# NSF-STU Workshop on Adaptive Control

Hägglund, Tore

1985

*Document Version:*
Publisher's PDF, also known as Version of record

Link to publication

*Citation for published version (APA):*
Hägglund, T. (Ed.) (1985). *NSF-STU Workshop on Adaptive Control*. (Research Reports TFRT-3175). Department of Automatic Control, Lund Institute of Technology (LTH).

*Total number of authors:*
1



LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

CODEN:LUTFD2/(TFRT-3175)/1-175/(1985)

# *NSF-STU Workshop*

# *on*

# *Adaptive Control*

9 - 11 July 1984

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY
APRIL 1985

# NSF-STU Workshop on Adaptive Control

Department of Automatic Control
Lund Institute of Technology
Lund Sweden

9 - 11 July 1984

Edited by Tore Hägglund

LUND INSTITUTE OF TECHNOLOGY
DEPARTMENT OF AUTOMATIC CONTROL
Box 725
S 220 07 Lund 7 Sweden

Document name
Final report

Date of issue
April 1985

Document number
CODEN:LUTFD2/(TFRT-3175)/1-175/(1985)

Supervisor

Sponsoring organization
The National Science Foundation (NSF)
The Swedish Board for Technical Development (STU)

Author(s)

Tore Hägglund

Title and subtitle

NSF-STU Workshop on Adaptive Control

Abstract

The National Science Foundation (NSF) and the Swedish Board for Technical Development (STU) have signed an agreement for cooperation. Both agencies are currently supporting research in adaptive control. Within this framework, a workshop was held at the Department of Automatic Control at the Lund Institute of Technology, Lund, Sweden, on July 9-11 1984. This report contains abstracts, copies of the viewgraphs and a summary of the discussions.

Key words

Classification system and/or index terms (if any)

Supplementary bibliographical information

ISSN and key title

ISBN

Language
English

Number of pages
175

Security classification

Recipient's notes



Distribution: The report may be ordered from the Department of Automatic Control or borrowed through the University Library 2, Box 1010, S-221 03 Lund, Sweden, Telex: 33248 lubbis lund.

# Introduction

The National Science Foundation (NSF) and the Swedish Board for Technical Development (STU) have signed an agreement for cooperation. Both agencies are currently supporting research in adaptive control. The purpose of the seminar was to bring the research in the supported projects together to provide a perspective of the field, its accomplishments and deficiencies and to find directions for future research.

The laboratory for Information and Decision Systems at the Massachusetts Institute of Technology, directed by Professor Sanjoy Mitter, and the Department of Automatic Control at Lund Institute of Technology, Lund, Sweden, directed by Professor Karl Johan Åström, have long maintained close informal contact on research problems of common interest. There is interest on both sides in formalizing this arrangement so that exchanges of faculty, staff (and possibly students) can take place. The workshop on Adaptive Control was organized within this framework.

The workshop program was discussed with Dr. Abraham Haddad, former Program Director of the Systems Theory and Operations Research Program of the National Science Foundation and Mr Ove Berkefelt Program Director for at the Swedish Board of Technical Development. They were in agreement with the aims and objectives of the workshop.

The workshop was held at the Department of Automatic Control at the Lund Institute of Technology, Lund, Sweden, on July 9 - 11, 1984 this week followed the IFAC Congress in Budapest.

The workshop was informal. There were 14 US and 20 Swedish participants. The formal presentations covered industrial needs and experiences, applications, stability theory, system identification, stochastic adaptive control, unmodeled dynamics and new directions as well as many informal discussions. The workshop was viewed very favourable from the participants who found it a stimulating intellectual experience.

A brief assessment of the field can be summarized as follows. There has been research in adaptive control for at least 30 years. The field is, however, still not in very good shape. There is a proliferation of ideas and techniques, and a lack of coherence. Recently there has, however, been some limited theoretical results. Adaptive techniques are also starting to be used in microprocessor based controllers. It appears as a good research field because theoretical results are badly needed to get insight, to structure the problem, and to unify the field. There is also a considerable industrial interest to use adaptive techniques in many different fields. Several new products have recently been announced. There are several strong research groups in the field, both in the United States and in Europe. The theoretical aspects have been emphasized in U.S. research. In Europ the theoretical research has however also been blended with practice. There are new application areas emerging, e.g. in robotics.

This report contains abstracts, copies of the viewgraphs and a summary of the discussions.

## Dedication

This report is dedicated to the memory of Dr. Howard Elliott a prominant researcher in adaptive control. He contributed significantly to the success of the workshop which was the last formal meeting in which he participated.

# Program

MONDAY - APPLICATIONS

9.00 Introduction and Welcome

Berkefelt
Cooperation between NSF and STU

9 - 12 Industrial Products

Egardt
The ASEA-Novatune system

Bengtsson
Experiences with the ASEA-Novatune

Åström
Automatic tuning of simple regulators

Bååth
The NAF - Autotuner

13 - 15 Applications

Stein
History and issues in adaptive flight control

Olsson, Rundqwist
Self-tuning control of dissolved oxygen concentration in activated sludge systems

Elliott
Adaptive pole placement for robots and servomechanisms

15 - 17 Discussion

Where do we stand with respect to applications? What algorithms are being used? What things work? What are the difficulties? What tricks are used?

Sternby
Some desirable features of industrial adaptive controllers

## TUESDAY - THEORY

8 - 10 Stability Theory

Wittenmark
Self-tuning regulator with increased prediction horizon

Morse
A universal control capable of stabilizing any single-input, single-output, minimum phase linear system of relative degree $\leq 2$

Byrnes
Adaptive stabilization of linear multivariable systems

Johansson
Lyapunov functions, cost functions and adaptive control

10 - 12 System Identification

Söderström
Instrumental variable methods for systems operating in closed loop with application to adaptive control

Solo
Adaptive spectral factorization

Ljung
Frequency domain properties of identified transfer functions

13 - 15 Stochastic Adaptive Control

Varaya
Multi-armed bandits

Kumar
On self-tuning to the optimal controller

Millnert
A comparison of some control strategies for systems with fast parameter variations

Hägglund
Recursive estimation of slowly time-varying parameters

15 - 17 Discussion
Where does the theory stand? What results are needed? Do the results cover the problems brought up by practice? What problems can we hope to solve?

## WEDNESDAY - ROBUSTNESS AND NEW DIRECTIONS

8 - 11 <u>Unmodeled Dynamics and Robustness</u>

Kokotovic
Robustness of (MRAS) adaptive control

Sastry
Parameter convergence in model reference adaptive control and its impact on robustness

Trulsson
On adaptive control with prescribed robustness properties

Rohrs
On living with the positive real condition

Bertsekas
Distributed asynchronous algorithms for deterministic and stochastic optimization

11 - 12 <u>Demonstration of Control Laboratory</u>

13 - 15 <u>New Directions</u>

Åström
Expert Control

Årzén
Experiments with Expert Control

15 - 17 <u>Discussion</u>
Future directions.

## List of Participants

| | |
|---|---|
| Gunnar Bengtsson | ASEA |
| Ove Berkefelt | STU |
| Dimitri Bertsekas | MIT |
| Torsten Bohlin | KTH |
| Christopher Byrnes | Harvard |
| Lars Bååth | NAF Controls |
| Bo Egardt | ASEA |
| Howard Elliott | UMass |
| Gene Franklin | Stanford |
| Ove Gradin | KTH |
| Ivar Gustavsson | ASEA |
| Per Hagander | LTH |
| Tore Hägglund | LTH |
| Rolf Johansson | LTH |
| Petar V Kokotovic | Illinois |
| P R Kumar | Maryland |
| Lennart Ljung | LiTH |
| Mille Millnert | LiTH |
| Sanjoy K Mitter | MIT |
| Stephen Morse | Yale |
| Gustaf Olsson | LTH |
| Charles Rohrs | Notre Dame |
| Lars Rundqwist | LTH |
| Shankar Sastry | Berkeley |
| Victor Solo | Purdue |
| Gunter Stein | MIT |
| Jan Sternby | Gambro |
| Torsten Söderström | Uppsala |
| Eva Trulsson | LiTH |
| Pravin Varaya | Berkeley |
| Björn Wittenmark | LTH |
| George Zames | McGill |
| Karl-Erik Årzén | LTH |
| Karl Johan Åström | LTH |

## Technical secretaries

| | |
|---|---|
| Anders Ahlén | Uppsala |
| Jan Peter Axelsson | LTH |
| Mats Lilja | LTH |
| Bengt Mårtensson | LTH |
| Per Persson | LTH |
| Mikael Sternad | Uppsala |
| Hussein Youlal | LTH |
| Lars Pernebo | Alfa Laval |
| Ulf Hagberg | Alfa Laval |

# Cooperation between NSF and STU

Ove Berkefelt

STU
Stockholm

Nearly three years ago NSF and STU reached a general agreement on cooperation in research. This agreement covers all scientific areas where STU and NSF are funding research projects.

For a small country like Sweden with limited resources in personnel and money it is of course difficult to find areas where our research is of adequate level and size compared to US. I am therefore very pleased to note that we have recently managed to arrange workshops in two areas within the field of electronics, computers and systems sciences. About two months ago we had a joint NSF-STU workshop on computer based vision and this week we will have this workshop on adaptive control.

It is interesting to compare these two areas. Computer based vision is a new science. By a generous budget we have managed to create a good scientific level in a short time, approximately 5 years.

Adaptive control on the other side has been built up gradually during a long time and I would say more thanks to excellent and devoted researchers than to a generous budget. In any case we have in adaptive control enough research results, enough researchers and enough applications to attract the interest from NSF and US researchers and this is in a time when, as I understand it, NSF is putting heavier conditions on research cooperation with other countries than earlier.

The purpose of this workshop is apart from the exchange of research results between the two countries to find out whether adaptive control is an area where we can find possibilities for future cooperation. A joint US-Swedish project on adaptive control would have a great chance to get funding at least from STU. Unfortunately my counterpart from NSF is not here so that we can hear NSF's opinion.

This workshop happens to be very suitably located in time. We are at present at STU planning a new national program in information technology. We are trying to establish which areas in systems and computer sciences where we can compare with other countries and where scientific results are likely to lead to industrial applications and progress. I hope this workshop will help in this respect.

Let me end by saying that I hope that our American guests will have a pleasant time at Lund for some days and that this workshop will lead to deeper contacts between Swedish and US researchers and to a future more or less formalised cooperation.

# The ASEA-Novatune system

Bo Egardt

ASEA AB
Västerås, Sweden

Egardt gave an overview of the system hardware, containing process interfaces etc.

The application program is written in a block oriented language. This is the industrial control engineers look at processes and control. Besides the selftuning regulator the language contains arithmetics, logics and other functions like PID.

The signal types in the language are integer and real, and there are modules like selectors, arithmetic operations, logic, delays, interface modules, filters, regulators etc.

All modules are available in the system library. The program is entered via a simple hand terminal or a standard terminal in a laboratory or at the installation site.

The programmer selects sampling intervals and priorities for the different control tasks. Except standard clock interrupt it's possible to use software interrupts or pulse counters to determine the sampling instants.

The system contains three different adaptive regulators built around the same algorithm. The difference between the regulators is the degree of flexibility offered to the user. The regulators are

| | |
|---|---|
| STAR1 | Basic, least complex regulator |
| STAR2 | Medium complex regulator |
| STAR3 | Complex regulator |

STAR3 is the most frequently used regulator. STAR2 and STAR3 both contain feedforward, but in STAR3 the number of parameters in the control law is selected by the user.

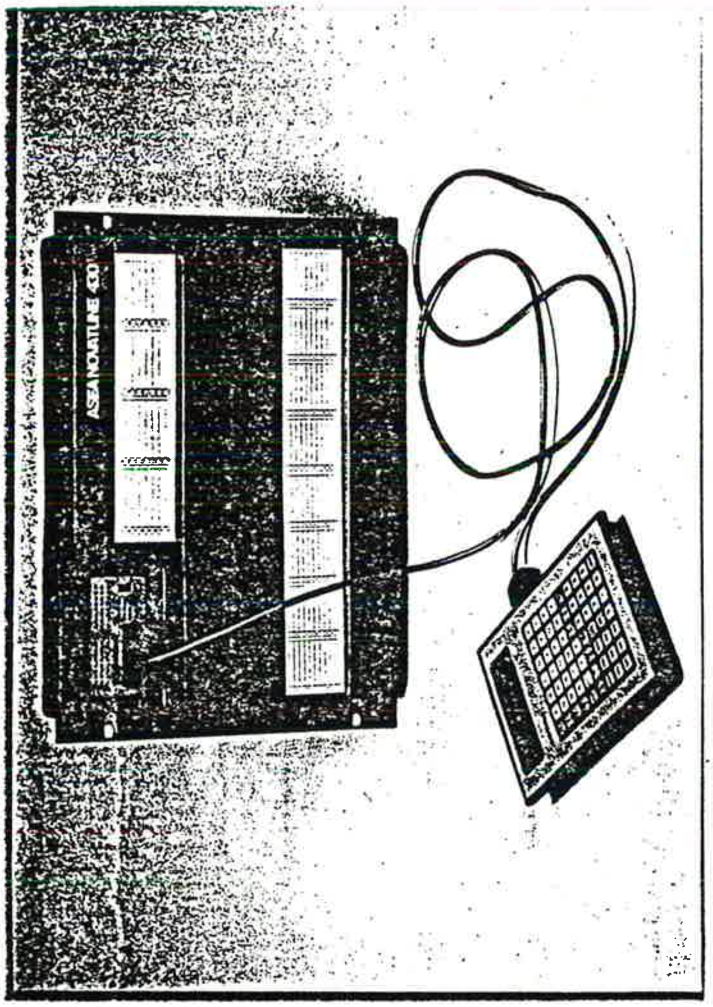
The signals connected to the STAR3 module are the manual control signal, the feedback signal (process output), the reference value and the feedforward signal. The control value can be given both absolute and incremental limits.

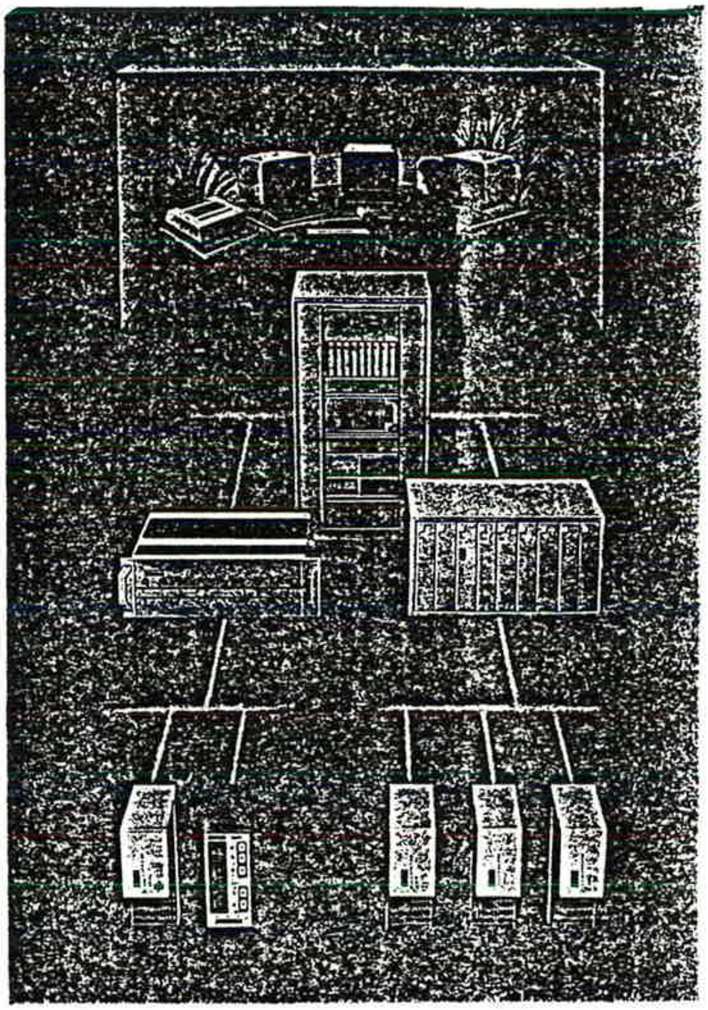
Integer signals determine the mode of the regulator, i.e. adaptation can be switched off, saved parameters can be restored etc. Unconnected inputs are given default values.

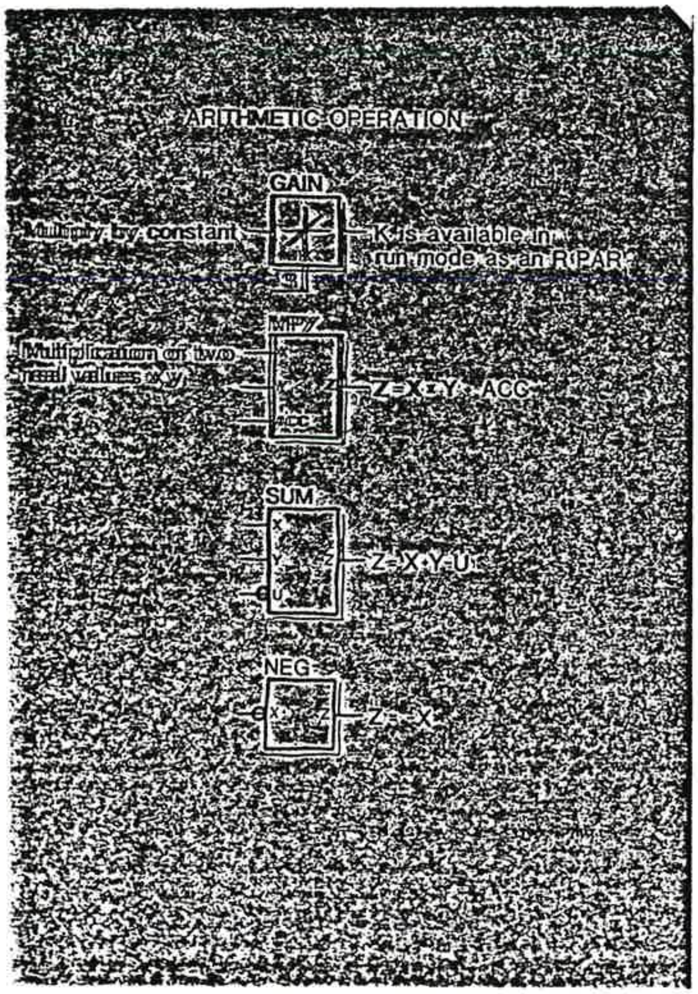
The regulator uses a minimum variance algorithm with least squares identification. Besides, one closed loop pole can be defined, the prediction horizon and the sampling interval can be chosen, and the integral action can be switched on and off. A penalty can be introduced on the control output.

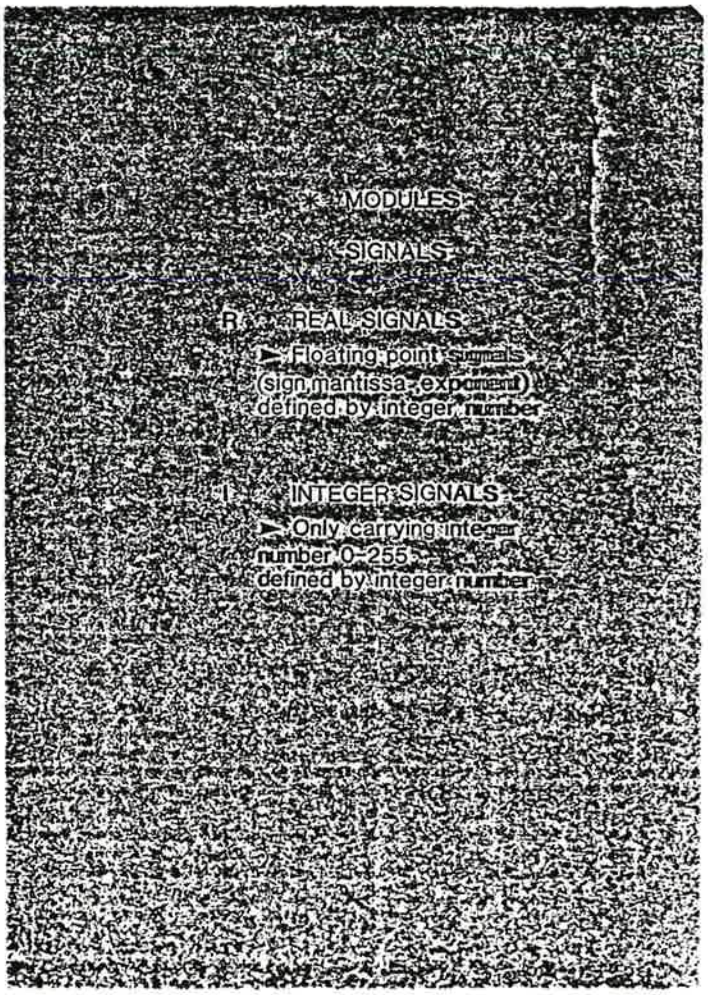
Reference:

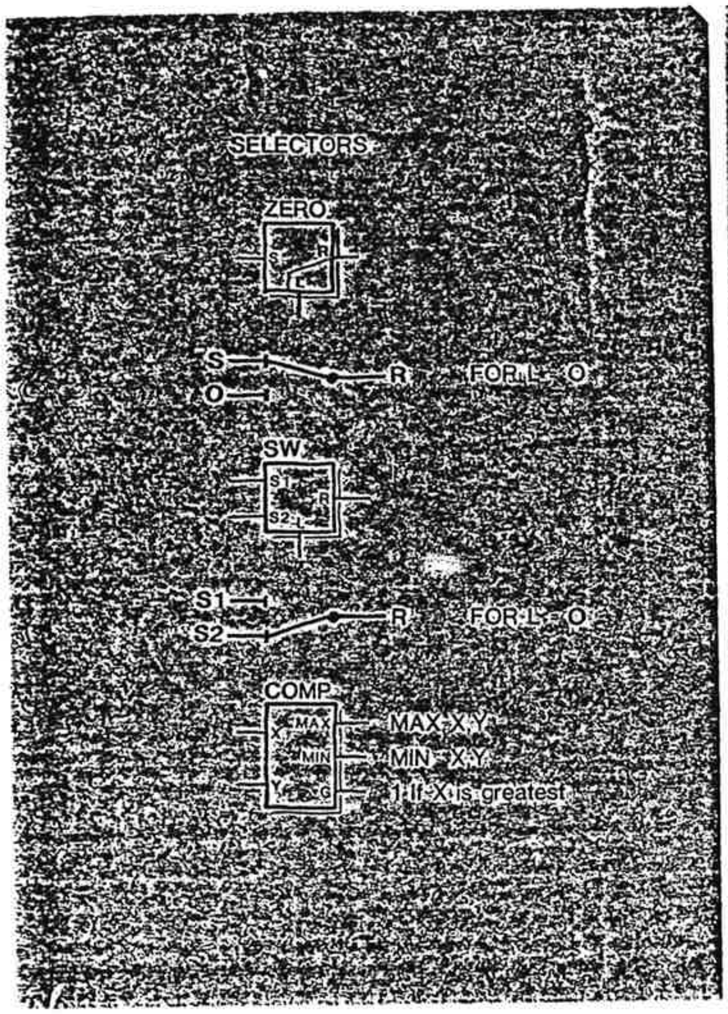
Bengtsson G, Egardt B (1984): Experiences with Self-Tuning Control in the Process Industry. Preprints of IFAC 9th World Congress, Budapest, Hungary, vol XI, 132-140.

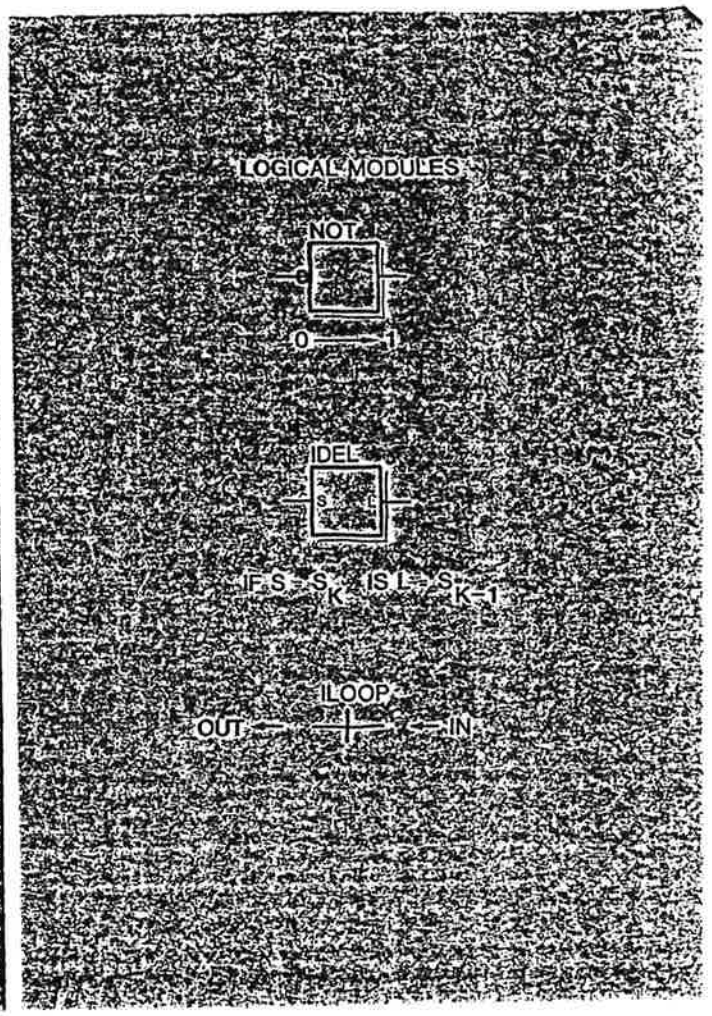

ASEA NOVATUNE 400

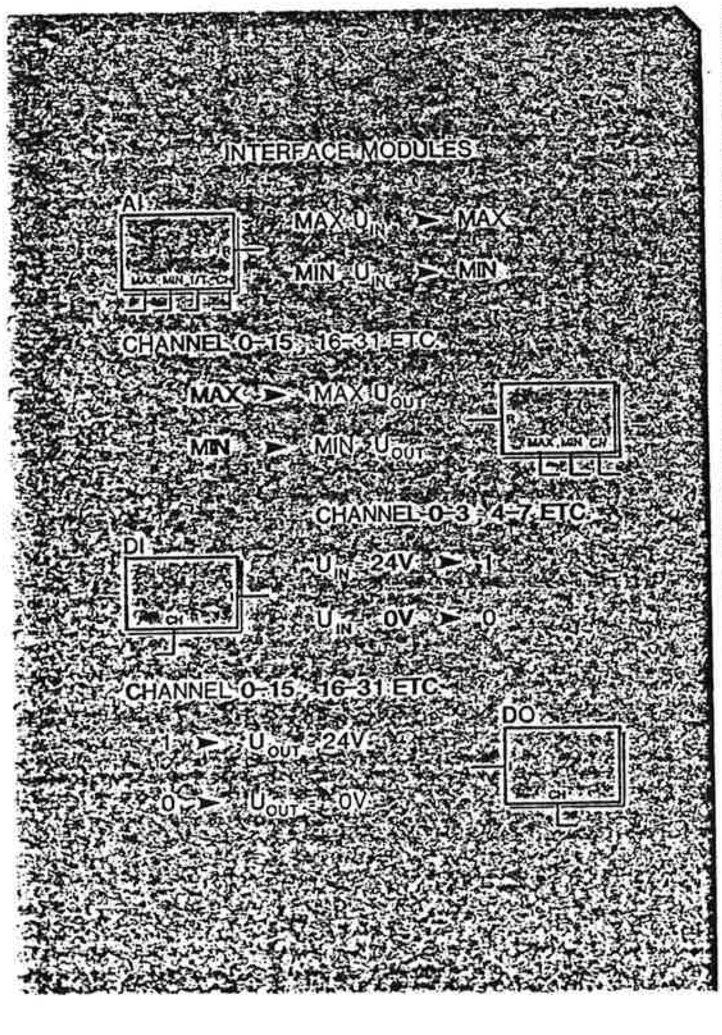


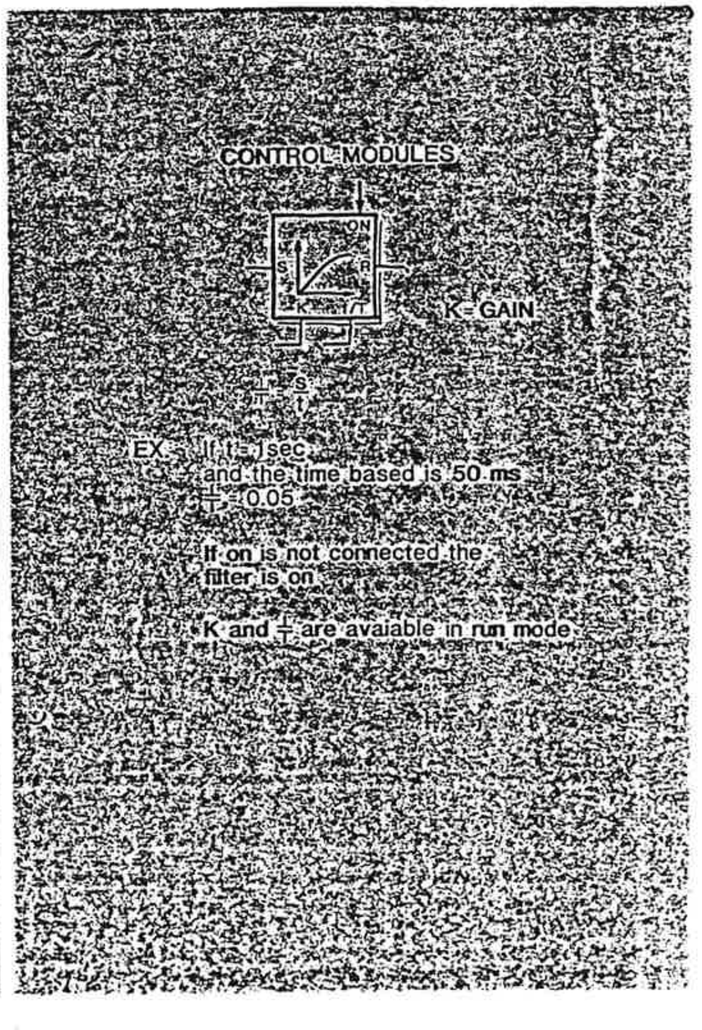

ARITHMETIC OPERATION
GAIN
Multiply by constant
K is available in
run mode as an R-PAR
MPZ
Multiplication of two
real values x,y
Z=X·Y·ACC
SUM
Z=X+Y-U
NEG
Z=-X

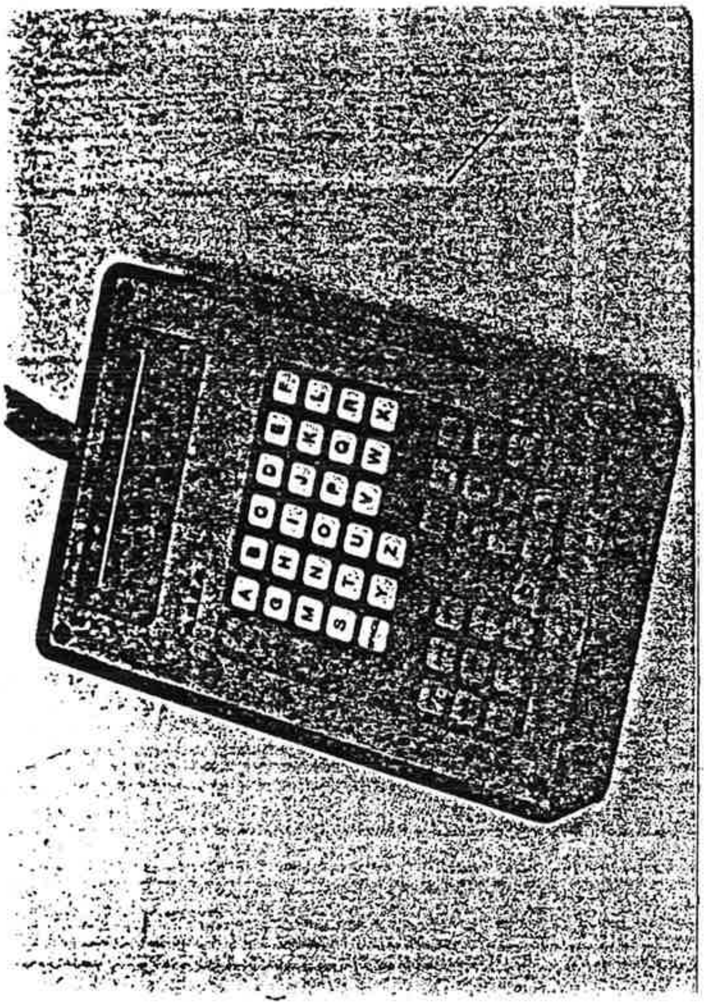

* MODULES
SIGNALS
R REAL SIGNALS
► Floating point signals
(sign,mantissa, exponent)
defined by integer number
I INTEGER SIGNALS
► Only carrying integer
number 0-255
defined by integer number


SELECTORS
ZERO
S2
R
L
S
O
R
FOR L = O
SW
S1
S2
R
L
S1
S2
R
FOR L = O
COMP
X
Y
MAX
MIN
G
MAX = X,Y
MIN = X,Y
1 If X is greatest


LOGICAL MODULES
NOT
e
0
1
IDEL
S
IF S = S K IS L = S K-1
ILOOP
OUT
IN


INTERFACE MODULES
AI
R
MAX, MIN, 1/T, CH
MAX U IN ➤ MAX
MIN U IN ➤ MIN
CHANNEL 0-15, 16-31 ETC.
MAX ➤ MAX U OUT
MIN ➤ MIN U OUT
MAX, MIN, CH
CHANNEL 0-3, 4-7 ETC.
DI
CH
U IN = 24V ➤ 1
U IN = 0V ➤ 0
CHANNEL 0-15, 16-31 ETC.
DO
CH
1 ➤ U OUT = 24V
0 ➤ U OUT = 0V


CONTROL MODULES
ON
S
R
K
1/T
K = GAIN
1/T = S/t
EX. If t = 1 sec
and the time based is 50 ms
1/T = 0.05
If on is not connected the
filter is on
K and 1/T are avaiable in run mode



## DEFINING A MODULE

| | |
|---|---|
| NEW | IDENT * |
| ① 1 | TYPE * |
| AO | R:IN * |
| ② 12 | R:PAR * |
| 5 | 2 * |
| 0 | I:PAR * |
| 0 | * |

① Module identity is only defined in a block I.E. 1 can be the first module even in block 2.

