk for both firms. The spread crucially depends on how much of this claim the buyer can expect to recover from the counterparty in case of a credit event. The quoting firm is likely to lower the charge for credit protection if its own credit risk increases. Arora et al. (2012) finds that counterparty risk is significantly priced in the CDS markets. This is particularly relevant in the crisis setting when market participants in the financial sector are all affected by the systemic risk. A lower price quoted by an counterparty with increased credit risk could result in a downward bias in estimated default probabilities. Giglio (2011) has shown that ignoring counterparty risk biases the estimates of default probability extracted from CDS spreads downwards.

Another observation is that the bankruptcy of Lehman Brothers triggered a domino effect in the banking system that the share price of firms in the sample plummeted and default risk increased. The systemic risk that may trigger the failure of the full banking system has

attracted full attention from the government and banking regulators. This study investigates banks individually one at a time, and the probability of multiple defaults is not explicitly modelled. A probability model that captures systemic default risk will provide a meaningful tool for market regulators. CDS price contains pairwise joint default risk for the reference entity and the quoting counterparty, but it is not possible to distinguish between the risk that comes from the reference entity and from the counterparty. However, bond prices are not affected by counterparty risk and reflect only individual default probabilities. Utilising combined information in CDS spreads and bond prices could be potentially useful in exploiting joint default probability for both firms, and ultimately joint default probabilities for several firms. However, this may pose a challenge to data availability as it would require counterparty-specific CDS quotes rather than the average of all dealers. Giglio (2011) attempts to establish such a model and constructs bounds to characterise multiple default risk for banks which captures the notion of systemic risk. Giglio (2011)'s results are derived from several modelling assumptions and affected by limitations in the data, nevertheless, it points out a meaningful direction for future research.

2.7 Appendix

Table 2.1: Sample data description

| Company | Data starting and ending (month) | | Default probs Sep-4-07 | | Ex-post firm status | | |
|-------------------|----------------------------------|--------|------------------------|--------|---------------------|--------|---|
| | Option | CDS | Option | CDS | | | |
| Lehman Brothers | Jan 05 | Sep 08 | Jan 05 | Sep 08 | 0.0046 | 0.0017 | Filed Ch11 bankruptcy Sep 15, 08 |
| Washington Mutual | Mar 07 | Sep 08 | Jan 07 | Sep 08 | 0.0093 | 0.0022 | Filed Ch11 bankruptcy Sep 26, 08 |
| Bear Stearns | Jan 06 | May 08 | Jan 06 | May 08 | 0.0034 | 0.0027 | Acquired by JPM May 29, 08 |
| AIG | Aug 07 | Sep 10 | Jan 07 | Sep 10 | 0.0022 | 0.0005 | Series of government bailout |
| Merrill Lynch | Jan 07 | Sep 08 | Jan 07 | Dec 08 | 0.0032 | 0.0016 | Acquired by BoA Sep 14, 08 |
| Wachovia | Jun 07 | Oct 10 | Jan 07 | Dec 08 | 0.0042 | 0.0006 | Acquired by WF Dec 31, 08 |
| Bank of America | Aug 07 | Sep 10 | Jan 07 | Sep 10 | 0.0013 | 0.0005 | Capital Purchase Program + government bailout |
| J.P. Morgan Chase | Jun 07 | Sep 10 | Jan 07 | Sep 10 | 0.0036 | 0.0006 | Capital Purchase Program |
| Citigroup | Jun 07 | Nov 09 | Jan 07 | Dec 09 | 0.0026 | 0.0005 | Capital Purchase Program + government bailout |
| Morgan Stanley | Feb 07 | Sep 10 | Jan 07 | Sep 10 | 0.0029 | 0.0013 | Capital Purchase Program |
| Goldman Sachs | Jan 07 | Sep 10 | Jan 07 | Sep 10 | 0.0026 | 0.0013 | Capital Purchase Program |
| Wells Fargo | Sep 07 | Sep 10 | Jan 07 | Sep 10 | 0.0040 | 0.0006 | Capital Purchase Program |

Table provides characteristic summary of each of the firms and their data range. Columns 2-5 describes the starting and ending month of data available for model estimation. Columns 6-7 outline option- and CDS-implied bankruptcy probabilities estimated on September 4, 2007, which is the earliest common start point of estimation. The last column describes ex-post knowledge on firm status during the financial crisis period.

Table 2.2: Descriptives of cross-sectional option data used to estimate the MLNbk model

| Panel A | | | | | | | | | | | | | | |
|-------------------|-----------------|-------|-------|-------|-------|-------|-------|-----|-------|--------|-----|-----|----------|----------------------|
| Company | 6-10 | 11-15 | 16-20 | 21-25 | 26-30 | 31-35 | 36-40 | 40+ | Mean | Median | Min | Max | Std.Dev. | Ave. life of options |
| | Lehman Brothers | 30 | 9 | 3 | 2 | 1 | 1 | 0 | 0 | 11.07 | 8.5 | 6 | 32 | 6.23 |
| Washington Mutual | 14 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 8.68 | 8 | 6 | 12 | 2.14 | 33.47 |
| Bear Stearns | 14 | 10 | 3 | 2 | 0 | 0 | 0 | 0 | 12.10 | 11 | 6 | 24 | 4.29 | 34.00 |
| AIG | 8 | 5 | 6 | 1 | 3 | 0 | 2 | 4 | 21.14 | 17 | 6 | 54 | 14.56 | 32.80 |
| Merrill Lynch | 9 | 9 | 2 | 4 | 0 | 0 | 0 | 0 | 13.13 | 12.5 | 6 | 24 | 5.36 | 34.04 |
| Wachovia | 12 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 8.76 | 8 | 6 | 15 | 2.82 | 34.29 |
| Bank of America | 17 | 15 | 5 | 0 | 0 | 0 | 0 | 0 | 11.73 | 11 | 7 | 19 | 3.40 | 34.29 |
| J.P. Morgan Chase | 7 | 14 | 8 | 6 | 5 | 0 | 0 | 0 | 16.28 | 14.5 | 6 | 30 | 6.45 | 34.30 |
| Citigroup | 20 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 9.25 | 8.5 | 6 | 15 | 2.56 | 34.39 |
| Morgan Stanley | 15 | 14 | 10 | 4 | 0 | 0 | 0 | 0 | 12.79 | 13 | 6 | 23 | 4.96 | 33.47 |
| Goldman Sachs | 4 | 6 | 9 | 11 | 7 | 4 | 3 | 1 | 23.18 | 22 | 9 | 55 | 9.34 | 34.20 |
| Wells Fargo | 11 | 14 | 9 | 3 | 0 | 0 | 0 | 0 | 13.32 | 14 | 6 | 22 | 4.76 | 34.12 |

| Panel B | | | | | | | |
|-------------------|--------|--------|--------|--------|--------|---------|--|
| | Q1 | Median | Q3 | Q4 | Mean | Std.Dev | |
| Lehman Brothers | 0.0014 | 0.0042 | 0.0085 | 0.1449 | 0.0113 | 0.0238 | |
| Washington Mutual | 0.0011 | 0.0018 | 0.0045 | 0.0167 | 0.0037 | 0.0043 | |
| Bear Stearns | 0.0057 | 0.0223 | 0.0395 | 0.1173 | 0.0296 | 0.0288 | |
| AIG | 0.0053 | 0.0236 | 0.1027 | 1.0680 | 0.1130 | 0.2158 | |
| Merrill Lynch | 0.0025 | 0.0095 | 0.0274 | 0.1167 | 0.0249 | 0.0329 | |
| Wachovia | 0.0003 | 0.0074 | 0.0077 | 0.0438 | 0.0018 | 0.0130 | |
| Bank of America | 0.0003 | 0.0013 | 0.0181 | 0.1500 | 0.0141 | 0.0293 | |
| J.P. Morgan Chase | 0.0010 | 0.0049 | 0.0153 | 0.1634 | 0.0151 | 0.0289 | |
| Citigroup | 0.0004 | 0.0011 | 0.0046 | 0.0473 | 0.0042 | 0.0091 | |
| Morgan Stanley | 0.0011 | 0.0032 | 0.0109 | 0.2898 | 0.0150 | 0.0449 | |
| Goldman Sachs | 0.0279 | 0.1127 | 2.5365 | 32.27 | 3.49 | 7.64 | |
| Wells Fargo | 0.0010 | 0.0031 | 0.0168 | 0.1428 | 0.0175 | 0.0335 | |

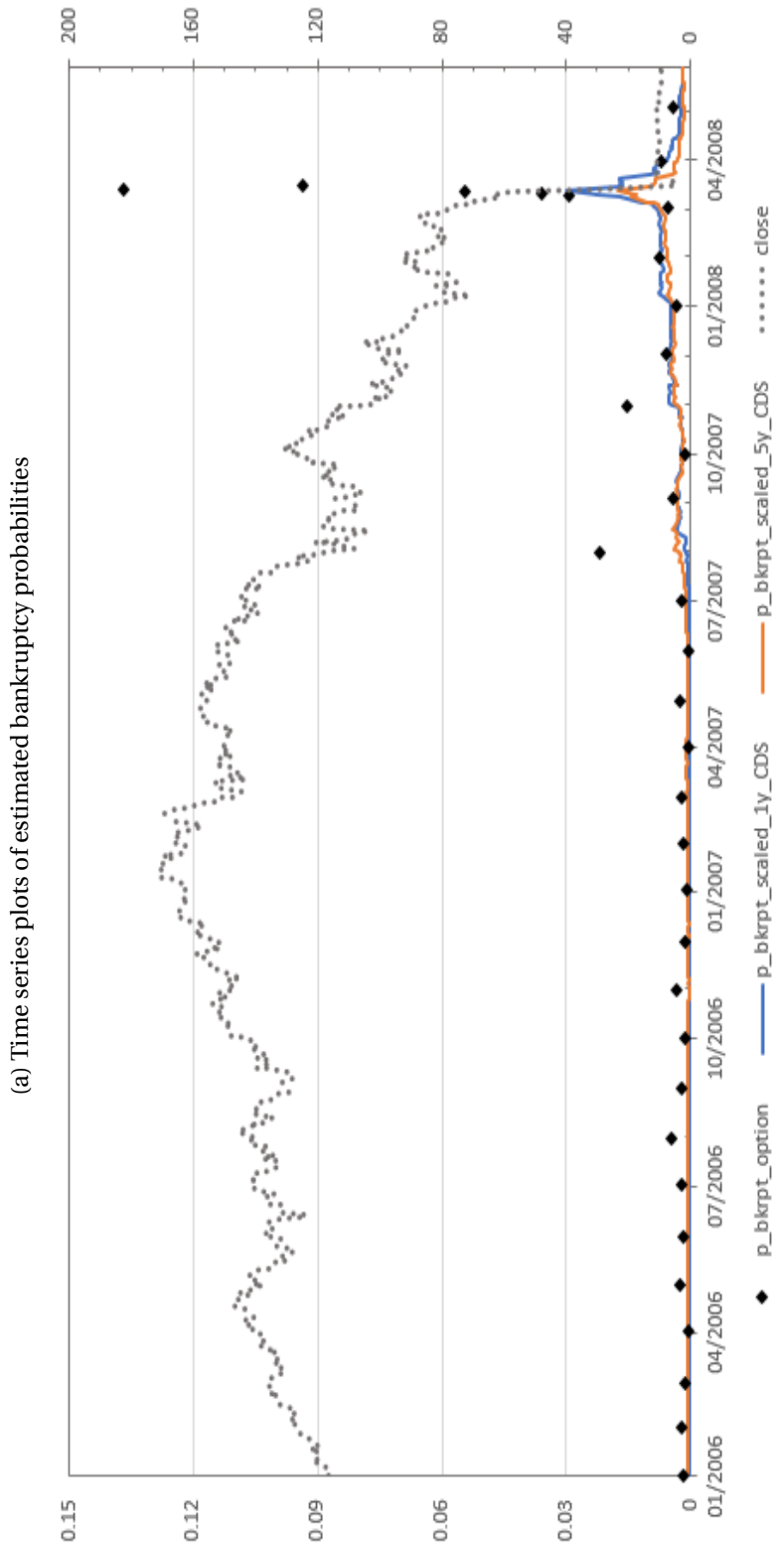
Table provides information on cross-sectional option data used to estimate the MLNbk model. Panel A details the number of options used in the range from 6 to 40+ for each firm for parameter estimation, and its summary statistics. Average life of options is reported as trading days. Panel B gives summary statistics for the G value defined by equation (2.5). Note that G values for Goldman Sachs are much higher than those for the rest of the sample. This is possibly due to GS's much higher stock price averaging \$165.14 across sample period (see Figure 2.15) and associated higher option prices. Since G values are proportional to option prices, higher option prices result in higher G values.

Table 2.3: Granger Causality tests for bankruptcy probabilities inferred from option prices and 1 year CDS spreads

| | Time series at level | | | | Time series at first difference | | | |
|-------------------|----------------------|-----------|------------|---------------|---------------------------------|-----------|------------|---------------|
| | Causality Direction | F -stat | p -value | Decision | Causality Direction | F -stat | p -value | Decision |
| J.P. Morgan Chase | $O \rightarrow C$ | 2.52 | 0.283 | Do not reject | $O \rightarrow C$ | 0.32 | 0.854 | Do not reject |
| | $C \rightarrow O$ | 1.68 | 0.433 | Do not reject | $C \rightarrow O$ | 0.94 | 0.626 | Do not reject |
| Bank of America | $O \rightarrow C$ | 34.78 | 0.000 | Reject | $O \rightarrow C$ | 18.24 | 0.000 | Reject |
| | $C \rightarrow O$ | 3.47 | 0.177 | Do not reject | $C \rightarrow O$ | 3.51 | 0.173 | Do not reject |
| Citigroup | $O \rightarrow C$ | 11.65 | 0.003 | Reject | $O \rightarrow C$ | 1.18 | 0.554 | Do not reject |
| | $C \rightarrow O$ | 8.78 | 0.012 | Reject at 2% | $C \rightarrow O$ | 4.55 | 0.103 | Do not reject |
| Goldman Sachs | $O \rightarrow C$ | 0.26 | 0.880 | Do not reject | $O \rightarrow C$ | 1.53 | 0.465 | Do not reject |
| | $C \rightarrow O$ | 10.95 | 0.004 | Reject | $C \rightarrow O$ | 53.79 | 0.000 | Reject |
| Morgan Stanley | $O \rightarrow C$ | 1.49 | 0.474 | Do not reject | $O \rightarrow C$ | 4.58 | 0.101 | Do not reject |
| | $C \rightarrow O$ | 2.41 | 0.300 | Do not reject | $C \rightarrow O$ | 0.71 | 0.700 | Do not reject |
| Wells Fargo | $O \rightarrow C$ | 13.77 | 0.001 | Reject | $O \rightarrow C$ | 1.12 | 0.572 | Do not reject |
| | $C \rightarrow O$ | 0.16 | 0.925 | Do not reject | $C \rightarrow O$ | 0.40 | 0.820 | Do not reject |

Table reports pairwise Granger Causality tests for bankruptcy probabilities inferred from option prices and 1 year CDS spreads. Tests are done for firms that are relatively healthy and survived the crisis. Tests are done for both level and first differenced time series. Causality directions are given as $O \rightarrow C$ ($C \rightarrow O$) for option (CDS) probabilities Granger Cause CDS (option) probabilities. 2 lags are included for the tests and F -statistic and p -values are reported in the table.

Figure 2.1: Bankruptcy probabilities inferred from option prices and CDS spreads for Bear Stearns



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

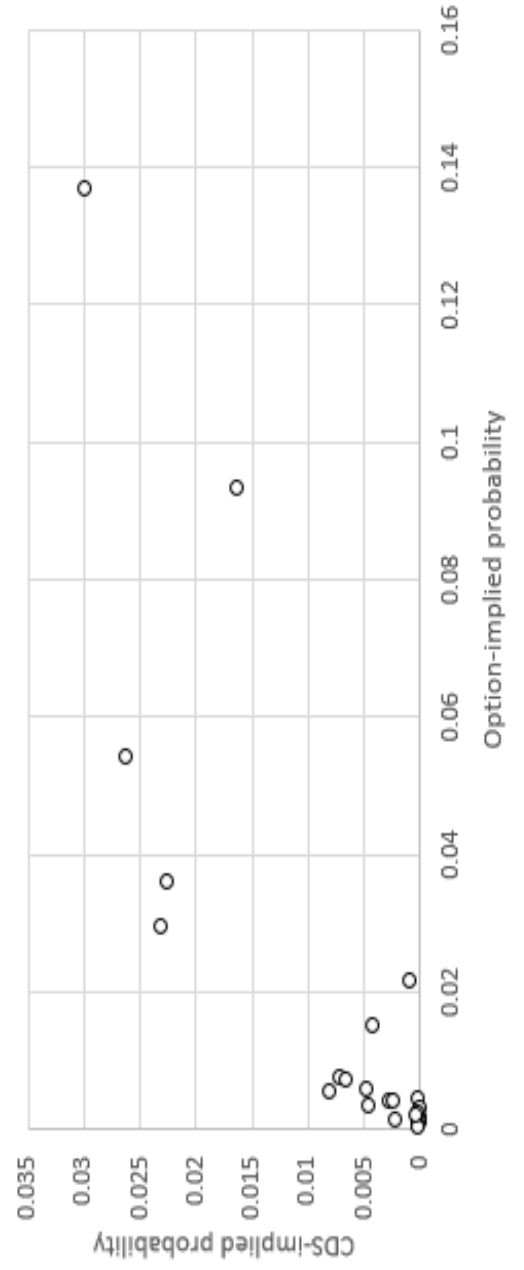


Figure 2.1: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Bear Stearns

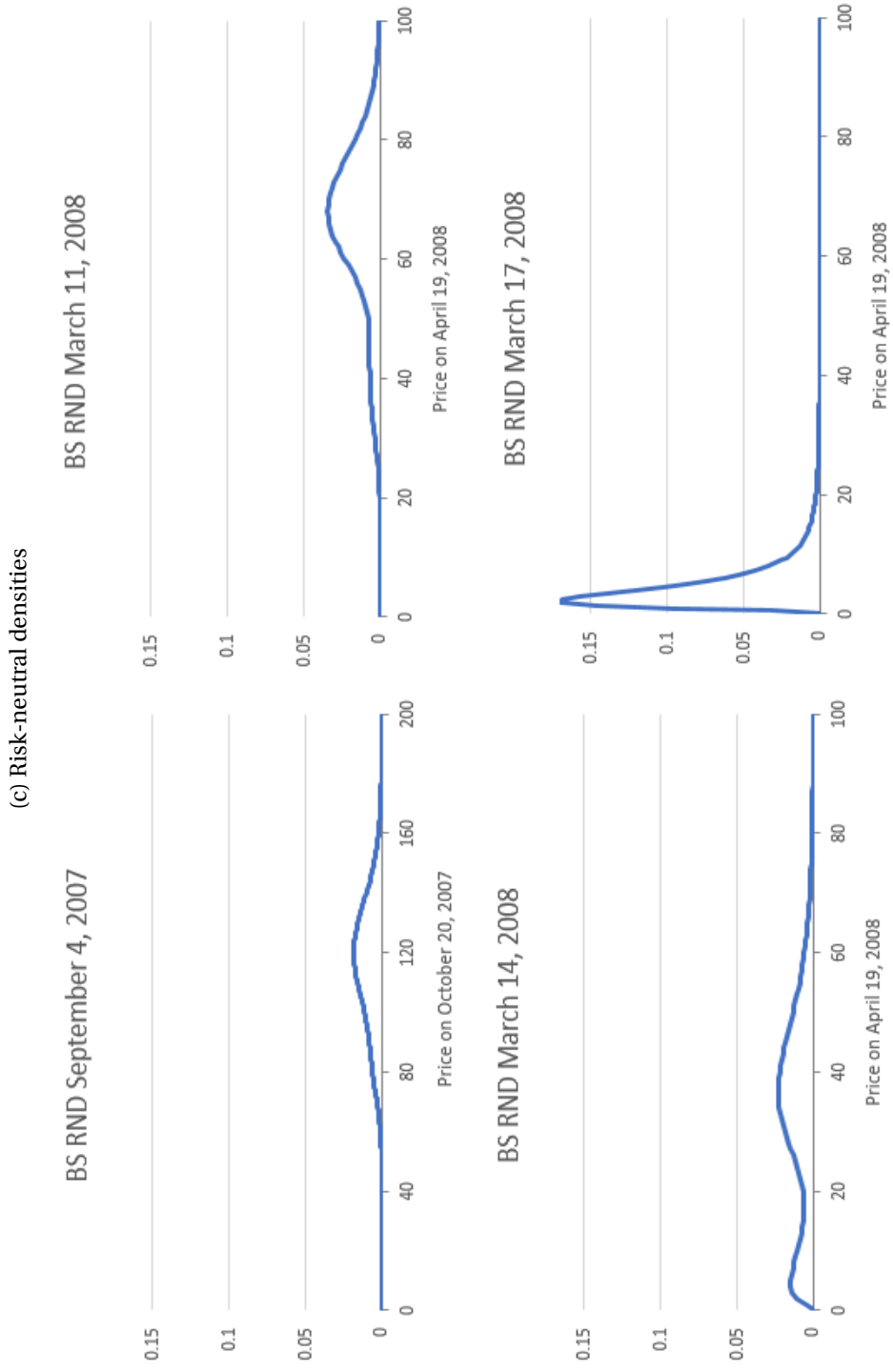
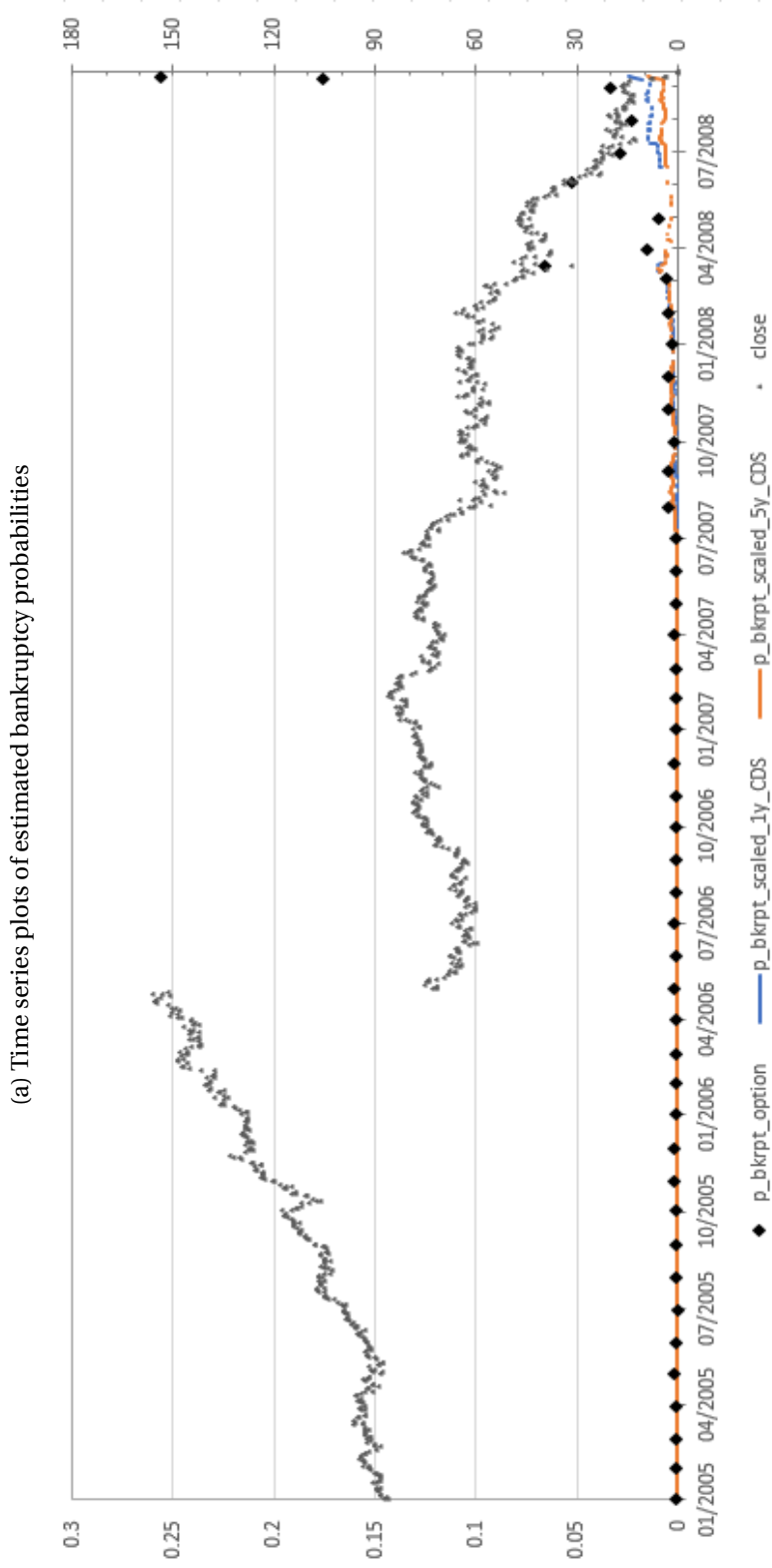


Figure 2.2: Bankruptcy probabilities inferred from option prices and CDS spreads for Lehman Brothers



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

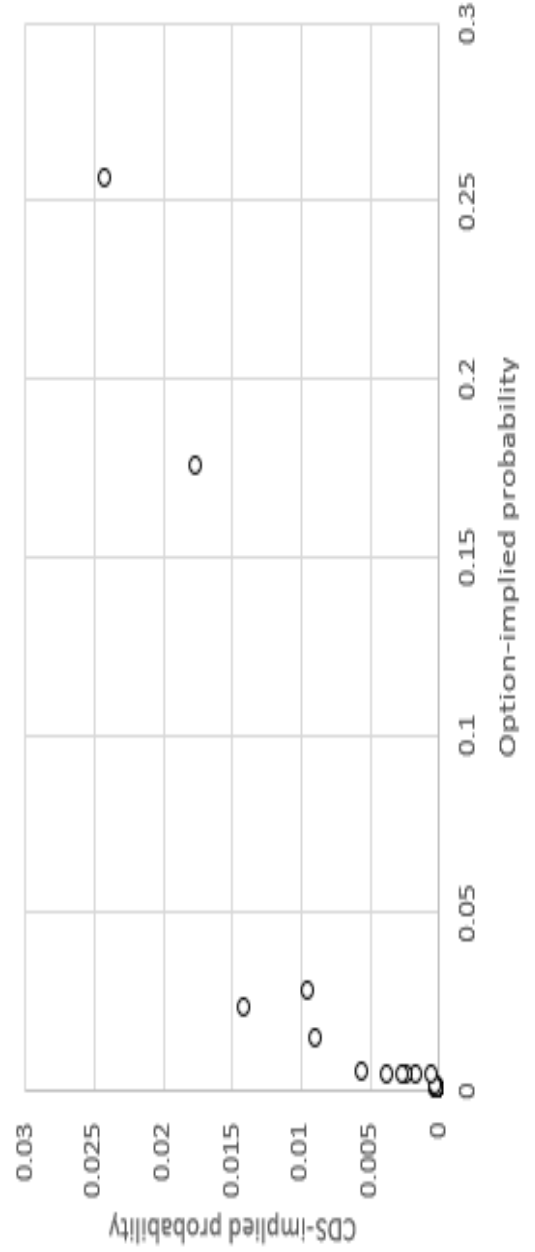


Figure 2.2: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Lehman Brothers

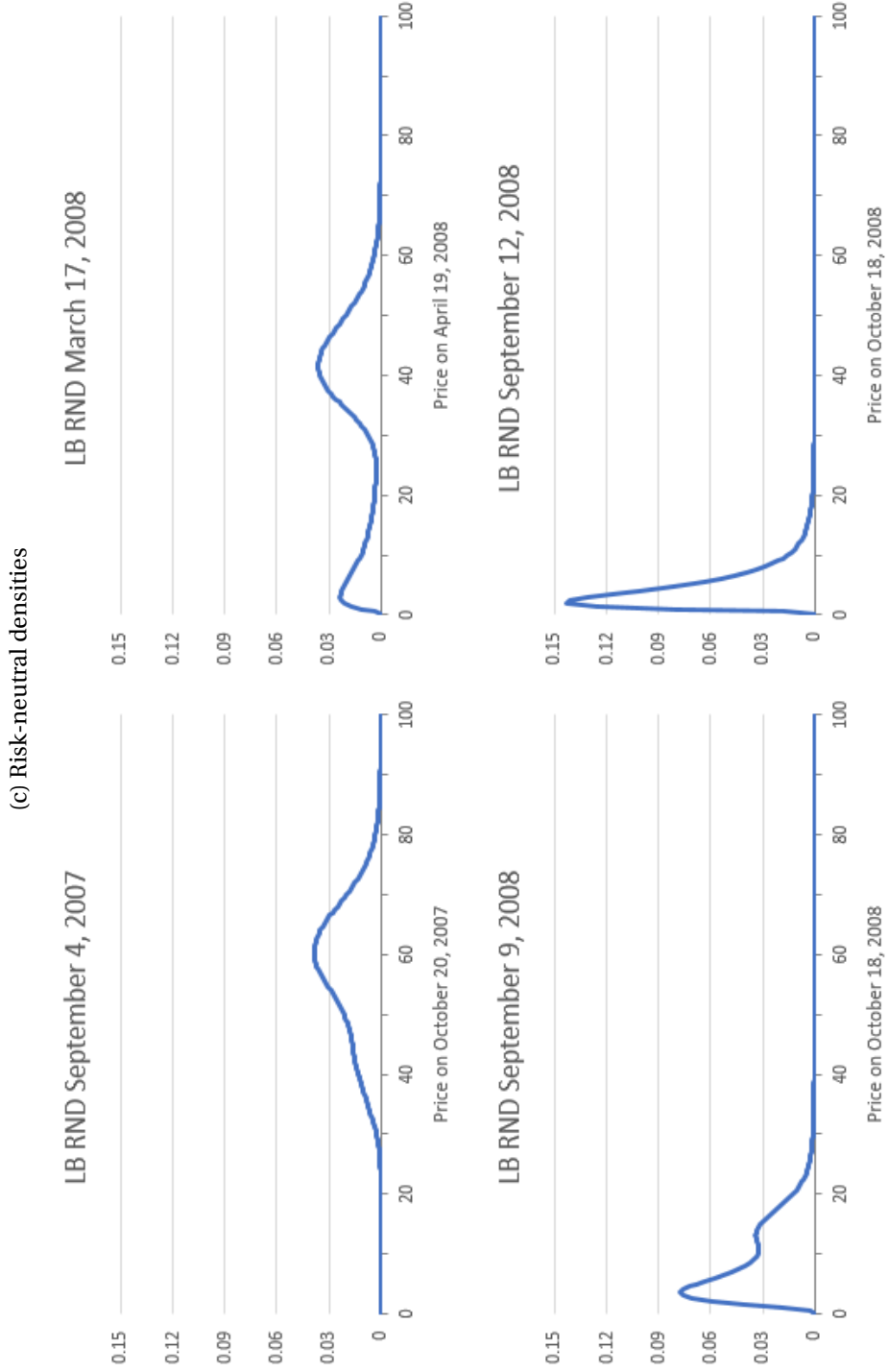
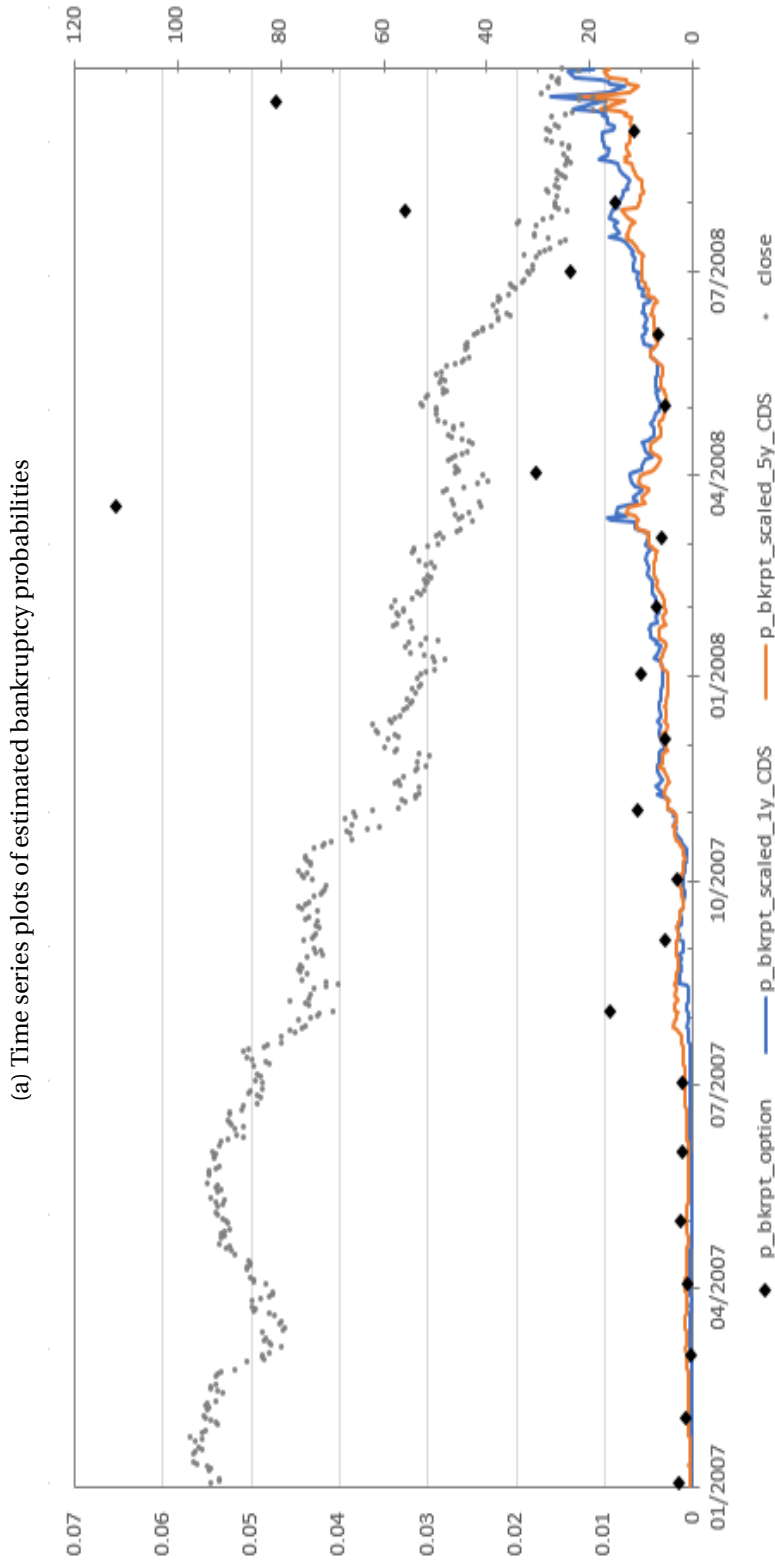


Figure 2.3: Bankruptcy probabilities inferred from option prices and CDS spreads for Merrill Lynch



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

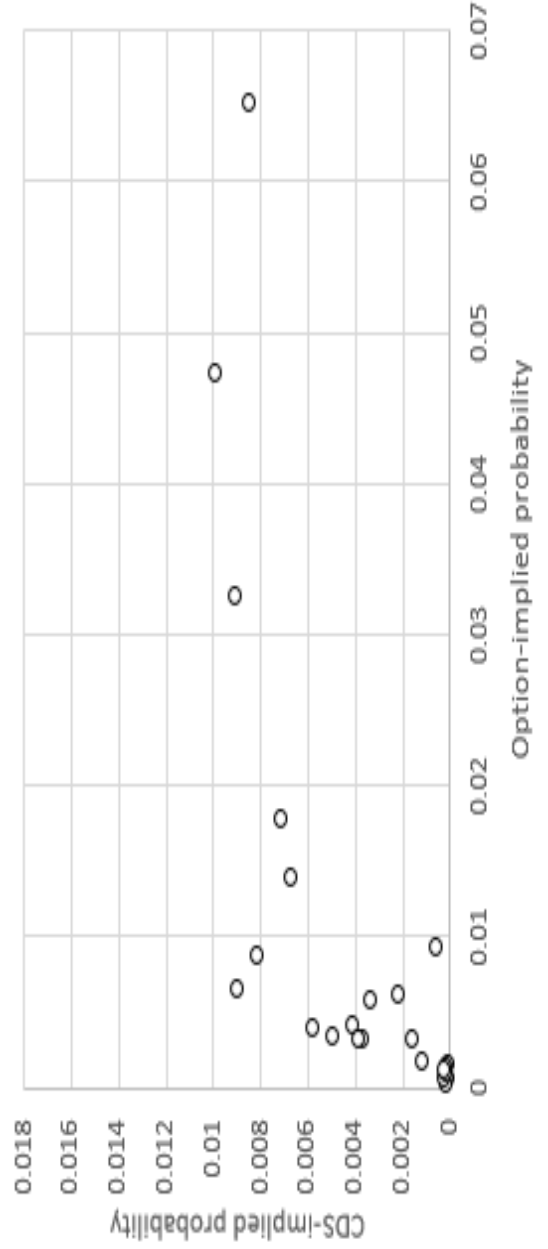


Figure 2.3: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Merrill Lynch

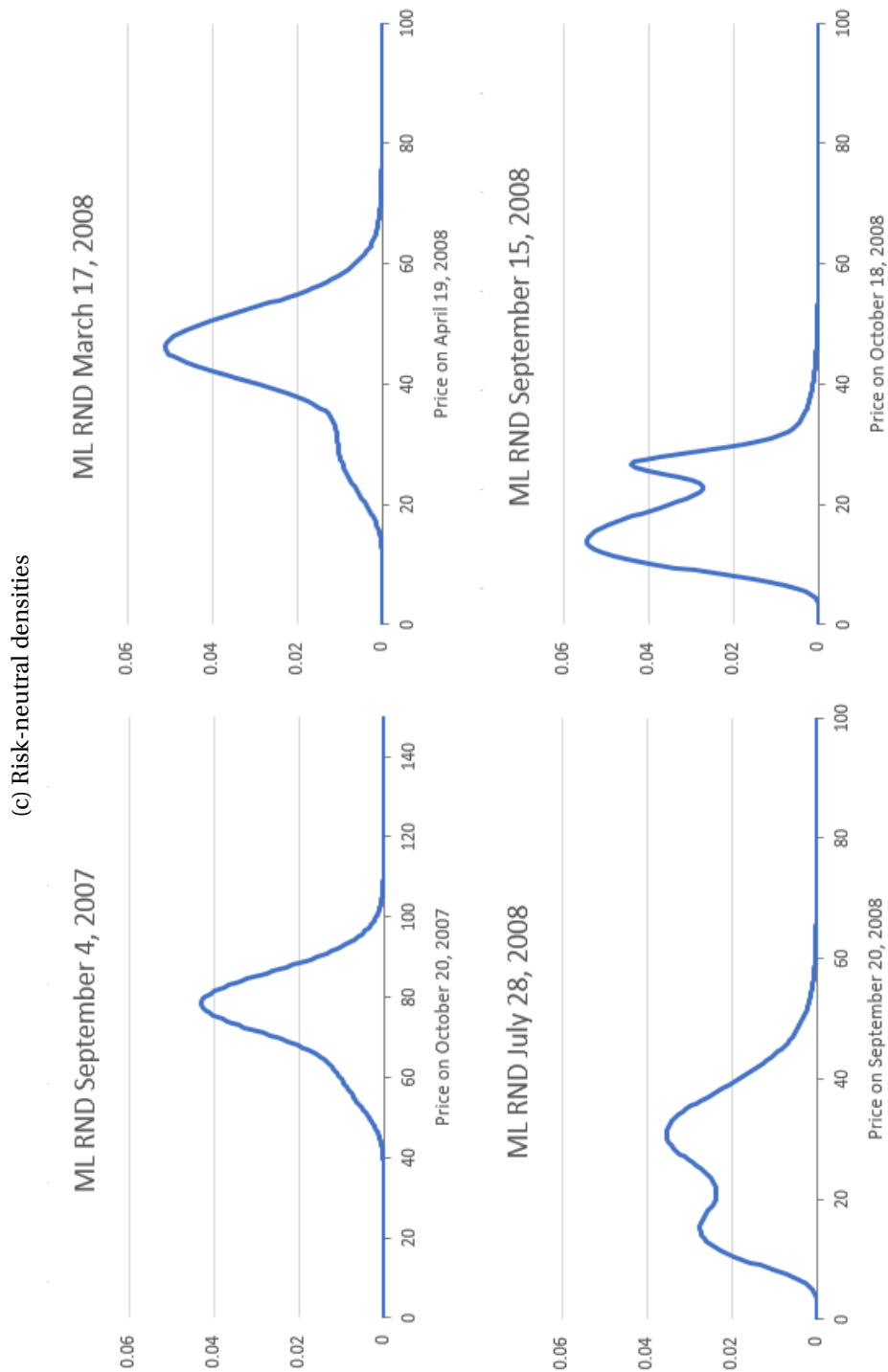
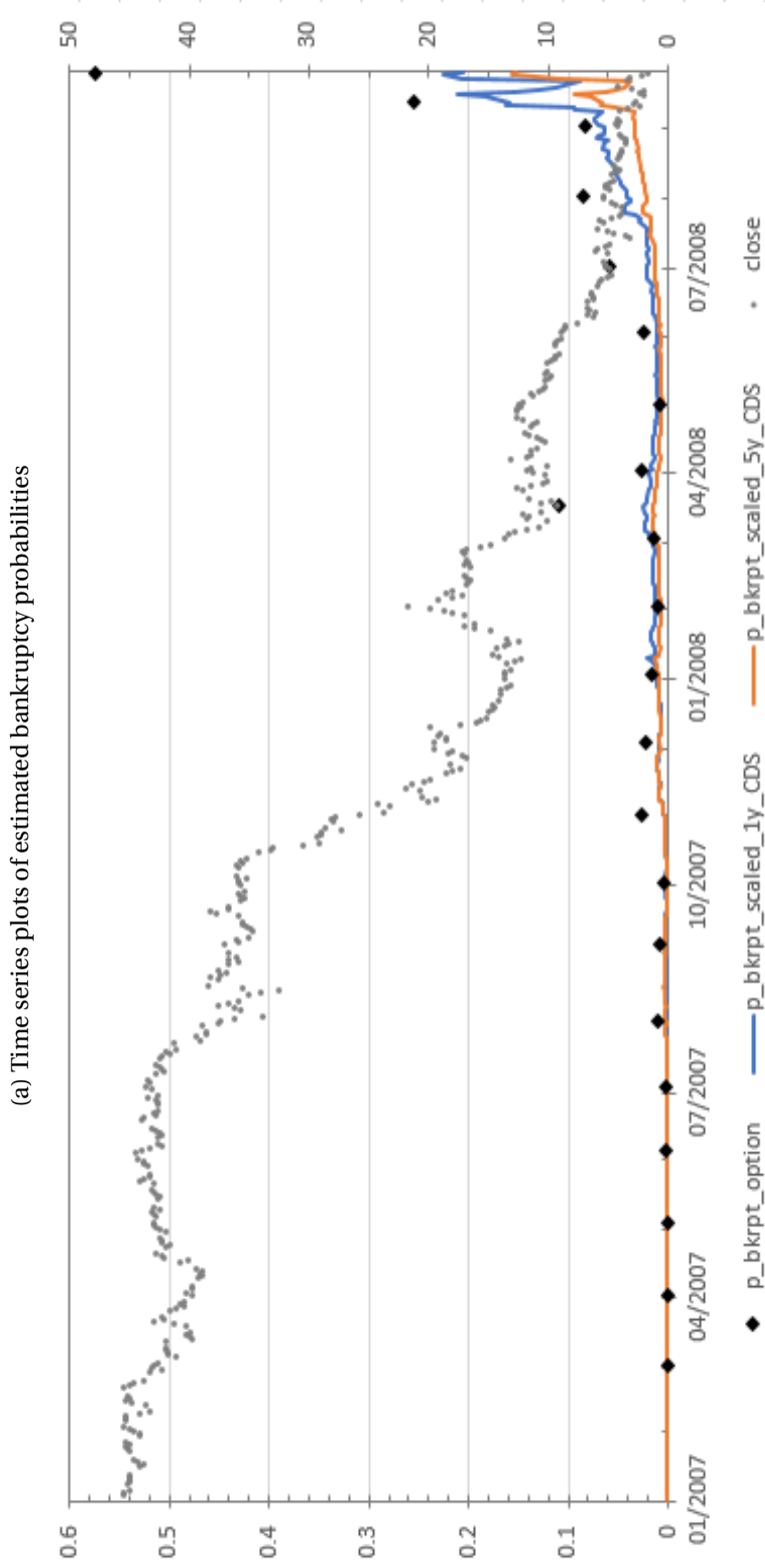


Figure 2.4: Bankruptcy probabilities inferred from option prices and CDS spreads for Washington Mutual



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

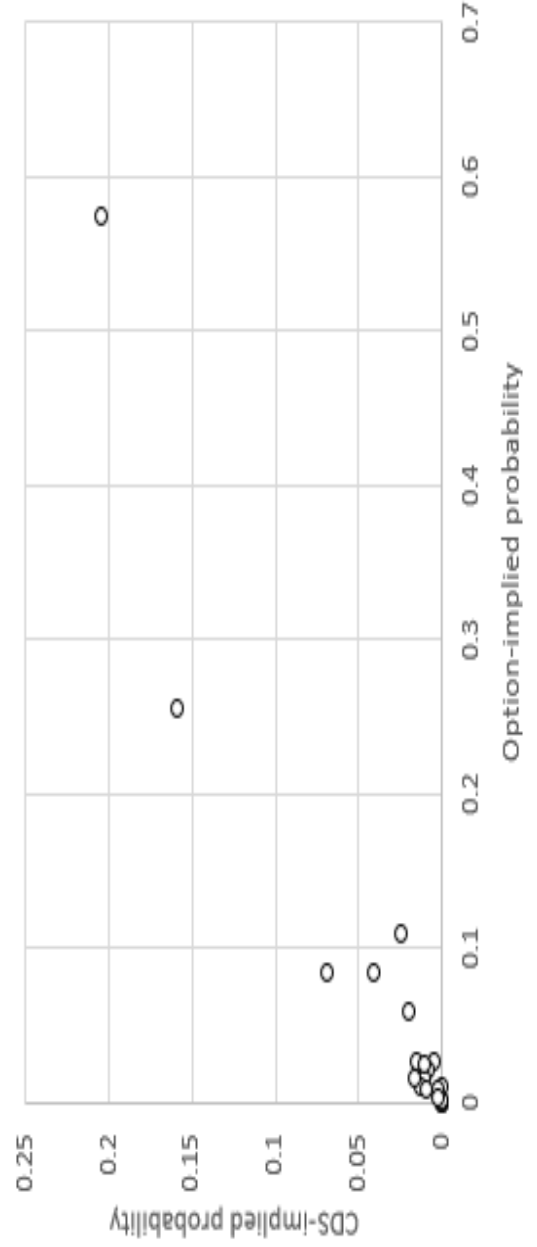


Figure 2.4: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Washington Mutual

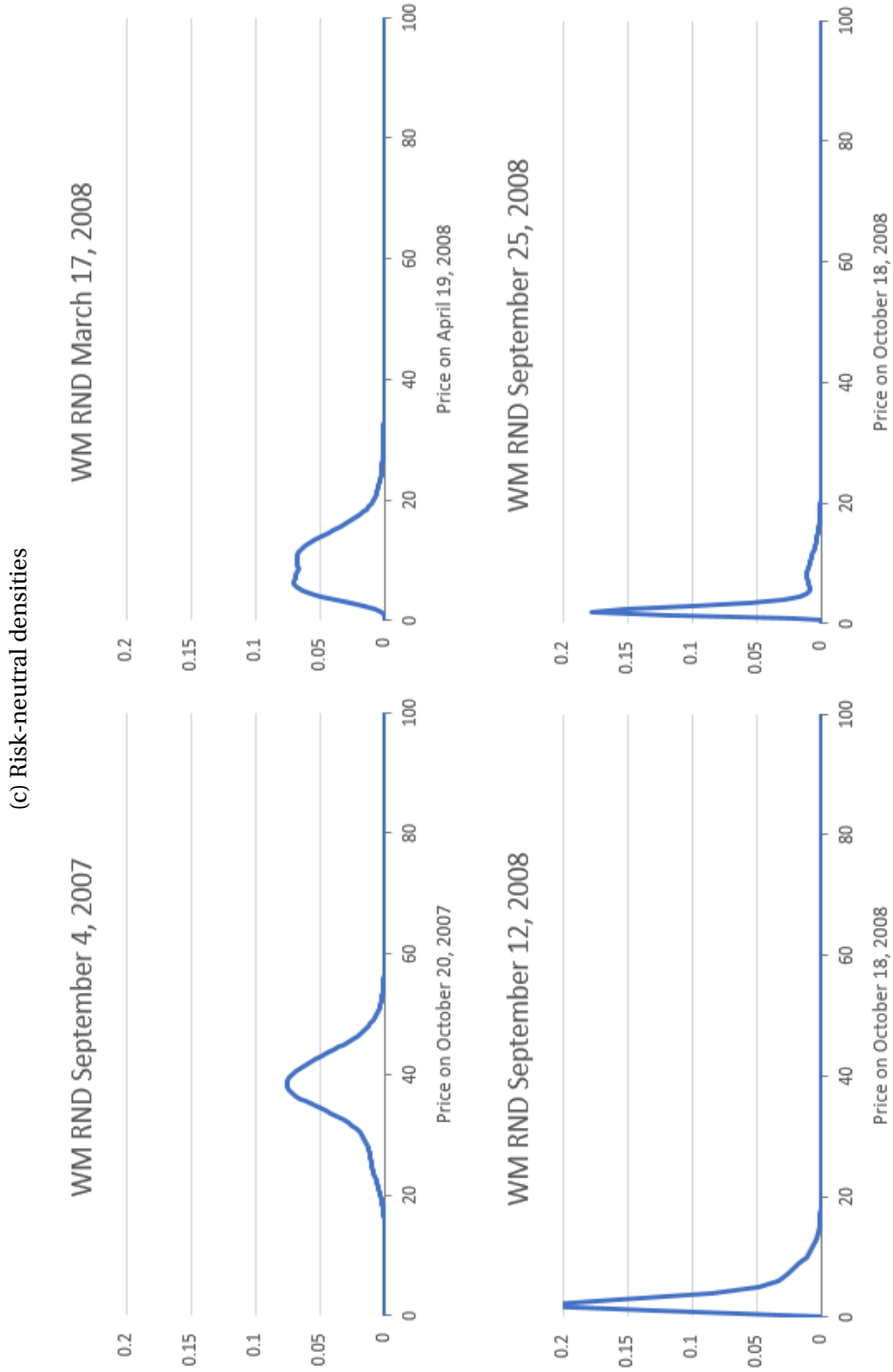
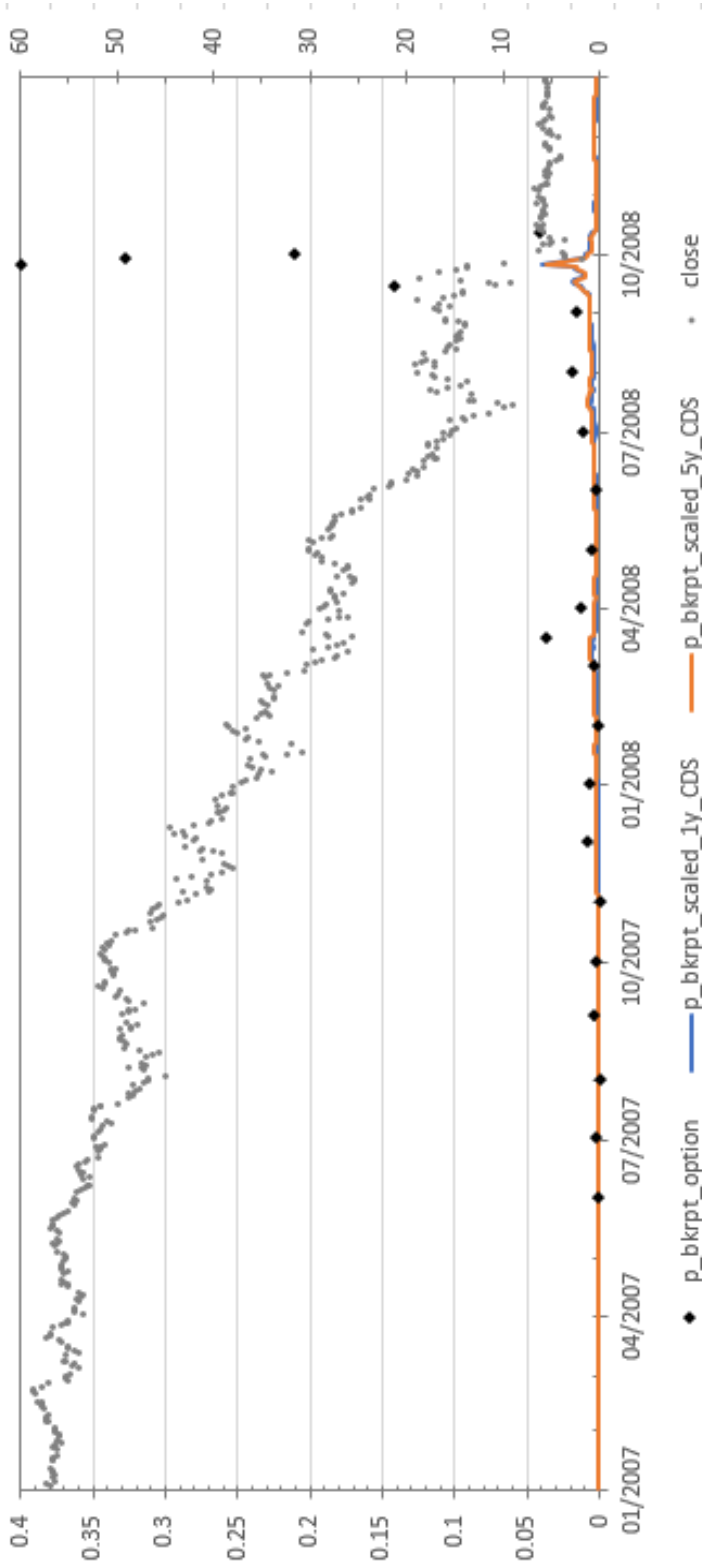


