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# Outage Analysis of Cognitive Two-Way Relaying Networks with SWIPT over Nakagami- $m$ Fading Channels

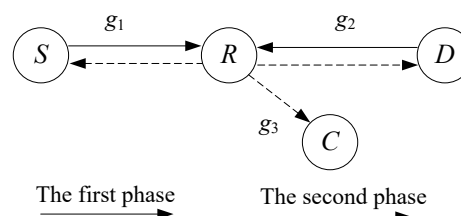
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Dear editor,

Simultaneous wireless information and power transfer (SWIPT) technique has attracted the attention of the research community, since it enables the wireless nodes to continually acquire energy from external sources in energy-limited environments [1–3]. In [4], the SWIPT was utilized in a cognitive two-way relaying network as an efficient means to improve energy and spectral efficiency. Assuming Rayleigh fading, the authors in [4] investigated the outage probability (OP) performance. It is well known that Nakagami- $m$  fading can well characterize the wireless propagation channel in many practical cases and span a wide range of fading scenarios via the  $m$  parameter, including the one-sided Gaussian distribution ( $m = 0.5$ ) and Rayleigh fading ( $m = 1$ ) as special cases. In our contribution, we generalize the analysis of [4] to the more general Nakagami- $m$  channel model. Specifically, we derive the exact expressions on OP for the two primary users and a tight upper bound on OP for the secondary user. It is shown that for the special case of  $m = 1$ , i.e., when the Nakagami- $m$  distribution becomes Rayleigh, our derived analytical expressions sim-

ply to the previously known expressions in [4]. Simulations are also performed to verify the correctness of our theoretical analysis.



**Figure 1** System model.

*System Model.* As illustrated in Figure 1, we consider a two-way cognitive amplify-and-forward (AF) relaying network, with two primary users,  $S$  and  $D$ , and two secondary users,  $R$  and  $C$ . Node  $R$  has its own information to broadcast to  $C$  and also acts as a relay to assist primary transmission. Assume that the two primary users  $S$  and  $D$  have fixed power supply,  $P_S$ , but no energy is provided to relay  $R$ . The whole communication takes place in two phases. In the first phase,  $S$  and  $D$  transmit their information to  $R$  simultaneously. In the

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second phase,  $R$  harvests energy from the part of its received signal from  $S$  and  $D$ , and employs the harvested energy to deliver the resulting information with a power gain, along with the message intended for  $C$ . It is also assumed that  $S$  and  $D$  can successfully decode the interference from secondary transmission.

Let  $g_1$ ,  $g_2$  and  $g_3$  represent the channel coefficients in  $S \leftrightarrow R$ ,  $R \leftrightarrow D$  and  $R \leftrightarrow C$  links, respectively. Since all channels undergo Nakagami- $m$  fading,  $|g_j|^2$  follows the Gamma distribution with fading parameter  $m_j$ , and mean power  $\Omega_j$ . Assuming the integer values of  $m_j$ , the probability density function (PDF) and cumulative distribution function (CDF) of  $|g_j|^2$  are given as [5]

$$f_{|g_j|^2}(x) = \frac{m_j^{m_j}}{\Omega_j^{m_j} (m_j-1)!} x^{m_j-1} \exp\left(-\frac{m_j}{\Omega_j} x\right), \quad (1)$$

and

$$F_{|g_j|^2}(x) = 1 - \exp(-m_j x / \Omega_j) \sum_{k=0}^{m_j-1} \frac{(m_j x / \Omega_j)^k}{k!}, \quad (2)$$

respectively, where  $j = 1, 2, 3$ .

