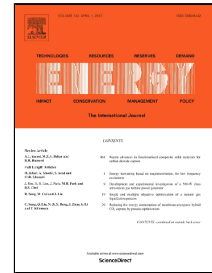


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- Analysis of the 3D unsteady aerodynamics of a Darrieus wind turbine blade in motion
- Highly spatially and temporally refined time-dependent simulations carried out with the COSA code
- One month calculation time on more than 16.000 processors on a IBM BG/Q cluster
- Detailed description of: tip losses, dynamic stall, vortex propagation and blade/wake interaction

Darrieus Wind Turbine Blade Unsteady Aerodynamics: a Three-Dimensional Navier-Stokes CFD assessment

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Abstract

Energized by the recent rapid progress in high-performance computing and the growing availability of large computational resources, computational fluid dynamics (CFD) is offering a cost-effective, versatile and accurate means to improve the understanding of the unsteady aerodynamics of Darrieus wind turbines, increase their efficiency and delivering more cost-effective and structurally sound designs.

In this study, a Navier-Stokes CFD research code featuring a very high parallel efficiency was used to thoroughly investigate the three-dimensional unsteady aerodynamics of a Darrieus rotor blade. Highly spatially and temporally resolved unsteady simulations were carried out using more than 16,000 processor cores on an IBM BG/Q cluster. The study aims at providing a detailed description and quantification of the main three-dimensional effects associated with the periodic motion of this turbine type, including tip losses, dynamic stall, vortex propagation and blade/wake interaction. Presented results reveal that the three-dimensional flow effects affecting Darrieus rotor blades are significantly more complex than assumed by the lower-fidelity models often used for design applications, and strongly vary during the rotor revolution. A comparison of the CFD integral estimates and the results of a blade-element momentum code is also presented to highlight strengths and weaknesses of low-fidelity codes for Darrieus turbine design.

The reported CFD results provide a valuable and reliable benchmark for the calibration of lower-fidelity models, which are still key to industrial design due to their very high execution speed.

Keywords

Darrieus wind turbine, unsteady Navier-Stokes simulations, CFD, tip flows

42 **Nomenclature**43 Latin symbols

44	AoA	angle of attack	
45	AR	aspect ratio	[-]
46	c	blade chord	[m]
47	C_t	torque coefficient	[-]
48	C_p	pressure coefficient	[-]
49	CP	power coefficient	[-]
50	BEM	Blade Element Momentum	
51	CFD	Computational Fluid Dynamics	
52	H	turbine height	[m]
53	k	turbulent kinetic energy	[m ² /s ²]
54	NS	Navier-Stokes	
55	p	static pressure	[Pa]
56	PDEs	Partial Differential Equations	
57	R	turbine radius	[m]
58	RANS	Reynolds-Averaged Navier-Stokes	
59	SST	Shear Stress Transport	
60	T	torque per unit length	[Nm]
61	TSR	tip-speed ratio	[-]
62	u, v, w	Cartesian components of local fluid velocity vector	[m/s]
63	U	magnitude of absolute wind speed	[m/s]
64	\underline{v}	local fluid velocity vector	[m/s]
65	\underline{v}_b	local grid velocity vector	[m/s]
66	VAWTs	Vertical-Axis Wind Turbines	
67	W	magnitude of relative wind speed	[m/s]
68	x, y, z	reference axes	
69	y^+	dimensionless wall distance	[-]

70

71 Greek symbols

72	ϑ	azimuthal angle	[deg]
73	μ_t	turbulent viscosity	[Kg/m/s]
74	ρ	air density	[kg/Nm ³]
75	Φ	computational domain diameter	[m]
76	Ψ	computational domain height	[m]
77	ω	specific turbulence dissipation rate	[1/s]
78	Ω	turbine revolution speed	[rad/s]

79

80 Subscripts

81	∞	value at infinity
82	ave	averaged value

83

84 **1. Introduction**85 *1.1 Background*

86 After most research on vertical-axis wind turbines (VAWTs) came to a standstill in the
 87 mid 90's [1], the Darrieus wind turbine [2] is receiving again increasing attention of both
 88 researchers and manufacturers [3-6]. For distributed wind power generation in the built
 89 environment [7], inherent advantages of this turbine type, such as performance insensitivity to

90 wind direction, generator often positioned on the ground, low noise emissions [8], enhanced
91 performance in skewed [9] or highly turbulent and unsteady flows [10-12], may outweigh
92 disadvantages, such as lower power coefficients and more difficult start-up, with respect to
93 typical horizontal axis machines. Moreover, in densely populated areas VAWTs are often
94 preferred to other turbine types because they are perceived as aesthetically more pleasant and
95 thus easier to integrate in the landscape [13]. The applicability of Darrieus wind turbines for
96 utility-scale power generation making use of floating platforms also appears to present
97 important benefits in terms of overall dynamic stability [14].

98 Historically, the aerodynamic performance analysis of these rotors has been carried out
99 with low-fidelity methods, like the Blade Element Momentum (BEM) theory [1,15-17] or
100 lifting line methods [18-19]. More recently, however, the intrinsic limitations of these models
101 made clear that higher-fidelity tools are needed in order to understand in greater depth the
102 complex unsteady aerodynamics of Darrieus rotors [20], such as the interaction of the blades
103 with macro vortices [21] or dynamic stall [22].

104 While experimental testing is often quite difficult and expensive, Navier-Stokes (NS)
105 Computational Fluid Dynamics (CFD) can be a versatile and accurate means to improve the
106 understanding of VAWT unsteady aerodynamics and achieve higher-performance, structurally
107 sound and more cost-effective Darrieus turbine designs. The use of NS CFD for simulating time-
108 dependent Darrieus turbine aerodynamics is rapidly increasing due to both the ongoing
109 development and deployment of more powerful high-performance computing hardware, such
110 as large clusters of multi- and many-core processors [23], and also the development of
111 computationally more efficient algorithms.

112

113 *1.2 Previous CFD studies on Darrieus VAWTs*

114 Early use of the Reynolds-Averaged Navier-Stokes (RANS) CFD technology for
115 Darrieus rotor aerodynamics for investigating the complex fluid mechanics of these machines
116 was based mostly on two-dimensional (2D) simulations (e.g. [24]). An extensive literature
117 review of 2D RANS CFD studies is provided by Balduzzi et al. [25], along with an overview
118 of the numerical settings frequently used for Darrieus rotor RANS studies and guidelines for
119 the optimal set-up of these simulations. Two-dimensional RANS analyses, suitably corrected
120 for three-dimensional (3D) effects, such as struts resistive torque and blade tip losses, have
121 been used to estimate the turbine overall performance [26] and predominantly 2D phenomena,
122 like virtual camber [27] and virtual incidence [28] effects, the influence of unsteady wind
123 conditions on turbine aerodynamics [29], the evolution of the flow field at start-up [30], and
124 turbine/wake interactions [31].

125 However, the use of 2D simulations to analyse the flow field past real rotors may result
126 in significant uncertainties, due to the difficulty of reliably quantifying complex 3D
127 aerodynamic features such as blade tip flows, their dependence on the blade tip geometry and
128 their impact on the overall efficiency as a function of the blade aspect ratio. Moreover, most
129 3D aerodynamic features of Darrieus rotor flows vary not only spatially (e.g. dynamic stall
130 decreases from midspan to the blade tips, as shown below), but also temporally during each
131 revolution. It is difficult to develop corrections to improve the predictions of time-dependent
132 2D CFD analyses, and the resulting uncertainty on unsteady loads may severely impair both
133 aerodynamic and structural (e.g. fatigue) assessments. Therefore 3D CFD simulations are key
134 to characterizing and quantifying the aforementioned 3D aerodynamic phenomena. This is
135 important for reducing the uncertainty associated with modeling such phenomena on the basis
136 of a relatively small amount of data referring to existing turbines, and assumptions based on
137 overly simplistic analytical models. However, large computational resources are needed for
138 such simulations, due to the high temporal and spatial grid refinement needed for accurately
139 resolving all design-driving aerodynamic phenomena.

140 Comparisons of 2D and 3D simulations are sometimes carried out considering test cases
141 for which experimental data are available, as in Howell et al. [32] and Lam et al. [33]. In [32]
142 the use of 2D RANS CFD led to relatively poor agreement with experiments: a maximum
143 power coefficient CP of 0.371 at a tip-speed ratio (TSR) of 2.4 was predicted against a
144 measured maximum CP of 0.186 at TSR=1.85. Using 3D simulations, the maximum power
145 coefficient was instead correctly predicted, although the shape of computed and measured
146 power curves presented significant differences. The $k-\varepsilon$ re-normalization group turbulence
147 model [34] was used for all simulations, but no detailed information on the mesh size was
148 provided, except for the indication of a high dimensionless wall distance ($y^+ \approx 10$) which
149 required the use of wall functions. Also in [35] notable discrepancies between the results of 2D
150 and 3D RANS simulations were observed: the 2D simulations predicted a maximum power
151 coefficient of 0.43 at TSR=4.5 against a maximum CP of 0.27 at TSR=1.85 predicted by the
152 3D analyses. A fairly coarse grid, consisting of 2.95 million elements was used for the 3D
153 simulations of the considered two-blade rotor, and the $k-\omega$ Shear Stress Transport (SST)
154 turbulence model [36] was used.

155 Gosselin et al. [37] used 2D and 3D RANS simulations with the $k-\omega$ SST turbulence
156 model to investigate the dependence of 3D effects on the blade aspect ratio, finding that an
157 aspect ratio of 7 led to a relative efficiency drop of 60 percent with respect to the 2D analysis.
158 Although the simulations were performed with very refined meshes (up to 700 nodes on each
159 airfoil), the article reports that no rigorous mesh-independence was obtained. Joo et al. [38]
160 analysed the aerodynamic characteristics of a two-blade rotor as a function of design
161 parameters such as solidity and optimal TSR using the 3D RANS model coupled to a realizable
162 $k-\varepsilon$ turbulence model. They used a computational mesh of 1.2 million elements and compared
163 their baseline results with available experimental data: a general overestimation of the power
164 curve was noticed, with a 10% discrepancy at the peak power. Moreover, significant shape
165 differences of computed and measured power curves were observed, particularly in the right
166 branch of such curves (i.e. for high TSR values), with the measured curve being steeper than
167 the computed one. A two-blade Darrieus rotor with straight blades was analysed by Li et al.
168 [35,39] by means of wind tunnel experiments and 3D RANS simulations using the $k-\omega$ SST
169 turbulence model. Numerical results compared favourably to experimental data in the left
170 branch (i.e. low TSR values) of the power curve, but CFD significantly overestimated power
171 for higher TSR, possibly due to excessively small distance between the rotor and the farfield
172 boundaries of the physical domain (about 2 rotor diameters from the rotor center). The
173 simulations used a grid with about 5 million elements, and predicted a peak power coefficient
174 of 0.24 at TSR=2.09 whereas experiments showed a peak CP of 0.18 at TSR=2.18.