Figure 2.5: Bankruptcy probabilities inferred from option prices and CDS spreads for Wachovia

(a) Time series plots of estimated bankruptcy probabilities



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

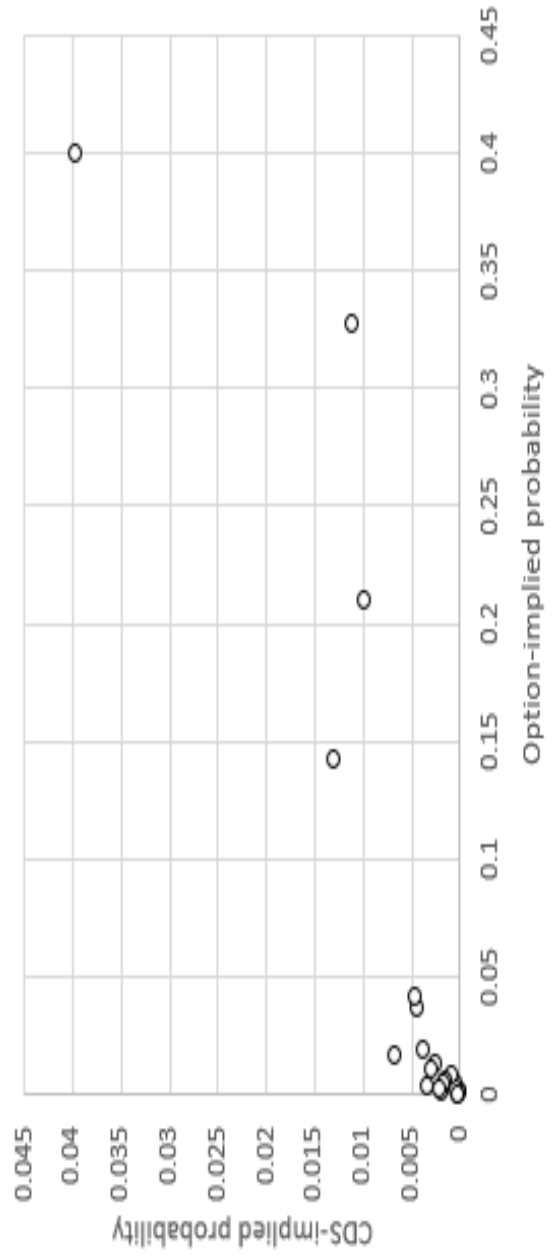


Figure 2.5: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Wachovia

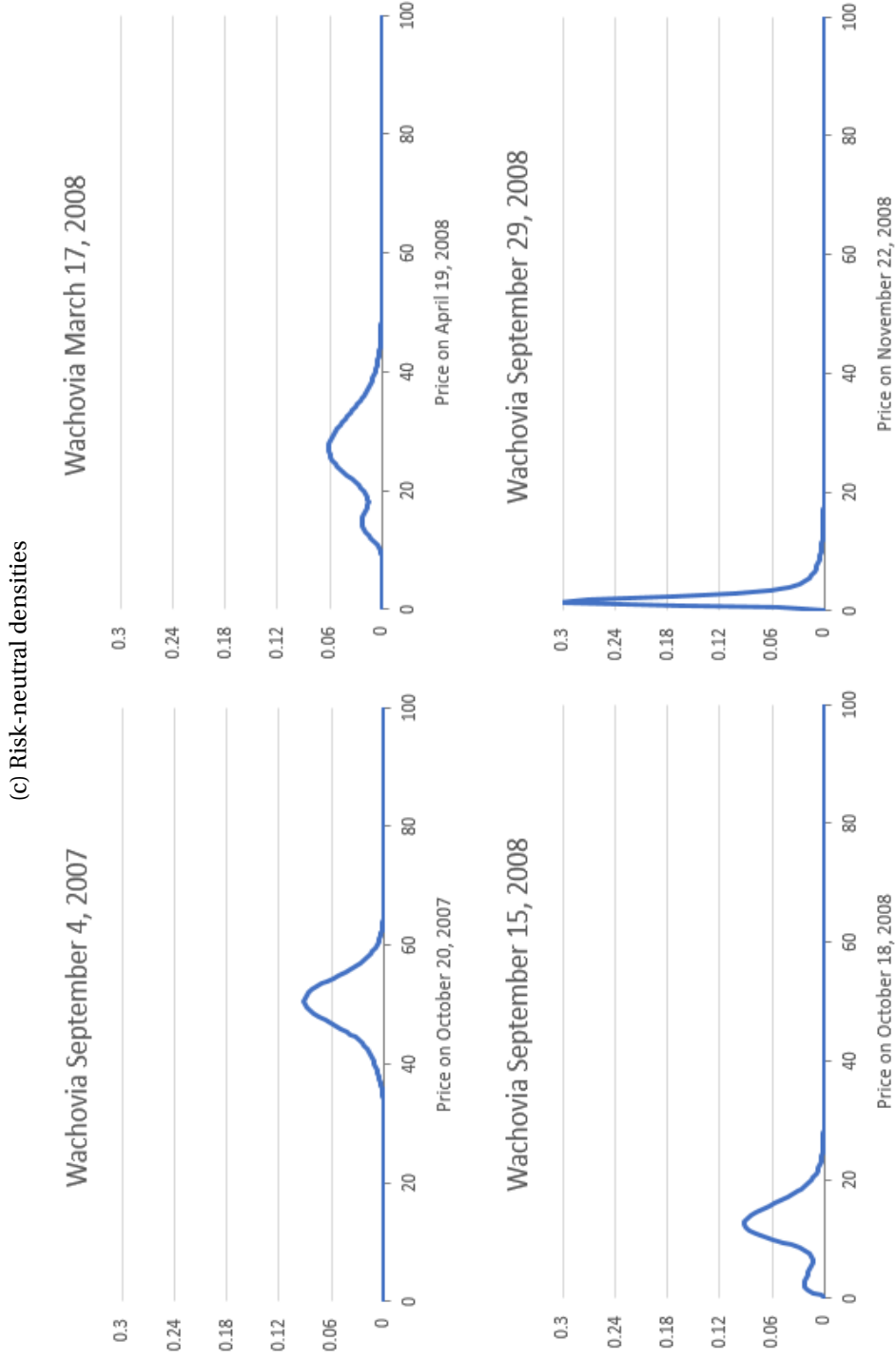
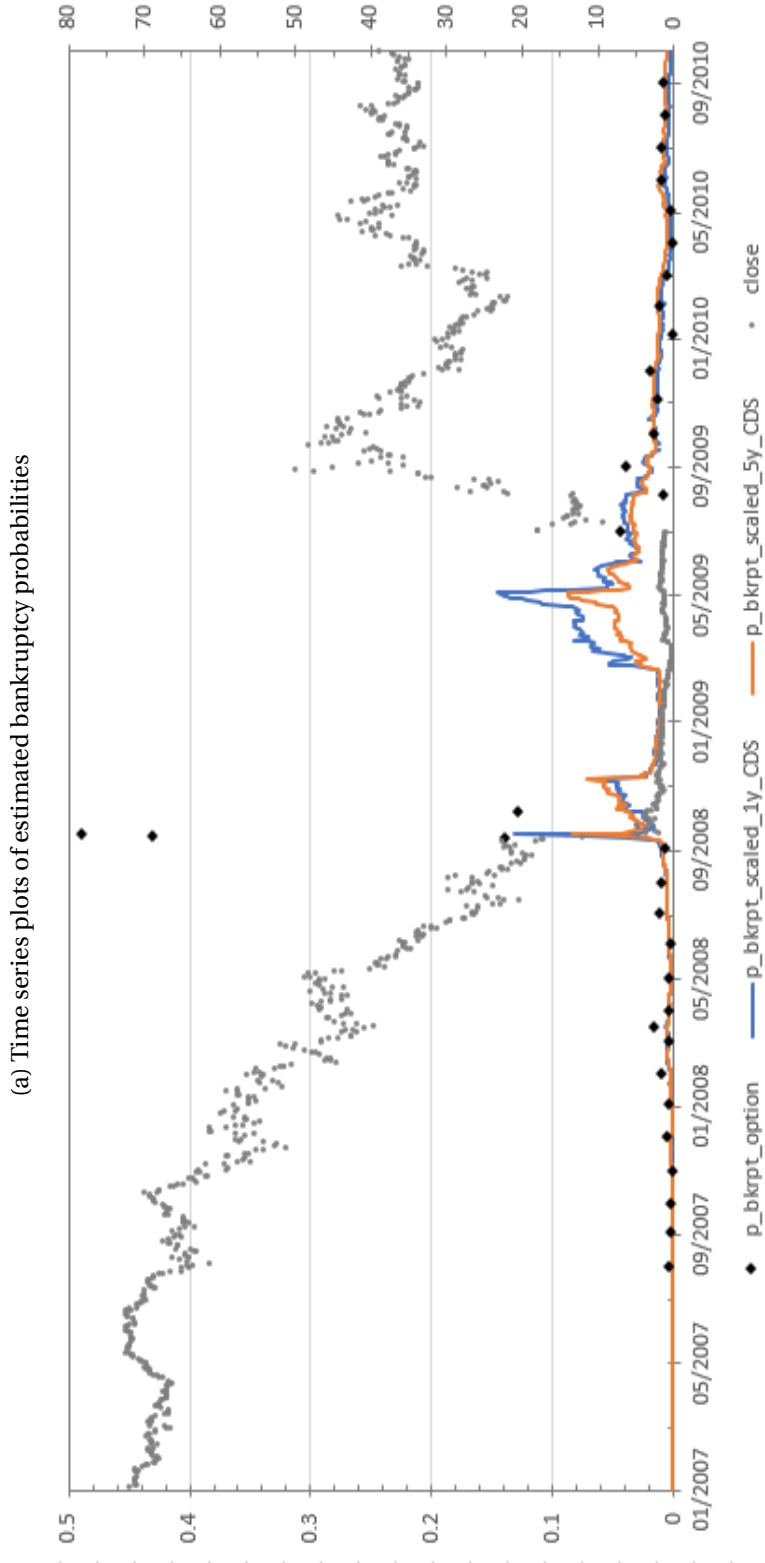


Figure 2.6: Bankruptcy probabilities inferred from option prices and CDS spreads for AIG



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

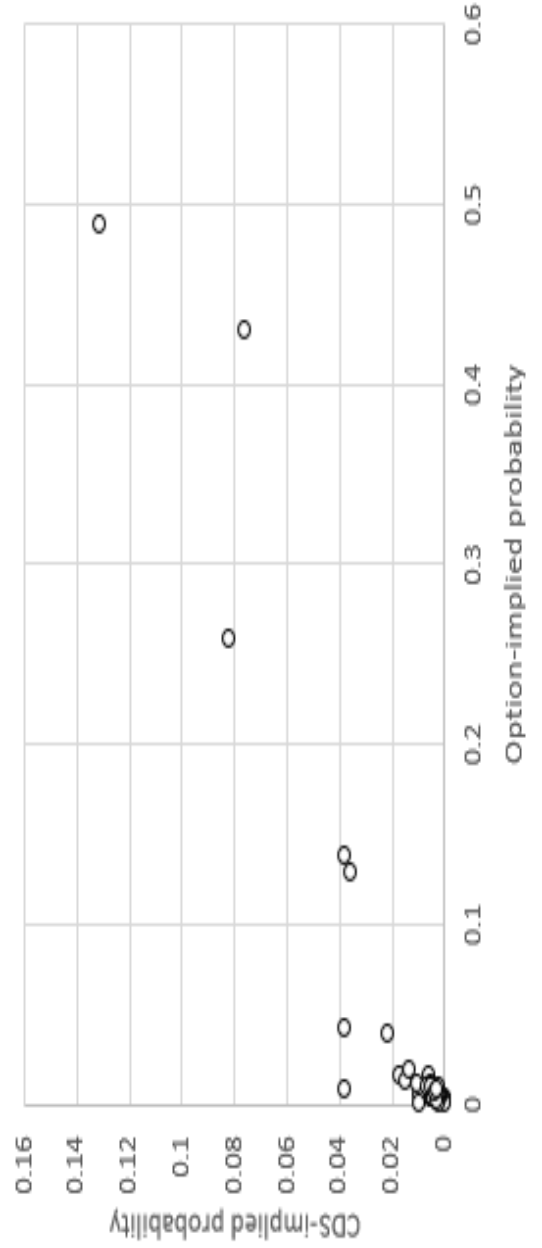


Figure 2.6: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for AIG

(c) Risk-neutral densities

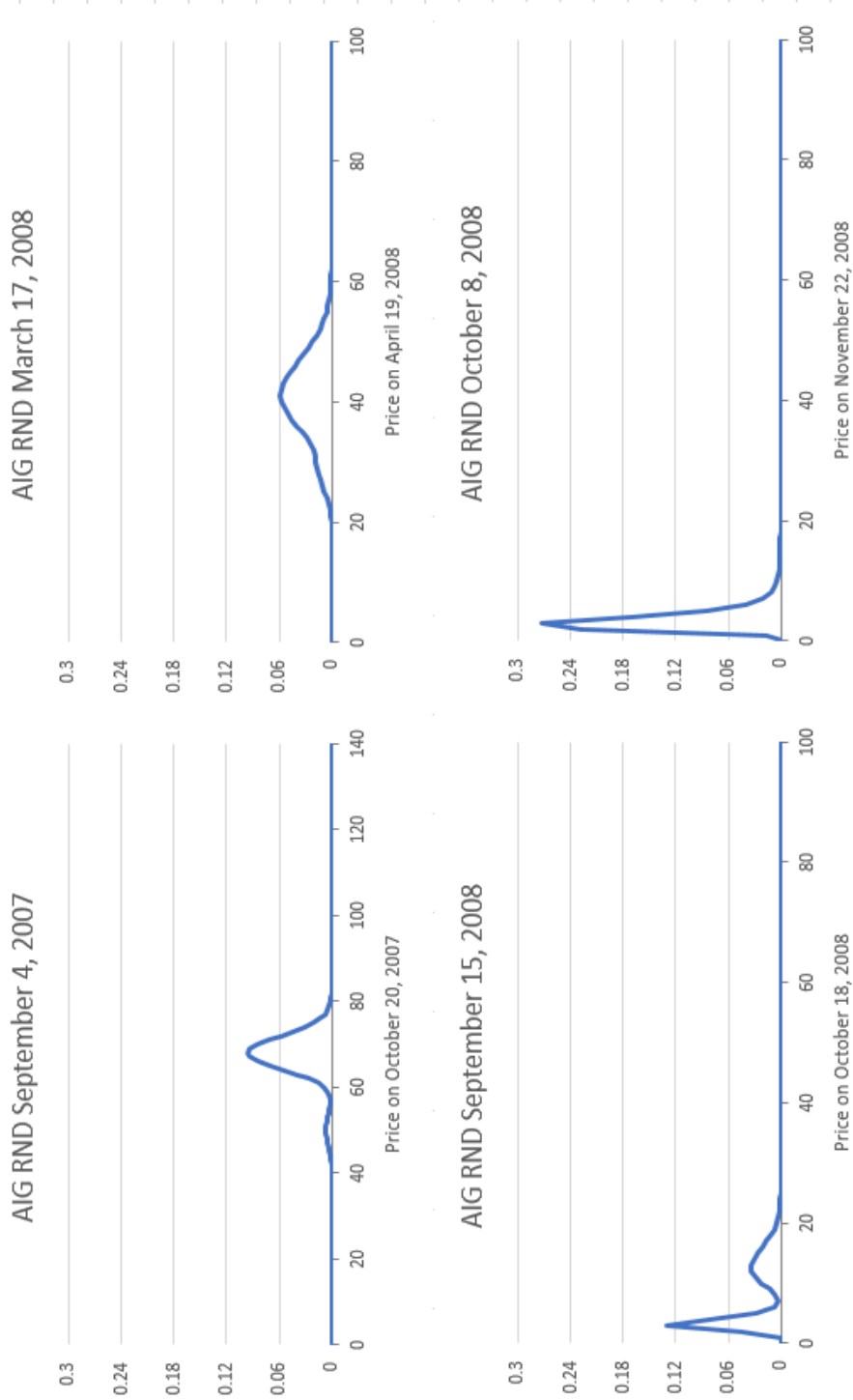
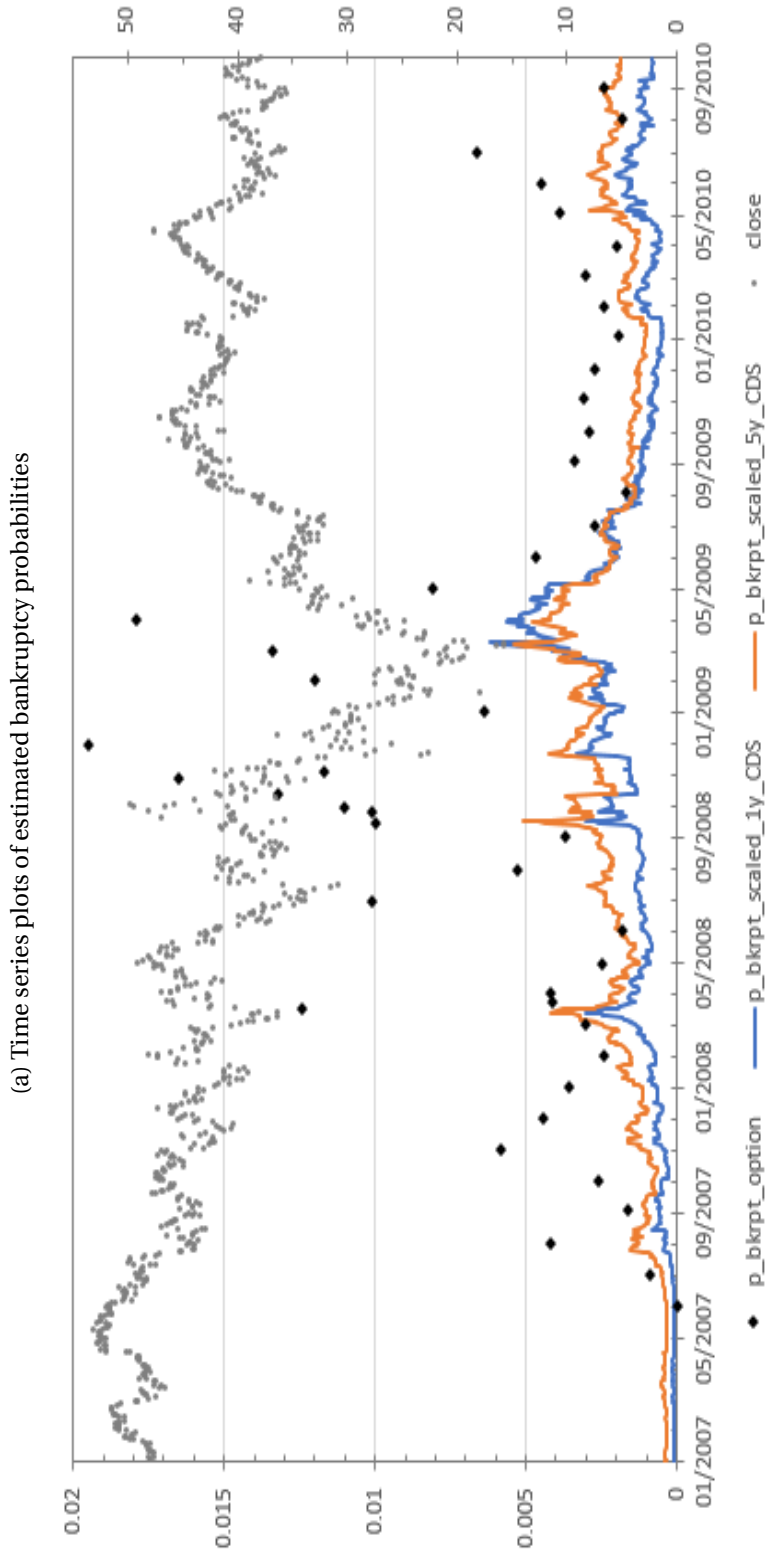


Figure 2.7: Bankruptcy probabilities inferred from option prices and CDS spreads for J.P. Morgan Chase



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

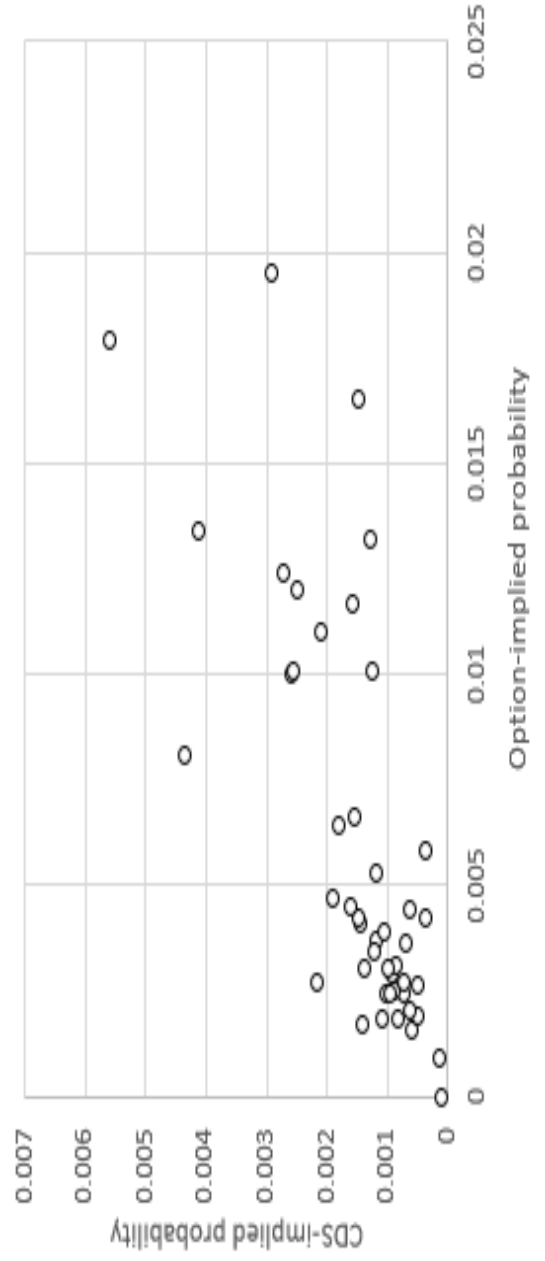


Figure 2.7: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for J.P. Morgan Chase

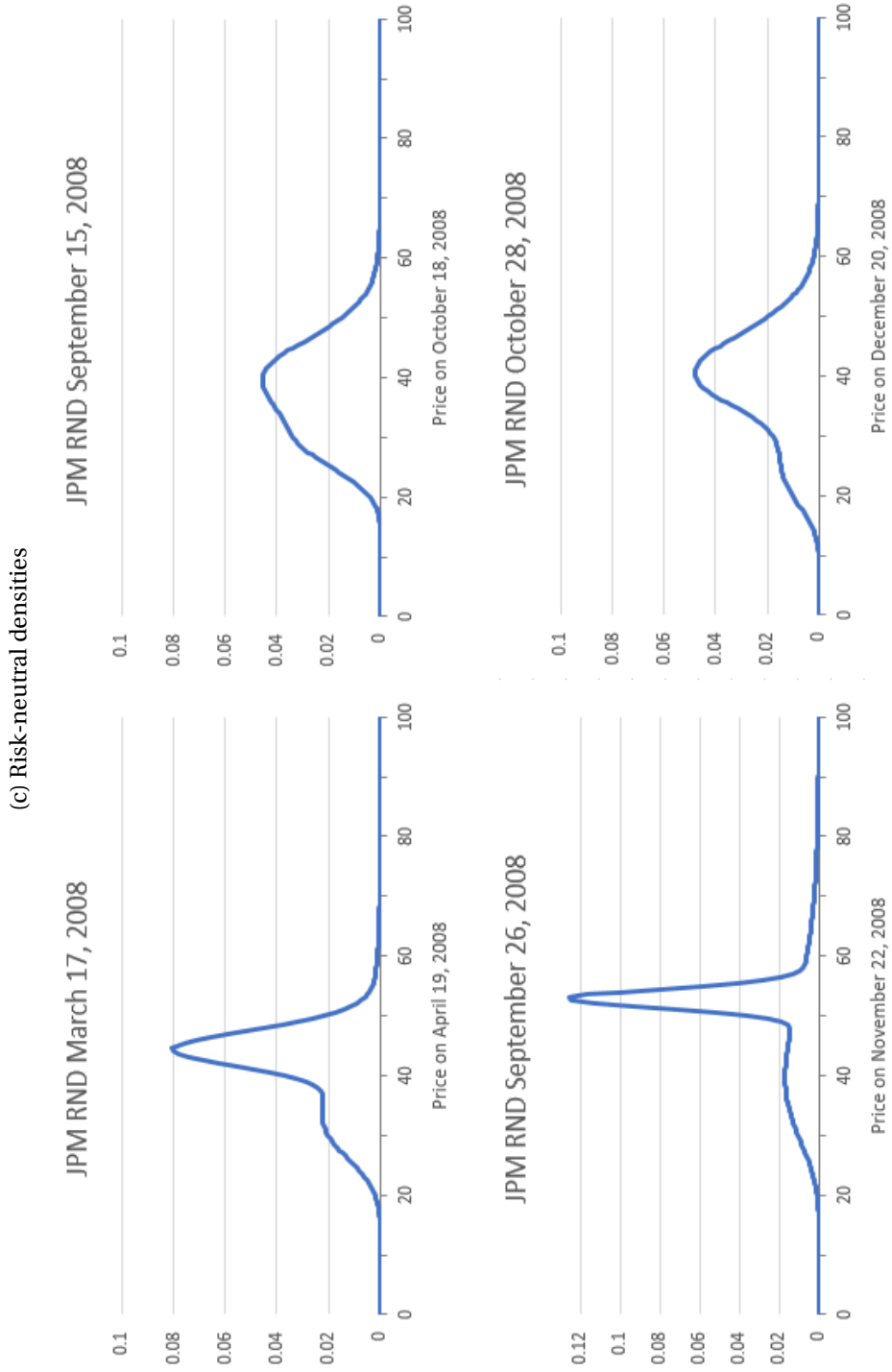
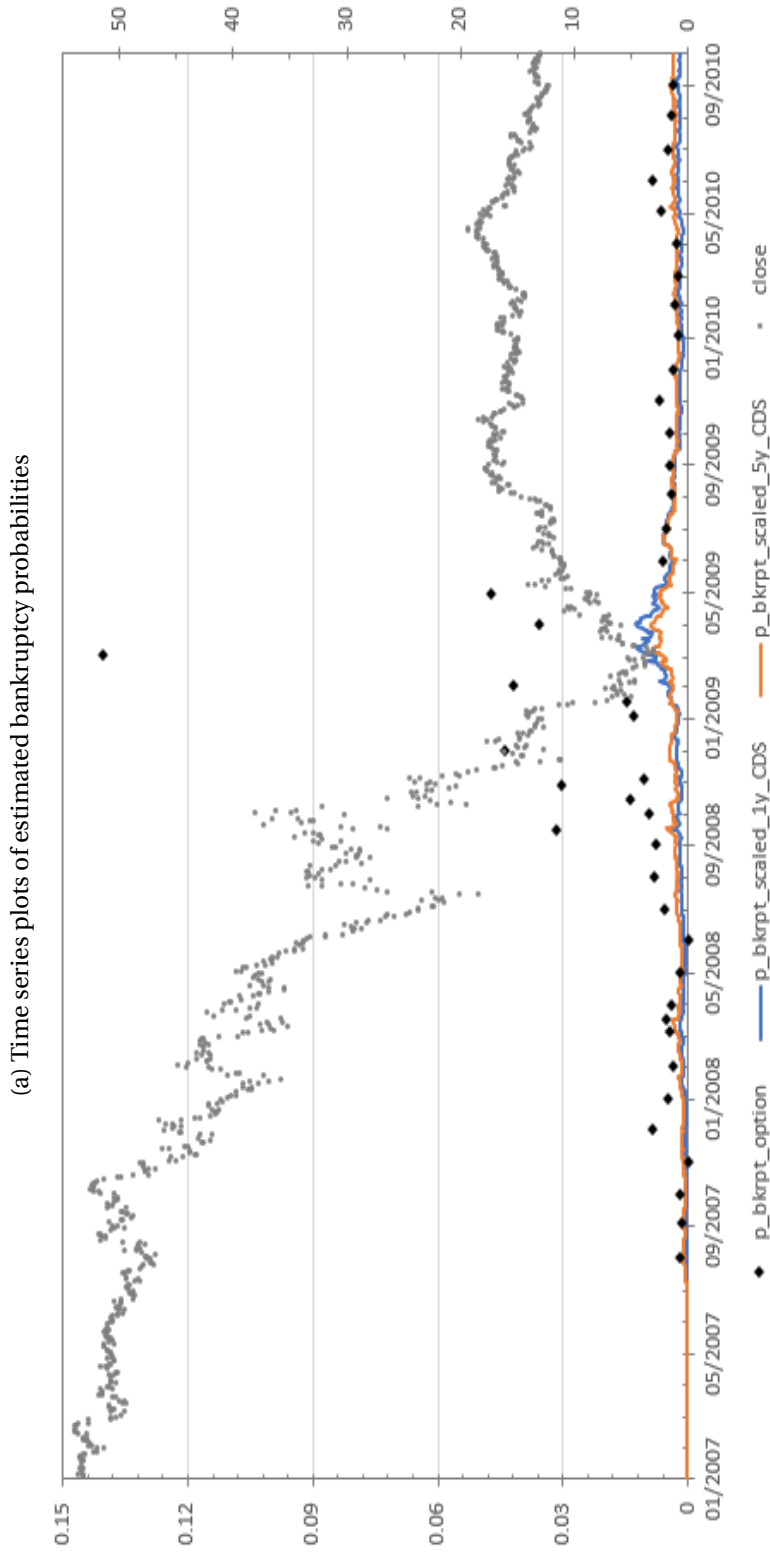


Figure 2.8: Bankruptcy probabilities inferred from option prices and CDS spreads for Bank of America



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

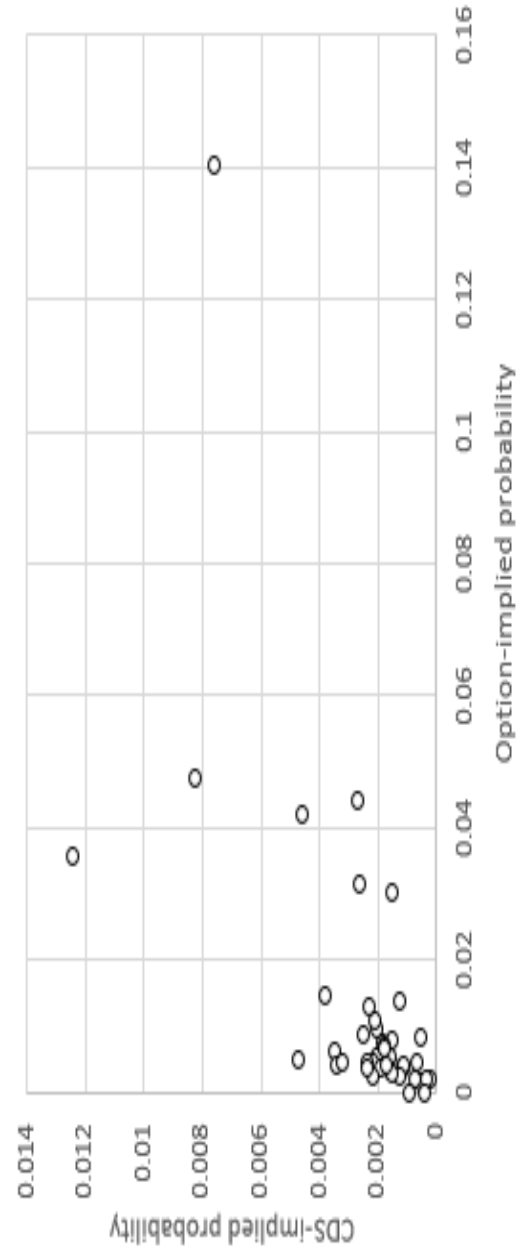


Figure 2.8: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Bank of America

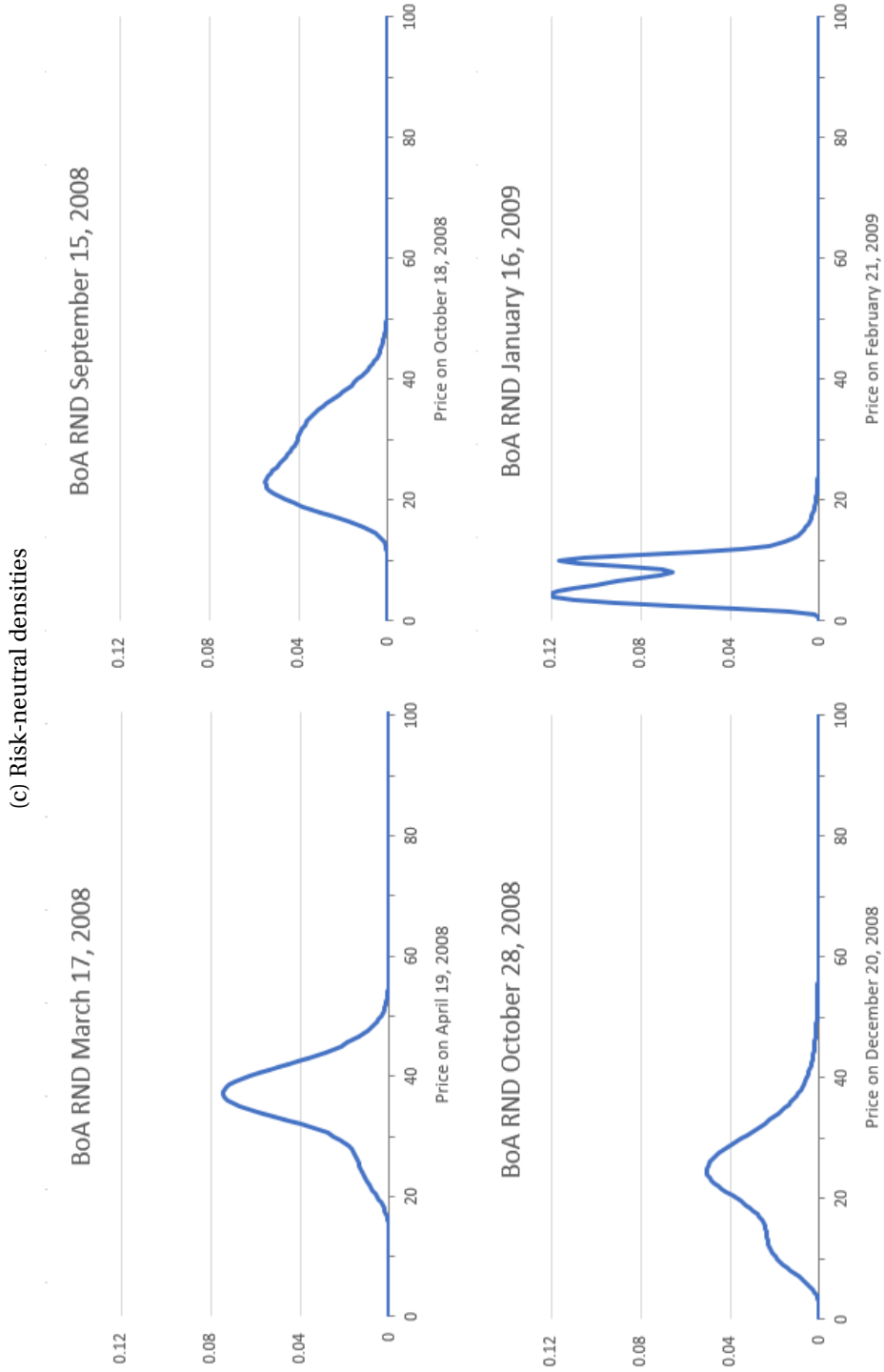
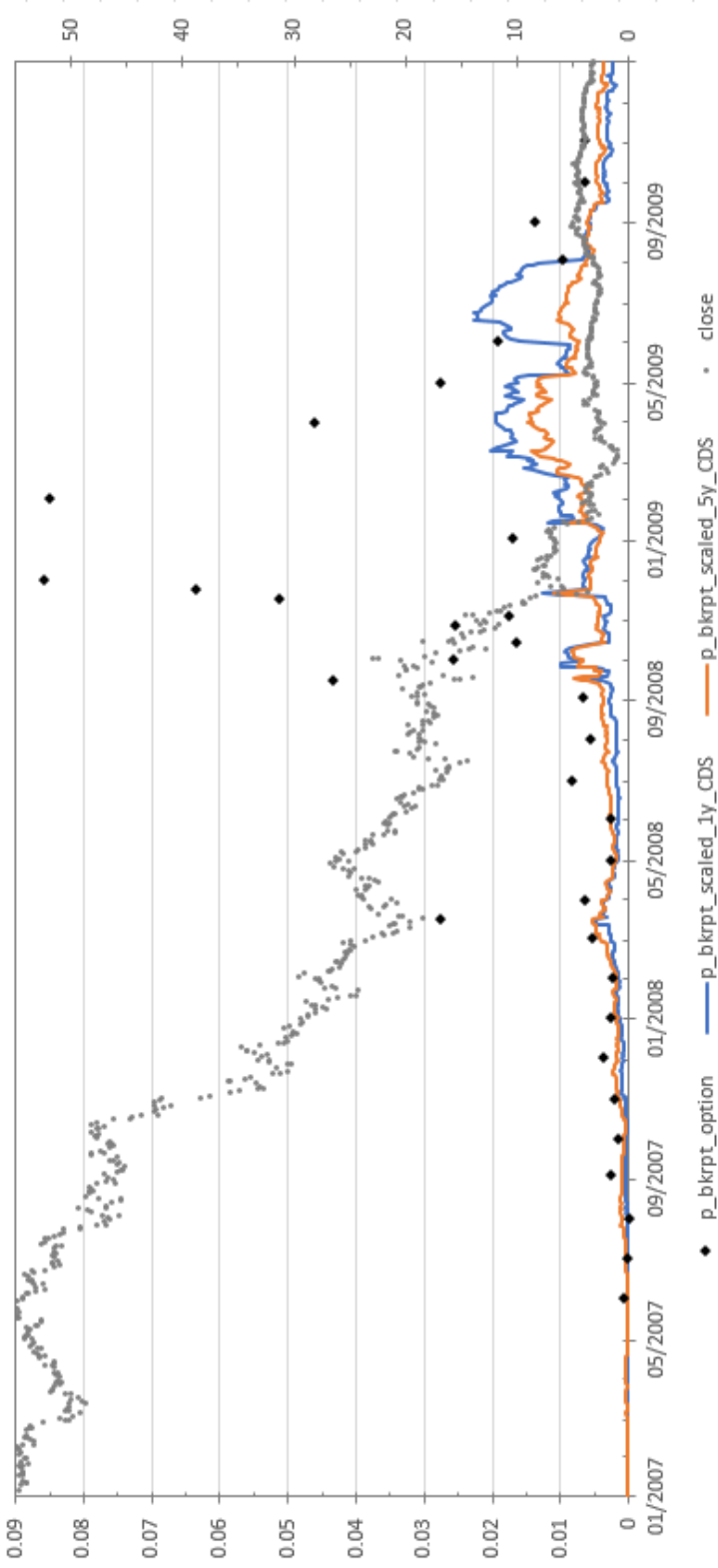


Figure 2.9: Bankruptcy probabilities inferred from option prices and CDS spreads for Citigroup

(a) Time series plots of estimated bankruptcy probabilities



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

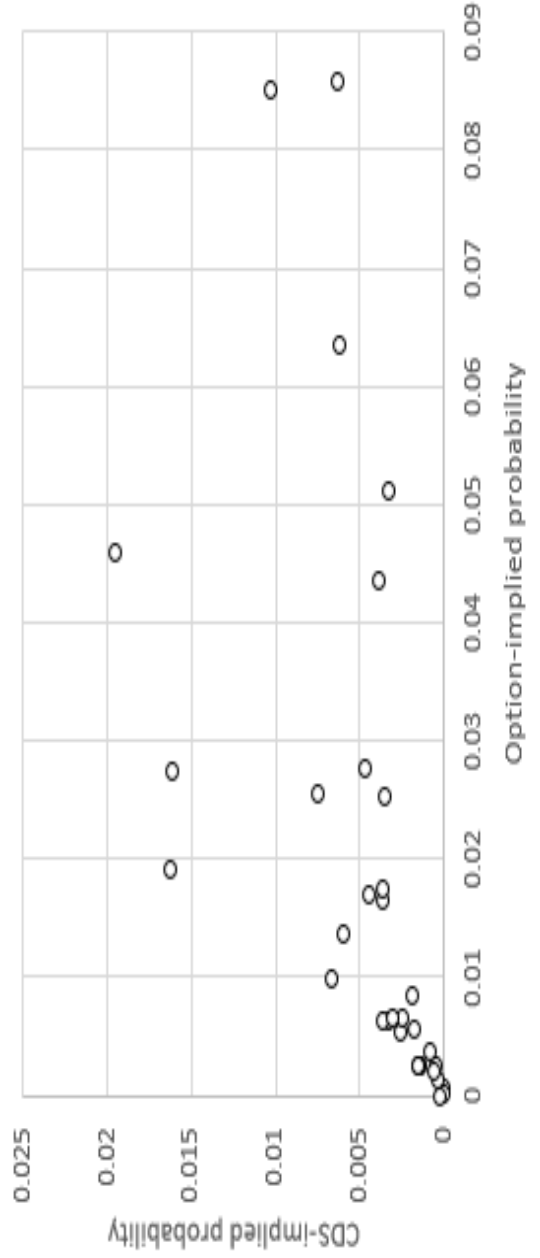


Figure 2.9: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Citigroup

(c) Risk-neutral densities

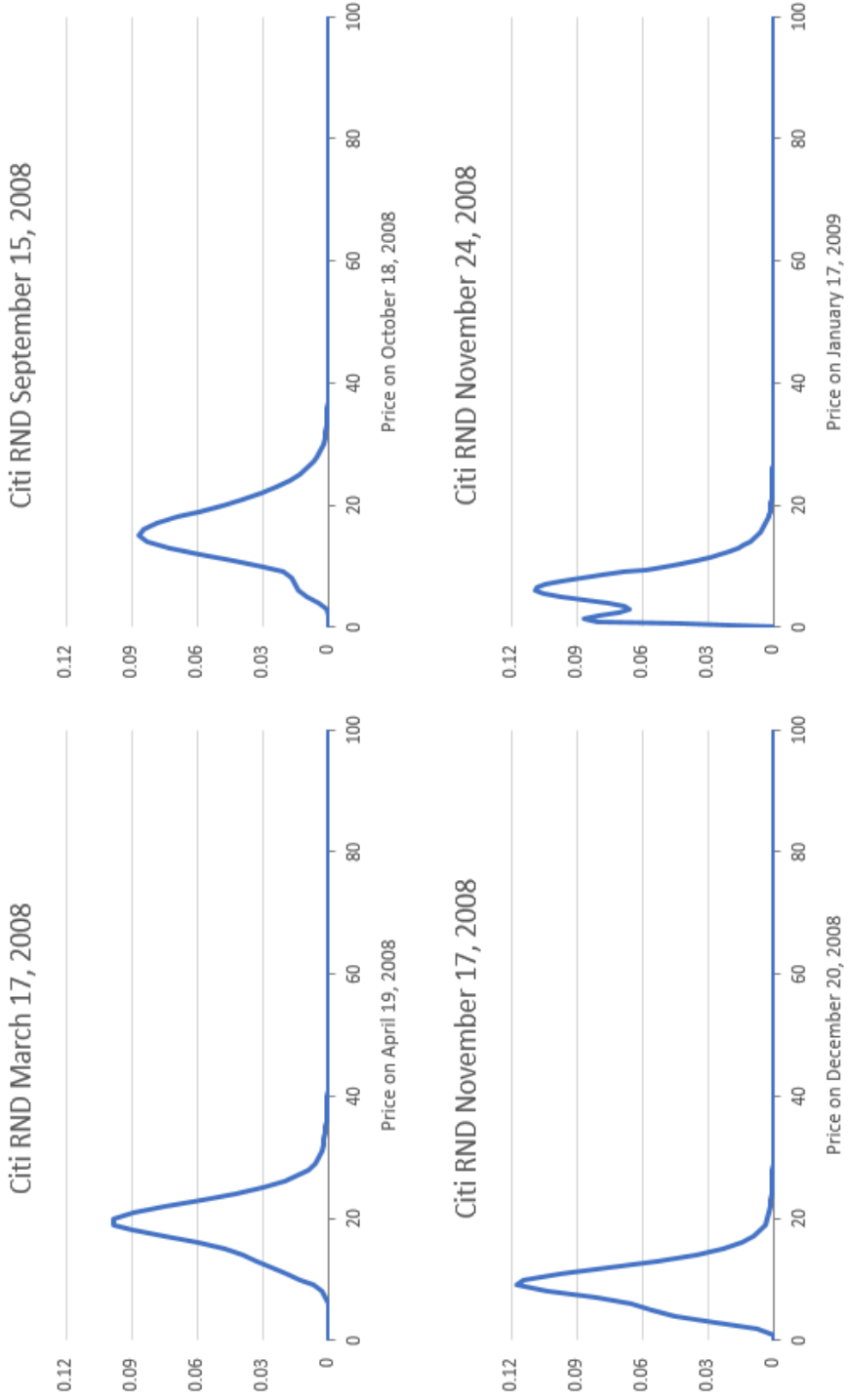
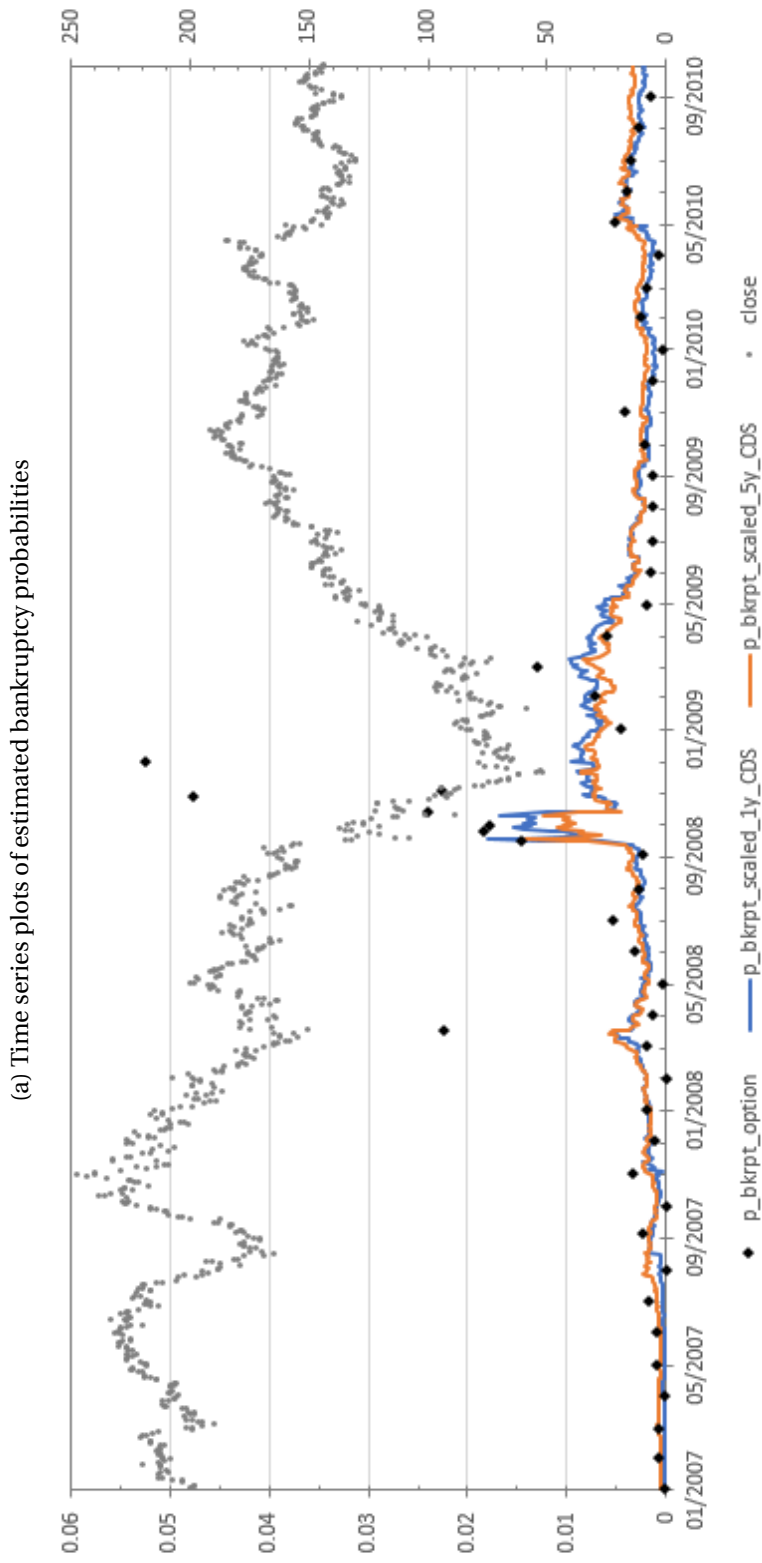