The instantaneous signal-to-noise ratios (SNRs) at  $S$ ,  $D$ , and  $C$  can be expressed as [4]

$$\gamma_S = \frac{\alpha \eta \lambda (1-\lambda) P_S |g_2|^2 |g_1|^2}{\alpha \eta \lambda |g_1|^2 \sigma_0^2 + (1-\lambda) \sigma_0^2}, \quad (3)$$

$$\gamma_D = \frac{\alpha \eta \lambda (1-\lambda) P_S |g_2|^2 |g_1|^2}{\alpha \eta \lambda |g_2|^2 \sigma_0^2 + (1-\lambda) \sigma_0^2}, \quad (4)$$

and

$$\gamma_C = \min \{\gamma_{S,C}, \gamma_{D,C}, \gamma_{C,C}\}, \quad (5)$$

respectively, where  $\sigma_0^2$  represents the additive noise power at all users,  $\lambda \in (0, 1)$  denotes the portion of information split for energy harvesting,  $\eta \in [0, 1]$  represents the energy conversion efficiency, and  $\alpha \in [0, 1]$  indicates the fraction of the harvested power to broadcast the remaining information. Besides,  $\gamma_{G,C}$ ,  $G \in \{S, D, C\}$ , denotes the instantaneous signal to interference plus noise ratio (SINR) at node  $G$  to decode secondary information intended for  $C$ . We have the following approximations, which are very accurate at high SNRs,

$$\gamma_{S,C} \approx \frac{\eta \lambda P_S |g_1|^2 (1-\alpha) (|g_1|^2 + |g_2|^2)}{\alpha \eta \lambda |g_1|^2 P_S |g_2|^2 + \sigma_0^2}, \quad (6)$$

$$\gamma_{D,C} \approx \frac{\eta \lambda P_S |g_2|^2 (1-\alpha) (|g_1|^2 + |g_2|^2)}{\alpha \eta \lambda |g_1|^2 P_S |g_2|^2 + \sigma_0^2}, \quad (7)$$

and

$$\gamma_{C,C} \approx \frac{(1-\alpha) \eta \lambda P_S |g_3|^2 (|g_1|^2 + |g_2|^2)}{\alpha \eta \lambda |g_3|^2 (P_S |g_1|^2 + P_S |g_2|^2) + \sigma_0^2}. \quad (8)$$

**Outage Probability Analysis.** The OP is the probability that the instantaneous SNR  $\gamma_G$  at user  $G$  falls below a predefined threshold  $t_G$ , i.e.,  $P_{\text{out}}^G = \Pr(\gamma_G < t_G)$ ,  $G \in \{S, D, C\}$ .

**Theorem 1.** The OP at  $S$  and  $D$  are given in closed-form as

$$P_{\text{out}}^S = 1 - \frac{m_2^{m_2} \exp(-et_S m_2 / a \gamma \Omega_2)}{\Gamma(m_1) \Gamma(m_2) \Omega_2^{m_2}} \sum_{p=0}^{m_1-1} \sum_{k=0}^{m_2-1} \frac{1}{p!} \\ \times \left(\frac{et_S}{a \gamma}\right)^{m_2-1-k} \binom{m_2-1}{k} \left(\frac{m_1 t_S}{a \gamma \Omega_1}\right)^p \\ \times \left(\frac{4m_1 t_S \Omega_1}{m_2 a \gamma \Omega_2}\right)^{\frac{n}{2}} K_n \left(\sqrt{\frac{4m_1 t_S m_2}{a \gamma \Omega_1 \Omega_2}}\right), \quad (9)$$

and

$$P_{\text{out}}^D = 1 - \frac{m_1^{m_1} \exp(-et_D m_1 / a \gamma \Omega_1)}{\Gamma(m_1) \Gamma(m_2) \Omega_1^{m_1}} \sum_{p=0}^{m_2-1} \sum_{k=0}^{m_1-1} \frac{1}{p!} \\ \times \left(\frac{et_D}{a \gamma}\right)^{m_1-1-k} \binom{m_1-1}{k} \left(\frac{m_2 t_D}{a \gamma \Omega_2}\right)^p \\ \times \left(\frac{4m_2 t_D \Omega_2}{m_1 a \gamma \Omega_1}\right)^{\frac{n}{2}} K_n \left(\sqrt{\frac{4m_1 t_D m_2}{a \gamma \Omega_1 \Omega_2}}\right), \quad (10)$$

respectively, where  $a = \alpha \eta \lambda$ ,  $b = (1-\alpha) \eta \lambda$ ,  $e = \alpha \eta \lambda / (1-\lambda)$ ,  $\gamma = P_S / \sigma_0^2$ ,  $n = k - p + 1$  and  $K_n(\cdot)$  is the modified Bessel function of the second kind and order  $n$  [6, Eq. (8.407)].