175 Alaimo et al. [40] carried out a comparative study of the aerodynamic performance of
176 three-blade Darrieus rotors using straight and helical blades. For the three-blade rotors they
177 analysed, they found that blade tip vortex flows significantly reduced the rotor performance
178 and that the use of helical blades significantly reduces this power loss. The largest grids used
179 for those 3D RANS analyses had about 10 million elements, and turbulence was modelled
180 using the $k-\varepsilon$ turbulence model with wall functions to enable the use of relatively coarse grids
181 and enhance numerical stability. De Marco et al. [41] performed 3D RANS simulations to
182 analyse the influence of the geometry of the blade supporting arms on the turbine performance.
183 Using grids featuring between 4 and 18 million elements and the $k-\omega$ SST model, they found
184 that inclined and aerodynamically shaped supporting arms can significantly increase the mean
185 power coefficient. Using computational meshes with up to 12 million elements and the $k-\omega$
186 SST model, Zamani et al. [42] characterized with 3D RANS simulations the impact of J-shaped
187 blades on Darrieus rotor torque and power characteristics at low and medium TSR values,
188 finding that this blade shape significantly increases the rotor performance in these regimes,
189 resulting in improved start-up characteristics. Orlandi et al. [43] used 3D RANS simulations

190 and the $k-\omega$ SST model to study the influence of skewed wind conditions on the aerodynamic
191 characteristics of a two-blade Darrieus turbine; to limit the computational burden of the 3D
192 analyses, grids with about 10 million elements were used.

193 A recent Navier-Stokes CFD study of a three-blade Darrieus rotor using pitching blades
194 to further improve the aerodynamic performance has made use of a Large Eddy Simulation
195 [44], an approach that can yield more accurate results than the RANS method. However,
196 resolved LES analyses require computational grids far larger than those needed for grid-
197 independent RANS analyses, and this makes the use of resolved 3D LES simulations even
198 more difficult than that of grid-independent 3D RANS analyses.

199 The aforementioned 3D CFD analyses highlighted new important aerodynamic
200 phenomena, but in almost all cases limited availability of computational resources imposed the
201 use of fairly coarse spatial and temporal refinement, and this may result in uncertainty due to
202 lack of complete grid-independence of the CFD solutions. Recent parametric analyses
203 investigating the impact of several numerical parameters on the computationally less
204 demanding 2D CFD analysis of Darrieus rotors [25,45-47] showed that the analysis reliability
205 - in terms of accuracy of both performance prediction and resolution of important flow
206 structures - is tremendously affected by the quality of the meshing and time-stepping strategies.
207 These studies highlighted that the minimum temporal and spatial refinement required to obtain
208 grid-independent solutions is quite high, due to the aerodynamic complexity of these unsteady
209 flows. Because of these constraints, the computational cost of reliable 3D unsteady NS analyses
210 of Darrieus rotor flows is extremely large due to the necessity of maintaining the high temporal
211 resolution indicated by the 2D parametric studies and a high level of spatial refinement both in
212 the grid planes normal to the rotor axis and the 3rd direction orthogonal to such planes.
213 Refinement in the 3rd direction is essential to reliably resolving 3D flow features. For example,
214 the parametric study of [25] showed that temporal and spatial grid-independent 2D RANS
215 analyses of a three-blade rotor require grids with at least 400,000 elements. To preserve the
216 same accuracy level in a 3D RANS simulation of the same turbine (modelling only half of the
217 rotor making use of symmetry boundary conditions on the plane at rotor midspan and ensuring
218 adequate refinement in the tip region) the mesh would consist of at least 90 million elements,
219 which is almost ten times the size of the finest meshes used in the 3D RANS studies of Darrieus
220 rotor flows published to date. Failing to maintain these refinement levels may reduce the
221 benefits achievable by using 3D RANS simulations to improve Darrieus rotor design.

222

223 *1.3 Study aim*

224 In this study, the COSA RANS research code, which features a very high parallel
225 efficiency, is used to investigate in great detail the 3D flow features of a rotating Darrieus rotor
226 blade and the impact of flow three-dimensionality on power generation efficiency. More
227 specifically, the study aims at providing a highly accurate analysis of the main 3D phenomena
228 occurring during each revolution of the considered one-blade rotor, including tip vortices,
229 dynamic stall and downstream vortex propagation, and to assess the impact of these phenomena
230 on the overall performance of this rotor. The use of a single rotating blade for the type of
231 analyses reported herein is not uncommon (see for example [40]). This set-up enables a better
232 understanding of individual key fluid mechanics phenomena adversely impacting loads and
233 energy efficiency of Darrieus rotors, and this information constitutes the first knowledge level
234 required to improve the design of these machines. Follow-on studies will focus on additional
235 3D fluid mechanic aspects resulting from the multi-blade environment making use of
236 computational resources larger than those used in the present study.

237 To maximize the analysis reliability, a time-dependent 3D simulation using very high
238 levels of spatial and temporal refinement is carried out using a large 98,304-core IBM BG/Q
239 cluster.

240 The presented test case is expected to be highly valuable to other research groups both to
 241 verify new CFD approaches and to calibrate lower-fidelity models (e.g. model based on lifting
 242 line theory and free vortex methods), which are key to industrial design due to their extremely
 243 low computational cost.

244 The paper is organized as follows. Section 2 presents the numerical methodology that
 245 has been followed in the study: sub-section 2.1 reports the governing equations solved by the
 246 COSA CFD code for the analysis of Darrieus rotor flows, sub-section 2.2 summarizes the main
 247 numerical features and previous work carried out with this code, and sub-section 2.3 provides
 248 the main features of the case study and describes the adopted numerical set-up. Section 3
 249 presents the main results of the 3D CFD analysis and compares them to those of the 2D analysis
 250 of the same case study, to highlight the impact of 3D effects. In Section 3 the results of the
 251 RANS CFD analyses are also compared with those of the blade element momentum theory to
 252 highlight strengths and weaknesses of this engineering approach. A summary of the study and
 253 concluding remarks are finally provided in Section 4.

254

255 2. Numerical methodology

256 2.1 Governing equations

257 The compressible NS equations are a system of 5 nonlinear partial differential equations
 258 (PDEs) expressing the conservation of mass, momentum and energy in a viscous fluid flow.
 259 Averaging these equations on the longest time-scales of turbulence yields the so-called RANS
 260 equations, which feature additional terms depending on the Reynolds stress tensor. Making use
 261 of Boussinesq approximation, this tensor has an expression similar to that of the laminar or
 262 molecular stress tensor, with the molecular viscosity replaced by a turbulent or eddy viscosity
 263 [48-49]. In the COSA CFD code, the eddy viscosity is computed by means of the two-equation
 264 k - ω SST turbulence model [36]. Thus, turbulent flows are determined by solving a system of 7
 265 PDEs.

266 Given a moving time-dependent control volume $C(t)$ with time-dependent boundary $S(t)$,
 267 the Arbitrary Lagrangian–Eulerian integral form of the system of the time-dependent RANS
 268 and SST equations in an absolute frame of reference is:

$$269 \frac{\partial}{\partial t} \left(\int_{C(t)} \mathbf{U} dC \right) + \oint_{S(t)} (\underline{\Phi}_c - \underline{\Phi}_d) \cdot d\underline{S} - \int_{C(t)} \mathbf{S} dC = 0 \quad (1)$$

270 where \mathbf{U} is the array of conservative variables defined as:

$$271 \mathbf{U} = [\rho \quad \rho \underline{v}' \quad \rho E \quad \rho k \quad \rho \omega] \quad (2)$$

272 The symbols ρ , \underline{v} , E , k and ω denote respectively fluid density, flow velocity vector of
 273 Cartesian components (u, v, w) , total energy per unit mass, turbulent kinetic energy per unit mass
 274 and specific dissipation rate of turbulent energy, and the superscript ' denotes the transpose
 275 operator. The total energy is defined as $E = e + (\underline{v} \cdot \underline{v})/2 + k$, where e denotes the internal energy
 276 per unit mass; the perfect gas law is used to express the static pressure p as a function of ρ , E ,
 277 k and the mean flow kinetic energy per unit mass $(\underline{v} \cdot \underline{v})/2$. The generalized convective flux vector
 278 is defined as:

$$279 \underline{\Phi}_c = \underline{E}_c \underline{i} + \underline{F}_c \underline{j} + \underline{G}_c \underline{k} - \underline{v}_b \mathbf{U} \quad (3)$$

280 where \underline{E}_c , \underline{F}_c and \underline{G}_c are respectively the x-, y- and z-component of $\underline{\Phi}_c$ and are given by:

$$\begin{cases}
 \mathbf{E}_c = [\rho u \quad \rho u^2 + p \quad \rho uv \quad \rho uw \quad \rho uH \quad \rho uk \quad \rho u\omega] \\
 \mathbf{F}_c = [\rho v \quad \rho uv \quad \rho v^2 + p \quad \rho vw \quad \rho vH \quad \rho vk \quad \rho v\omega] \\
 \mathbf{G}_c = [\rho w \quad \rho uw \quad \rho vw \quad \rho w^2 + p \quad \rho wH \quad \rho wk \quad \rho w\omega]
 \end{cases} \quad (4)$$

in which $H=E+p/\rho$ is the total enthalpy per unit mass. The vector \underline{v}_b is the velocity of the boundary S, and the flux term $-\underline{v}_b \mathbf{U}$ is its contribution of the boundary motion to the overall flux balance.

The expressions of the diffusive fluxes $\underline{\Phi}_c$ and the turbulent source term \mathcal{S} appearing in Eq. (1) can be found in [48] and [50].

2.2 COSA CFD code

COSA is a compressible density-based finite volume code that solves the system of PDEs corresponding to Eq. (1) using structured multi-block grids. The code features a steady flow solver, a time-domain (TD) solver for the solution of general unsteady problems [48-49], and a harmonic balance solver for the rapid solution of periodic flows [50-52]. The second-order space discretization of the convective fluxes of both the RANS and the SST equations uses an upwind scheme based on Van Leer's MUSCL extrapolations and Roe's flux difference splitting. The second order discretization of all diffusive fluxes is instead based on central finite-differencing. The space-discretized RANS and SST equations are integrated in a fully-coupled fashion with an explicit solution strategy based on full approximation scheme multigrid featuring a four-stage Runge-Kutta smoother. Convergence acceleration is achieved by means of local time-stepping and implicit residual smoothing. For general time-dependent problems, the TD equations are integrated using a second order accurate dual time-stepping approach.