Figure 2.10: Bankruptcy probabilities inferred from option prices and CDS spreads for Goldman Sachs



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

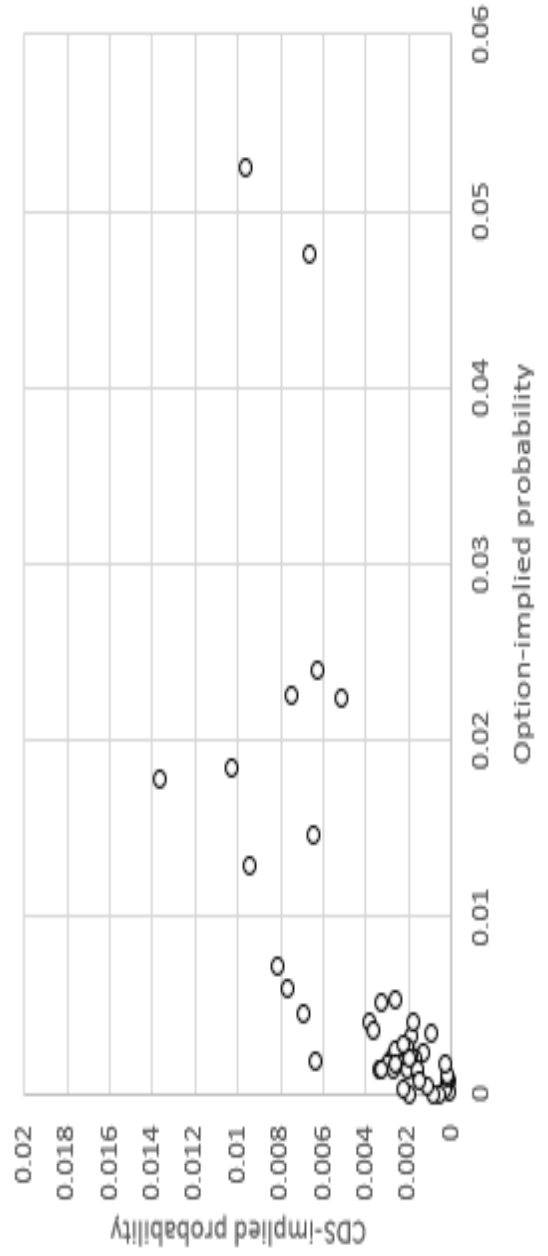


Figure 2.10: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Goldman Sachs

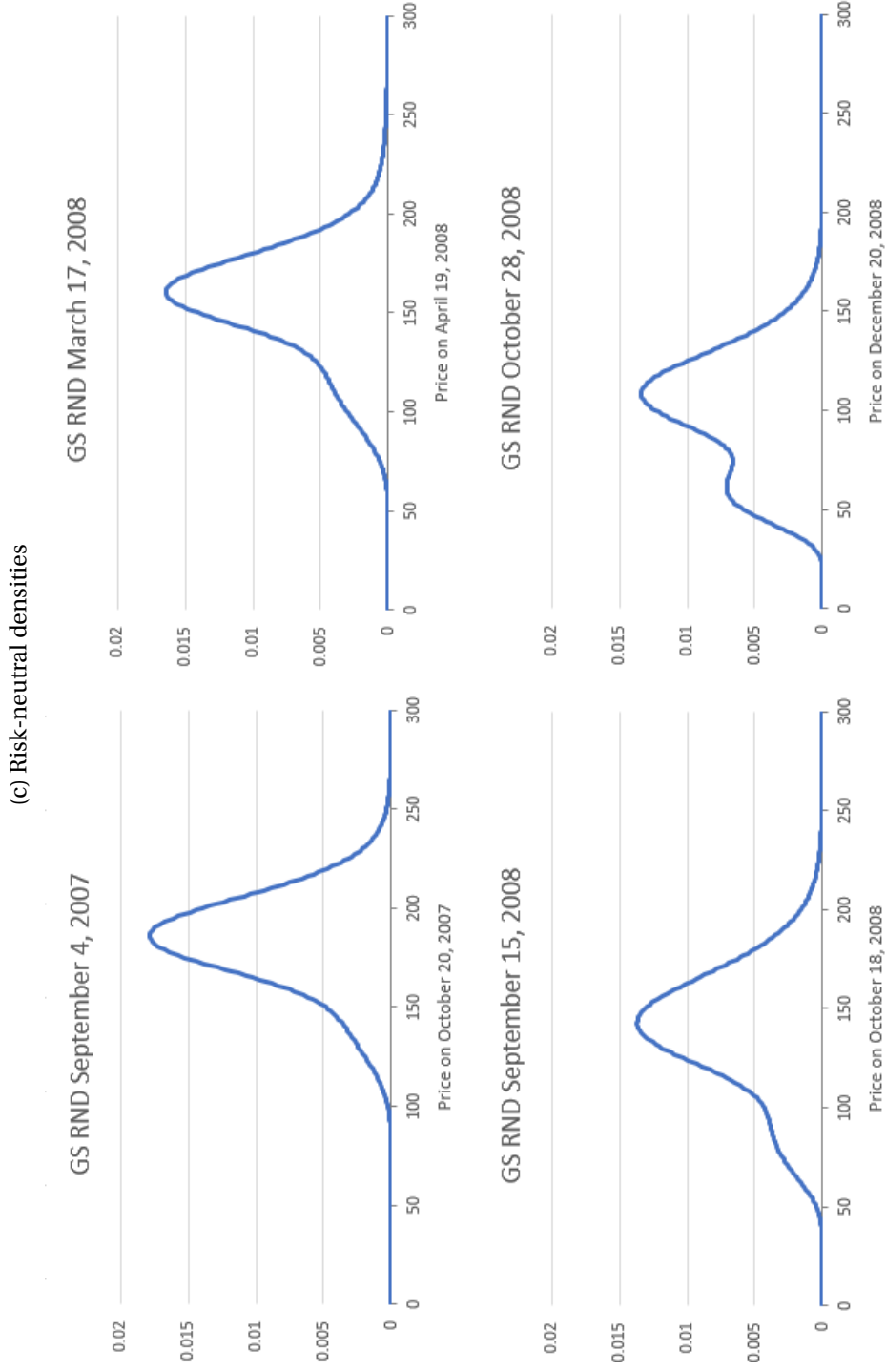


Figure 2.1.1: Bankruptcy probabilities inferred from option prices and CDS spreads for Morgan Stanley

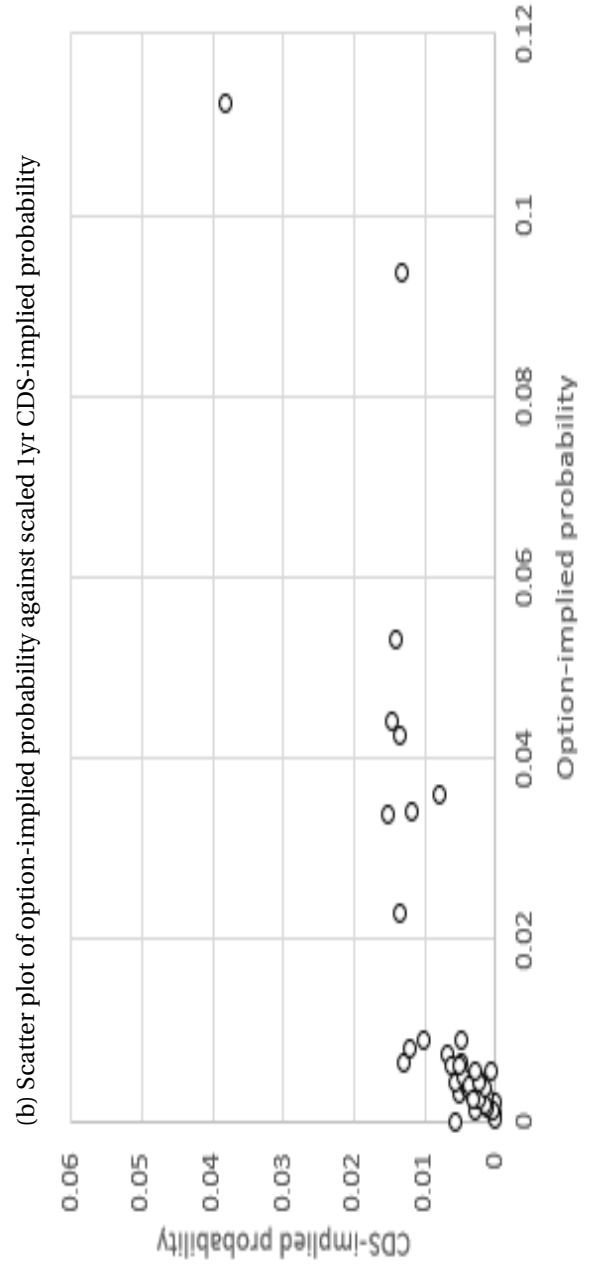
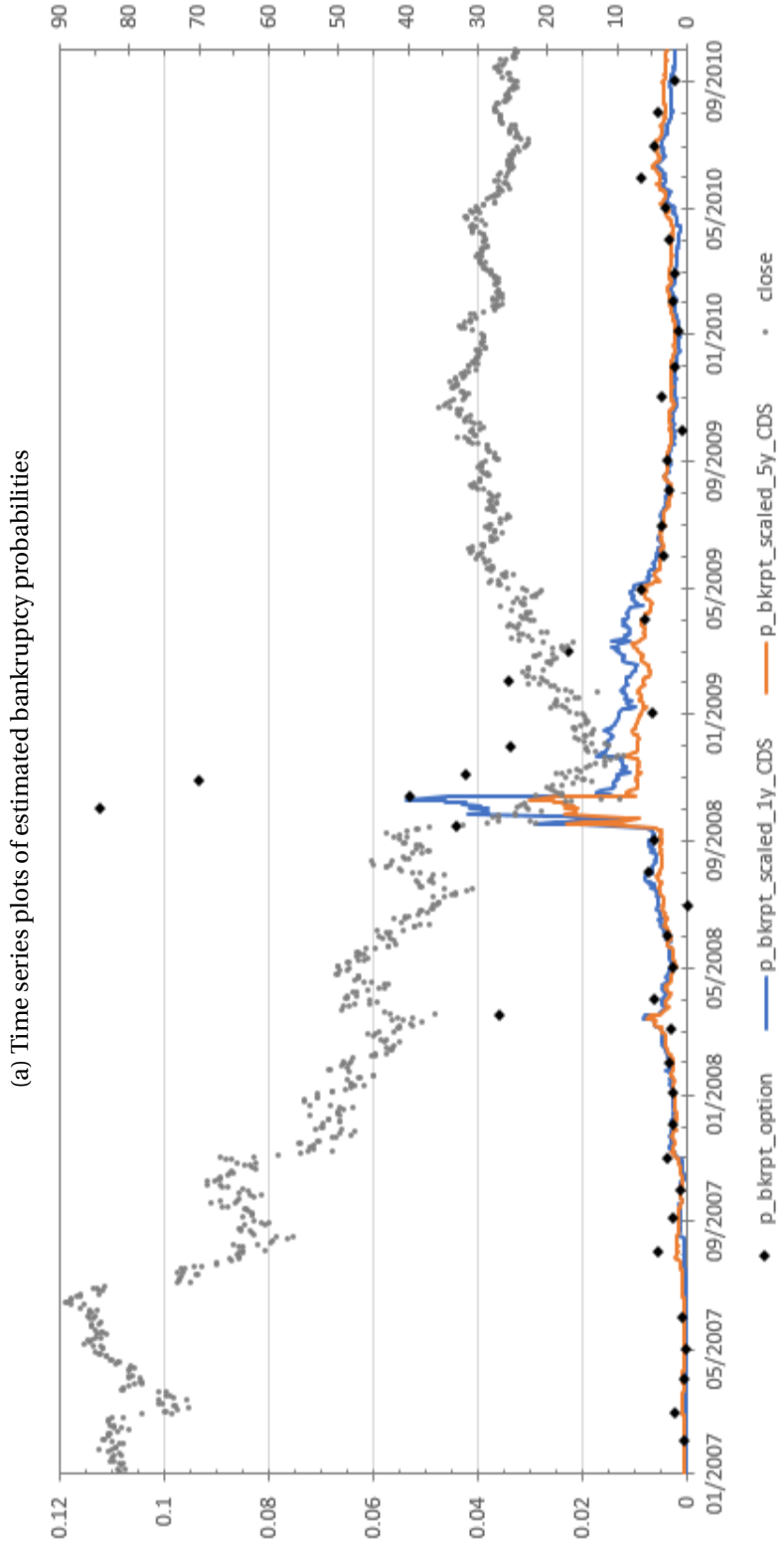


Figure 2.11: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Morgan Stanley

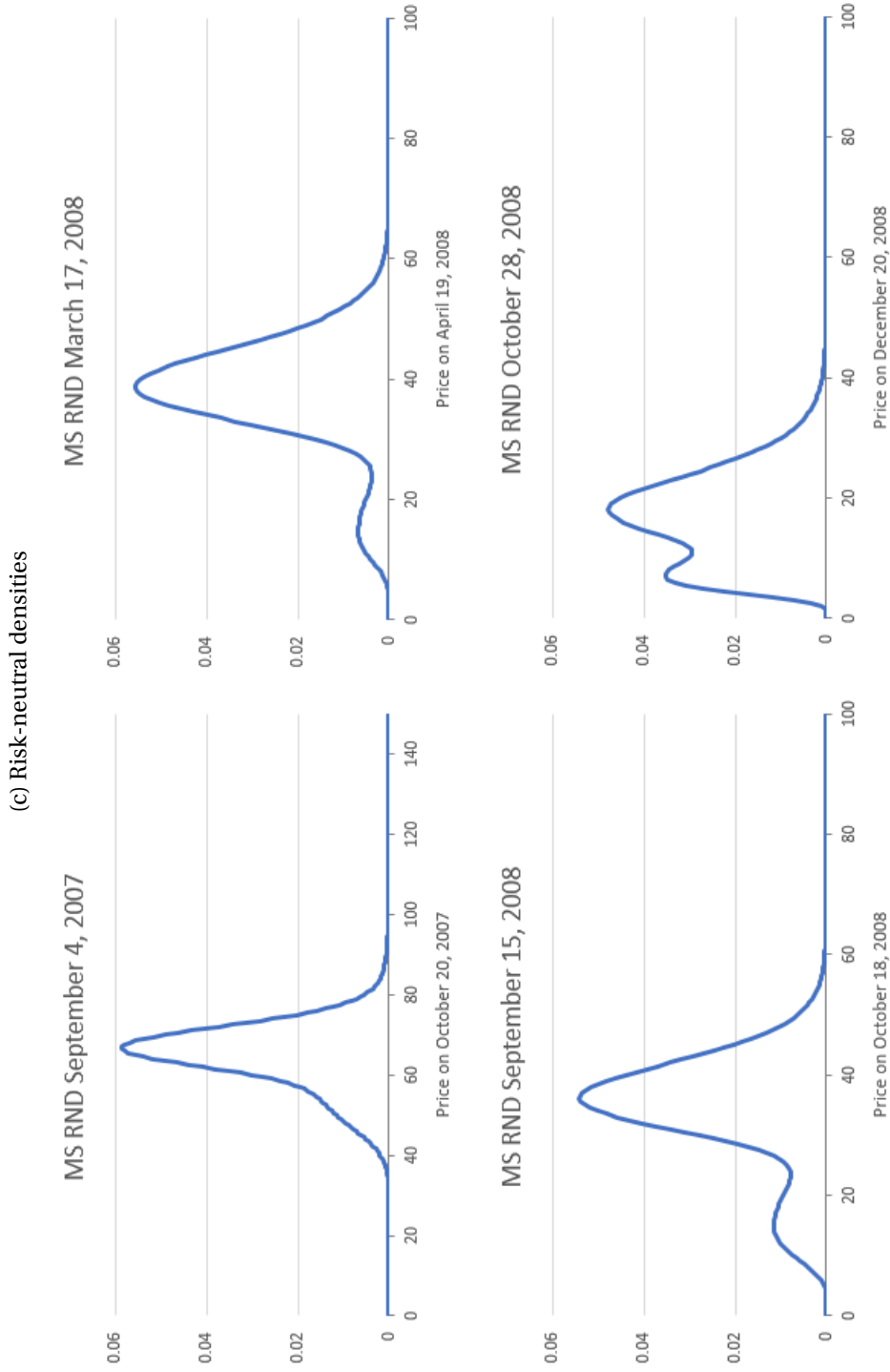
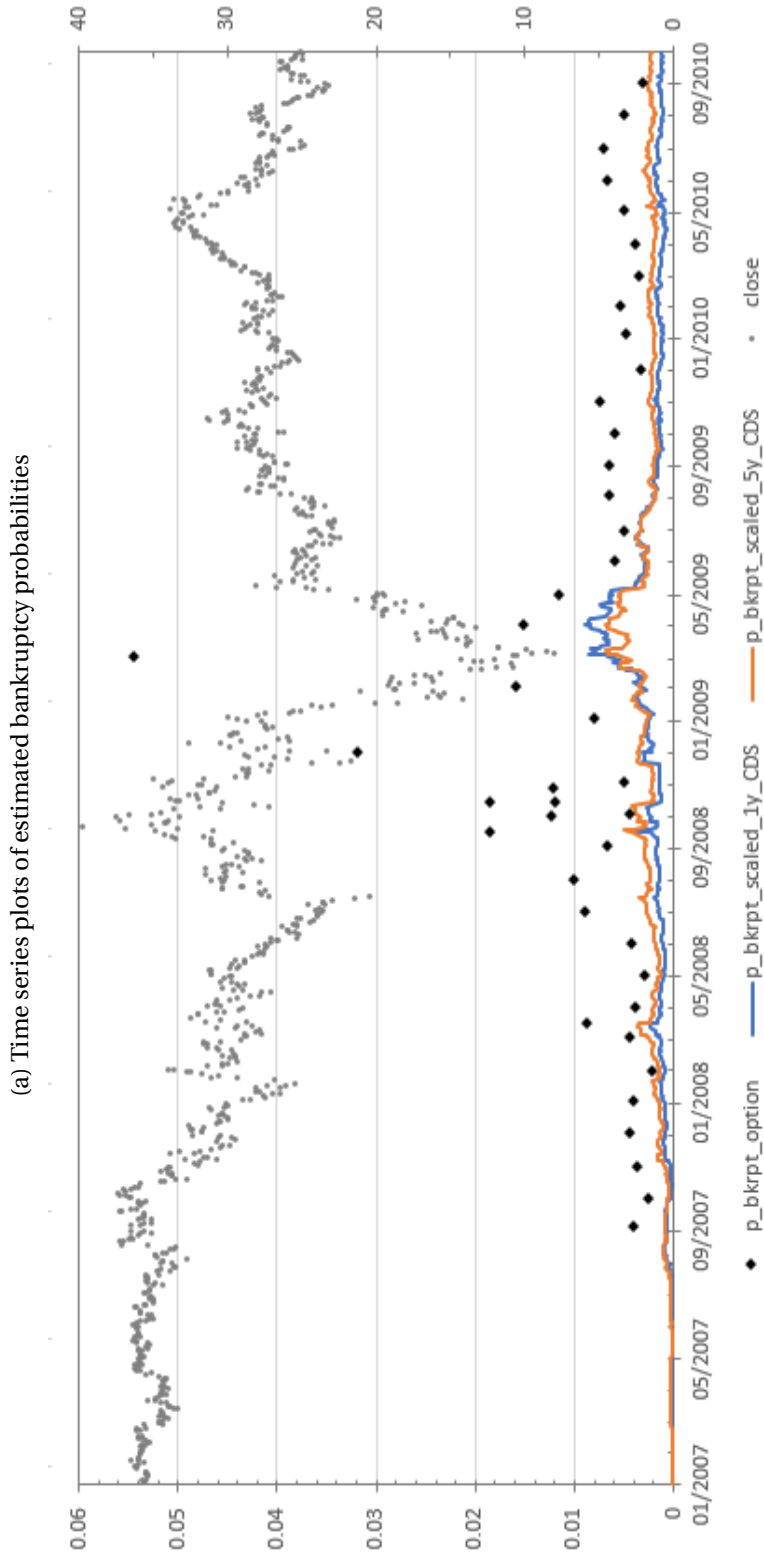


Figure 2.12: Bankruptcy probabilities inferred from option prices and CDS spreads for Wells Fargo



(b) Scatter plot of option-implied probability against scaled 1yr CDS-implied probability

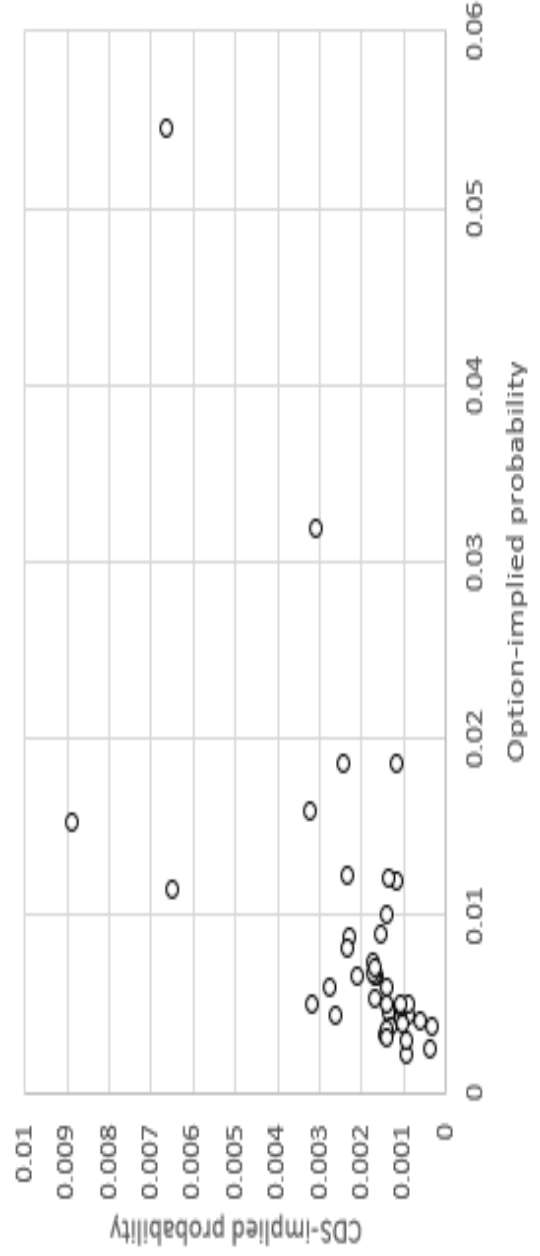


Figure 2.12: Continued Bankruptcy probabilities inferred from option prices and CDS spreads for Wells Fargo

(c) Risk-neutral densities

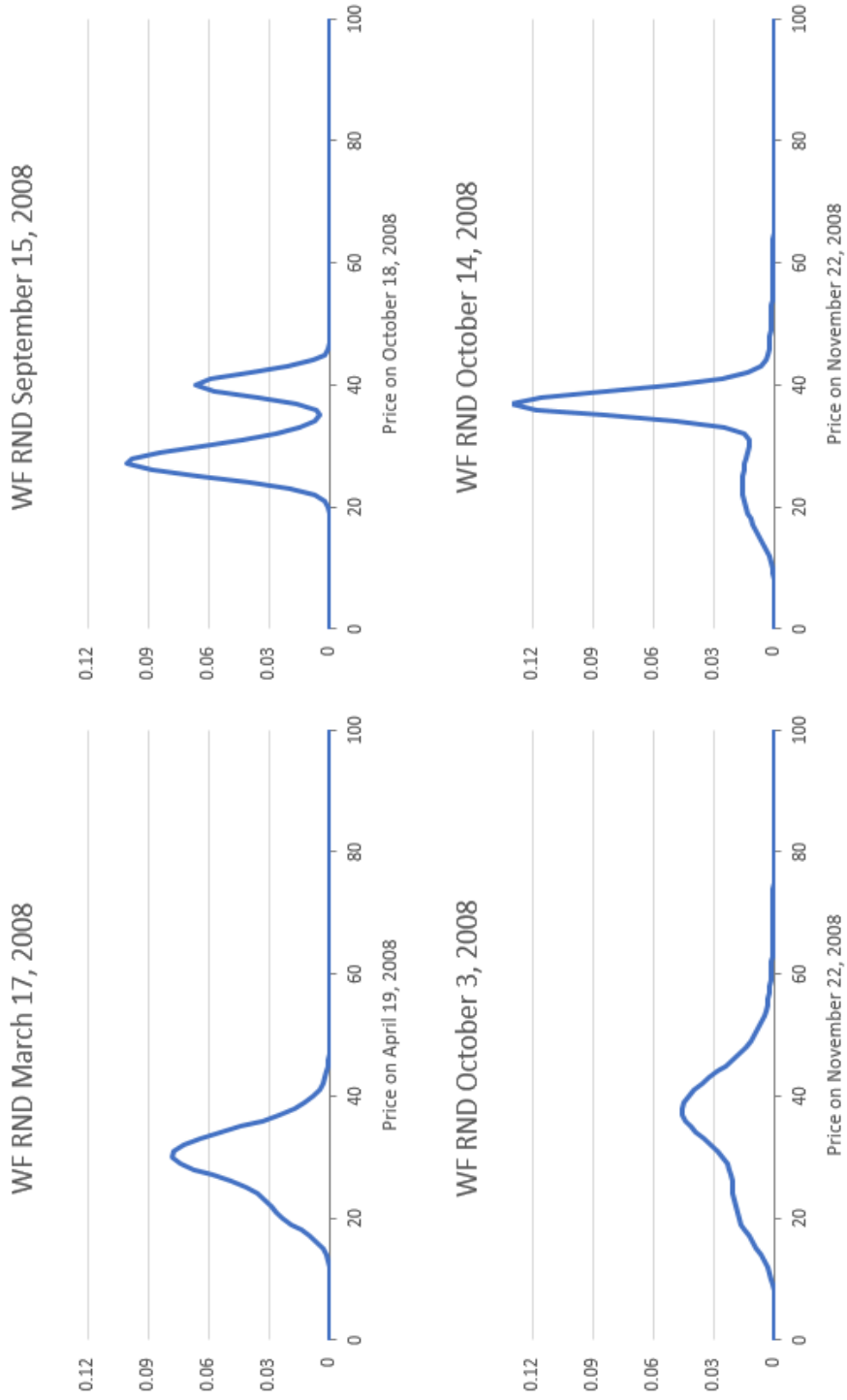
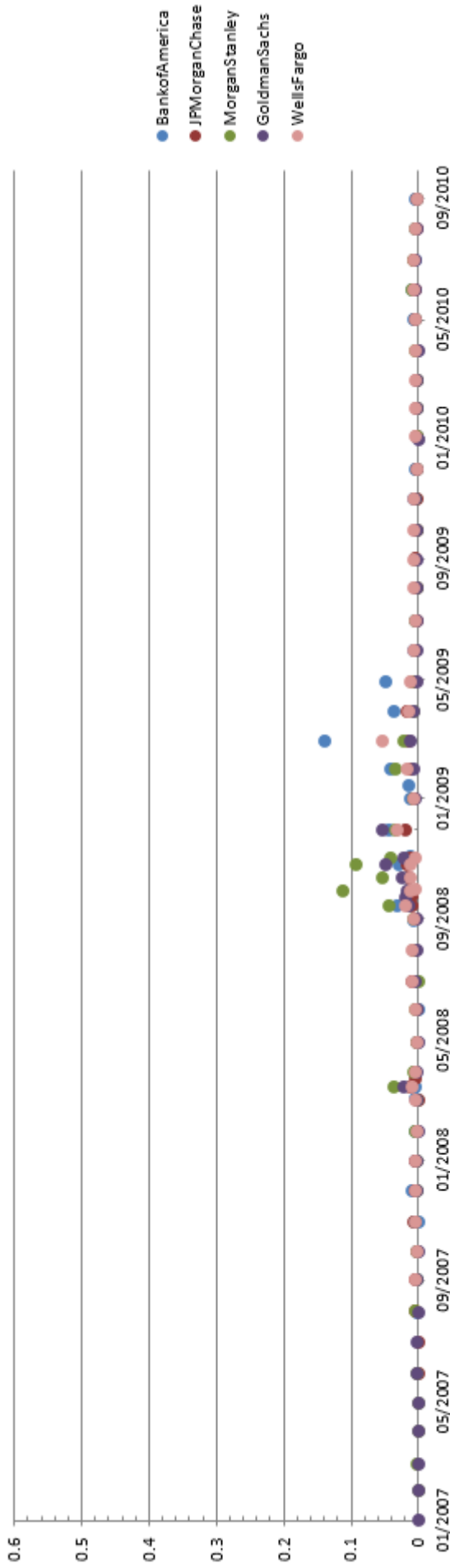


Figure 2.13: Bankruptcy probabilities inferred from option prices across healthy firms and distressed firms

(a) Bankruptcy probabilities inferred from option prices across healthy firms



(b) Bankruptcy probabilities inferred from option prices across distressed firms

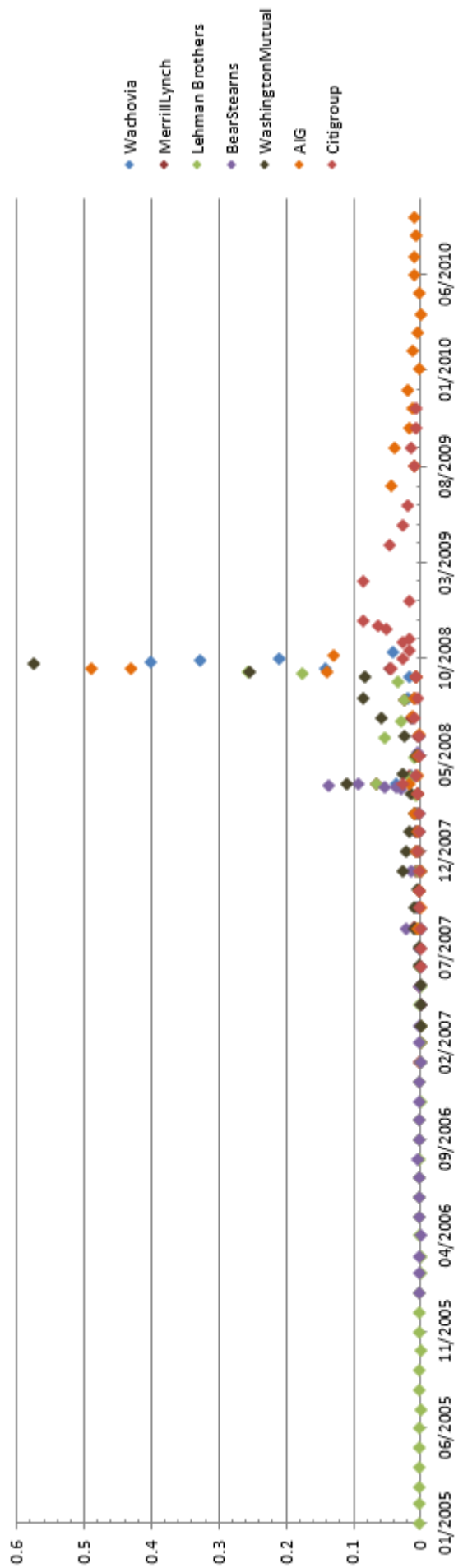
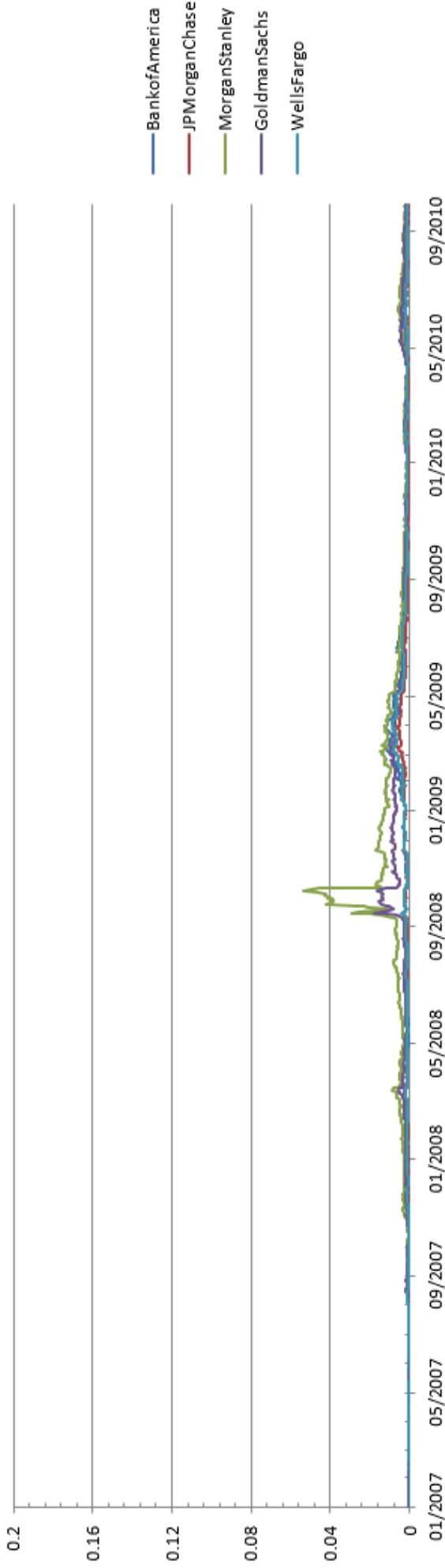


Figure 2.14: Bankruptcy probabilities inferred from option prices across healthy firms and distressed firms

(a) Bankruptcy probabilities inferred from 1 year CDS spreads across healthy firms



(b) Bankruptcy probabilities inferred from 1 year CDS spreads across distressed firms

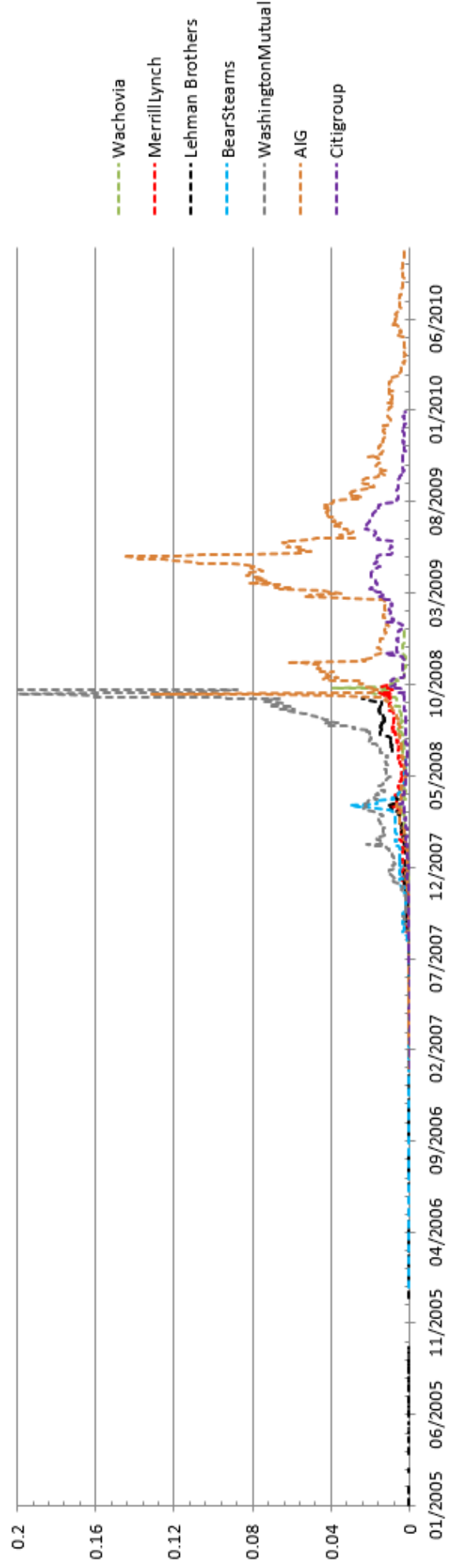
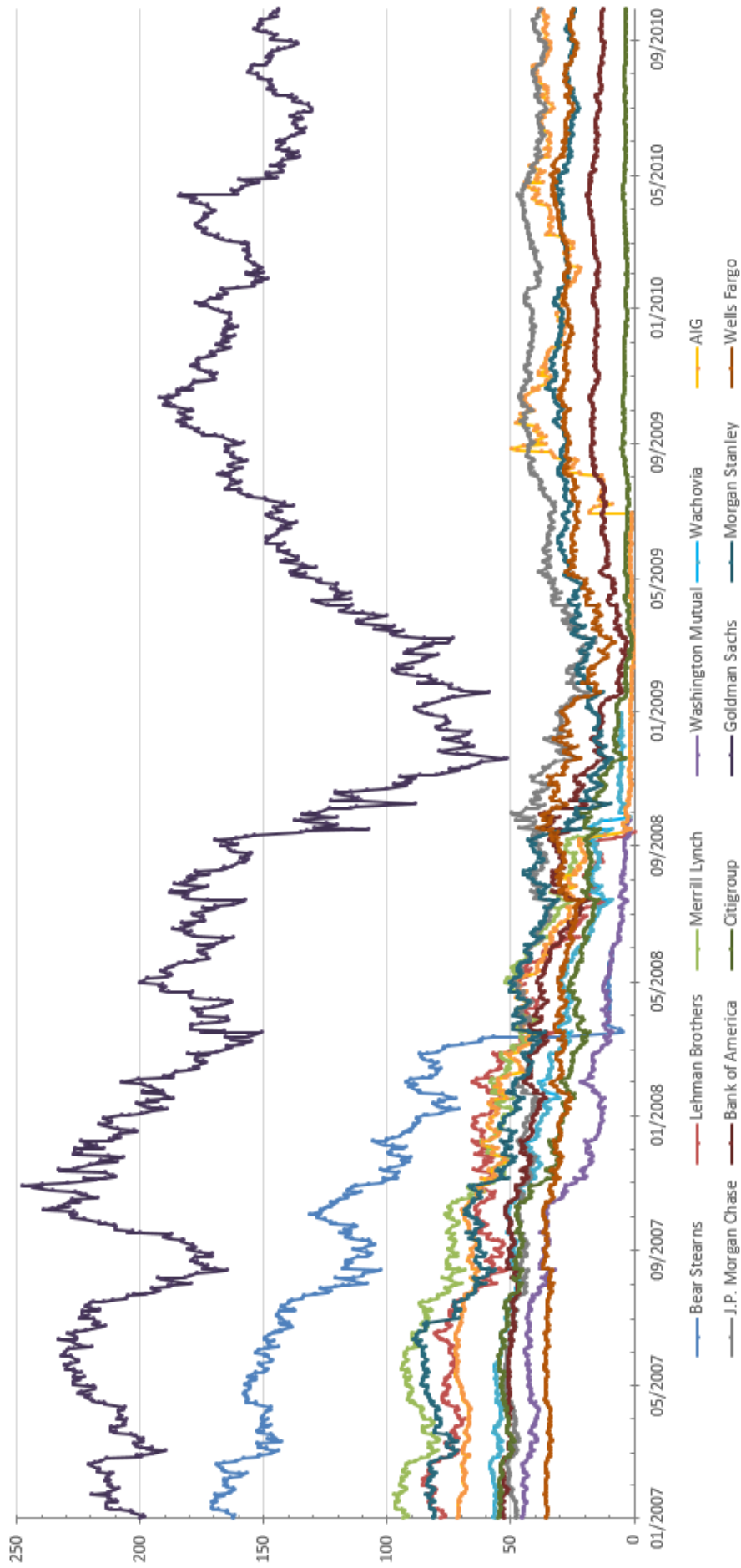


Figure 2.15: Times series plots of stock prices for all firms



Chapter 3

Model Free Expectation of Implied Volatility and International Spillovers

3.1 Introduction

Over the past fifty years, financial markets throughout the world have become more open to foreign investors, and national markets have become more closely connected. Academic research on international market integration has come full circle. Early papers focus on portfolio diversification where investors can benefit from less-than-perfect correlations among returns on international investments. The surge in international portfolio investment activities spawned increasing linkages between national markets and intensified interdependency between countries with more trades and investments. A potential by-product of this rapid growth in capital flows is whether it has changed how global asset returns move together. A series of studies started to investigate lead-lag relationships between international stock markets. While some find strong contemporaneous relationships, others conclude no significant interrelationship.

Research and public interest on this pursuit were heightened during stressful market events, such as the International Crash of October 1987, the 1997 Asian financial crisis, and

more recently the 2008 Global financial crisis. Stock markets around the world fell uniformly. These events cast doubts on the very benefit of international diversification as how could world markets fall simultaneously when the underlying economies are fundamentally different? This question has engendered an immense literature researching return and volatility spillovers across markets. Some studies focus on measures of return and volatility spillover, others try to link these spillovers to economic fundamentals, or theorize that they come from market contagion. However, none of these efforts provided a definitive answer to the questions raised by the Crash of October 1987. One general conclusion from these papers is that the capital flows that sought to exploit international diversification benefit have intensified linkages across markets. These linkages in turn engender a systemic vulnerability to liquidity gaps and market shocks whose impact was once contained within national boundaries.

Later in the early 2000s, researchers sought to revisit this topic by investigating spillovers in terms of implied volatility. Implied volatility is directly extracted from an option price, and is considered a forward looking measure as it reflects market expectation of future volatility over the remaining life of an option. Empirical studies have documented its information superiority over historical measures of volatility. It thus serves as a natural alternative to further investigate this topic.

The development of volatility indices around the world paved the way for studies on implied volatility spillover. A volatility index provides a consensus market measure of risk of the underlying asset over a fixed period. The Chicago Board Option Exchange (CBOE) became the first to introduce the volatility index VIX (now VXO), and many of the world's other exchanges have followed so. The methodologies in constructing volatility indices around the world are in general a replication of that for the new VIX developed by the CBOE. However, as pointed out in Jiang and Tian (2007), the calculated volatility index using the CBOE methodology contains substantial errors, and these errors translate to significant economic

consequence if not accounted for.

This study first addresses the issues with the CBOE methodology, and provides an alternative in constructing a volatility index. I calculate volatility indices for 15 markets around the world, and study implied volatility spillover between these markets. I employ vector autoregressive analysis, impulse response functions and forecast error variance decomposition to study the dynamic interdependence between these markets. The general conclusion is consistent with those in ex-post measurement studies. I find that the U.S. market is unambiguously the dominant source of uncertainty in the world. Correlation between markets largely depends on geographical proximity. Developed markets have bigger impact on developing markets. The findings support the notion of informationally efficient international stock markets, in that information transmitted from one market to another is processed within a maximum of two days.

This study is organized as follows. Section 3.2 provides a comprehensive review on spillover studies distinguishing between historical measure and the risk-neutral forward looking measure. Section 3.3 provides an overview of the CBOE methodology in constructing the new VIX, and its associated issues. I then provide an alternative measure in constructing volatility index. Data is described in this section and an empirical analysis of calculated volatility indices is given. Section 3.4 details empirical analysis of spillover effects and section 3.5 concludes. A list of Figures and Tables are in Appendix in section 3.6.

3.2 Literature review

The studies of international market linkages took off from early papers documenting the benefits of international portfolio diversification. Works of Grubel (1968), Levy and Sarnat (1970), and Solnik (1974) contributed to the mean-variance portfolio investment strategy by showing that further diversification benefits could be achieved by expanding a national

portfolio to include stocks from other countries. The key insight delivered by these papers is a shift in investment policy to countries with the lowest possible correlation with the home country from countries with stronger economic ties with the home country.

The implications of international asset allocation strategy engendered a surge in international portfolio investments. As capital flows became freer across borders, the linkages between markets grew stronger. Seemingly unrelated economies have grown increasingly interdependent through foreign investments. Since Grubel's work (1968), a series of studies analysed the lead-lag relationship between international stock markets. Examples are Granger and Morgenstern (1970), Agmon (1972), and Hiliard (1979). These early empirical results are mixed. While Agmon (1972) finds strong contemporaneous relationship between the U.S. market and others, Granger and Morgenstern (1970) conclude that there is little or no interrelationship between different stock markets. Hiliard (1979) uses daily data and revealed close relationship among countries that are not apparent in Granger and Morgenstern (1970) who use weekly data, but finds that inter continental prices are unrelated. The main concern of these studies on early data from the 1960s to early 1970s is to show that the interdependency among markets is less pronounced across countries than within a country, but little is revealed about the structure of interdependency between markets.

Eun and Shim (1989) rose to this occasion and hence initiated studies of international equity market spillover. Early studies of stock market spillover literature employs regression analysis to study return or price spillover effects. With the development of GARCH models which intend to capture time varying volatilities, researchers started to investigate volatility spillovers across the world using a variety of GARCH specifications. These studies which employ historical measures prevail for two decades since the 1990s. It was not until early in the twenty first century that a forward looking measure emerged in the literature in studying international spillover effects. Implied volatility, with widely documented superiority of in-

formation content over ex-post measures of volatilities (see e.g. Blair et al. (2001)), became a tool in providing new evidence on international market integration. The remainder of this section first summarises related literature using ex-post measures, including return spillover and volatility spillover, followed by a group of growing literature focused on forward looking measure in studying international stock market linkages.

3.2.1 International stock market spillovers under the historical measure

3.2.1.1 Return spillover – regression framework

The findings in the studies of Grubel (1968), Granger and Morgenstern (1970), Agmon (1972), and Hiliard (1979) etc. in the 1970s reveal evidence of comparable interdependence of share price movements across and within countries. Eun and Shim (1989) then investigated, upon the established evidence, the interdependence structure of international stock markets. They employed a vector autoregressive (VAR) model on nine markets (including Australia, Canada, France, Germany, Hong Kong, Japan, Switzerland, the United Kingdom, and the United States) using daily returns of stock market indices from 1980 to 1985. Their focus was on addressing issues (1) how much movements in one stock market could be explained by innovations in other markets; (2) how influential is the U.S. market on other markets; and (3) how rapidly price movements in one market transmitted to other markets. They employ VAR system and its structural analysis – impulse response functions and forecast error variance decomposition – to answer these questions. The findings are that the U.S. market is by far the most influential market in the world; no single market's innovations can explain its own variance; impulse responses results indicate that all markets respond to U.S. shocks, mostly strongly with a one-day lag, and tapers off thereafter. Eun and Shim (1989) concluded that the evidence supports the notion of informationally efficient markets.

Von Furstenberg and Jeon (1989) conducted a principal component analysis on the world's four largest equity markets – U.S., Germany, Great Britain, and Japan during 1986-1988, focusing particularly on the correlation of price movements of 1987 post-crash periods using daily stock returns. They employ a VAR model and impulse response analysis and forecast error variance decomposition to study the leadership of news transmission between these markets. The ordering of the variables respects the time zone of each market. They also advance each market to allow for a chance to appear at the top of the ordering. Impulse response analysis revealed an increased but not sustained influence of Japan on other markets. It also revealed that innovations of the British market had a longer-lasting effect on other markets after crash than before, but possibly due to the expansion of the London Market during the year preceding the crash. The authors also attempted to link the increased post-crash price co-movements to economic fundamentals, but the evidence is rather weak.

Koch and Koch (1991) model daily returns for an extended sample which includes eight countries for the years 1972, 1980, and 1987. Their sample includes Japan, Australia, Hong Kong, Singapore, Switzerland, West Germany, United Kingdom, and United States. They employ a block-recursive simultaneous equations model to study the contemporaneous and lead-lag relationships between these markets. Their results conclude in general that international markets have grown more interdependent, and the high degree of market efficiency is revealed by rapid adjustment to news – most adjustments are completed within 24 hours. The results further show that the interdependence between markets is largely observed within time-zone region where trading hours overlap. Japan has grown to have more impact on other markets over time whereas U.S. market's influence has waned and starts to respond to other markets.

Malliaris and Urrutia (1992) investigated causal relationships between six equity market indices by means of Granger Causality. Sample includes New York S&P 500, Tokyo Nikkei,

London FT-30, Hong Kong Hang Seng, Singapore Straits Times, and Australia all Ordinaries. Data was divided into before, during, and after the crash of October 1987, which starts May 1987 and ends March 1988. The empirical results show that practically no lead-lag relationships were detected for before and after crash period, however, bidirectional and unidirectional causality was observed during the month of the crash. The causal results indicate that Tokyo played a passive role during the crash, whereas New York failed to confirm its alleged leading role. Contemporaneous causality was also examined by including current values in the right hand side of Granger Causality equations. Little contemporaneous causality was observed for pre-crash period, but an increased causality was detected during and after the month of the crash.

Studies of international stock market linkages have found conflicting evidence. Arshana-palli and Doukas (1993) address methodological problems associated with the VAR model that might explain the difference. Empirical studies in the 1980s have shown non-stationary evidence of national stock indices. To make time series stationary, the common practice is to take the first difference. The authors argue that taking first difference may filter out potential important information about long-run trends among non-stationary stock indices. They use theory of cointegration and error correction analysis to test linkages between Japan, France, Germany, the U.K., and the U.S. for the period between 1980 and 1990. Their results report strong linkages of France, Germany and U.K. with the U.S. for post-crash period, but the same cannot be found for the pre-crash period. They also find no evidence of linkages between Japan and the U.S., and the Japanese stock market is not related to the European markets, which suggests that the Japanese market is not fully integrated with the world's other major stock markets.

Copeland and Copeland (1998) studied the lead-lag structure of daily returns of 29 countries in three regions (Americas, Europe, and Pacific) by industry groups between 1992 and

1997. They conduct pairwise OLS regression with a null assumption that the U.S. as the largest market in the world is the lead economy for most industries. The regressions are done for regional, country, and industry groups. They found strong contemporaneous relationships among regional markets operating at the same time. Americas lead Europe and Pacific by one day and no significant lags beyond one day, suggesting that information adjustments are processed within one day. They also developed a simple trading strategy to test whether economic gain is exploitable from statistically significant lead and lags, however, the results are at best attainable by trading houses but not by individuals.

Becker et al. (1990) examine intraday data of the S&P 500 index and the Nikkei 225 index between 1985 and 1988 to study the synchronization of price movements between the world's largest stock markets. The use of open-to-close, close-to-open returns marked a significant departure from earlier studies that relied on close-to-close returns. They found high correlation between the open to close returns of U.S. stocks in the previous trading day and the Japanese market performance in the current period. The U.S. market explains about 7%-25% of variation in the Japanese market open-to-close returns, and 11%-18% variation in Japan's overnight returns. In contrast, Japan has a small impact, about 1% on the U.S. market performance. High correlation between markets' open-to-close returns violates an efficient market hypothesis. Simulated trading strategies were successful in predicting up and down movements in Japan, but profits from these strategies were eliminated immediately when transaction costs and taxes in Japan were taken into account.