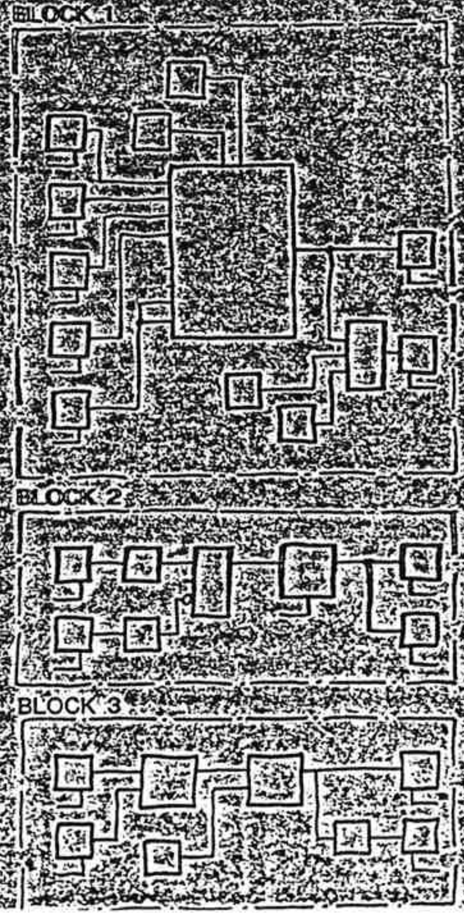
② Signals are general I.E. 1 can only be used in this block

## DEFINING A BLOCK

| | |
|---|---|
| PGM | > |
| ① NEW | IDENT> |
| 1 | TYPE> |
| ② TIME | PERIOD> |
| ③ 4 | PRIORITY> |
| ④ 1 | * |

| | | | |
|---|---|---|---|
| ① | NEW | ► | New block is defined |
| | OLD | ► | Allready defined block is asked for |
| | DEL | ► | Block is going to be deleted |
| ② | TIME | ► | Time clock exeuted block |
| | SC1 | ► | Soft clock 1 is executing the block (connected to sint module) |
| ③ | 4 | ► | 4 x base execution time |

## APPLICATION PROGRAM



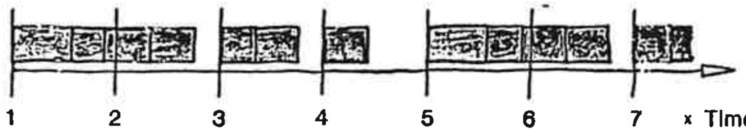
④ PRIORITY

EXAMPLE

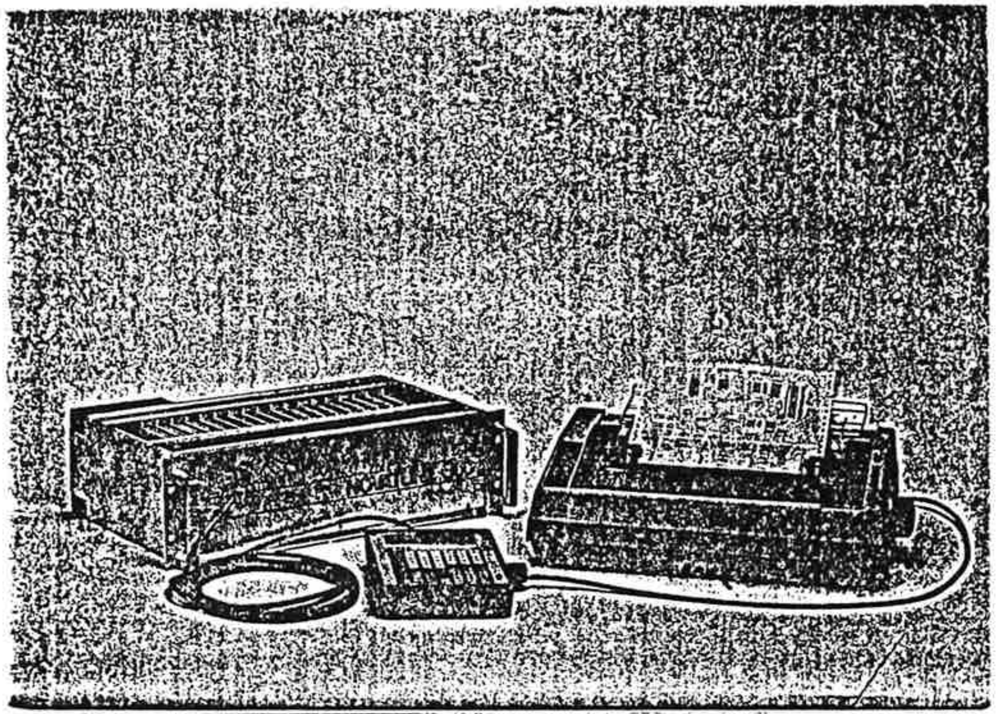
BLOCK 1 PRIORITY 1 PERIOD 4

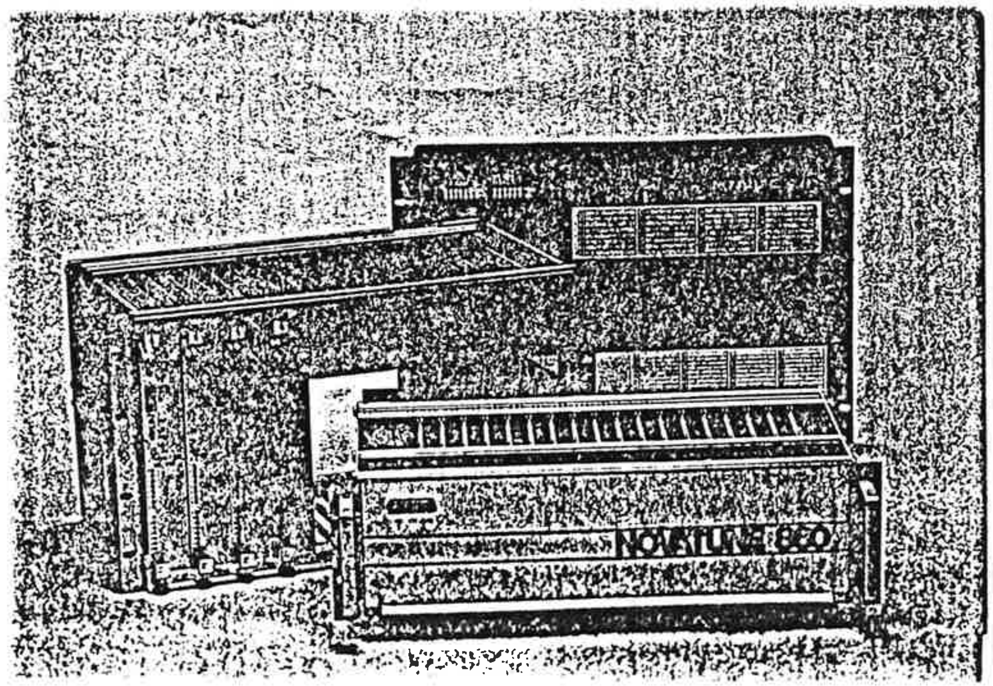
BLOCK 2 PRIORITY 2 PERIOD 2

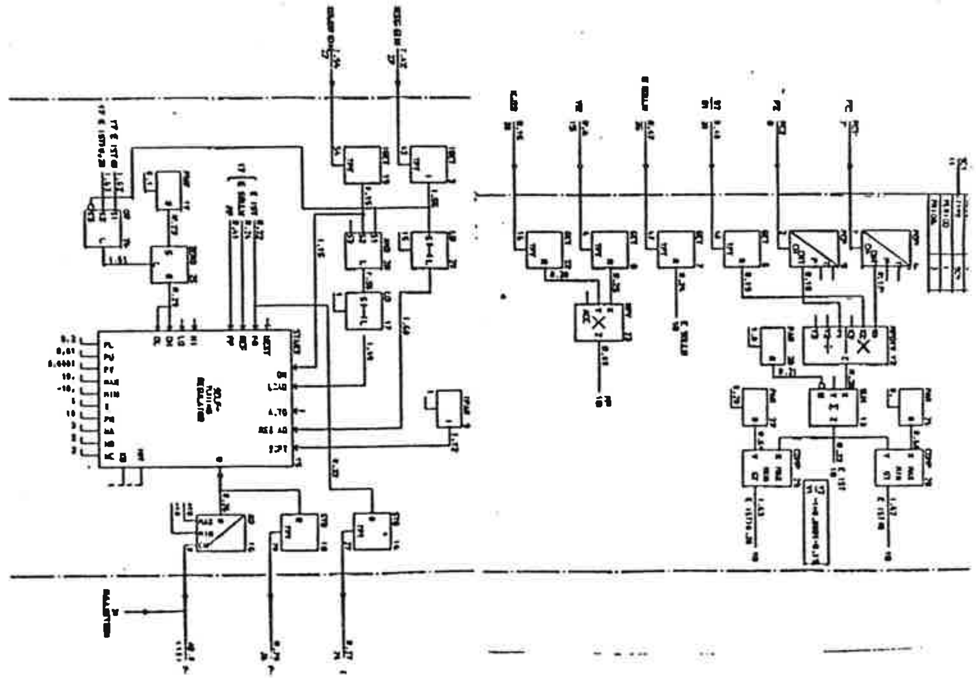
BLOCK 3 PRIORITY 3 PERIOD 1

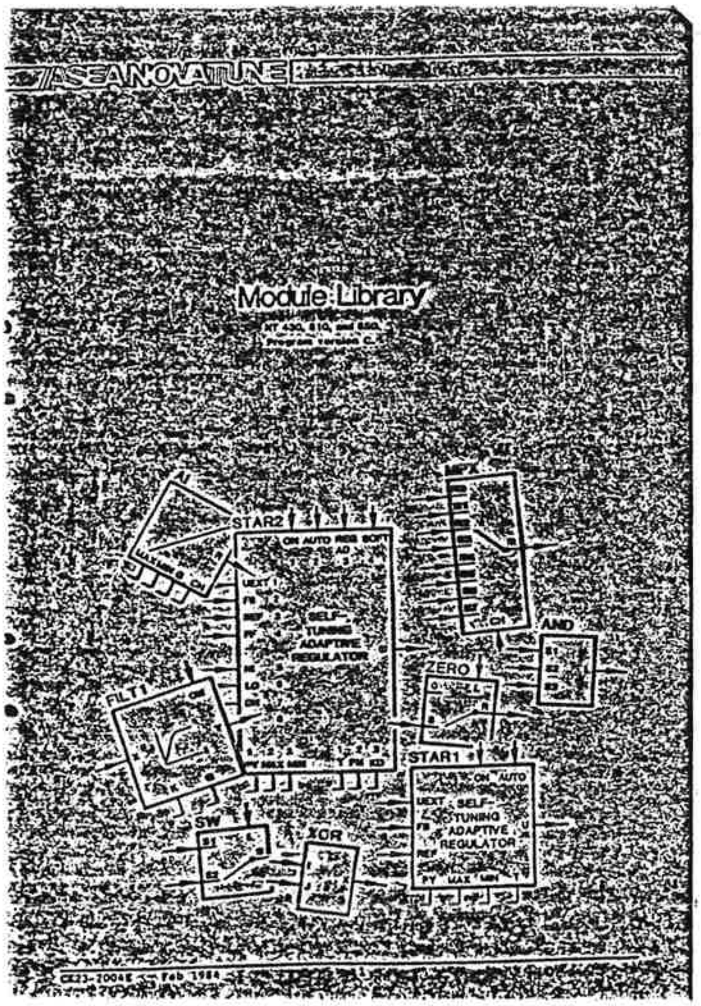


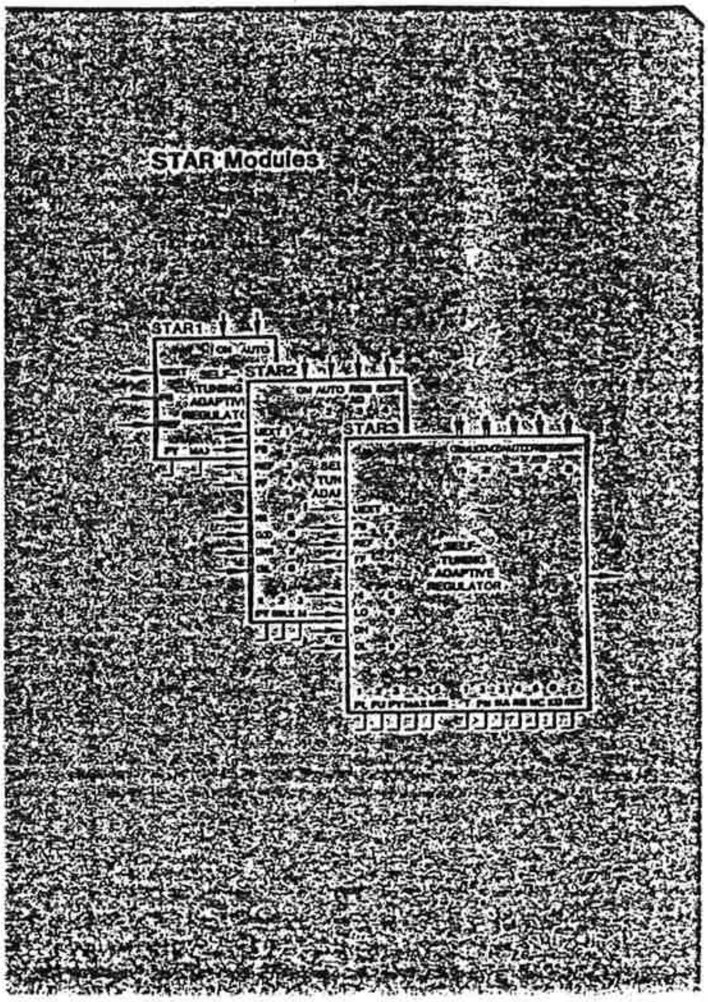

EXECUTION TIME

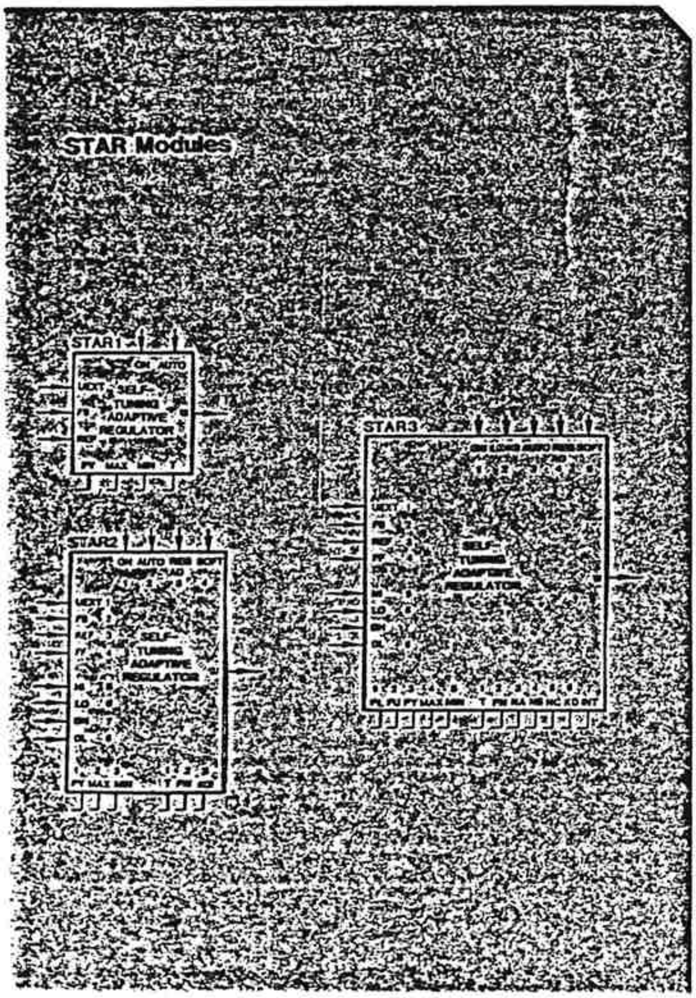


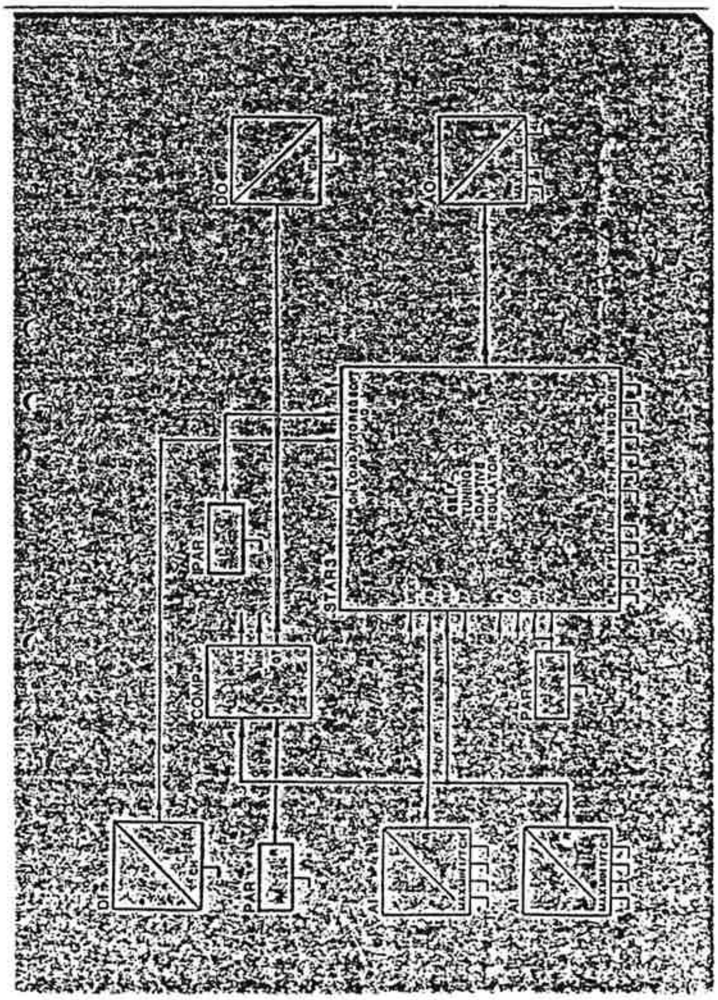





ASEA NOVATUNE
Module Library
STAR2
SELF-TUNING ADAPTIVE REGULATOR
ZERO
AND
STAR1
SELF-TUNING ADAPTIVE REGULATOR
SW
XOR


STAR Modules
STAR1
STAR2
STAR3
SELF-TUNING ADAPTIVE REGULATOR


STAR Modules
STAR1
SELF-TUNING ADAPTIVE REGULATOR
STAR2
SELF-TUNING ADAPTIVE REGULATOR
STAR3
SELF-TUNING ADAPTIVE REGULATOR



# Experiences with the ASEA-Novatune

Gunnar Bengtsson

ASEA AB
Västerås, Sweden

A number of feasibility studies for adaptive controls have been made at Asea during the past 10 years. The adaptive system Novatune was announced in 1982. A lot of experience has been gained during the past years use of it. About 70 systems are currently in operation. The systems cover a wide range of applications in paper mills, steel mills, boilers, waste water treatment, building automation, and looms. The installations of Novatunes is currently increasing rapidly. To do this it has been necessary to train a number of people in the commissioning of the system. A series of courses has been designed to give the proper background for customers and ASEA engineers.

There are several reasons why the Novatune system has got a good acceptance. In many process industries ordinary PID regulators are frequently badly tuned. As a result of this they are oftened switched to manual mode. There are a number of critical control loops where there are tangible economical benefits by reducing variations in quality variables. Experience indicates that reductions in standard deviations by a factor of two compared with a welltuned PID is quite common. The reason for this is that there frequently are time delays which the Novatune handles better than PID. The improvements in comparisons with poorly tuned PID are of course more favorable. Improvements in variances with an order of magnitude have been found in several cases when feedforward can be applied. Effective use of feedforward requires however good models which have to be updated. It is thus a good case for adaptive control.

Two applications are described in some detail, a cold rolling mill and a chemical reactor.

The rolling mill is a typical batch process there are roughly speaking three phases, startup, full speed operation and breaking. The Novatune was applied to the gauge control loop. The screw position was controlled using feedback from a gauge sensor after the rolls and by feedforward from a gauge sensor in front of the rolls. The main disturbances are variations in gauge and hardness. The time constants and the time delay varies with a factor of 25 over the operation range. The variations in the time delay are handled by having a speed sensor and by introducing length as the independent variable instead of time. The actual sampling period in the regulator will thus vary with speed from 40 ms at full speed to several seconds at slow speeds. The adaptive regulator performed significantly better than a conventional PID regulator with feedforward.

The first Novatune application was made in connection with temperature control in a chemical reactor. The temperature fluctuations were reduced by an order of magnitude mainly due to feedforward operation. The application is critical with respect to safety and production. The possibility of storing parameters which will give a safe performance of the closed loop system and reinitializing the adaptation using these parameters was incorporated in the system. This application is described in more detail in [1].

The key problem areas that have been found have to do with nonlinearities like friction, dead zone and hysteresis. It may be a good idea to have more flexible ways of modeling these in the regulator. The unstable zeros which appear when the sampling rate is increased is another problem of practical importance.

Reference:

[1] Bengtsson G, Egardt B (1984): Experiences with Self-Tuning Control in the Process Industry. Preprints of IFAC 9th World Congress, Budapest, Hungary, vol XI, 132-140.

## Adaption to changing dynamics

## Adaptive feedforward


## Dead-Time

**ASEA NOVATUNE**

are now running in several plants . . .

- Pulp Mill
- Pulp Drier
- Winder
- Chemical Reactor
- Skin Pass Mill
- Cold Rolling Mill
- Rotary Kiln Drier
- . . .

REFERENCE LIST - 1984-03-10

STEEL/METALLURGIC

| | | |
|---|---|---|
| Krupp Bochum (Germany) | - | Skin Pass Mill |
| Grängesberg (Sweden) | - | Strip Tension |
| Sandvik (Sweden) | - | AGC |
| SSAB (Sweden) | - | Mould Level Control |
| Falk (Italy) | - | Induction Furnace |
| Arvedi ( ) | - | - " - |

PULP/PAPER

| | | |
|---|---|---|
| Mörrum (Sweden) | - | Pulp Drying |
| Edet ( ) | - | Consistency Control |
| EKA (France) | - | Retention Control |
| Hylte (Sweden) | - | Roll Trimmer |
| Kvarnsveden (Sweden) | - | - " - |
| St. Regis (USA) | - | - " - |
| Boise Caslade (Canada) | - | - " - |
| Bowater (Canada) | - | - " - |
| Albury (Australia) | - | - " - |
| Mondi (South Africa) | - | - " - |
| Hallsta (Sweden) | - | - " - |
| Follum (Norway) | - | - " - |

FOOD

| | | |
|---|---|---|
| SSA (Sweden) | - | Beet Pulp Dryer |
| African Products (South Africa) | - | Maize Drying |

CHEMICAL

| | | |
|---|---|---|
| Berol (Sweden) | - | Chemical Reactor |
| Trelleborg (Sweden) | - | Rubber |
| Cementa (Sweden) | - | Cement Klinker |
| Berol (Sweden) | - | Total Instrumentation |
| Boliden (Sweden) | | Total Instrumentation |
| Esmanord (Sweden) | - | - " - |

BOILER

| | | |
|---|---|---|
| Roskilde (Denmark) | - | Boiler Control |
| Munusso (Sweden) | - | Total Instrumentation |

WATER

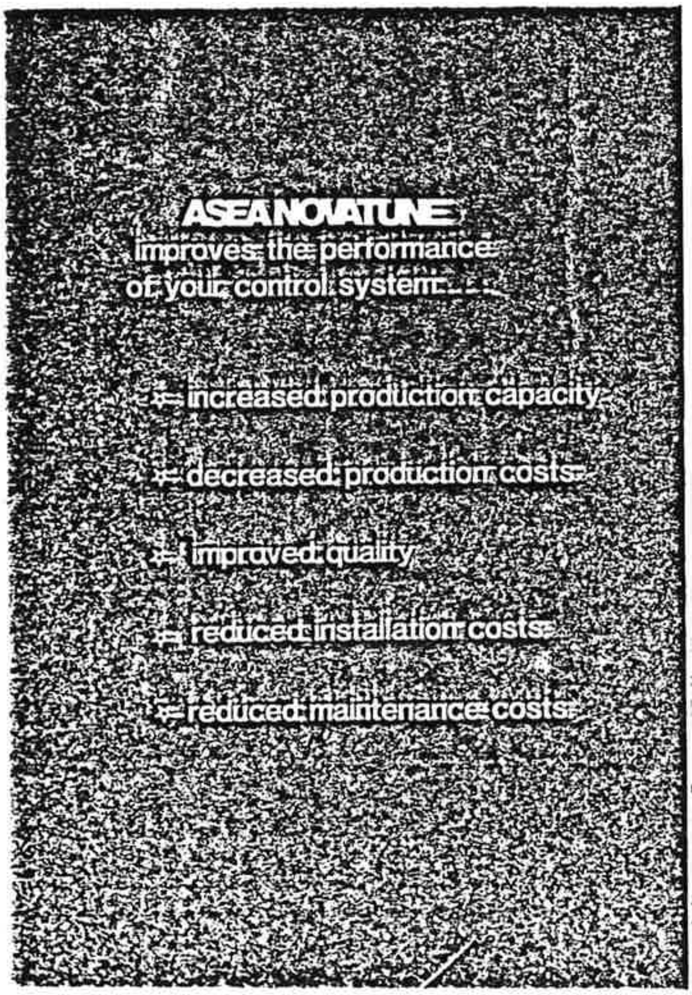
| | | |
|---|---|---|
| Käppala (Sweden) | - | Waste Water Control |

BUILDING AUTOMATION

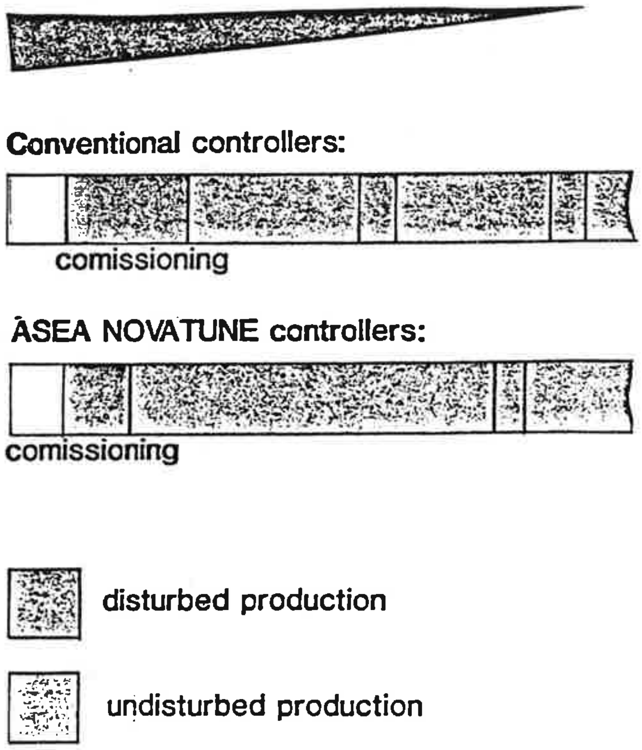
| | | |
|---|---|---|
| Huddinge Hospital (Sweden) | - | Total Instrumentation |

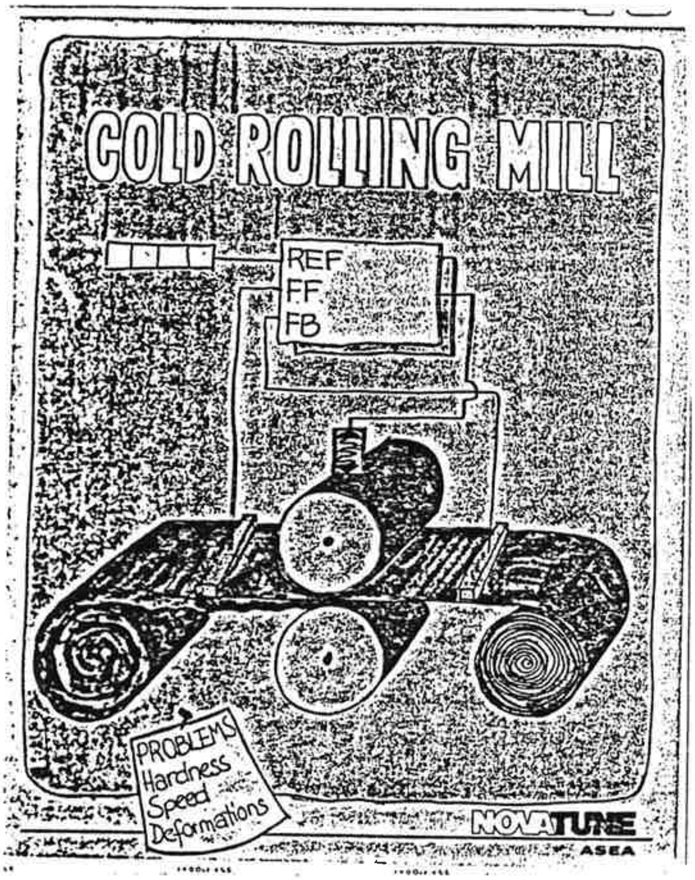
OEM

| | | |
|---|---|---|
| Alrnult (Global) | - | Loom Control |




## Reduced Installation Costs





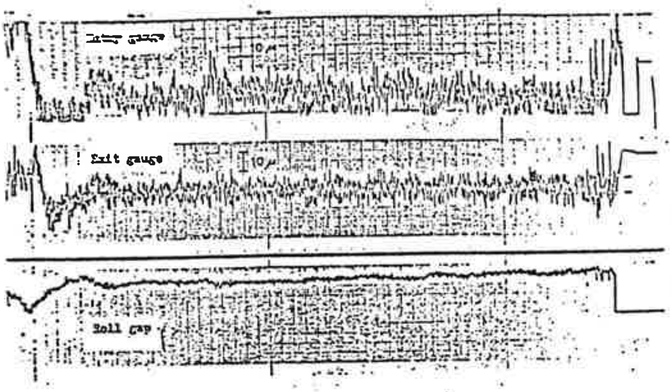




Figure 4 Rolling with a well-tuned conventional regulator

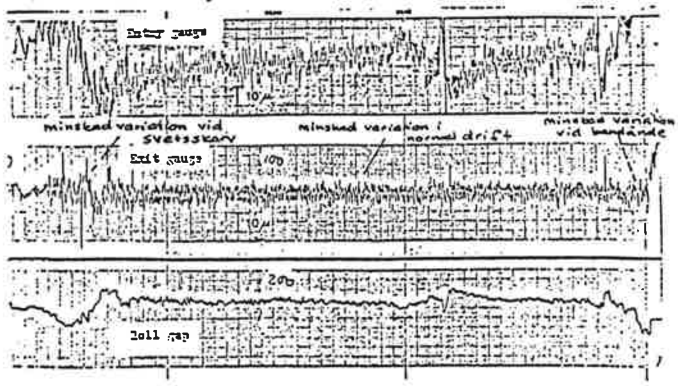



Figure 5 Rolling with a self-adaptive regulator

61638/338B



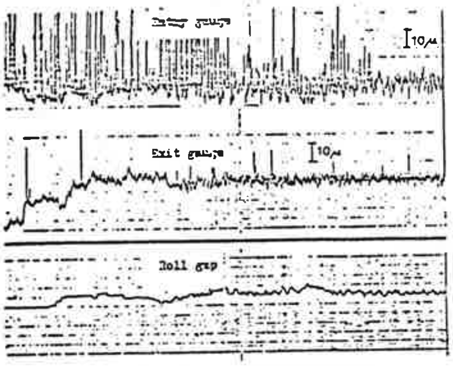



Figure 8 Self-adaptive regulator with severe disturbances in the entry gauge due to mechanical interaction