Comprehensive information on the numerical methods used by COSA and thorough validation analyses are reported in [50,52] and other references cited therein. For unsteady problems involving oscillating wings and cross-flow open rotors such as the Darrieus turbines, COSA solves the governing equations in the absolute frame of reference using body-fitted grids. In the case of Darrieus rotors in open field operation this implies that the entire computational grid rotates about the rotational axis of the turbine. The suitability of COSA for the simulation of Darrieus wind turbines has been recently assessed through comparative analyses with both commercial CFD codes and experimental data [53-54].

2.3 Case study and computational model

The selected case study is a one-blade H-Darrieus rotor using the NACA 0021 airfoil. The blade chord ($c=0.0858$ m), the blade length ($H=1.5$ m) and the rotor radius ($R=0.515$ m) were set equal to those used in the case-study of [24]; the blade was attached to the spoke at midchord according to the original 3-blade model of [24]. The decision of simulating a single blade was based both on physical considerations and on hardware limitations. First, a one-blade model is sufficient to investigate all the desired 3D flow structures that lead to an efficiency reduction of a finite blade. At the same time, the use of a single blade allows one to isolate and analyze fundamental aerodynamic phenomena of finite-length blade aerodynamics, removing additional aerodynamic effects due to multiple blade/wake interactions occurring in a multi-blade rotor. From a practical viewpoint, the need of ensuring an adequate level of spatial refinement both in the grid planes normal to the rotor axis and in the axial direction would have required a grid with more than 100 million elements for a three-blade rotor, which was beyond the resources available for this project.

325 To further reduce the computational cost of the 3D simulation, the central symmetry of
326 H-Darrieus rotors was exploited, enabling to simulate only one half of the rotor flow, thus
327 halving computational costs. Consequently, the aspect ratio (AR) of the simulated blade portion
328 is 8.74 which is half that of the actual blade. The modeled blade portion was contained in a
329 cylindrical domain (Fig. 1) of radius $\Phi=240R$, a value chosen to guarantee a full development
330 of the wake, based on the sensitivity analyses reported in [53]. The domain height was set to
331 $\Psi=2.53H$, corresponding to half the height of the wind tunnel where the original 3-blade model
332 was tested [24,54]; measured data from these tests were previously used for validating the
333 robustness of the RANS CFD methodology [26,53] also used in the present study.

334 The 3D structured multi-block grid (2D and 3D views are reported in Fig. 2) was obtained
335 with the software ANSYS® ICFM® by first generating a 2D mesh past the airfoil using the
336 optimal mesh settings identified in [47,49], and then extruding this mesh in the spanwise (z)
337 direction and filling up with grid cells the volume between the blade tip and the upper (circular)
338 farfield boundary. The far-field boundary condition enforced on the lateral (cylindrical)
339 boundary and the upper boundary of the domain is based on suitable combinations of one-
340 dimensional Riemann invariants and user-given freestream data, namely pressure, density and
341 velocity components. The sub-set of these far-field data combined with suitable Riemann
342 invariants depends on whether the fluid stream enters or leaves the computational domain at
343 the considered boundary point (the code detects automatically inflow and outflow points of the
344 boundaries at each iteration). The complete definition of this far-field boundary condition is
345 provided in [55]. On the blade surface, a no-slip condition is enforced. Since the equations are
346 solved in the absolute frame of reference, this requires imposing that the fluid velocity at the
347 blade surface equals the velocity of the blade surface itself at the considered wall point, where
348 pressure and density are extrapolated from the interior domain. The 2D grid section normal to
349 the z-axis and containing the airfoil (Fig. 2(a)) consisted of 4.3×10^5 quadrilateral cells. The
350 airfoil was discretized with 580 nodes and the first element height was set to $5.8 \times 10^{-5}c$ to
351 guarantee a dimensionless wall distance y^+ lower than 1 throughout the revolution. As
352 recommended in [25], a fairly high mesh refinement of both leading and trailing edge regions
353 was adopted (Fig. 2(b)), and a high refinement in the airfoil region within one chord from the
354 airfoil surface was also used to resolve the separated flow regions at high angle of attack (AoA)
355 [27]. After extrusion in the z direction, 80 grid layers in the half-blade span were formed (Fig.
356 2(c)), with progressive grid clustering from midspan to tip to ensure an accurate description of
357 tip flows. A fairly high grid refinement was also adopted in the whole tip region above the
358 blade in order to capture the flow separation and the tip vortices. The final mesh consisted of
359 64 million hexahedral cells.

360 The rotor flow field was computed by solving the system of governing equations
361 corresponding to Eq. (1), that is by solving the RANS and SST equations in the absolute frame
362 of reference. In such frame, the entire body-fitted grid rotates past the rotor axis, the additional
363 flux components due to the grid motion is accounted for by the term $-\mathbf{v}_b \mathbf{U}$ appearing in Eq. (3),
364 and no sliding surface is required.

365 To keep computational costs within the limits of the available resources, only one
366 operating condition was simulated, corresponding to a tip-speed ration (TSR) of 3.3. This
367 condition corresponds to the same revolution speed already analyzed by some of the authors
368 for the 3-blade turbine in [53]. For a 1-blade rotor, this TSR corresponds to a different point of
369 the rotor power curve. The operating condition corresponding to this TSR, however, was
370 considered of particular interest also for the 1-blade rotor because, also in this case, a) it
371 corresponds to fairly high efficiency and thus a regime at which the rotor is expected to work
372 more often than at other TSRs, and b) it features several complex aerodynamic phenomena
373 (e.g. stall and strong tip vortices) posing a significant modelling challenge for the CFD analysis.
374 Figure 3 displays the power coefficient at $TSR=3.3$ evaluated with the CFD analysis reported

375 below on the expected power curve, which was calculated with a computationally more
 376 affordable code based on Lifting Line Theory coupled to a free vortex wake model. The model
 377 was successfully tuned on this case-study in [56] and thus it is expected to provide a power
 378 curve prediction fairly consistent with the CFD analysis reported below.

379 The free-stream wind speed was $U=9.0$ m/s. The turbulence farfield boundary conditions
 380 were a turbulent kinetic energy (k) based on 5% turbulence intensity and a characteristic length
 381 of 0.07 m.

382 The 3D and 2D simulations reported below were performed with the time-domain solver
 383 of COSA. The 3D simulation was run on an IBM BG/Q cluster [57] featuring 8,144 16-core
 384 nodes for a total of 98,304 cores. Exploiting the outstanding parallel efficiency of COSA, the
 385 simulation could be carried out using about 16,000 cores. This required partitioning the grid
 386 into 16384 blocks using in-house utilities, and this operation was performed starting from a
 387 grid with fewer blocks generated with the ANSYS® ICEM® grid generator. All grid blocks had
 388 identical number of cells to optimize the load balance of the parallel simulation. Using a time-
 389 discretization yielding 720 steps per revolution, the simulation needed 12 revolutions to
 390 achieve a fully periodic state. The flow field was considered periodic once the difference
 391 between the mean torque values of the last two revolutions was smaller than 0.1% of the mean
 392 torque in the revolution before the last. The wall-clock time required for this 3D simulation
 393 was about 653 hours (27.2 days).

394

395 2.4 Grid and time-step sensitivity analyses

396 One of the key elements of this study is that the 3D calculation was carried out using a
 397 high level of spatial and temporal resolution. The 3D grid used to carry out the analyses
 398 reported in Section 3 was obtained by extruding in the third direction the 4.3×10^5 -element 2D
 399 grid described above, and such grid was shown to provide accurate and grid-independent
 400 results in [53].

401 To assess the impact of using coarser spatial and temporal refinement on the computed
 402 solution, the considered flow regime was also simulated using only 360 steps per revolution
 403 and a coarser 3D grid with 8 million elements, obtained from the 64million element fine grid
 404 by removing every second line in all three directions.

405 The periodic profiles of the instantaneous torque coefficient C_t obtained with the coarse
 406 and fine grids are compared in Fig. 4, and the definition of C_t is provided by Eq. (5), in which
 407 T denotes the instantaneous torque on the entire blade, U_∞ and ρ_∞ denote respectively the far-
 408 field wind speed and the air density, c is the blade chord, and H is the overall blade length. The
 409 angular position $\vartheta=0^\circ$ corresponds to the blade leading edge facing the oncoming wind and
 410 entering the upwind half of its revolution.

411

$$411 \quad C_t = \frac{T}{\frac{1}{2} \rho_\infty U_\infty^2 c^2 H} \quad (5)$$

412 The comparison shows that differences between the two predictions occur over most
 413 parts of the period, particularly around the maximum values of C_t . These discrepancies are
 414 caused by differences in the prediction of strength and timing of stall on the airfoils and under-
 415 resolved wakes and wake/blade interactions when using the coarse grid. The position of the
 416 curve peak (maximum C_t in the upwind region of rotor trajectory) predicted by the coarse grid
 417 has an error of about 3 degrees in azimuthal coordinates, leading to a shift of the curve in the
 418 range between $\vartheta=90^\circ$ and $\vartheta=300^\circ$. Such discrepancies, reported in Fig. 4 also as the difference
 419 between the coarse and fine grid profiles normalized by the revolution-averaged mean torque

420 of the fine grid (curve labeled “% variation”) result in the mean torque coefficient obtained
 421 with the coarse grid being 3.2 percent higher than that obtained with the fine grid. As discussed
 422 in the following, this difference corresponds to nearly 40 percent of the energy efficiency loss
 423 due to finite blade length effects. This highlights the importance of using a fine grid for this
 424 type of analyses.

425 The impact of the mesh refinement on the resolution of some of the 3D flow phenomena
 426 occurring during the revolution are examined in Fig. 5. This figure shows the extent of the
 427 vortices generated at the blade tip at $\vartheta=80^\circ$ predicted with the two meshes. The red and blue
 428 vortices represent the regions of ascending and descending flow, respectively. The higher
 429 dissipation of the coarse mesh leads to an under-prediction of the downstream propagation of
 430 the vortex, which is reduced from about three chords (Fig 5(b)) to less than two chords (Fig
 431 5(a)). The coarse grid under-estimation of the tip effects contributes to the overestimation of
 432 the torque highlighted in Fig. 4. The vorticity contours at midspan when the blade is at $\vartheta=315^\circ$
 433 are compared in Fig. 6 to assess the resolution of the free convection of vorticity in the
 434 downstream region. With the finer mesh the wake is resolved more sharply, thus fulfilling
 435 essential prerequisites for adequately resolving blade-wake interactions in the downwind part
 436 of the revolution. The under-resolution of the wake in the downwind rotor region contributes
 437 to the higher torque produced by the blade when interacting with the wake shed in the upstream
 438 trajectory. The impact of all these vortical phenomena on the rotor performance is even higher
 439 in multi-blade rotors, due to higher number of interactions (and thus energy loss events) per
 440 revolution.