3.2.1.2 Return and Volatility spillover – GARCH framework

The crash of October 1987 was a uniform fall of national stock markets around the world, despite the differences in economic fundamentals, market structure, liquidity, etc. A Large number of commentaries and reports generated by this event have not answered the ques-

tion why markets fall in such uniformity. With the development of advanced computational technology, researchers explore more methodologically rigorous approaches to uncover both return and volatility spillover across national markets in an attempt to address partly the unanswered question.

Hamao et al. (1991) was one of the first studies to employ a GARCH-in-mean framework to study the transmission mechanism in conditional first and second moments. Their sample include index returns of FTSE 100, S&P 500, and Nikkei 225 during the period of 1985 to 1988. They separate open-to-close return and close-to-open return to isolate the spillover effect. First, they study the spillover effect of open-to-close return for a given market. They estimate a GARCH(1,1)-M model for all markets and take the squared residual of the conditional mean equation, as volatility surprise, from the most recently open foreign market, and append it to the conditional variance equation of the domestic market. In parallel, they also estimate another model appending volatility surprise of both foreign markets to the domestic market to examine separate volatility effects from both foreign markets. The models are estimated for the full sample and pre-crash period from the start to September 30, 1987. The results show statistically significant positive foreign volatility surprise for all three markets for the full sample estimation, but significant volatility spillover for pre-crash period is only found from U.S. to Japan. The inclusion of a second foreign volatility surprise does not seem to diminish the effect of the first foreign market. The authors conclude that Japan is most sensitive to volatility spillover effects from foreign markets, while U.S. and U.K. are moderately, if at all, sensitive. The spillover asymmetry across national stock markets coincide with evidence documented in Eun and Shim (1989) discussed earlier.

Hamao et al. (1991) also consider spillover effects in the conditional mean by including the open-to-close return of the most recent traded foreign market, and preserving the volatility surprise in the conditional variance equation. The models are estimated for the

full sample, and pre-crash sample. Same asymmetry effect is observed for mean spillover, while significant spillover effects are observed from U.S. to Japan, U.K. to U.S., none is observed from Japan to U.K.. They also estimate a noon-to-close return model for the U.S. market to explain the observed largest significant spillover effect from U.S. to U.K., possibly due to one-hour overlapping of trading between the two markets. The effect is eliminated in the conditional mean once overlapping is accounted for, but volatility spillover effects in conditional variance are still manifest.

Theodossiou and Lee (1993) provide additional insight on international stock market interdependency by extending the study of Hamao et al. (1991) in a number of ways. They use weekly data, rather than intra-day data, and expand the sample to include Canada and Germany for an extended period from 1980 to 1991. Unlike Hamao et al. (1991) estimating GARCH-M for each market separately, Theodossiou and Lee (1993) estimate the model in a multivariate framework under the assumption of constant conditional correlation over time. The results for mean spillover show statistically significant effects from U.S. to U.K., Canada, and Germany. Geographic proximity accounts for bigger impact of U.S. past return on Canada than that of Germany and U.K.. Interestingly Japan has a negative spillover effect on Germany. To the extent the significant spillover effects violate efficient market hypothesis, effectively forecasting stock market returns cannot be achieved as the explanatory power of the model is as low as 5%. The results for conditional variance equation show that past values of conditional variance are highly significant for all markets, indicating volatility persistence. Own-volatility spillover is found for U.S., Germany, and Japan, but not for Canada and U.K., suggesting that the conditional volatility is imported from abroad for these two markets. Significant cross-volatility spillover effects are found from the U.S. to all four markets, from Germany to Japan, and from U.K. to Canada. This is consistent with findings in Eun and Shim (1989) that the U.S. is unambiguously the leading source of uncertainty.

Bae and Karolyi (1994) assert that the true magnitude and persistence of spillover effects across national markets can be significantly understated if the return and volatility generating processes are mis-specified. Specifically, the leverage effect which distinguishes asymmetric effect of past bad news and good news plays an important role in capturing the true dynamics of return and volatility. Bae and Karolyi (1994) extend the GARCH framework to allow for asymmetric effect of negative and positive foreign market return shocks for volatility. They focus on the joint dynamics of open-to-close and close-to-open returns for S&P 500 and Nikkei 225 between 1988 and 1992. The authors use a two-stage estimation to study the volatility spillover for a basic GARCH model, and two asymmetric GARCH specification – GJR-GARCH developed by Glosten et al. (1993) and partially non-parametric GARCH (PNP-GARCH) model introduced by Engle and Ng (1993). Their evidence shows that the asymmetric parameter is statistically significant and its appearance strengthens volatility spillover effect from one market to the other. In general bad news from both domestic and foreign markets have a much larger effect on the subsequent return volatility than good news.

Koutmos and Booth (1995) further extend research of asymmetric effects on international stock market linkages by employing a multivariate EGARCH model developed by Nelson (1991). Using open-to-close returns, they study the price and volatility transmission mechanism between New York, London, and Tokyo from 1986 to 1993. Results show significant return spillover from New York to Tokyo and from Tokyo to London and New York, but not from London to New York, which is possibly due to the high correlation that is accounted for by the 2 hour overlapping effect between London and New York. Volatility spillovers are more extensive and reciprocal. They found significant volatility spillover from New York to Tokyo and London, from London to New York and Tokyo, and from Tokyo to London and New York. Consistent with evidence in Bae and Karolyi (1994), volatility transmission mechanism is asymmetric; in all instances adverse news have a far greater impact on the next mar-

ket session than good news. Koutmos (1996) also employs the same multivariate EGARCH framework to study the linkages between four major European markets – Germany, France, U.K. and Italy. Similar evidence of multi-directional spillover effects are found in both return and volatility, as well as asymmetric effects. The author concludes that the European markets are integrated in a sense that they respond to both domestic and foreign news.

While most studies focus on markets across the globe with no or few overlapping trading hours, some studies look directly at markets with synchronous trading. Susmel and Engle (1994) employ a univariate GARCH model with asymmetry feature to study news transmission between London and New York, two markets having 2 hour simultaneous trading. They use high frequency hourly data so that they can examine return and volatility spillover at specific hourly periods as well as the behaviour at the open of the New York market. Data is from January 1987 to February 1989. The findings are as follows, (1) there is no return spillover between the two markets in non-overlapping periods, with only two exceptions. (2) There is no strong evidence of volatility spillover. The most significant volatility spillover occurs at New York opening time, and lasts for only one to two hours. (3) The inclusion of the crash period October 1987 does not seem to alter spillover effects between markets.

Karolyi (1995) uses a bivariate GARCH model and a VAR model to examine the short-run dynamics of return and volatility between the U.S. and Canadian stock markets. These two markets have perfect synchronous trading hours, which circumvents the problem of disentangling the confounding effects of non-synchronous trading hours. Combining simulated impulse response analysis of both models, the results show that return spillovers are sensitive to the dynamics of conditional volatility. He found diminishing effect of return spillover from U.S. to Canada in the latter part of 1980s. He also found that S&P 500 returns have a larger impact on non-inter-listed TSE 30 stock returns than inter-listed returns, suggesting that investment barriers might be an important factor to consider in studying international

stock markets co-movement.

Early studies typically focused on the volatility spillovers among developed markets. The interest in studying international stock markets linkages has gradually and naturally extended to regional and emerging markets as they become more important. To the degree of integration of these markets with developed markets, implications can be drawn on portfolio selection and asset allocation for investors. To name a few, studies examine linkages between developed markets and Asian markets include Ng (2000), Worthington and Higgs (2004), Chanchaoenchai and Dibooglu (2006), Chuang et al. (2007), Beirne et al. (2010), Wang and Wang (2010), Kang and Yoon (2011), and Li and Giles (2015). Bellotti and Williams (2005) investigate volatility transmission between emerging markets, between Asia and Latin America. Majdoub and Mansour (2014) study volatility spillover between the U.S. and five Islamic equity markets. Other studies which focus on emerging European markets include Tse et al. (2003) on the U.S. and Poland, Li and Majerowska (2008) on Poland and Hungary, and Booth et al. (1997) on Scandinavian markets.

It is observed with regularity that stock markets behaviour displays notable similarities during crisis periods, financial markets volatilities increase sharply and spill over across markets, despite that the underlying economies are fundamentally different. What are the determinants of return and volatility spillover, or is spillover simply driven by irrational panics? While this strand of literature is largely preoccupied with the measurement of international spillovers, the next wave of studies seek to establish factors that drive spillovers.

One strand of literature attempts to link return and volatility spillover to fundamental economic factors, such as interest rate, exchange rate, inflation etc. Macroeconomic news was also addressed on the impact of covariance dynamics between markets. This linkage is generally established by an equilibrium model of returns. The main conclusion from this strand of literature drawn is that macroeconomic announcements do not seem to affect co-

movements between stock markets in a meaningful way, and that economic variables explain only a small fraction of international market comovements.

The other strand of literature explores the idea of market contagion. Contagion, as defined by Claessens et al. (2001), is the spread of market disturbances (mostly on the downside) from one market to the other, a process observed through comovements in exchange rates, stock prices, sovereign spreads and capital flows. The source of contagion could come from real and financial linkages between markets, as well as from pure irrational responses, such as herd behaviour and loss of confidence etc. Gagnon and Karolyi (2006) provide an excellent review on market contagion literature and spillover studies linked to economic fundamentals. A detailed description of these papers are beyond the scope of this study, therefore I will limit the quantity of references to the above mentioned.

3.2.2 International stock market spillovers under the forward looking measure

Early studies of international stock market integration have shown that stock markets are highly integrated in terms of realised returns and realised variance. Relatively little is documented in the literature about the notion of implied volatility spillover. The price of an option contains information about the market's consensus expectation of expected risk-neutral volatility of the underlying asset over the life of the option. The superiority of the information content of forward looking implied volatility over ex-post measures of volatility has been widely documented (Fleming et al. (1995), Blair et al. (2001), etc.). Since volatility plays a key role in derivative pricing and portfolio management, how markets are linked with one another in terms of expectation of future volatility opens up an important area for investigation.

The seminal work of Whaley (1993) formally established the field of constructing a volatil-

ity index from a basket of option prices. Chicago Board Option Exchange (CBOE), became the first exchange to introduce the volatility index VIX (now the VXO) in 1993, which quickly became the benchmark risk measure of the U.S. equity market. Following the example of CBOE, other exchanges in the world in succession developed their respective volatility indices. The Deutsche Börse introduced VDAX in 1994, and French Marché des Options Négociables de Paris (MONEP) introduced two volatility indices VX1 and VX6 in 1997. CBOE revamped the VXO and launched the new VIX using a model free approach in 2003. NYSE Euronext published VFTSE for FTSE 100 in 2008. Many more volatility indices were published and further revised subsequently for the more developed markets. Siriopoulos and Fassas (2009) provides an extensive review on the world volatility indices both publicly available and proposed by academic researchers.

Motivated by traditional stock market studies of integration, and the availability of newly developed volatility indices, researchers shifted their attentions to the examination using forward looking measure. Since the literature is relatively small, the references listed hereafter are in chronological order.

Gemmill and Kamiyama (2000) was one of the first papers that formally studies international implied volatility spillover. They calculate implied volatility index for Nikkei 225, S&P 500, and FTSE 100 index options from 1985 to 1995. A correlation analysis show that Nikkei implied volatility is less well-correlated with other markets, and Granger Causality test further shows that Nikkei implied volatility does not influence any other markets, but itself is influenced by the U.S.. The authors conclude that the results are generally consistent with those found using ex-post measure literature – both show significant spillovers emanating predominantly from the U.S.. The paper also investigates spillovers in terms of implied skewness, however they found no clear evidence of skewness spillover across markets, but there is weak evidence of long-term U.S. impact on U.K. skewness.

Aboura (2003) examines daily observations of volatility index of the CAC40, DAX30, and S&P 100 from 1994 to 1999, namely the French VX1, German VDAX, and the VXO. They employ VAR framework and its structural analysis, i.e. impulse response function analysis, forecast variance error decomposition, and Granger Causality tests to test implied volatility transmission. They found that the U.S. is clearly the most influential market as it explains about 9% and 10% error variance of the French and German market, while the latter two can only explain 1% of that of the U.S. market. French VX1 and German VDAX responds strongly to the shock in the U.S. within the first two days and dies down thereafter.

Skiadopoulos (2004) constructed a volatility index GVIX of the Greek market using FTSE/ASE-20 index options and futures, and studied the contemporaneous spillover effects with the U.S. market volatility index VXO and VXN (volatility index on NASDAQ). Their data include 554 observations from October 10, 2000 to December 31, 2002. By examining the properties of GVIX and its relationship with the underlying stock market, the author found that the stock market has predictive power of future movements of GVIX while the reverse is not true. They performed unidirectional regressions to test for implied volatility spillovers from the U.S. market to the emerging market. They found contemporaneous spillovers from the U.S. market, but no lead-lag effect.

Wagner and Szimayer (2004) reason that since implied volatility as a risk measure not only reflects ex-ante risk expectations but also has an immediate impact on traded option prices, shocks in implied volatility are crucial in understanding the risk. They model shocks as jumps in implied volatility, and proposed a mean reverting model that allows for Poisson jumps in implied volatility to estimate jump risk and spillover jump events. They investigate the dynamic behaviour of daily implied volatility of the U.S. and Germany as measured by VIX and VDAX from the period 1992 to 2002. They find significant positive jumps occurring for both markets, and the magnitude of jumps in VDAX is twice as big as those in VIX. Fur-

thermore, they find most of these jumps are caused by country-specific events rather than by global events. Only 4 spillover shocks are detected during the 11 years of the sample period.

Nikkinen and Sahlström (2004) analysed the implied volatility index of the U.S., U.K., Germany and Finland from 1996 to 2000. They use VAR analysis and Granger Causality test to investigate transmission of uncertainty. They chose generalised impulse developed by Pesaran and Shin (1998) in the impulse response analysis, as generalised impulse do not depend on the ordering of the variables. The results show that the U.S. is the leading source of uncertainty as changes in uncertainty in the U.S. market are transmitted to the other markets. Impulse responses suggest that U.S. leads other markets by one day and shocks originated in other markets are absorbed within one day. In the European market, Germany appears to be the leading source of uncertainty, and the Finnish market, as a representative of smaller markets, appears to be less integrated.

In a similar context, Nikkinen et al. (2006) study linkages of market expectation of future volatilities derived from currency options. Their data include daily implied volatilities on three major European currencies – GBP, EUR, and CHF, quoted against USD. The sample period used is from January 2001 to September 2003. Vector autoregressive modelling is applied to ascertain the causal dynamics between currency volatilities. The results show that the market expectation of future exchange rate volatilities are highly linked among major European currencies. In particular, the Euro appears to be the dominant currency as its implied volatility has a major impact on other currency volatilities, and not vice versa.

Äijö (2008) extends the studies of stock market integration by looking at the term structure linkages of implied volatilities. Previous studies have found that term structure of implied volatility provides useful information about expectation of future volatility over different horizons. This study adds to the literature by investigating the term structure linkages of three European markets volatility indices – VDAX, VSMI, and VSTOXX. The underlying stock

indices are Germany Dax, Swiss SMI, and European blue chip Dow Jones EuroStoxx50. The author employs vector autoregressive analysis and its structural analysis. Consistent with findings in previous studies, the author finds that the implied volatility term structures are highly correlated with each other in the European market. The term structure of each index varies a lot over time, and VDAX seems to show a leading role in the European market.

The literature has shown, though limited in volume, that the vector autoregressive analysis and its structural analysis is the widely adopted methodology in studying implied volatility spillover effects. More papers follow the same framework but using extended samples are Badshah (2009) and Siriopoulos and Fassas (2009). By the same spirit, as in ex-post measure spillover studies, there are papers look into the linkages of implied volatility between developed markets and emerging market (see Narwal et al. (2012)) for India and developed countries. The general findings of this body of research is that world stock markets are highly correlated with each other, with the correlation heightened with geometric proximity. The developed markets seem to have larger impact on smaller markets, and U.S. appears to be the dominant source of uncertainty.

3.3 Construction of model free implied volatility indices

In 1993, the Chicago Board Options Exchange (CBOE) introduced a stock market volatility index called the VIX (now VXO), also known as the "investor fear gauge". It was designed to measure the market's expectation of 30-day volatility and quickly became the benchmark risk measure of the U.S. equity market. The original VIX was constructed based on the Black-Scholes-Merton option pricing model (BSM hereafter) and is calculated using at-the-money options on the S&P 100 index. Ten years later in 2003, the CBOE made several significant changes to the original VIX and introduced a revamped VIX. The new VIX is based on the S&P 500 index, which was a timely switch as the S&P 500 has becoming more important as

the tracking index for the U.S. equity market. The construction of the new VIX no longer depends on any pricing model, instead, it is based on the concept of fair value of future variance developed by Demeterfi et al. (1999) and is calculated directly from market observables. The new VIX is model free and reflects market expectation of volatility over the next 30 calendar days.

The new VIX has become a core instrument for risk management since its transformation. It underlies a number of volatility derivatives such as VIX futures, options, and variance swaps which followed its introduction. The market for volatility derivatives has since been growing rapidly. Many exchanges across the world started to develop their own respective volatility index following the steps of the CBOE. Early attempts similar to the construction of the old VIX are VDAX, and VX1 and VX6 calculated respectively by the Deutsche Börse and the MONEP (Marché des Options Négociables de Paris). The method uses at-the-money options and is model dependent. These volatility indices have then later been redesigned following the update of the new VIX.

Up to date, the volatility indices for AEX and CAC40 are calculated and disseminated by Euronext¹. FTSE Group calculates and manages volatility indices for FTSE 100 and FTSE MIB. Eurex, also created a family of volatility indices: VDAX-NEW², VSMI, and VSTOXX³. NIKKEI Inc. introduced its volatility index for NIKKEI 225 index in November 2010 and has retroactively calculated the index on the end-of-day basis to June 1989. The Korea Exchange has developed a volatility index that suits the Korean market situation and published it in April 2009. Volatility index for Hang Seng equity index was launched by Hang Seng Indexes Company Limited in February 2011 and backdated to January 2001. In 2002, the Montréal Exchange introduced the Implied Volatility index (MVX) based on the old VIX methodology,

¹Table 3.6 lists each volatility index with its respective underlying equity index and index constituents.

²The original VDAX is still calculated and disseminated. The discussion of volatility index on DAX throughout this study refers to the new VDAX with Bloomberg ticker VIX.

³VSTOXX is the volatility index on DJ EURO STOXX 50, which consists of 50 largest sector leaders based on market capitalization in the Eurozone.

and only eight years later in October 2010 did they revise the method to reflect the construction of the new VIX. The history of the new volatility index VIXC for the Canadian stock market started in October 2010 and the original MVX stopped calculation and dissemination thereafter⁴.

While the construction of the new VIX is more appealing than its predecessor, there are flaws in the associated formula, as claimed by Jiang and Tian (2007). In their paper, they demonstrate that the CBOE procedure in constructing implied volatility leads to biases in the calculated values because the formula is not a perfect representation of the theoretical fair value of future variance, which in turn results in approximation errors. Any bias against the true model free implied volatility will render significant economical consequences. Since the VIX methodology prevails across the world's markets in creating volatility indices, it is important to re-evaluate the method and address the problems accordingly.

In the next section I first give a detailed description of the CBOE methodology in constructing the new VIX and its associated problems pointed out by Jiang and Tian (2007). I then provide a solution, proposed by Jiang and Tian (2007), with modification to fix the problem. Section 3.3.4 describes data used in this study and is followed by some empirical comparison with the CBOE procedure presented in section 3.3.5.

3.3.1 CBOE procedure for constructing VIX

Before I come to the description of VIX construction, the theoretical underpinning of the new VIX within the broader context of model free implied volatility (MFIV) should be appreciated. Breeden and Litzenberger (1978) have laid the pioneer foundation for subsequent research on extracting risk-neutral densities and model free variance from option prices. The concept of model free implied variance, which rose from the development of variance

⁴Volatility indices calculated and disseminated by organised exchanges or relevant parties revealed here may not be exhaustive. The coverage is limited to the interest of data available.

swaps, appeared in as early as Neuberger (1994), which is further developed by Demeterfi et al. (1999), Britten-Jones and Neuberger (2000), Bakshi et al. (2003), and Jiang and Tian (2005). Specifically, the new VIX is based on the concept of fair value of future variance developed by Demeterfi et al. (1999) (DDKZ variance hereafter).

$$V_{ddkz} = \frac{2}{T} \left\{ rT - \left[\frac{S_0}{S_*} \exp(rT) - 1 \right] - \ln\left(\frac{S_*}{S_0}\right) \right\} + \frac{2}{T} \left\{ \exp(rT) \int_0^{S_*} \frac{P(K, T)}{K^2} dK + \exp(rT) \int_{S_*}^{\infty} \frac{C(K, T)}{K^2} dK \right\} \quad (3.1)$$

where $C(K, T)$ and $P(K, T)$ are European call and put prices with strike price K and maturity T , r is the risk-free rate, S_0 is the current asset price, and S_* is the reference asset price that is close to the at-the-money forward price which often marks the boundary between liquid and illiquid options. The future evolution of the underlying asset is assumed to follow a diffusive process – this means no jumps are allowed. Assume the stock pays no dividends. Special case when S_* equals futures price F , $S_* = F = S_0 e^{rT}$, the first term in equation (3.1) is eliminated.

As explained in "The CBOE volatility index – VIX (2009)", the generalised formula used in the VIX calculation from market prices is:

$$\sigma_T^2 = \frac{2}{T} \sum_{i=L}^{i=U} \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2, \quad (3.2)$$

where T is time to expiration (in years), F is forward index level, and r is risk-free rate. K_0 is the first strike below the forward index level F . K_i is the i^{th} out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both call and put if $K_i = K_0$. $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike price K_i . ΔK_i is the increment in strike prices calculated as:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

ΔK_i for the highest strike is simply the difference between the highest strike and the next lower strike, and likewise for the lowest strike price, it is the difference between the lowest strike and the next higher strike.

For each contract month, the forward level F is determined by identifying the strike price at which the absolute difference between the call and put quotes are the smallest, and the forward level is then implied by put-call parity

$$F = \text{Strike Price} + e^{rT}(\text{Call price} - \text{Put Price})$$

Then K_0 is identified as the first strike price immediately below F . Options with non-zero bid quotes are selected in the order of strike price starting at K_0 . Out-of-the-money options are selected by choosing call options with strike price greater than K_0 and put options with strike price smaller than K_0 . After encountering two consecutive strike prices with zero bid, the selection process ends and no further options are included. Let K_L and K_U be the lowest and highest strike price selected according to the selection rule. At strike price K_0 , the average of the mid quote from the call and put option is used.

Equation (3.2) is intended to approximate equation (3.1) which is theoretically exact. Equation (3.2) measures implied variance over horizon T , whereas σ_{VIX}^2 is intended to measure the risk-neutral expected variance over a constant 30-day horizon. Since there are generally no options which expire exactly in 30 calendar days, two option series that are closest to 30-day expiry need to be considered. The near-term series must have at least one week to expiry to mitigate concerns of liquidity and microstructure effects. When the near-term options have less than a week to expiration, the VIX calculation rolls to the second and third maturity contracts. The implied variance is calculated for both maturities, and the desired VIX variance with the 30-day maturity is then linearly interpolated between the two available maturities

$$\sigma_{VIX}^2 = \frac{365}{30} [\omega T_1 \sigma^2(T_1) + (1 - \omega) T_2 \sigma^2(T_2)], \quad (3.3)$$

where

$$\omega = \frac{T_2 - 30/365}{T_2 - T_1}.$$

VIX is expressed as $\sigma_{VIX} \times 100$.

3.3.2 Problems associated with the CBOE procedure and solutions

It shows clearly from equation (3.1) that the model free implied variance is defined as an integral of weighted option prices over an infinite range of strike prices, but the actual construction procedure of the VIX used by the CBOE represents a discrete sum of weighted option prices plus a correction term. The approximation of an integral by a discrete proxy induces bias if care is not taken in minimizing implementation errors. As demonstrated by Jiang and Tian (2007), there are mainly two sources of approximation errors in the CBOE procedure which may bias the calculated implied volatilities⁵.

The first type of approximation errors is due to the limited availability of strike prices. If options are available for all strike prices, then it is straightforward to calculate the integral using numerical methods. But in reality, there is only a finite number of options actually traded in the market. Let K_U and K_L be the highest and lowest strike prices available for a given maturity, then truncation errors arise from the CBOE procedure as an infinite range of strike prices is replaced by a finite range between K_L and K_U :

$$\int_0^{K_0} \frac{P(K, T)}{K^2} dK + \int_{K_0}^{\infty} \frac{C(K, T)}{K^2} dK \approx \int_{K_L}^{K_0} \frac{P(K, T)}{K^2} dK + \int_{K_0}^{K_U} \frac{C(K, T)}{K^2} dK.$$

⁵Jiang and Tian (2007) report four types of approximation errors embedded in the CBOE procedure, some of which are unlikely to be economically significant. For the relevance of this study and possible remedies addressed to the problem, I mainly discuss two types of approximation errors.

The size of the truncation error is

$$\delta_{trunc} = -\frac{2}{T} \exp(rT) \left[\int_0^{K_L} \frac{P(K, T)}{K^2} dK + \int_{K_U}^{\infty} \frac{C(K, T)}{K^2} dK \right].$$

The negative sign indicates a downward bias in the calculated variance.

As pointed out by Jiang and Tian (2007), truncation error may vary substantially over time as the truncation interval $[K_L, K_U]$ is not fixed for all maturities. The reason lies in the way the CBOE filters out problematic options. Any option with a zero bid price is excluded from the VIX calculation. Options outside two consecutive strike prices with zero bid quotes are also excluded from the calculation. Since options with zero bid quotes are considered illiquid and they tend to cluster at extreme strikes (far away from at-the-money strikes), exclusion of options at such arbitrary boundaries would induce a significant reduction of the number of options included in the calculation, which in turn leads to a downward bias of the calculated value.

The second type of approximation errors is the discretization error due to CBOE's rather unusual numerical integration:

$$\int_{K_L}^{K_U} \frac{P(K, T)}{K^2} dK + \int_{K_0}^{K_U} \frac{C(K, T)}{K^2} dK \approx \sum_{i=L}^{i=U} \frac{\Delta K_i}{K_i^2} Q(K_i, T).$$

Although numerical integration errors can be minimised by using sufficiently finely partitioned strike prices, the actual partition used in the CBOE procedure is based on listed strike prices which are typically quite coarse. The size of the discretization error is

$$\delta_{disc} = \frac{2}{T} \exp(rT) \left\{ \sum_{i=L}^{i=U} \frac{\Delta K_i}{K_i^2} Q(K_i, T) - \left[\int_{K_L}^{K_U} \frac{P(K, T)}{K^2} dK + \int_{K_0}^{K_U} \frac{C(K, T)}{K^2} dK \right] \right\}.$$

The discretization error leads to an overestimation of the calculated variance. RHS of equa-

tion (3.1) requires the computation of the following

$$\int_0^{S_*} \frac{P(K, T)}{K^2} dK + \int_{S_*}^{\infty} \frac{C(K, T)}{K^2} dK.$$

The integrand function reaches a peak at $K = S_*$ and declines monotonically on both sides as K goes away from S_* ; it is discontinuous at $K = S_*$ whenever $S_* \neq F$. While S_* is an arbitrary value close to the forward level which generally does not coincide with any listed strike price, the option price at $K = S_*$ needs to be estimated. A relatively small error in the estimated option value may lead to a sizeable approximation error in the calculated variance since the integrand peaks sharply at S_* . Figure 3.10 illustrates BSM model simulated option prices plot against strike prices. Option price $Q(K_i)$ reaches a peak at $K = S_*$ (equal to 1 in this example) and declines sharply on both sides as K goes away from S_* . The option price $\hat{Q}(K_i)$ at K_0 in CBOE procedure is calculated as the average price of the call and put option at strike price K_0 . Jiang and Tian (2007) have shown that $\hat{Q}(K_i)$ tends to overestimate $Q(K_i)$. The larger the gap between K_0 and S_* , the bigger the overestimation.

Jiang and Tian (2007) performed Monte Carlo simulation to illustrate the magnitude of the approximation errors in the CBOE procedure. They choose a base set of parameters that are consistent with typical market conditions, and vary one parameter at a time while keeping others constant to examine the size of each respective approximation error. They show that the truncation error leads to an underestimation of the true volatility, and the errors are likely to rise sharply when volatilities are high. The discretization error leads to overestimation of the true volatility and declines as strike price increment increases. Both approximation errors are negligible when the truncation interval is sufficiently large and the differences between strike prices are sufficiently small.

They further analyse the approximation errors under a more realistic setting by using actual data on a given day. They show that total approximation error from the CBOE procedure

ranges from -4.4% to 2.6% of the true volatility, or -197 to +25 index basis points. One index basis point is worth \$10 per VIX futures. These approximation errors translate into -\$1970 to +\$250 per futures contract which is clearly economically significant. Although negative truncation errors tend to offset positive discretization errors when volatilities are low, negative errors tend to dominate the total error when the market experience a sharp increase in volatility, leading to underestimation of the true volatility, precisely when a more accurate measure of VIX is needed. Thus, a fix to the CBOE drawback is desired.

Jiang and Tian (2007) propose a simple smoothing method in constructing model free implied variance to mitigate approximation errors in the CBOE procedure. The general idea is to construct a smooth implied volatility function using an interpolation-extrapolation scheme. Suppose N strike prices are listed for trading on a given day with maturity T . Let $C_M(K_i, T)$ and $P_M(K_i, T)$ be market price of a call and a put option respectively with strike price K_i . Let $0 < K_L = K_1 < K_2 < \dots < K_N = K_U < \infty$. The implied volatilities $\sigma(K_i, T)$ for $i = 1, 2, \dots, N$ can be obtained from market option prices using the BSM model. Only out-of-the-money options are used in constructing implied volatility functions.

The implied volatility functions are constructed for the listed available strike prices between K_L and K_U . To construct the implied volatility function over the entire range of strike prices $(0, +\infty)$, N known implied volatilities must be used for interpolation and extrapolation. A nonparametric approach is adopted by Jiang and Tian (2007) in fitting the known implied volatilities exactly. Interpolation using cubic splines is first implemented between listed strike prices to construct a smooth function of implied volatilities. Flat extrapolation is then implemented outside the range of the lowest and highest strike price to construct an extension of the implied volatility function on the tails. Once the implied volatility functions are constructed for the entire range of the integral, option prices can be obtained using the BSM model. The model free implied variance can be thus calculated accordingly. Imple-

mentations of Jiang and Tian (2007) style interpolation methods are used in other papers, e.g. Taylor et al. (2010).

3.3.3 Construct implied variance – the BKM methodology

Calculation of implied variance based on equation (3.1) requires the use of S_* , a selected value, which in applications complicates calculation when dividends need to be taken into account⁶. To simplify calculation I adopt the model free implied variance proposed by Bakshi et al. (2003) (BKM hereafter) which applies directly to the use of spot asset prices, and also respects the continuum of options required for implied variance calculation. Let S_0 be the current asset price, and $C(K, T)$ and $P(K, T)$ be call and put option prices at strike price K with maturity T . The implied variance is given by

$$V_{BKM} = e^{rT} \left(\int_{S_0}^{\infty} \frac{2(1 - \ln \frac{K}{S_0})}{K^2} C(K, T) dK + \int_0^{S_0} \frac{2(1 + \ln \frac{S_0}{K})}{K^2} P(K, T) dK \right). \quad (3.4)$$

A continuum of options prices are needed to compute the integrals. I adopt Jiang and Tian (2005)'s interpolation and extrapolation method with modifications in achieving this. For the interpolation part, Jiang and Tian (2005) use cubic splines to fit the known implied volatilities. The output of the cubic spline fitting is a smooth function with a first and second order derivatives $f'(K)$ and $f''(K)$ at every strike price in the interval. Smooth as its advantage, spline method may overfit data and produces oscillations if the known implied volatilities are not smooth. Instead, I employ the piecewise cubic Hermite interpolating polynomials⁷ (pchip hereafter) to do the interpolation. Pchip constructs the interpolant almost the same way as the splines, except that the slopes are different. Splines choose the slopes such that

⁶Equation (3.1) gives the correct expected variance when dividends are paid only when $S_* = F$.

⁷Piecewise cubic Hermite interpolating polynomials and cubic splines are part of the computational routines for piecewise polynomial interpolation. See https://uk.mathworks.com/help/matlab/ref/pchip.html?s_tid=gn_loc_drop for detailed description and comparison between them.

the first and second order derivatives are all continuous, while it may not be the case for pchip at the second order derivative. The slopes for pchip are chosen in a way that the function preserves the shape of the data and respects monotonicity. This means that on intervals where data are monotonic, pchip provides no overfitting of data.

Following the implementation procedure of DeMiguel et al. (2009), I fit pchip with moneyness (K/S) as an independent variable and implied volatility as a function value to obtain interpolated implied volatilities within the known range of moneyness. I then extrapolate using the boundary strike prices to construct the tail distribution of implied volatilities. Following Carr and Wu (2009) and Jiang and Tian (2005), I implement a flat extrapolation scheme assuming that the implied volatility function is constant beyond the strike prices at both ends. Once the implied volatility function is constructed, option prices at any required moneyness level can be translated from the corresponding implied volatility using BSM model. Then the model free implied variance can be calculated following equation (3.4) for each maturity contract, and then linearly interpolated for the 30-day expiry.

3.3.4 Data

Data used for this study is the Ivy DB Global Indices database from OptionMetrics. This comprehensive database covers the world's major equity indices and provides end-of-day historical prices for index options and their associated underlying instruments. The database includes data for 27 equity indices covering 17 countries and regions around the world. As the interest of this study is to examine the dynamic relationship among world's major markets, I intend to include as much coverage as possible. Initial screening of the database gives 17 indices covering non-repetitive countries and regions. A closer look into each index options finds that some data are problematic and thus excluded from the study⁸. Table 3.1

⁸Options for the Australian stock index S&P/ASX 200 show multiple prices of calls and puts with the same strike price for year 2007 and 2009. I am not able to distinguish between these options as to which is correctly priced for the information given. In addition, I find irrational prices sitting around for deep in- and out- of-

shows 15 equity indices I include in this study and their respectively availability of option data⁹.

To calculate model free implied variance given by equation (3.4), option data needs to be filtered. For each index, I include options with strictly positive bid prices¹⁰, and discard options with missing implied volatilities or Black-Scholes (BS) delta, which occurs for options with intrinsic value above mid price and when options have non-standard settlement. Option series with maturity shorter than 7 days and longer than 300 days are excluded to minimise pricing anomalies. Interest rates are zero coupon rates obtained from the zero_curve file in the database, and they are linearly interpolated to match each option maturity.

I select out-of-the-money options, namely, calls with BS delta below 0.5 and puts with delta above -0.5. I define moneyness by K/S , following Jiang and Tian (2005), options with extreme moneyness beyond [0.7, 1.3] are excluded due to extremely high implied volatilities. I fit pchip to known implied volatilities points within the interval to obtain the interpolated implied volatility function and then extrapolate beyond the moneyness to obtain tail functions. I fill a total of 1001 grid points in the moneyness range from 1/3 to 3 for the implied volatility function. As suggested by Jiang and Tian (2005), truncation errors are virtually zero when truncation points are beyond 3.5 standard deviations from $K/S = 1$. The discretization increment is below 0.01, which is a sufficiently fine partition for the discretization error to be negligible.

Once implied volatility function is obtained for the range of moneyness, option prices for each moneyness (or strike price) can be translated from the BSM model through interpolated implied volatilities. Subsequently, the model free implied variance for each equity

the-money options throughout year 2013 and 2014, which I believed are either mis-priced or mis-reported by the data provider. Thus, the Australian index is excluded from examination. The other index excluded is TAIEX index of Taiwan, whose options history is only available until 2008.

⁹Note that the raw data is incomplete when provided by OptionMetrics where data for all indices for March, April, and May are missing for years 2005, 2006, and 2007.

¹⁰Bid, ask price are defined as the best closing bid, ask price across all exchanges. Where bid ask prices are not available for some of the indices options, last price, defined as the closing trade price or the settlement price of the option, is used as a proxy for the bid price.

index on each maturity can be calculated. The implied variance of 30-day horizon can be obtained using two nearest maturities by linear interpolation, and implied volatility is taken as the square root of the implied variance. The time series of constant 30-day expectation of implied volatility is termed volatility index for each underlying equity index.

3.3.5 Empirical results

First I compare my calculation of implied volatilities with those calculated using the CBOE procedure. Table 3.6 provides a brief overview of publicly available volatility indices that are calculated and disseminated by organised exchanges or relevant party. As can be seen that volatility indices are available for most of the equity indices under examination, except for BEL 20,¹¹ HELSINKI 25, and IBEX 35. The methodology to construct volatility index revealed by each respective calculation body is almost identical to the CBOE procedure in constructing VIX. Trivial differences to accommodate the local market lie in the selection of options where the choice of rolling convention or the correction term are not exact replication. A slight deviation from the CBOE procedure is the method used for Hang Seng volatility index. The method replicates exactly the first term of equation (3.2) but leaves out the second term, which was claimed to reflect its market trading characteristics. The outcome of this choice would result in higher implied volatility than it otherwise would. FTSE calculates volatility index for both FTSE 100 and FTSE MIB. Its methodology deviates from the CBOE procedure in that they employ a slightly different numerical integration scheme. Options are partitioned into three contiguous groups. Simpson's rules with unequal-interval is used to approximate the integral. The weights given to each option price is a weighted combination of the nearest three strikes as opposed to the simple difference between two adjacent strike prices as in the CBOE procedure, but the method does not solve material problems

¹¹The equity index BEL 20 is under management by Euronext Brussels. They do not seem to provide volatility index for BEL 20 any more despite that there has been a record for doing so.

with truncation interval and discretization issues.

Table 3.7 reports empirical results of implied volatility calculated using the BKM method compared with those under CBOE procedure. The publicly available volatility indices are obtained through Bloomberg. Implied volatility calculated using BKM method is denoted *bkm_* its respective equity index or *IV_bkm* for general referral. Bloomberg implied volatility is denoted *IV_bb*. Data length of each comparison is subject to data availability of both the Bloomberg source and the calculated values. Comparisons are made for one to one date matching between *IV_bb* and *IV_bkm*. Dates with missing values are excluded in both sources. Numbers in the last four columns are in percentages.

Column 2 of Table 3.7 reports the number of total observations of each pair under comparison. Columns 4 and 5 report percentages of observations that the value of *IV_bkm* is higher (lower) than *IV_bb*. It shows that 9 out of 12 implied volatility series calculated using the BKM method produce more higher values than those under CBOE procedure, ranging from 50.80% to 99.92%. Only three series produce less higher values, with the lowest being 25.44% for *IV_Euro STOXX 50*. It thus lends support to the claims of Jiang and Tian (2007) that the CBOE procedure tends to underestimate the true volatility. Note that here I do not disentangle between the truncation errors and discretization errors. The numbers are for the total effect. It may well be the case that the two errors offset each other as their signs are opposite. But in most cases values of Bloomberg sources are lower than those of my calculation, implying that truncation errors are more dominant leading to the underestimation of the CBOE methodology.

Columns 6 and 7 report mean values for each comparison that *IV_bkm* exceeds (is below) *IV_bb*¹². It shows that the mean value that *IV_bkm* exceeds *IV_bb* ranges from +1.15% to +4.81%, with the exception of +29.04% for *IV_S&P/TSX 60*. The mean value that *IV_bkm* is below *IV_bb* ranges from -3.93% to -0.17%. It summarises the magnitude of approximation

¹²Values in column 6 and 7 are calculated as the average of $\frac{IV_bkm - IV_bb}{IV_bb}$.

errors produced by the CBOE procedure. It can be seen that the range of errors are rather symmetric on both directions (except for one). My results are similar in magnitude to the example illustrated by Jiang and Tian (2007) using one day of market data.

The empirical results are odd for IV_S&P/TSX 60. Results show that implied volatilities calculated using my method are significantly higher than those provided by Bloomberg, see Figure 3.2. Only one data point has a value smaller than that of IV_bb. The average magnitude of values exceeding IV_bb is +29.04%, much higher than that for the rest of the comparisons. The Montréal Exchange calculates VIXC exactly the same way as VIX with minor changes in the rolling convention and calculation of risk free rate. Figure 3.2 shows that the values are gently above IV_bb at the beginning of the series, but deviate significantly starting July 2012. Obvious oscillation is shown for VIXC for the first half of 2014, where IV_S&P/TSX 60 shows similar pattern but with bigger magnitude. A few individual spikes are shown on the latter part of the series. It is not clear why IV_S&P/TSX 60 deviates substantially from IV_bb. Figure 3.3 shows one example of a ‘well behaved’ calculated volatility index which is closely aligned to IV_bb, namely, IV_S&P 500. Table 3.8 presents summary statistics of calculated implied volatility series together with those of Bloomberg source.

Table 3.9 provides information on near- and next-term maturity options used to calculate IV_bkm. Columns (days) report average maturities in calendar days. Columns (nopt) and (range(nopt)) report the average number, and the range, of options observed for pchip linear interpolation. The average minimum and maximum moneyness over time for each index is given in columns (min) and (max), followed by two columns of the range of the minimum and maximum moneyness. The average minimum and maximum moneyness for near-term (next-term) maturity options across sample range from 0.72 to 0.86 and 1.10 to 1.23 (0.73 to 0.86 and 1.10 to 1.25). The range of minimum and maximum moneyness given by available strike prices after options are filtered illustrates possible truncation intervals

used in the CBOE procedure. Options beyond the truncation interval are ignored, which leads to truncations errors.

Volatility indices are designed to reflect the market's expectation of future volatility over 30 days, and are closely followed by market participants. It is essential to the pricing of volatility derivatives, such as future, options on volatility index, and variance swaps. It is also important for estimating variance risk premium. A correctly constructed volatility index is thus the key to ensure the accurate pricing of volatility derivatives. The CBOE procedure in constructing VIX leads to substantial biases in the calculated values due to approximation errors. Thus, a robust method providing a reliable volatility index is called for in building investors confidence.

3.4 Implied volatility spillover effect

This part of the study discusses international market integration through implied volatility transmission. Early studies have shown that the stock market is highly integrated in terms of realised returns and realised volatilities. Since volatility plays a key role in derivative pricing and portfolio management, it is important to learn about whether its future expectation across different markets are linked with each other. Volatility indices provide good tools as they are designed for risk management purposes. Since they are directly extracted from option prices, they are forward looking and contain timely information regarding the future development of the market. The linkage between markets provides market participants a view ex-ante as opposed to ex-post using returns or realised variance. I will be able to see whether market participants have different views towards future volatility development in different markets. The interaction between markets may answer the question whether one market drives another, which may have important implications on asset allocation and risk management. It may also be useful for option traders and portfolio managers in implement-

ing innovative trading strategies in the option market.

Implied volatilities used in this part of analysis are calculated from section 3.4. I employ vector autoregressive models and its structural analysis, namely, impulse response functions and forecast error variance decomposition to analyse the dynamic inter relationship between markets. Section 4.1 describes how data is structured. Section 4.2 gives a preliminary glance at linkages between markets under contemporaneous correlation. Model specification and empirical results follows.

3.4.1 Data

The analysis of transmission effects is done for different groups. Since data includes countries covering three continents, and I am interested in the dynamic relationships of the variables within a continent as well as across continents, I group implied volatilities into Asia, European area, and North America according to their respective geographic locations, and a cross-continent group, Global. The analysis within each group requires a common time window for all variables subject to data availability of implied volatilities calculated in section 3.4. Table 3.2 shows a description of the grouping of the variables and their time window.

Groups Asia and North America are intuitively comprised of countries and regions in their respective continent. European area is comprised of countries in continental Europe as well as the U.K.. Eurozone (represented by the STOXX 50 index) is excluded from the analysis of European area, as it essentially represents a collection of companies of the eurozone, which might induce spurious effects when examined together with its component countries. For Global analysis I consider one country or region as a representative of each continent to avoid calculation overload. Japan is chosen as the representative of Asia as it is one of the most important markets in the world and has the highest market capitalization among the

three countries in its group. Eurozone represents the continental European area¹³, and U.K. is picked as it is one of the most influential markets in the world.

Time frame for each group depends on data availability. The longest shared period within each group is chosen as its study period. Though calculated implied volatilities of European countries generally have a longer history, there is a portion of data missing for some consecutive periods in 2008 for the Dutch market. Thus common period for the European area is chosen to start in March 2009 where periods of very high volatilities due to the financial crisis are excluded.

3.4.2 Preliminary discussion

Given that each market operates in different time zones, it is necessary to understand the operating hours of one market relative to another on the same scale. Figure 3.1 shows information of option market trading hours for each country in Tokyo time¹⁴, which is the first market to open on a given day. The time zone of Tokyo is UTC + 9. As can be seen in the figure the Asian markets are the first to open, then followed by the European markets and lastly North American markets. Japanese market closes at 15:00 before any European market opens, while the U.S. market opens at 23:30 with about 2 hours operating concurrently with the European markets.

Table 3.10 shows contemporaneous correlations for the differenced implied volatility series¹⁵ among markets for each group. Group B shows that pairwise correlation among European areas are much higher than those in Asia and North America. Correlations are more than one half for most pairs in the European region except for those with Finland. Correla-

¹³Only Swiss market in the sample is not accounted for by Eurozone.

¹⁴Note that Japan does not use daylight saving time, whereas North American markets and the European markets observe summer time. Relative trading hours discussed in this analysis is based on standard time (winter time).

¹⁵Differenced implied volatility series are used for the following study of spillover analysis. Reasons for such choice is detailed in the next section.

tions between Finland and other European countries are between 0.31 and 0.45, suggesting that the Finnish market might be less integrated in the European area. Second to Finland is Italy which has relatively low correlations with other European countries. Correlation coefficients are lowest at 0.31 with Finland and highest at only 0.56 with France, suggesting Italy might have less influence on other countries in the region. The highest pairwise correlations are among U.K., France, Germany, and Switzerland. These four markets are the largest markets in the European area in terms of their market capitalization. High correlations among them may reflect fast information flow as larger markets tend to be more liquid.

Group A shows that the intra-regional pairwise correlations tend to be higher than inter-regional correlations. The highest being 0.80 between U.K. and Eurozone; the lowest being 0.13 between Japan and U.S.. This pattern of contemporaneous correlations is consistent with what I expect from the structure of time zone differences between markets. Geographically the closer the markets the higher the correlation, while the further apart between markets, the lower the correlation. This may also reflect the degree to which markets are integrated. The more integrated two markets are, the more strongly one market movement may be correlated with the other.

Developments in the Japanese market does not seem to influence much on the U.S. market, while correlations between the U.S. market and two European markets are higher, which may reflect the fact that the U.S. market and the European markets simultaneously operate for about 2 hours before the latter close. The low correlation between the Japanese market and the U.S. market seems to suggest that U.S. influences Japan. If U.S. were to be influenced by Japan, then the U.S. market would have responded to the Japanese market, which closes before U.S. market opens, on the same day. This in turn would result in a higher correlation between the two markets. Contrary to results reported in Eun and Shim (1989) that the U.S. and Canada exhibits the highest correlation for residual returns, my results show that the

correlation of changes in implied volatility between these two markets are moderate.

3.4.3 VAR model specification

To analyse the transmission of implied volatility, vector autoregressive analysis (VAR) and its structural analysis are employed. The VAR model, popularised by Sims (1980), treats all variables as a priori endogenous, and describes the dynamic evolution of its component variables from their common history. The general mathematical form of the VAR(p) model is:

$$\mathbf{Y}_t = \mathbf{c} + \Pi_1 \mathbf{Y}_{t-1} + \Pi_2 \mathbf{Y}_{t-2} + \cdots + \Pi_p \mathbf{Y}_{t-p} + \boldsymbol{\epsilon}_t \quad (3.5)$$

where $\mathbf{Y}_t = (y_{1t}, y_{2t}, \dots, y_{kt})'$ denotes a $(k \times 1)$ vector of times series variables of interest. \mathbf{c} is a k dimensional constant vector. Π_i are $(k \times k)$ coefficient matrices, and $\boldsymbol{\epsilon}_t$ is an $(n \times 1)$ unobservable vector of random variables with zero mean and covariance matrix Σ . $\boldsymbol{\epsilon}_t$ is serially uncorrelated but may be contemporaneously correlated. p is lag length. Applied to the Global group variables, the VAR(p) system is¹⁶:

$$\begin{aligned} US_t &= c^{US} + \sum_{i=1}^p A_{1i}^{US} US_{t-p} + \sum_{i=1}^p B_{1i}^{UK} UK_{t-p} + \sum_{i=1}^p C_{1i}^{Euro} Euro_{t-p} + \sum_{i=1}^p D_{1i}^{JP} JP_{t-p} + \epsilon_t^{US} \\ UK_t &= c^{UK} + \sum_{i=1}^p A_{2i}^{US} US_{t-p} + \sum_{i=1}^p B_{2i}^{UK} UK_{t-p} + \sum_{i=1}^p C_{2i}^{Euro} Euro_{t-p} + \sum_{i=1}^p D_{2i}^{JP} JP_{t-p} + \epsilon_t^{UK} \\ Euro_t &= c^{Euro} + \sum_{i=1}^p A_{3i}^{US} US_{t-p} + \sum_{i=1}^p B_{3i}^{UK} UK_{t-p} + \sum_{i=1}^p C_{3i}^{Euro} Euro_{t-p} + \sum_{i=1}^p D_{3i}^{JP} JP_{t-p} + \epsilon_t^{Euro} \\ JP_t &= c^{JP} + \sum_{i=1}^p A_{4i}^{US} US_{t-p} + \sum_{i=1}^p B_{4i}^{UK} UK_{t-p} + \sum_{i=1}^p C_{4i}^{Euro} Euro_{t-p} + \sum_{i=1}^p D_{4i}^{JP} JP_{t-p} + \epsilon_t^{JP} \end{aligned}$$

Here US_t , UK_t , $Euro_t$ and JP_t denote the implied volatility measures of U.S., U.K., Eu-

¹⁶All following illustrations of model specification are given for group Global, and the same methodology applies to the rest of the groups where discussions of empirical results are in subsequent sections.

rozone, and Japan, with

$$\Pi_1 = \begin{bmatrix} A_{11} & B_{11} & C_{11} & D_{11} \\ A_{21} & B_{21} & C_{21} & D_{21} \\ A_{31} & B_{31} & C_{31} & D_{31} \\ A_{41} & B_{41} & C_{41} & D_{41} \end{bmatrix}, \dots, \Pi_p = \begin{bmatrix} A_{1p} & B_{1p} & C_{1p} & D_{1p} \\ A_{2p} & B_{2p} & C_{2p} & D_{2p} \\ A_{3p} & B_{3p} & C_{3p} & D_{3p} \\ A_{4p} & B_{4p} & C_{4p} & D_{4p} \end{bmatrix}.$$

The VAR model requires variables generated from a stationary process. The econometrics analysis can be done either in levels or in changes. However, the choice is not easy for this particular data. The ADF unit root tests shown in Table 3.3 indicate that the time series of implied volatility levels for U.K., Eurozone, and Japan are stationary at the 1% level, whereas U.S. level implied volatility is only stationary at 5% level. Time series of implied volatility changes are stationary at all conventional levels for all countries. First order autocorrelation in level volatilities are high, being 0.982, 0.983, 0.977 and 0.981 respectively for the U.S., U.K., Eurozone and Japan. This is intuitive as volatilities are known to be persistent and have a long memory feature. However, the high serial correlations would give a highly autocorrelated error term, which may induce model mis-specification. The results suggest that an analysis in differences may be necessary. Thus, I examine how a change in implied volatility of one market at its close impacts another.