61638/338B



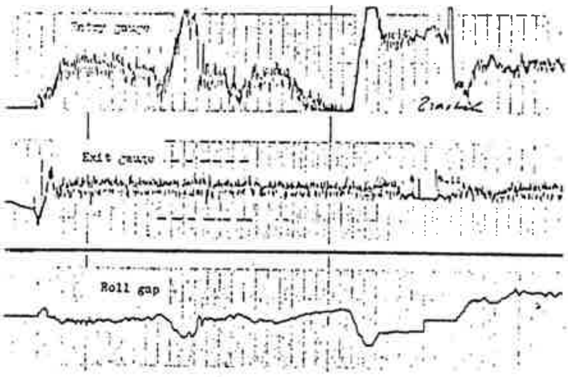



Figure 6 The first pass with the self-adaptive regulator controlling a strip with large deviation in the entry gauge. The tolerance limits (in terms of deviation) after the ninth pass are marked out

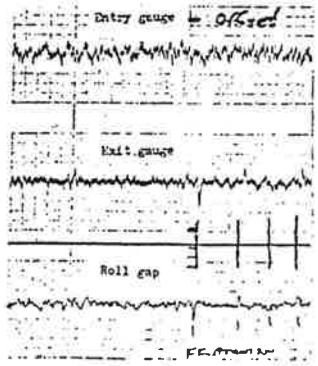
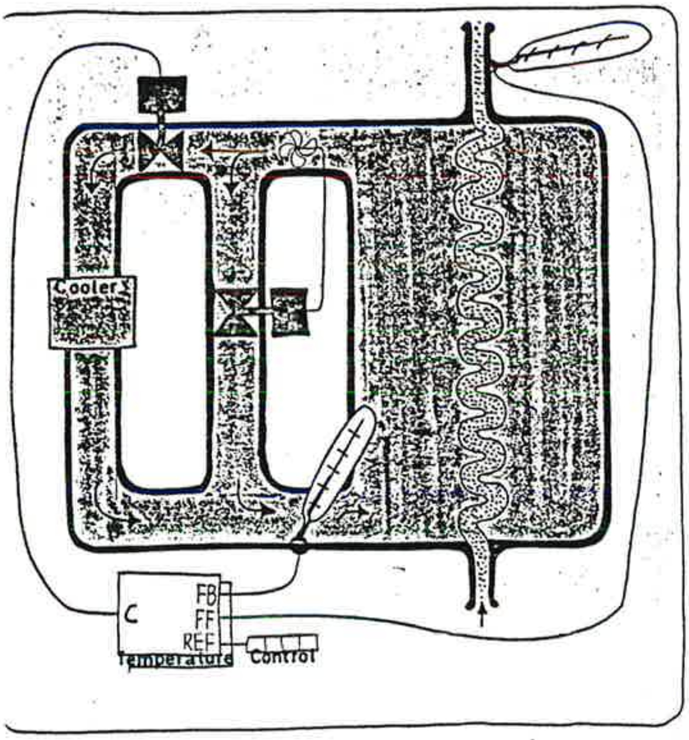
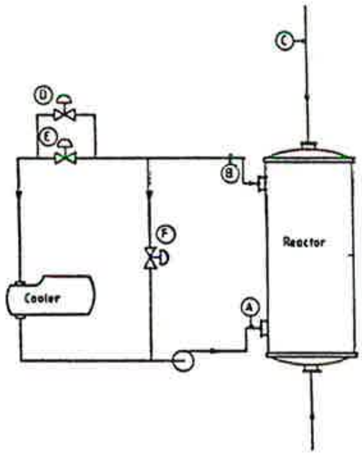



Figure 7 Self-adaptive regulator with off-set disturbance on the entry gauge measurement. The regulator reduces its feedforward

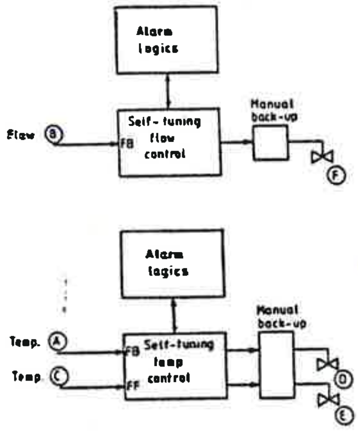
# Chemical Reactor

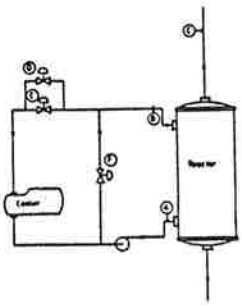



## Reactor Design



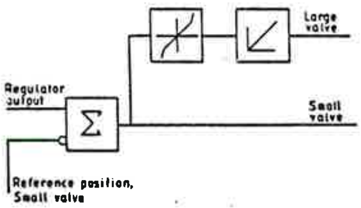

## Reactor Control System Structure



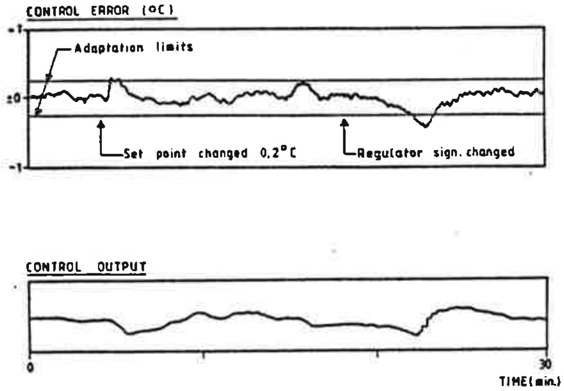
## Reactor Control

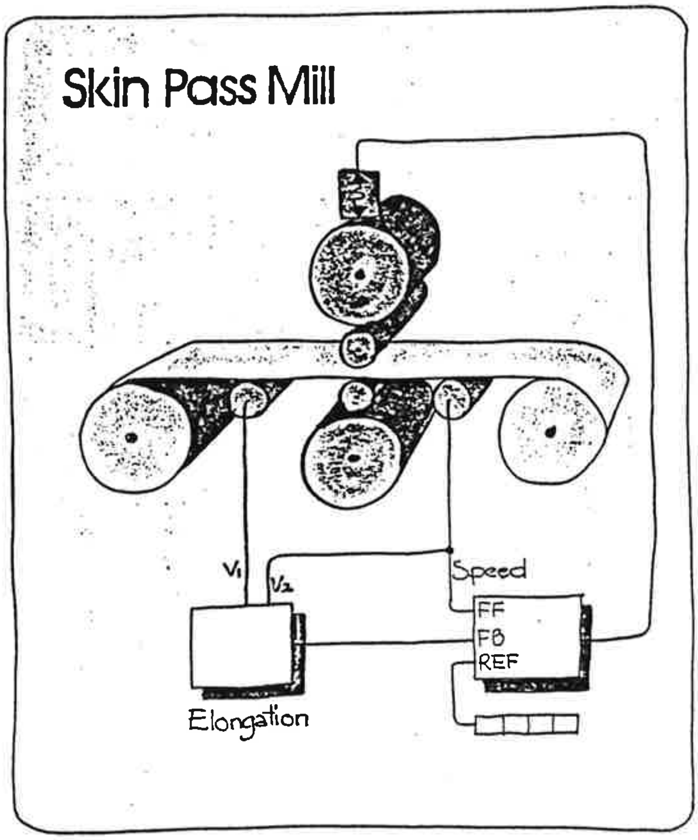
### Combining parallel valves

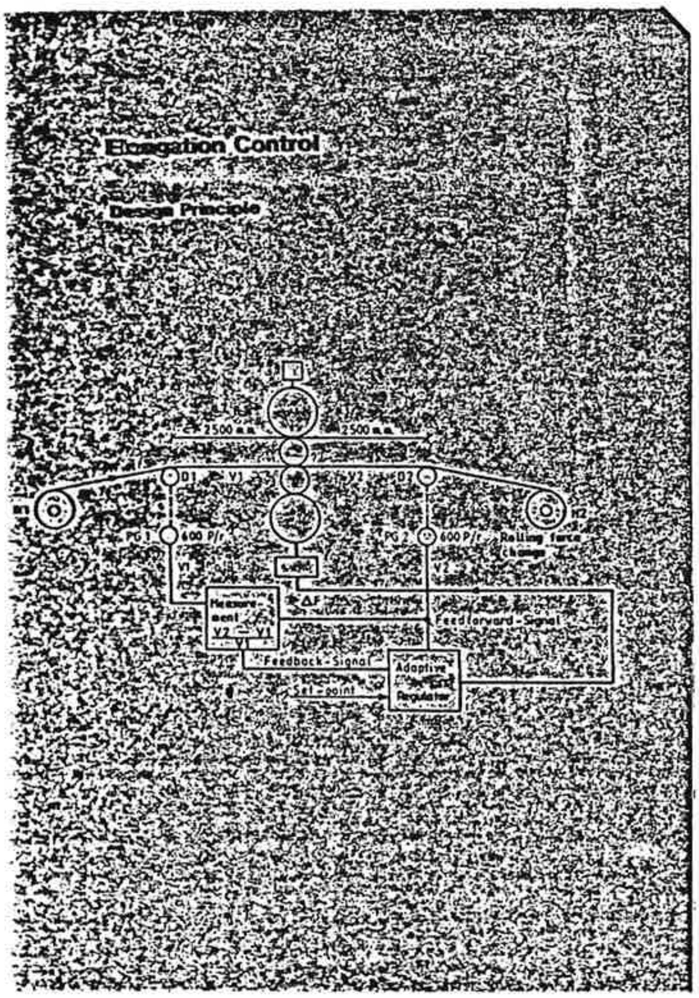


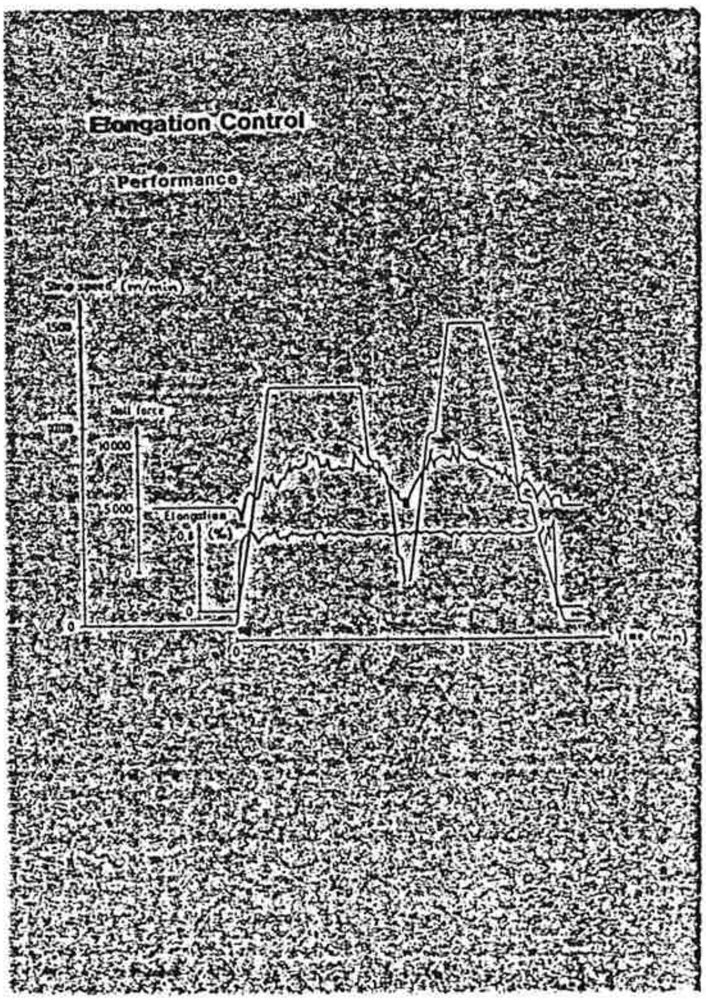
## Reactor Control

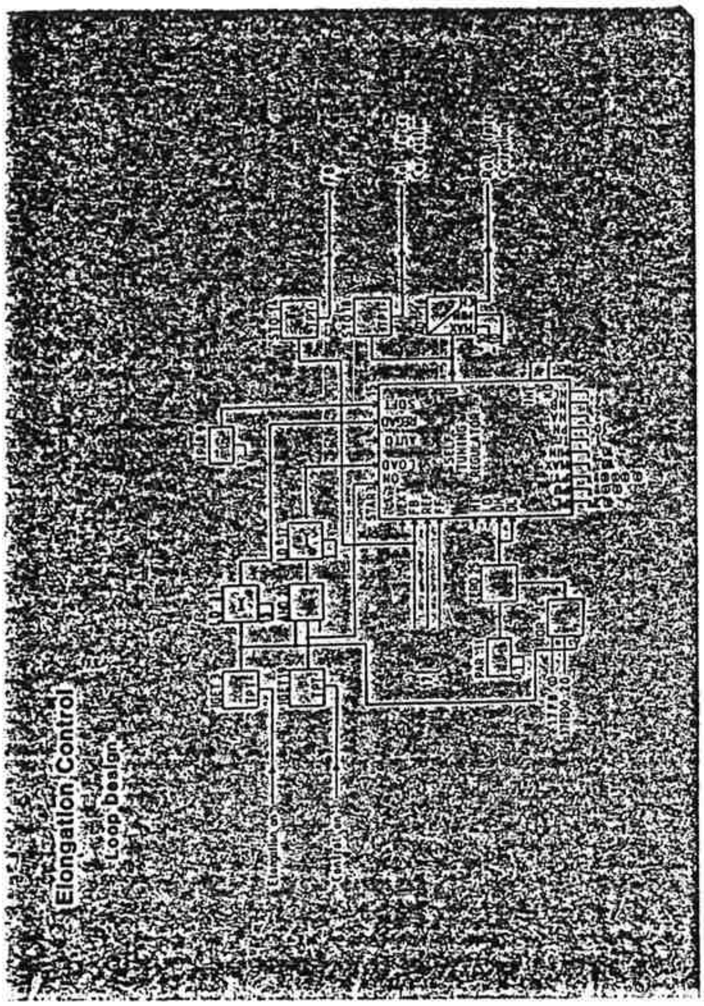
### Performance



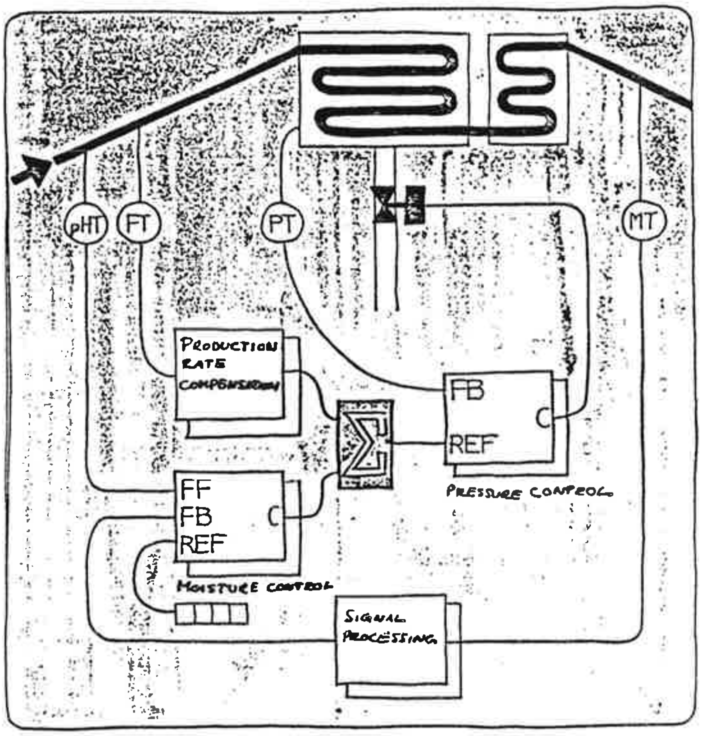



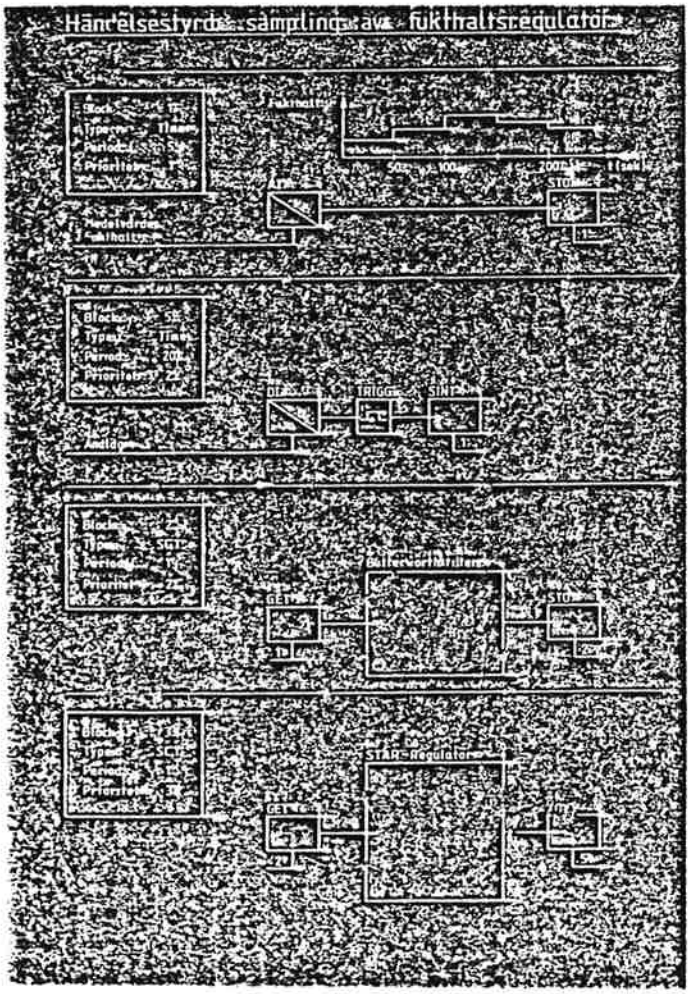



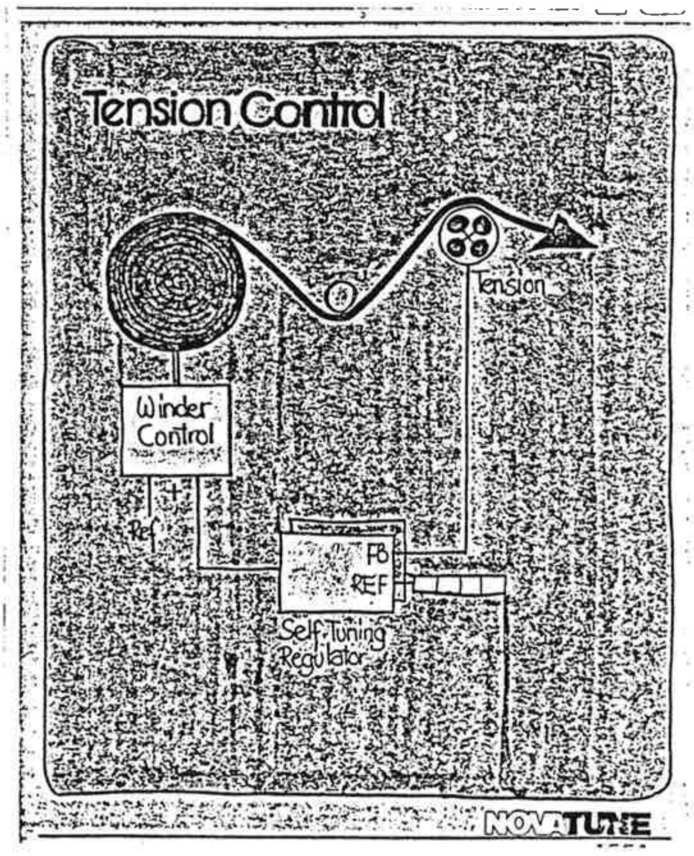



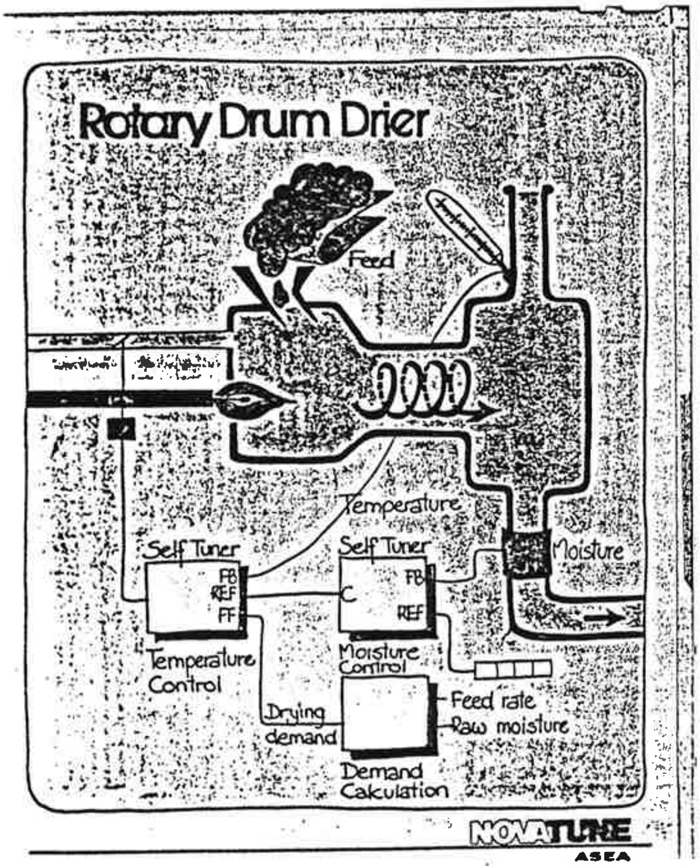

Elongation Control
Loop Design

# PULP DRIER


pHT
FT
PT
MT
PRODUCTION RATE COMPENSATION
FB
REF
C
PRESSURE CONTROL
FF
FB
REF
C
MOISTURE CONTROL
SIGNAL PROCESSING


Händelsestyrd sampling av fukthaltsregulator


Tension Control
Tension
Winder Control
Ref
FB
REF
Self-Tuning Regulator
NOVATUNE



Rotary Drum Drier
Feed
Temperature
Self Tuner
FB
REF
FF
Temperature Control
Self Tuner
C
FB
REF
Moisture Control
Moisture
Feed rate
Raw moisture
Drying demand
Demand Calculation
NOVATUNE
ASEA

# Automatic tuning of simple regulators

K.J. Åström and T. Hägglund

Department of Automatic Control
Lund Institute of Technology
Lund, Sweden

Abstract.

Procedures for automatic tuning of regulators of the PID type are described. The methods are based on a simple identification method which gives critical points on the Nyquist curve of the open loop transfer function. The key idea is a scheme which provides automatic excitation of the process which is nearly optimal for estimating the desired process characteristics. The methods proposed are primarily intended to tune simple regulators of the PI(D) type. In such applications they will of course inherit the limitations of the PI(D) algorithms. They will not work well for problems where more complicated regulators are required. The proposed algorithms may be used in several different ways. They may be incorporated in single loop controllers to provide an option for automatic tuning. They may also be used to provide a solution to the long-standing problem of safe initialization of more complicated adaptive or self-tuning schemes. In contrast to other methods based on self-tuning control, they do not require apriori information about time scales.

References

Åström, K. J. and T. Hägglund. Automatic Tuning of Simple Regulators with Specifications on Phase and Amplitude Margins. Proceedings IFAC Workshop on Adaptive Systems in Control and Signal Processing. San Francisco 1983.

Åström, K. J. and T. Hägglund. Automatic Tuning of Simple Regulators with Specifications on Phase and Amplitude Margins. Automatica 20, 645-651, 1984.

Åström, K. J. and T. Hägglund. Automatic Tuning of Simple Regulators. Proceedings IFAC 9th World Congress, Budapest, Hungary. 1984.

# AUTOMATIC TUNING OF SIMPLE REGULATORS

K.J. Åström
LUND – SWEDEN

1. INTRODUCTION
2. THE BASIC IDEA
3. CONDITIONS FOR OSCILLATION
4. REGULATOR DESIGN
5. PRACTICAL ISSUES
6. EXPERIMENTS
7. CONCLUSIONS

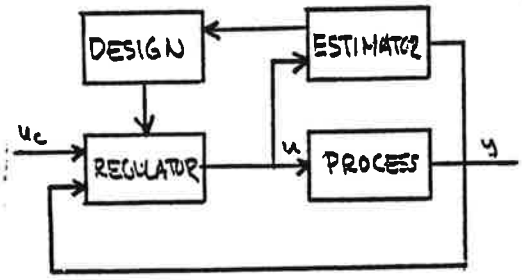


## INTRODUCTION

- BACKGROUND
  - ADAPTIVE CONTROL RESEARCH
    - ROBUSTNESS
    - UNMODELED DYNAMICS
    - PRIOR INFORMATION
  - REACTIONS FROM INDUSTRY
- ADAPTATION VS TUNING
- THE PID STRUCTURE
- STR & MRAS APPROACHES
  - LS+MU
  - LS+PP
  - ELS+LQG
- A NEW APPROACH

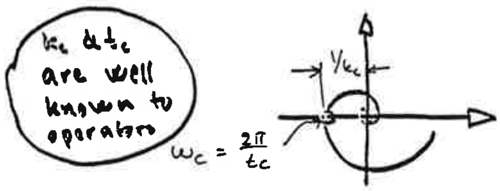


## SELF-TUNING CONTROL




## THE BASIC IDEA

- DESCRIBE THE PROCESS IN TERMS OF CRITICAL GAIN $k_c$ AND CRITICAL PERIOD $t_c$.



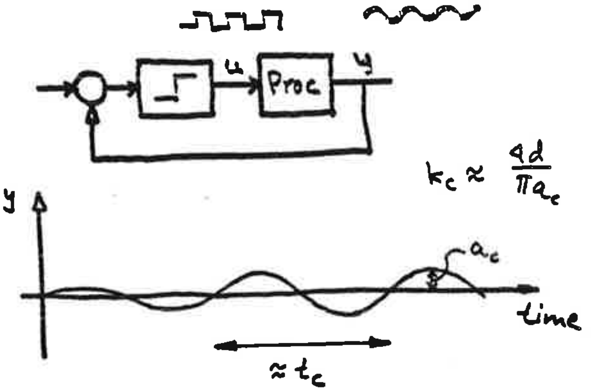

- FIND METHODS FOR DETERMINING $k_c$ AND $t_c$.
- APPLY DESIGN METHOD BASED ON $k_c$ AND $t_c$

EX: ZIEGLER NICHOLS

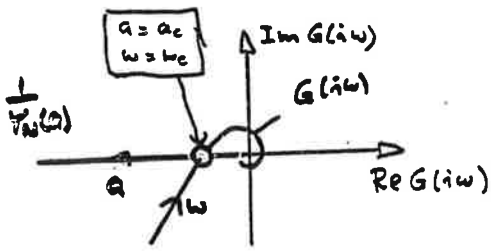
$$k = \frac{k_c}{2}, \quad T_i = \frac{t_c}{2}, \quad T_d = \frac{t_c}{8}$$

## DETERMINATION OF $k_c$ & $t_c$

RELAY CONTROL




$k_c \approx \frac{4d}{\pi a_c}$



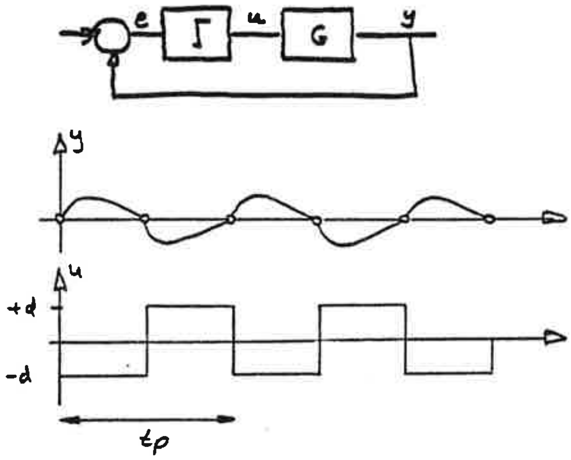



## PROPERTIES

- AUTOMATIC GENERATION OF NEAR OPTIMAL INPUT
- GOOD POSSIBILITIES FOR CONTROLING THE OUTPUT AMPLITUDE DURING THE EXPERIMENT
- SAFE PROCEDURE FOR STABLE SYSTEMS



## CRITERIA FOR OSCILLATIONS




SAMPLE AT $t_p/2$

$$Z(u) = \frac{1}{z+1}$$

THEOREM: $H(-1) = 0$

APPROXIMATION

$$k_c \approx \frac{4d}{\pi a}$$



## THE DESCRIBING FCN APPROX.

$$H(e^{sh}) = \sum_{m=-\infty}^{\infty} \frac{1}{h(s+im\omega_s)} \left(1 - e^{-h(s+im\omega_s)}\right) G(s+i$$

$\omega_s = \frac{2\pi}{h}$, $sh = i\pi$ gives

$$H(-1) = \sum_{n=-\infty}^{\infty} \frac{2}{i(\pi+2n\pi)} G\left(i\,\frac{\pi+2n\pi}{h}\right)$$

$$= \sum_{n=0}^{\infty} \frac{4}{\pi(1+2n)} \operatorname{Im}\left\{G\left(\frac{\pi+2n\pi}{h} i\right)\right\}$$

$$\approx \frac{4}{\pi} \operatorname{Im}\left\{G\left(i\frac{\pi}{h}\right)\right\}$$

## EXAMPLE 1

$$G(s) = \frac{1}{s} e^{-sT}$$

$$H(z) = \frac{zT + (h-T)}{z-1}$$

$$H(-1) = 0 \Rightarrow h = 2T$$

Period $T_p = 2h = \underline{\underline{4T}}$

$$\arg G(i\omega_c) = -\frac{\pi}{2} - \omega_c T = -\pi$$

$$\Rightarrow \omega_c = \frac{\pi}{2T} \qquad T_p = \frac{2\pi}{\omega_c} = \underline{\underline{4T}}$$

6C

## EXAMPLE 2

$$G(s) = \frac{1}{s(s+1)(s+a)}$$

$$\arg G(i\omega_c) = 0 \Rightarrow \omega_c = \sqrt{a}$$

$$H(-1) = -\frac{h}{2a} + \frac{1}{a-1}\left[\frac{1-e^{-h}}{1+e^{-h}} - \frac{1}{a^2}\frac{1-e^{-ah}}{1+e^{-ah}}\right]$$

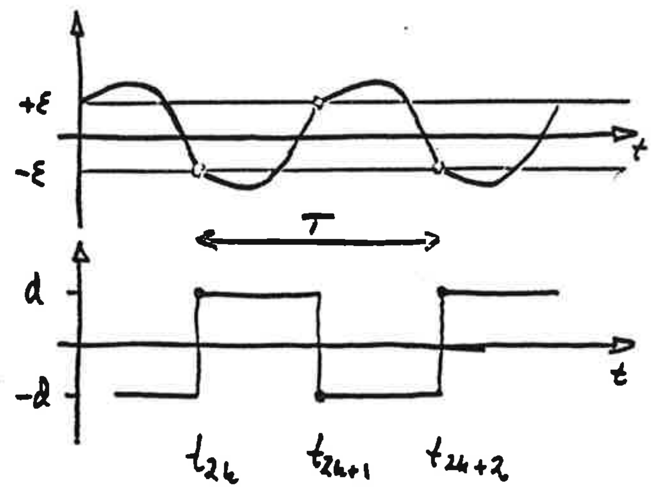
$$H(-1) = 0 \Rightarrow h \approx \frac{2\sqrt{3}}{\sqrt{a}}$$

$$T_p = \frac{2\pi}{\sqrt{a}} = \frac{6.28}{\sqrt{a}} \quad \text{approx}$$

$$T_p \approx \frac{4\sqrt{3}}{\sqrt{a}} = \frac{6.92}{\sqrt{a}}$$

6D

## CONDITIONS FOR PERIODIC SOLUTIONS




$$\mathcal{Z}\{u\} = \frac{d}{z+1} \qquad \mathcal{Z}\{y\} = -\frac{\varepsilon}{z+1}$$

$$H\left(\frac{T}{2}, -1\right) = -\frac{\varepsilon}{d}$$

6E

## EXAMPLE:

$$G(s) = \frac{b}{s+a} \qquad a, b > 0$$

$$H(\tau, z) = \frac{b(1-e^{-a\tau})}{z - e^{-a\tau}}$$

$$H(\tau, -1) = -\frac{b(1-e^{-a\tau})}{1+e^{-a\tau}} = -\frac{\varepsilon}{d}$$

$$T = 2\tau = -\frac{2}{a}\ln\frac{bd-\varepsilon}{bd+\varepsilon} \approx \frac{4\varepsilon}{abd}$$

6F

## EXAMPLE

$$\frac{dx}{dt} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \omega \end{bmatrix} u$$

$$y = [1 \quad 0] x$$

$$H(\tau, z) = \frac{z - 2\cos\omega\tau + 1}{z^2 - 2z\cos\omega\tau + 1}$$

$$H(\tau, -1) = -\frac{\cos\omega\tau}{1+\cos\omega\tau} = -\frac{\varepsilon}{d}$$

$$T = 2\tau = \frac{2}{\omega} \arccos \frac{\varepsilon}{d-\varepsilon}$$

$$\varepsilon = 0 \Rightarrow T = \frac{2}{\omega} \cdot \frac{\pi}{2} = \frac{\pi}{\omega}$$

66

## EXAMPLE:

$$G(s) = \frac{1}{s^2}$$

$$H(\tau, s) = \frac{\tau^2}{2} \frac{z+1}{(z-1)^2}$$

$$H(\tau, -1) = 0$$

- NO PERIODIC SOLUTION WITH HYSTERESIS
- PERIODIC SOLUTIONS WITH ARBITRARY PERIOD WITH IDEAL RELAY

6?