*Proof.* Define  $X = |g_1|^2$ ,  $Y = |g_2|^2$ . We have

$$F_{\gamma_S}(t_S) = \int_0^\infty \Pr\left(y < \frac{t_S e x + t_S}{a \gamma x}\right) f_X(x) dx. \quad (11)$$

Substituting (1) and (2) into (11), and employing [6, Eq.(3.471.9)], (9) is obtained. Similarly, (10) can be obtained.

**Theorem 2.** The OP at  $C$ ,  $P_{\text{out}}^C = 1$ , if  $\tau_C \triangleq 1/t_C \leq a/b$ . Otherwise,  $P_{\text{out}}^C$  can be upper bounded as

$$P_{\text{out}}^C \leq 1 - B \sum_{i=1}^2 \sum_{k=0}^{m_3-1} \sum_{p=0}^{m_i-1} \sum_{q=0}^{p-k} \sum_{r=0}^{p-k-q} (-1)^{m_i-1+p-r} \\ \times \frac{(m_3-1)!}{k!} \binom{m_i-1}{p} \binom{p-k-q}{r} \left(\frac{1}{b \tau_C - a}\right)^{k-p+k-q} \\ \times \binom{p-k}{q} \left(\frac{\Omega_3-1}{m_3}\right)^{m_3-k} \left(\frac{1}{\gamma}\right)^{p-r+1} a^r S_i \zeta_i \\ - B \sum_{k=0}^{m_3-1} \sum_{p=0}^{m_2-1} \sum_{q=0}^{p-k} \sum_{r=0}^{p-k-q} \sum_{s=0}^{m_2+m_1-2-p+r} \frac{(m_3-1)!}{k!} \\ \times \binom{m_2-1}{p} (-1)^{m_2-1-p} \left(\frac{1}{\gamma}\right)^{2p+k-q+1} \left(\frac{1}{b \tau_C - a}\right)^k \\ \times \binom{p-k}{q} \binom{p-k-q}{r} \exp\left(-\frac{m_2 X_0}{\Omega_2} - \frac{m_1 X_0}{\Omega_1}\right) \\ \times \left(\frac{\Omega_3-1}{m_3}\right)^{m_3-k} X_0^{p-k-q-r+s} \frac{(A+r)!}{r!} \left(\frac{\Omega_1}{m_1}\right)^{A+1-r}$$

$$\times S_2 \quad (12)$$

where

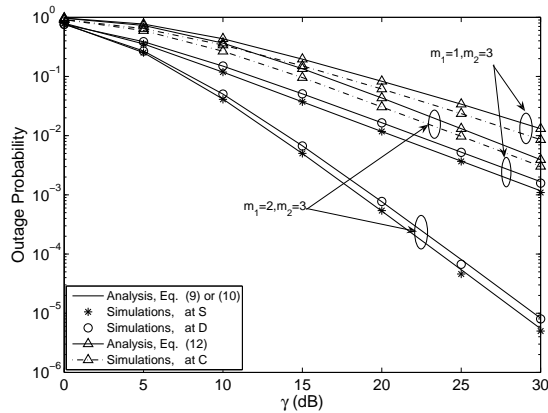
$$\zeta_i = \left( \frac{m_i}{\gamma \Omega_i (b \tau_C - a)} \right)^{\frac{A}{2}} X_0^{\frac{2+A}{2}} \exp \left( -\frac{m_i}{2\gamma \Omega_i (b \tau_C - a)} X_0 \right) \times W_{-\frac{2+A}{2}, \frac{A+1}{2}} \left( \frac{m_i}{\gamma \Omega_i (b \tau_C - a)} X_0 \right), \quad (13)$$