The VAR model is estimated with OLS. In order to estimate the model, the number of lags p needs to be determined, however, it is never an empirically easy issue. Longer lag lengths lessen the chance of mis-specification, but result in the loss of more degrees of freedom. Akaike (AIC), Schwarz (SIC), and Hannan-Quinn (HQ) information criteria, final prediction error (FPE), and modified LR test are used to select the lag order. As mentioned in Lütkepohl (2005), the lag order obtained with information criteria depends to some extent on the choice of the initial guess. Choosing a small lag, an appropriate model may not be in the set

of possibilities and choosing a large number may result in a large spurious value. Using a moderate value seems to be a sensible strategy. The same as Äijö (2008), I choose $p = 8$ lags to start with.

Table 3.4 shows the determination of lag order given by aforementioned information criteria. While AIC, FPE, and modified LR test suggest lag length of 8 to be appropriate for the VAR(p) model, SIC and HQ suggest optimal lag length 7. Model adequacy can be further checked by some formal tests. If the number of lags is appropriate, there should be no autocorrelation left in the residuals. Portmanteau and Breusch-Godfrey-LM tests are standard tools for checking residual autocorrelation in VAR models. However, the multivariate portmanteau test and LM test collectively reject the null hypothesis of no serial correlation up to lag 12. The Q-stats are as big as a few multiples of its respective degrees of freedom. Higher order of lags have been tried and the same rejection of no serial correlation results. This result indicates possible serious autocorrelation in the VAR system which would bias estimates of coefficients and standard errors in the regression.

However, the cross correlograms in Figure 3.4 show that there is no obvious pattern in the residuals up to lag 7, but some significant values at lag 10. While there is no theory suggesting autocorrelation occurs in latter lags but not lower lags, it appears that for the underlying data, the changes of implied volatility 10 days apart tend to coincidentally move in the same direction. This leads us to believe that failing the test of no serial correlation may be due to the heteroskedastic feature of the data rather than actual autocorrelation. AR roots table shows that all inverse roots have modulus less than one and lie inside the unit circle, which means the estimated VAR system is stable, and which further confirms that 7 lags is adequate in capturing the system dynamics. I proceed on to structural analysis with the VAR(7) model.

3.4.4 Impulse response analysis

3.4.4.1 Theory

The general VAR(p) model has many parameters, and it may be difficult to interpret coefficients due to complex interactions and feedback effects between variables in the model. As a result, the dynamic properties of a VAR system are often summarised through some structural analysis. Two main types of structural analysis are impulse response functions and forecast error variance decompositions. I will employ both analyses to study the dynamics in the VAR system.

Impulse response analysis traces the effects of a shock to one endogenous variable on to the other variables in the VAR system. It moreover traces the speed and persistence of the shocks, and therefore enables the examination of the time structure of the transmission. I consider in this study generalized impulse response functions¹⁷ as opposed to traditional impulse response in that the generalized responses do not require orthogonalization of shocks and are invariant to the ordering of the variables in the VAR system. Since there is no economic theory underpinning the causal relationship between country level implied volatilities, it is reasonable to use generalised impulse response functions to study the inter-relationship between them without any a priori assumption.

Under the assumption of covariance-stationarity, equation (3.5) can be rewritten as the infinite moving average representation,

$$\mathbf{Y}_t = \omega + \sum_{i=0}^{\infty} \mathbf{A}_i \epsilon_{t-i} \quad (3.6)$$

Where the $k \times k$ coefficient matrices \mathbf{A}_i can be obtained by recursive substitution,

$$\mathbf{A}_i = \Pi_1 \mathbf{A}_{i-1} + \Pi_2 \mathbf{A}_{i-2} + \dots + \Pi_p \mathbf{A}_{i-p}, \quad i = 1, 2, \dots,$$

¹⁷See Pesaran and Shin (1998).

with $\mathbf{A}_0 = \mathbf{I}_k$ and $\mathbf{A}_j = \mathbf{0}$ when $j < 0$.

An impulse response can be described as the effect of a hypothetical $k \times 1$ vector of shocks of size $\delta = (\delta_1, \dots, \delta_k)'$ hitting the system at time t compared with a base-line profile at time $t + n$, given the system's history. Denoting the known history of the system up to time $t - 1$ by the information set Ω_{t-1} , the generalised impulse response function of \mathbf{Y}_t at horizon n , is defined by

$$\mathbf{GI}_Y(n, \delta, \Omega_{t-1}) = E(\mathbf{Y}_{t+n} | \epsilon_t = \delta, \Omega_{t-1}) - E(\mathbf{Y}_{t+n} | \Omega_{t-1})$$

The appropriate choice of hypothesized vector of shocks, δ , is central to the properties of the impulse response function. The generalised impulse response function shocks one element of ϵ_t , say the j th element, and integrates out the effects of other shocks using an assumed or historically observed distribution of errors. Then we have

$$\mathbf{GI}_Y(n, \delta_j, \Omega_{t-1}) = E(\mathbf{Y}_{t+n} | \epsilon_{jt} = \delta_j, \Omega_{t-1}) - E(\mathbf{Y}_{t+n} | \Omega_{t-1}). \quad (3.7)$$

Assuming ϵ_t has a multivariate normal distribution with mean vector zero and covariance matrix $\Sigma = (\sigma_{ij})$, the conditional expectation is

$$E(\epsilon_t | \epsilon_{jt} = \delta_j) = (\sigma_{1j}, \sigma_{2j}, \dots, \sigma_{kj})' \sigma_{jj}^{-1} \delta_j = \Sigma \mathbf{e}_j \sigma_{jj}^{-1} \delta_j$$

where \mathbf{e}_j is an $k \times 1$ selection vector with unity at its j th element and zeros elsewhere. Hence, the generalised impulse response of the effect of a shock in the j th equation on \mathbf{Y}_{t+n} at time t is given by

$$\left(\frac{\mathbf{A}_n \Sigma \mathbf{e}_j}{\sqrt{\sigma_{jj}}} \right) \left(\frac{\delta_j}{\sqrt{\sigma_{jj}}} \right), \quad n = 0, 1, 2, \dots$$

Setting $\delta_j = \sqrt{\sigma_{jj}}$, the scaled generalised impulse response function is the $k \times 1$ vector:

$$\psi_j(n) = \frac{1}{\sqrt{\sigma_{jj}}} \mathbf{A}_n \Sigma \mathbf{e}_j, \quad n = 0, 1, 2, \dots \quad (3.8)$$

3.4.4.2 Results for the Global group

Figure 3.5 presents the results of impulse response analysis with 2 standard error bounds. The results show that given a shock to any variable in the system, all other variables will have a positive contemporaneous response to that shock, but the magnitude of responses is different for each variable. A shock in the change of implied volatility in Japan only gives rise to about a half unit response of change of implied volatility for U.S., U.K., and Eurozone, while those responses are much stronger towards a shock in the U.S., U.K., or Eurozone. The responses are more than 1 unit. Japan clearly has a lagged response to the Western markets. It has a very small response to the U.S. market on day 1 but has an increased response on day 2. This could be explained by the fact that Japan is on the far East of the globe, and there is about 8.5 hours difference between the close of the Japanese market and the open of the U.S. market. Any information not captured on day 1 is flowing into day 2 when Japanese markets opens about 3 hours after the U.S. market closes, which gives Japan a stronger reaction on day 2 than day 1.

Similarly Japan has a lagged response to the two European markets, but the responses on day 2 are decreasing possibly because any news in these two markets has been communicated with the U.S. market during their 2 hours trading overlap, and is captured by the responses to the U.S. market. While the impulse responses of Japan to the U.S., U.K., and Eurozone are positive on the next day, those of U.S., U.K., and Eurozone are corrected on day 2. This indicates that the responses of U.S., U.K., and Eurozone to the shocks of Japan are incorporated within one day but Japan has a lagged response on the following day, which

suggests that the U.S., U.K., and Eurozone markets lead the Japanese market by one day. Analogously, the U.K. and Eurozone market have a lagged response to the U.S. market while the U.S. markets responds to the two European markets only on day 1, which indicates that the U.S. market leads the U.K. and Eurozone market by one day. The figure shows clearly that the impact of the shocks are positive on day one or day two, and start to decay thereafter, which suggests that these markets are efficient in that international news are processed within two days.

The time sequence of markets is not arbitrary. On a given date, Japan is at the top, Eurozone and U.K. come in the middle, and U.S. at the bottom; simply because the Japanese market is the first to open and U.S. is the last to close. But one can argue that which market comes first depends on where one cuts into the chain, and we should expect to result in the same transmission mechanism under different ordering. Hence, I advance U.S. and European market each to the top of the chain by appropriately lagging the other variables.

Figure 3.6 shows the impulse responses when the change of implied volatilities are grouped as US_t , $Eurozone_{t+1}$, UK_{t+1} and JP_{t+1} . It can be seen that US_t has a lagged response to UK_{t+1} and $Eurozone_{t+1}$ with an increased next day response. This is because the two continental markets are grouped such that they are at the two far ends of the chain, that UK_{t+1} and $Eurozone_{t+1}$ have an immediate impact on US_{t+1} but inevitably not much on US_t . The pattern of the impulse responses of Japan to shocks in the U.K. and Eurozone is the same as in the previous discussion meaning that the U.K. and Eurozone markets lead Japanese market by one day. But JP_{t+1} does not have a lagged response to shocks in US_t as JP_{t+1} sits immediately after US_t , of which information is processed contemporaneously. Likewise for U.K. and Eurozone, the impact of shock to U.S. on day t is fully processed on day $t + 1$, suggesting that U.S. leads U.K. and Eurozone market by one day.

In Figure 3.7, the variables are arranged as $Eurozone_t$, UK_t , US_t , and JP_{t+1} . Results

show that JP_{t+1} has a stronger impact on $Eurozone_{t+1}$ and UK_{t+1} than on $Eurozone_t$ and UK_t . Putting Japan on the bottom of the chain helps us to see that the shocks to U.S., U.K., and Eurozone on day t is fully incorporated by Japan on day $t + 1$, which suggests that U.S., U.K., and Eurozone market leads Japanese market by one day. Similarly, $Eurozone_t$ and UK_t have a lagged response to US_t , which suggests that the U.S. market leads U.K. and Eurozone market by one day. In summary, all three sets of impulse responses tend to agree that the Japanese market is a follower in this global setup while U.S. is the dominant leader in implied volatility spillover. The European markets sit in the middle where it follows the U.S. and leads Japan.

The aforementioned argument can be further supported by a closer look at the VAR regressions. Table 3.5 shows the R^2 of each component regression equation under a VAR(3)¹⁸ specification, as well as the breakdown of lags included in the estimation. The table reports results of all three groups under different ordering of the variables. Group one shows that when variables are grouped on the same timeline day t , lags up to 3 periods are included in the estimation for each of the variable. The Japan equation has the highest $R^2 = 0.28$ whereas U.S. has the lowest at 0.06. When U.S. is advanced to the top of the timeline and JP, Eurozone, and U.K. are lagged one day in the second group, R^2 changed significantly. The lags included in the U.S. equation are $t - 1, t - 2$ and $t - 3$ while those included in JP, Eurozone, and the U.K. equation are $t, t - 1$ and $t - 2$. R^2 increased substantially for the U.S. equation to 0.44 from previous 0.06 by including $Eurozone_t, UK_t$, and JP_t in the estimation, indicating that changes of implied volatility of $Eurozone_t, UK_t$, and JP_t have a significant impact on the changes of implied volatility of U.S. on day t . Omitting the lag of US_t in the equation of $Eurozone_{t+1}, UK_{t+1}$, and JP_{t+1} significantly reduced the explanatory power.

¹⁸Different ordering of the variables requires distinct VAR specification. The system is best described by a VAR(7), VAR(3), and VAR(3) respectively for each group. But in order to disentangle the marginal effect of inclusion(exclusion) of one specific lag in a particular equation, I employ a VAR(3) model for all groups for consistent comparison.

R^2 decreased to 0.04, 0.04, and 0.14 respectively, indicating that US_t has a strong influence on $Eurozone_{t+1}$, UK_{t+1} , and JP_{t+1} . In the third group when Japan is ordered the last of the chain, results show that R^2 of Japan equation reduced to 0.05 when $Eurozone_t$, UK_t , and US_t are not included in the equation, indicating $Eurozone_t$, UK_t , and US_t have a significant impact on JP_{t+1} . The results show consistent evidence that the U.S. market impacts other markets on the following day, and the two European markets impact Japanese market on the following day.

3.4.5 Forecast error variance decomposition

While impulse response functions trace the effect of shocks from one endogenous variable on other variables in system, variance decomposition ascertains the proportion of innovations in other variables in explaining forecast variances of one variable. It provides the relative importance of each random innovation in affecting the variables in the VAR system. Decomposing the error variance requires uncorrelatedness of the innovations in the VAR system. Since ϵ_t can be contemporaneously correlated, it is desirable to orthogonalise the shocks. The traditional approach uses Cholesky decomposition of Σ such that

$$\mathbf{P}\mathbf{P}' = \Sigma,$$

where \mathbf{P} is an $k \times k$ lower triangular matrix. Then equation (3.6) can be rewritten as

$$\mathbf{Y}_t = \omega + \sum_{i=0}^{\infty} (\mathbf{A}_i \mathbf{P}) (\mathbf{P}^{-1} \epsilon_{t-i}) = \omega + \sum_{i=0}^{\infty} (\mathbf{A}_i \mathbf{P}) \xi_{t-i}$$

Such that $\xi_t = \mathbf{P}^{-1}\epsilon_t$ are orthogonalised, and that $\mathbf{E}(\xi\xi') = \mathbf{I}_k$. Then, the proportion of n -step ahead forecast error variance of variable i which is accounted for by variable j is given by

$$\theta_{ij}(n) = \frac{\sum_{l=0}^n (\mathbf{e}_i' \mathbf{A}_l \mathbf{P} \mathbf{e}_j)^2}{\sum_{l=0}^n (\mathbf{e}_i' \mathbf{A}_l \Sigma \mathbf{A}_l' \mathbf{e}_i)} \quad i, j = 1, 2, \dots, k.$$

Note that by the law of total variance $\sum_{j=1}^k \theta_{ij}(n) = 1$.

Figure 3.8 reports the results of variance decomposition for the four markets¹⁹. Forecast periods are for 1 to 10 days ahead. Each entry in column 3 to 6 denotes the percentage of forecast error variance of the panel variable explained by column variables. It can be seen that substantial amounts of interactions are shown across continents. The variance of one market is accounted for by its own innovations as well as innovations in other markets. The results indicate that U.S. market is the most influential one in the world. Its own innovations accounts for about 98% of its own variance while the other three markets collectively account for 2%. On the other hand, innovations in the U.S. market accounts for a significant portion of variances in other markets, 38%, 32%, and 22% respectively for the U.K., Eurozone, and Japanese market. The U.K. market turns out to be the second most influential market in that it explains 31% of Eurozone market's variance and 6% of Japanese market's variance. The Japanese market appears to be a follower in the international markets. Innovations in the Japanese market explain only about 1% of the variances of the other three markets.

¹⁹Cholesky decomposition of Σ implies that the shock to the second variable does not affect the first variable contemporaneously, but both shocks on the first and second variable can have a contemporaneous effect on the second variable. Hence, variance decomposition could be sensitive to the ordering of the variables. The order of the variable in this analysis is U.S., U.K., Eurozone, and Japan, which is determined by its relative importance from impulse response analysis. However, when I change the order of the variables, the U.S. still emerges as the most dominant market.

3.4.6 Empirical results for groups European area, Asia, and North America

In this section I briefly discuss the findings of impulse response analysis for the European area, Asia and North America²⁰. Figure 3.9 shows impulse responses for the European area. Each individual graph shows combined impulse responses of one variable to shocks to all other variables. The results reveal some patterns of regional effect. It can be seen that all markets respond to a shock in another market in almost an identical fashion. They accommodate the shocks in other markets within one day, which indicate that the European markets have a high degree of integration and possible free flow of information. However, there is one exception in this group, Italy. The graph shows that Italy has a rather sluggish response to shocks in all other markets. It continues to react noticeably on the following day, and the adjustment is completed in day 2. It suggests that Italy acts as a follower in the European region.

Further examination reveals the interesting position of the Finnish market. The bottom right corner graph shows that the responses of the Finnish market are collectively smaller than any other market to exogenous innovations. The size of the responses is about 0.5. The rest of the graphs show at the same time that responses of all other markets to shocks to the Finnish market is likewise the smallest. The weak interaction with other European markets suggests that the Finnish market is rather independent and less integrated in the European region, which confirms its low correlation with other European markets. A similar conclusion is drawn in Nikkinen and Sahlström (2004).

The results also show that, the U.K., German, French, and Swiss markets, being the bigger markets in the European region, have a much stronger influence on the rest of the smaller

²⁰I do not perform forecast error variance decomposition analysis for each regional market. Regional markets are highly correlated with one another and exhibit feedback effects among each other. It is thus hard to identify the sequence of information flow, which renders the ordering of variables difficult.

markets, as smaller markets react strongly to bigger markets while not vice versa. It suggests that any news emerges in the bigger markets will be quickly transmitted to the smaller markets, while news in the smaller markets tend to be local which will not have a big impact outside its own market. Clearly, bigger markets play a more dominant role in producing information in the region.

Results of groups Asia and North America²¹ reveal the same regional pattern. All shocks are absorbed within the same day, indicating that regional markets are highly integrated. However, the U.S. market does not appear to be dominant over the Canadian market, as the responses of the Canadian market to the U.S. market is of the same magnitude as those of the U.S. market to the Canadian market.

3.5 Conclusions

This study adds to the literature of international stock market integration by investigating spillover effects across markets through implied volatility. Earlier studies have focused on ex-post measures, such as realised returns and realised volatility in establishing linkages between markets. Implied volatility, extracted directly from option prices, is considered as a forward looking measure which contains a rich source of information about the risk of the underlying asset until the expiration of the option contract. By nature implied volatilities are more informative than realised volatilities, and provide an attractive alternative to study linkages between markets.

The world's volatility indices on national equity indexes are publicly calculated and disseminated by organised exchanges. The method generally follows that of the CBOE in calculating the VIX, which is subject to estimation errors pointed out in Jiang and Tian (2007). This study constructs volatility indices for 15 markets in the world. The method follows the BKM

²¹Figures for groups Asia and North America are not reported to preserve space.

methodology in calculating model free implied variance implemented through a modified curve-fitting method suggested by Jiang and Tian (2007). The proposed method addresses the issue of the truncation error and the discretization error associated with the CBOE procedure. To the best of my knowledge, this is the first study that calculates model free implied volatilities for a large sample of national equity markets, and makes a direct comparison with the CBOE method. The results provide empirical evidence that the CBOE procedure induces substantial estimation errors. The errors are dominated by the truncation error which leads to underestimated implied volatilities.

Implied volatility spillover effects are examined through vector autoregressive analysis and impulse response functions and forecast error variance decomposition. The results show that the U.S. market is unambiguously the dominant source of uncertainty in the world. It leads the rest of the markets in the world by one day. Japan has the least impact on the western markets in the global group. Markets within geographical proximity show higher correlations than with markets across continents. In general, the empirical results support the notion of informationally efficient international stock markets, in that information transmitted from one market to another is processed within one or two days, with information flow within one continent processed contemporaneously with no lags. The potential predictability of the direction from movement of one market to the other may have important implications for traders and asset managers. An increase in implied volatility in the U.S. might have an impact on the potential movement of implied volatility in other markets, thus signalling possible trades for a short window.

This study also opens up areas for further investigation in terms of revisiting the methodology of vector autoregressive analysis. As reported in section 4.3, the multivariate portman-teau tests reject the null hypothesis of serial correlation, possibly due to heteroskedasticity embedded in volatility series. Inference on models when residuals do not behave like white

noise could potentially be misleading. A possible remedy to overcome this issue could be applying univariate GARCH modelling to each individual volatility index to account for the heteroskedasticity effect, and then extract the residuals of each series for the spillover investigation.

3.6 Appendix

Table 3.1: Equity indices underlying option data

| Equity Index | Country/Region | Option data history |
|------------------|----------------|---------------------|
| S&P 500 | U.S. | Jan 2004 – Dec 2014 |
| S&P/TSX 60 | Canada | Jul 2007 – Dec 2014 |
| SMI | Switzerland | Jan 2004 – Dec 2014 |
| FTSE 100 | U.K. | Jan 2004 – Dec 2014 |
| NIKKEI 225 | Japan | May 2004 – Dec 2014 |
| KOSPI 200 | South Korea | May 2004 – Dec 2014 |
| HANG SENG | Hong Kong | Jan 2006 – Dec 2014 |
| CAC 40 | France | Jan 2004 – Dec 2014 |
| DAX | Germany | Jan 2004 – Dec 2014 |
| AEX | Netherlands | Jan 2007 – Dec 2014 |
| FTSE MIB | Italy | Jan 2007 – Dec 2014 |
| BEL 20 | Belgium | Jan 2004 – Dec 2014 |
| HELSINKI 25 | Finland | Jan 2004 – Dec 2014 |
| IBEX 35 | Spain | Jan 2008 – Dec 2014 |
| DJ EURO STOXX 50 | Eurozone | Jan 2004 – Dec 2014 |

Table 3.2: Group description

| Group | Countries | Time window |
|---------------|---|-------------------------|
| Global | U.S., U.K., Eurozone, Japan | 06/08/2004 – 18/12/2014 |
| Asia | Japan, South Korea, Hong Kong | 04/01/2006 – 18/12/2014 |
| North America | U.S., Canada | 03/07/2007 – 18/12/2014 |
| European area | U.K., Germany, France, Netherlands Belgium, Spain, Italy, Switzerland, Finland | 02/03/2009 – 18/12/2014 |

Table 3.3: ADF unit root tests

| | levels | <i>p</i> -value | Changes | <i>p</i> -value |
|----------|--------|-----------------|---------|-----------------|
| U.S. | -3.24 | 0.018 | -24.14 | 0.00 |
| U.K. | -3.57 | 0.006 | -24.35 | 0.00 |
| Eurozone | -3.87 | 0.002 | -25.59 | 0.00 |
| Japan | -3.61 | 0.006 | -22.43 | 0.00 |

Table reports Augmented Dickey-Fuller unit root tests with five lags and without a time trend. The tests are done for levels as well as changes in implied volatility.

Table 3.4: Lag order selection for VAR(*p*) model

| Lag | LogL | LR | FPE | AIC | SC | HQ |
|-----|-----------|---------|--------|--------|--------|--------|
| 0 | -19755.71 | NA | 47.19 | 15.21 | 15.21 | 15.21 |
| 1 | -19150.63 | 1207.82 | 29.99 | 14.75 | 14.80 | 14.77 |
| 2 | -18999.14 | 301.94 | 27.02 | 14.65 | 14.73 | 14.68 |
| 3 | -18894.81 | 207.61 | 25.24 | 14.58 | 14.70 | 14.62 |
| 4 | -18839.82 | 109.25 | 24.50 | 14.55 | 14.70 | 14.61 |
| 5 | -18802.30 | 74.44 | 24.10 | 14.53 | 14.72 | 14.60 |
| 6 | -18717.30 | 168.37 | 22.85 | 14.48 | 14.71 | 14.56 |
| 7 | -18642.57 | 147.79 | 21.84 | 14.44 | 14.70* | 14.53* |
| 8 | -18614.11 | 54.19* | 21.63* | 14.43* | 14.72 | 14.53 |

Table reports lag order selection give by Akaike, Schwarz, and Hannan-Quinn information criteria, final prediction error (FPE), and modified LR test. * indicates lag order selected by the criterion.

Table 3.5: R^2 of VAR regressions

| Timeline | t | t | t | t | t | t+1 | t+1 | t+1 | t+1 | t | t | t | t+1 |
|----------|------|----------|------|------|------|------|----------|------|-----|----------|------|------|------|
| | JP | Eurozone | U.K. | U.S. | U.S. | JP | Eurozone | U.K. | | Eurozone | U.K. | U.S. | JP |
| Lags | t-1 | t-1 | t-1 | t-1 | | t | t | t | | | | | t |
| | t-2 | t-2 | t-2 | t-2 | t-1 | t-1 | t-1 | t-1 | | t-1 | t-1 | t-1 | t-1 |
| | t-3 | t-3 | t-3 | t-3 | t-2 | t-2 | t-2 | t-2 | | t-2 | t-2 | t-2 | t-2 |
| | | | | | t-3 | | | | | t-3 | t-3 | t-3 | |
| R^2 | 0.28 | 0.13 | 0.16 | 0.06 | 0.44 | 0.14 | 0.04 | 0.04 | | 0.21 | 0.26 | 0.11 | 0.05 |

Table 3.6: Publicly calculated and disseminated volatility indices

| Underlying asset | Volatility index_Bloomberg ticker | Calculation body | Index constituents | Calculation method |
|------------------|--|--------------------|--------------------------------|---|
| S&P 500 | VIX | CBOE | 500 large-cap US stocks | VIX methodology |
| S&P/TSX 60 | VIXC | Montréal Exchange | 60 large-cap Canadian stocks | VIX methodology |
| SMI | V3X | Swiss Exchange | 20 blue-chip Swiss stocks | VIX methodology |
| FTSE 100 | VFTSE | FTSE Group | 100 large-cap UK stocks | VIX methodology with modified numerical integration |
| NIKKEI 225 | VNKY | NIKKEI Inc. | 225 Japanese stocks | VIX methodology |
| KOSPI 200 | VKOSPI | Korea Exchange | 200 Korean stocks | VIX methodology |
| HANG SENG | VHSI | Hang Seng indexes | 50 HK stocks | VIX methodology suppressing correction term |
| CAC 40 | VCAC | Euronext Paris | 40 large-cap French stocks | VIX methodology |
| DAX | VIX | Deutsche Borse | 30 blue-chip German stocks | VIX methodology |
| AEX | VAEX | Euronext Amsterdam | 25 Dutch stocks | VIX methodology |
| FTSE MIB | IVMIB30 | FTSE Group | 40 most-traded Italian stocks | VIX methodology with modified numerical integration |
| Euro STOXX 50 | V2X | Eurex Exchange | 50 largest stocks in Euro area | VIX methodology |
| BEL 20 | No publicly available volatility index | - | - | - |
| HELSINKI 25 | No publicly available volatility index | - | - | - |
| IBEX 35 | No publicly available volatility index | - | - | - |

Table 3.7: Comparison of volatility indices calculated using BKM procedure and those publicly calculated and disseminated

| Volatility index | Obs. | Time window | IV_bkm > IV_bb | IV_bkm < IV_bb | Mean IV_bkm > IV_bb | Mean IV_bkm < IV_bb |
|------------------|------|-----------------|----------------|----------------|---------------------|---------------------|
| IV_S&P 500 | 2688 | Jan 04 – Dec 14 | 99.37 | 0.63 | +3.90 | -0.73 |
| IV_S&P/TSX 60 | 1302 | Oct 09 – Dec 14 | 99.92 | 0.08 | +29.04 | -0.17 |
| IV_SMI | 2704 | Jan 04 – Dec 14 | 42.90 | 57.10 | +1.61 | -2.59 |
| IV_FTSE 100 | 2714 | Jan 04 – Dec 14 | 76.11 | 23.89 | +2.6 | -1.46 |
| IV_NIKKEI 225 | 2551 | May 04 – Dec 14 | 50.80 | 49.20 | +1.63 | -1.88 |
| IV_KOSPI 200 | 2586 | May 04 – Dec 14 | 78.38 | 21.62 | +3.14 | -1.26 |
| IV_HANG SENG | 2166 | Jan 06 – Dec 14 | 61.54 | 38.46 | +4.06 | -1.70 |
| IV_CAC 40 | 2705 | Jan 04 – Dec 14 | 68.69 | 31.31 | +4.81 | -3.70 |
| IV_DAX | 2722 | Jan 04 – Dec 14 | 37.62 | 62.38 | +1.27 | -3.76 |
| IV_AEX | 1907 | Jan 07 – Dec 14 | 57.84 | 42.16 | +2.68 | -2.42 |
| IV_FTSE MIB | 1135 | Apr 10 – Dec 14 | 54.01 | 45.99 | +2.14 | -3.93 |
| IV_Euro STOXX 50 | 2720 | Jan 04 – Dec 14 | 25.44 | 74.56 | +1.15 | -3.46 |

Table provides comparisons between volatility indices calculated using the BKM method and the CBOE method. Column 2 gives pairwise number of observations. Columns 4 and 5 show the proportion of observations that the BKM implied volatility is higher (lower) than the CBOE implied volatility. Columns 6 and 7 show the average proportion difference between IV_bkm and IV_bb for when BKM implied volatility is higher (lower) than the CBOE implied volatility, calculated as $\frac{IV_bkm - IV_bb}{IV_bb}$.

Table 3.8: Descriptive statistics for calculated volatility indices and Bloomberg source

| | S&P 500 | S&P/TSX 60 | SMI | FTSE 100 | NIKKEI 225 | KOSPI 200 | HANG SENG | CAC 40 | DAX | AEX | FTSEMIB | EURO STOXX 50 |
|---------------|---------|------------|-------|----------|------------|-----------|-----------|--------|-------|-------|---------|---------------|
| <u>IV_bkm</u> | | | | | | | | | | | | |
| Mean | 20.53 | 21.38 | 18.09 | 19.57 | 25.16 | 23.37 | 26.35 | 22.58 | 21.84 | 23.24 | 27.58 | 22.77 |
| Std. Dev. | 10.02 | 4.70 | 8.41 | 9.23 | 10.72 | 10.73 | 14.57 | 9.35 | 8.94 | 11.14 | 7.89 | 9.45 |
| Skewness | 2.41 | 1.11 | 2.94 | 2.39 | 2.79 | 2.41 | 2.28 | 1.98 | 2.43 | 2.15 | 1.67 | 2.23 |
| Kurtosis | 10.02 | 4.62 | 15.05 | 11.36 | 13.93 | 11.40 | 10.32 | 8.94 | 11.52 | 9.19 | 5.57 | 11.13 |
| Min | 10.27 | 10.99 | 5.11 | 8.95 | 11.23 | 9.83 | 11.47 | 5.39 | 4.41 | 10.35 | 15.97 | 5.10 |
| Q1 | 14.06 | 18.01 | 13.22 | 13.48 | 18.41 | 16.85 | 19.96 | 16.13 | 16.29 | 15.68 | 22.18 | 16.43 |
| Median | 17.37 | 20.65 | 15.59 | 16.80 | 23.57 | 20.28 | 20.44 | 20.62 | 19.47 | 19.60 | 24.99 | 20.49 |
| Q3 | 23.38 | 24.44 | 19.89 | 22.64 | 17.71 | 26.22 | 31.81 | 25.74 | 24.38 | 27.24 | 30.10 | 25.81 |
| Max | 83.65 | 43.26 | 92.30 | 91.96 | 98.67 | 97.63 | 137.45 | 88.38 | 86.35 | 91.73 | 64.89 | 109.34 |
| <u>IV_bb</u> | | | | | | | | | | | | |
| Mean | 19.80 | 16.95 | 18.21 | 19.22 | 25.12 | 22.79 | 25.39 | 22.02 | 22.30 | 23.17 | 27.78 | 23.31 |
| Std. Dev. | 9.71 | 4.67 | 8.23 | 8.90 | 10.29 | 9.88 | 12.24 | 8.73 | 9.02 | 11.05 | 7.70 | 9.46 |
| Skewness | 2.40 | 1.20 | 2.80 | 2.31 | 2.57 | 2.39 | 1.82 | 2.04 | 2.40 | 1.93 | 1.51 | 2.03 |
| Kurtosis | 10.46 | 4.53 | 13.82 | 10.68 | 12.25 | 11.36 | 6.94 | 9.04 | 10.95 | 7.44 | 4.90 | 8.79 |
| Min | 9.89 | 7.80 | 9.24 | 9.10 | 11.06 | 10.21 | 11.53 | 9.24 | 11.65 | 10.39 | 16.57 | 9.46 |
| Q1 | 13.50 | 13.98 | 13.46 | 13.22 | 18.44 | 16.81 | 17.02 | 16.10 | 16.66 | 15.58 | 22.44 | 16.96 |
| Median | 16.69 | 15.89 | 15.67 | 16.49 | 23.36 | 20.09 | 20.34 | 19.88 | 19.83 | 19.49 | 25.08 | 21.00 |
| Q3 | 22.73 | 19.08 | 20.04 | 22.31 | 27.76 | 25.60 | 30.94 | 25.00 | 24.69 | 27.25 | 30.63 | 26.42 |
| Max | 80.86 | 36.71 | 84.90 | 78.69 | 92.03 | 89.30 | 104.29 | 78.05 | 83.23 | 81.22 | 64.17 | 87.51 |

Table 3.9: Option information on calculating BKM volatility indices

| Index | Time | Near-term maturity options | | | | | Next-term maturity options | | | | | | | | |
|---------------|-----------|----------------------------|------|--------------|------|------|----------------------------|-------------|------|------|--------------|------|------|-------------|-------------|
| | | days | nopt | range (nopt) | min | max | range (min) | range (max) | days | nopt | range (nopt) | min | max | range (min) | range (max) |
| S&P 500 | 2004-2014 | 23 | 72 | 5-162 | 0.76 | 1.11 | 0.70-0.98 | 1.05-1.30 | 44 | 76 | 6-173 | 0.73 | 1.14 | 0.70-0.93 | 1.05-1.30 |
| S&P/TSX 60 | 2007-2014 | 21 | 44 | 4-89 | 0.83 | 1.15 | 0.70-1.10 | 0.96-1.30 | 51 | 40 | 6-83 | 0.84 | 1.14 | 0.70-1.10 | 0.98-1.30 |
| SMI | 2004-2014 | 22 | 54 | 6-88 | 0.77 | 1.18 | 0.70-1.01 | 0.99-1.30 | 49 | 56 | 7-87 | 0.76 | 1.19 | 0.70-1.01 | 0.99-1.30 |
| FTSE 100 | 2004-2014 | 21 | 48 | 9-89 | 0.75 | 1.19 | 0.70-0.95 | 1.02-1.30 | 51 | 44 | 23-77 | 0.74 | 1.20 | 0.70-0.89 | 1.05-1.30 |
| NIKKEI 225 | 2004-2014 | 22 | 23 | 4-65 | 0.74 | 1.23 | 0.70-0.94 | 1.04-1.30 | 51 | 24 | 4-62 | 0.73 | 1.25 | 0.70-0.90 | 1.08-1.30 |
| KOSPI 200 | 2004-2014 | 21 | 22 | 5-43 | 0.86 | 1.11 | 0.70-0.99 | 1.03-1.30 | 51 | 21 | 7-43 | 0.86 | 1.11 | 0.70-1.00 | 1.03-1.30 |
| HANG SENG | 2006-2014 | 21 | 52 | 26-81 | 0.72 | 1.23 | 0.70-0.87 | 0.94-1.30 | 51 | 50 | 19-80 | 0.74 | 1.23 | 0.70-0.91 | 1.06-1.30 |
| CAC 40 | 2004-2014 | 22 | 32 | 5-54 | 0.75 | 1.20 | 0.70-0.99 | 1.01-1.30 | 55 | 28 | 4-51 | 0.74 | 1.22 | 0.70-0.93 | 0.90-1.30 |
| DAX | 2004-2014 | 24 | 32 | 4-91 | 0.84 | 1.13 | 0.70-1.05 | 0.91-1.30 | 41 | 46 | 5-92 | 0.78 | 1.18 | 0.70-1.11 | 0.95-1.30 |
| AEX | 2007-2014 | 20 | 26 | 7-82 | 0.76 | 1.22 | 0.70-1.00 | 1.02-1.30 | 50 | 20 | 5-34 | 0.75 | 1.23 | 0.70-0.96 | 1.02-1.30 |
| FTSE MIB | 2007-2014 | 20 | 29 | 4-72 | 0.77 | 1.21 | 0.70-0.99 | 0.97-1.30 | 48 | 21 | 4-41 | 0.76 | 1.23 | 0.70-0.99 | 0.93-1.30 |
| EURO STOXX 50 | 2004-2014 | 24 | 24 | 4-68 | 0.83 | 1.15 | 0.70-1.05 | 0.82-1.30 | 41 | 31 | 4-73 | 0.77 | 1.19 | 0.70-1.12 | 0.86-1.30 |
| BEL 20 | 2004-2014 | 21 | 17 | 5-39 | 0.83 | 1.16 | 0.70-1.05 | 1.00-1.30 | 51 | 14 | 6-32 | 0.81 | 1.18 | 0.70-1.01 | 1.00-1.30 |
| HELSINKI 25 | 2004-2014 | 21 | 23 | 6-49 | 0.86 | 1.10 | 0.70-1.03 | 1.00-1.30 | 51 | 22 | 6-47 | 0.86 | 1.10 | 0.70-1.03 | 1.10-1.30 |
| IBEX 35 | 2008-2014 | 21 | 36 | 10-66 | 0.78 | 1.18 | 0.70-0.98 | 0.99-1.30 | 51 | 38 | 18-70 | 0.78 | 1.19 | 0.70-0.93 | 1.04-1.30 |

Table provides information on near- and next- term maturity options used to calculate BKM volatility indices. Columns (days) report average maturities in calendar days. Columns (nopt) and (range(nopt)) report the average number, and the range, of options observed for linear interpolation. Moneyness is defined as K/S . The average minimum and maximum moneyness over time for each index is given in columns (min) and (max), followed by two columns of the range of the minimum and maximum moneyness. The range of moneyness between the minimum and maximum illustrates possible truncation intervals given by filtered available strike prices used in the CBOE method.

Table 3.10: Correlation coefficients for differenced implied volatility series

| <u>Group A</u> | | | | | | | | | |
|----------------|-------------|-------------|-----------|---------|---------|-------|-------|-------------|------|
| Global | JP | Eurozone | U.K. | U.S. | | | | | |
| JP | 1 | | | | | | | | |
| Eurozone | 0.38 | 1 | | | | | | | |
| U.K. | 0.43 | 0.80 | 1 | | | | | | |
| U.S. | 0.13 | 0.51 | 0.55 | 1 | | | | | |
| <u>Group B</u> | | | | | | | | | |
| European area | Netherlands | Belgium | France | Germany | Finland | Spain | Italy | Switzerland | U.K. |
| Netherlands | 1 | | | | | | | | |
| Belgium | 0.71 | 1 | | | | | | | |
| France | 0.87 | 0.72 | 1 | | | | | | |
| Germany | 0.83 | 0.67 | 0.88 | 1 | | | | | |
| Finland | 0.41 | 0.38 | 0.45 | 0.40 | 1 | | | | |
| Spain | 0.69 | 0.56 | 0.74 | 0.68 | 0.40 | 1 | | | |
| Italy | 0.51 | 0.44 | 0.56 | 0.52 | 0.31 | 0.55 | 1 | | |
| Switzerland | 0.83 | 0.67 | 0.86 | 0.83 | 0.42 | 0.69 | 0.54 | 1 | |
| U.K. | 0.84 | 0.69 | 0.86 | 0.81 | 0.40 | 0.67 | 0.50 | 0.82 | 1 |
| <u>Group C</u> | | | | | | | | | |
| Asia | Japan | South Korea | Hong Kong | | | | | | |
| Japan | 1 | | | | | | | | |
| South Korea | 0.56 | 1 | | | | | | | |
| Hong Kong | 0.40 | 0.41 | 1 | | | | | | |
| <u>Group D</u> | | | | | | | | | |
| North America | U.S. | Canada | | | | | | | |
| U.S. | 1 | | | | | | | | |
| Canada | 0.47 | 1 | | | | | | | |

Figure 3.1: Option trading hours for foreign markets on the same calendar day (in Tokyo time)

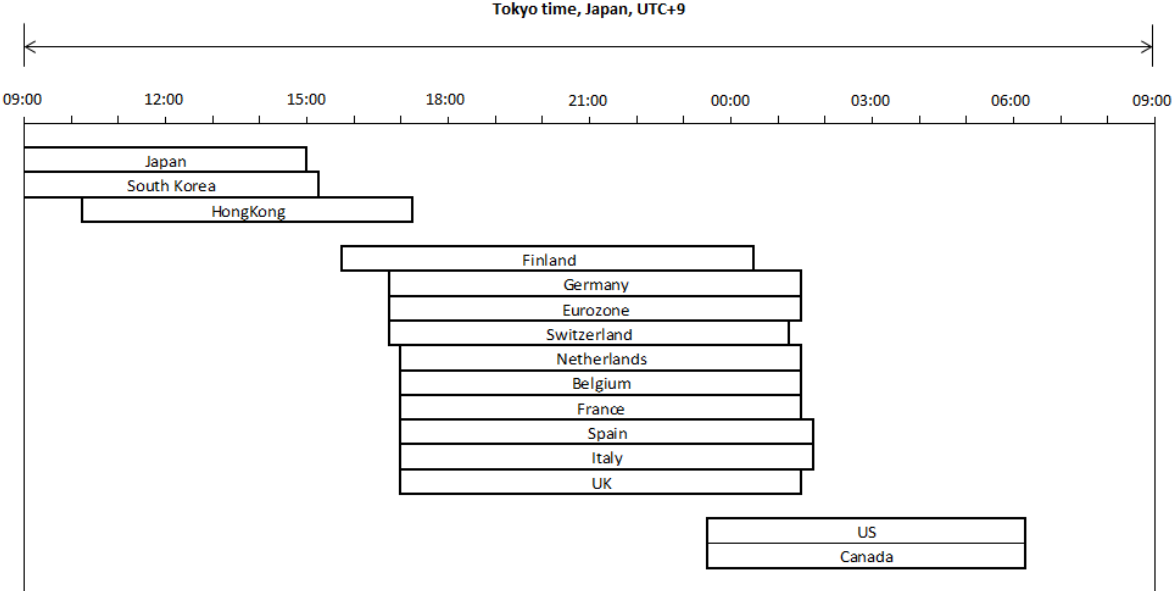


Figure 3.2: IV_S&P/TSX 60 vs. VIXC

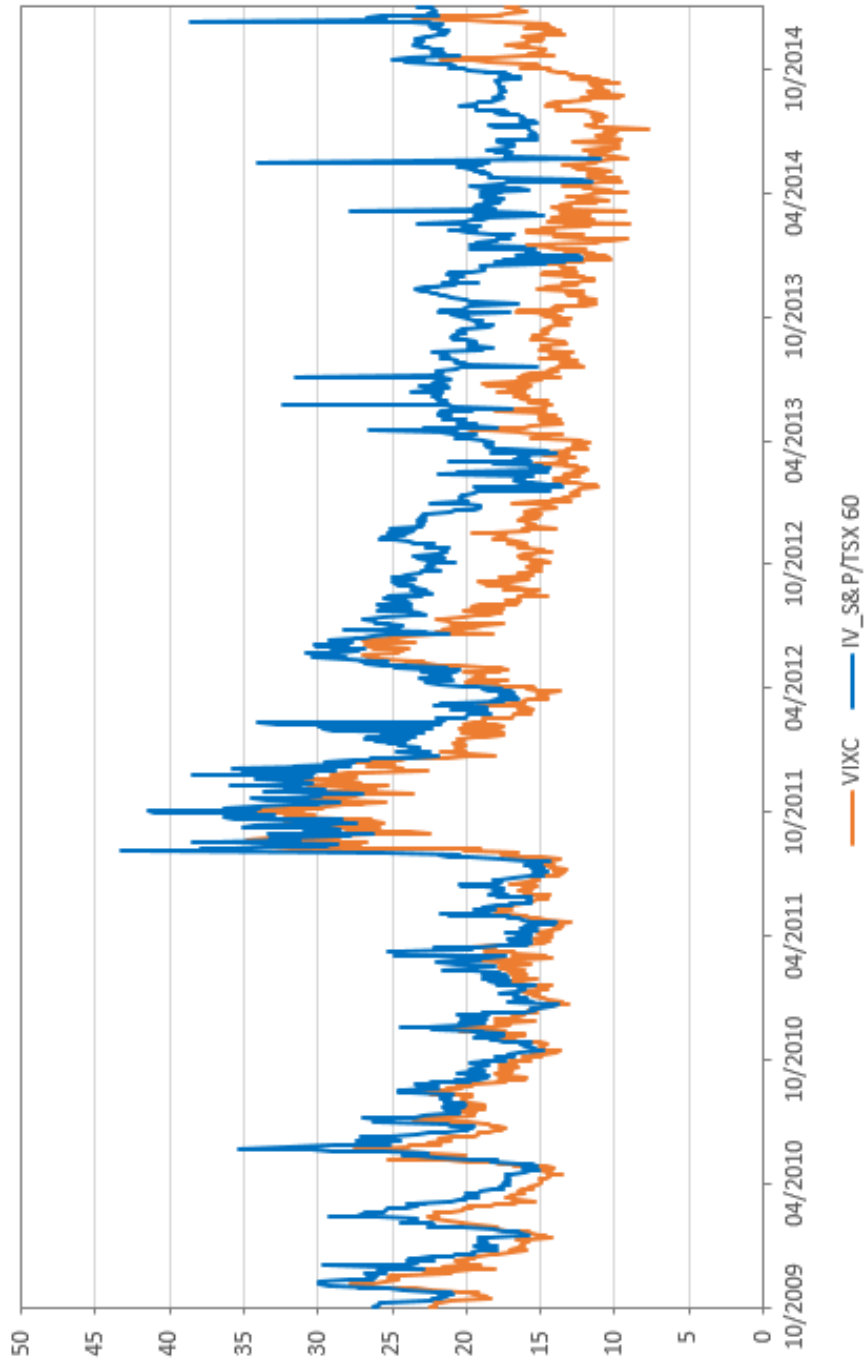


Figure 3.3: IV_S&P 500 vs. VIX

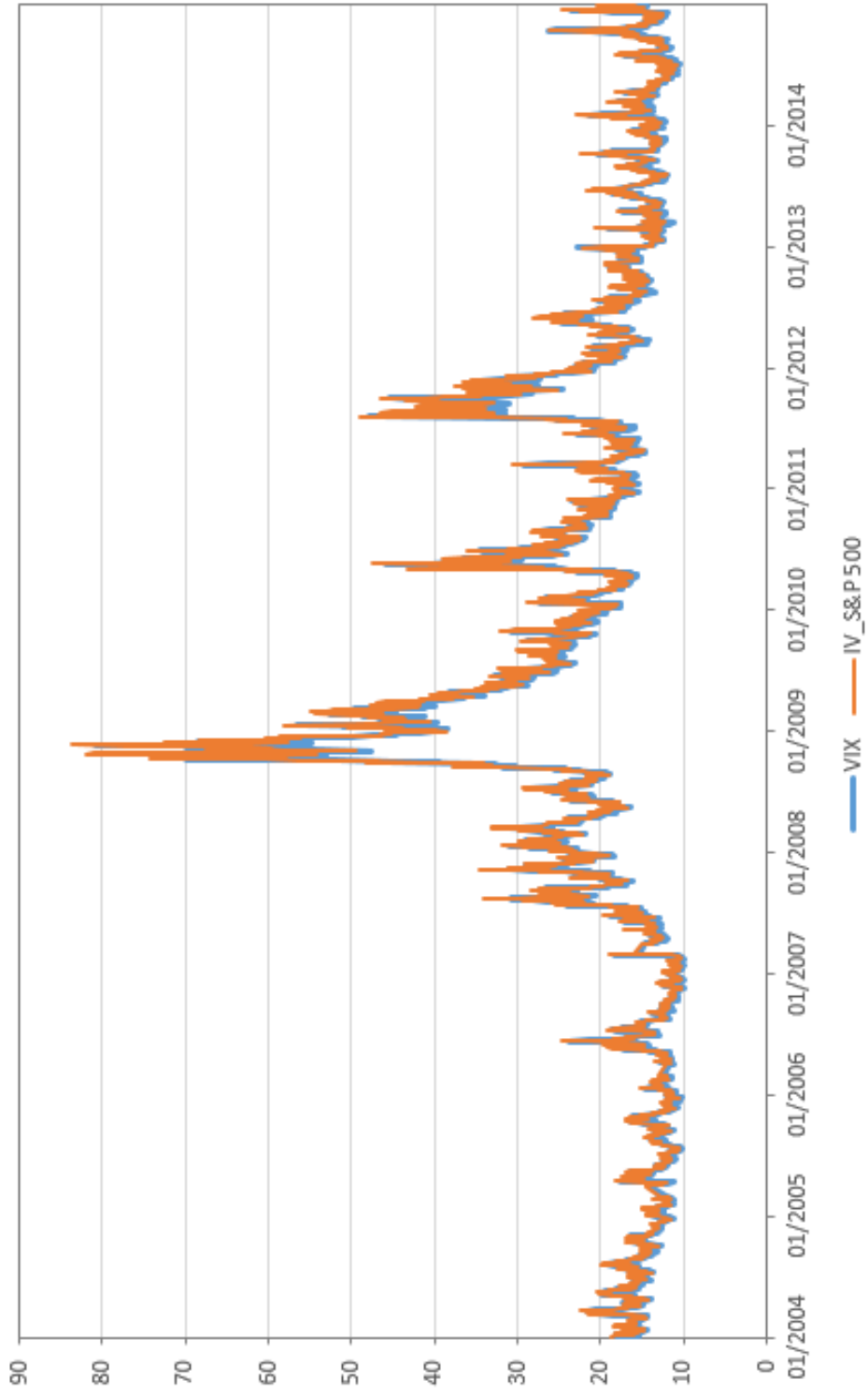


Figure 3.4: System autocorrelations with 2 Std.Err. bounds, for the Global group

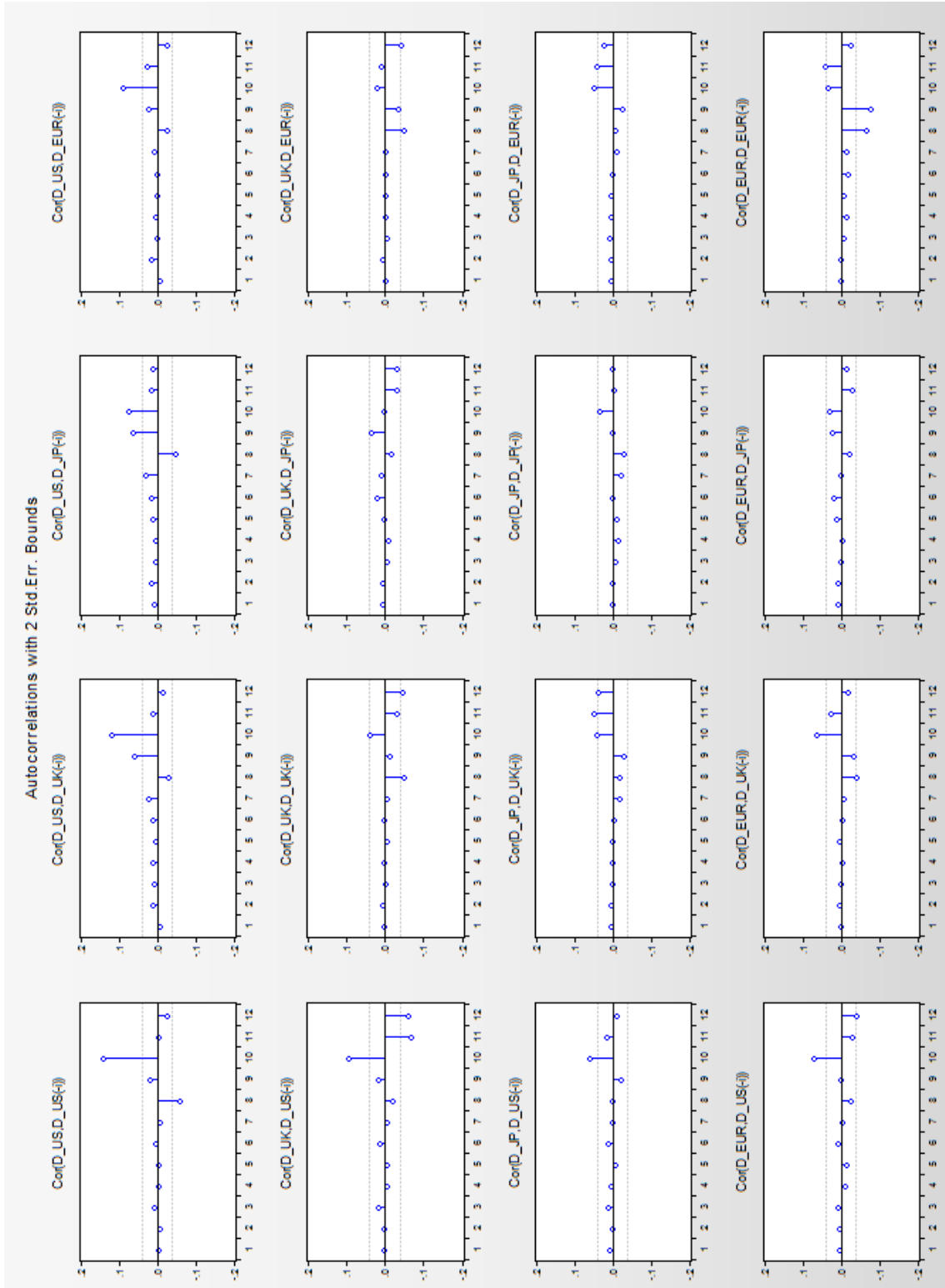


Figure 3.5: Impulse responses to one S.D. innovations with 2 Std.Err. bounds, for the Global group.
 Variable ordering: $US_t, Eurozone_t, UK_t, JP_t$.