441

442 3. Results and discussion

443 Figure 7(a) reports the instantaneous torque coefficient per unit length (C_{tz}) at different
 444 span lengths along the blade (0 percent and 100 percent correspond to midspan and tip,
 445 respectively). The instantaneous torque coefficient per unit length C_{tz} is defined by Eq. (6).
 446 Here T_z denotes the instantaneous torque per unit blade length at the considered spanwise
 447 position.

448

$$C_{tz} = \frac{T_z}{\frac{1}{2}\rho_\infty U_\infty^2 c^2} \quad (6)$$

449 Figure 7(b) reports three torque profiles. The profile labelled *2D* refers to the results of a
 450 2D simulation of the same rotor, and corresponds to the “ideal” torque of a blade with infinite
 451 span, i.e. without any secondary effects at the blade tip. This 2D simulation was carried out
 452 using a mesh equal to the midspan section of the 3D fine mesh and the same numerical
 453 parameters of the 3D simulations. The torque profile labelled “0%” is the torque per unit blade
 454 length at midspan of the finite-length rotor, whereas the torque profile labelled *3D* is the overall
 455 torque coefficient C_t of the 3D rotor defined in Eq. (7). The result obtained by using this
 456 definition is identical to that obtained by using Eq. (5).

457

$$C_t = \frac{2}{H} \int_0^{\frac{H}{2}} C_{tz} dz \quad (7)$$

458 Examination of these profiles reveals several important facts. Firstly, the ideal 2D torque
 459 and the 3D torque profiles are characterized by similar patterns, including the occurrence of
 460 two relative maxima, one in the upwind the other in the downwind regions, and also similar
 461 azimuthal positions of both maxima: the maximum torque in the upwind portion of the

462 revolution is located at $\vartheta \approx 88.5^\circ$ and the maximum torque in the downwind portion of the
 463 revolution is located at $\vartheta \approx 257^\circ$ in both cases. This behaviour is in line with the analyses of both
 464 Lam [33] and Alaimo [40], which showed that the periodic torque profiles obtained with 2D
 465 and 3D simulations differ significantly for their amplitudes but have comparable shapes. Figure
 466 7(b) also highlights that the differences between the 2D torque profile and that at midspan of
 467 the 3D rotor are negligible, highlighting that 3D flow effects due to tip flows do not reach this
 468 position.

469 Examination of all profiles of Fig. 7(b) shows that the effects of blade finite-length effects
 470 are very small when the blade loading is low, i.e. when the angle of attack is low ($0^\circ < \vartheta < 40^\circ$ and
 471 $130^\circ < \vartheta < 210^\circ$): in these portions of the revolution, the 2D and both 3D curves are almost
 472 superimposed. When the incidence increases, the blade load also increases and the blade starts
 473 experiencing stall. Figure 8 reports the top view of the vorticity contours at midspan at three
 474 azimuthal positions to examine the onset of stall in the upwind zone. At $\vartheta = 70^\circ$ a small
 475 separation region forms on the suction side of the blade. At $\vartheta = 80^\circ$ the blade stall has become
 476 significant, since the flow is detached from the blade. At the position of torque peak a large
 477 region of the suction surface is affected by stall. Consequently, the torque loss due to tip effects
 478 also increases because the strength of tip vortex flow increases with the flow incidence. The
 479 same behaviour can be seen also in the downwind zones. Closer inspection of the 2D and mean
 480 3D CP curves shows that these effects are strongest in the upwind region of the period, where
 481 a maximum difference of 9.7 percent between the torque peaks occurs.

482 Examining the torque profiles at the spanwise positions considered in Fig. 4(a), some
 483 additional observations can be made:

- 484 • The torque profiles of the blade sections at 20%, 40% and 50% semispan are almost
 485 identical, indicating that at least half of the blade is characterized by a predominantly 2D
 486 flow with negligible impact of tip flow effects;
- 487 • The torque profiles of the blade sections at 60%, 70% and 80% show a progressive
 488 reduction of the torque peak, down to -14 percent with respect to the midspan section.
 489 The remainder of the torque curve is less affected, especially in the downwind zone;
- 490 • The torque profiles of the blade sections at 90%, 95% and 97.5% show that at these
 491 positions, 3D effects are strong throughout the whole revolution. Notably, in the regions
 492 of positive torque production, the efficiency is remarkably reduced;
- 493 • In proximity of the blade tip (99%), almost no positive contribution to the torque output
 494 is given, due to the large load reduction;
- 495 • The azimuthal position of the torque peak occurs later in the cycle as one moves towards
 496 the tip, with a 5° shift between the 0% and 97.5% sections. This can be explained with a
 497 reduction of the incidence angle (downwash), as shown below. The experiments of Li et
 498 al. [35] highlight the same trend and show that the aforementioned shift is even more
 499 pronounced for a turbine with a very low aspect ratio ($AR=4.5$).

500 To compare the CFD prediction of the impact of finite blade effects on turbine
 501 performance to that of the widespread low-fidelity BEM theory, Fig. 9 compares the 2D and
 502 mean 3D torque profiles obtained with NS CFD and the corresponding estimates obtained with
 503 the VARDAR research code, a state-of-the-art BEM code developed at the University of
 504 Florence [6,17-18] using the ubiquitous Leicester-Prandtl model for the finite-wing correction
 505 [58]. The two BEM profiles of Fig. 9 differ in that one includes tip flow corrections and the
 506 other does not. Examination of these profiles shows that the reduction of the torque peak in the
 507 upwind portion of the revolution predicted by the CFD analyses is in good agreement with that
 508 estimated with the simplified tip flow model included in the BEM theory, and the shapes of the
 509 CFD and BEM torque profiles are in a qualitatively good agreement. Conversely, the patterns
 510 of the torque curves in the downwind portion of the revolution predicted by the BEM and CFD
 511 analyses are significantly different, and the torque reduction due to blade finite length predicted

512 by the BEM analysis is higher than predicted by CFD. This comparative analysis highlights
513 the potential of using CFD also for further improving the predictions of low-fidelity
514 engineering tools, which are key to Darrieus rotor industrial design due to their extremely small
515 computational requirements.

516 To provide a different quantitative perspective of the impact of tip losses, Fig. 10
517 compares the CFD and BEM profiles of mean torque coefficient per unit length. For each blade
518 height the mean value is obtained by averaging the profiles of Fig. 4(a) over one revolution.
519 The figure also reports the constant mean torque values of the 2D and 3D simulations for both
520 the CFD and BEM models. All curves are normalized with respect to the mean 2D torque
521 coefficient. One sees that the mean blade performance is almost unaffected by tip-effects up to
522 approximately 70% semispan. More specifically, it is found that tip flow effects adversely
523 affect the performance of the blade for a span length of approximately $2.6c$ (yellow zone in
524 Fig. 10). In terms of aggregate data, the tip effects yield a reduction of the rotor torque of 8.6%
525 with respect to the 2D calculation with virtually infinite span. This can be seen as an equivalent
526 reduction of the actual blade's height of $0.75c$ for each half blade (red colored zone in Fig. 10).
527 Such a correction factor needs to be accounted for when estimating the turbine performance by
528 means of 2D simulations.

529 The observations above are in accordance with the findings of Li et al. [35] in terms of
530 performance drop as a function of the distance from the tip. Their experiments showed that at
531 55% semispan, corresponding to a distance of $1.0c$ from the tip, the torque peak is greatly
532 reduced. At this blade height, they found a CP reduction of 40% over the midspan value at
533 $TSR=2.2$ and 60% at $TSR=2.5$, corresponding to an equivalent reduction of the actual blade's
534 height by $1.8c$ and $2.7c$, respectively. Other analyses focused on estimating the mean power
535 reduction due to finite blade length effects through comparisons of 2D and 3D CFD analyses
536 [32,33,37,40], but their results are not directly comparable with the present study due to the
537 use of different aspect ratio, rotor solidity, TSR, airfoil geometry and number blades.. Overall,
538 the equivalent height reduction can vary from $0.8c$ for a NACA 0022 three-blade rotor at
539 $TSR=1.3$ [32] up to $5c$ for a NACA 0018 two-blade rotor at $TSR=4.5$ [33].

540 To investigate in greater detail the 3D phenomena accounting for energy efficiency
541 reduction, the Mach contours and streamlines at the angular position of maximum separation
542 ($\vartheta=120^\circ$) are examined in Fig. 11(a). Different spanwise sections are considered to analyse the
543 flow pattern alterations from midspan to the blade tip.

544 In the central portion of the blade (from midspan to about 70% semispan) the streamlines
545 are contained in planes orthogonal to the blade axis, indicating a predominantly 2D flow
546 character, and a fairly large region of separated flow in the rear of the suction side. Closer to
547 the tip (90% semispan) the downwash due to the tip flow reduces the effective AoA with
548 respect to that at midspan, and the extension of the stall region is thus reduced. The skin friction
549 lines and contours of the z velocity component (w) on the blade suction surface reported in Fig.
550 11(b) show the extension of the region affected by downwash. Near the tip, the flow on the
551 pressure side is no longer able to follow the blade profile, and travels over the tip due to the
552 pressure difference between the pressure side and the suction side. The tip vortex flow is
553 responsible for the downwash velocity component and therefore for the incidence variation
554 along the span, in accordance with the theory of finite wings [58]. It is noted that the finite
555 wing effects occurring in Darrieus rotors are more complex than those encountered in fixed
556 finite wings. This is primarily because of the flow curvature associated with the circular
557 trajectory of the blade, and also the flow nonlinearities due to dynamic stall.