## HOW TO DETERMINE $t_c$ & $a$

ZERO CROSSINGS & PEAK DETECTION

ZERO CROSSINGS & CORRELATION

LEAST SQUARES

$$\Sigma [y(t) - b y(t-h) + y(t-2h)]^2 \; Min$$

$$b = 2\cos(h/t_c)\cdot 2\pi$$

$$\Sigma [y(t) - a_1 \sin\omega t - a_2 \cos\omega t]^2$$

$$\omega = 2\pi / t_c$$

## ZIEGLER - NICHOLS RULE 1

CRITICAL GAIN $K_c$

" - " PERIOD $T_c$

| | $K$ | $T_I$ | $T_D$ |
|---|---|---|---|
| P | $0.5K_c$ | | |
| PI | $0.4K_c$ | $0.8T_c$ | |
| PID | $0.6K_c$ | $0.5T_c$ | $0.12T_c$ |

MODIFIED RULES:

BETTER DAMPING?



8

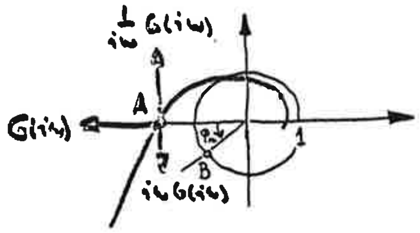
## CONTROL DESIGN

- AMPLITUDE MARGIN DESIGN

$$G_R(s) = k\left[1 + \frac{1}{sT_i} + sT_d\right]$$
$$= k\left[1 + \frac{1}{sT_i}(1 + s^2 T_i T_d)\right]$$

$$k = \frac{k_c}{A_m}, \quad T_d = \frac{1}{\omega_c^2 T_i}, \quad T_i \text{ arbitrary}$$

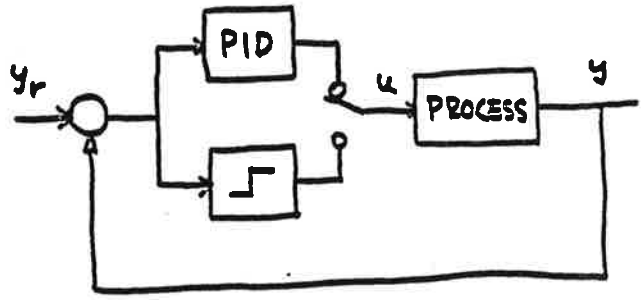
- PHASE MARGIN DESIGN




PICK $k$, $T_i$ AND $T_d$ TO MOVE A TO B



## REGULATOR STRUCTURE






## PRACTICAL ISSUES

- MEASUREMENT NOISE
- ADJUSTMENT OF AMPLITUDE OF OUTPUT
- SATURATION OF ACTUATORS
- WHAT PRIOR INFORMATION IS NEEDED?
- HYSTERESIS



## EXPERIMENTS

- GOALS
  - WHEN & HOW WILL IT WOR
  - USER REACTION
- MEANS
  - LSI 11/03, Apple II, Intel 8086
  - Analog computer simulation
  - Laboratory processes
  - Industry
    - Flow
    - Temperature
    - Level
    - Composition
- RESULTS

# PI - AUTO-TUNER

$$G(s) = \frac{1}{(1+0.25s)^4}$$

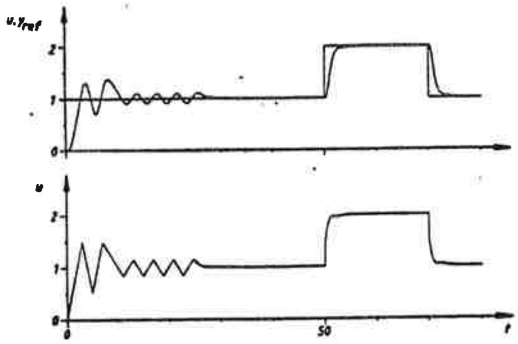


Figure 8. - Simulation of a PI auto-tuner applied to a process with the transfer function $G(s) = 1/(1+0.25s)^4$.



# VARIATIONS IN PROCESS GAIN

$$G(s) = \frac{k}{(1+0.25s)^4}$$

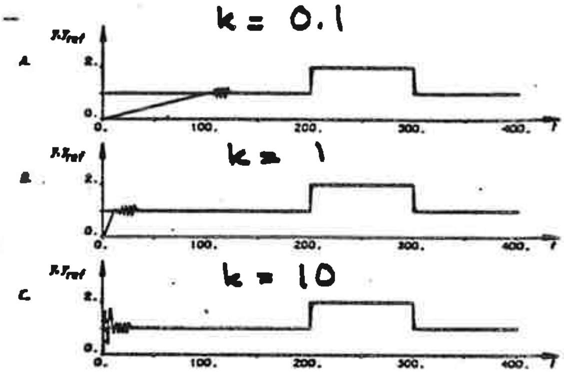



Figure 9. - Simulation of a PI auto-tuner applied to a process with the transfer function $G(s) = k/(1+0.25s)^4$. The process gain k is 0.1, 1 and 10 in A, B, and C respectively.



# VARIATIONS IN PROCESS TIME CONSTANTS

$$G(s) = \frac{1}{(1+sT)^4}$$

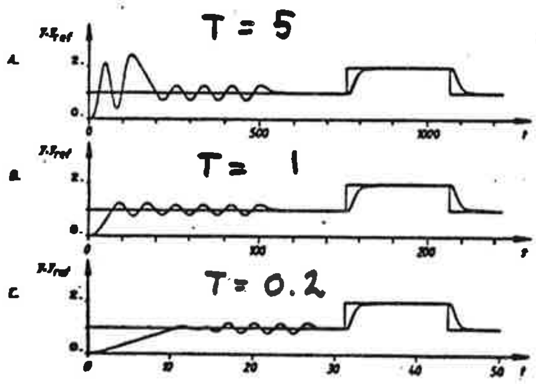



Figure 10. - Simulation of a PI auto-tuner applied to a process with the transfer function $G(s) = 1/(1+sT)^4$. The process time constant T is 5, 1 and 0.2 in A, B, and C respectively.



# EFFECTS OF NOISE

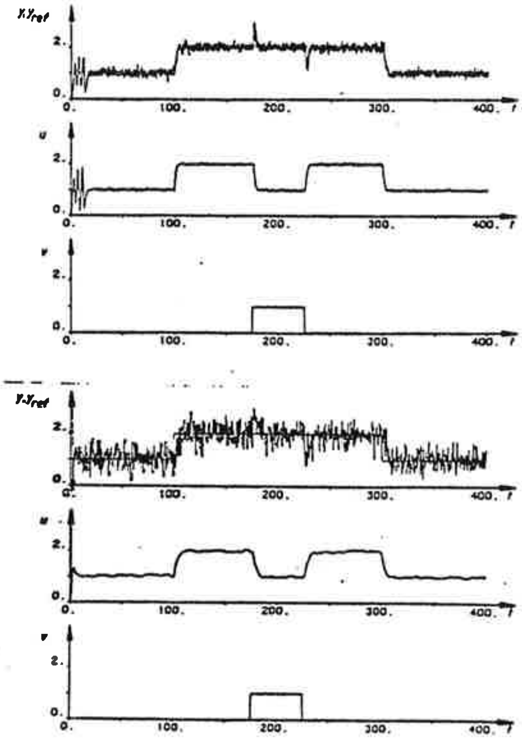
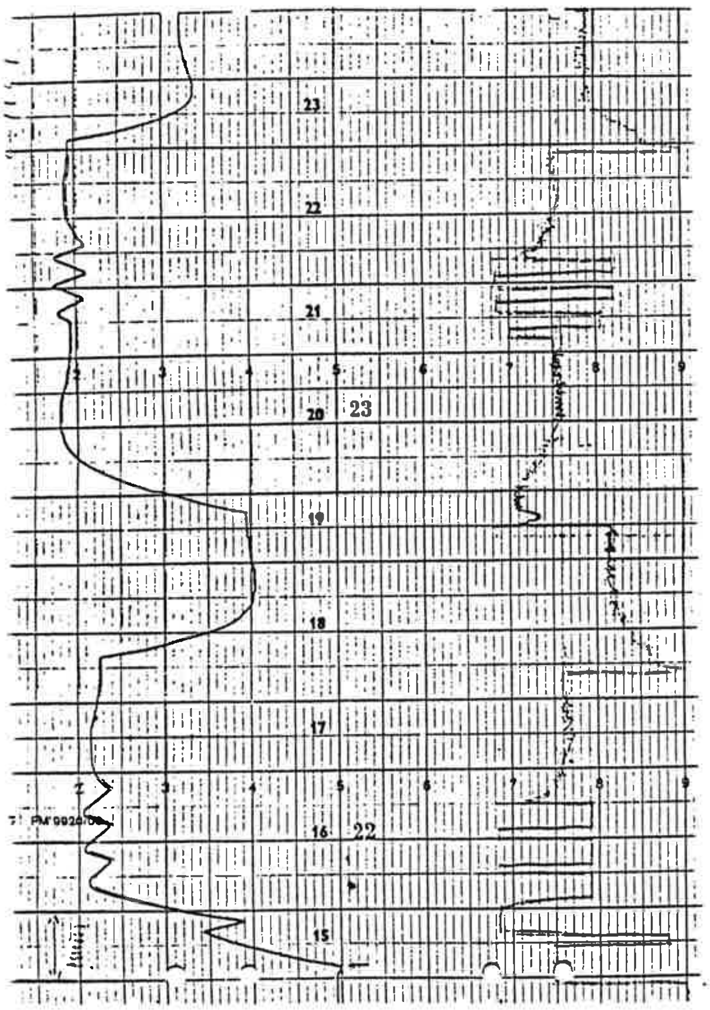
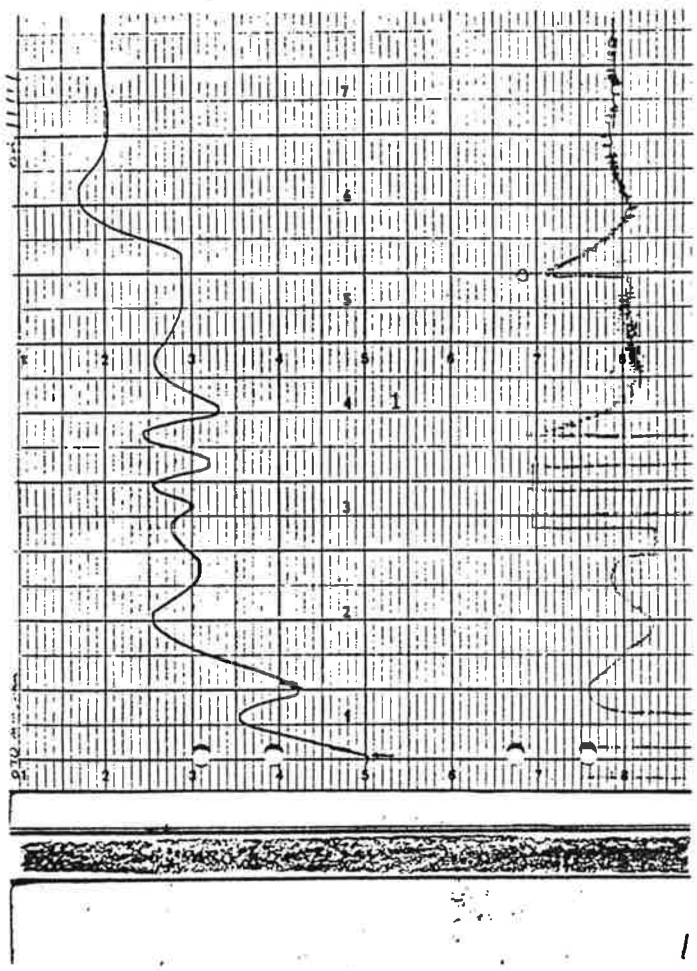


Figure 12. - Simulation of the system with variable noise level. The standard deviation of the measurement noise is 0.1 and 0.3 in A and B respectively.





## CONCLUSIONS

- A SIMPLE IDEA
- SEEM TO WORK WELL
- ROBUST
- IMPLICATIONS FOR MORE GENERAL ADAPTIVE SYSTEMS
- GETS YOU IN THE BALL PARK
- RESEARCH ISSUES

## AUTOMATIC TUNING OF SIMPLE REGULATORS

K. J. Åström
LUND - SWEDEN

1. INTRODUCTION
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# The NAF - Autotuner

Lars Bååth

NAF Controls AB
Stockholm

Among other products for instrumentation and control NAF Control AB manufactures the Control and Information system NAF-Unic. As a special version of the software of the systems NAF-Unic S SDM-10 and NAF-Unic S SDM-20 the NAF-Autotuner can be obtained.

The NAF-Unic S SDM-20 system consists of one central unit, one or two color, screens and function keyboards, one alphanumeric keyboard for program development, one printer and one tape recorder. The system has the capacity of handling 30 analog and 30 digital inputs, 16 digital and 16 analog outputs and up to 45 PID-controllers. With 45 PID-loops the system loop time is 250 ms. The expanded version can handle 240 inputs and 128 output signals.

The system can be programmed with function modules such as PID-controllers, a deadtime controller, limiters, alarm modules, adders, multipliers, logical modules and so on. In total there are about 30 different modules. The software of the system can contain 220 function modules. The system also contains a PLC-system which is integrated with the analog control system. It is possible to program the system on line.

A major goal in the design of this system has been high system security. Several measures have been taken to achieve this, for example

- All PC-boards are duplicated
- Checksum calculation of configuration in RAM and code in EPROM
- A triple serial bus for internal communication is used
- Self diagnostic routines for the inputs and outputs

The NAF-Unic S SDM-20 and SDM-10 systems also have incorporated an autotuner to help the operator in tuning the PID- controllers. A major design effort has been to simplify operation of the autotuner. All 45 PID regulator loops can use the Autotuner. The Autotuner has been developed by NAF-Controls in collaboration with Karl Johan Åström and Tore Hägglund at the Department of Automatic Control at Lund Institute of Technology. The principle of the Autotuner is to replace the normal PID-controller by a relay controller. A system with a relay controller starts to oscillate. If the period and amplitude of this oscillation are measured, the critical gain ($k_c$) and the critical period time ($T_c$) can be computed, because the describing function of the relay is known. When $k_c$ and $T_c$ are known a PID-controller can easily be designed.

The use of NAF-autotuner is very simple. Tuning can be started on operator command or by a digital signal. When no previous tuning is done, the operator must bring the process up to desired reference value in MAN-mode and start tuning.

If there is a controller that already has been tuned, just start tuning. Tuning can be interrupted by setting the controller in MAN- or AUTO-mode. When tuning is ready, the controller is automatically set in AUTO-mode with new PID-parameters.

## SDM-20. System Features and Capacity



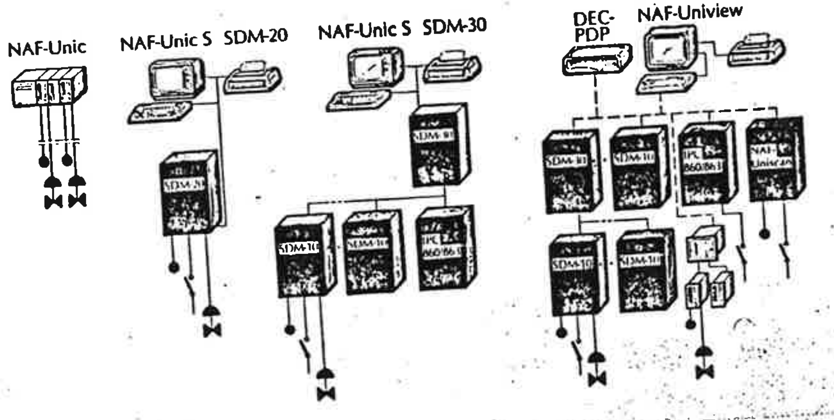
- handles both analogue and digital signals
  - 60 input signals — analogue or digital
  - 32 output signals — analogue or digital
- for up to 45 control loops
- supervision of up to 8 external single loop controllers
- optional signal can be connected to a pen-recorder for long-term trends
- system loop time, 250 ms
- software with 220 function modules
- advanced PLC-system for interlocking and sequence control
- expandable to 240 input and 120 output signals
- Very high system security
- PLC-system is integrated with analoge control-system by means of 256 digital signals used for communication
- On line programming

NAF-Unic S
on the spot computing



## NAF Control and Information System





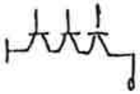

NAF-Unic S
on the spot computing



## Major Design Goal

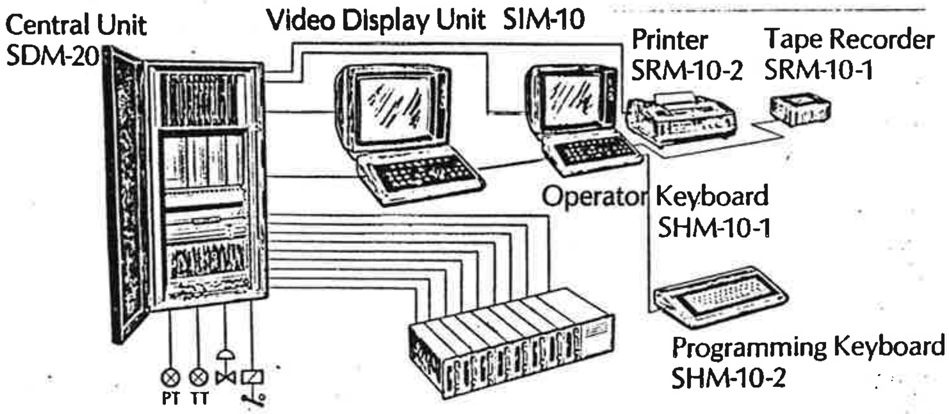
**System Security**

- Serial bus (trippled)
- Backup on all PCB:s
- PCB:s with continuous selfcheck function and watch-dog
- All outputs overload protected
- 24 V DC supply with battery backup available
- Alarm on PCB:s, VDU and printer on error
- Checksum calculation of configuration in RAM and code in EPROM



## NAF-Unic S SDM-20
### with options



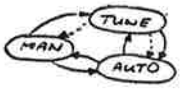



- up to eight single loop controllers can be connected to the system as well
- optional signal can be connected to a pen-recorder via the alphanumeric keyboard

NAF-Unic S
on the spot computing

# NAF-AUTOTUNER in Unic-S

- Major design effort : Simplify operation of AUTOTUNER
- Unic-S incorporates 45 PID controllers with AUTOTUNER




## OPERATION

- Tuning can be started on operator-command or by a digital signal
- Start of tuning is only permitted if control error is sufficiently small

First time tuning : Slowly bring the process up to desired reference value, in MAN-mode, and start Tuning.

Previous tuning : Start Tuning

- Tuning ready => Controller automatically set in AUTO with new PID-parameters
- Tuning can be interrupted by setting controller in MAN- or AUTO-mode

TUNE → MAN => Control signal returns to its value prior to start of tuning

TUNE → AUTO => Controller returns to AUTO-mode with old PID-parameters

# NAF - AUTOTUNER

NAF-Unic S

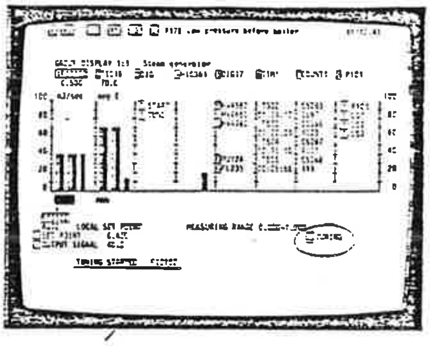


Fig. 2 Group display with controller selected for tuning

| No. | Parameter | Note | Value | New value |
|---|---|---|---|---|
| 1 | Function block | | PID | |
| 2 | Circuit reference | | ****** | |
| 3 | Input signal | AI 1 | ****** | |
| 4 | Measuring range, high | | ****** | |
| 5 | Measuring range, low | | ****** | |
| 6 | P gain | | 0.100000 | |
| 7 | TI integration time | | 1000.0 | |
| 8 | TD delivery time | | 0.000000 | |
| 9 | TD differentiation time | | 0.250000 | |
| 10 | Output signal | AO | | |
| 11 | Remote set point | AI 2 | | |
| 12 | Start local set point | | | |
| 13 | Set point limiting, high | | | |
| 14 | Set point limiting, low | | | |
| 15 | Store current set point | | | |
| 16 | DI blocking, I section | DI 1 | | |
| 17 | DI blocking, controller | DI 2 | | |
| 18 | Output signal, controller blocking (%) | | | |
| 19 | Output signal, zero deviation (%) | | | |
| 20 | DO remote/local | DO 1 | | |
| 21 | DO auto/manual | DO 2 | | |
| 22 | Direct/reversed (D/R) | | D | |
| 23 | Start remote/local | | L | |
| 24 | Start auto/manual | | M | |
| 25 | Start output signal (%) | | | |
| 26 | [illegible] | | N | |
| 27 | [illegible] | DI 3 | | |
| 28 | [illegible] | DO 4 | | |

# History and issues in adaptive flight control

Gunter Stein

Honeywell Systems Research Center
Minneapolis, USA

This paper attempts to answer three questions: Why are there no adaptive flight control systems in modern aircrafts? Why has adaptive theory been irrelevant to all attempted designs? What is happening to change this around?

Inspection of aircraft physics reveals that the linearized aircraft dynamics depend typically on three parameters, dynamic pressure, Mach number and angle of attack. If these parameters are known it is straight forward to design flight control systems which accomplishes desired goals. Two conmeting alternative designs are discussed: the adaptive schemes and systems based on air data scheduling. The history is described from the point of view of competition between these schemes. An important aspect is that the control systems are flight critical in new aircraft. This means that rigid safety measures are required. The space shuttle is another example. It requires four identical control channels plus a fifth backup channel. These channels are not adaptive. The adaptive systems described include the ones used in the X15, F-111 and the F-8.

The tradeoff between performance and stability are reviewed in order to discuss quantitatively the influence of plant uncertainty on feedback design. The plant uncertainties are separated into structured uncertainty which corresponds to the rigid body dynamics and unstructured uncertainties which correspond to flexure modes. Adaptive control can only deal with the structured part. Some assumptions on the unstructured dynamics must be made beforehand. It is interesting that the frequency ranges are surprisingly similar for a wide range of aircrafts. The airframe typically has resonances at 40 rad/s, the nonstationary aerodynamics around 60 rad/s actuator dynamics is around 12 rad/s, presampling filters and delays around 100 rad/s.

It is concluded that adaptive control can help only with the structured model errors but that it must work in the presence of unstructured model uncertainties. Some knowledge about the unstructured model errors must be available. This knowledge must be included in the design of the adaptive system.

The main conclusion is that the adaptive control has lost the competition against gainscheduling for regular flight control. Some potential problems where adaptive control may apply are when airdata is impractical (small missiles and rentry vehicles), where air data will not work (flexure and flutter control).

It is suggested that some work in the adaptive field is devoted to understand the engineering fixes that are made to make the systems work and particularly that unstructured uncertainties are considered.

References

[1] G. Stein (1980): Adaptive Flight Control: A Pragmatic View. In Narendra and Monopoli (eds) Applications of Adaptive Control. Academic Press, New York 1980.

[2] IEEE Transaction on Automatic Control AC-22, Mini-issue on NASA's advanced control law program for the F-8 DFBW aircraft.

# HISTORY AND ISSUES IN ADAPTIVE FLIGHT CONTROL

GUNTER STEIN

NSF - STU

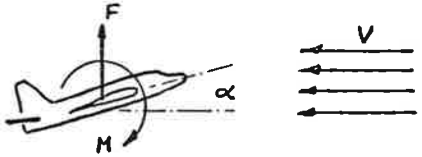
WORKSHOP ON ADAPTIVE CONTROL

JULY 1984

## THREE QUESTIONS

- WHY ARE THERE NO ADAPTIVE FLIGHT CONTROL SYSTEMS?
- WHY HAS ADAPTIVE THEORY BEEN IRRELEVANT TO ALL ATTEMPTED DESIGNS?
- WHAT IS HAPPENING TO CHANGE THIS?

## A LITTLE AIRPLANE PHYSICS

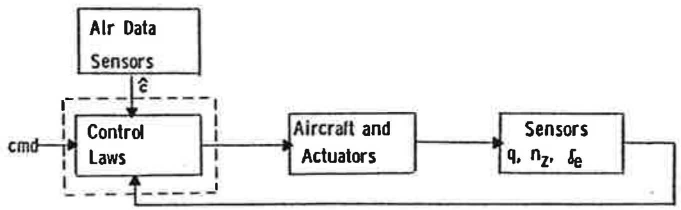



- FORCES/MOMENTS PROPORTIONAL TO DYNAMIC PRESSURE: $\bar{q} = \rho V^2/2$ 100:1
- FORCES/MOMENTS MOVE WITH MACH: $M = V/a$ 10:1
- SOME FORCE/MOMENT SLOPES CHANGE WITH ANGLE-OF-ATTACK: $\alpha = \tan^{-1} V_y/V_x$ 5:1

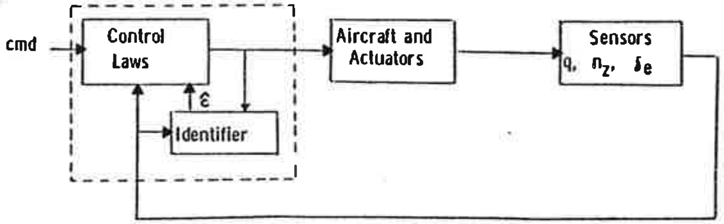
$$\dot{x} = A(\bar{q},M,\alpha)x + B(\bar{q},M,\alpha)u$$

## COMPETING FLIGHT CONTROL ALTERNATIVES

CONVENTIONAL AIR-DATA-SCHEDULED CONTROL




ADAPTIVE ALTERNATIVE



## HISTORY OF COMPETITION

1960
- POOR AIRDATA SYSTEMS

  SEVERAL ADAPTIVE APPLICATIONS

  - X-15
  - F-111
  - F-101 TESTS
  - F-4 TESTS

  } ALL CERTAINTY EQUIVALENCE

1970
- MUCH IMPROVED AIRDATA

  NO ADAPTIVE APPLICATIONS

  'TEEN FIGHTERS
  SHUTTLE

- RE-EVALUATION

  REASONS
  - MODERN THEORY
  - DIGITAL IMPLEMENTATION
  - RELIABILITY
  - UNSOLVED PROBLEMS

- NASA F-8C PROGRAM

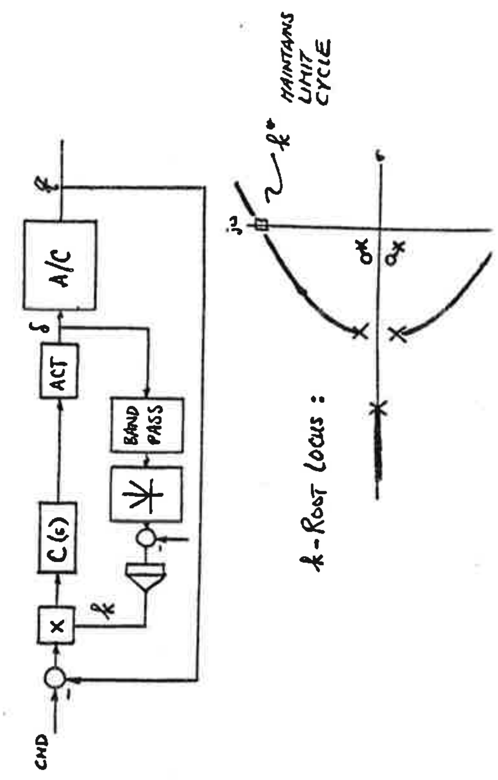
  DESIGN
  SIMULATOR EVAL
  FLIGHT TEST

1980

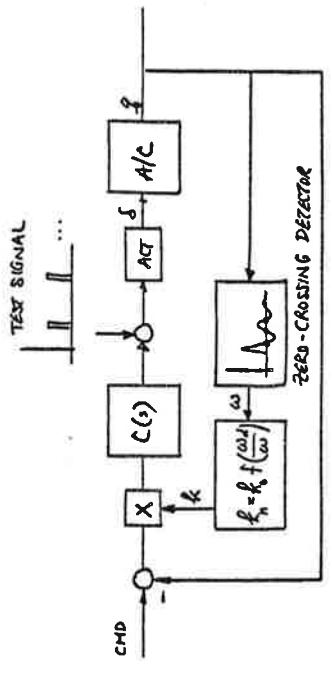
- FLUTTER TESTS ?