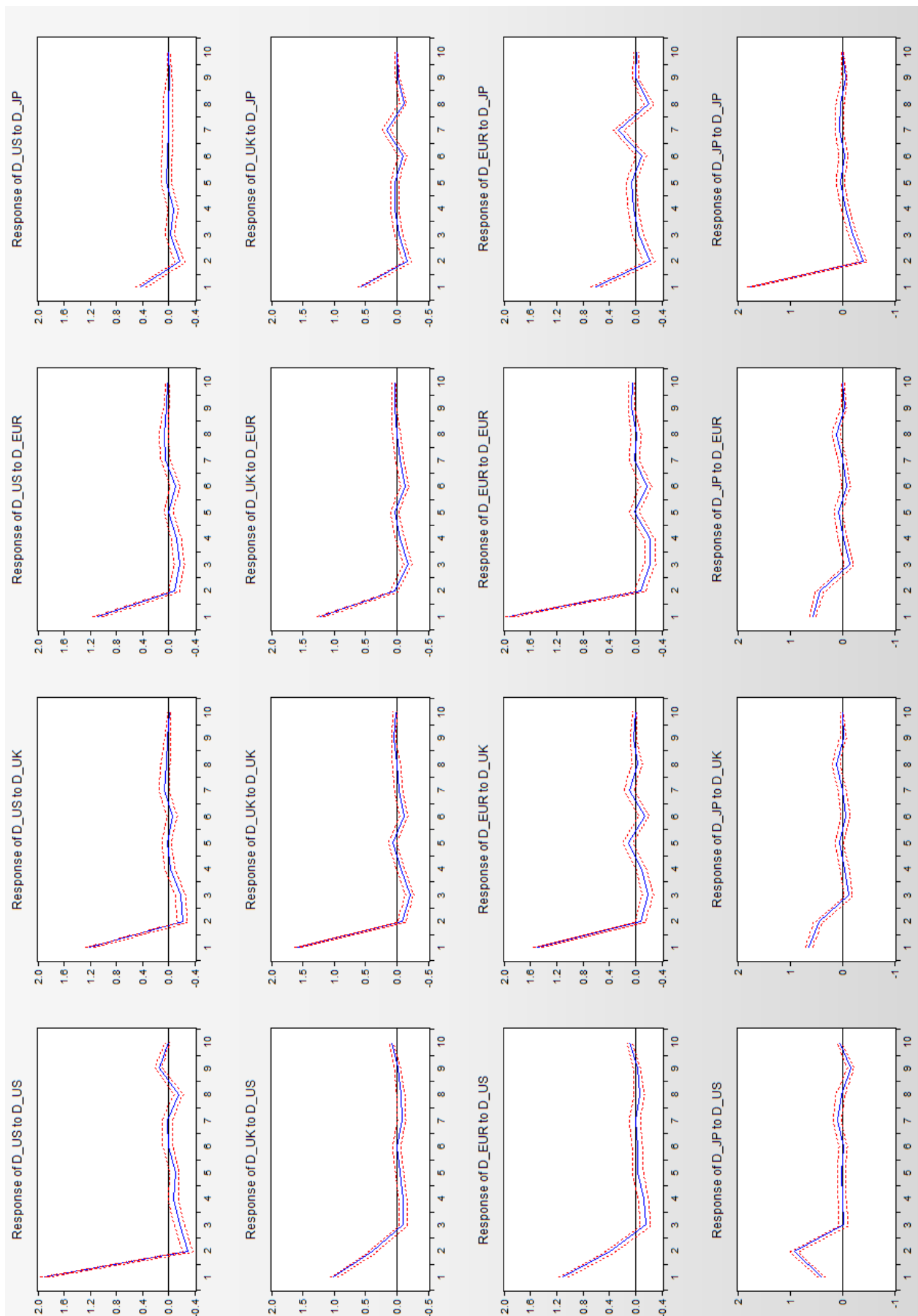


Figure 3.6: Impulse responses to one S.D. innovations with 2 Std.Err. bounds, for the Global group.
 Variable ordering: $US_t, Eurozone_{t+1}, UK_{t+1}, JP_{t+1}$.

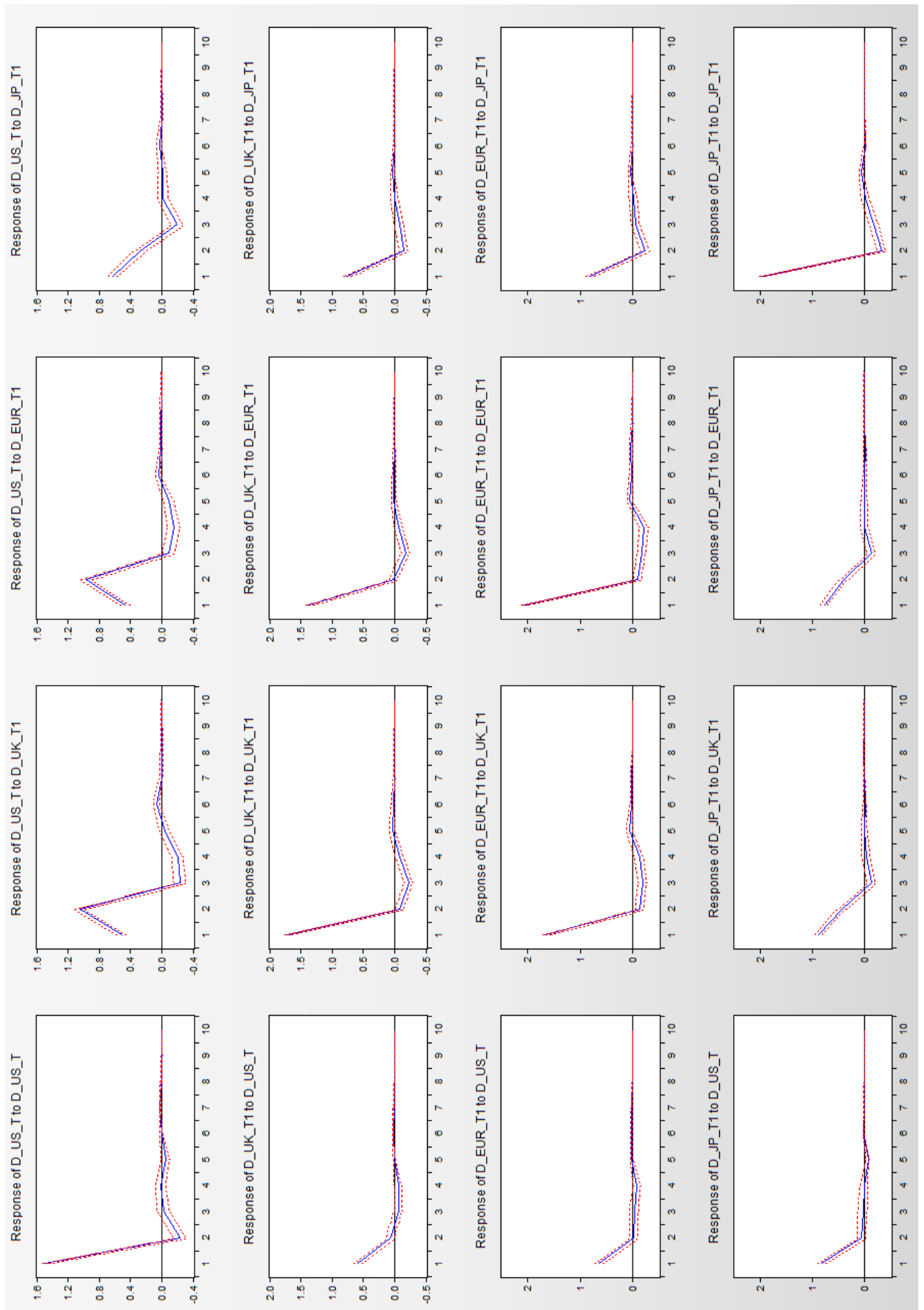


Figure 3.7: Impulse responses to one S.D. innovations with 2 Std.Err. bounds, for the Global group.
 Variable ordering: $Eurozone_t, UK_t, US_t, JP_{t+1}$.

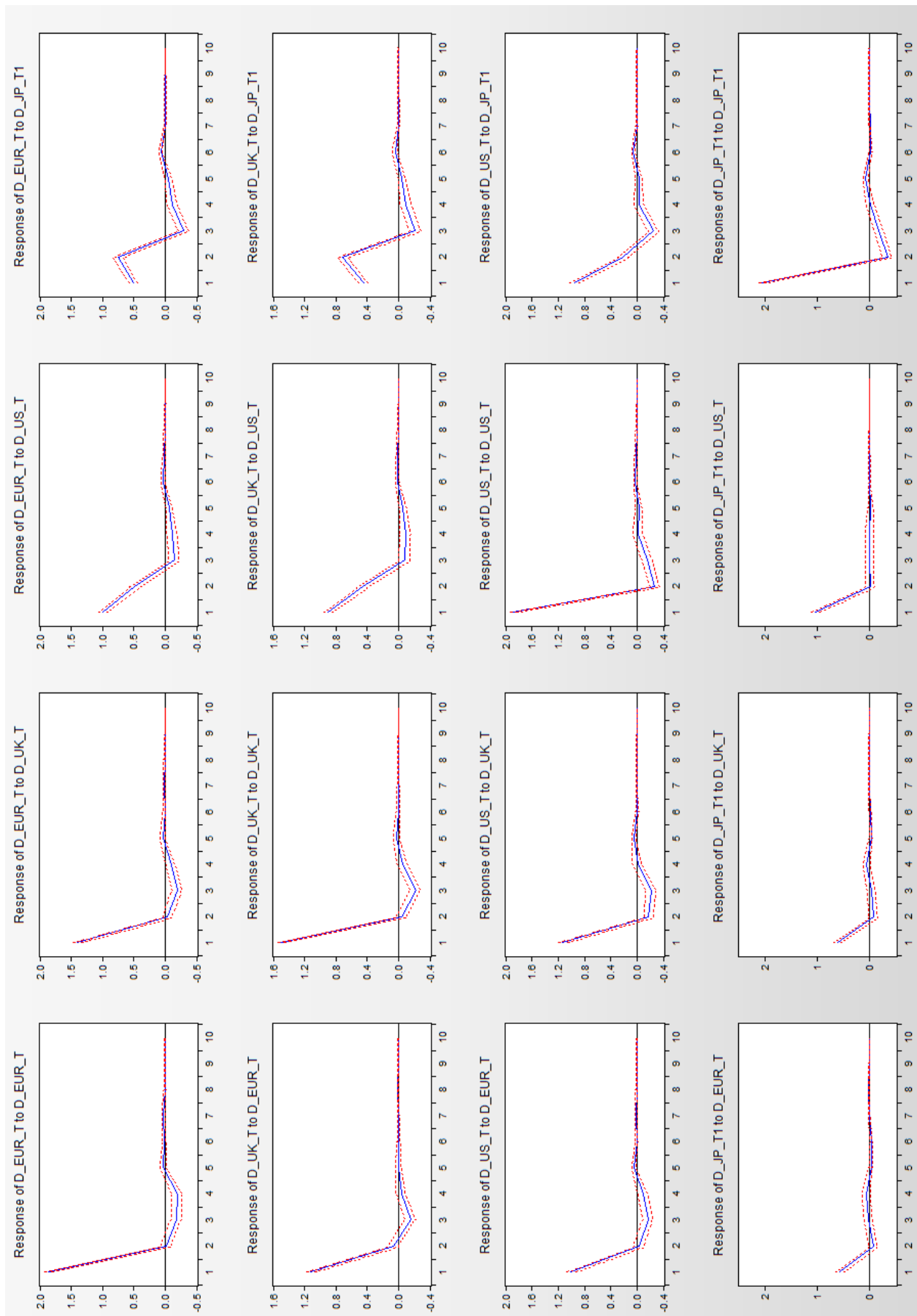


Figure 3.8: Forecast error variance decomposition for the Global group

| Variance Decomposition of D_US: | | | | | |
|---------------------------------|----------|----------|----------|----------|----------|
| Period | S.E. | D_US | D_UK | D_EUR | D_JP |
| 1 | 1.927345 | 100.0000 | 0.000000 | 0.000000 | 0.000000 |
| 2 | 1.966853 | 98.51479 | 0.094045 | 0.829172 | 0.561992 |
| 3 | 1.980319 | 97.94273 | 0.654239 | 0.817934 | 0.585099 |
| 4 | 1.986138 | 97.38966 | 0.653230 | 1.273252 | 0.683862 |
| 5 | 1.989641 | 97.22732 | 0.755402 | 1.271121 | 0.746157 |
| 6 | 1.990108 | 97.22686 | 0.755254 | 1.272054 | 0.745829 |
| 7 | 1.990295 | 97.21804 | 0.756013 | 1.278927 | 0.747020 |
| 8 | 1.990310 | 97.21683 | 0.756125 | 1.279558 | 0.747483 |
| 9 | 1.990320 | 97.21614 | 0.756175 | 1.279957 | 0.747727 |
| 10 | 1.990328 | 97.21572 | 0.756170 | 1.280385 | 0.747722 |

| Variance Decomposition of D_UK: | | | | | |
|---------------------------------|----------|----------|----------|----------|----------|
| Period | S.E. | D_US | D_UK | D_EUR | D_JP |
| 1 | 1.605062 | 38.97427 | 61.02573 | 0.000000 | 0.000000 |
| 2 | 1.725587 | 39.09942 | 59.97105 | 0.040784 | 0.888752 |
| 3 | 1.740554 | 38.69724 | 60.36923 | 0.040542 | 0.892990 |
| 4 | 1.744118 | 38.85939 | 60.12342 | 0.044517 | 0.972682 |
| 5 | 1.747923 | 38.89189 | 59.98621 | 0.094336 | 1.027561 |
| 6 | 1.748044 | 38.89404 | 59.98158 | 0.096933 | 1.027443 |
| 7 | 1.748229 | 38.90237 | 59.96976 | 0.098149 | 1.029714 |
| 8 | 1.748300 | 38.89926 | 59.96539 | 0.104918 | 1.030433 |
| 9 | 1.748307 | 38.89905 | 59.96500 | 0.104972 | 1.030983 |
| 10 | 1.748313 | 38.89896 | 59.96466 | 0.105301 | 1.031077 |

| Variance Decomposition of D_EUR: | | | | | |
|----------------------------------|----------|----------|----------|----------|----------|
| Period | S.E. | D_US | D_UK | D_EUR | D_JP |
| 1 | 1.960533 | 31.82740 | 29.37400 | 38.79860 | 0.000000 |
| 2 | 2.077059 | 32.27703 | 31.86362 | 34.85414 | 1.005210 |
| 3 | 2.086976 | 32.52230 | 31.90397 | 34.57200 | 1.001735 |
| 4 | 2.101377 | 32.45561 | 31.49015 | 34.92707 | 1.127164 |
| 5 | 2.104759 | 32.49165 | 31.53717 | 34.81574 | 1.155439 |
| 6 | 2.105006 | 32.50483 | 31.52993 | 34.80956 | 1.155678 |
| 7 | 2.105551 | 32.51097 | 31.51365 | 34.82011 | 1.155269 |
| 8 | 2.105589 | 32.50981 | 31.51487 | 34.81912 | 1.156193 |
| 9 | 2.105599 | 32.50958 | 31.51457 | 34.81936 | 1.156495 |
| 10 | 2.105616 | 32.50946 | 31.51405 | 34.82001 | 1.156476 |

| Variance Decomposition of D_JP: | | | | | |
|---------------------------------|----------|----------|----------|----------|----------|
| Period | S.E. | D_US | D_UK | D_EUR | D_JP |
| 1 | 1.801573 | 4.413166 | 8.092184 | 0.265604 | 87.22905 |
| 2 | 2.104278 | 22.41527 | 6.564736 | 0.232473 | 70.78752 |
| 3 | 2.116871 | 22.26137 | 6.838600 | 0.317347 | 70.58268 |
| 4 | 2.117924 | 22.26023 | 6.832665 | 0.367861 | 70.53924 |
| 5 | 2.122414 | 22.56026 | 6.807450 | 0.391186 | 70.24111 |
| 6 | 2.122679 | 22.56133 | 6.819964 | 0.391104 | 70.22760 |
| 7 | 2.122911 | 22.57499 | 6.819539 | 0.393138 | 70.21233 |
| 8 | 2.122968 | 22.57458 | 6.819213 | 0.397588 | 70.20862 |
| 9 | 2.122985 | 22.57488 | 6.820046 | 0.397595 | 70.20748 |
| 10 | 2.122987 | 22.57495 | 6.820033 | 0.397597 | 70.20742 |

Figure 3.9: Generalised impulse responses for group European area

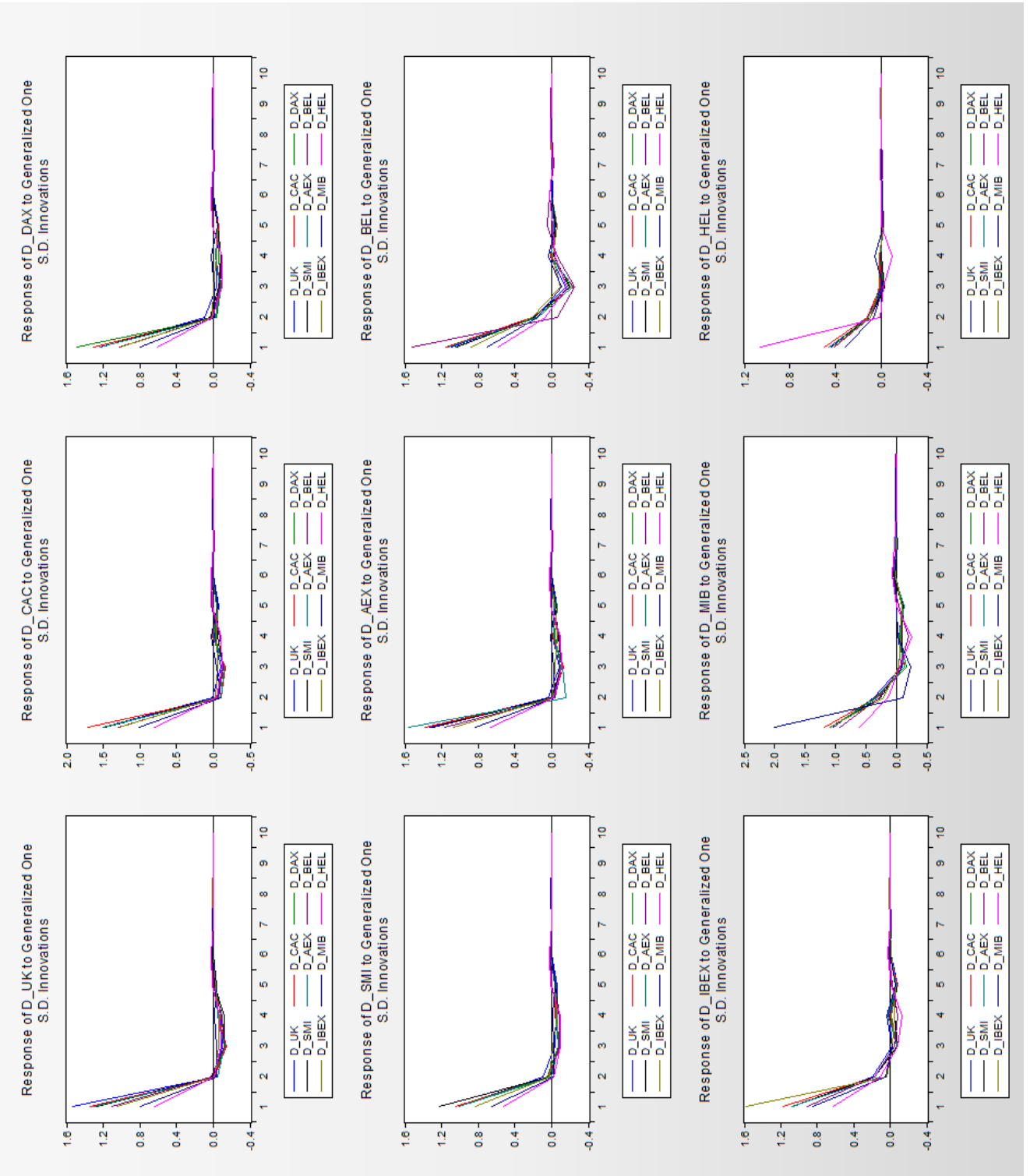
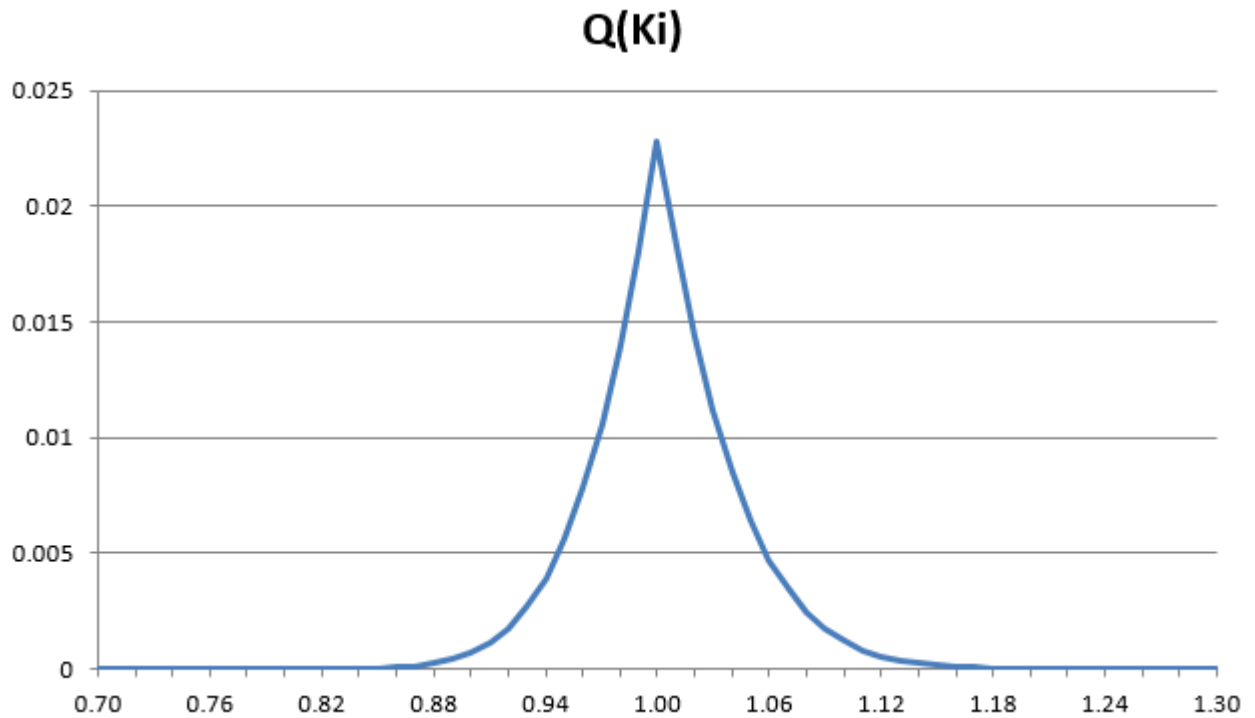


Figure 3.10: Numerical intergration method used in the CBOE procedure

Figure illustrates option prices against strike prices simulated from the BSM model for a base set of parameters. The asset price S is 1, The lowest and highest strike price K_L and K_U are 0.7 and 1.3. The increment between strik prices is 0.01. Volatility σ of the underlying asset is 0.2. Risk free rate and dividend yield are set to 0. Option maturity T is 30 calendar days.



Chapter 4

Variance Risk Premiums and International Spillover Effects

4.1 Introduction

It has been well documented in the literature that return variance on financial assets is stochastic. When investing in a security, investors face not only risk about the return as captured by return variance, but also uncertainty about return variance itself. An understanding of how investors deal with uncertainty about return variance is imperative for derivative pricing, risk management, and asset pricing in general.

In recent research, the difference between realised variance and risk-neutral expected value of realised variance has been interpreted as a proxy for the variance risk premium. Researchers have shown in theoretical work that the risk-neutral expectation of future variance can be well approximated by a combination of options across strike prices with the same maturity. In line with this theoretical development, the Chicago Board of Options Exchange (CBOE) developed the new volatility index (the VIX) that approximates the 30-day implied variance on the S&P 500 index, thus making the risk-neutral expected value of 30-day variance almost an observable quantity. On the other hand, theories on computing realised

variance from historical returns at different frequency have long been established. Thus, calculating the realised measure minus the risk-neutral measure makes variance risk premium almost a directly observable quantity.

In this study, I study the variance risk premiums for the U.S., the U.K., the Eurozone, and Japan for the period from 2004 to 2014. I construct the variance risk premium for each market from their representative equity index – the S&P 500 index, the FTSE 100 index, the DJ STOXX 50 index, and the NIKKEI 225 index. I find that variance risk premiums across markets are on average negative with negative skewness and large kurtosis. There is significant variation in the time series and the magnitude of variance risk premium is high when markets experience disruptive events. Negative risk premiums suggest that investors view an increase in return variance as unfavourable and are willing to accept negative returns on a long position in a variance swap to hedge against the upward movement in return variance.

Since the variance risk premium represents some sort of aggregate market uncertainty, it is important to understand whether and how this uncertainty from one country is transmitted to another. The channel through which variance risk premium manifest in the global market may have important implications in asset allocation and risk management. There are studies documenting variance risk premium's return predictability on domestic and foreign stock markets (Bollerslev et al. (2014), Londono (2015)), but they do not provide a direct linkage between variance risk premiums across markets. To the best of my knowledge, this study is the first to investigate the dynamic relationships between variance risk premiums across markets.

To investigate the transmission of variance risk premiums across borders, I employ the vector autoregressive model and its structural analysis. Analysis is done in subsamples for before, around, and after the 2008 financial crisis. Empirical results show that the U.S. is unambiguously the leading source of uncertainty in the global market, and no other country

or a combination of countries have predictive power on the U.S.. The Japanese market seems to be less integrated with the rest of the markets due to its low correlation as well as the role as a follower. Information flow from one market to another is fully processed within two days, but it may take longer to be digested during the crisis period.

The remainder of the study is organised as follows. Section 4.2 provides a review on related studies on the variance risk premium¹. Section 4.3 lays out the theoretical methodology in constructing variance risk premium. Section 4.4 describes data used to compute realised and risk-neutral variance. Section 4.5 provides empirical analysis on variance risk premium dynamics and spillover effects. Section 4.6 concludes. A list of Figures and Tables are in Appendix in section 4.7.

4.2 Literature review

Early studies focus on documenting the evidence that volatility risk is priced. This strand of literature bases its methodology on option pricing theory by specifying appropriate return and stochastic volatility dynamics and calibrate it to option data. The main purpose of these papers is to parametrize the data generating process for stock returns and volatility to reconcile with the inconsistencies found empirically when relying on Black-Scholes assumptions, and volatility risk premia implicitly modelled in the risk-neutral underlying dynamics is captured as a by-product from the estimation. These structural models are estimated with either Bayesian methods or efficient methods of moments which require much computational effort. This strand of literature is extensive, and prominent examples are Guo (1998), Bates (2000), Pan (2002), Benzoni (2002), Chernov and Ghysels (2000), Jones (2003), and Eraker (2004), among others.

¹A review on spillover analysis is covered in Chapter 3 section 3.2 literature review.

Guo (1998) investigates empirically the risk-neutral variance process and the market price of variance risk implied in foreign currency option market. He specified Heston (1993) stochastic volatility model and calibrated it to dollar/mark option prices from 1987 to 1992. From the estimated parameters in the risk-neutral variance process combined with sample variance, as proxy for physical long-run mean variance, an estimate of variance risk premium can be obtained through appropriate transformation between the risk-neutral and physical measure of variance process. The author finds that the variance risk premium is not zero and time varying; estimates are negative from 1987 to 1990 and positive from 1991 to 1992 on a yearly basis. However, the estimates of variance risk premium are negative for 1991 and 1992 if longer period of sample variance is used. The magnitude of estimated variance risk premiums imply that the compensation for variance risk is statistically significant as well as economically important.

Much of the attention on estimating stochastic volatility models has been on the S&P 500 index and its options, and much of the model choice has been Heston (1993) stochastic volatility model due to its analytical tractability and computational advantage of closed-form solution in derivative pricing. Chernov and Ghysels (2000) propose a procedure that jointly estimates the objective and risk-neutral measure of underlying dynamics using simultaneously the underlying asset price and a set of option information. The method involves first fitting a semi-nonparametric density of the underlying asset returns using market data, and then simulates asset and option prices and calibrates the option pricing model to fit the conditional density of the market data dynamics. The same methodology was adopted by Jones (2003) and Eraker (2004). Benzoni (2002) claims that simultaneously using both derivatives and underlying data to estimate stochastic volatility models would limit the length of underlying return sample due to limited trading of option contract. Instead he applies a two-stage estimation procedure which allows him to combine a large sample of index returns

and a small sample of option prices. In the first stage, historical daily index returns are used through a simulated method of moments to estimate the parameters of the model, and in the second stage a sample of option prices together with a simulation methodology is used to estimate the risk adjustments. Bates (2000) and Pan (2002) extend afore-mentioned pure diffusion settings by incorporating state-dependent price jumps to allow for an explicit understanding of the role played by volatility and jump risk premia. Most recently, Wu (2011) uses tick data on S&P 500 index futures to construct several quadratic variation estimators and combine variance swap rates inferred from options to study variance dynamics. In this strand of literature, while some studies do not provide insight on the relative importance of volatility risk premia, some find significant negative volatility risk premia that are economically important for option pricing.

Coval and Shumway (2001) examine the theoretical and empirical nature of option returns in the context of the broader asset pricing theory. The Black-Scholes framework assumes that any risk associated with the variance of the underlying asset returns is not priced as the return volatility is constant. This implies that options are redundant assets and options positions that are delta neutral should earn on average the risk free rate. Coval and Shumway (2001) construct a zero-beta straddle on the S&P 500 index from 1990 to 1995, which includes a combination of long positions in calls and puts having offsetting variances with the index. They derive an expression for the straddle return based on call option beta and found that this market neutral straddles on the S&P 500 index receive on average -3% returns per week. The results are robust to non-synchronous trading, measurement error in option beta, and changes in sample period and frequency. The authors conclude that systematic stochastic volatility may be priced in the returns of option index.

Bakshi and Kapadia (2003a) investigate whether the volatility risk premium is negative in index option markets by examining profit and loss arising from a dynamically delta-hedged

portfolio with a long call position hedged by a short position in the stock. The rationale underlying the analysis is to infer whether volatility risk is priced by looking at whether the delta-hedged gains are zero. If volatility risk is priced, then the sign and magnitude of portfolio gains is determined by volatility risk premium. Data includes a sample of daily short term call and put options on the S&P 500 index from 1988 to 1995. The authors perform both cross-sectional (across strikes) and time series (at-the-money) tests on delta-hedged portfolios and find negative volatility risk premiums in option prices. Further tests suggest that although jump risk explains some portion of delta-hedged gains for short dated options, the volatility risk premium is the predominant factor that justifies the losses incurred on the delta-hedged portfolios. Bakshi and Kapadia (2003b) extend their research to study the implications of market volatility risk premium on 25 individual equity options. They find that, first, implied volatilities are larger than realised volatilities for individual equity options, but the differences between the two measures are smaller for individual options than for index options; second, delta-hedged gains for individual equity options are more negative than positive, and smaller than those for index options. Their results suggest that the volatility risk premium in individual equity options is negative and lower than that in index options.

Some authors construct a variance risk premium based on the notion of a variance swap, which is an over-the-counter contract that pays the difference between the realised variance over a given period and the fixed variance swap rate. Since variance swaps cost zero to enter, the variance swap rate represents the risk-neutral expected value of the realised variance. Works of Carr and Madan (1998), Demeterfi et al. (1999), Britten-Jones and Neuberger (2000), and Jiang and Tian (2005) have shown theoretical advances in synthesizing variance swap rates using European option contracts. The realised variance can be obtained relatively easily using historical returns of the underlying asset. Thus variance risk premium can be handily quantified through a model free approach.

Bondarenko (2004) employs a model free approach to study variance risk premium and links it to the performance of hedge funds. He uses daily options prices on the S&P 500 index futures for the period from 1988 to 2000 to infer the price of a variance contract. Realised variance is calculated from the S&P 500 futures using a non-standard expression of discretely-sampled variance such that it is able to accommodate a more general assumption of the underlying asset process that may include jumps. He finds that variance risk is priced, and its premium is negative and large in economic terms. Consistent with the previous literature, the author finds that the market variance is negatively correlated with the market return, but the negative correlation only explains a portion of negative variance risk premium. The remaining portion can be interpreted as the compensation for a pure variance risk. This paper further establishes a link between negative variance risk premium and hedge fund performance. The results show that hedge fund returns are negatively correlated with variance returns. Hedge funds profits from selling variance contracts and variance risk account for a significant portion of hedge fund average returns.

Carr and Wu (2009) synthesize variance swap rates using a large options data set, and investigates historical behaviour of variance risk premiums on five stock indexes and 35 individual stocks. Their sample includes European options on the S&P 500 index, the Dow Jones Industrial Index, and the NASDAQ-100 Index, and American options on the S&P 100 index, the QQQ (NASDAQ-100 tracking stocks), and 35 individual stocks for the period from 1996 to 2003. They synthesize variance swap rate at a fixed 30-day horizon, and obtain corresponding annualized 30-day realised variance using daily futures prices. Statistical analysis shows that the sample averages of variance risk premium are negative for all indexes and most of the individual stocks. If an investor is going long a variance swap contract on the S&P 500 index and holds to maturity, the average return on a \$100 investment is -\$2.74. It is clear that investor are willing to accept negative returns to go long on variance swap con-

tracts on stock indexes to hedge away upward movement in index return variance. On the other hand, variance risk premiums on individual stocks show large cross-sectional variations, which suggests that the market does not price variance risk in individual stocks, but only prices a systematic variance risk in the market portfolio. The authors further perform some tests to investigate whether a classic asset pricing model can explain negative variance premiums. They find that the well-documented negative correlation between index returns and volatility only generates a small portion of the negative variance risk premium, and Fama-French factors cannot account for strong negative variance risk premium either.

Todorov (2010) adopts a semi-parametric framework in modelling both stochastic volatility and jumps associated with variance risk, to learn about the dynamic dependencies in the premium. The model assumes that the futures price of the S&P 500 index has three components: a drift term, a continuous martingale, and a jump martingale, where the stochastic variance that drives the time variation of the continuous martingale is modelled as a sum of a continuous component and a jump component. The author left the drift term, as well as parametric form for jumps in price and stochastic volatility, purposely unspecified due to the lack of agreement on the parametric models of these variables in the literature. Realised variance and realised jumps are constructed from 5-minutes high frequency returns on the S&P 500 index from 1990 to 2002. Parameters of the stochastic volatility model are estimated by matching moments implied by the model to those inferred from high-frequency returns. Risk-neutral expectations of future variance are inferred from the synthesized variance swap rate using a portfolio of out-of-the-money options on the S&P 500 index. The variance risk premium is obtained by differencing realised with risk-neutral measures. The empirical evidence suggests that the variance premium is not only big but also varies significantly over time. The time variation of variance risk premium depends on both continuous component of stochastic volatility and past price jumps. Further tests show that jumps play an important

role in explaining variance risk premium. The estimated variance risk premium increases with a big market jump and slowly reverts to its long-run mean thereafter.

Another strand of literature studies the notion of variance risk premium within a consumption-based asset pricing framework². The basic setup involves a standard endowment economy with Epstein-Zin-Wein recursive preferences, and extends to incorporate stochastic volatility of consumption growth volatility. The variance risk premium, as a proxy for macroeconomic uncertainty risk driven solely by the consumption uncertainty risk, is the difference between the implied variance and the realised variance. Zhou (2009) describes that there is a fundamental link between the option-implied volatility risk premium and the variance risk premium in consumption-based asset pricing frameworks. For the former, stochastic volatility can only be priced if its innovation is correlated with the underlying equity return innovation. There is ample empirical evidence (i.e. papers mentioned earlier estimating parametric stochastic volatility models) documenting negative correlations between equity volatility and equity returns, such that the volatility risk premium is negative and serves as a hedging tool for investors. In the consumption-based framework, however, there is no assumption about statistical correlation between the volatility and consumption innovation. The representative agent is endowed with a preference for earlier resolution of uncertainty, and a stochastic volatility of volatility. Under such setup the variance risk premium embedded in equity markets must be positive to compensate for bearing more risk. Zhou (2009) claims that "the positive variance risk embedded in underlying asset is entirely consistent with the negative volatility risk premium implied from option prices".

Bollerslev et al. (2009) (hereafter BTZ) is one of the founding papers in this strand of literature that, followed by many others, leads to a new branch of research on asset return pre-

²Variance risk premiums are conventionally defined as the difference between the realised variance under the P measure and the Q measure, which are often negative. Nevertheless, there are papers study variance risk premiums from a consumption-based asset pricing framework, where variance risk premiums are interpreted as the difference between the Q measure and the P measure, which are often positive. Negative variance risk premiums are consistent with positive variance risk premiums multiplied by a factor of -1 . In this study, I focus on the variance risk premium under the traditional interpretation.

dictability linked to variance risk premium. In the model setup, the volatility of consumption growth rate and the volatility of volatility are governed by square root type processes, through which the time-varying volatility risk premium is generated. The asset return derived from the equilibrium model solutions can be shown to increase with endowment volatility and the volatility of volatility, reflecting the compensation for bearing volatility risk. The model-implied equity premium is composed of two terms, the time-varying volatility of consumption growth, and the stochastic volatility of volatility, of which the latter represents a fundamentally different source of risk that dominates the variation of the equity premium, which in turn constitutes the true volatility risk premium. The difference between the risk-neutral expected variance and the actual variance effectively isolates this factor and should serve as an useful predictor for asset returns. Monthly data for the new VIX index is used for quantifying risk-neutral variance. Realised variance are based on summation of 78 intraday five-minute returns on the S&P 500 index. The sample period is from January 1990 to December 2007. Time series analysis shows that the variance risk premium is almost always positive, with spikes occurring during 1997-2002 period. The variance risk premium is then used, in addition to other tradition predictor variables such as P/E ratio, price-dividend ratio, default spread, term spread, risk-free rate, and the consumption-wealth ratio (CAY), to forecast returns over different horizons. The empirical results show that the variance risk premium is able to predict returns on the S&P 500 index, and the highest predictability is at quarterly horizon, where it dominates that afforded by other predictor variables.

Zhou (2009) extends the theoretical foundation and empirical findings of BTZ to examine return predictability of variance risk premium of the S&P 500 index on equity returns, bond returns, and credit spreads for the period from 1990 to 2008. The additional predictor variables for equity returns, bond returns, and credit spreads are P/E ratio, forward spread, and short rates respectively. The regression results for different return horizons show that

the variance risk premium has a significant forecast power alone, and in combination with other predictor variables in explaining excess returns in all three markets, and the return predictability peaks around one-to-four months, and declines thereafter. This suggests that the risk premia across markets co-move in the short run, which may be driven by a common risk factor proxied by the variance risk premium. The author provides theoretical and calibration evidence based on the BTZ framework and found that the stylised model can provide qualitative justification for equity and bond risk premia, but the parameter setting is not rich enough to simultaneously explain credit spread puzzle.

Mueller et al. (2011) empirically examine the predictability of the market variance risk premium on bond risk premia. Similar to previously mentioned studies, the risk-neutral expected variance is measured by the new VIX index. The realised measure adopted in this study uses a HAR-RV (Corsi (2009)) model augmented by lags of implied variance. Data for regression analysis runs from 1990 to 2010 on a monthly frequency. The results show that variance risk premium is statistically significant for short term T-bill excess returns from 2-6 months with 1-5 months holding period, and the forecasting power is negligible for long term treasury bonds at 2-5 years with a 1 year holding period. The short run predictability is robust with the inclusion of other well-established bond risk premium predictors. The authors propose a potential explanation for the short-run predictability by extending BTZ's economy with stochastic dynamics in the inflation process. The calibrated results show that the inflation process alone, with or without stochastic volatility, is not able to reproduce the size of bond risk premia, but combined with consumption growth uncertainty, leads to rich bond risk premia.

Han and Zhou (2012) study the relationship between returns and variance risk premium on 500 U.S. individual stocks from 1996 to 2009. The sample included is a subset sample of stocks with traded options in CRSP database that have high market cap, high institutional

ownership, high analyst coverage, and tilt more towards growth stocks. The authors find that the monthly variance risk premium is significantly positive for two-thirds of the stocks, and it exhibits substantial cross-sectional variation. The variance risk premiums are significantly related to the sensitivity of stock returns to common risk factors, such as Fama-French factors and other proxies for systematic volatility factors. Stocks with higher risk exposure to systematic market volatility tend to have lower returns, and on the other hand, low risk exposure stocks serve as useful hedges for systematic volatility risk. Stocks with high variance risk premium outperform those with low variance risk premium. A value-weighted portfolio that is long the stocks ranked in the top quintile of variance risk premium and short those in the bottom quintile has an average return of 1.84% per month.

Wang et al. (2013) study predictability of variance risk premium on CDS spreads. Their sample include monthly observations of 382 U.S. entities that are not in the government, financial, or utility sectors. Data runs from 2001 to 2011. Regression results show that the variance risk premium displays a significant predictive power on firm-level CDS spreads in the presence of other credit risk determinants as well as firm-specific variables, and such predictability cannot be substituted by the latter. In addition, firm-level variance risk premium dominates market variance risk premium and implied variance in capturing macroeconomic uncertainty embedded in CDS spreads. The predictive power of variance risk premium on CDS spreads increases as firm's credit quality deteriorates, and is unchanged before and after the 2008 financial crisis.

Bollerslev et al. (2014) extend existing empirical evidence of return predictability based on monthly U.S. data to an international setting. Markets include France, Germany, Japan, Switzerland, the Netherlands, Belgium, the U.K., and the U.S.. Variance risk premiums are calculated for each market on the respective market equity index for the period from 2000 to 2011. Statistical analysis shows that the variance risk premiums are almost all positive

across markets, and display remarkable coherence in distributions. Regressing multi-period returns on respective country variance risk premiums results in similar humped-shaped coefficient estimates across countries, however, the degree of predictability is not as strong as that previously reported for the U.S. market. This suggests that there may be a worldwide systematic common risk factor priced in the market as opposed to country-specific risk factor. The authors proceed to construct a global variance risk premium proxy, measured by market-cap weighted-average of individual country variance risk premium. The regression results show stronger findings for all countries with a peak of predictability around four-month horizon.

Londono (2015) conducts similar analysis for an representative index for the Euro area, as well as the U.S., the U.K., Japan, Germany, Switzerland, the Netherlands, and France for the sample between 2000 and 2012. Consistent with the finding in Bollerslev et al. (2014), the author finds that except the predictability found in the U.S. market, every other country's variance risk premium is not a useful predictor for domestic stock returns. Further investigation shows that the U.S. variance risk premium has predictive power for foreign market returns. The predictability pattern is similar to that found for the U.S. market, which exhibits hump-shaped pattern and is maximised around 3-6 months horizon. The predictability is robust to several measures of variance risk premiums, and is also confirmed in an extended sample of countries where it is not possible to calculate variance risk premium for the representative stock index. To rationalise the predictive power of the U.S. variance risk premium on international stock returns, the author proposes a two-country general equilibrium model based on BTZ's single-country setup. The additional feature in this two-country setup is that it allows macroeconomic uncertainty to be transmitted across countries. The interaction between the transmission of uncertainty shows that if consumption growth of one country is exposed to macroeconomic uncertainty originated in a foreign country, then the uncertainty-driver

country's variance risk premium plays a significant role in explaining the time variation of stock returns in both countries.

The literature on variance risk premium is ever evolving, and this review is by no means exhaustive. Other related studies include Bollerslev et al. (2011) which exploits the linkages between the objective and risk-neutral expectation of integrated volatility to construct a risk aversion index and links it to a set of macro-finance state variables. Aït-Sahalia and Kimmel (2007) estimate a stochastic volatility model and volatility risk premium using maximum likelihood method based on closed-form approximations to the true likelihood function. Bollerslev and Todorov (2011) exploit the structure of jump tails and finds fears for rare events account for a large fraction of variance risk premium. Trolle and Schwartz (2010) investigate variance risk premium in crude oil and natural gas. In this study, I examine variance risk premiums for a set of countries under the conventional measure, as the difference between the P measure and the Q measure.

4.3 Construction of Variance Risk Premiums

I construct variance risk premium following the argument proposed in Carr and Wu (2009) in the notion of a variance swap. The payoff of a variance swap is given by the difference between the realised variance over the life of the contract and a fixed variance swap rate,

$$payoff = [RV_{t,T} - SW_{t,T}]L,$$

where $RV_{t,T}$ denotes realised variance, $SW_{t,T}$ is variance swap rate predetermined at time t , and L denotes notional value of the swap contract. No arbitrage requires that the variance

swap rate $SW_{t,T}$ equals the risk-neutral expected value of realised variance

$$SW_{t,T} = E^Q[RV_{t,T}].$$

Carr and Wu (2009) has shown that $E^Q[RV_{t,T}]$ can be approximated by the continuum of European out-of-the-money option prices with the same maturity date. Assuming an appropriate pricing kernel and no arbitrage, the variance risk premium is quantified by

$$VRP_{t,T} \equiv RV_{t,T} - SW_{t,T}.$$

Following this idea, I approximate $E^Q[RV_{t,T}]$ as the model free option-implied variance $IV_{t,T}$ for the representative equity index.

The methodology to calculate model free implied variance is described in Chapter 3³. Applied to Bakshi et al. (2003) method, the implied variance is given by

$$IV_{t,T} = E^Q[RV_{t,T}] = e^{rT} \left(\int_{S_0}^{\infty} \frac{2(1 - \ln \frac{K}{S_0})}{K^2} C(K, T) dK + \int_0^{S_0} \frac{2(1 + \ln \frac{S_0}{K})}{K^2} P(K, T) dK \right), \quad (4.1)$$

where S_0 is the current asset price, and $C(K, T)$ and $P(K, T)$ are call and put option prices at strike price K with maturity T . Using the appropriate option prices and calculation methods, the annualised IV is obtained for each country with 30-day expiry.

Corresponding to each 30-day implied variance, the annualised 30-day realised variance in Carr and Wu (2009) is calculated as:

$$RV_{t,t+30} = \frac{365}{30} \sum_{i=1}^{i=1} \left(\frac{F_{t+i,t+30} - F_{t+i-1,t+30}}{F_{t+i-1,t+30}} \right)^2,$$

³For details of the methodology in calculating the model free implied-variance see Chapter 3, section 3.3.

where $F_{t,T}$ denotes the time t forward price with expiry date T . Realised variance, in high-frequency econometrics, is a measure of the sum of finely-sampled squared returns over a fixed time interval during the trading day. It is a consistent estimator of the corresponding quadratic return variation when returns are sampled at increasingly higher frequency and microstructure effects can be ignored. Empirical studies have demonstrated that this type of measure produces more accurate ex-post measures of actual variance than measures based on daily or coarser frequency data (Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002a,b)). However, since the standard market practice to calculate the realised variance in a variance swap contract is using daily returns, I proceed with daily data to quantify variance risk premium. The variance risk premium calculated by Carr and Wu (2009) represents an actual measure based on ex-post knowledge of the realised variance. I propose a measure of realised variance based on conditional variance expectations that provides an expected measure of the variance risk premium over T .

The conditional variance model is proposed by Corsi (2009), termed Heterogeneous, Autoregressive model of Realised Volatility (HAR-RV). Financial data show well-known stylised facts such as strong persistent autocorrelations of squared and absolute returns that last for long periods (long-memory⁴), fat tails of return distribution, etc., which pose challenges to standard econometric models. The HAR-RV model is an approximate long-memory conditional volatility model which is able to reproduce these features that standard GARCH and stochastic volatility models can not. Stemming from the idea of Heterogeneous Market Hypothesis presented by Müller et al. (1993), which recognizes the presence of heterogeneity across traders, Corsi (2009) concentrates on the heterogeneity that originates from the difference in the time horizons. Typically, the financial markets are composed of participants with different trading frequencies: high frequency for market makers and speculators, and relatively low frequency for institutional investors. The main idea is that market participants

⁴See Baillie (1996) for a review on long memory processes.

with different time horizons perceive, react to, and cause different types of volatility components, which is shown as volatility cascade generated by heterogeneous market structure. Motivated by this idea, Corsi (2009) developed an additive cascade model of different volatility components each of which is generated by the actions of different types of market participants. This model incorporates a simple AR-structure in the realised volatility over different horizons, and is able to reproduce the observed volatility persistence as well as other main stylised facts of financial data.