558 To quantify the impact of these effects, it is convenient to examine the curves of the
559 torque coefficient per unit length at midspan and 90% semispan (Fig. 12). The percentage
560 difference between the two curves (i.e. the torque coefficient difference between the curves at
561 each azimuthal angle divided by the revolution-averaged torque coefficient at midspan) is also

562 reported to quantify the dependence of the torque variation on the azimuthal position. A notable
563 torque reduction occurs in the interval $40^\circ < \vartheta < 130^\circ$. In addition, a large and sudden torque
564 reduction occurs towards the end of the revolution, in the interval $315^\circ < \vartheta < 340^\circ$, a range in
565 which the AoA is decreasing and goes below the value yielding stall. Also, an inversion in the
566 expected trend is noticed close to $\vartheta = 150^\circ$, where the tip section performs better than the
567 midspan section: the torque of the section at 90% semispan is about 10 percent higher than that
568 at midspan. According to the finite wing theory, the lift should be in fact always reduced in
569 proximity of the tip. Therefore, the inversion at $\vartheta = 150^\circ$ cannot be explained with this theory
570 alone. This occurrence and the sudden torque loss of the tip section towards the end of the
571 revolution are analysed in further detail below.

572 To investigate the origin of the sudden torque reduction at the blade tip in the interval
573 $315^\circ < \vartheta < 340^\circ$, isosurfaces of the turbulent kinetic energy field at selected azimuthal positions
574 are examined in Fig. 13. The color scale is based on the intensity of the velocity component
575 along the z-axis (w). Three azimuthal positions of the blade are considered: $\vartheta = 60^\circ$, $\vartheta = 180^\circ$ and
576 $\vartheta = 315^\circ$. During the upwind half of the revolution ($\vartheta = 60^\circ$) the tip vortex is strong, since the
577 vertical component of velocity is fairly high. A high turbulence region is then generated from
578 the blade tip. At $\vartheta = 180^\circ$, the region of high turbulent kinetic energy corresponding to the tip
579 vortex is increased in size and length, and is still associated with large values of w . This strong
580 vortex detaches from the blade, is convected by the wind, and is re-encountered by the blade
581 at $\vartheta = 315^\circ$. The blade interaction with this vortex induces a more pronounced reduction of the
582 torque with respect to the 2D case, where this effect is absent.

583 To investigate the reasons for the higher torque of the 90% section over the midspan
584 section at $\vartheta = 150^\circ$, top views of the streamlines at $\vartheta = 150^\circ$ and $\vartheta = 48^\circ$ are examined in Fig. 14.
585 The position $\vartheta = 48^\circ$ is selected because this is the other angular position of the upwind half of
586 the revolution experiencing the same AoA of $\vartheta = 150^\circ$. Streamlines on both the pressure and
587 suction sides of the blade are visualized at four different span locations. At $\vartheta = 48^\circ$ the
588 downwash effect is visible: moving from midspan to the tip, the incidence of the oncoming
589 flow decreases and the air stream after the trailing edge is more aligned to the airfoil chord.
590 This phenomenon is not very pronounced due to the low loading on the blade at this angular
591 position. At $\vartheta = 150^\circ$, moving from midspan to the tip, the incidence of the oncoming flow is
592 progressively reduced similarly to what seen at $\vartheta = 48^\circ$. However, the flow pattern on the suction
593 side of the central portion of the blade is significantly different from that at $\vartheta = 48^\circ$, despite the
594 fact that the AoA is similar in the two cases. A large separation region exists at $\vartheta = 150^\circ$ due to
595 stall. Due to the finite wing length, a strong modification of this flow pattern is observed
596 moving towards the tip: from 70% semispan, the flow is attached due to lower downwash-
597 induced loading and is more aligned to the airfoil chord after the trailing edge.

598 The observations above can be explained by a combined effect of downwash and
599 dynamic stall. From $\vartheta = 0^\circ$ to $\vartheta = 90^\circ$ the AoA increases and stall in the central blade portion
600 occurs between $\vartheta = 70^\circ$ and $\vartheta = 80^\circ$. The dominant effect is that of the downwash which reduces
601 the AoA to the outer portion of the blade. When the AoA reaches its maximum towards $\vartheta = 90^\circ$,
602 the central portion of the blade experiences high level of stall. From $\vartheta = 90^\circ$ to $\vartheta = 180^\circ$ the AoA
603 decreases but the central portion of the blade remains stalled due to delay of the flow in
604 readjusting to the decreasing incidence (a distinctive feature of dynamic stall). However, the
605 outer sections of the blade remain stall-free, and this is the reason why at $\vartheta = 150^\circ$ the torque of
606 the tip sections is higher than that of the midspan section, whereas the opposite is observed at
607 $\vartheta = 48^\circ$.

608 Fig. 15 presents an analysis of the same type of that of Fig. 14 for the angular positions
609 $\vartheta = 210^\circ$ and $\vartheta = 300^\circ$. Both positions belong to the downwind portion of the rotor trajectory and
610 are characterized by a comparable AoA. However at $\vartheta = 210^\circ$ the AoA is increasing whereas at
611 $\vartheta = 300^\circ$ the AoA is decreasing. One notices that the streamline pattern at $\vartheta = 210^\circ$ is similar to

612 that at $\vartheta=48^\circ$. At $\vartheta=300^\circ$ the streamline patterns from midspan to tip are the same as those at
 613 $\vartheta=210^\circ$. The similarity of the flow patterns at these two positions is due to the fact that no stall
 614 occurs in the downwind portion of the rotor trajectory.

615 Fig. 16 depicts the blade streamlines at $\vartheta=315^\circ$, the position at which the tip vortex
 616 interacts with the outboard portion of the blade in its downwind trajectory, as highlighted in
 617 Fig. 13. One observes a sudden deviation of the oncoming flow in the tip region with respect
 618 to the flow direction at midspan. Such deviation is due to the blade-vortex interaction, which
 619 prevails over the effects due to downwash.

620 All aforementioned results can be more quantitatively described by evaluating the
 621 pressure coefficient (C_p) distributions and the vorticity contours along the blade. The pressure
 622 coefficient used in this study is defined by Eq. (7), where p denotes the static pressure at the
 623 airfoil surface. Due to the difficulty of properly defining the actual relative wind speed at each
 624 blade height, the relative flow velocity w_{th} used to calculate C_p neglects the induced velocity
 625 and is computed using the vectorial sum of the absolute free-stream velocity and entrainment
 626 velocity ΩR .

627 Fig. 17 reports the C_p profiles at different blade heights for three key angular positions:
 628 maximum loading ($\vartheta=80^\circ$), inversion of torque of midspan and tip sections ($\vartheta=150^\circ$) and
 629 maximum loading in the downwind half of the revolution ($\vartheta=240^\circ$). The objective of this
 630 analysis is to highlight the impact of downwash and stall at different angular positions.

$$631 \quad C_p = \frac{p - p_\infty}{\frac{1}{2} \rho_\infty w_{th}^2} \quad (7)$$

632 At $\vartheta=80^\circ$ the blade is subject to high loading (high AoA and high relative speed). The
 633 top subplot of Fig. 17 confirms that, in these conditions, 3D flow effects affect almost 40
 634 percent of the blade (from the tip to 60% semispan), since moving from midspan to the tip, the
 635 C_p profile at 60% already shows a slight loading reduction with respect to midspan. Closer to
 636 the tip, the suction side of the blade is characterized by an almost constant pressure, indicating
 637 that this blade portion generates a small lift. As a result, the torque of the tip sections is
 638 substantially lower than that of the midspan section, and the torque becomes negative at 97.5%
 639 midspan, as shown in Fig. 7. At $\vartheta=240^\circ$ (middle subplot of Fig. 16) the AoA is high but the
 640 relative speed magnitude is lower than at $\vartheta=80^\circ$. In these conditions 3D flow effects affect only
 641 the last 20 percent of the semispan (i.e. from 80% semispan to tip): significant differences in
 642 the C_p profiles with respect to the midspan values are observed only on the last 10 percent of
 643 the blade, where the loading becomes significantly smaller than at midspan. Unlike at the two
 644 angular positions just discussed, a strong flow separation due to stall occurs at $\vartheta=150^\circ$ (bottom
 645 subplot of Fig. 17). This is highlighted by the pressure profiles at 0% and 60% semispan, which
 646 feature a fairly shallow slope on the suction side. In this circumstance, the lower AoA at the
 647 tip sections induced by the tip vortex-related downwash results in the flow past such tip sections
 648 remaining attached and these sections outperforming the midspan region of the blade.

649 The evolution of the vorticity contours at different blade span heights is presented in Fig.
 650 18. In the upwind half of the revolution, the two positions $\vartheta=40^\circ$ and $\vartheta=140^\circ$ are of particular
 651 interest. Although at these two positions the torque profiles along the blade are comparable
 652 (see Fig. 7(a) and Fig. 7(b)), the vorticity patterns and thus the flow field are remarkably
 653 different. On the other hand, moving to the downwind half of the revolution, one sees that the
 654 vorticity patterns around the blade are quite similar at all azimuthal positions. These patterns
 655 are in line with the previous analyses of streamlines and pressure coefficient profiles.

656 Figure 19 reports the top view of the vorticity contours at four different span locations at
 657 the two aforementioned azimuthal positions and highlights the vorticity differences in greater
 658 detail. At $\vartheta=40^\circ$ the vorticity contours are very similar, moving from midspan to tip, whereas

659 at $\vartheta=140^\circ$ the large separation region due to stall is clearly visible along a large central portion
660 of the blade.

661

662 4. Conclusions

663 A 3D time-accurate Reynolds-averaged Navier-Stokes CFD analysis of an aspect ratio
664 17.5 blade rotating in Darrieus-like motion has been presented. Special attention was paid to
665 the description of 3D flow effects and their impact on the energy efficiency of Darrieus rotor
666 blades. This was accomplished also by comparative analyses of 3D and 2D CFD analyses. The
667 presented 3D CFD results were obtained with a highly refined analysis using a grid with 64
668 million elements and time-marching the flow field to a periodic state using 720 time-steps per
669 revolution. A 3D mesh sensitivity analysis was also presented. The main outcomes of the
670 analysis can be summarized as follows:

- 671 a) 3D flow effects due to finite blade length reduce the mean power of the considered
672 17.5 aspect ratio blade by 8.6 percent with respect to the torque of the corresponding
673 infinite blade. Such mean torque reduction corresponds to a reduction of the effective
674 blade length of $1.5c$ ($0.75c$ for each half blade).
- 675 b) A strong interaction between the tip-vortex released in the upwind portion of the
676 blade revolution and the blade traveling in the downwind region occurs at $\vartheta=315^\circ$,
677 and this yields an additional reduction of the outboard blade sections in this region of
678 the revolution.
- 679 c) Finite blade length effects do not modify significantly the overall shape of the blade
680 torque profile over the revolution with respect to the torque profile of the
681 corresponding infinite blade;
- 682 d) For given azimuthal position, the torque profile along the blade height varies
683 substantially from midspan to tip, and the pattern of these variations strongly depends
684 on the azimuthal position; i.e. on the magnitude of the relative velocity of the
685 oncoming flow and its local angle of attack;
- 686 e) The mean torque reduction predicted by the 3D CFD analysis and that of a state-of-
687 the-art BEM analysis using tip loss corrections is comparable, but the profiles of the
688 blade torque in the downwind portion of the revolution differ significantly. The
689 reliability of BEM analyses may be improved by using 3D CFD results to develop
690 azimuthal position-dependent tip loss corrections;
- 691 f) The 3D grid sensitivity analysis highlighted that the use of a coarser grid, with size
692 comparable to those used in most 3D Darrieus studies to date, may yield uncertainty
693 levels in the prediction of tip vortex flows, blade/wake/tip vortex interactions, and
694 dynamic stall timings and strength. All these phenomena affect torque and power
695 generation. The mean power predicted by a typical coarse grid and the fine grid of
696 this study differed by more than 3 percent, and significantly larger differences are
697 expected for multi-blade rotors due to higher number of blade/wake/tip vortex
698 encounters per revolution.