  CONCEPTS TAC OCT 77
  - MULTIPLE MODEL
  - MODEL REFERENCE
  - CERTAINTY EQUIVALENCE (MLE / LQ)

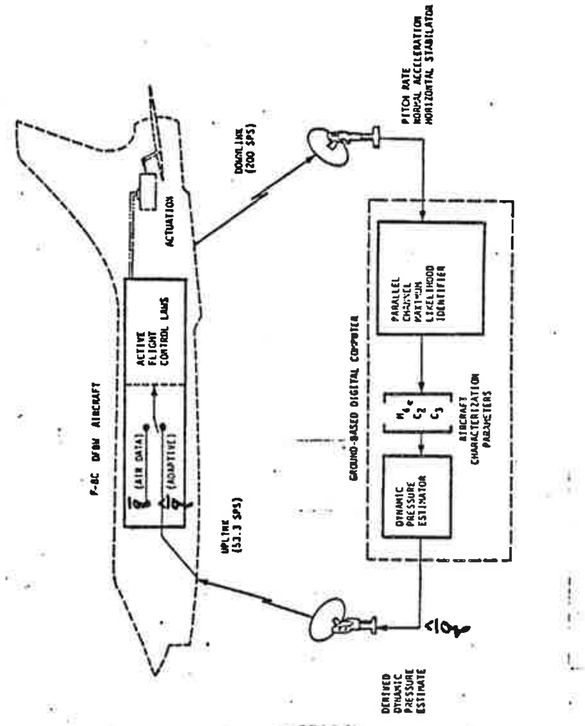
## X-15 LIMIT CYCLE SYSTEM



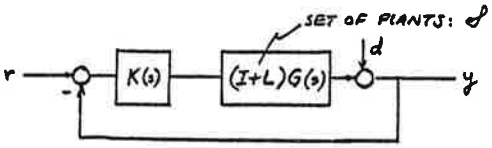

## F-111 EXPLICIT IDENTIFIER

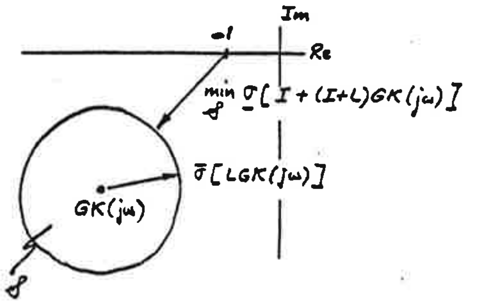



## F-8C EXPLICIT IDENTIFIER



## THE BASIC FEEDBACK PROBLEM




- FOR PERFORMANCE

  $\min_{\mathcal{S}} \underline{\sigma}\left[I+(I+L)GK(j\omega)\right]$ LARGE AT SOME ω'S

- FOR STABILITY

  $\min_{\mathcal{S}} \underline{\sigma}\left[I+(I+L)GK(j\omega)\right] \neq 0$ AT ALL ω'S

## RESULTING FUNDAMENTAL "FACT OF LIFE"

GOOD FEEDBACK PERFORMANCE

( IE. $GK >> I$ OVER A GIVEN FREQUENCY RANGE )

IS POSSIBLE IF AND ONLY IF

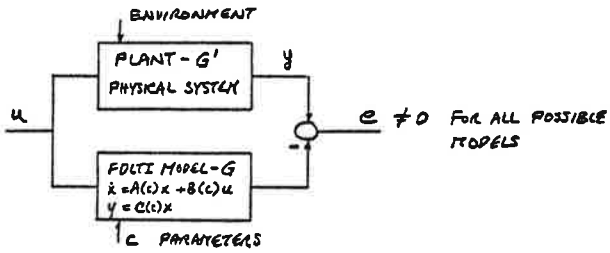
MODEL UNCERTAINTIES ARE SUFFICIENTLY SMALL

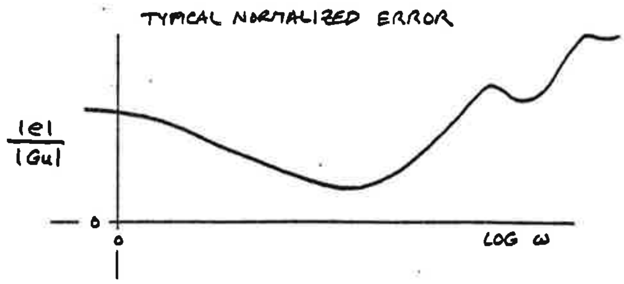
( IE. $\bar{\sigma}[L] < 1$ IN THE SAME FREQUENCY RANGE )

WE HAVE TWO OPTIONS

- KNOW OUR PLANT BEFORE HAND, OR
- LEARN IT AS WE GO

## UNCERTAINTIES IN FDLTI MODELS



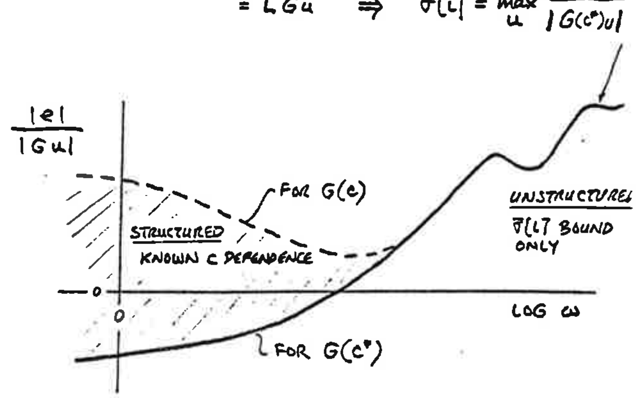




## TWO TYPES OF ERRORS

STRUCTURED - ERRORS WHICH CAN BE ELIMINATED BY ADJUSTING PARAMETERS c TO BEST MATCH THE PLANT

UNSTRUCTURED - ERRORS WHICH REMAIN

$$e = G'u - G(c^*)u$$
$$= (I+L)Gu - Gu$$
$$= LGu \Rightarrow \bar{\sigma}[L] = \max_u \frac{|e|}{|G(c^*)u|}$$



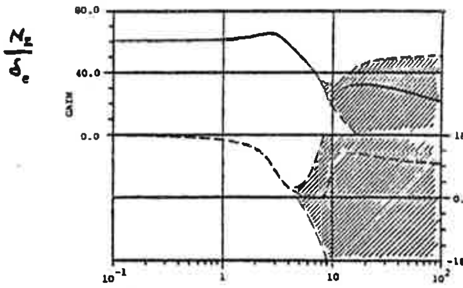

SOME MECHANISMS : NEGLECTED DYNAMICS
- ACTUATOR SERVOS
- RESONANCES
- DIFFUSIONS

TIME DELAYS
CONE-BOUNDED NONLINEARI
ETC

## MORE "FACTS OF LIFE"

- ADAPTIVE CONTROL CAN HELP WITH STRUCTURED ERRORS ONLY
- ADAPTIVE CONTROL MUST WORK IN THE PRESENCE OF UNSTRUCTURED ERRORS
- WE MUST KNOW (OR ASSUME) THE $\bar{\sigma}[L]$ CURVE BEFORE HAND
  - FREQ RANGE OVER WHICH GK CAN BE LARGE
  - BOUNDS ON GK OUTSIDE THAT RANGE

## F-8C UNSTRUCTURED UNCERTAINTIES




ω RAD/SEC

- ACTUATOR SERVOS ~ 12 R/S
- AIRFRAME STRUCTURAL RESONANCES ~ 40 R/S
- UNSTEADY AERODYNAMICS ~ 60 R/S
- SAMPLING DELAYS / PREFILTERS ~ 100 ,

## ADAPTIVE CONTROL TASKS

TASK 1

LEARN $C^*$ (EXPLICITLY OR IMPLICITLY)

TASK 2

IMPLEMENT CONTROL LAW WITH GK CONSISTENT WITH $\bar{\sigma}[L]$ CURVE AND WITH EXPECTED $C^*$ ERRORS

TASK 3

PROTECT AGAINST UNSTRUCTURED ERRORS

IE. DON'T LET THEM CONFUSE THE LEARNING PROCESS

DON'T MAKE GK LARGE WHERE $\bar{\sigma}[L] > 1$

## ADAPTIVE CONTROL SOLUTIONS VIA STOCHASTIC OPTIMIZATION

OPTIMIZATION PROBLEM

GIVEN $\dot{x} = A(c)\,x + B(c)\,u + \xi$

$y = C(c)\,x + \eta$

FIND $u = F\{\, y(\tau),\ \tau \le t \,\}$

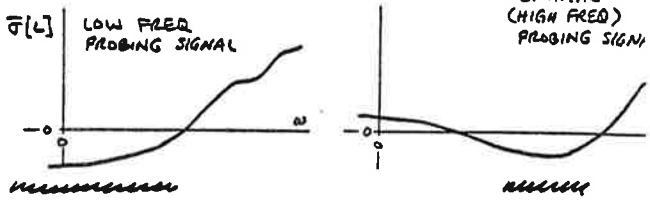
TO MINIMIZE

$$J = \mathop{E}_{\xi,\eta,c} \left\{ \frac{1}{T} \int_0^T L(x,u)\,dt \right\} \qquad T \to \infty$$

FEATURES : **THEORETICALLY OPTIMAL**

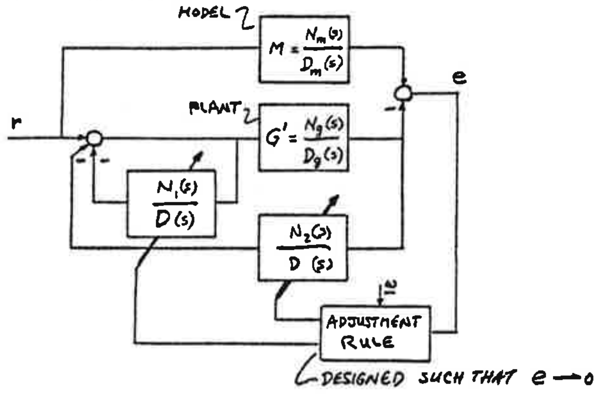
- GENERALLY UNSOLVABLE
- TASKS 1 & 2 ARE OPTIMALLY BLENDED (DUAL EFFEC
- TASK 3 IS NOT ADDRESSED

SOLUTIONS ARE (PROBABLY) EASILY CONFOUNDED BY UNSTRUCTURED UNCERTAINTIES



## ADAPTIVE CONTROL SOLUTIONS VIA STABILITY THEORY

MODEL REFERENCE STRUCTURE




FEATURES: **THEORETICALLY GLOBALLY STABLE**

- TASK 1 ACHIEVED IMPLICITLY BY ADJUSTING $N_1, N_2, D$ TO CANCEL PLANT

$$\frac{N_g D}{D_g(N_1+D) + N_g N_2} = \frac{N_m}{D_m}$$

- TASK 2 ACHIEVED BY APPROPRIATE SELECTION OF $M(s)$
- TASK 3 NOT ADDRESSED

SOLUTIONS KNOWN TO BE CONFOUNDED BY UNSTRUCTURED ERRO[RS]

## ASSUMPTIONS REQUIRED FOR STABILITY/CONVERGENCE PROOFS

PLANT: $G'(s) = \frac{N_g(s)}{D_g(s)} \longrightarrow \frac{k}{s^m}$ AS $|s| \to \infty$

ASSUMPTIONS:

(1) $N_g(s)$ IS STABLE

(2) $D_g(s)$ HAS KNOWN MAX DEGREE

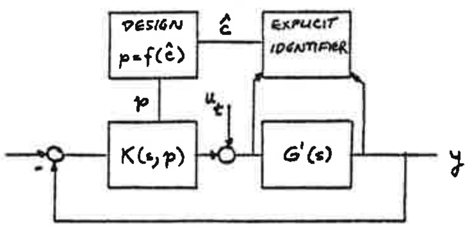
(3) $k$ HAS KNOWN SIGN

(4) RELATIVE DEGREE $m$ IS KNOWN

IMPLICATIONS:

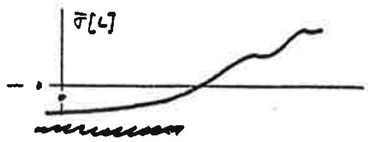
- ZERO UNSTRUCTURED UNCERTAINTIES
- HIGH GAIN FEEDBACK WITHOUT ADAPTATION WILL ACHIEVE MODEL FOLLOWING

## ADAPTIVE CONTROL SOLUTIONS VIA CERTAINTY EQUIVALENCE




FEATURES: **NO THEORETICAL PROPERTIES**

- TASK 1: ACHIEVED BY EXPLICIT IDENTIFICATION OF 'BEST FIT' C VECTOR
- TASK 2: ACHIEVED BY PRE-DESIGNED COMPENSATOR $K(s,p)$ WITH $p = f(\hat{c})$
- TASK 3: ACHIEVED BY
  - 'BEST FIT' FUNCTION AND TEST SIGNAL $u_t$ CONFINED TO DESIRED FREQ RANGE




  - $K(\cdot)$ AND $f(\cdot)$ CONSTRAINED TO MAKE GK LARGE ONLY AS ALLOWED BY $\bar{\sigma}[L]$

## F-8C PROGRAM CONCLUSIONS

- ADAPTIVE FLIGHT CONTROLS STILL CAN'T BEAT AIRDATA
  - MOST 'MODERN' CONCEPTS DID NOT WARRANT FLIGHT TESTING
  - THE MLE CONCEPT BARELY MATCHES AIRDATA PERFORMANCE
- ADAPTIVE FLIGHT CONTROLS NEED TEST SIGNALS (SUFFICIENTLY RICH INPUTS)
  - PILOTS HATE THESE
- ADAPTIVE FLIGHT CONTROL DESIGNERS SHOULD TURN TO PROBLEMS BEYOND THE CAPABILITY OF AIRDATA

## POTENTIAL ADAPTIVE PROBLEMS

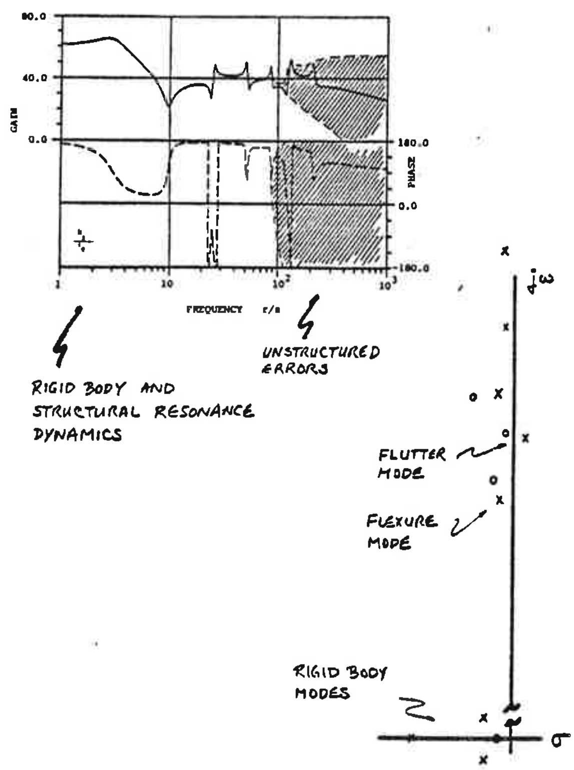
| | STATUS |
|---|---|
| **WHERE AIRDATA IS IMPRACTICAL** | |
| • SMALL MISSILES | ADAPT SYSTEMS EXIST |
| • REENTRY | BLENDED AERO/INERTIAL DATA WINS OUT |
| **WHERE AIRDATA WON'T WORK** | |
| • DIRECT FORCE MODES | IN-FLIGHT CAL WINS OUT |
| • FLEXURE/FLUTTER CONTROL | WIND TUNNEL TESTS LOOK GOOD |
| **PIE-IN-THE-SKY** | |
| • SURFACE RECONFIGURATION | DHYB |
| • LARGE SPACE STRUCTURES | |

## FLEXURE/FLUTTER CONTROL

- AIRPLANES HAVE SPEED LIMITS

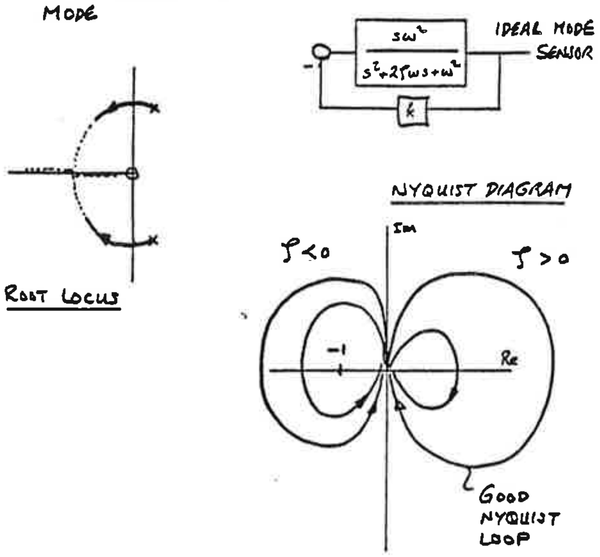

- ACTIVE CONTROL CAN EXTEND LIMITS BY AUGMENTING AEROELASTIC DAMPING
  - DYNAMICS BETWEEN 10 - 1000 R/SEC
  - HIGHLY DETAILED, POORLY KNOWN
  - NO VIABLE CONTROL SOLUTIONS TODAY

## FLEXURE/FLUTTER MODELS

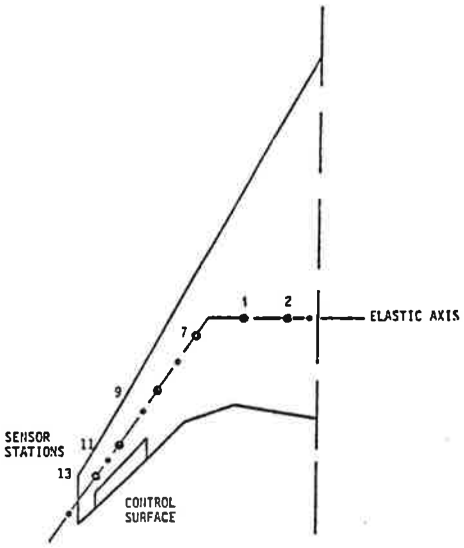



## IDEALLY FLEXURE/FLUTTER CONTROL IS EASY

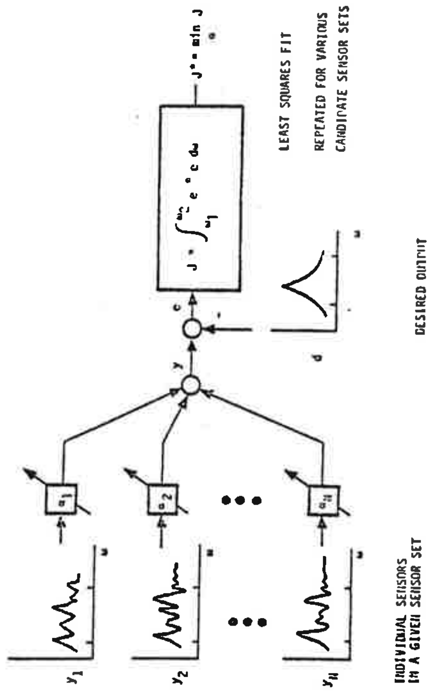
- SENSE EACH RESONANCE INDIVIDUALLY
- CLOSE SIMPLE RATE LOOPS AROUND EACH MODE



## IDEAL MODE SENSORS ARE SYNTHESIZED FROM VARIOUS REAL SENSORS




## IDEAL SENSOR SYNTHESIS




## ADAPTATION IN FLUTTER CONTROL ?

1) FLUTTER DETECTION

   IS A MODE UNSTABLE ?
   IF SO, WHICH ONE ?

2) CONTROL LAW DESIGN

   GAIN & PHASE COMPENSATION WITH
   GOOD SENSOR SYNTHESIS AVAILABLE

3) ON-LINE SENSOR SYNTHESIS

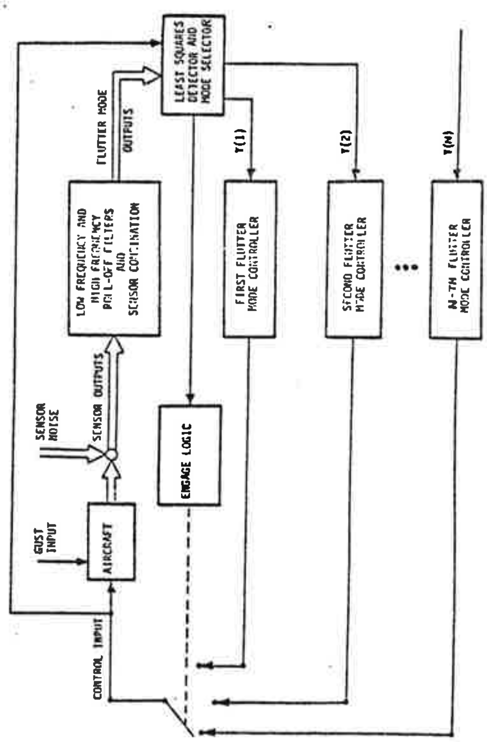
MAIN ISSUE

SPEED OF RESPONSE

STATE OF THE ART

- STEPS (1) & (2) FEASIBLE IN SIMULATIONS AND WIND TUNNEL TESTS
- STEP (3) IS CURRENT RESEARCH

## SIMPLE ADAPTIVE FLUTTER CONTROL



RAW ACCELEROMETERS

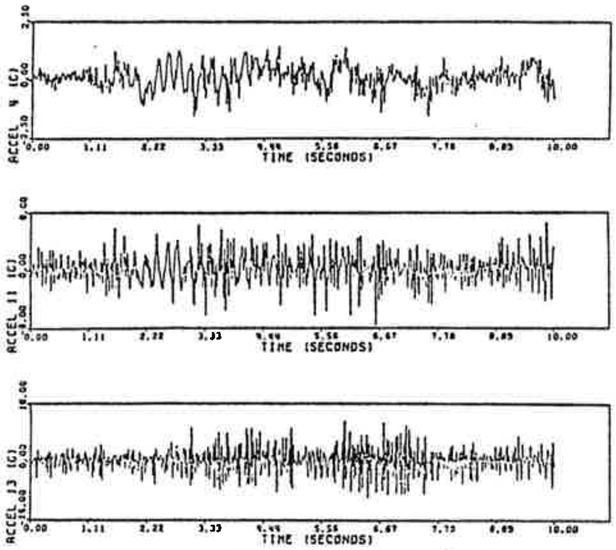



Figure 33. Transient Responses for Run 7 of Table 13 (MLE, Case 1, 1.3 $V_f$) (continued)

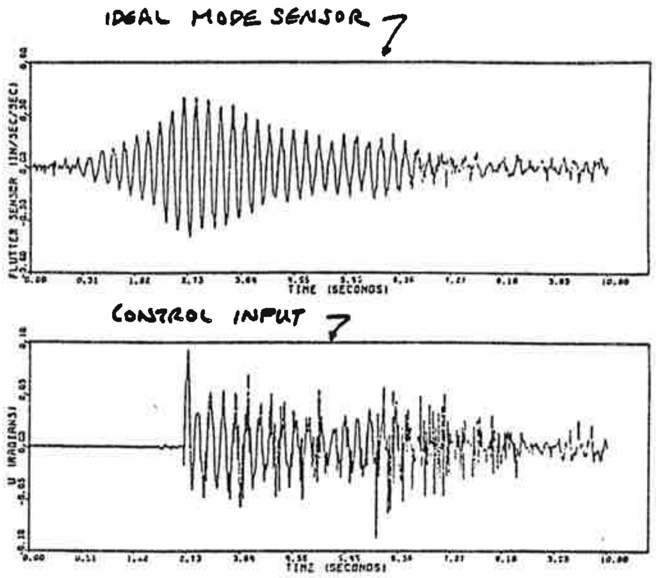



Figure 33. Transient Responses for Run 7 of Table 13 (MLE, Case 1, 1.3 $V_f$) (continued)

## PROGRESS IN ADAPTIVE THEORY

- OLD ENGINEERING FIXES GET SERIOUS THEORETICAL ATTENTION
  - DEADZONES — Pederson, Orlicki
  - RETARDATION — Kreisselmeier, Ioannou, Kaufman
  - SAMPLE RATES — Astrom etal, Rohrs
- NEW APPLICATION OF OLD THEORETICAL TOOLS
  - AVERAGING ANALYSES — Astrom, Krause
- REFORMULATIONS
  - INCLUSION OF UNSTRUCTURED UNCERTAINTIES — Kosut, Johnson

## CONCLUSIONS

BAD NEWS

WE LOST THE COMPETITION TO SENSOR BUILDERS FOR ALL CONVENTIONAL FLIGHT CONTROL PROBLEMS

GOOD NEWS

THERE ARE A FEW PROBLEMS WHERE OUR TECHNOLOGY OFFERS HOPE

BAD NEWS

WE CAN'T SOLVE THESE PROBLEMS (YET)

GOOD NEWS

THE THEORY IS COMING ALONG TO HELP !

# Self tuning control of the dissolved oxygen concentration in activated sludge systems

Gustaf Olsson and Lars Rundqwist

Department of Automatic Control
Lund Institute of Technology
Lund, Sweden

The activated sludge process is recognized as the most common and major unit process for the reduction of organic waste. An overview of of the control problems is found in [1].

A fully structured model of an activated sludge system is very complex and includes the following phenomena,

* the degradation of degradable pollutants, containing both organic carbon, phosphorus and nitrogen;
* cell growth and basal metabolism;
* the oxygen requirements of the system;
* the flow regime in the aeration basin;
* the representation of the settler and clarifier performance;
* the effect of secondary parameters, such as temperature, pH and toxic or inhibitory substances.

The goal for the reactor operation is of course to degrade the degradable pollutants. However, the operation must be such that organisms with the preferred floc formation are produced, thus giving desired clarification and thickening properties. Otherwize the operation of the system will fail, even if the degradation is efficient.

The DO concentration is an essential variable of the activated sludge process. It has a significant influence on both the plant operation economy and on the biological activity, and consequently on the quality of the effluent water.

A detailed derivation of the equations can be found in [1]. In a dispersed plug flow reactor the resulting DO dynamics can be described by

$$\frac{\partial c}{\partial t} = E\frac{\partial^2 c}{\partial z^2} - v\frac{\partial c}{\partial z} + k_L a(c^s - c) - R \qquad (1)$$

where

$z$=length along the reactor

$c(z,t)$=dissolved oxygen concentration

$c^s$=dissolved oxygen saturation concentration

$k_La$=oxygen mass transfer rate= $f(u)$, $u$ = air flow rate

$E$=dispersion coefficient

$v$=stream velocity

$R$=oxygen uptake rate (respiration rate)

The phenomena influencing the DO concentration have widely different timescales.

The control of DO as a physical variable does not require any in-depth knowledge of the microbial dynamics. The problem of finding the right DO set-point has been discussed e.g. in [3].

There are some important reasons for self-tuning control. The oxygen transfer rate is approximately proportional to the control signal, the air flow rate. Therefore a self-tuner can compensate for different "time constants" at different operating levels. Moreover, the oxygen transfer rate is time varying on a day-to-day time scale. The respiration R varies on an hourly time scale and is the main reason for control. However, R is interesting to know for other reasons. This can be part of the estimation scheme of a self-tuner. If the fact is used, that $k_La$ and R vary in different time scales they can be estimated simultaneously.

Since last year full scale experiments of DO control have been performed at the Käppala wastewater treatment plant at Lidingö, outside Stockholm. The plant serves the northern suburbs of Stockholm and has a flow rate of about 2-4 $m^3$/sec. A Novatune controller has been installed to take care of both the DO control loop and the air production system.

The figure shows some important features of the control. A constant setpoint of the DO concentration is given to the STR, sampled every 10 minutes. The DO sensor is fed into the controller and the control signal is cascaded with a local analog controller to adjust a throttle valve for the air flow rate. A limiting switch tells the STR if the valve saturates.

The pressure control is currently based on constant pressure set-point. It is kept via guide vanes on 3 of the 6 compressors. This does not give full control authority, and discontinuities of the control signals cannot be avoided. A pressure optimized will be added as soon as a sensor of the throttle valve angle can be measured. Then the pressure will be minimized so as to keep the valve as open as possible.

The results hitherto are encouraging but the evaluations of the biological properties due to control have just started. An expansion of the control to the whole activated sludge system will be made during the Fall of 1984.

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## SELF TUNING CONTROL OF THE DISSOLVED OXYGEN CONCENTRATION IN ACTIVATED SLUDGE SYSTEMS

GUSTAF OLSSON

LARS RUNDQWIST

## C O N T E N T

→ INTRODUCTION

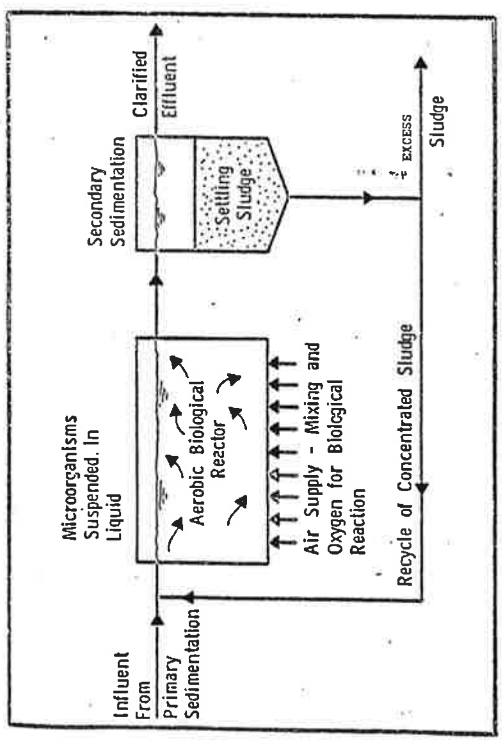
DISSOLVED OXYGEN DYNAMICS

WHY SELF TUNING CONTROL ?

THE KÄPPALA PLANT

EVALUATION OF RESULTS

## THE ACTIVATED SLUDGE PROCESS




## THE ACTIVATED SLUDGE PROCESS

INFLUENT WATER CONTAINS

- BIODEGRADABLE POLLUTANTS
- NON_BIODEGRADABLE POLLUTANTS
  - CHEMICALS
  - TOXIC MATERIAL
  - INERT MATERIAL

MICROORGANISMS REACT WITH POLLUTANTS AND OXYGEN

TO FORM MORE CELL MASS
CARBON DIOXIDE
WATER

OXYGEN IS SUPPLIED FROM DIFFUSERS

## INCENTIVES FOR CONTROL

- RISING OPERATING COSTS ( 600 % BETWEEN 1971 AND 1984)
  - ENERGY (AIR SUPPLY; PUMPING)
  - CHEMICALS
  - PERSONNEL
- STRICTER EFFLUENT CONTROL
- TIME VARYING INFLUENT
  - HYDRAULICS
  - CONCENTRATION
  - COMPOSITION

## POSITIVE DRIVING FORCES

- BETTER PROCESS KNOWLEDGE
- IMPROVED INSTRUMENTS
- COMPUTER COSTS AND COMPUTER PERFORMANCE
- BETTER CONTROL METHODS

## CONTENT

- INTRODUCTION
- → DISSOLVED OXYGEN DYNAMICS
- WHY SELF TUNING CONTROL ?
- THE KAPPALA PLANT
- EVALUATION OF RESULTS

## THE DISSOLVED OXYGEN CONCENTRATION

| | |
|---|---|
| ECONOMY | - MINIMIZE THE CONCENTRATION |
| SET POINT | - FORMATION OF DIFFERENT ORGANISMS |
| | - LIMITATION OF GROWTH |
| MIXING | - DO CONCENTRATION AND MIXING ARE COUPLED |
| | - FLOC FORMATION |
| MULTI REACTOR SYSTEM | - ANOXIC AND OXIC ZONES IN SERIES |
| PROFILES OF DO | - CAN GENERALLY NOT CONTROL THE PROFILE ALONG THE REACTOR |

## DISSOLVED OXYGEN DYNAMICS

IN COMPLETE MIX REACTOR - MASS BALANCE

$$\frac{dc}{dt} = \frac{Q_{in}}{V} \cdot c_{in} - \frac{Q_{out}}{V} \cdot c + \alpha \cdot u\,(c^{*} - c) - R$$

OXYGEN TRANSFER          RESPIRATION

OXYGEN TRANSFER

$\alpha$ VARIES SLOWLY (DAYS - WEEKS)

RESPIRATION

R DEPENDS ON BIOLOGICAL GROWTH AND MAINTENANCE

VARIES QUICKLY (MINUTES - HOURS)

## CONTENT


INTRODUCTION

DISSOLVED OXYGEN DYNAMICS

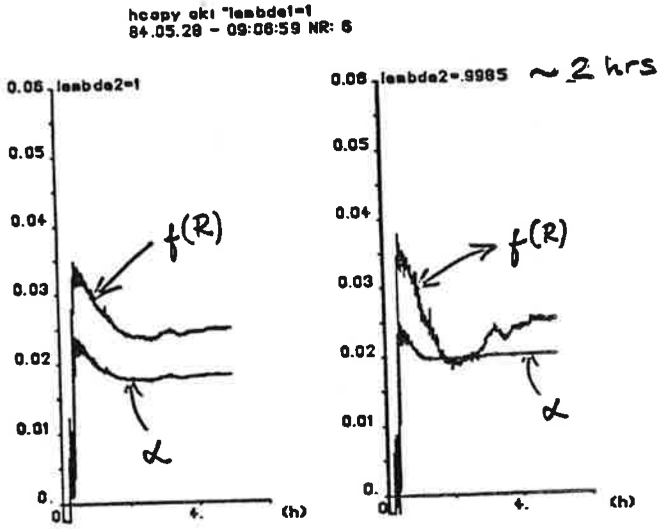
→ WHY SELF TUNING CONTROL ?