Following the idea of Corsi (2009), the realised variance over a desired horizon h is given by

$$RV_{t,t+h} = RV_{t+1} + RV_{t+2} + \dots + RV_{t+h}, \quad h = 1, 2, 3, \dots$$

By definition $RV_{t+i-1,t+i} \equiv RV_{t+i}$. Hence, a weekly realised variance at time t is given by

$$RV_{t,t+5} = RV_{t+1} + RV_{t+2} + RV_{t+3} + RV_{t+4} + RV_{t+5},$$

and a monthly realised variance is

$$RV_{t,t+22} = RV_{t+1} + RV_{t+2} + \dots + RV_{t+22},$$

where $h = 5$ and 22 counts trading days as a proxy for weekly and monthly horizons. The proposed cascade HAR-RV model is represented by

$$RV_{t,t+h} = \beta_{0,h} + \beta_{d,h}RV_{t-1,t} + \beta_{w,h}RV_{t-5,t} + \beta_{m,h}RV_{t-22,t} + \epsilon_{t,t+h}. \quad (4.2)$$

Equation (4.2) can be seen as a three-factor stochastic variance model, which predicts the next h -period ahead value based on the past realised variance over daily, weekly, and monthly horizons. Of course the realised variance over other horizons can be easily included in the

model as additional explanatory variables, but daily, weekly, and monthly measures afford a natural economic interpretation.

Hence, the variance risk premium, in a simpler notation, is given by

$$VRP_{t,T} = E^P[RV_{t,t+h}] - IV_{t,T}, \quad (4.3)$$

with h (in trading days) matching T (defined in years). The difference measures the profit and loss per dollar from a long variance swap contract and holding it to maturity. Dividing both sides by $IV_{t,T}$ captures the return from the investment,

$$RVRP_{t,T} = \frac{E^P[RV_{t,t+h}] - IV_{t,T}}{IV_{t,T}}. \quad (4.4)$$

4.4 Data

For the empirical analysis of variance risk premium spillover effects, I consider variance risk premiums of four major markets across the globe – The U.S., the U.K., the Eurozone, and Japan. The representative equity index for each market is the S&P 500 index, FTSE 100 index, DJ EURO STOXX 50 index, and NIKKEI 225 index. Data under examination spans from 2004 to 2014.

Option prices used to calculate $IV_{t,T}$ are obtained from Ivy DB Global Indices database from OptionMetrics, as described in section 3.3.4, Chapter 3. Option prices are filtered to include options with strictly positive bid prices, or last prices (defined as the closing trade price or the settlement price of the option) whenever bid prices are not available. Options with missing implied volatilities or Black-Scholes delta are discarded. Option series with maturities shorter than 7 days and longer than 300 days are excluded. Interest rates are zero coupon rates obtained within the database and are linearly interpolated to match option

maturities.

Calculation of equation (4.1) requires option prices for a continuum of strikes, but option strikes traded in the market are limited. To minimise the estimation error in equation (4.1) I select out-of-the-money options, whose implied volatilities are fitted and interpolated and extrapolated to obtain a smooth implied volatility function. Option prices for a full range of strikes are then translated from the Black-Scholes model through this implied volatility function. $IV_{t,T}$ for a 30-day horizon are then linearly interpolated using two nearest maturities⁵.

Daily equity index prices are needed to calculate $RV_{t,T}$. Data is obtained from Datastream. Denote by P_t the closing price of the index, daily realised variance is calculated from squared returns:

$$RV_{t,t+1} \equiv RV_{t+1} = \left(\frac{P_{t+1} - P_t}{P_t} \right)^2.$$

To match the 30-day horizon with $IV_{t,T}$, the HAR-RV model considered is

$$RV_{t,t+22} = \beta_0 + \beta_d RV_{t-1,t} + \beta_w RV_{t-5,t} + \beta_m RV_{t-22,t} + \epsilon_{t,t+22}, \quad (4.5)$$

where 22 trading days approximates 30 calendar days, and

$$RV_{t-1,t} \equiv RV_t = \left(\frac{P_t - P_{t-1}}{P_{t-1}} \right)^2,$$

$$RV_{t-5,t} = RV_{t-4} + RV_{t-3} + RV_{t-2} + RV_{t-1} + RV_t,$$

$$RV_{t-22,t} = RV_{t-21} + RV_{t-20} + \dots + RV_t.$$

Table 4.1 reports OLS estimates of the HAR-RV regressions of equation (4.5) for each of the markets. The standard errors reported in parentheses are based on a Newey-West correction allowing for serial correlation of up to an order of 44 days, as suggested in Andersen et al. (2007). The estimates of coefficients show that most of the estimates are highly statistically

⁵For details of $IV_{t,T}$ calculation see section 3.3, Chapter 3.

significant, but with a rather weak significance for all daily coefficients. The estimates confirm highly persistent volatility dependence. The monthly variance component appears to have a relatively bigger influence on the predictions than daily and weekly components for all markets, except Japan. Interestingly, daily variance components seem to have a relatively weak impact, and two of the coefficient estimates across sample are statistically insignificant at the 10% level.

Corsi (2009) examined S&P 500 futures from 1990 to 2007 for realised volatility using the HAR-RV model for one-day prediction. Realised volatilities aggregated over three different horizons are all highly significant, and the importance of the market components decrease with the horizon of the aggregation. The results show that daily volatility components appear to have a stronger impact on the predictions than weekly and monthly components. Realised variance in his paper is constructed by tick-by-tick spot logarithmic middle prices of bid and ask quotes. Similar to Corsi (2009), Andersen et al. (2007) studied same data on realised volatility but spans from 1986 to 1999 with tick-by-tick high-frequency data. They estimated the HAR-RV model for $h = 1, 5,$ and 22 with an additional jump component in the regression. They found that the relative importance of the daily volatility component decreases from the daily to the weekly to the monthly regressions, whereas the monthly volatility component tends to be relatively more important for the longer-run monthly regressions, which echoes my findings.

The insignificant coefficient estimates may be due to the high level of noise contained in daily close price which is used to construct realised variance, as opposed to intra-day data. The noisier estimation of the daily realised variance induces weak significance of the daily volatility component, while weekly and monthly realised variances, being averages over longer periods, arguably contain less noise and more information on the volatility process and, hence, receive higher weights from the model.

Figure 4.1 shows time series of HAR-RV expectations of annualised realised variance for each of the markets. Each of the series clearly exhibits a high degree of serial correlation. As can be confirmed in the autocorrelation plots in Figure 4.2. High values of Q-stats of Ljung-Box test rejects the null hypothesis of no autocorrelation. It shows that the HAR-RV model is able to reproduce the persistence in empirical data. Table 4.2 provides descriptive statistics for the four sets of expected realised variances. All exhibit patterns of low level of variance in calm market conditions, and spikes during market turmoil, the most obvious being the 2008 financial crisis.

4.5 Empirical results

4.5.1 Variance risk premium dynamics

Table 4.2 presents descriptive statistics for each market for the realised variance and volatility and implied variance and volatility respectively in Panel A and B. Panel C presents summary statistics for annualised variance risk premiums and return variance risk premiums given by equation (4.3) and (4.4). Data is from May 2004 to December 2014. The averages of the variance risk premiums are negative for all countries, with similar magnitude. Variance risk premiums show large kurtosis ranging from 30 for the U.S. to 75 for the Eurozone, and sometimes large skewness. The distribution of return variance risk premiums appear to be close to normal. Carr and Wu (2009) examined variance risk premiums for 5 stock indexes and 35 individual stocks. The mean, standard deviation, skewness, and kurtosis for variance risk premium on the S&P 500 index for a sample from January 1996 to February 2003 are -0.0274 , 0.0363 , -1.44 , and 17.86 respectively. Results of my sample share similar magnitude with those reported in Carr and Wu (2009), but with higher kurtosis.

The mean estimate of variance risk premiums represents the average profit or loss for

a variance swap contract. If an investor holds a long 30-day variance swap contract with a notional of \$1 on the S&P 500 index to maturity, the average payoff for the sample period is \$-0.0144. Similar losses result for the other three countries. On the other hand, the return variance risk premium gives the return of going long a variance swap contract and hold it to maturity. The average return for the U.S., U.K., Eurozone, and Japan are -19.4%, -14.7%, -5.30%, and -1.80% respectively. Despite the different scaling, it is clear that investors are willing to accept negative returns on the long swap contract, hence, shorting variance swap contracts on these four markets will generate positive returns for this sample period.

Figure 4.3 plots time series of variance risk premiums and return variance risk premiums for the four markets. The figure suggests that variance risk premiums (hereafter VRP) display significant time variation. Volatility of the variance risk premium is smallest for the U.K. at 3.2% and biggest for Japan at 6.1%. The graphs show common periods of VRP spikes, the most obvious being around the 2008 financial crisis, where not surprisingly, the minimum and maximum values occurred during this period. VRPs are small with little variation prior to 2007 for all countries, and began to increase starting around the sub-prime mortgage crisis in early 2007. The second episode of spikes, but with smaller magnitudes, occurred around May 2010, known as the flash crash for the United States trillion dollar stock market crash, and its influence quickly spread over to the other markets. The third common episode of spikes occurred around August 2011 when stock prices drop sharply across the world. This was due to the fears of contagion of the European sovereign debt crisis as well as the downgrade of the credit rating of the United States.

Despite the similarities of the behaviour of variance risk premiums between the four markets, there appears an unusual spike uniquely for Japan in March 2011. On Friday March 11, 2011, a 9.0 magnitude earthquake hit Japan and triggered powerful tsunami around the area. Immediately after the earthquake, the tsunami disabled the power supply and cooling

of three Fukushima Daiichi reactors, which lead to a nuclear meltdown in the following three days. The Fukushima nuclear accident was the largest nuclear disaster since 1986 Chernobyl. The variance risk premium was large and negative the following Monday, followed by consecutive large and positive variance risk premiums for the next 10 trading days. Stock market fell after panic selling as fears of nuclear disasters intensified. Realised volatility shot up significantly as the market became more disturbed. Another episode of large VRPs unique to Japan occurred in May 2013, when NIKKEI 225 fell by 7%, its worst intra-day fall since the 2011 earthquake. This was attributed to the investors being rattled by weak economic data from China and indications that the U.S. Federal Reserve may start reducing its bond-buying program as early as June.

Table 4.3 shows that variance risk premiums are highly correlated across countries (numbers in parenthesis show return variance risk premium correlations between markets), all of which are above 0.5. The highest correlation is between the U.K. and the Eurozone, being 0.85, and this is not surprising as these two markets are geographically the closest with no non-overlapping trading hours. In contrast, Japan has a relatively weak correlation with the U.S. (0.51) and the Eurozone (0.52).

4.5.2 Spillover analysis

To study the spillover effects of variance risk premiums between countries, vector autoregressive (VAR) models are employed. As described in Chapter 3 section 3.4.3, the VAR model, popularised by Sims (1980), is used to capture linear interdependencies among multiple time series. A VAR(p) model takes the general form:

$$\mathbf{Y}_t = \mathbf{c} + \Pi_1 \mathbf{Y}_{t-1} + \Pi_2 \mathbf{Y}_{t-2} + \cdots + \Pi_p \mathbf{Y}_{t-p} + \boldsymbol{\epsilon}_t \quad (4.6)$$

Where $\mathbf{Y}_t = (y_{1t}, y_{2t}, \dots, y_{kt})'$ denotes a $(k \times 1)$ vector of times series variables of interests.

\mathbf{c} is a k dimensional vector. Π_i are $(k \times k)$ coefficient matrices, and ϵ_t is an $(k \times 1)$ unobservable vector of random process with zero mean and covariance matrix Σ . ϵ_t is serially uncorrelated but may be contemporaneously correlated. p is lag length. Applying to the sample variables, the VAR(p) system is:

$$\begin{aligned}
US_t &= c^{US} + \sum_{i=1}^p A_{1i}^{US} US_{t-p} + \sum_{i=1}^p B_{1i}^{UK} UK_{t-p} + \sum_{i=1}^p C_{1i}^{Euro} Euro_{t-p} + \sum_{i=1}^p D_{1i}^{JP} JP_{t-p} + \epsilon_t^{US} \\
UK_t &= c^{UK} + \sum_{i=1}^p A_{2i}^{US} US_{t-p} + \sum_{i=1}^p B_{2i}^{UK} UK_{t-p} + \sum_{i=1}^p C_{2i}^{Euro} Euro_{t-p} + \sum_{i=1}^p D_{2i}^{JP} JP_{t-p} + \epsilon_t^{UK} \\
Euro_t &= c^{Euro} + \sum_{i=1}^p A_{3i}^{US} US_{t-p} + \sum_{i=1}^p B_{3i}^{UK} UK_{t-p} + \sum_{i=1}^p C_{3i}^{Euro} Euro_{t-p} + \sum_{i=1}^p D_{3i}^{JP} JP_{t-p} + \epsilon_t^{Euro} \\
JP_t &= c^{JP} + \sum_{i=1}^p A_{4i}^{US} US_{t-p} + \sum_{i=1}^p B_{4i}^{UK} UK_{t-p} + \sum_{i=1}^p C_{4i}^{Euro} Euro_{t-p} + \sum_{i=1}^p D_{4i}^{JP} JP_{t-p} + \epsilon_t^{JP}
\end{aligned}$$

Where US_t , UK_t , $Euro_t$ and JP_t denotes measures of variance risk premiums of the U.S., the U.K., the Eurozone, and Japan.

The main assumptions of VAR models are that time series \mathbf{Y}_t are (weakly) stationary with time invariant mean and covariance matrix, and that ϵ_t are serially uncorrelated innovation processes that follow multivariate normal distributions. However, daily variance risk premiums are a result of linear combination of daily *IVs* and *RVs* looking 30 calendar days ahead in the future. By construction, therefore, any two observations within 22 trading days apart will contain overlapping information, and thus induce serial correlation. This can be confirmed by Ljung-Box test statistics reported in and plots of autocorrelation functions in Figure 4.4.

Portmanteau test Q -statistic can be used to detect serial correlation on the level series and serial dependence on the squared series. Ljung and Box (1978) Q -statistic is $Q(m) = T(T+2) \sum_{l=1}^m \frac{(\hat{\rho}_l)^2}{T-l}$. Where $\hat{\rho}_l$ is lag- l sample autocorrelation. The null hypothesis is H_0 :

$\rho_1 = \dots = \rho_m = 0$ against alternative $H_a : \rho_i \neq 0$ for some $i \in \{1, \dots, m\}$. Under appropriate assumptions $Q(m)$ is asymptotically chi-square distributed with m degrees of freedom.

Table 4.4 reports Ljung-Box tests for serial correlation on variance risk premiums and return variance risk premiums in squared brackets. Tests statistics are reported for 10, 15, and 20 degrees of freedom. As can be seen that variance risk premiums for all countries are serially correlated with large Q -statistics. P -values of the test statistics shown in parenthesis are essentially zero. The Q -statistics for return variance risk premiums are bigger than those for dollar amount variance risk premiums, suggesting possible more severe serial correlation in return variance risk premiums. Further evidence can be seen in the autocorrelation function in Figure 4.4. It suggests that variance risk premiums for four countries are serially correlated.

While a simple AR structure of each component variable in the VAR system may or may not capture serial correlation, serial dependence of the time series will remain if not accounted for, and its effect will feed into the residuals causing failure of model validity checks, hence render inference on the VAR model invalid. In fact, full sample VRPs and subsamples of VRPs have been fit using the VAR model. Both statistics of multivariate Portmanteau tests and White heteroskedasticity tests are large and significant with p -values practically equal to zero, suggesting that there is serious autocorrelation and heteroskedasticity in the residuals. The results are not made better by increasing the order of autoregressive terms. Hence it suggests that the VAR model does not provide a good fit directly to variance risk premiums, and steps must be taken to account for both effects.

ARCH models provide a natural remedy to capture such characteristics in the time series. ARCH stands for autoregressive conditional heteroskedasticity. Changes in the scale of a variable give rise to heteroskedasticity. In the pioneering work of Engle (1982), a stochastic process is defined whose variables have conditional mean zero and conditional variance

given by a linear function of previous squared variables. The variable of interest is the return of an asset, and the variance of the return is conditional on the information in previous returns. Subsequent research has provided many alternative functions to allow conditional mean to vary through time, and to allow for more components in the conditional mean to capture dynamics of different asset classes.

There is a multitude of ARCH specifications, and the best known being GARCH (generalised ARCH) from Bollerslev (1986). Letter ‘G’ stands for generalised by including a lagged variance term in the conditional variance equation. Let r_t be the log return of an asset at time t , with conditional mean μ_t and conditional variance h_t conditional on information set available until time $t - 1$. Then the formal definition of GARCH(p, q) is given by

$$r_t = \mu_t + \sqrt{h_t}\epsilon_t, \quad (4.7)$$

$$h_t = \omega + \sum_{i=1}^p \alpha_i (r_{t-i} - \mu_i)^2 + \sum_{j=1}^q \beta_j h_{t-j} \quad (4.8)$$

where $\{\epsilon_t\}$ is a sequence of i.i.d. random variables with mean zero and variance 1. ϵ_t is often assumed to follow a standard normal distribution. Other distributions such as standardized student-t distribution and generalized error distribution are also popular candidates. Constraints are $\omega > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$. The unconditional variance $E[h_t] = \frac{\omega}{1-\alpha-\beta}$ is finite if and only if $\alpha_i + \beta_i < 1$, assuming $p = q = 1$, whereas the conditional variance h_t evolves over time.

The GARCH(1,1) model with conditional normal distribution is the most popular ARCH specification in empirical research, particularly in modelling daily returns in their ability to capture stylized facts of daily returns. The estimation of parameters is easily obtained by maximum likelihood methods. Considerable empirical investigations have been applied to financial markets covering equities, foreign exchange, and interest rates. A review of theory and empirical evidence up to the 1990s can be found in Bollerslev et al. (1992). In this study,

I apply GARCH(1,1) models to variance risk premiums⁶.

As can be seen from Figure 4.3, the 10 years time series of variance risk premiums exhibit distinct patterns in different periods, with time varying volatility and high and low level of volatility clustering, it is thus difficult to adequately capture the dynamics of the time series with possible structural breaks using one GARCH equation⁷. In order to better describe the characteristics of variance risk premiums, I divide the full sample into three subsamples to investigate them individually.

The subsamples are pre-crisis May 2004 – April 2006, crisis September 2008 – March 2009, and post-crisis January 2012 – November 2014. Subsamples are chosen such that there is no obvious structural break during each period. The construction of realised variance incorporates overlapping information from its history, thus produces serial correlation up to 22 observations. This autocorrelation is naturally present in variance risk premiums and needs to be accounted for. I introduce appropriate $AR(m)$ structure into the mean equation to capture this effect. Parameters of the mean equation and variance equation are obtained simultaneously by maximum likelihood estimation.

Table 4.6 presents GARCH fitting of subsamples of variance risk premiums for each market. The autoregressive terms in the mean equation are indicated as $AR()$, and the numbers in the parenthesis indicate the location of the lags. All lags included are statistically significant. Parameter estimates of $\alpha + \beta$ are very close to 1, suggesting that the volatilities of daily variance risk premiums are highly persistent. Empirical estimates of the sum of α and β sometimes exceed 1 if parameters are not constrained. There are three cases in the subsamples return estimates of $\alpha + \beta > 1$ without parameter constraints – the pre-crisis Japan, crisis Eurozone, and post-crisis Japan subsample. One way to get around this problem is by es-

⁶The following analysis in GARCH modelling and spillover analysis is done for the dollar measure of variance risk premiums, since the Ljung-Box Q -statistics are smaller than those for the return variance risk premiums.

⁷GARCH specification with various combination of p and q together with AR structure in the mean equation have been explored to model the full sample variance risk premiums, however none is able to fully capture the dynamics. The Ljung-Box test on standardized residuals for serial correlation is statistically significant.

timating an integrated GARCH (IGARCH) model introduced by Engle and Bollerslev (1986), which imposes a constraint that $\alpha + \beta = 1$. But, under this specification, the unconditional variance is infinite, which seems hard to justify for variance risk premium series.

An alternative approach to estimate parameters of GARCH(1,1) model is known as variance targeting, introduced by Engle and Mezirich (1996). This is an useful technique to reduce the dimension of optimisations in maximum likelihood estimation. It involves a two-step estimation where, the first step sets the unconditional variance $E[h_t]$ equal to the sample variance calculated from the data. ω is then parametrised by $E[h_t](1 - \alpha - \beta)$. In the second step, two parameters α and β can be estimated by maximum likelihood. This method has been recommended in Christoffersen (2003) and Hull (2011). Shephard and Sheppard (2010) also applied it to high-frequency based volatility models. I apply this method for the three subsamples in estimating the parameters.

To check the validity of the GARCH model, the standardized residuals series are examined. Let a_t be the residuals of the mean equation, then $a_t = r_t - \mu_t$. Standardized residuals are given by $\tilde{a}_t = \frac{a_t}{\sqrt{h_t}}$. The Ljung-Box Q-statistic of \tilde{a}_t can be used to check the adequacy of the mean equation and that of \tilde{a}_t^2 for the volatility equation. Table 4.6 reports Ljung-Box Q-statistics up to 20 lags adjusted for degrees of freedom with p -values reported in parenthesis. As can be seen, the Q-statistics for both \tilde{a}_t and \tilde{a}_t^2 are small and p -values are large for most of the samples, indicating that there is no serial correlation or heteroskedasticity in the residuals, hence the fitted GARCH models are adequate. Further evidence can be seen from the plots of standardized residuals in Figure 4.5, which look like white noise processes.

However, subsample pre-crisis Japan rejects the null hypothesis of Ljung-Box test on \tilde{a}_t^2 at 10% level, suggesting there is remaining ARCH effect in the standardized residuals. But note that the p -value of Q(19) is 0.77, suggesting there may be minor correlations at lag 20 for squared standardized residuals, but not up to 19 lags. The same is the case with post-

crisis Eurozone, where $Q(20)$ gives p -value 0.02 while 0.25 for $Q(17)$. This could be driven by occasional extreme values or level shifts of variance risk premiums in the respective periods.

To study the spillover effects of variance risk premiums using the VAR model without compromising VAR assumptions, I feed standardized residuals of fitted GARCH model into the VAR analysis. Since standardized residuals behave more or less like a white noise process, I expect no or little significant self-explanatory lags for each market. Any significant other-explanatory lags indicate explanatory power of one market to another, hence the spillover effect. VAR models are fit for the three subsample periods to study the dynamics of different periods. Empirical results are now reported separately for each subsample commencing with the most recent subsample.

4.5.2.1 Post-crisis spillover

Table 4.7 reports estimates of the vector autoregression model for the post-crisis subsample. A VAR(2) is estimated according to Akaike (AIC), Schwarz (SIC), Hannan-Quinn (HQ), and final prediction error (FPE) information criteria. The first column reports regression results of US on the lags of the US, UK, Eurozone, and Japan. The results show that none of the coefficient estimates are statistically significant, indicating that no lags of other markets, as well as own lags of US, have explanatory power on US. This is also confirmed by the very low R^2 of 0.015 and F -statistic equals 1.342. Likewise for Japan, there is no significant self-explanatory lags, but a significant lag of US at lag 1. For UK and Eurozone however, there is significant self-explanatory power at lag 1, at 5% and 1% level respectively, reflecting possible information not captured at lower lags in GARCH fitting. It is worth noting that lags of US are highly significant in the regressions of UK, Eurozone, and Japan, indicating that the history of US have significant explanatory power on other three markets, suggesting possible lead-lag relationship.

The dynamic properties of a VAR(p) model is often summarised through some structural analysis. Impulse response analysis traces the effects of a shock to one endogenous variable on the other variables in the VAR system. If there is a reaction of one variable to an impulse in another variable, there may be causal effect from the latter to the former. It moreover traces the speed and persistence of the shocks, and therefore enables the examination of the time structure of the transmission. I consider in this study generalised impulse response functions proposed by Pesaran and Shin (1998) as opposed to the orthogonalized impulse response function in that the generalised impulses are invariant to the ordering of the variables. Since there is no economic theory underpinning the causal relationship between variance risk premiums, it is reasonable to use generalised impulse response functions to study the inter-relationship between them without a priori assumptions.

Figure 4.6 shows impulse responses to one standard deviation innovations of four markets. Column one shows responses of US, UK, Eurozone, and Japan to one standard deviation shock to the US. Similarly, columns two to four show responses of each country to one shock to UK, Eurozone, and Japan respectively. The vertical axis shows the magnitude of the responses, and the horizontal axis shows the time line of responses. It can be seen from the figures in the first column that UK, Eurozone, and Japan all have lagged responses to a shock to the US. They not only respond contemporaneously but also one day beyond. The effect of a shock to US on UK and Eurozone decreases on the second day and tapers out thereafter. The magnitude of the contemporaneous response is about one half. Japan on the other hand, shows little contemporaneous response on day one, but an increased response on the next day, and tapers out thereafter. This is possibly due to the time zone difference between US and Japan, where there is about 8.5 hours difference between the close of the Japanese market and the open of the US market⁸. On the same calendar day US should have

⁸For details of opening and closing time of these four markets see Chapter 3 Figure 3.1 and discussions thereafter.

no impact on Japan. But the Japanese market reopens about 3 hours after US market closes. Any information not captured on day one along with new information is reflected immediately on day two, which shows a much stronger reaction than day one. There also seems to be some residual response on day three. Information in figures in the first column suggests that US leads UK and Eurozone by one day, and Japan by two days.

Figures in columns two and three show responses to a shock to UK and Eurozone respectively. It is not surprising that UK and Eurozone respond to each other contemporaneously as the two markets operate at almost the same time. Despite 6 hours difference between the opening of US and European markets, US responds to UK and Eurozone with no lagged effect. This is possibly due to two hours overlapping trading hours before the European markets close, and all information successfully communicated to the US market. However, Japan reacts to both European markets very weakly, and with lagged effects up to three days. UK and Eurozone lead Japan by two days. Column four shows responses of three western markets to a shock from Japan. It is clear that all responses are very small, if not none, indicating that Japan does not have a big impact on western markets.

Besides impulse response analysis, the Granger Causality test is another popular structural analysis for VAR models. The Granger Causality, proposed in Granger (1969), refers to a statistical concept whether one time series is useful in predicting another. If past values of X contains information useful in predicting Y above and beyond past values of Y alone, then time series X is said to Granger Cause time series Y.

Table 4.8 reports pairwise Granger Causality test and tests whether an endogenous variable can be treated as exogenous. For each equation in the VAR, the output displays Wald test statistics for the joint significance of each of the other lagged endogenous variables in that equation. For example in the first VAR equation, test on Excluded(UK) tests for the joint significance of UK lag 1 and lag 2. The null hypothesis is that the coefficient of UK lag 1

equals that of UK lag 2 equals to zero. The χ^2 statistic and p -value indicate that the null cannot be rejected, suggesting that the history of UK up to 2 lags does not 'Granger cause' the US. Similar conclusions can be drawn for the joint significance of Japan lagged values, where test for Eurozone is marginally rejected at 10% significance level, but is not rejected at a 5% level. The statistic in the last row (All) is the statistic for joint significance of all other lagged endogenous variables in the equation. It can be seen that the test statistic is insignificant, suggesting that the history of UK, Eurozone and Japan do not Granger Cause US. Likewise for the VAR equation for UK, Eurozone and Japan, lags of US are statistically significant for all three markets, implying uni-directional Granger Causal effect from the US market. But none can be explained singly by any other market than the US.

It can be seen from both impulse response analysis and Granger Causality tests that for the post-crisis subsample period, the US market is the dominant source of uncertainty in the world where the European markets and the Japanese market are strongly impacted. As opposed to findings in the 1980s and 1990s studies in volatility spillovers, the Japanese market is no longer the leader of information distribution, but rather a follower. The global markets are highly integrated between the western countries as information is incorporated contemporaneously with a maximum one day of delay, whereas it takes up to three days for Japan to fully digest it.

The validity of the estimated VAR model can be checked by the examinations of the residuals of the system. Table 4.9 reports autocorrelation test and heteroskedasticity test. The top panel shows Portmanteau tests for autocorrelation up to 12 lags. The null hypothesis is that there is no residual autocorrelations up to h lags. Each column reports lags, Q-statistic, p -value, and degrees of freedom respectively. It can be seen that the p -values are large for all lags suggesting that the null hypothesis of no residual autocorrelations cannot be rejected.

The bottom panel reports White tests for residual heteroskedasticity. The test regression

is run by regressing each cross product of the residuals on the levels and squares of the original regressors and test the joint significance of the regression. The output shows both joint test and each individual component. It shows clearly that the null hypothesis of no residual heteroskedasticity cannot be rejected. Since we cannot reject the null hypothesis of no residual autocorrelation and heteroskedasticity, it can be concluded that the VAR model is not mis-specified, and the inference on the structural analysis is valid.

4.5.2.2 Crisis spillover

The same as the post-crisis spillover analysis, I employ the same methodology analysing the crisis period. The crisis period spans about six months from breakout of the 2008 global financial crisis to the end of the first quarter in 2009, covering the most volatile periods since the collapse of Lehman Brothers. Since it is the crash episode that market participants worry about the most, and that world's markets tend to be heavily impacted by one another, it is important to understand the interrelations between these markets. Any lead-lag effect may have important implications on asset allocation across markets.

Table 4.10 reports vector autoregression estimates for crisis period. A VAR(1) is fitted and is considered adequate by residual checks in Table 4.12. The output shows that the lag of US has significant explanatory power on all other markets, and no other lags is significant in the VAR system, indicating that none of UK, Eurozone, and Japan's one period history is able to provide useful information in predicting any market.

Figure 4.7 shows impulse response functions for a crisis subsample. The impulse responses show very similar pattern to those of post-crisis period. UK, Eurozone, and Japan have lagged response to a unit shock to the US, and the impact of the shock lasts two to three days. The responses are positive on the first two days, with a minor negative correction on day three. Responses of UK and Eurozone show a decreasing trend whereas Japan has the

biggest response on day two due the time zone effect.

US responds little to Japan. UK and Eurozone have one day lagged response to Japan with a significant negative response on the second day. It suggests that given one unit shock in Japan, UK and Eurozone show about -0.1 response to that shock. On the other hand, Japan shows a lagged response to shocks to UK and Eurozone. The bi-direction response may suggest a feedback effect between Japan and European markets as they process new information.

Compared to post-crisis results, responses in the crisis period take longer to die down. Information between some of the markets is fully processed by up to three days, reflecting the fact that there is explosive information flow and great uncertainty during market turmoil periods. Traders take caution in executing orders when they are unclear about the market direction and face mixed signals, which may result in a delayed reaction to a particular information. However, despite this, the US is unambiguously the leading source of uncertainty during the crisis period.

This can be confirmed by the Granger Causality tests in Table 4.11. US has uni-directional explanatory power on all other markets, but no market is able to predict US. However, past history of all other markets has predictive ability in explaining UK, Eurozone, and Japan. The p -values of exclusion test for All for UK, Eurozone, and Japan are 0.002, 0.003, and 0 respectively, suggesting that all other lagged variables are jointly significant in predicting these markets.

4.5.2.3 Pre-crisis spillover

Lastly, I investigate spillover effects for pre-crisis subsample. Data ranges from May 2004 to April 2006, where markets are generally quiet with little volatility around the world. Vector autoregressive estimates in Table 4.13 show that again US has predictive power on other

markets, and not vice versa. Lagged value of UK seems to have significant explanatory power on Japan.

The impulse responses analysis for pre-crisis subsample (see Figure 4.8) is almost identical to those of post-crisis subsample, and hence the interpretation. US leads UK, Eurozone, and Japan by one day; responds weakly to UK and Eurozone; almost no response to Japan. UK, and Eurozone lead Japan by one day, and respond very little to Japan. It is worth noting that in this period Japan has a subtly increased response to UK on the second day. Granger Causality test confirms that besides the uni-directional causality from US to other markets, UK has significant predictive power on Japan. The model's adequacy can be confirmed by Table 4.15.

To summarise, the VAR model together with structural analysis, impulse response functions and Granger Causality tests, on pre-crisis, crisis, and post-crisis subsamples of variance risk premiums show that the US is unambiguously the leading source of uncertainty in the global market, and no other single country or a combination of countries have predictive power on the US. The western markets generally have a leading impact on Japanese market, especially during low volatility periods, where western countries lead Japan by one day. However, during market turmoil periods, there appears to be complex feedback effect between the European markets and the Japanese market, where no clear leadership can be identified during this periods. The Japanese market does not have much impact on the western countries. The global markets are highly integrated in that information flow from one market to another is fully processed within two days, but in crisis period it may take longer as greater uncertainty exists in the market.

4.6 Conclusions

In this study, I analysed variance risk premiums in the notion of a variance swap. The variance swap rate represents the risk-neutral expectation of realised variance. The difference between the realised measure and risk-neutral measure quantifies the variance risk premium. I calculate the risk-neutral expected variance using a set of option prices, and construct realised variance using the HAR-RV model based on daily data. Variance risk premiums are calculated for the U.S., the U.K., the Eurozone, and Japan for the period from 2004 to 2014. Variance risk premiums across markets are on average negative with negative skewness and large kurtosis. Similar patterns are observed for all countries that the magnitude of variance risk premiums are high when equity markets fall. It shows clear evidence that investors are willing to accept negative returns to go long a variance swap to hedge away the increase in return variance.

To analyse the spillover effects of variance risk premiums across markets, I employ a vector autoregressive methodology and its structural analysis. Since the time series of variance risk premiums exhibit significant time variation, I divide the full sample into pre-crisis, crisis, and post-crisis subsamples to accommodate statistical assumptions required by VAR analysis. Empirical spillover results show that the U.S. is unambiguously the distributor of uncertainty in the global market, and no other country is able to predict the U.S.. Japan seems to have little impact on western markets as indicated by low correlation and the role as a follower. In general, the results show that these markets are highly integrated in that new arrived information is fully processed within two days, but in crisis period it may take longer due to greater uncertainty.

In this study I fit univariate GARCH models to each subsample/country and extract standardized residuals for each country as endogenous variables fed into the VAR model to address autocorrelation and heteroskedasticity in VRP data. A potential alternative method to

address concerns on overlapping returns could be to construct daily exposure to VRP, as discussed in Kozhan et al. (2013). This involves engaging in a variance swap and delta hedging it. Instead of holding it to maturity, sell it the next day. This procedure can be repeated every day, and ignoring transaction costs, it gives the daily exposure to variance risk premium.

Variance risk premiums have been well researched for the U.S. market, however, little is documented in the international setting. A number of studies have observed that the variance risk premium has predictability on the U.S. equity market returns. Bollerslev et al. (2014) validated the return predictability pattern for the U.S. and extended the regressions to seven other countries (including France, Germany, Japan, Switzerland, the Netherlands, Belgium, and the United Kingdom). Their empirical results show that the return predictability pattern observed for the U.S. market generally holds true for most of the other markets, but the degree of predictability is attenuated. The interesting results prompt the authors to investigate whether the variance risk is priced at a global level rather than a country-specific level. They then construct a global variance risk premium proxy as a market capitalization weighted average of each individual country variance risk premium (the largest weight around 60% is assigned to the U.S.), and regress each market's returns on the global variance risk premium. They find stronger predictability for all markets in the sample, and the results are robust with alternative measures of variance risk premiums. In a similar spirit, Londono (2015) finds that the U.S. variance risk premium has predictive power on international stock returns while other markets' own variance risk premiums do not for domestic stock market returns. In his two-country setup in explaining U.S. return predictability on international stock returns, he finds that the macroeconomic uncertainty is transmitted across countries, and the uncertainty-originated country's variance risk premium plays a key role in explaining both countries' stock returns.

The empirical results of the dynamic interaction between international variance risk pre-

miums revealed in this study provide support to the economic interpretation shared by the two studies (Bollerslev et al. (2014) and Londono (2015)). Variance risk premium can be interpreted as a measure of aggregate risk aversion or aggregate economic uncertainty. The lead-lag relationship between international variance risk premiums, given that the underlying economies are fundamentally different, suggests that the country-level uncertainty may be exposed to a global risk factor, potentially largely captured by the U.S. market risk. Similar interpretation is drawn in the foreign exchange market, as in Della Corte et al. (2016), the currency variance risk premium contains a compensation for a local risk as well as a global risk. An alternative interpretation may be explored from the contagion literature, where behavioural arguments are employed to explain excessive correlations between markets. All of these different economic mechanisms likely play some role in generating dynamic relationships between international variance risk premiums. I will leave it for future research to provide further empirical evidence.

4.7 Appendix

Table 4.1: HAR-RV regressions

$$RV_{t,t+22} = \beta_0 + \beta_d RV_{t-1,t} + \beta_w RV_{t-5,t} + \beta_m RV_{t-22,t} + \epsilon_{t,t+22}$$

| | <u>U.S.</u> | <u>U.K.</u> | <u>Eurozone</u> | <u>Japan</u> |
|-----------|-----------------------|-----------------------|-----------------------|-----------------------|
| β_0 | 0.0107*** (0.0032) | 0.0115*** (0.0025) | 0.0180*** (0.0036) | 0.0300*** (0.0057) |
| β_d | 0.0091 (0.0143) | 0.0184* (0.0105) | 0.0157 (0.0109) | 0.0336** (0.0152) |
| β_w | 0.3445** (0.1401) | 0.2496** (0.1199) | 0.1624** (0.0679) | 0.3328*** (0.1054) |
| β_m | 0.3753*** (0.1342) | 0.3970*** (0.0648) | 0.4640*** (0.0583) | 0.1191 (0.0765) |
| R^2 | 0.5644 | 0.4710 | 0.4287 | 0.3063 |

Table reports OLS estimates of the HAR-RV regressions for 22 days ahead annualised realised variance predictions for the U.S., the U.K., the Eurozone, and Japan. Daily realised variances are squared returns of daily close prices spanning the period from January 2004 to December 2014. Standard errors reported in parentheses are based on a Newey-West correction allowing for serial correlation of up to order 44. R^2 of regressions for each of the market is reported in the last row.

***, **, * denotes significance level at 1%, 5%, and 10% respectively.

Table 4.2: Descriptive statistics of realised variance and volatility, implied variance and volatility, and variance risk premium

| <u>Panel A</u> | | | | | | | | |
|----------------|-----------------------|--------|----------|--------|------------------------------|--------|----------|--------|
| | Realised variance | | | | Realised volatility | | | |
| | U.S. | U.K. | Eurozone | Japan | U.S. | U.K. | Eurozone | Japan |
| Mean | 0.040 | 0.035 | 0.051 | 0.059 | 0.179 | 0.173 | 0.213 | 0.233 |
| Median | 0.022 | 0.023 | 0.037 | 0.047 | 0.149 | 0.150 | 0.191 | 0.216 |
| Maximum | 0.736 | 0.534 | 0.522 | 0.932 | 0.858 | 0.731 | 0.723 | 0.965 |
| Minimum | 0.013 | 0.013 | 0.020 | 0.031 | 0.113 | 0.113 | 0.143 | 0.175 |
| Std.Dev | 0.063 | 0.043 | 0.048 | 0.058 | 0.090 | 0.071 | 0.074 | 0.069 |
| Skewness | 5.575 | 5.977 | 4.461 | 8.139 | 3.434 | 3.429 | 2.744 | 4.861 |
| Kurtosis | 40.567 | 48.200 | 29.238 | 87.546 | 17.642 | 19.068 | 12.759 | 36.127 |
| <u>Panel B</u> | | | | | | | | |
| | Implied variance | | | | Implied volatility | | | |
| | U.S. | U.K. | Eurozone | Japan | U.S. | U.K. | Eurozone | Japan |
| Mean | 0.055 | 0.049 | 0.063 | 0.077 | 0.209 | 0.199 | 0.231 | 0.255 |
| Median | 0.031 | 0.030 | 0.043 | 0.055 | 0.176 | 0.172 | 0.207 | 0.235 |
| Maximum | 0.728 | 0.870 | 1.201 | 0.967 | 0.853 | 0.933 | 1.096 | 0.983 |
| Minimum | 0.010 | 0.008 | 0.010 | 0.013 | 0.100 | 0.089 | 0.098 | 0.113 |
| Std.Dev | 0.074 | 0.062 | 0.070 | 0.094 | 0.106 | 0.095 | 0.097 | 0.110 |
| Skewness | 4.316 | 4.881 | 5.079 | 5.064 | 2.396 | 2.336 | 2.230 | 2.828 |
| Kurtosis | 23.861 | 37.833 | 48.452 | 34.612 | 10.331 | 11.002 | 10.846 | 14.247 |
| <u>Panel C</u> | | | | | | | | |
| | Variance risk premium | | | | Return variance risk premium | | | |
| | U.S. | U.K. | Eurozone | Japan | U.S. | U.K. | Eurozone | Japan |
| Mean | -0.014 | -0.014 | -0.012 | -0.018 | -0.194 | -0.147 | -0.053 | -0.018 |
| Median | -0.007 | -0.005 | -0.003 | -0.004 | -0.217 | -0.174 | -0.086 | -0.074 |
| Maximum | 0.328 | 0.157 | 0.207 | 0.448 | 1.374 | 1.539 | 1.560 | 2.044 |
| Minimum | -0.348 | -0.396 | -0.717 | -0.628 | -0.795 | -0.838 | -0.808 | -0.827 |
| Std.Dev | 0.034 | 0.032 | 0.036 | 0.061 | 0.275 | 0.307 | 0.318 | 0.401 |
| Skewness | -1.888 | -4.411 | -5.406 | -3.692 | 0.592 | 0.659 | 0.701 | 0.965 |
| Kurtosis | 30.409 | 40.495 | 75.771 | 29.645 | 3.458 | 3.531 | 4.040 | 4.688 |

Table provides descriptive statistics for realised variance, implied variance, variance risk premium, and return variance risk premium defined by equation (4.5), (4.1), (4.3), and (4.4) respectively. Summary statistics are also reported for realised volatility and implied volatility calculated as the square root of the realised variance and implied variance. Results are reported for 2594 daily observations from 2004 to 2014.

Table 4.3: Pairwise correlations of (return) variance risk premiums

| Correlation | U.S. | U.K. | Eurozone | Japan |
|-------------|-------------|-------------|-------------|-------|
| U.S. | 1 (1) | | | |
| U.K. | 0.68 (0.63) | 1 (1) | | |
| Eurozone | 0.59 (0.65) | 0.85 (0.81) | 1 (1) | |
| Japan | 0.51 (0.39) | 0.70 (0.56) | 0.52 (0.52) | 1 (1) |

Table reports pairwise variance risk premium (return variance risk premium) correlations for the period from May 2004 to December 2014.

Table 4.4: Ljung-Box test on [return] variance risk premiums

| | U.S. | U.K. | Eurozone | Japan |
|-------|---------------------|----------------------|---------------------|----------------------|
| Q(10) | 4055 [8666] (0) | 8151 [11951] (0) | 5012 [10689] (0) | 12579 [14913] (0) |
| Q(15) | 5676 [11113] (0) | 10135 [15614] (0) | 6585 [13865] (0) | 17826 [21008] (0) |
| Q(20) | 7126 [13196] (0) | 12141 [18836] (0) | 7762 [16213] (0) | 22277 [26434] (0) |

Table reports Ljung-Box test statistics on variance risk premiums [return variance risk premiums] from May 2004 to December 2014 for serial correlation. The $Q(m)$ -statistics follow a chi-squared distribution with m degrees of freedom. Test statistics reported are for degrees of freedom 10, 15, and 20. Values in parenthesis are p -values of $Q(m)$. All p -values are < 0.0001 .

Table 4.5: GARCH fitting for subsample variance risk premiums

| | US | UK | Eurozone | Japan |
|---|----------------------------|--------------------------|--------------------------------|-------------------------------|
| <u>Pre-crisis: May 11, 2004 – Apr 28, 2006, 451 observations</u> | | | | |
| GARCH specification | AR(1,2,5,6)+ GARCH(1,1) | AR(1,2)+ GARCH(1,1) | AR(1,2)+ GARCH(1,1) | AR(1,2,5,6,11)+ GARCH(1,1) |
| $\alpha + \beta$ | 0.941 | 0.773 | 0.966 | 0.955 |
| $Q_{\tilde{a}_t}(20)$ | 7.15 (0.97) | 11.20 (0.89) | 16.12 (0.58) | 19.73 (0.18) |
| $Q_{\tilde{a}_t^2}(20)$ | 16.86 (0.40) | 17.41 (0.50) | 11.19 (0.89) | 23.03 (0.08) |
| <u>Crisis: Sep 02, 2008 – Mar 31, 2009, 146 observations</u> | | | | |
| GARCH specification | AR(1)+ GARCH(1,1) | AR(1)+ GARCH(1,1) | AR(1)+ GARCH(1,1) | AR(1,2,5,6)+ GARCH(1,1) |
| $\alpha + \beta$ | 0.990 | 0.966 | 0.950 | 0.944 |
| $Q_{\tilde{a}_t}(20)$ | 16.94 (0.59) | 21.50 (0.31) | 14.00 (0.78) | 13.99 (0.60) |
| $Q_{\tilde{a}_t^2}(20)$ | 10.49 (0.94) | 14.62 (0.75) | 17.38 (0.56) | 4.45 (0.99) |
| <u>Post-crisis: Jan 03, 2012 – Nov 28, 2014, 732 observations</u> | | | | |
| GARCH specification | AR(1,5,6)+ GARCH(1,1) | AR(1,5,6)+ GARCH(1,1) | AR(1,5,6,13,14)+ GARCH(1,1) | AR(1,2,4,5,6)+ GARCH(1,1) |
| $\alpha + \beta$ | 0.820 | 0.995 | 0.996 | 0.971 |
| $Q_{\tilde{a}_t}(20)$ | 21.65 (0.20) | 23.54 (0.13) | 22.71 (0.09) | 15.95 (0.39) |
| $Q_{\tilde{a}_t^2}(20)$ | 11.92 (0.81) | 18.06 (0.39) | 29.04 (0.02) | 5.65 (0.99) |

Table presents GARCH fitting with AR structure in the mean equation to subsamples of variance risk premiums for each market. Subsamples are selected for pre-crisis, crisis, and post-crisis period. Parameters of subsamples pre-crisis Japan, crisis Eurozone, and post-crisis Japan are estimated using variance targeting technique to mitigate estimation difficulty, as the sum of α and β exceeds one without constraints. Ljung-Box Q-statistics on standardized residuals adjusted for degrees of freedom are reported for each subsample with p -value reported in the parenthesis.

Table 4.6: Vector Autoregression Estimates for post-crisis subsample

| | US | UK | Eurozone | Japan |
|----------------|--------------------|-----------------------|-----------------------|-----------------------|
| US(-1) | -0.043 (-0.989) | 0.260 (6.157)*** | 0.253 (6.012)*** | 0.256 (5.825)*** |
| US(-2) | -0.024 (-0.538) | 0.134 (3.042)*** | 0.112 (2.559)** | 0.044 (0.971) |
| UK(-1) | 0.052 (0.910) | -0.140 (-2.499)** | 0.035 (0.628) | -0.083 (-1.420) |
| UK(-2) | 0.060 (1.047) | -0.010 (-0.182) | -0.067 (-1.199) | 0.015 (0.256) |
| Eurozone(-1) | -0.075 (-1.294) | -0.081 (-1.433) | -0.242 (-4.296)*** | 0.100 (1.701) |
| Eurozone(-2) | -0.124 (-2.155) | -0.094 (-1.671) | -0.054 (-0.965) | 0.084 (1.437) |
| Japan(-1) | 0.011 (0.301) | 0.005 (0.148) | 0.038 (1.061) | -0.068 (-1.829) |
| Japan(-2) | 0.040 1.117 | -0.007 -0.198 | 0.006 0.170 | 0.015 0.413 |
| C | -0.002 (-0.042) | -0.105 (-2.863)*** | -0.098 (-2.684)*** | -0.101 (-2.635)*** |
| R^2 | 1.5% | 6.1% | 6.6% | 8.0% |
| F -statistic | 1.342 | 5.813 | 6.396 | 7.829 |

Table reports Vector Autoregression estimates for post-crisis subsample. First column reports number of lags included in the regression. Column two to five reports estimates of coefficients with t-statistic in the parenthesis. R^2 and F -statistic is shown at the bottom of the table. *** and ** indicate coefficient estimates are statistically significant at 1% and 5% level.

Table 4.7: VAR Granger Causality test for post-crisis subsample

| <u>Dependent variable: US</u> | | | |
|-------------------------------------|--------|----|-------|
| Excluded | Chi-sq | df | Prob. |
| UK | 1.644 | 2 | 0.440 |
| Eurozone | 5.447 | 2 | 0.066 |
| Japan | 1.309 | 2 | 0.520 |
| All | 6.770 | 6 | 0.343 |
| <u>Dependent variable: UK</u> | | | |
| Excluded | Chi-sq | df | Prob. |
| US | 42.088 | 2 | 0 |
| Eurozone | 4.075 | 2 | 0.130 |
| Japan | 0.064 | 2 | 0.969 |
| All | 43.833 | 6 | 0 |
| <u>Dependent variable: Eurozone</u> | | | |
| Excluded | Chi-sq | df | Prob. |
| US | 38.633 | 2 | 0 |
| UK | 2.155 | 2 | 0.340 |
| Japan | 1.141 | 2 | 0.565 |
| All | 45.975 | 6 | 0 |
| <u>Dependent variable: Japan</u> | | | |
| Excluded | Chi-sq | df | Prob. |
| US | 33.926 | 2 | 0 |
| UK | 2.275 | 2 | 0.321 |
| Eurozone | 4.175 | 2 | 0.124 |
| All | 60.799 | 6 | 0 |

Table reports Granger Causality tests for post-crisis subsample. Each equation tests for the joint significance of excluded variables in explaining the dependent variable. A probability of zero rejects the null that the coefficient estimates of the Excluded variable(s) equal to zero at all conventional levels, suggesting that the Excluded variable(s) 'Granger cause' the dependent variable.

Table 4.8: Validity check for post-crisis VAR estimation

| <u>VAR Residual Portmanteau Tests for Autocorrelations</u> | | | |
|--|-------------|-------------|-----|
| Lags | Q-statistic | Probability | df |
| 1 | 0.096 | NA* | NA* |
| 2 | 0.731 | NA* | NA* |
| 3 | 9.841 | 0.875 | 16 |
| 4 | 28.882 | 0.625 | 32 |
| 5 | 48.314 | 0.460 | 48 |
| 6 | 58.128 | 0.683 | 64 |
| 7 | 73.332 | 0.688 | 80 |
| 8 | 101.326 | 0.335 | 96 |
| 9 | 117.064 | 0.353 | 112 |
| 10 | 131.489 | 0.398 | 128 |
| 11 | 142.607 | 0.517 | 144 |
| 12 | 153.440 | 0.631 | 160 |

| <u>VAR Residual Heteroskedasticity Tests</u> | | |
|--|--------|-------------|
| <u>Joint test</u> | | |
| Chi-sq | df | Probability |
| 125.837 | 160 | 0.979 |
| <u>Individual components</u> | | |
| Dependent | Chi-sq | Probability |
| res1*res1 | 19.745 | 0.232 |
| res2*res2 | 16.198 | 0.439 |
| res3*res3 | 17.963 | 0.326 |
| res4*res4 | 7.768 | 0.956 |
| res2*res1 | 17.427 | 0.359 |
| res3*res1 | 14.461 | 0.564 |
| res3*res2 | 13.205 | 0.658 |
| res4*res1 | 12.408 | 0.715 |
| res4*res2 | 4.956 | 0.996 |
| res4*res3 | 5.939 | 0.989 |

Table reports validity tests for the estimated VAR model. Top panel reports Portmanteau tests for residual serial correlation and bottom panel reports White tests for residual heteroskedasticity.