699 Future work will include investigating 3D flow effects at different tip-speed ratios,
700 particularly the lower ones, at which the impact of dynamic stall is expected to be more
701 pronounced than at the considered regime, and extending this analysis to multi-blade turbines,
702 to assess in detail all aspects of wake/blade interactions. This type of high-fidelity analyses
703 provides valuable data for validating and further improving the reliability of low-fidelity tools
704 such as BEM codes and codes based on lifting line theory and free vortex transport methods.
705 Due to their extremely high execution speeds, these engineering tools are of crucial importance
706 to improving the design of future small and large Darrieus turbines.

707

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719

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FIGURES

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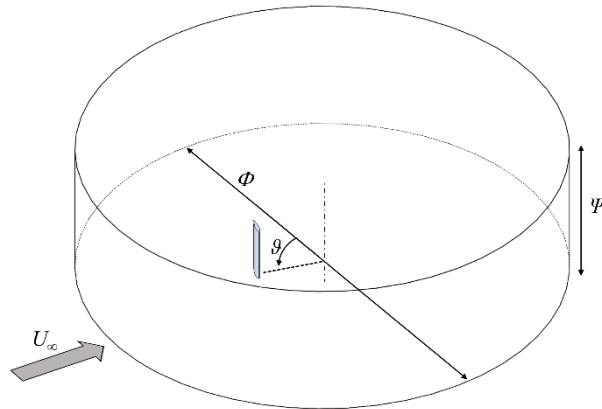


Figure 1 - Computational domain.

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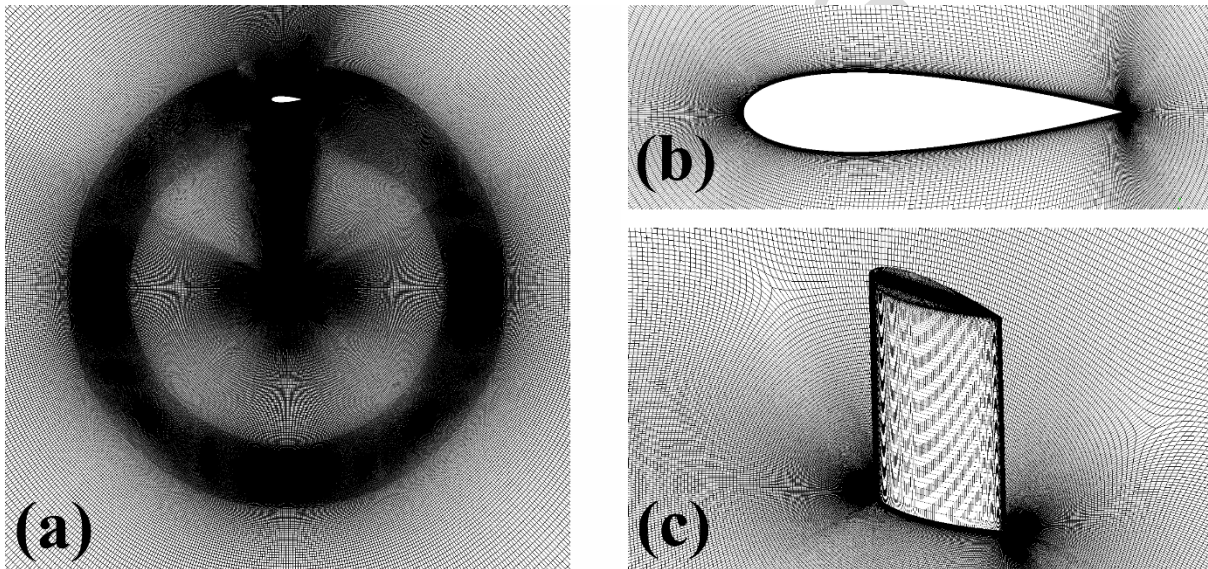


Figure 2 - Some details of the computational mesh.

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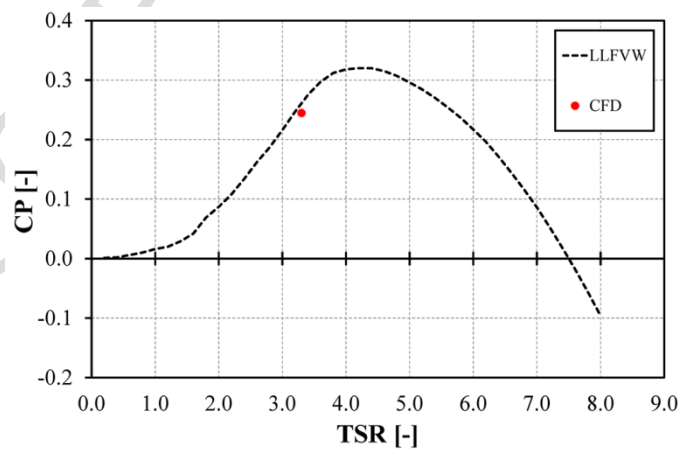


Figure 3 - Attended power curve of the 1-blade model.

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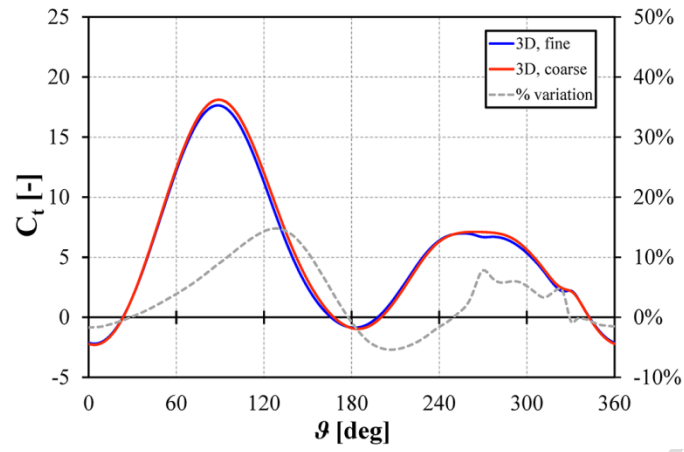


Figure 4 - Differences in the torque profile between the selected mesh and a coarser one.

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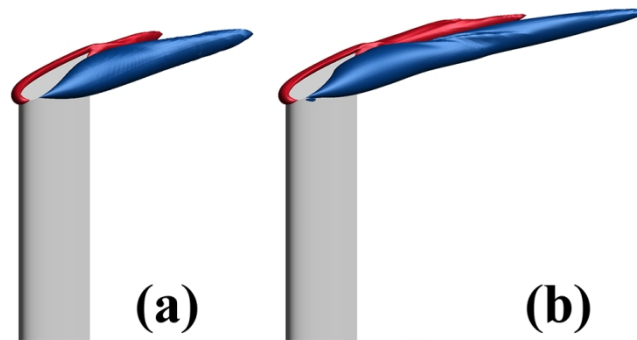
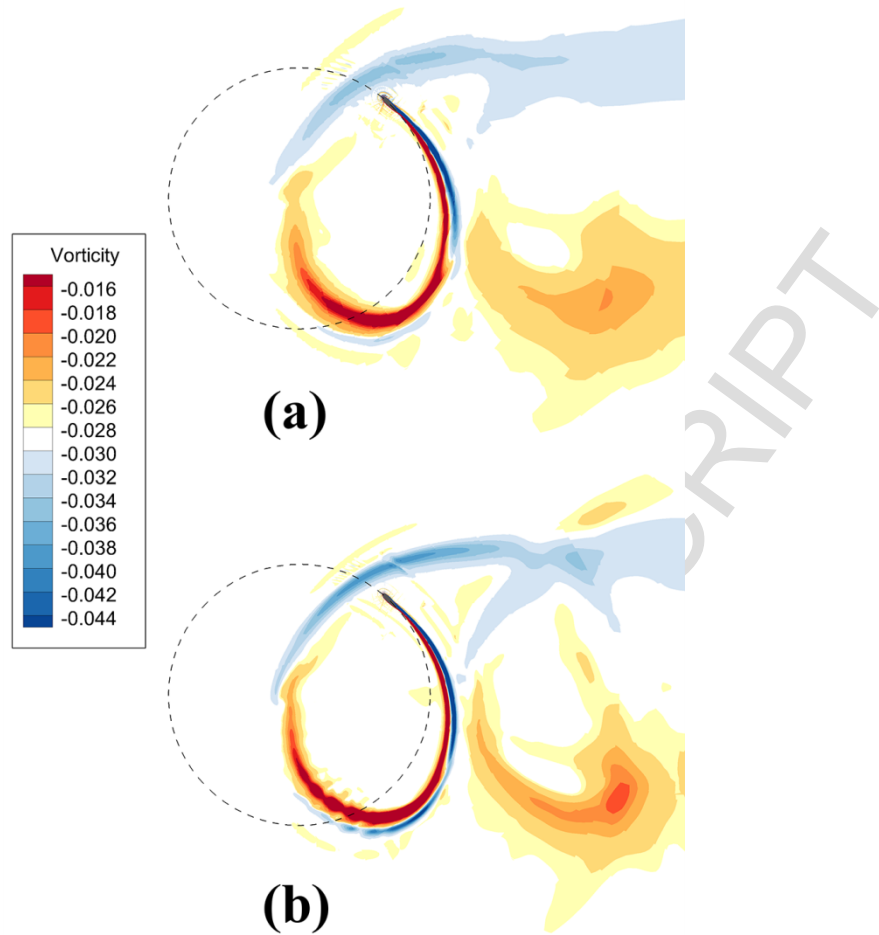