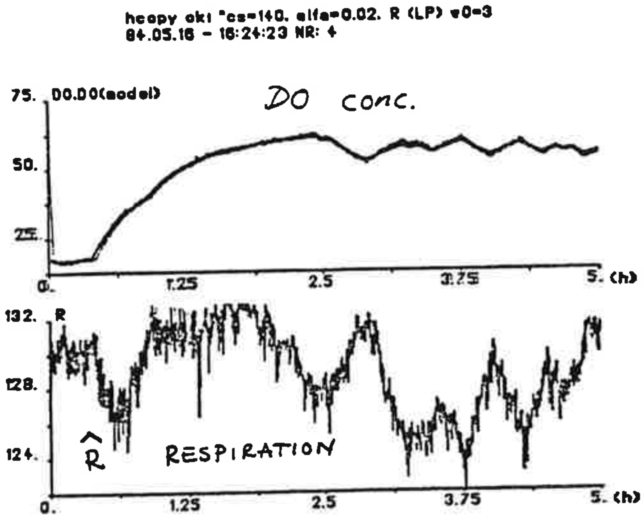
THE KAPPALA PLANT

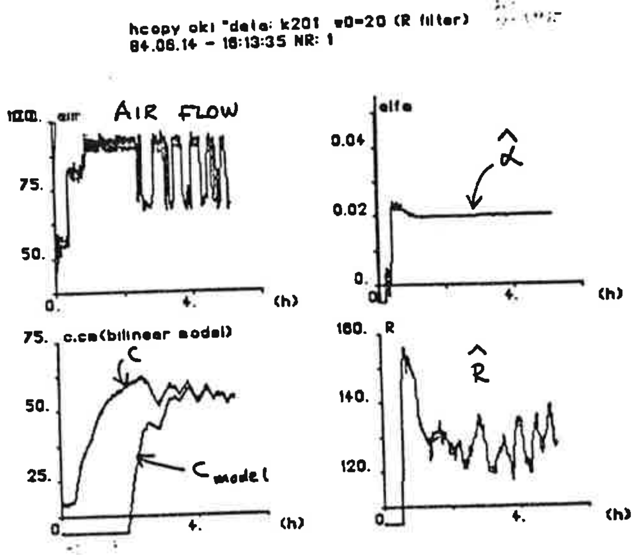
EVALUATION OF RESULTS


## WHY SELF-TUNING CONTROL OF DO?

- BILINEAR SYSTEM - "TIME CONSTANT" VARIES WITH OPERATING LEVEL
- $\alpha$ VARIES SLOWLY
- WANT TO KNOW THE RESPIRATION FOR OTHER PURPOSES; ESTIMATE FROM THE MASS BALANCE!
- FEED FORWARD SIGNALS CAN BE INTRODUCED FROM INFLUENT (COD; TOC )






## C O N T E N T

INTRODUCTION

DISSOLVED OXYGEN DYNAMICS

WHY SELF TUNING CONTROL ?

THE KÄPPALA PLANT

EVALUATION OF RESULTS

## T H E  K Ä P P A L A  P L A N T

- SERVES THE NORTHERN PART OF STOCKHOLM
- ABOUT 500 000 PERSONS
- INFLUENT FLOW RATE 2-4 $M^3$/SEC
- SIX PARALLELL BASINS
- ELECTRIC BILL ABOUT 1 MKR/YEAR

C O N T E N T

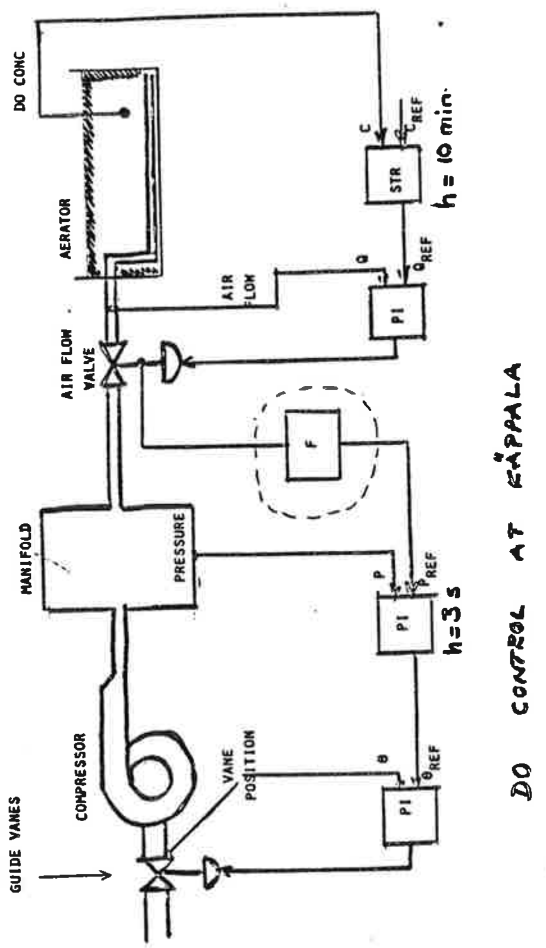

INTRODUCTION

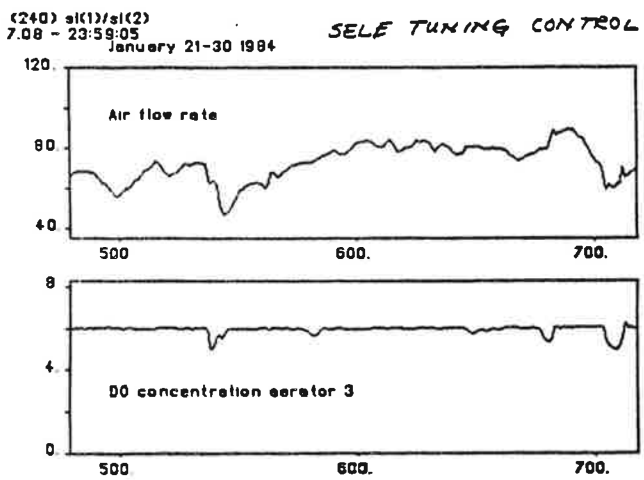
DISSOLVED OXYGEN DYNAMICS

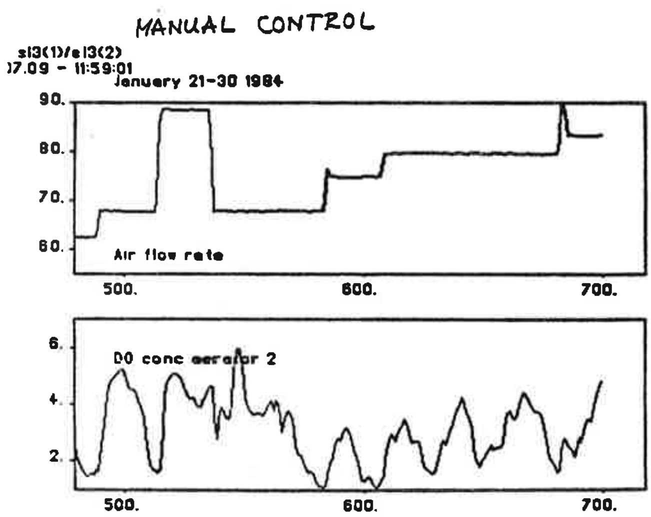
WHY SELF TUNING CONTROL ?

THE KÄPPALA PLANT

➡ EVALUATION OF RESULTS

## EVALUATION OF RESULTS

- CONTROL AUTHORITY
  - DISCONTINUITIES WHEN BLOWERS SWITCH ON/OFF
  - CERTAIN RANGES DIFFICULT
- THROTTLE VALVE
  - POSITION HAS TO BE MEASURED
- FINDING THE PROPER PRESSURE
- WATER QUALITY
  - BEING EVALUATED BY INSTITUTE OF SURFACE CHEMISTRY
- DETECTION OF TOXIC INPUTS
- EXPANSION TO ALL SIX REACTORS

# Adaptive pole placement for robots and servomechanisms

H. Elliott and R. Spada

Department of Electrical and Computer Engineering
University of Massachusetts
Amherst, Ma. 01003 USA

Robots or mechanical manipulators are multi-degree of freedom systems whose dynamics are often nonlinear. Furthermore, because such systems pick up and move loads of various shapes and weights, the dynamics can also be viewed as time varying. The difficulties of working with nonlinear dynamics and time variation lead researchers to consider adaptive control as an alternative. Many recent papers have applied adaptive control schemes originally developed for unknown linear dynamics to such systems[1]-[4]. In this presentation we will begin by showing that this is a potentially dangerous design methodology. That is, adaptive controllers of this type will not perform as expected if the nonlinearities are dominant.

With this as motivation, we then develop a new approach to adaptive control of manipulators where the controller adapts to load changes but not nonlinearities. Rather the nonlinearities are included in design of the adaptive control law. The approach first uses nonlinear feedback to cancel the effect of the nonlinearities. Linear feedback is then used to place closed loop poles. The controller is intended for digital computer implementation and is discrete in nature. Most importantly, the design is based upon discrete time model for the nonlinear dynamics which is derived using Euler approximations for derivatives.

As a step toward understanding the stability and performance properties of such an adaptive scheme, one must first determine the stability and performance properties of the corresponding fixed computer control algorithm designed using the same Euler approximation model. To this end, we restrict our attention to the case of a single link or single degree of freedom manipulator. Assuming torque to be supplied by a D.C. motor, this boils down to design of a D.C. servo system. However, because the link represents an asymmetric load rotating in a gravitational field the resulting model is still nonlinear. In the case where the link rotates in a horizontal plane the model becomes linear. By considering this simple case of computer control of a linear servo, it is shown that fixed computer control algorithms for pole placement, designed using Euler approximation models are stable for a very wide range of model parameters, sample rates and closed pole locations.

Furthermore, it is shown that considerable performance improvements can be obtained when designing zero cancelling controllers such as are used in model matching. In particular the problems which arise because sample data models have zeros on or near the unit circle can be avoided. Since many adaptive control schemes are based upon fixed zero cancelling control strategies these results are also of potential importance in the design of adaptive sample data systems.

During the course of the presentation, simulations will be presented demonstrating the potential performance of the new nonlinear adaptive control algorithm on a three degree of freedom manipulator. In addition experimental results will be presented for a hardware implementation on a one degree of freedom servo system in both the linear and nonlinear cases.

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H. Elliott, T. Depkovich, J. Kelly, B. Draper, "Nonlinear Adaptive Control of Mechanical Linkage Systems with Application to Robotics," Proceedings of 1983 ACC, June, 1983, San Francisco, pp. 1050-1055.

G.C. Goodwin and K.S. Sin, <u>Adaptive Filtering Prediction and Control</u>, Prentice Hall, Englewood Cliffs, 1984.

# ADAPTIVE POLE PLACEMENT FOR ROBOTS AND SERVO MECHANISMS

by

H. Elliott  R. Spada
Dept of ECE
Univ of Mass.
Amherst, Ma 01003

## OUTLINE OF PRESENTATION

I. INTRODUCTION

II. LINEAR VS NONLINEAR ADAPTIVE CONTR
Case Study - 1 DOF

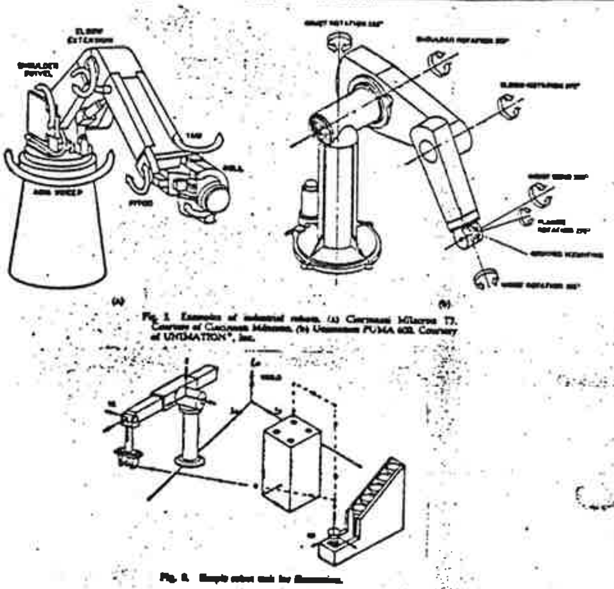
III. GENERAL ALGORITHM FOR CONTROL OF MANIPULATORS
2 DOF Case

IV. 3 DOF Case and Engineering simplifications

V. Issues in Stability Analysis

VI

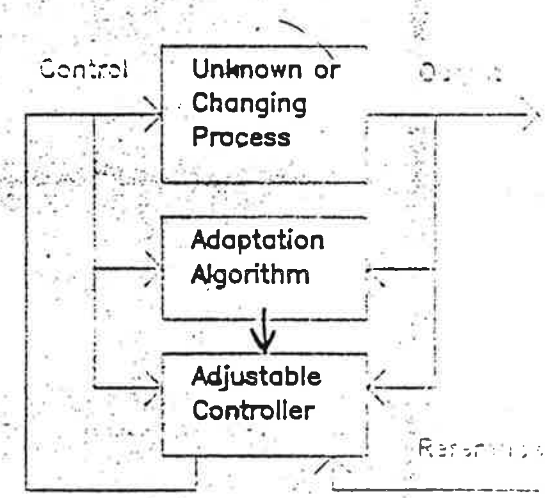
VII. Experimental Results



Problems In Manipulator Control

* Changing Dynamics
  load changes
  environmental changes
* Nonlinearities
  coordinate transformations
  dynamics

Why Adaptive Control?




* Changing Dynamics
* ~~[illegible]~~

## General Form of Nonlinear Dynamics

$$J(y)\ddot{y} + K\dot{y} + C(y,\dot{y}) + G(y) = u$$

- $y$ = vector of joint angles
- $u$ = vector of motor torques
- $J(y)$ = inertia matrix
- $K$ = damping matrix
- $C(y,\dot{y})$ = coriolis and centrifugal terms
- $G(y)$ = gravitational terms

## Control Problem

- Find $u(t)$ so that $y$ tracks reference trajectory $r$

## General Approach

Let control signal have two components:

$$u(t) = n(t) + l(t)$$

$n(t)$ — adaptively cancels nonlinearities

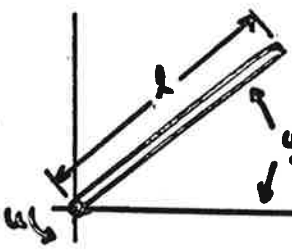
$l(t)$ — adaptively implements linear control strategy

## Advantages

[illegible]

# II. Comparison of Two Approaches

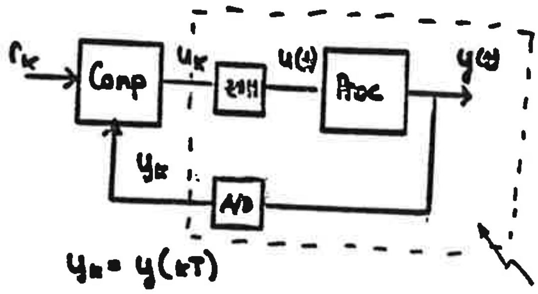
Single-Link Case



$J = \frac{ml^2}{3}$

$G = mgl$

$$J\ddot{y} + K\dot{y} + G\cos y = u$$




$y_k = y(kT)$

$u(t) = u_k \quad t \in [kT, (k+1)T)$

discrete model

$$\dot{y} \cong (y_k - y_{k-1})/T$$

$$\ddot{y} \cong (y_{k+1} - 2y_k + y_{k-1})/T^2$$

$$\frac{J}{T^2}(y_{k+1} - 2y_k + y_{k-1}) + \frac{K}{T}(y_k - y_{k-1}) + G\cos y_k = u$$

$$\boxed{\phi_k^T \theta = u_k}$$ Nonlinear Model

$$\phi_k^T = [y_{k+1}, y_k, y_{k-1}, \cos y_k]$$

$$\theta^T = \left[\frac{J}{T^2}, \frac{K}{T} - \frac{2J}{T^2}, \frac{J}{T^2} - \frac{K}{T}, G\right]$$

$$\boxed{\tilde{\phi}_k^T \tilde{\theta} \cong u_k}$$ Linearized Model

$$\tilde{\phi}_k^T = [y_{k+1}, y_k, y_{k-1}], \text{[illegible]}]$$

$$\tilde{\theta} = ?$$

## Deadbeat (Minimum Variance) Control

Assume $y^*_k$ = seq. to be tracked
know one step ahead
$(r_k = y^*_{k+1})$

Choose

$$\boxed{u_k = \phi_k^{*T}\theta} \qquad k \geq 0$$

$$\phi_k^{*T} = [\, y^*_{k+1}, y_k, u_{k-1}, \cos y_k ]$$

Guarantees $y_k = y^*_k \quad k \geq 1$

For linearized model

$$\boxed{u_k = \tilde{\phi}_k^{*T}\tilde{\theta}}$$

$$\tilde{\phi}_k^{*T} = [y^*_{k+1}, y_k, y_{k-1}], u_{k-1}]$$

## Adaptive Implementation

$$\boxed{\begin{aligned} u_k &= \phi_k^{*T}\theta_k \\ u_k &= \tilde{\phi}_k^{*T}\check{\theta}_k \end{aligned}} \qquad \begin{aligned} \theta_k &= Est(\theta) \\ \tilde{\theta}_k &= Est(\tilde{\theta}) \end{aligned}$$

## Least Squares Estimation

$$e_k = u_k - \phi_k^T\theta_k \quad (0 = u_k - \phi_k^T\theta)$$

$$\theta_k = \theta_{k-1} + \frac{P_{k-2}\,\phi_{k-1}e_{k-1}}{\lambda + \phi_{k-1}^T P_{k-2}\phi_{k-1}}$$

$$P_{k-1} = \left[P_{k-2} + \frac{P_{k-2}\,\phi_{k-1}\phi_{k-1}^T P_{k-2}}{\lambda + \phi_{k-1}^T P_{k-2}\phi_{k-1}}\right]\frac{1}{\lambda}$$

$P_{-1} = \gamma I \quad \gamma > 0$

$0 < \lambda \leq 1$ (exponential weighting)

linear case $\phi_k, e_k \rightarrow \tilde{\phi}_k, \check{e}_k$

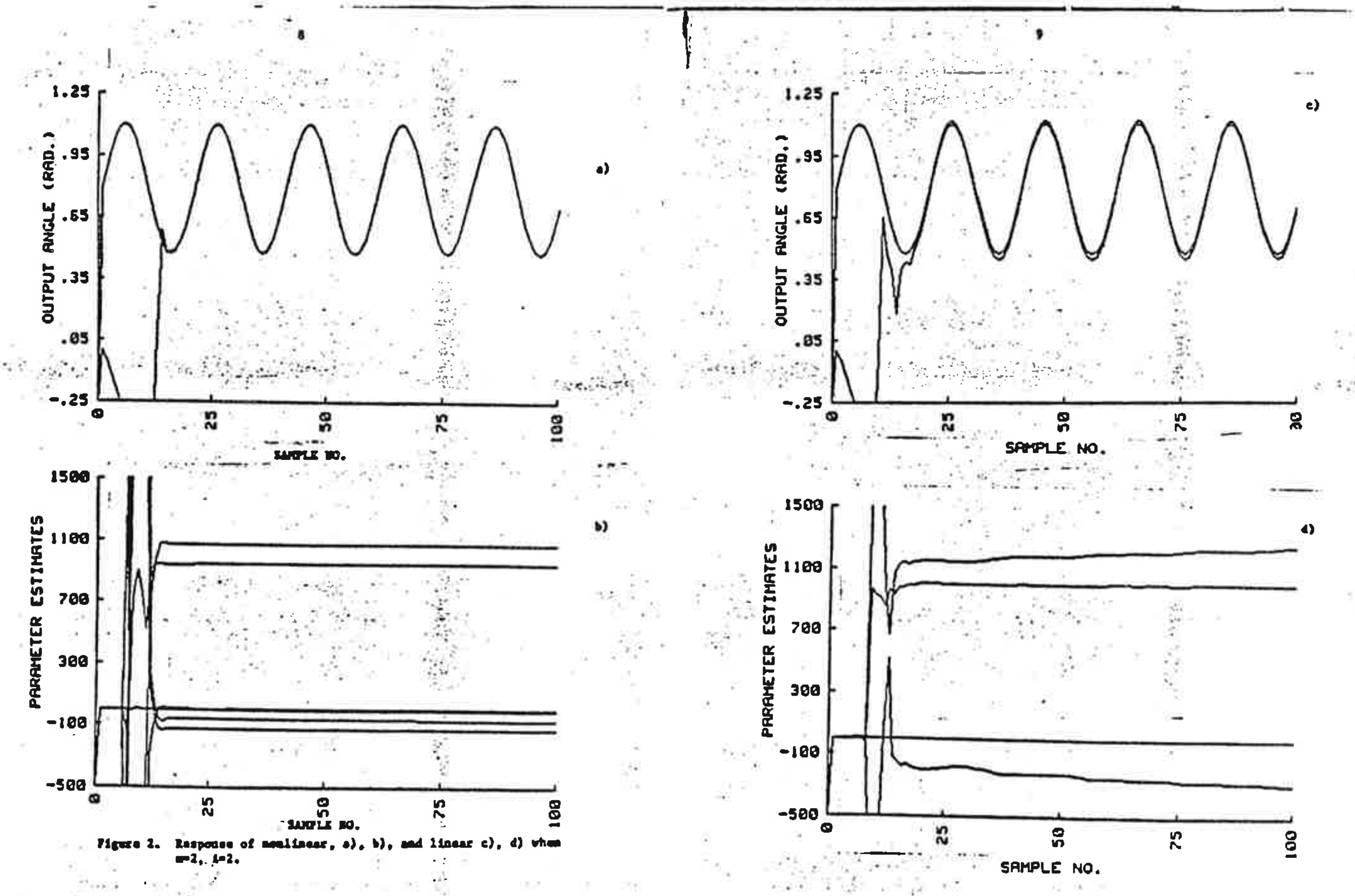



Figure 2. Response of nonlinear, a), b), and linear c), d) when m=2, λ=2.

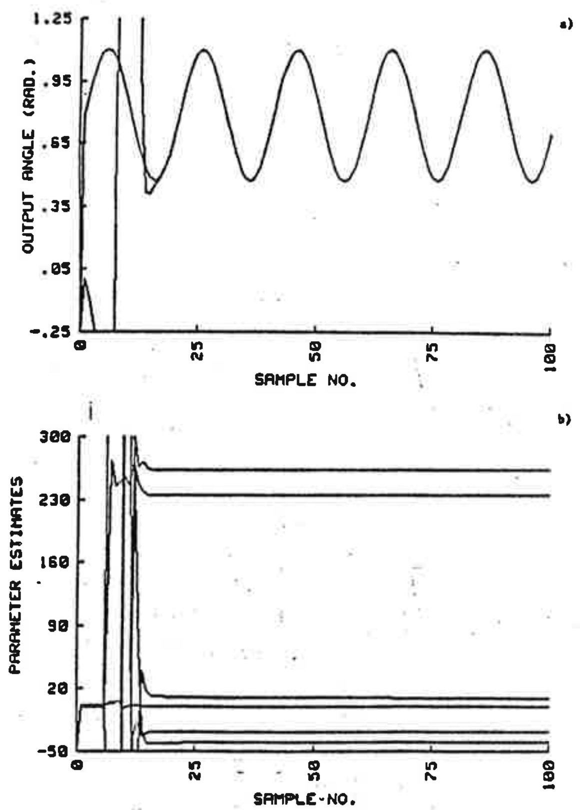
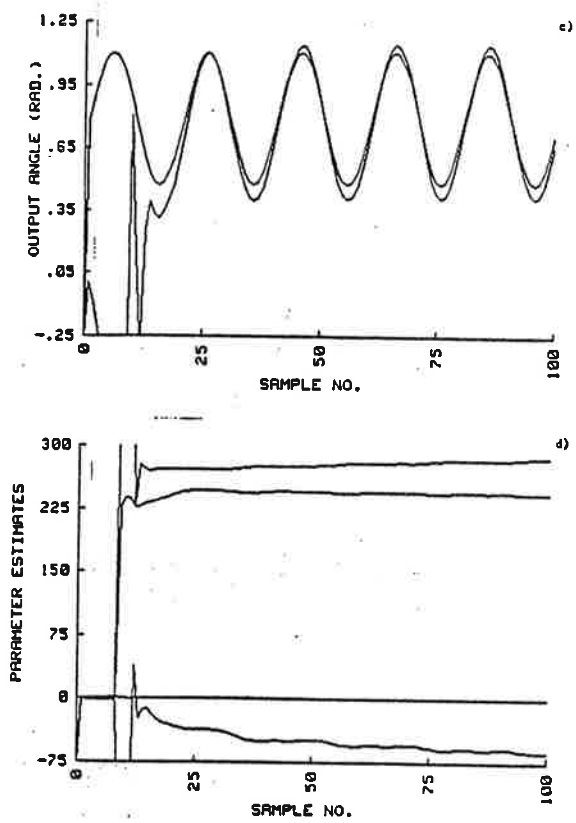



Figure 3. Response of nonlinear a), b), and linear c), d) when m=2, l=1.







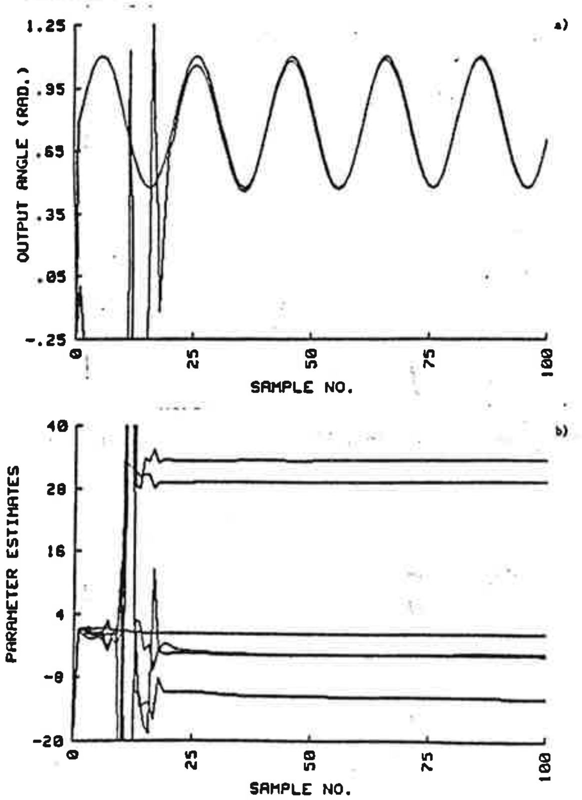
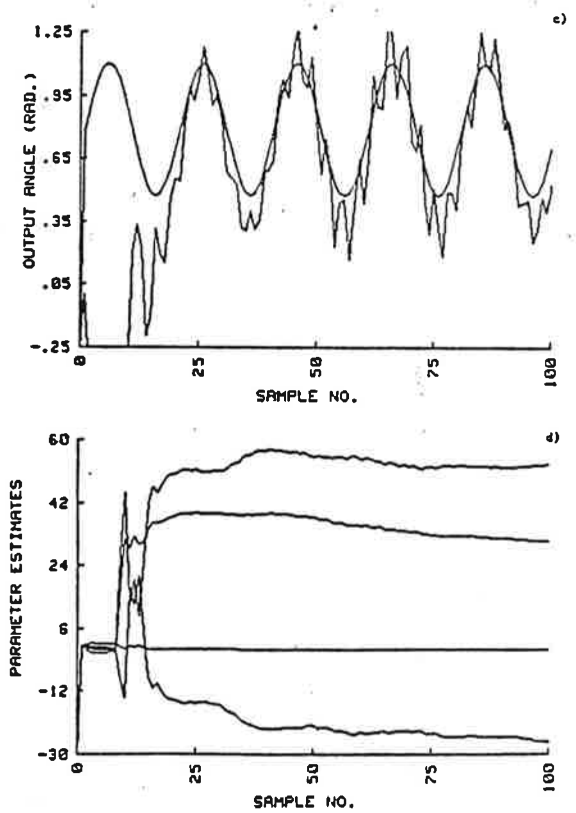




Figure 4. Response of nonlinear a), b) and linear c), d) when m=2, $l = \frac{1}{2}$.