$A = m_i + m_3 - 2 - p$ ,  $X_0 = \sqrt{1/\gamma/(2b\tau_C - a)}$ , with  $W_{f,h}(z)$  denoting the Whittaker function [6, Eq. (9.22)], and

$$S_i = 2 \left( \frac{\gamma m_3 \Omega_i}{2 \Omega_3 (b \tau_C - a) m_i} \right)^{\frac{v}{2}} K_v \left( \sqrt{\frac{2 m_3 m_i}{\gamma \Omega_3 (b \tau_C - a) m_i}} \right), \quad (14)$$

where  $v = q + 1$  and  $B = \frac{m_1^{m_1} m_2^{m_2} m_3^{m_3}}{\Omega_1^{m_1} \Omega_2^{m_2} \Omega_3^{m_3} \Gamma(m_1) \Gamma(m_2) \Gamma(m_3)}$ .  
**Proof.** Refer to Appendix A.

Note that for the case of Rayleigh fading, i.e.,  $m_1 = m_2 = m_3 = 1$ , Eqs. (9), (10) and (12) reduce to [4, Eq. (23)], [4, Eq. (24)] and [4, Eq. (27)], respectively.

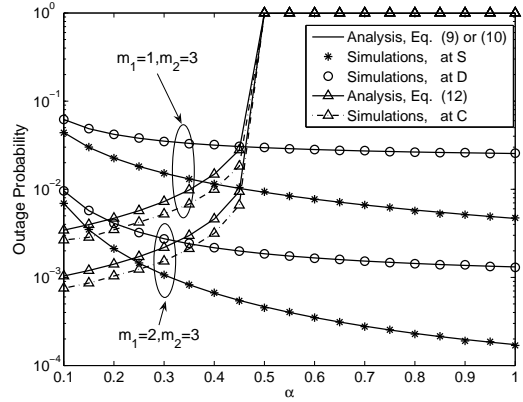


**Figure 2** OP against the  $\gamma$ , when  $\Omega_1 = \Omega_2 = \Omega_3 = 8$  dB,  $\eta = 1$ ,  $\lambda = 0.25$ ,  $\alpha = 0.24$ , and  $t_S = t_D = t_C = 3$ .

**Performance evaluation results.** Figure 2 shows the OP at  $S$ ,  $D$  and  $C$  against  $\gamma$  when fading parameter  $m_1 = m_3 = 1, m_2 = 3$  and  $m_1 = m_3 = 2, m_2 = 3$ . From it, we observe that the numerical results obtained from (9) and (10) perfectly match well with simulations. Besides, the analysis results of (12) we obtained are very close to the simulation results. Figure 3 depicts the OP against power split coefficient  $\alpha$  when fading parameter  $m_1 = m_3 = 1, m_2 = 3$  and  $m_1 = m_3 = 2, m_2 = 3$ . It can also be found that our analysis results of (9), (10) and (12) show a good agreement with their corresponding simulated ones. Besides, it can be seen that with the increase of  $\alpha$ , the OP of primary users become smaller while the OP of secondary user increases, which is consistent with the definition of  $\alpha$ . In addition, Figures 2 and 3 demonstrate under the SWIPT protocol, although there is no

extra power provided for node R, the system can also acquire a reasonable OP performance.

**Conclusion.** Exact expressions for the OP of primary users and a tight upper bound on the OP of secondary user for a two-way cognitive AF relaying system operating over Nakagami- $m$  fading channels and employing SWIPT have been derived. Numerical results accompanied with monte-carlo simulations have verified the accuracy of the proposed mathematical analysis.



**Figure 3** OP against  $\alpha$ , when  $\Omega_1 = \Omega_2 = \Omega_3 = 4$  dB,  $\gamma = 20$  dB,  $\eta = 1$ ,  $\lambda = 0.65$ ,  $t_S = t_D = 3$ , and  $t_C = 1$ .

**Supporting information** Appendix A. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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