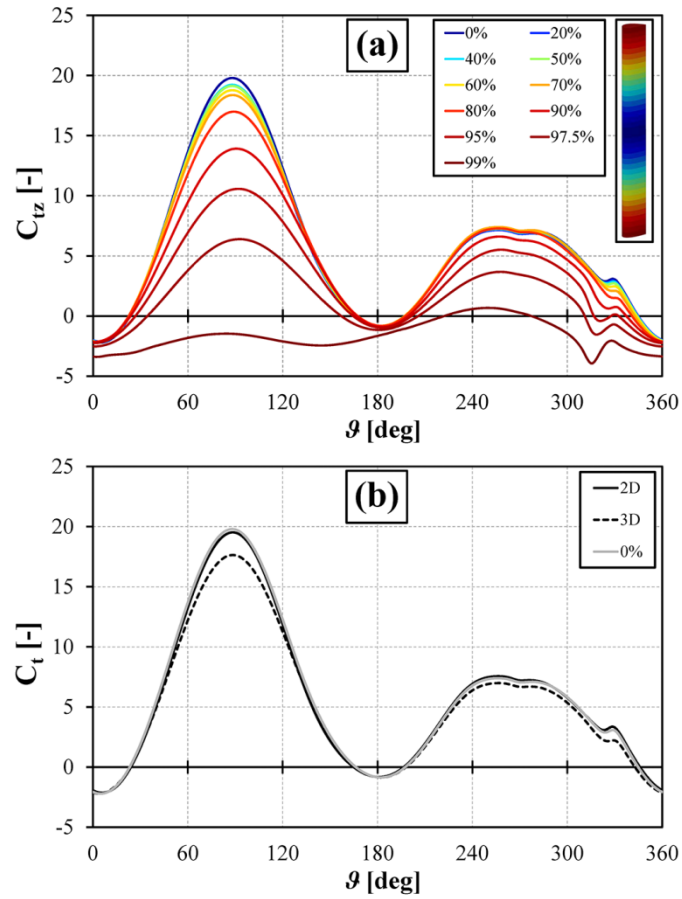
Figure 5 – Tip vortices generated at $\theta=80^\circ$: (a) coarse mesh; (b) selected mesh.

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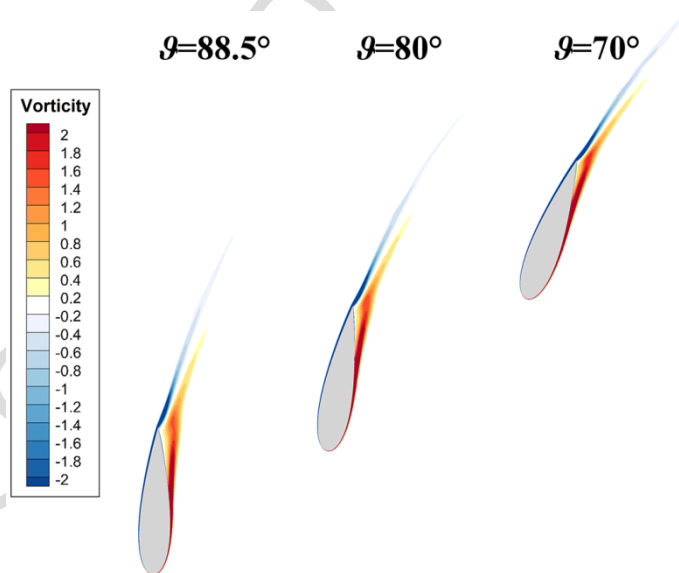


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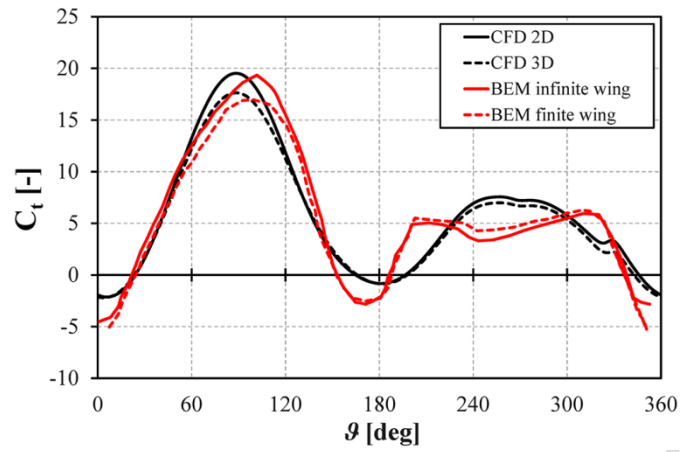
Figure 6 - Vorticity contours at midspan at $\theta=315^\circ$: (a) coarse mesh; (b) selected mesh.



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Figure 7 - Torque coefficient vs azimuthal angle: (a) variation at different span lengths; (b) 2D simulations compared to the 3D profile at midspan and average 3D profile.

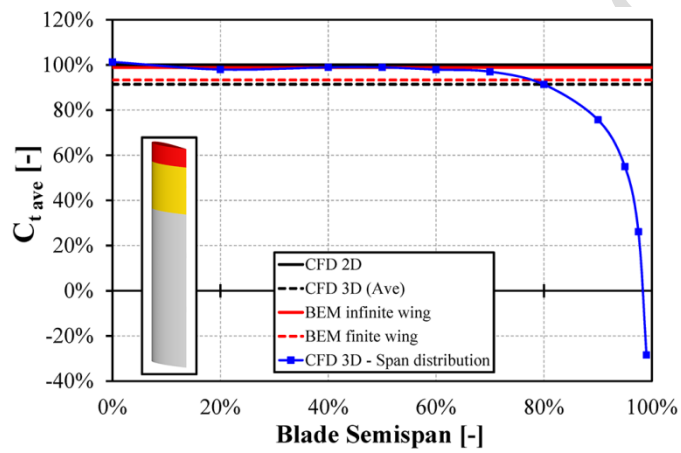


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Figure 8 - Vorticity contours at midspan: $\vartheta = 70^\circ$, $\vartheta = 80^\circ$ and $\vartheta = 88.5^\circ$.



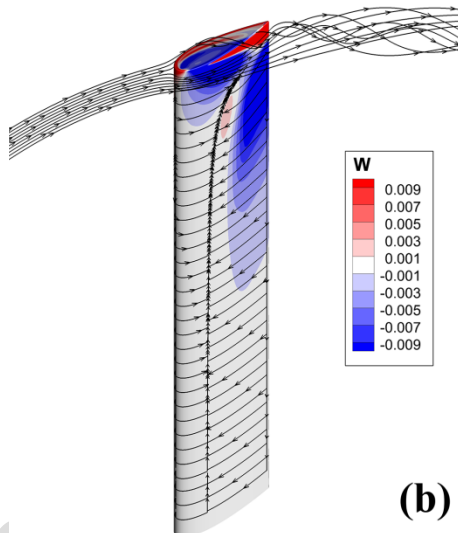
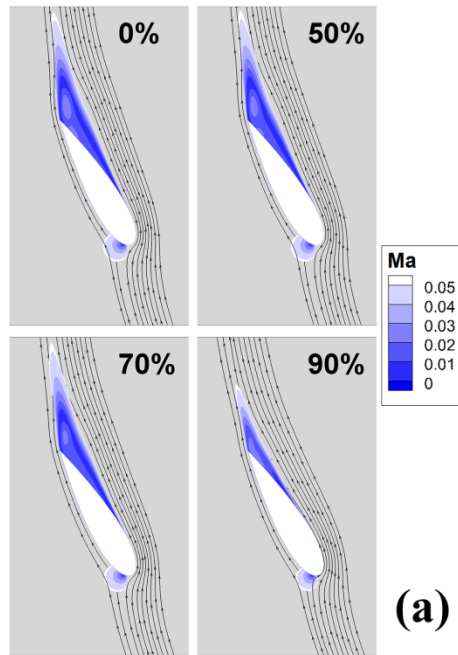
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Figure 9 - Torque coefficient profiles: 2D and 3D CFD data vs. BEM simulations either including or neglecting the finite-wing effects.



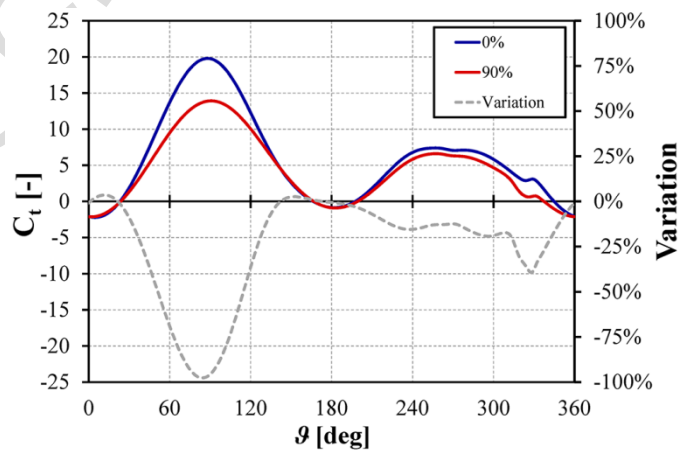
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Figure 10 - Torque coefficient distribution along the semispan.



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Figure 11 – Downwash effect at $\beta=120^\circ$: (a) Mach contours and streamlines at different semispan locations; (b) flow streamlines in the tip region, skin friction lines and z velocity component on the blade suction surface.



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Figure 12 - Comparison of torque coefficient curves at 0 and 90 percent semispan.

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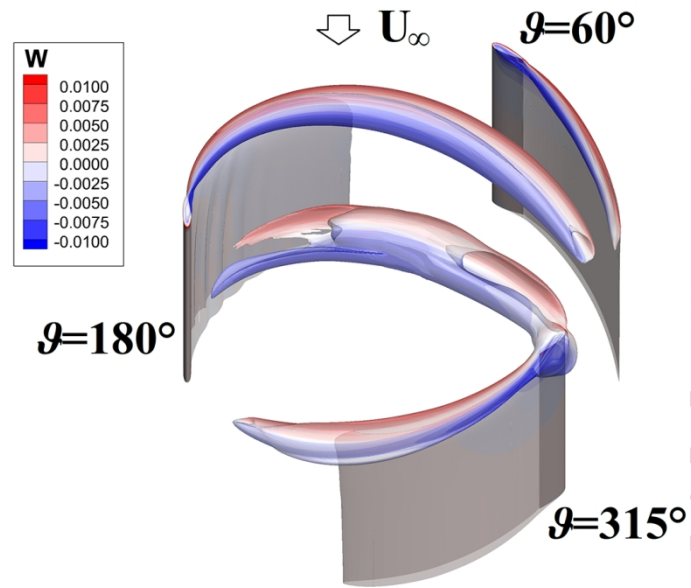
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Figure 13 - Isosurfaces of turbulent kinetic energy k colored with the contour of w .