## III. CONTROL OF TWO LINKS

$$\begin{bmatrix} d_{11} & d_{12}\cos(y_{2k}-y_{1k}) \\ d_{21}\cos(y_{2k}-y_{1k}) & d_{22} \end{bmatrix} A_{k+1}$$

$$+ \begin{bmatrix} d_{13} & 0 \\ 0 & d_{23} \end{bmatrix} V_k + \begin{bmatrix} d_{14} & 0 \\ 0 & d_{24} \end{bmatrix} S_k$$

$$+ \begin{bmatrix} d_{15} & 0 \\ 0 & d_{25} \end{bmatrix} Z_k = U_k$$

$$\boxed{J(Y_k)A_{k+1} + D_3V_k + D_4S_k + D_5Z_k = U_k}$$

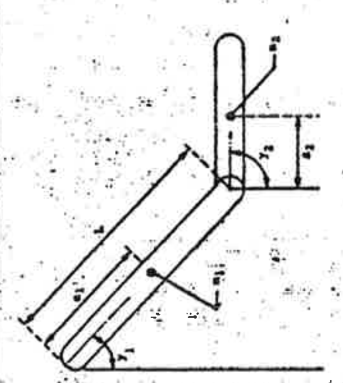
$$A_{k+1} = \begin{bmatrix} y_{1k+1} - 2y_{1k} + y_{1k-1} \\ y_{2k+1} - 2y_{2k} + y_{2k-1} \end{bmatrix} \times \frac{1}{T^2} = \begin{bmatrix} a_{1k+1} \\ a_{2k+1} \end{bmatrix}$$

$$V_k = \begin{bmatrix} y_{1k} - y_{1k-1} \\ y_{2k} - y_{2k-1} \end{bmatrix} \times \frac{1}{T} = \begin{bmatrix} V_{1k} \\ V_{2k} \end{bmatrix}$$

$$S_k = \begin{bmatrix} \sin y_{1k} \\ \sin y_{2k} \end{bmatrix} = \begin{bmatrix} S_{1k} \\ S_{2k} \end{bmatrix}$$

$$Z_k = \begin{bmatrix} \sin(y_{1k}-y_{2k})(y_{2k}-y_{2k-1})^2 \\ \sin(y_{2k}-y_{1k})(y_{1k}-y_{1k-1})^2 \end{bmatrix}$$

### Equations for Two-Link Planar Manipulator



$$\begin{bmatrix} (J_1 + m_1a_1^2 + m_2L^2) & m_2a_2L\cos(y_2 - y_1) \\ m_2a_2L\cos(y_2 - y_1) & J_2 + m_2a_2^2 \end{bmatrix} \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} + \begin{bmatrix} K_1 & \dot{y}_1 \\ K_2 & \dot{y}_2 \end{bmatrix}$$

$$+ \begin{bmatrix} (m_1a_1 + m_2L)g\sin y_1 \\ m_2a_2g\sin y_2 \end{bmatrix} + \begin{bmatrix} m_2a_2L\sin(y_1 - y_2) \\ m_2a_2L\sin(y_2 - y_1) \end{bmatrix} \begin{matrix} \dot{y}_2^2 \\ \dot{y}_1^2 \end{matrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

(Inertial Coupling; Viscous Friction; Gravity; Centifugal)



Can also be written as

$$\boxed{\begin{aligned} \phi_{1k}^T \theta_1 &= U_{1k} \\ \phi_{2k}^T \theta_2 &= U_{2k} \end{aligned}}$$

$$\theta_i^T = [d_{i1}, d_{i2}, \ldots, d_{i5}]$$

$$\phi_{ik} = [a_{ik+1}, a_{jk+1}, V_{ik}, S_{ik}, Z_{ik}]$$

$j=2$ if $i=1$
$j=1$ if $i=2$

### Typical Linear Model

$$Y_{k+1} + A_1Y_k + A_2Y_{k-1} = B_1U_k + B_2U_{k-1}$$

In simplest case
- $A_i$, $B_i$ diagonal
- 4 Parameters / row

General Case
- 8 Parameters / row

### MODEL MATCHING CONTROL

Desired Closed Loop System

$$Y_{k+1} + E_1Y_k + E_2Y_{k-1} = H_1R_k + H_2R_{k-1}$$

For $E_1 = E_2 = H_2 = 0$ $H_1 = I$ $R_k = Y^*_{k+1}$
get Deadbeat

Re-write Process Model $(A_{k+1} = Y_{k+1} - 2Y_k + Y_{k-1})$

$$Y_{k+1} = T^2 J^{-1}(Y_k)\left[U_k - D_3V_k - D_4S_k - D_5Z_k + \frac{1}{T^2}J(Y_k)(2Y_k - Y_{k-1})\right]$$

$$U_k = D_3V_k + D_4S_k + D_5Z_k + \frac{1}{T^2}J(Y_k)\left[(Y_{k-1} - 2Y_k) - E_1Y_k - E_2Y_{k-1} + H_1R_k + H_2R_{k-1}\right]$$

## Adaptive Implementation

1) Estimate $\theta_{ik}$ using the errors

$$e_{ik} = \phi_{ik}^T \theta_{ik} - U_{ik} \quad (0 = \phi_{ik}^T \theta_i - U_{ik})$$

and two parallel sequential least squares estimators (one for $\theta_{1k}$ one for $\theta_{2k}$)

Two estimation problems of size 5 (vs 4 or 8 for linear)
4 or 3

2) Use $\theta_{ik}$ to generate

$$J_k(Y_k) = Est(J(Y_k))$$
$$D_{ik} = Est(D_i) \quad i=3,4,5$$

Note: No divides of time varying quantities

3) Use

$$U_k = D_{3k}V_k + D_{4k}S_k + D_{5k}Z_k + \frac{1}{T^2} J_k(Y_k) [ Y_{k+1} - 2Y_{k-2} - E_1 Y_k - E_2 Y_{k+1} + H_1 R_k + H_2 R_{k+1} ]$$

## V. SIMULATION

Model

$$Y_{k+1} - Y_k + .29 Y_{k-1} = .29 R_k$$

$R_k$ = d.c. offset + sinusoids

T = .02

Link lengths = 1 foot

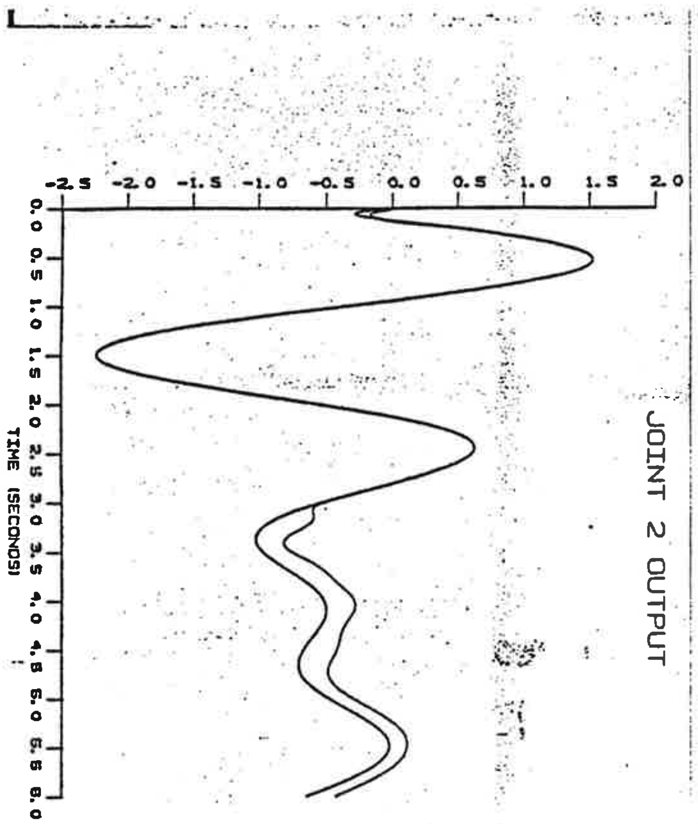
Masses = 1 lb mass   $l_2$ → 10 lbs at 3 sec.

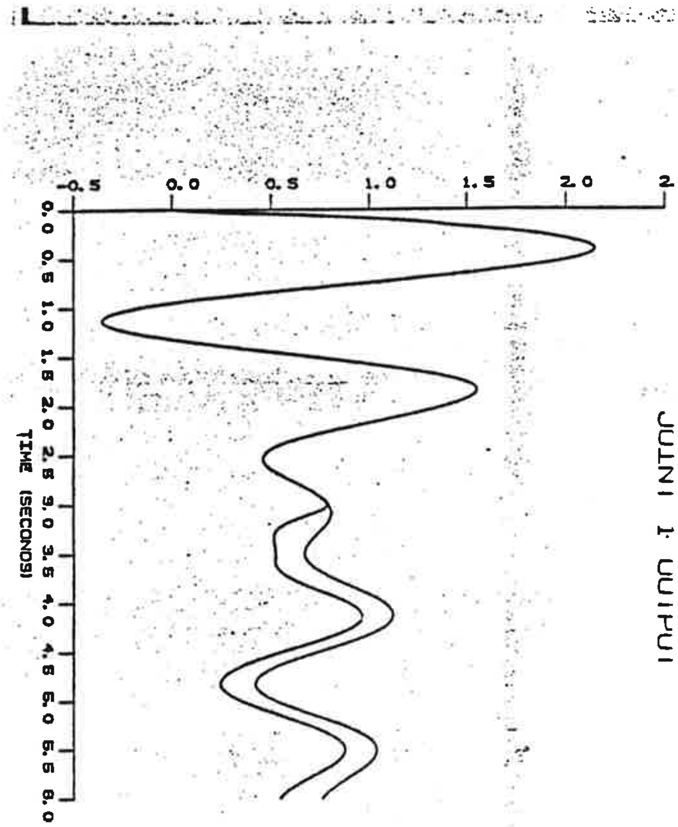
Damping = 1.5 ft-lb-sec

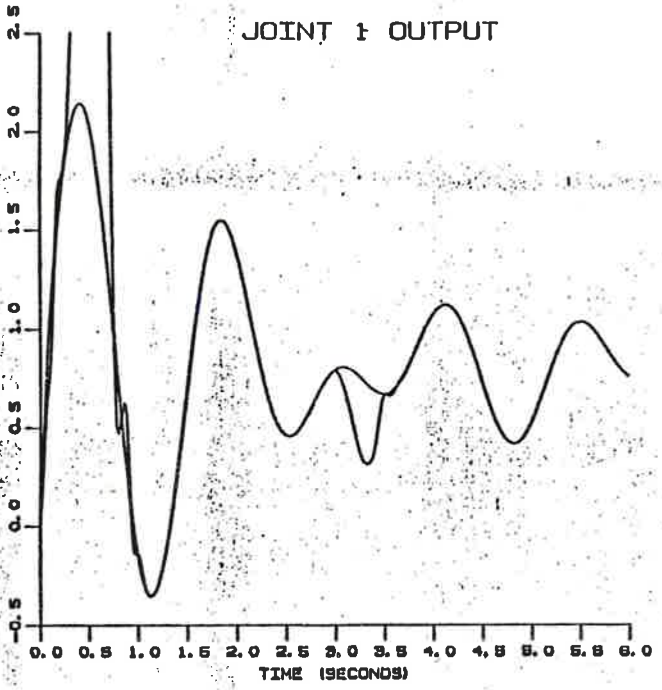
$\lambda = .95$

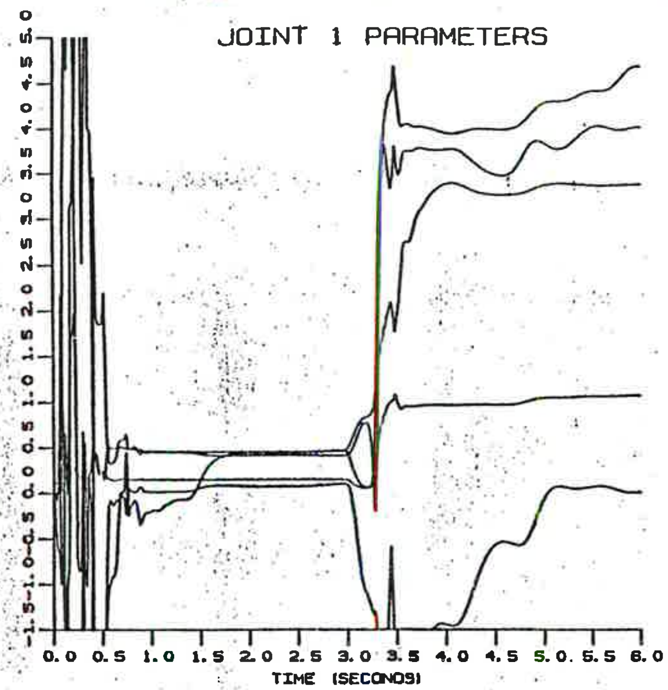
$\theta_{10} = [1 0 0 0 0]$

$\theta_{20} = [0 1 0 0 0]$




JOINT 1 OUTPUT
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0
TIME (SECONDS)
-0.5 0.0 0.5 1.0 1.5 2.0 2.5


JOINT 1 PARAMETERS
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0
TIME (SECONDS)
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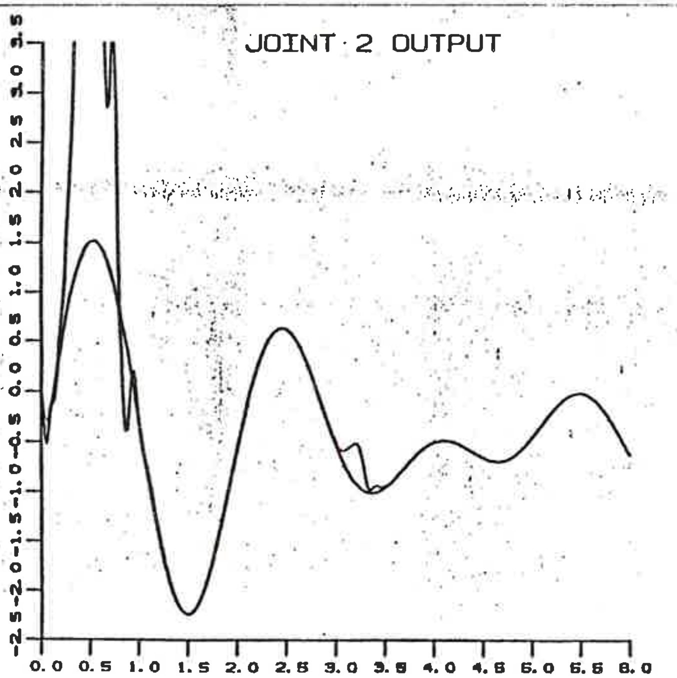
JOINT 2 OUTPUT
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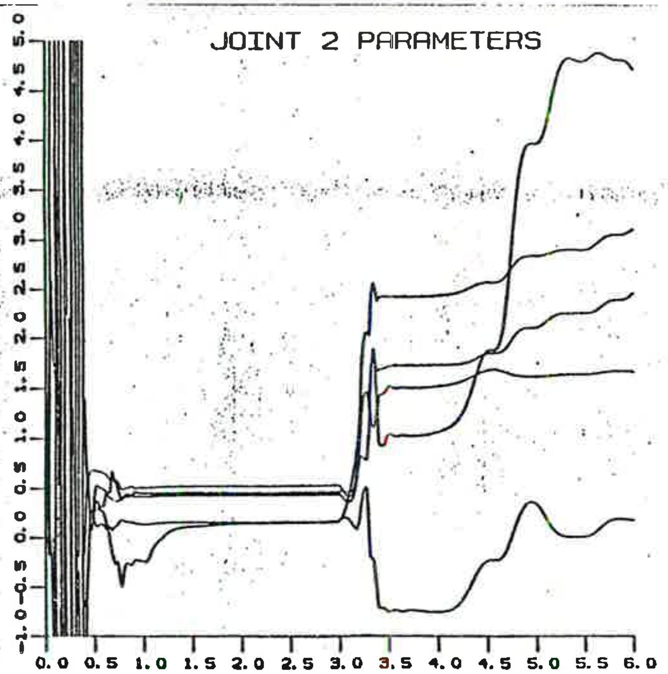
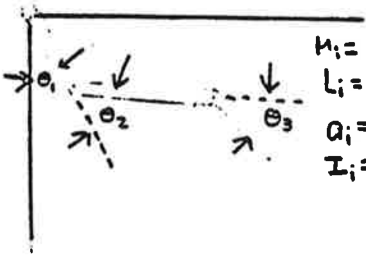

JOINT 2 PARAMETERS
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0
-1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0

## I. THREE LINK CASE:




$M_i$ = mass link i
$L_i$ = length link i
$a_i$ = dist. to c.g. link i
$I_i$ = mom. Inertia link i

**Eq 1**

$$D_{11}\ddot{\theta}_1 + D_{12}\ddot{\theta}_2 + D_{13}\ddot{\theta}_3 + D_{14}\left[c\theta_2(2\ddot{\theta}_1+\ddot{\theta}_2) - s\theta_2(\dot{\theta}_2^2+2\dot{\theta}_1\dot{\theta}_2)\right]$$
$$D_{15}\left[c(\theta_2+\theta_3)(2\ddot{\theta}_1+\ddot{\theta}_2) - s(\theta_2+\theta_3)(\dot{\theta}_1^2+\dot{\theta}_2^2+2(\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_1\dot{\theta}_3+\dot{\theta}_1\dot{\theta}_3)\right]$$
$$D_{16}\left[c\theta_2(2\ddot{\theta}_1+2\ddot{\theta}_2+\ddot{\theta}_3) - s\theta_2(\dot{\theta}_3^2+2\dot{\theta}_1\dot{\theta}_3+2\dot{\theta}_2\dot{\theta}_3)\right]$$
$$+ D_{17}\dot{\theta}_1 + D_{18}s\theta_1 + D_{19}s(\theta_1+\theta_2) + D_{1,10}s(\theta_1+\theta_2+\theta_3) = T_1$$

**Eq 2**

$$D_{21}\left[\ddot{\theta}_1+\ddot{\theta}_2\right] + D_{22}\ddot{\theta}_3 + D_{23}\left[c\theta_2(\ddot{\theta}_1) - s\theta_2(\dot{\theta}_1^2)\right]$$
$$+ D_{24}\left[c(\theta_2+\theta_3)(\ddot{\theta}_1) - s(\theta_2+\theta_3)(\dot{\theta}_1^2)\right]$$
$$+ D_{25}\left[c\theta_3(2\ddot{\theta}_1+2\ddot{\theta}_2+\ddot{\theta}_3) - s\theta_3(\dot{\theta}_3^2+2\dot{\theta}_1\dot{\theta}_3+2\dot{\theta}_2\dot{\theta}_3)\right]$$
$$+ D_{26}\dot{\theta}_3 + D_{27}s(\theta_1+\theta_2) + D_{28}s(\theta_1+\theta_2+\theta_3) = T_2$$

**Eq 3**

$$D_{31}\left[\ddot{\theta}_1+\ddot{\theta}_2+\ddot{\theta}_3\right] + D_{32}\left[c(\theta_2+\theta_3)\ddot{\theta}_1 - s(\theta_2+\theta_3)\dot{\theta}_1^2\right]$$
$$+ D_{33}\left[c\theta_3(\ddot{\theta}_1+\ddot{\theta}_2) - s\theta_3(\dot{\theta}_1^2-\dot{\theta}_3^2)\right]$$
$$+ D_{34}\dot{\theta}_3 + D_{35}s(\theta_1+\theta_2+\theta_3) = T_3$$

**Eq 1**

$D_{11} = M_1a_1^2 + M_2(L_1^2+a_2^2) + M_3(L_1^2+L_2^2+L_3^2) + I_1+I_2+I_3$

$D_{12} = M_2a_2^2 + M_3(L_2^2+a_3^2) + I_2+I_3$

$D_{13} = M_3a_3^2 + I_3$

$D_{14} = M_2L_1a_2 + M_3L_1L_2$

$D_{15} = M_3L_1a_3$

$D_{16} = M_3L_2a_3$

$D_{17} = K_1$

$D_{18} = M_1a_1g + M_2L_1g + M_3L_1g$

$D_{19} = M_2a_2g + M_3L_2g = gD'_{23}$

$D_{1,10} = M_3a_3g$

**Eq 2**

$D_{22} = D_{13}$

$D_{23} = D_{14} = L_1(M_2a_2+M_3L_2) = L_1D'_{23}$

$D_{24} = D_{15}$

$D_{25} = D_{16}$

$D_{26} = K_2$

$D_{27} = gD'_{23}$

$D_{28} = D_{1,10}$

**Eq 3**

$D_{31} = D_{13}$

$D_{32} = D_{15} = L_1(M_3a_3) = L_1D'_{32}$

$D_{33} = D_{16} = L_2D'_{32}$

$D_{34} = K_3$

$D_{35} = D_{1,10} = gD'_{32}$

## NET RESULT

**Estimate:**

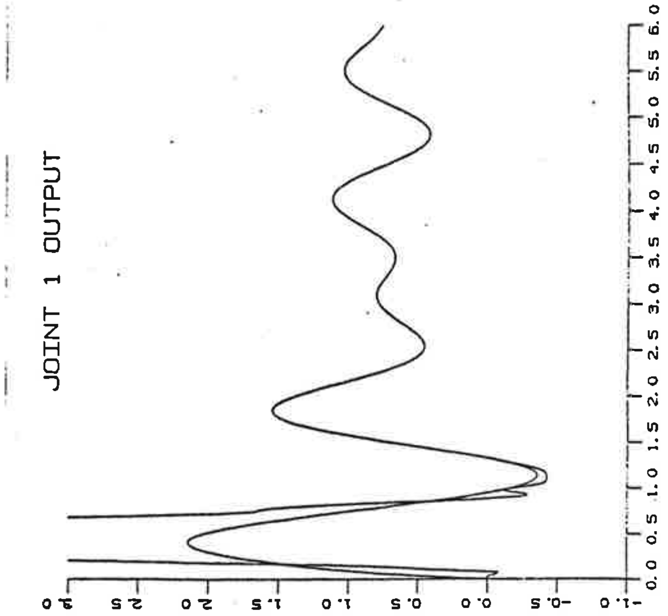
$\Theta_1 = [D_{11}, D_{17}, D_{17}]$

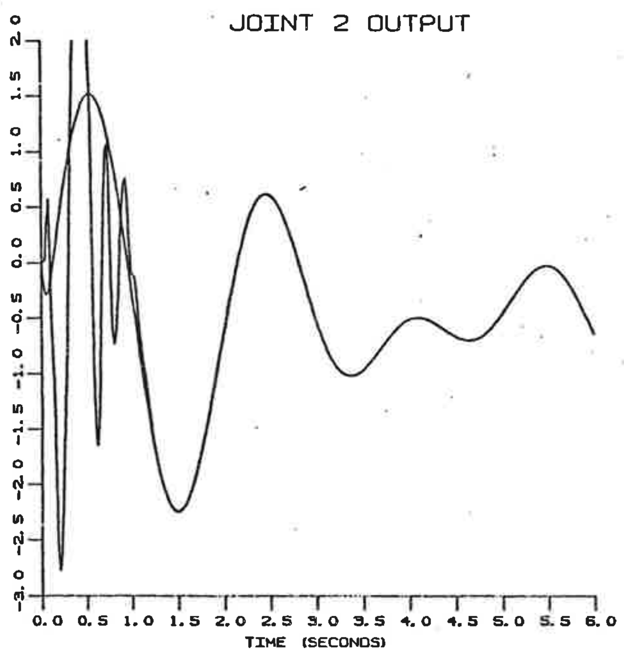
$\Theta_2 = [D_{21}, D'_{23}, D_{26}]$

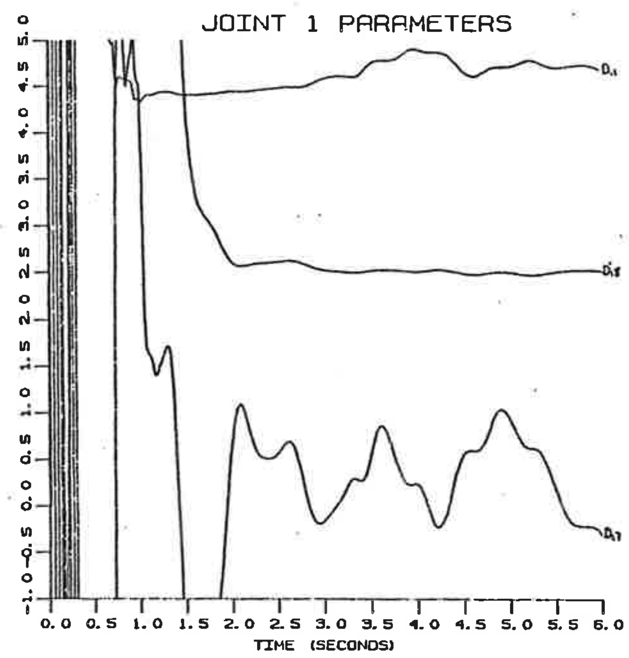
$\Theta_3 = [D_{31}, D'_{32}, D_{34}]$

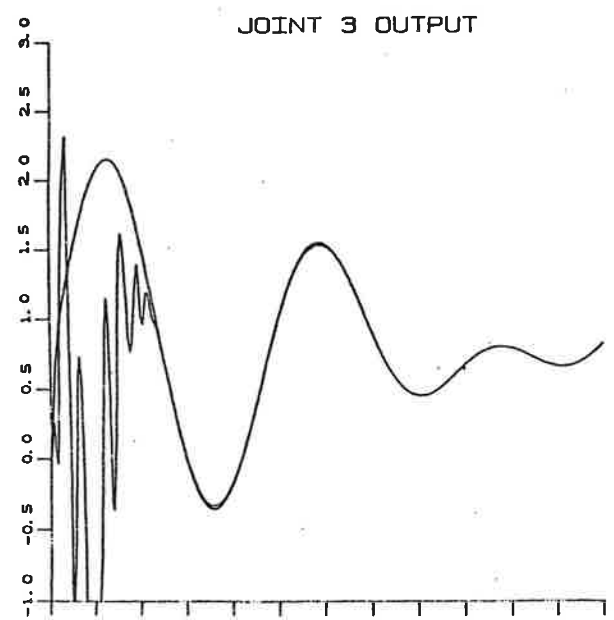
$X_i^T\Theta_i = Y_i$ → **Measurable Signals**

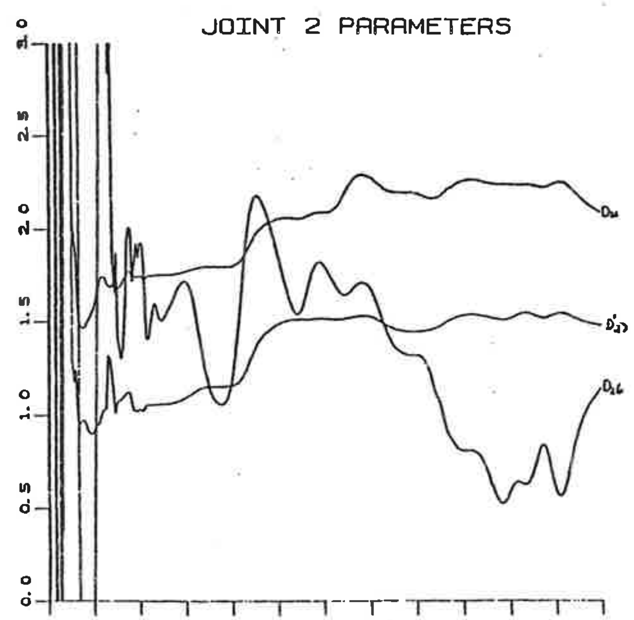
CONTROL STRATEGY
as before




JOINT 2 OUTPUT
TIME (SECONDS)


JOINT 1 PARAMETERS
TIME (SECONDS)


JOINT 3 OUTPUT


JOINT 2 PARAMETERS

## VI. DIGITAL CONTROLLER DESIGN USING EULER APPROX FOR SERVOS

Euler Approximation

$T\ddot{y} + K\dot{y} + \ldots$ [illegible]

$\ddot{y} \sim (y_{k+2} - 2y_{k+1} + y_k)/T^2$

$\dot{y} \sim (y_{k+1} - y_k)/T$

$y \sim y_k$

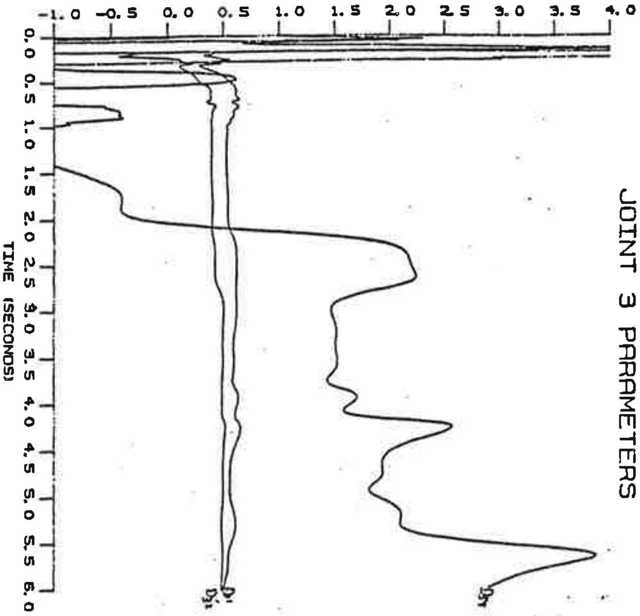
Assume for simplicity ...

① $\ddot{y} + \alpha\dot{y} = \alpha u$  D.C. servo

⇓ Euler Approx

$y_{k+2} + (\alpha T-2)y_{k+1} + (1-\alpha T)y_k = \alpha T^2 u_k$

| |
|---|
| [illegible] |
| ② NO ZEROS |




Recall Usual Approach

$\ddot{y} + \alpha\dot{y} = \alpha u$

⇕ STEP-INVARIANT APPROX

| |
|---|
| [illegible] ③ |

IF $u(t) = u_k$, $t \in [kT, (k+1)T)$ then $y(kT) = y_k$

ONE ZERO USUALLY NEAR $z=-1$

MODEL MATCHING CONTROL BASED ON ②

$$u_k = \frac{1}{\alpha T^2}(\alpha T - 2 - A_{m1})y_k + \frac{1}{\alpha T^2}(1-\alpha T - A_{m2})y_{k-1} + \frac{1}{\alpha T^2}(B_{m1}r_k + B_{m2}r_{k-1})$$

$$(y_{k+1} + A_{m1}y_k + A_{m2}y_{k-1} = B_{m1}r_k + B_{m2}r_{k-1})$$

POLE PLACEMENT USING ERROR FEEDBACK

$$u_k = \frac{-1}{\alpha T^2}(\alpha T - 2 - A_{m1})\varepsilon_k + \frac{-1}{\alpha T^2}(1-\alpha T - A_{m2})\varepsilon_{k-1}$$

$$\varepsilon_k = r_k - y_k$$

Usual Approach

Use ... [illegible] $T(q)$ ...

Model Matching

$AL + BP = A_m B'$

$T = B_m$

$A_m = 1 + A_{m1}q^{-1} + A_{m2}q^{-1}$

$B_m = B_{m1}q^{-1} + B_{m2}q^{-2}$

$A = 1 + a_1 q^{-1} + a_2 q^{-2}$

$B = b_1 q^{-1} + b_2 q^{-2} = b_1 B'$

Pole Placement

$AL + BP = A_m$

$T = P$

Questions:

Will new design be stable?

look at closed loop pole locations when Euler Based Controller Applied to Step-Inv. Model

How does new design perform comparitively?