Table 4.9: Vector Autoregression Estimates for crisis subsample

| | US | UK | Eurozone | Japan |
|----------------|--------------------|---------------------|---------------------|---------------------|
| US(-1) | 0.041 (0.441) | 0.318 (3.555)*** | 0.303 (3.259)*** | 0.552 (6.843)*** |
| UK(-1) | -0.176 (-1.252) | -0.172 (-1.276) | -0.155 (-1.102) | -0.118 (-0.970) |
| Eurozone(-1) | 0.169 (1.289) | 0.050 (0.395) | 0.062 (0.476) | -0.018 (-0.159) |
| Japan(-1) | 0.050 (0.566) | -0.096 (-1.123) | -0.130 (-1.466) | -0.062 (-0.800) |
| R^2 | 0.015 | 0.094 | 0.088 | 0.252 |
| F -statistic | 0.732 | 4.852 | 4.516 | 15.856 |

Table reports Vector Autoregression estimates for crisis subsample. First column reports number of lags included in the regression. Column two to five reports estimates of coefficients with t-statistic in the parenthesis. R^2 and F -statistic is shown at the bottom of the table. *** indicate coefficient estimates are statistically significant at 1% level.

Table 4.10: VAR Granger Causality test for crisis subsample

| <u>Dependent variable: US</u> | | | |
|-------------------------------------|--------|----|-------|
| Excluded | Chi-sq | df | Prob. |
| UK | 1.568 | 1 | 0.211 |
| Eurozone | 1.662 | 1 | 0.197 |
| Japan | 0.321 | 1 | 0.571 |
| All | 2.366 | 3 | 0.500 |
| <u>Dependent variable: UK</u> | | | |
| Excluded | Chi-sq | df | Prob. |
| US | 12.640 | 1 | 0.000 |
| Eurozone | 0.156 | 1 | 0.693 |
| Japan | 1.262 | 1 | 0.261 |
| All | 14.981 | 3 | 0.002 |
| <u>Dependent variable: Eurozone</u> | | | |
| Excluded | Chi-sq | df | Prob. |
| US | 10.619 | 1 | 0.001 |
| UK | 1.213 | 1 | 0.271 |
| Japan | 2.148 | 1 | 0.143 |
| All | 14.032 | 3 | 0.003 |
| <u>Dependent variable: Japan</u> | | | |
| Excluded | Chi-sq | df | Prob. |
| US | 46.830 | 1 | 0.000 |
| UK | 0.940 | 1 | 0.332 |
| Eurozone | 0.025 | 1 | 0.874 |
| All | 49.418 | 3 | 0.000 |

Table reports Granger Causality tests for crisis subsample. Each equation tests for the joint significance of excluded variables in explaining the dependent variable. A probability of zero rejects the null that the coefficient estimates of the Excluded variable(s) equal to zero at all conventional levels, suggesting that the Excluded variable(s) 'Granger cause' the dependent variable.

Table 4.11: Validity check for crisis VAR estimation

| <u>VAR Residual Portmanteau Tests for Autocorrelations</u> | | | |
|--|-------------|-------------|-----|
| Lags | Q-statistic | Probability | df |
| 1 | 3.734 | NA* | NA* |
| 2 | 29.629 | 0.020 | 16 |
| 3 | 35.994 | 0.287 | 32 |
| 4 | 51.882 | 0.325 | 48 |
| 5 | 67.279 | 0.366 | 64 |
| 6 | 103.235 | 0.041 | 80 |
| 7 | 113.976 | 0.102 | 96 |
| 8 | 128.032 | 0.143 | 112 |
| 9 | 140.308 | 0.216 | 128 |
| 10 | 154.313 | 0.263 | 144 |
| 11 | 162.064 | 0.440 | 160 |
| 12 | 192.980 | 0.181 | 176 |

| <u>VAR Residual Heteroskedasticity Tests</u> | | |
|--|--------|-------------|
| <u>Joint test</u> | | |
| Chi-sq | df | Probability |
| 96.464 | 80 | 0.101 |
| <u>Individual components</u> | | |
| Dependent | Chi-sq | Probability |
| res1*res1 | 3.632 | 0.889 |
| res2*res2 | 1.874 | 0.985 |
| res3*res3 | 7.006 | 0.536 |
| res4*res4 | 8.981 | 0.344 |
| res2*res1 | 4.030 | 0.854 |
| res3*res1 | 4.556 | 0.804 |
| res3*res2 | 4.124 | 0.846 |
| res4*res1 | 12.408 | 0.134 |
| res4*res2 | 14.336 | 0.073 |
| res4*res3 | 18.484 | 0.018 |

Table reports validity tests for the estimated VAR model. Top panel reports Portmanteau tests for residual serial correlation and bottom panel reports White tests for residual heteroskedasticity.

Table 4.12: Vector Autoregression Estimates for pre-crisis subsample

| | US | UK | Eurozone | Japan |
|----------------|--------------------|----------------------|---------------------|----------------------|
| US(-1) | -0.006 (-0.116) | 0.268 (5.512)*** | 0.231 (5.108)*** | 0.150 (3.225)*** |
| UK(-1) | 0.075 (1.205) | -0.126 (-2.100)** | -0.021 (-0.383) | 0.184 (-3.187)*** |
| Eurozone(-1) | -0.097 (-1.417) | 0.043 (0.652) | -0.099 (-1.610) | -0.053 (-0.833) |
| Japan(-1) | 0.010 (0.206) | -0.002 (-0.036) | 0.039 (0.853) | -0.011 (-0.234) |
| R^2 | 0.001 | 0.068 | 0.058 | 0.057 |
| F -statistic | 0.117 | 10.692 | 8.930 | 8.818 |

Table reports Vector Autoregression estimates for pre-crisis subsample. First column reports number of lags included in the regression. Column two to five reports estimates of coefficients with t-statistic in the parenthesis. R^2 and F -statistic is shown at the bottom of the table. *** and ** indicate coefficient estimates are statistically significant at 1% and 5% level.

Table 4.13: VAR Granger Causality test for pre-crisis subsample

| <u>Dependent variable: US</u> | | | |
|-------------------------------------|--------|----|-------|
| Excluded | Chi-sq | df | Prob. |
| UK | 1.453 | 1 | 0.228 |
| Eurozone | 2.008 | 1 | 0.157 |
| Japan | 0.042 | 1 | 0.837 |
| All | 2.246 | 3 | 0.523 |
| <u>Dependent variable: UK</u> | | | |
| Excluded | Chi-sq | df | Prob. |
| US | 30.384 | 1 | 0.000 |
| Eurozone | 0.425 | 1 | 0.514 |
| Japan | 0.001 | 1 | 0.971 |
| All | 33.195 | 3 | 0.000 |
| <u>Dependent variable: Eurozone</u> | | | |
| Excluded | Chi-sq | df | Prob. |
| US | 26.096 | 1 | 0.000 |
| UK | 0.147 | 1 | 0.702 |
| Japan | 0.728 | 1 | 0.394 |
| All | 26.596 | 3 | 0.000 |
| <u>Dependent variable: Japan</u> | | | |
| Excluded | Chi-sq | df | Prob. |
| US | 10.400 | 1 | 0.001 |
| UK | 10.154 | 1 | 0.001 |
| Eurozone | 0.694 | 1 | 0.405 |
| All | 29.535 | 3 | 0.000 |

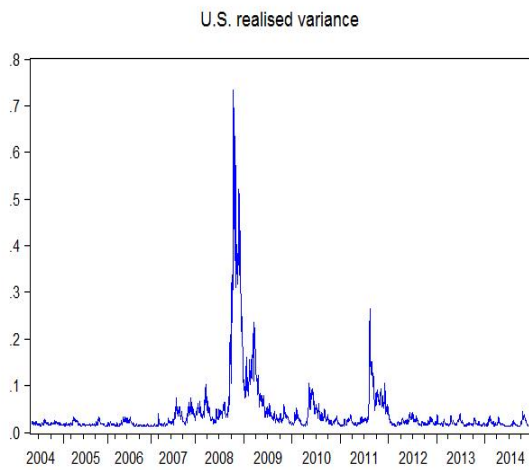
Table reports Granger Causality tests for pre-crisis subsample. Each equation tests for the joint significance of excluded variables in explaining the dependent variable. A probability of zero rejects the null that the coefficient estimates of the Excluded variable(s) equal to zero at all conventional levels, suggesting that the Excluded variable(s) 'Granger cause' the dependent variable.

Table 4.14: Validity check for pre-crisis VAR estimation

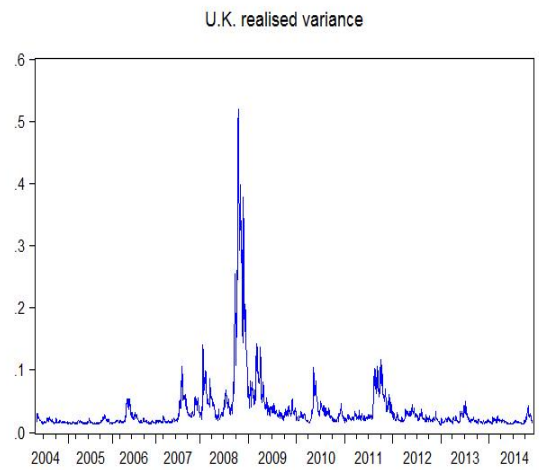
| <u>VAR Residual Portmanteau Tests for Autocorrelations</u> | | | |
|--|-------------|-------------|-----|
| Lags | Q-statistic | Probability | df |
| 1 | 0.914 | NA* | NA* |
| 2 | 9.394 | 0.896 | 16 |
| 3 | 23.293 | 0.869 | 32 |
| 4 | 38.717 | 0.828 | 48 |
| 5 | 54.572 | 0.794 | 64 |
| 6 | 67.115 | 0.848 | 80 |
| 7 | 83.448 | 0.816 | 96 |
| 8 | 98.055 | 0.823 | 112 |
| 9 | 107.577 | 0.892 | 128 |
| 10 | 129.748 | 0.797 | 144 |
| 11 | 141.006 | 0.858 | 160 |
| 12 | 159.454 | 0.809 | 176 |

| <u>VAR Residual Heteroskedasticity Tests</u> | | |
|--|--------|-------------|
| <u>Joint test</u> | | |
| Chi-sq | df | Probability |
| 88.816 | 80 | 0.234 |
| <u>Individual components</u> | | |
| Dependent | Chi-sq | Probability |
| res1*res1 | 3.605 | 0.891 |
| res2*res2 | 6.223 | 0.622 |
| res3*res3 | 7.287 | 0.506 |
| res4*res4 | 19.489 | 0.013 |
| res2*res1 | 2.769 | 0.948 |
| res3*res1 | 6.682 | 0.571 |
| res3*res2 | 7.070 | 0.529 |
| res4*res1 | 7.181 | 0.517 |
| res4*res2 | 12.543 | 0.129 |
| res4*res3 | 14.374 | 0.073 |

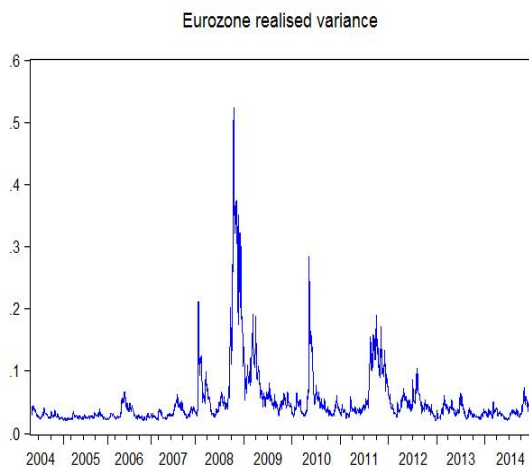
Table reports validity tests for the estimated VAR model. Top panel reports Portmanteau tests for residual serial correlation and bottom panel reports White tests for residual heteroskedasticity.



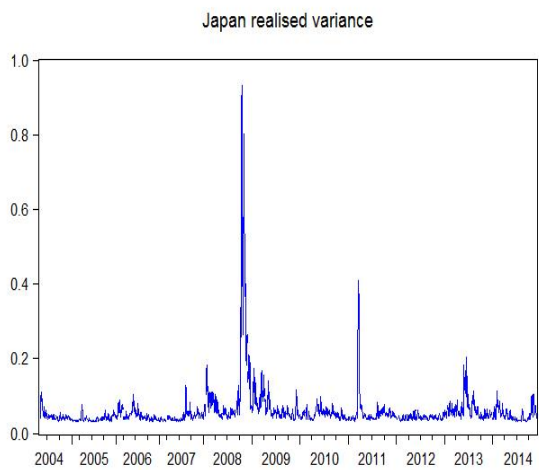
(a)



(b)

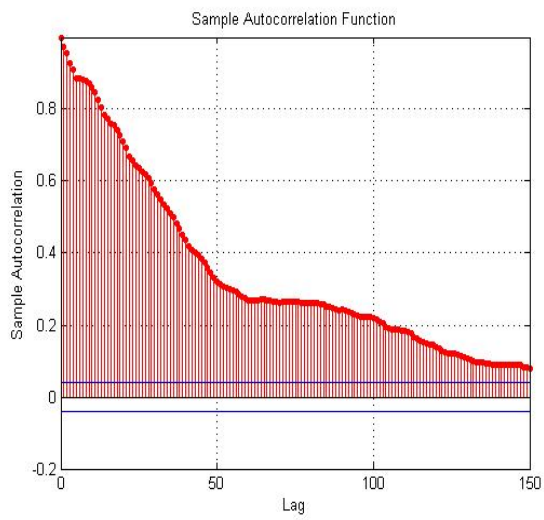


(c)

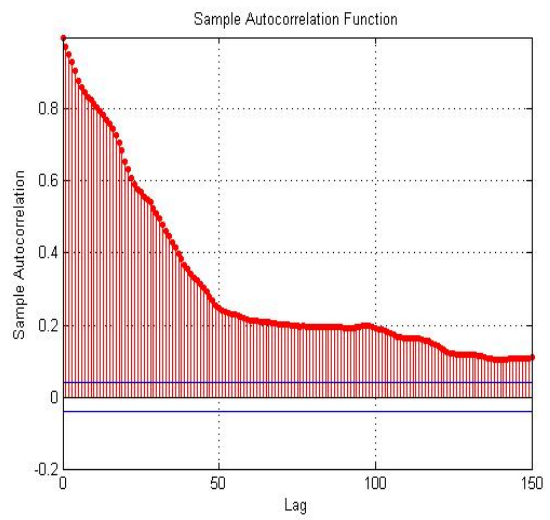


(d)

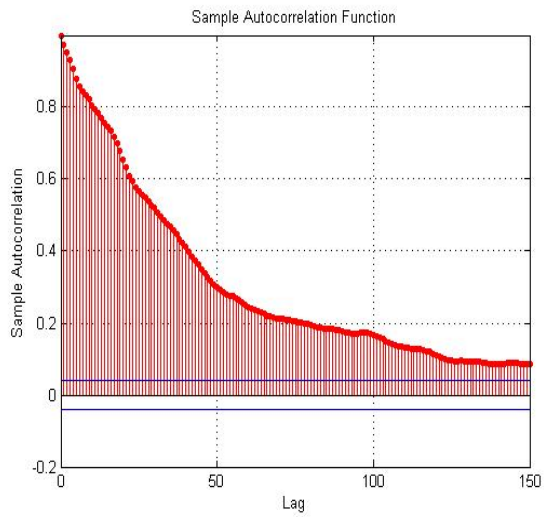
Figure 4.1: Time series plots of HAR-RV annualised expected realised variance



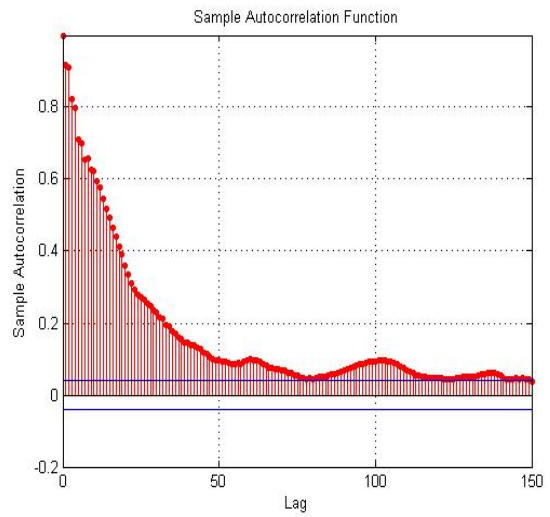
(a) U.S.



(b) U.K.

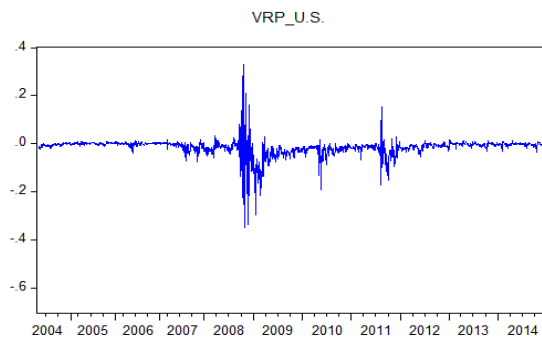


(c) Eurozone

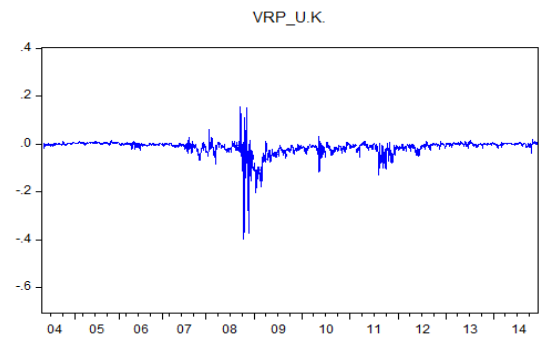


(d) Japan

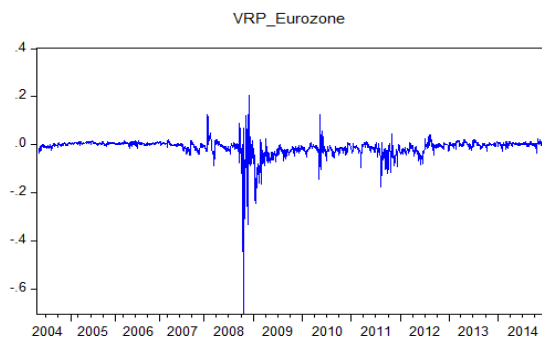
Figure 4.2: Autocorrelation plots of HAR-RV realised variance



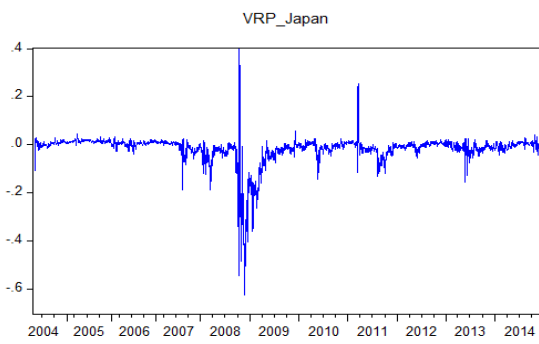
(a)



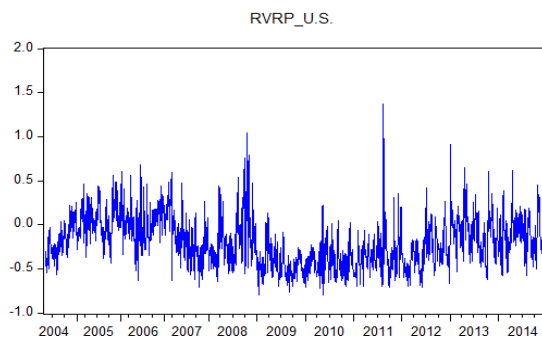
(b)



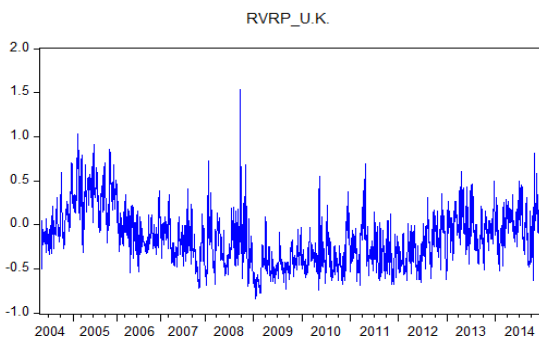
(c)



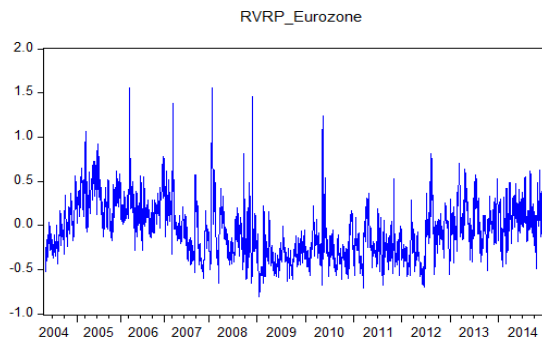
(d)



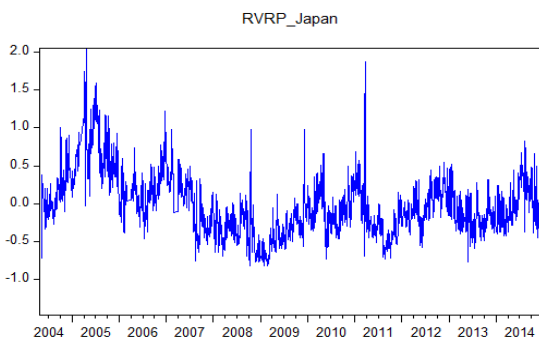
(e)



(f)

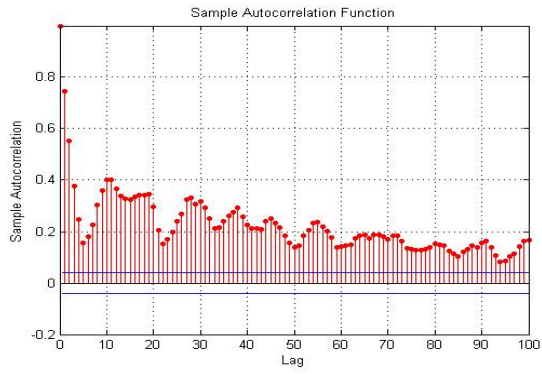


(g)

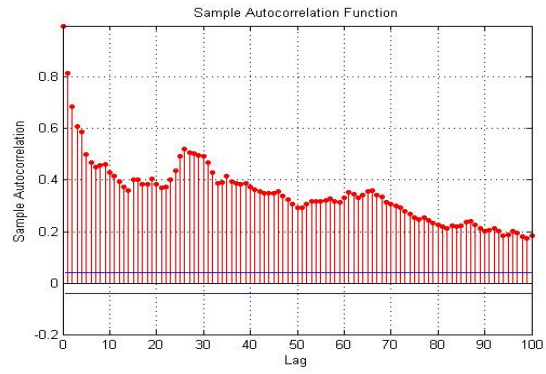


(h)

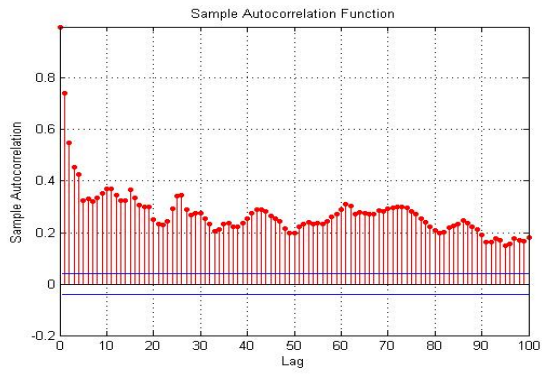
Figure 4.3: Variance risk premiums



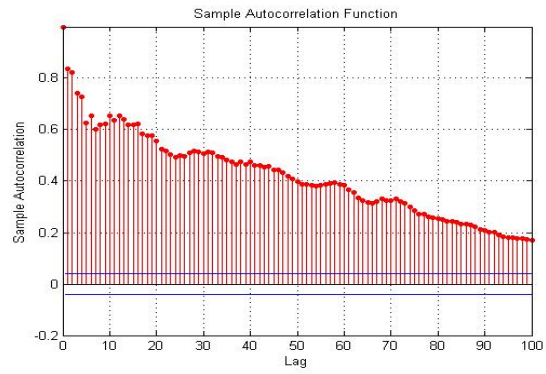
(a) US VRP



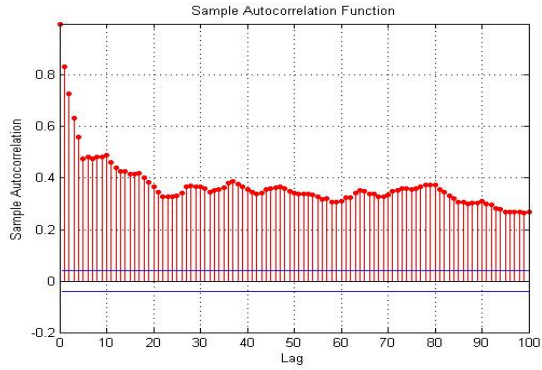
(b) UK VRP



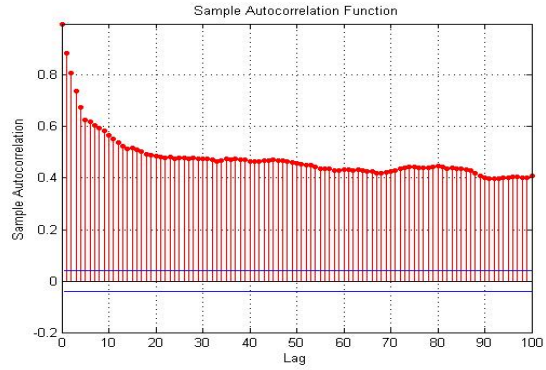
(c) Eurozone VRP



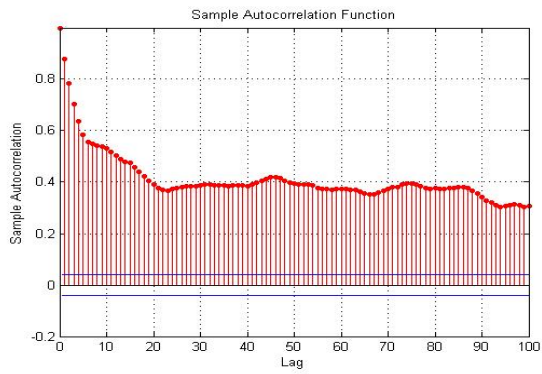
(d) Japan VRP



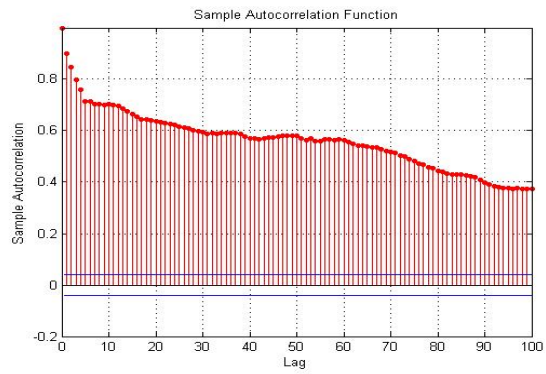
(e) US RVRP



(f) UK RVRP

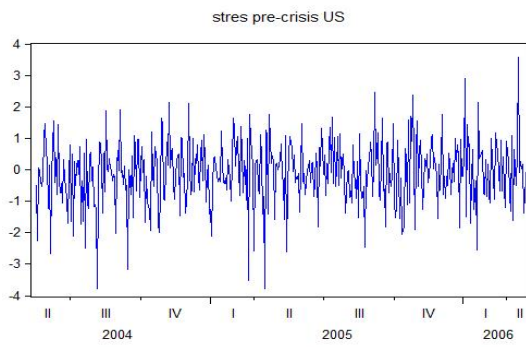


(g) Eurozone RVRP

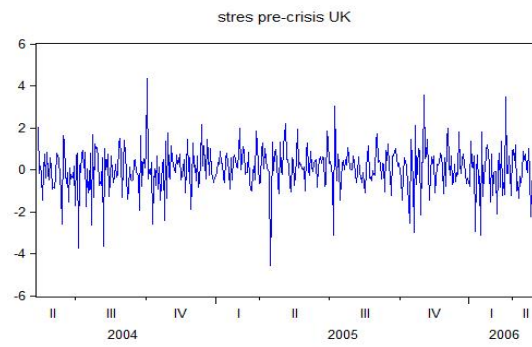


(h) Japan RVRP

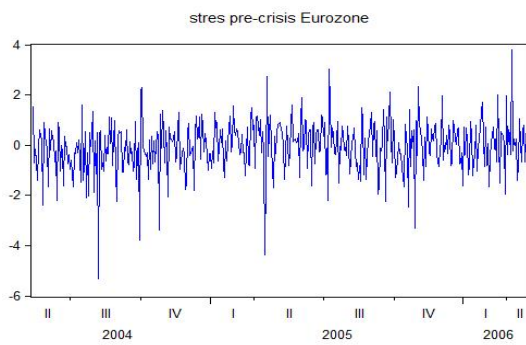
Figure 4.4: Autocorrelation plots of variance risk premiums



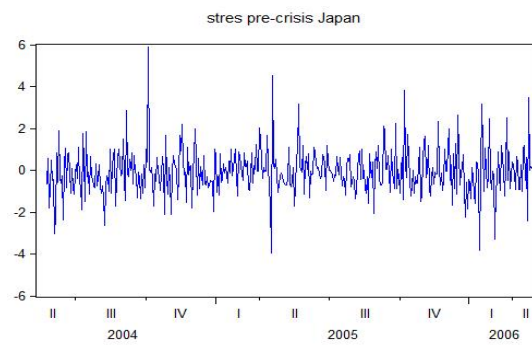
(a) standardized residuals pre-crisis US



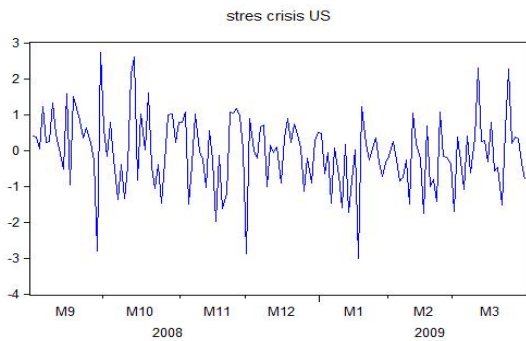
(b) standardized residuals pre-crisis UK



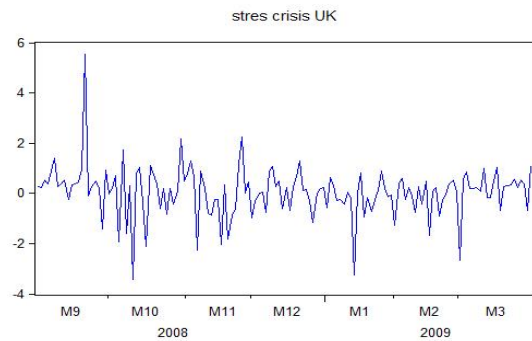
(c) standardized residuals pre-crisis Eurozone



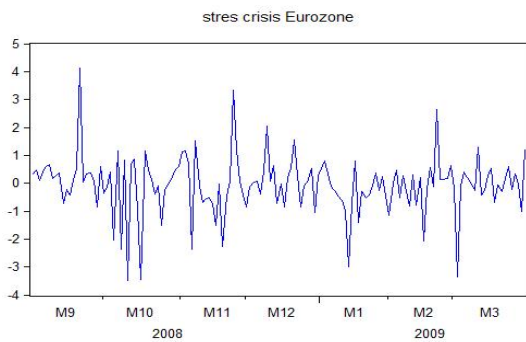
(d) standardized residuals pre-crisis Japan



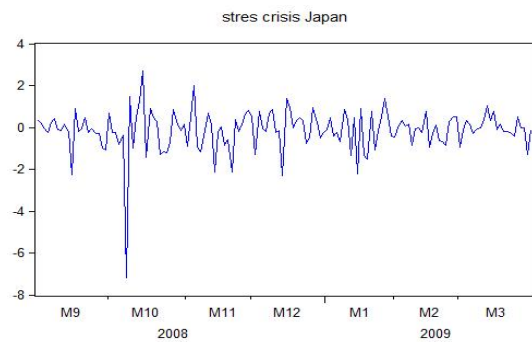
(e) standardized residuals crisis US



(f) standardized residuals crisis UK

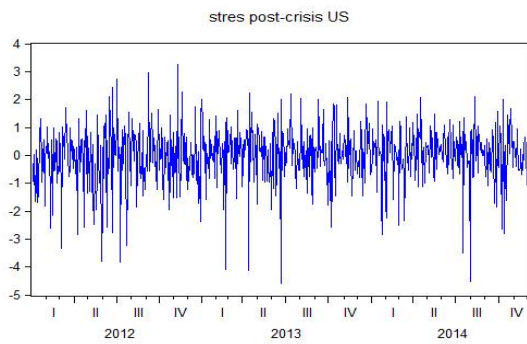


(g) standardized residuals crisis Eurozone

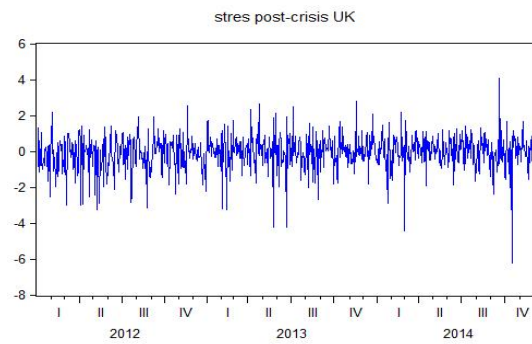


(h) standardized residuals crisis Japan

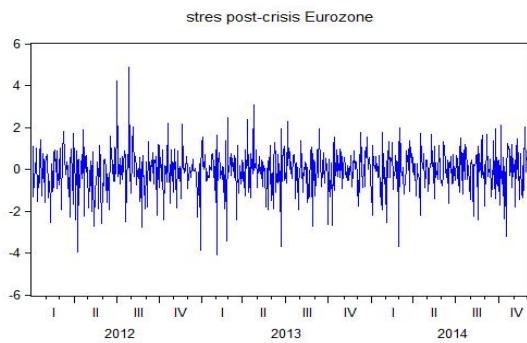
Figure 4.5: Standardized residuals plots of subsamples



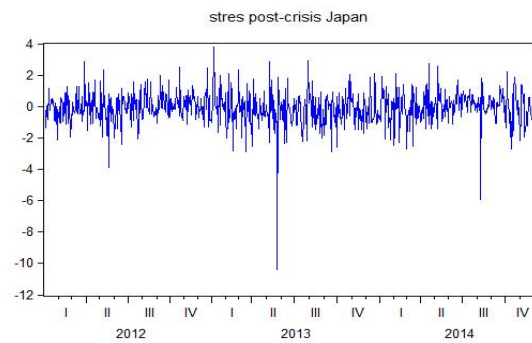
(i) standardized residuals post-crisis US



(j) standardized residuals post-crisis UK



(k) standardized residuals post-crisis Eurozone



(l) standardized residuals post-crisis Japan

Figure 4.5: Continued standardized residuals plots of subsamples

Figure 4.6: Impulse response analysis for post-crisis subsample

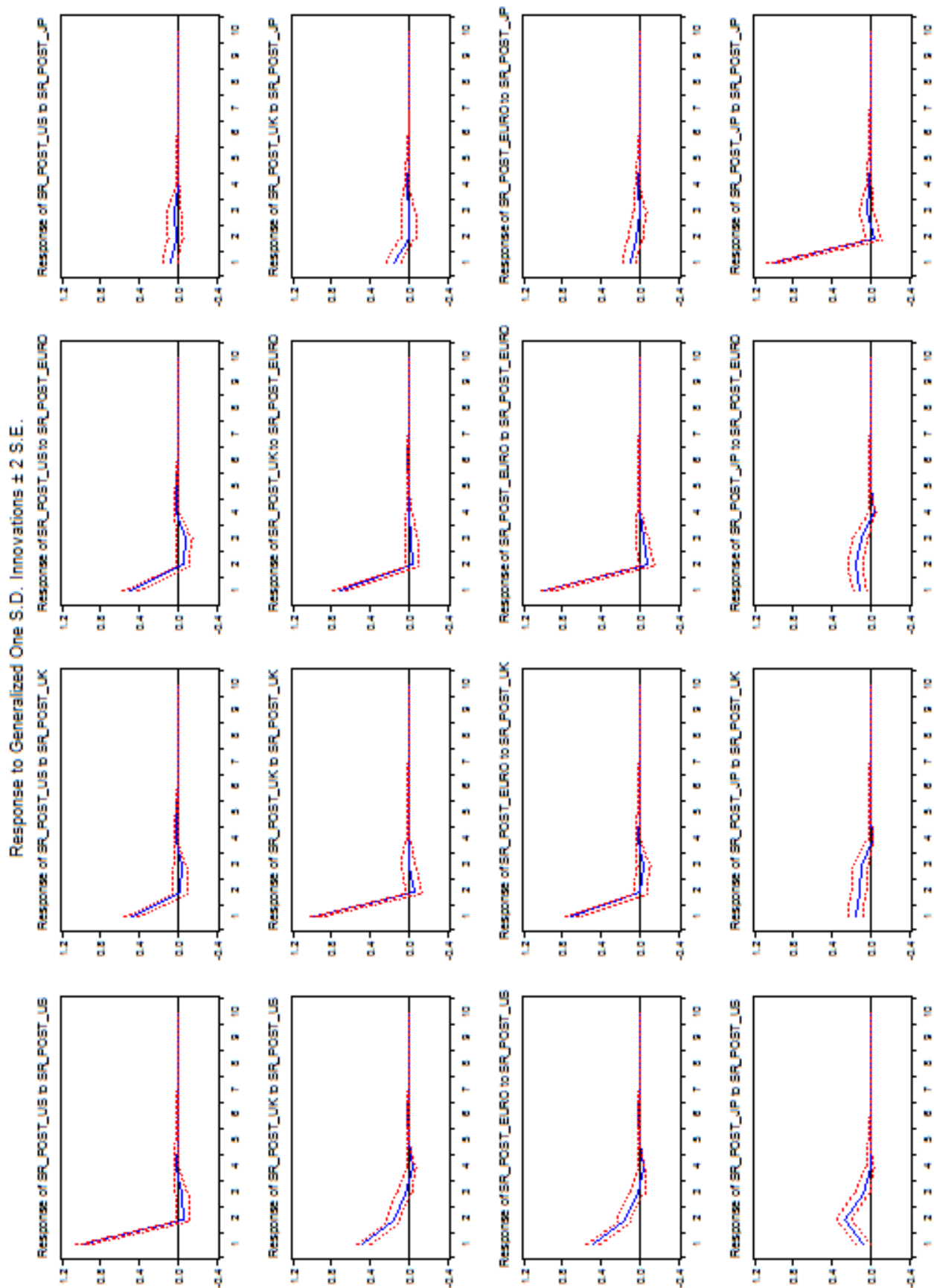


Figure 4.7: Impulse response analysis for crisis subsample

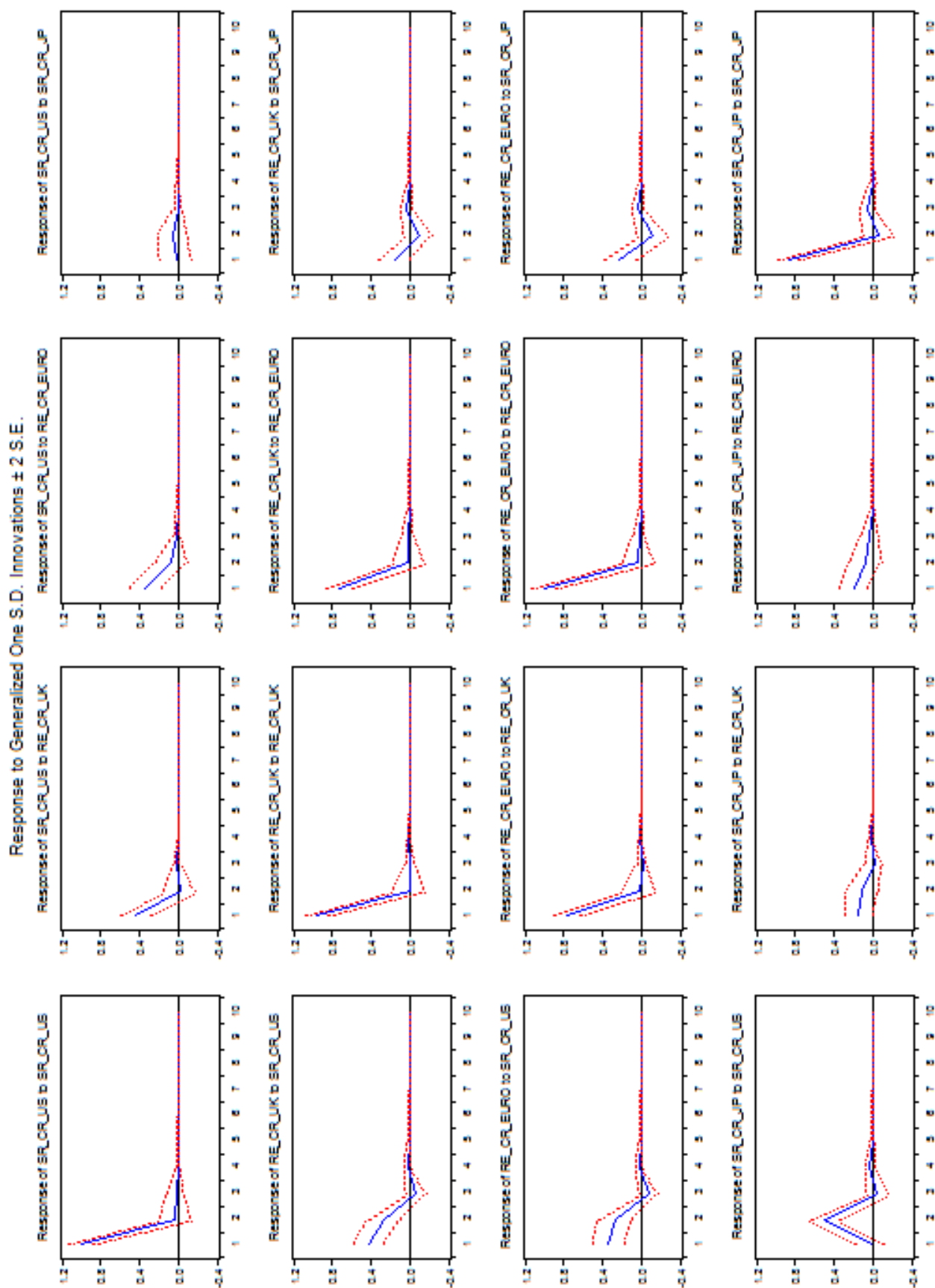
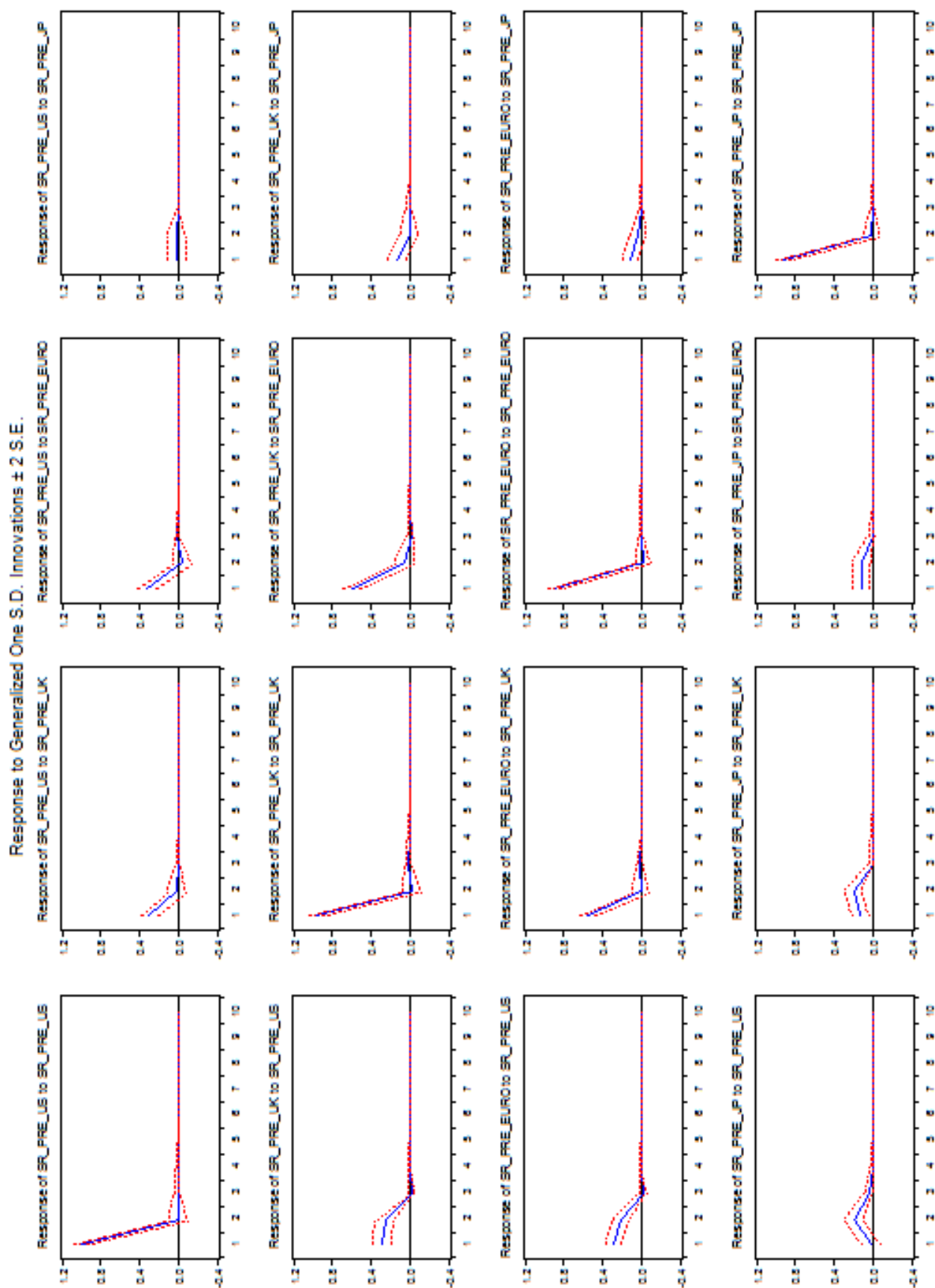


Figure 4.8: Impulse response analysis for pre-crisis subsample



Chapter 5

Concluding remarks

This thesis contains three individual essays on inferring information from financial derivative instruments. The thesis begins with an introductory chapter, which outlines the main ideas of each chapter as well as the structure of the thesis.

Chapter 2 presents empirical analysis on estimating bankruptcy probabilities from option prices and CDS spreads for 12 U.S. financial institutions. Assuming the risk-neutral density for the underlying asset to be a mixture of two lognormals augmented with a probability of bankruptcy, option-implied bankruptcy probabilities are inferred from calibrating the risk-neutral density to the observed option prices. CDS-implied bankruptcy probabilities are inferred from a non-linear equation that is set based on the fair valuation of a CDS contract. Comparisons between option-implied and CDS-implied bankruptcy probabilities show that the two sources provide complementary information in assessing bankruptcy risk. The derivative markets provide useful information ex-ante.

In the fixed income markets, modelling default probabilities embedded in bonds, CDS, and other fixed income instruments has become standard routines in the real world practice, whereas using option information to estimate default rates is not well established. Option-implied information bears the advantage that it can be used to infer default rates for shorter

horizons given flexible option maturities, while CDS and bonds can only provide a default estimate for the next few months or up to one year. Certainly, option contracts would serve as an appealing instrument for market practitioners to assess near-term default rate, and the magnitude of default probabilities and risk-neutral densities discovered in this study will help to provide reliable signals regarding a firm's financial status, especially during turbulent market conditions.

Motivated by the information superiority contained in option contracts, chapters 3 and 4 examine spillover effects in an international market integration setting based on option-implied information. Chapter 3 focuses on implied volatility spillovers and chapter 4 concentrates on variance risk premium spillovers. The first part of chapter 3 details the construction of volatility indices for a sample of equity markets around the world. The method to construct volatility indices follows the steps of the CBOE procedure, but modified to address the estimation errors associated with the CBOE procedure. This is the first study that calculates volatility indices for a large sample of national equity markets, and makes a comparison with the CBOE method. The results provide empirical evidence that the CBOE method induces estimation errors for all markets examined. In chapter 4, variance risk premiums are constructed by the difference between the risk-neutral expectation of realised variance, proxied by the implied variance extracted from options, and realised variance under the physical measure, calculated based on the HAR-RV model.

Spillover effects of implied volatility and variance risk premium are investigated using vector autoregressive analysis. Issues of serial correlation and heterogeneity are addressed and different steps are taken in each chapter to mitigate the impact of these issues. Empirical results show remarkable consistency in international markets lead-lag relationship between implied volatilities and variance risk premiums. The results support the notion of informationally efficient international stock markets, in that information flow across bor-

ders is quickly processed with little delay. The empirical findings are in general consistent with those found in international finance literature which studies spillover effects based on historical measures, e.g., return and volatility. Measures constructed from information extracted from option contracts serve as appealing alternatives as they provide forward looking information.

Volatility spillovers have been studied extensively in the literature. Many have attributed these spillovers to intensified international linkages across countries, or economic fundamentals; others have theorized they come from market contagion. However, there is little documentation on variance risk premium spillovers. This is the first study to provide empirical evidence on dynamic interactions between international variance risk premiums. The limited studies on variance risk premia around the world have focused on the stock market return predictability from the U.S. market variance risk premium. In this study, empirical results have shown a leading role of the U.S. variance risk premium on other markets. This provides supporting evidence to the findings in these studies. The economic interpretation is that the country-level risk aversion or economic uncertainty, as proxied by the variance risk premium, may be exposed to a global factor which potentially largely captured by the U.S. market risk. While other explanations from behavioural arguments may be plausible in generating dynamic interactions between international variance risk premiums, this is left for corroboration in future research.

In summary, this thesis provides views on information inferred from the derivatives markets from different perspectives, and it shows that the financial derivative instruments contain useful forward looking information. Stemming from the empirical evidence on equity markets, future research could potentially focus on inferring information from derivatives on other markets i.e., exchange rates, commodities etc., and investigate linkages across different asset classes.

In this thesis, information from the derivatives markets has been extracted and analysed predominantly on developed markets and western markets. This is primarily due to data availability and subject to the development of each country's derivatives markets. However, one cannot talk about globalisation and correlations between international markets without putting developing markets into perspective. For example, China, India, and Brazil, as the most populous countries in the world, are growing very fast, and they will have a huge impact on the development of the derivatives markets throughout the world in the years to come.

Take China as an example, it is the world's second-largest economy, and it is growing rapidly every year on average around 7%. According to the world investment report, China has been one of the world's largest foreign direct investment recipients for the past few years. This can be attributed to the comparatively low labour costs as well as being the biggest internal market with 1.3 billion potential customers. No doubt there will be shift in investment focus from the western countries to China as it becomes more important in the world economy. However, the derivatives markets in China are underdeveloped. China has considerable tradings in commodity derivatives and growing futures markets, but credit derivative products and options are very limited. It lacks a well-established legal framework and is complicated with weak supervision from different regulators. Since derivatives are important tools for transferring risk from one entity to another, it is important for China to further develop its derivatives markets not only to meet the demand of all kinds of investors but also to be fully integrated in the international financial society. This argument goes with all developing economies which are playing more and more important roles in shaping the world's economy.

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