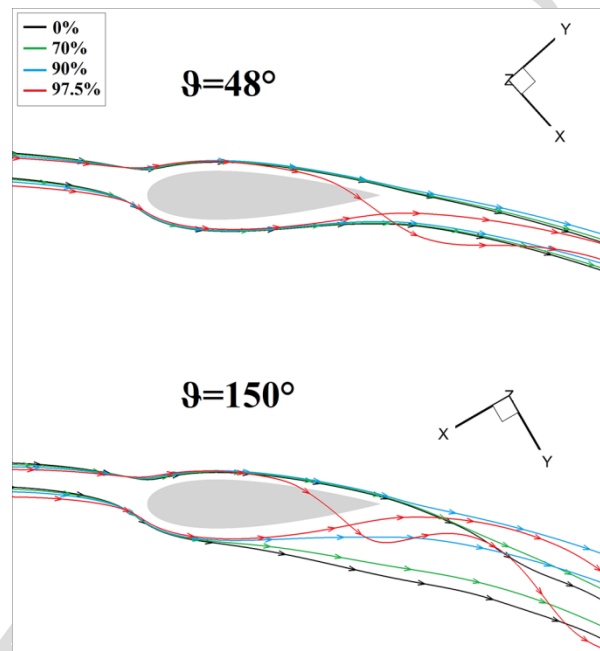
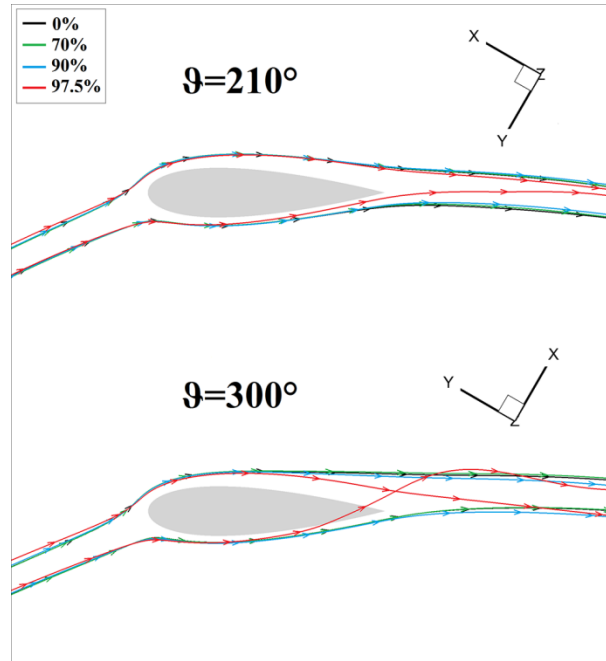
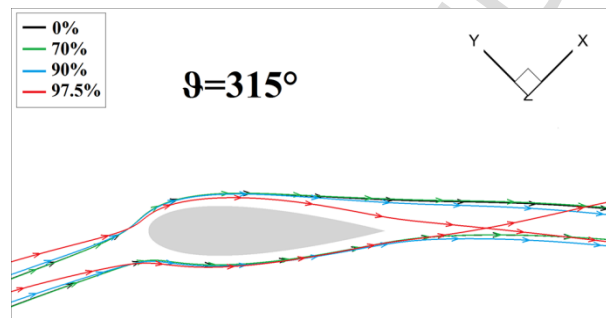
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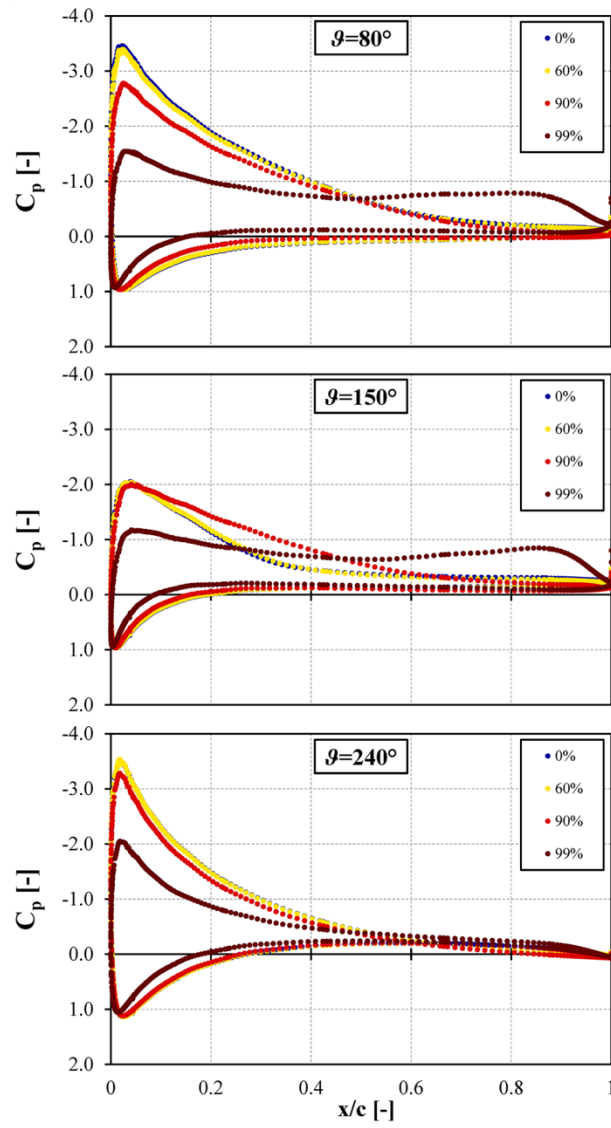
Figure 14- Streamlines at different span lengths: $\theta=48^\circ$ and $\theta=150^\circ$.



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Figure 15 - Streamlines at different span lengths: $\theta = 210^\circ$ and $\theta = 300^\circ$.



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Figure 16 - Streamlines at different span lengths: $\theta = 315^\circ$.



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Figure 17 - Pressure coefficient profiles at different span lengths: $\vartheta=80^\circ$, $\vartheta=150^\circ$ and $\vartheta=240^\circ$.

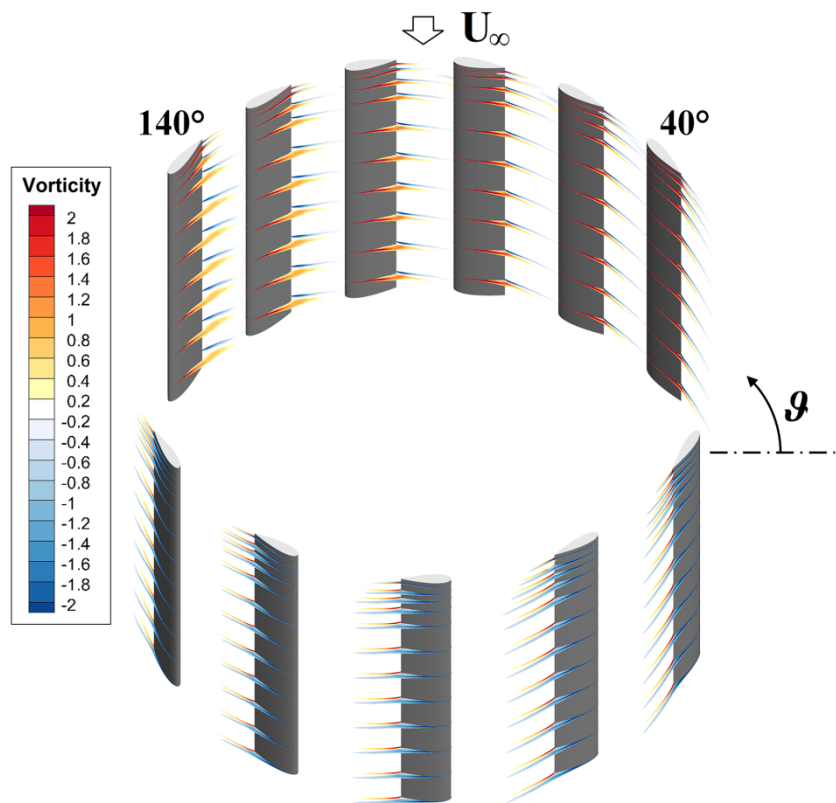
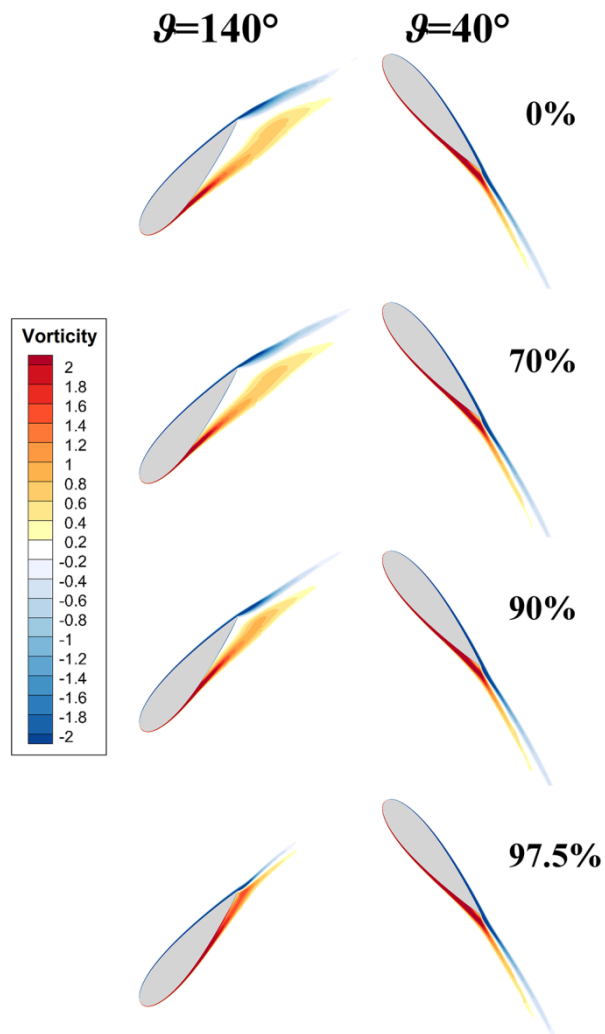


Figure 18 - Vorticity contours at different span lengths during the revolution.

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Figure 19 - Vorticity contours at different semispan locations: $\vartheta=40^\circ$ and $\vartheta=140^\circ$.