Simulate

[illegible]

1. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]

```
MOTOR TIME CONTANT=  5                        MOTOR GAIN=  25

DESIRED MODEL POLES ARE:   .3  .3

T= .002
CONTROLLER GAINS:    -13900                9000                  4900
REAL= .376830187955     IMAGINARY=-.824720370892      MAGNITUDE= .906732971011
REAL= .543700344403     IMAGINARY=-2.31217889532E-12  MAGNITUDE= .543700344403
REAL= .376830187955     IMAGINARY= .824720370894      MAGNITUDE= .906732971013

T= .005
CONTROLLER GAINS:    -2200                 1416                  784
REAL= .376680075482     IMAGINARY=-.814746917561      MAGNITUDE= .897608165593
REAL= .540143298732     IMAGINARY=-4.83711947924E-12  MAGNITUDE= .540143298732
REAL= .376680075483     IMAGINARY= .814746917565      MAGNITUDE= .897608165598

T= .01
CONTROLLER GAINS:    -540                  344                   196
REAL= .376686802506     IMAGINARY=-.798156891204      MAGNITUDE= .882579951147
REAL= .533966589102     IMAGINARY=-3.70394027834E-12  MAGNITUDE= .533966589102
REAL= .376686802507     IMAGINARY= .798156891208      MAGNITUDE= .882579951146

T= .02
CONTROLLER GAINS:    -130                  81                    49
REAL= .377699083553     IMAGINARY=-.765086825326      MAGNITUDE= .853237626927
REAL= .520574906266     IMAGINARY= 6.32789346228E-11  MAGNITUDE= .520574906266
REAL= .377699083542     IMAGINARY= .765086825263      MAGNITUDE= .853237626865

T= .05
CONTROLLER GAINS:    -18.4                 10.56                 7.84
REAL= .389848619384     IMAGINARY=-.666674339586      MAGNITUDE= .772293092742
REAL= .469169135366     IMAGINARY=-3.5605861706E-9    MAGNITUDE= .469169135366
REAL= .389848619808     IMAGINARY= .666674343146      MAGNITUDE= .77229309603

T= .1
CONTROLLER GAINS:    -3.6                  1.64                  1.96
REAL= .45450640687      IMAGINARY=-.514336384372      MAGNITUDE= .686380353867
REAL= .31400747615      IMAGINARY=-1.88270316324E-8   MAGNITUDE= .31400747615
REAL= .454506401728     IMAGINARY= .514336403199      MAGNITUDE= .686380036457

T= .2
CONTROLLER GAINS:    -.4                   -.09                  .49
REAL= .63158612952      IMAGINARY=-.401744736225      MAGNITUDE= .748531811005
REAL=-.0424445943679    IMAGINARY= 2.70323476899E-11  MAGNITUDE= .0424445943679
REAL= .63158612955      IMAGINARY= .401744736198      MAGNITUDE= .748531811016

T= .5
CONTROLLER GAINS:    .176                  -.2544                .0784
REAL=-.242461317643     IMAGINARY= 2.00457817012E-10  MAGNITUDE= .242461317643
REAL= .801496638335     IMAGINARY= .324651786525      MAGNITUDE= .864751781586
REAL= .80149663769      IMAGINARY=-.324651786725      MAGNITUDE= .864751781063

T= 1
CONTROLLER GAINS:    .144                  -.1636                .0196
REAL= .884148034108     IMAGINARY= .262202388101      MAGNITUDE= .9222081319
REAL=-.184587853035     IMAGINARY= 9.12800846236E-10  MAGNITUDE= .184587853035
REAL= .884148030295     IMAGINARY=-.262202389013      MAGNITUDE= .922208128504

T= 2
CONTROLLER GAINS:    .086                  -.0909                .0049
REAL= .936529052133     IMAGINARY= .201382831278      MAGNITUDE= .957936172311
 AL=-.0990087999541     IMAGINARY= 2.12253659798E-10  MAGNITUDE= .0990087999541
 AL= .936529052145      IMAGINARY=-.20138283149       MAGNITUDE= .957936172368
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```
RED MODEL POLES ARE:   .6  .6

.002
ROLLER GAINS:    -7900               6300                 1600
= .441428732919     IMAGINARY=-.493650219054      MAGNITUDE= .66223097558
= .713505770843     IMAGINARY= 1.90847827396E-9   MAGNITUDE= .713505770843
= .441428711559     IMAGINARY= .493650217145      MAGNITUDE= .662230959919

.005
ROLLER GAINS:    -1240               984                  256
= .440494688536     IMAGINARY=-.481553906652      MAGNITUDE= .652632925648
= .7100296198       IMAGINARY=-4.20957122248E-11  MAGNITUDE= .7100296198
= .44049468856      IMAGINARY= .481553906694      MAGNITUDE= .652632925695

.01
ROLLER GAINS:    -300                236                  64
= .439321565244     IMAGINARY=-.460933970349      MAGNITUDE= .636760208171
= .703758943416     IMAGINARY=-6.67293599669E-10  MAGNITUDE= .703758943416
= .439321565627     IMAGINARY= .460933971017      MAGNITUDE= .636760208918

.02
ROLLER GAINS:    -70                 54                   16
= .4386538438       IMAGINARY=-.417530845732      MAGNITUDE= .605598218143
= .688910463765     IMAGINARY=-6.93143698292E-9   MAGNITUDE= .688910463765
= .438653847954     IMAGINARY= .417530852664      MAGNITUDE= .605598225931

.05
ROLLER GAINS:    -8.8                6.24                 2.56
= .468278237297     IMAGINARY=-.24809018887       MAGNITUDE= .529937023937
= .588797417922     IMAGINARY=-3.18867768971E-10  MAGNITUDE= .588797417922
= .468278236825     IMAGINARY= .248090189189      MAGNITUDE= .52993702367

.1
ROLLER GAINS:    -1.2                .56                  .64
= .0969196313609    IMAGINARY=-4.38259234459E-10  MAGNITUDE= .0969196313609
= .690887118601     IMAGINARY= .209457018814      MAGNITUDE= .721939923663
= .690887118096     IMAGINARY=-.209457018376      MAGNITUDE= .721939923053

.2
ROLLER GAINS:    .2                  -.36                 .16
= .791540882985     IMAGINARY=-.212454879276      MAGNITUDE= .819557225071
=-.141626436961     IMAGINARY=-3.14192966201E-9   MAGNITUDE= .141626436961
= .791540883381     IMAGINARY= .212454882418      MAGNITUDE= .819557226268

.5
ROLLER GAINS:    .272                -.2976               .0256
= .885808738823     IMAGINARY= .183341300308      MAGNITUDE= .904583414713
=-.259205356212     IMAGINARY= 4.26923651651E-10  MAGNITUDE= .259205356212
= .885808735639     IMAGINARY=-.183341300735      MAGNITUDE= .904583411681

1
ROLLER GAINS:    .168                -.1744               .0064
= .933545222183     IMAGINARY= .149518767353      MAGNITUDE= .945443040934
=-.18722052227      IMAGINARY=-2.14357645026E-15  MAGNITUDE= .18722052227
= .933545222183     IMAGINARY=-.149518767353      MAGNITUDE= .945443040934

2
ROLLER GAINS:    .092                -.0936               .0016
= .963685474736     IMAGINARY= .115043592306      MAGNITUDE= .970528063678
=-.0993213725191    IMAGINARY= 8.9994491563E-12   MAGNITUDE= .0993213725191
= .963685474486     IMAGINARY=-.115043592315      MAGNITUDE= .970528063411
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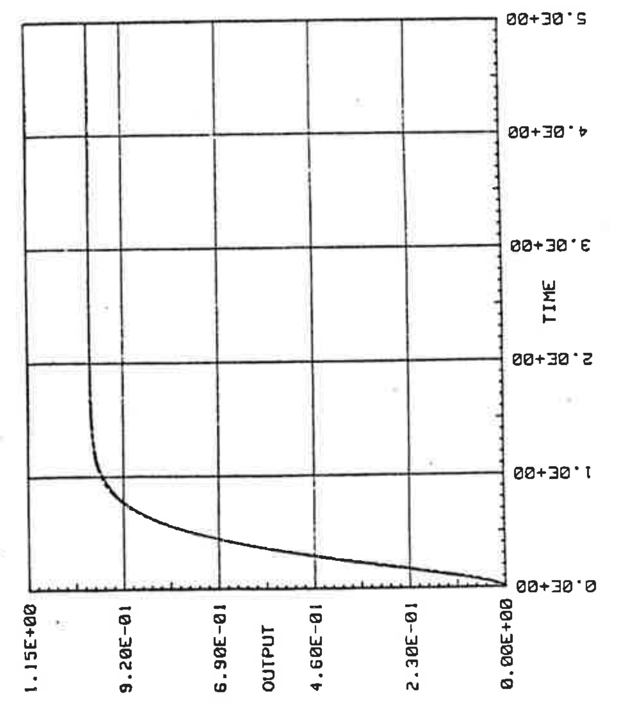
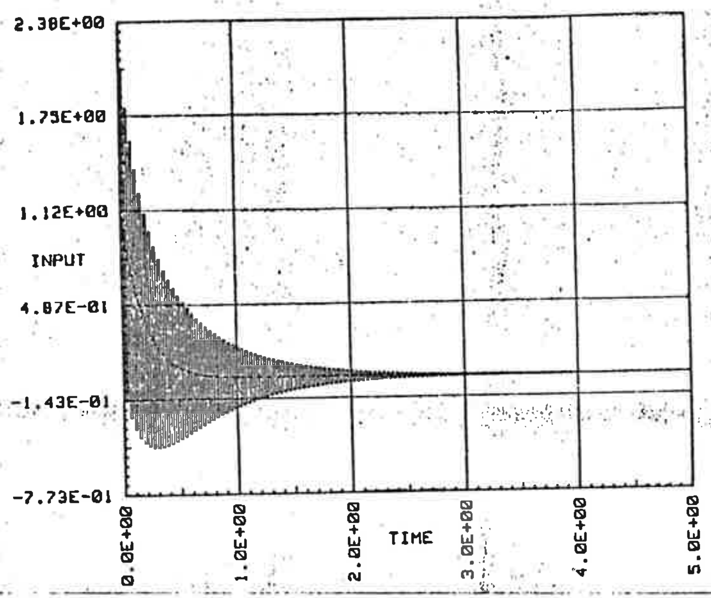
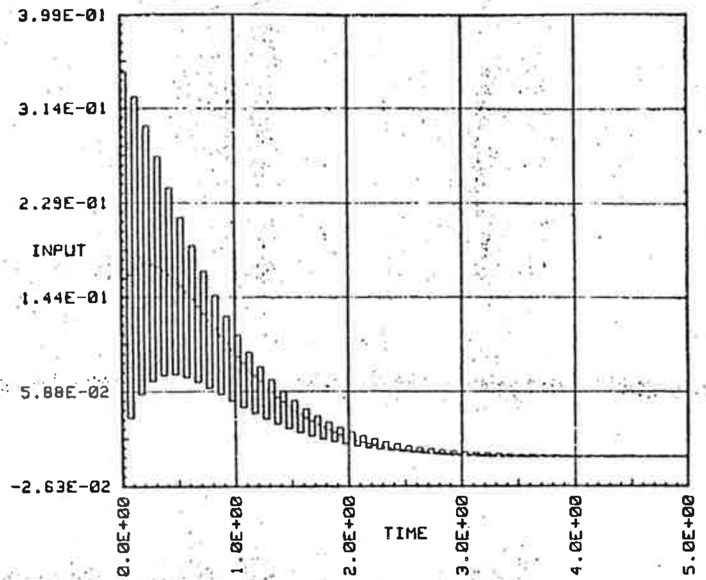
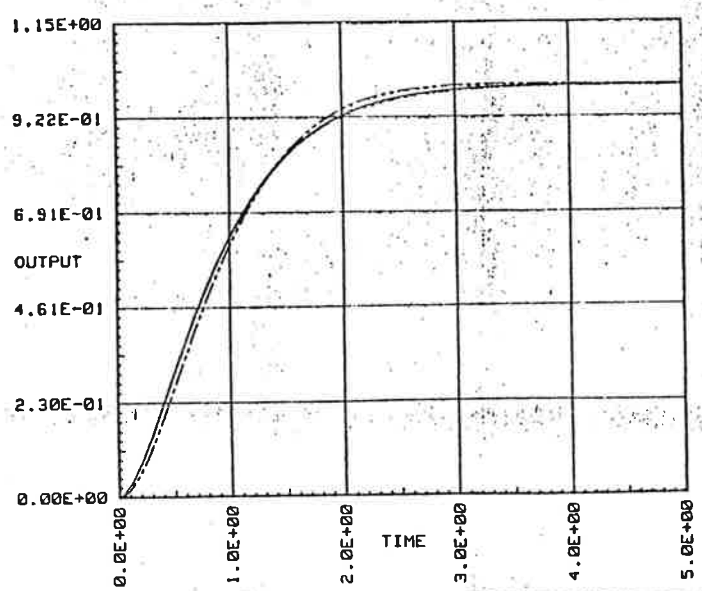




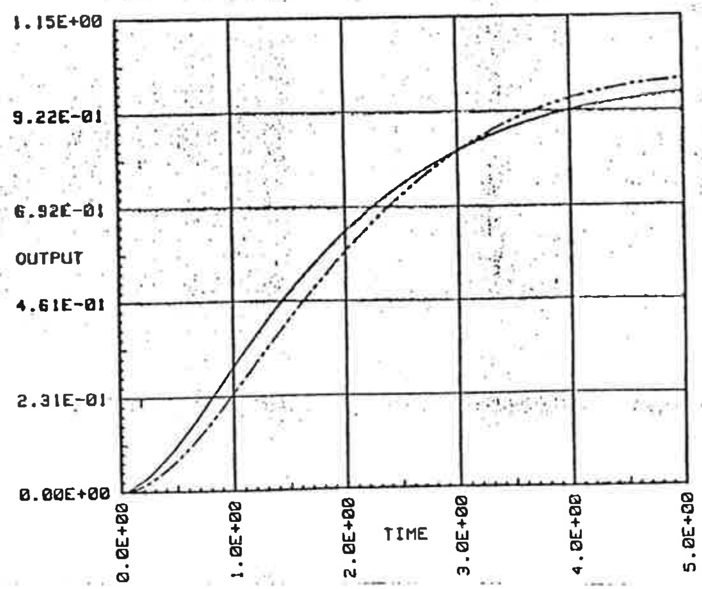
2.30E+00
1.75E+00
1.12E+00
INPUT
4.87E-01
-1.43E-01
-7.73E-01
0.0E+00
1.0E+00
2.0E+00
TIME
3.0E+00
4.0E+00
5.0E+00

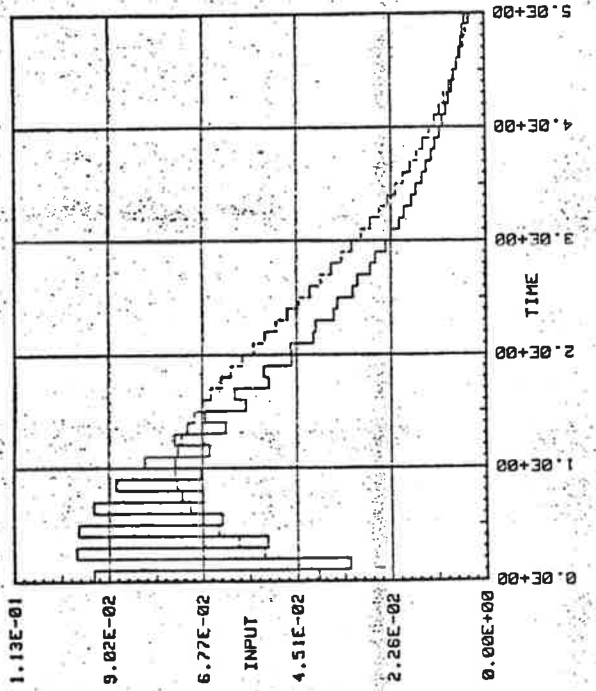
3.99E-01
3.14E-01
2.29E-01
INPUT
1.44E-01
5.88E-02
-2.63E-02
0.0E+00
1.0E+00
2.0E+00
TIME
3.0E+00
4.0E+00
5.0E+00

1.15E+00
9.22E-01
6.91E-01
OUTPUT
4.61E-01
2.30E-01
0.00E+00
0.0E+00
1.0E+00
2.0E+00
TIME
3.0E+00
4.0E+00
5.0E+00

1.15E+00
9.22E-01
6.92E-01
OUTPUT
4.61E-01
2.31E-01
0.00E+00
0.0E+00
1.0E+00
2.0E+00
TIME
3.0E+00
4.0E+00
5.0E+00




## V. STABILITY ANALYSIS

Can we justify methodology with any Theory?

[illegible]

1. Can we apply [illegible] the adaptive control [illegible] to show good [illegible] controlling nonlinear [illegible] model?

2. [illegible] design adaptive [illegible] controllers using [illegible] model [illegible]

---

THEOREM:

If the system to be controlled is the [illegible] approx. model, then the system is globally stable, i.e.

[illegible]

from the [illegible] conditions

[illegible]

and

[illegible]

Provided

$|\det J_k(y_k)| > \varepsilon \quad \forall \; k \geq 0$

Proof: Complete [illegible] (Chen [illegible])

Outline of proof:

Use analysis methodology of [illegible] and Caines, i.e. must show

$\|y_k\| \leq K_1 + K_2 [illegible]$

$e_k$ = Pred. error

Can show:

① [illegible]

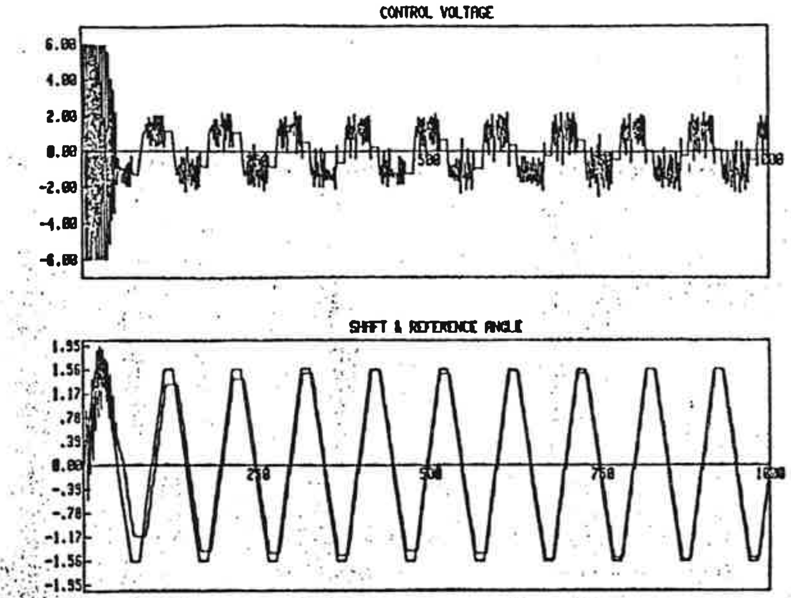
$\Rightarrow$ [illegible]

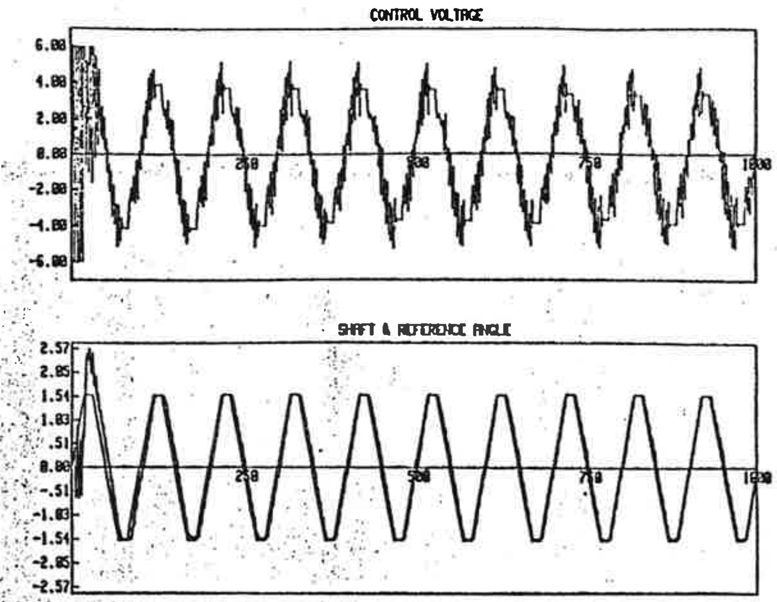
② [illegible]

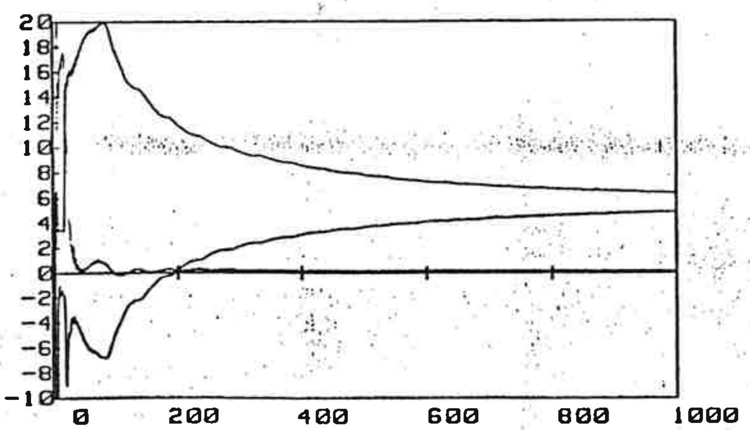
$\Rightarrow$ [illegible]

③ Problem: If # of DOF $\geq 2$ then

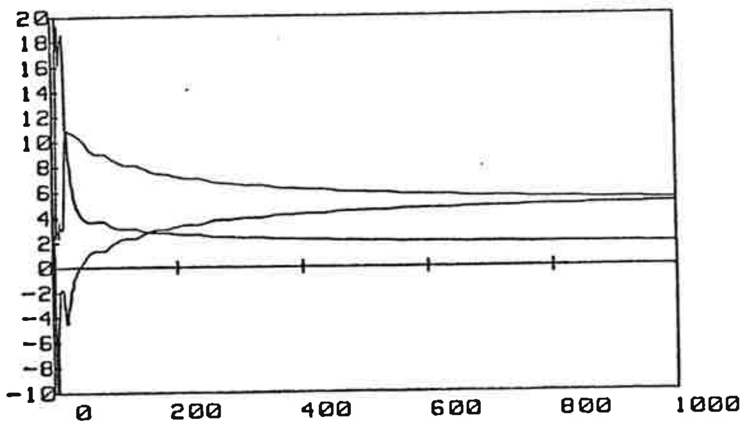
$\phi_k \sim y_k^2$



PARAMETERS VS. SAMPLE NO.
SAMPLE PERIOD = .1 SECONDS
___A = 4.870E+00___K = 6.311E+00
___G/10 = 1.188E-01



PARAMETERS VS. SAMPLE NO.
SAMPLE PERIOD = .1 SECONDS
___A = 4.952E+00___K = 5.224E+00
___G/10 = 1.793E+00

# Some desirable features of industrial adaptive controllers

Jan Sternby

Gambro AB
Lund, Sweden

Abstract

After a short introduction, three different physical processes will be described, for which adaptive control have been tried or considered. They are: a level control for a tank in a pulp production plant, an autopilot for ships, and a pressure and flow control system for a medical treatment system (artificial kidney). It will be discussed why adaptive control could be useful in these cases, and how it really works in two of them with certain algorithms.

Each process and its operators represent different demands on the adaptive control system such as robustness, variations in model structure, interpretability of identified parameters. These demands will be discussed for each process separately. This results in a small list of some desirable features of industrial adaptive controllers.

## Some desirable features of industrial adaptive controllers

- Introduction
- Level control
- Autopilot
- Dialysis machine
- Summary

## Level control

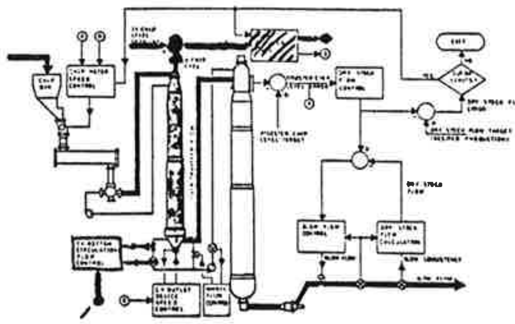


Fig. 1 - Principiell bild av ett massakokeri.

- Simple level control with unknown actuator dynamics
- Time delay 2-3 min.
- On PID control, $T_s = 5$ sec., no feed forward
- Digital controller gives setpoint to analog controller

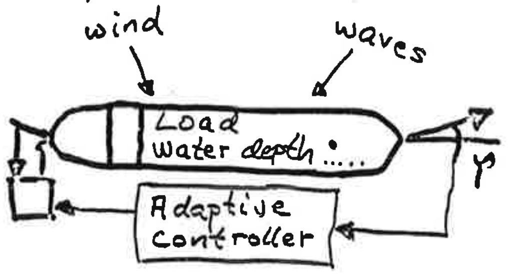
## Algorithm:

* Minimum Variance
* Least Squares Est. ($b_0$ fixed)
* Feed forward (from chip feed)
* $T_s = 1$ min.
* $k = 3, 5, 6$
* $\lambda = 0.96 - 0.995$

## Experiance:

* PID structure (NB = 0) robust, works well
* Difficult to adjust k if NB ≠ 0
* Desirable to be able to compare parameters with PID control
* Excess of parameters causes covariance blow-up (feed forward)

## Autopilot




* Adaptive controller gives set point for rudder machinery (nonlinear, partly unknown)
* Integrator + Time Constant (≈ time to go 1 ship length) + bias (wind.... )
* Varying noise conditions
* Constant ship parameters

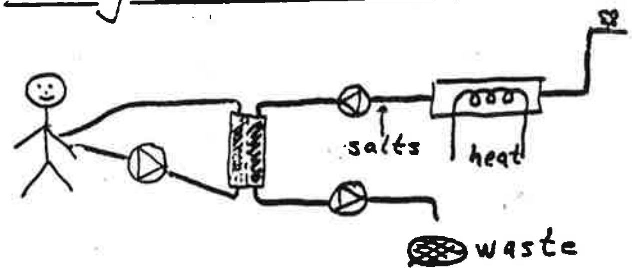
## Algorithm:

* Optimal LQ control
* Spectral factorization
* ELS estimation
* Forced integrator
* tr $P \leq$ constant
* $NA = 1$ (+ known integrato
* time delay $\leq NB$ (= 3)
* $T_s = 5$ sec. ( 3 sec.)
* $\lambda = 0.99 - 0.9999$

## Experiances:

* Integral action important (quick)
* Natural criterion good as motivation
* Essential to be able to handle different levels of noise
* Quarterly sea difficult – – should separate models for dynamics & disturbance
* Would like to fix physical parameters

## Dialysis machine




* Control of:
  - Temperature
  - Conductivity
  - → Flow
  - → Pressure
* Simple time constant 0.5 – 5 sec.
* Dynamics vary with:
  - Aging of pumps (Gain
  - Filter type (time con:
  - (Flow)
  - Blood composition (s

## Demands on adaptive control

- SAFETY!
  No risk for sudden or slow loss of control accuracy.
  → Robustness
- Should identify filter = once/treatment
  compensate for pump aging = = long term drift
  ⇒ separate known models and unknown parts
  Some physical parameters fixed
- PI(D)-structure good for servicability.

## Some desirable features

* Safety
→ * Avoid manual tuning of critical parameters (k..
→ * Robustness for model errors (e.g. rudder machinery
* PID-structure (N8 = 0) robust (?) and well-know
* Automatic detection of parameter excess (Covariance blow-up)
* Possibility to fix physical parameters
→ * Separation dynamics- -disturbance
→ * Capability to handle varying noise levels

* MUST cope with non-minimum phase

* Include analog prefilter in self-tuner

→ * Feedforward essential

# Self-tuning regulator with increased prediction horizon

Karl Johan Åström and Björn Wittenmark

Department of Automatic Control
Lund Institute of Technology
Lund, Sweden

The basic self-tuning regulator based on least squares estimation and minimum variance control is not suited for systems with zeros outside the unit circle. In the presentation it is shown that the basic algorithm often can be used also for this type of systems by increasing the prediction horizon of the regulator.

Examples are given which show how the algorithm can be used for non-minimum phase systems. Possible convergence points are discussed and some new results concerning local convergence of the algorithm are presented.

The new insight into the properties of the algorithm explains why it can be applied to a wide range of practical problems provided that some design parameters are correctly chosen.

Reference

Wittenmark B., Åström K.J. (1984): Practical issues in the implementation of self-tuning control. Automatica 20, 595-605, 1984.

1

# SELF-TUNING REGULATOR WITH INCREASED TIME HORIZON

* INTRODUCTION
* THE BASIC ALGORITHM
* INCREASED PREDICTION HORIZON
* ANALYSIS
* SIMULATIONS
* CONCLUSIONS

2

## THE BASIC ALGORITHM

PROCESS

$$A^*(q^{-1})y(k) = B^*(q^{-1})u(k-d_0) + C^*(q^{-1})e(k)$$

ALGORITHM

LS: $y(k+d) = R^*(q^{-1})u(k) + S^*(q^{-1})y(k)$

MV: $u(k) = -\frac{S^*}{R^*}y(k)$

3

## PROPERTIES

If convergence then

1. $r_y(\tau) = 0 \quad \tau = d_0, \ldots, d_0 + \deg S^*$

   $r_{yu}(\tau) = 0 \quad \tau = d_0, \ldots, d_0 + \deg R^*$

2. Sufficiently complex regulator
   ⇒ Minimum variance control

Convergence if
* Minimum phase plant
* $\frac{1}{C(z)} - \frac{1}{2}$ strictly positive real

Local stability if

$$C(z_i) > 0$$

for all $z_i$ such that $B(z_i) = 0$

4

## INCREASED PREDICTION HORIZON

Use $d > d_0$ in the estimation

What will happen?

* Good thing to do in practice
* Property 1 still valid but with $d_0$ replaced by $d$, Wittenmark (1973)
* Can handle some non minimum phase processes

5

## ANALYSIS

$$A(q)\,y(k) = B(q)\,u(k) + C(q)\,e(k)$$

$$d_0 = \deg A - \deg B$$

$$R(q)\,u(k) = -S(q)\,y(k)$$

Closed loop system

$$y(k) = \frac{CR}{AR+BS}\,e(k) = \frac{F}{q^{\alpha}}\,e(k)$$

MA process

Pole placement interpretation

Minimum variance ( $B^+ = B$, $A_m = q^{d_0-1}$, $A_0 = C$

$R = BF$

$AF + S = q^{d_0-1}C \quad \Rightarrow \quad y(k) = \frac{BF}{Bq^{d_0-1}}\,e(k)$

$\deg R = n-1$

$\deg S = n-1$

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## INCREASED TIME HORIZON

Assume $d = n - \alpha + 1$.

$$AR + BS = q^{\alpha}C$$

$$\Rightarrow \quad y(k) = \frac{CR}{Cq^{n-1}}\,e(k)$$

$\deg R = n-1$

$\deg S = n-1$

Comparison

* Same degree of regulator
* No cancellation of process zeros, compare Åström (1970)
* Closed loop system is MA of order $\alpha = n-1$

7

## RESULTS

1. If convergence then there is a possible convergence point that corresponds to MA process of order n-1
2. Local convergence can be analyzed

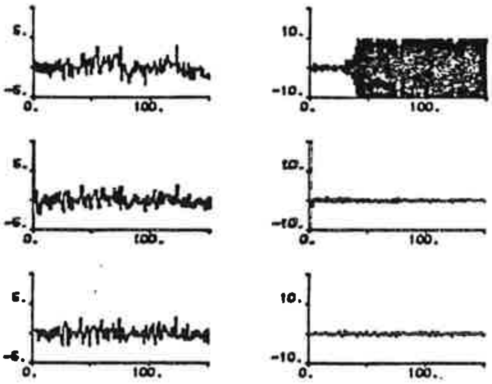
8

## SIMULATIONS

EXAMPLE 1

$$y(k) - y(k-1) = u(k-1) + 1.1\,u(k-2) + e(k) - 0.5\,e(k-1)$$

Output    Input




d = 1

d = 2

Optimal

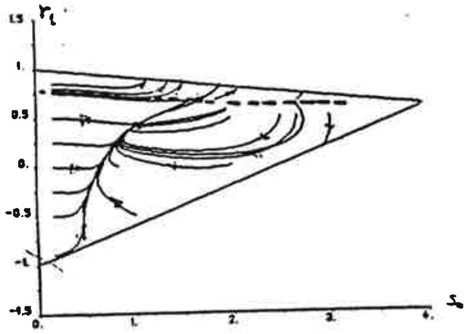
# EXAMPLE 2

$$\dot{y} = u(t-\tau)$$

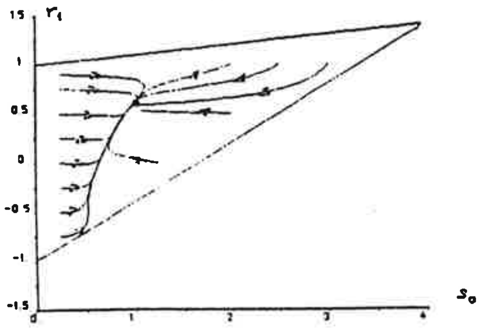
$$y(kh+h) = y(kh) + (h-\tau)u(kh) + \tau\, u(kh-h) + e(kh+h) + c\,e(kh)$$

Non minimum phase if $\tau > h/2$

Simulation of ODE for the LS case




$\tau = 0.4$



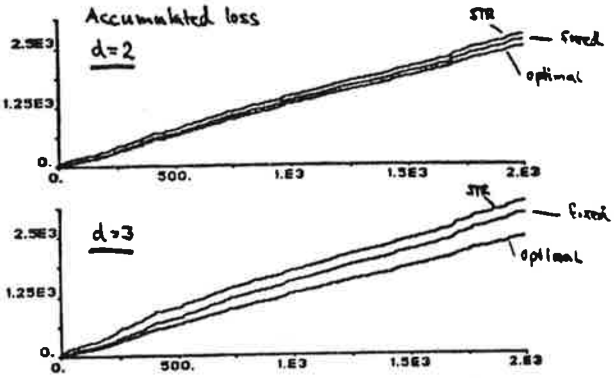

$\tau = 0.6$

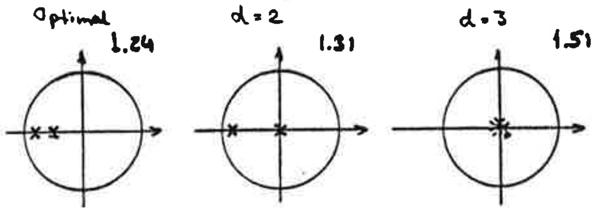


# EXAMPLE 3

$$A(z) = z(z-1)(z-0.5)$$
$$B(z) = (z+2)(z+0.8)$$
$$C(z) = z^2(z-0.7)$$

$$u(k) = -\frac{s_0 + s_1 q^{-1}}{1 + r_1 q^{-1} + r_2 q^{-2}}\, y(t)$$




+ C - polynomial

# A universal control capable of stabilizing any single-input, single-output, minimum phase linear system of relative degree ≤ 2

A.S. Morse

Deptartment of Electrical Engineering
Yale University
New Haven, USA

Within the past few years there have been developed several smooth dynamical controllers, not requiring "sufficiently rich" probing signals, which are capable of stabilizing any process which can be modelled by a linear system with transfer function of the form

$$T(s) = g\frac{\alpha(s)}{\beta(s)}$$

where g is a nonzero constant, and $\alpha(s)$ and $\beta(s)$ are monic, coprime polynomials, provided it can be assumed that

1/ $\alpha(s)$ is a strictly stable polynomial (i.e., $\Sigma$ is minimum phase)
2/ a bound $n \geq$ degree $\beta(s)$ is known
3/ the relative degree $n^* =$ degree $\beta(s)$ - degree $\alpha(s)$ is known exactly
4/ the sign of g is known

Since the preceding assumptions are very restrictive, there is ample motivation to see if controller structure can in some way be modified so that at least some of these assumptions can be avoided.

Prompted by this, we have just discovered that the 7-dimensional control system consisting of sensitivity function $\theta = [\theta_u, \theta_y, y]'$, where

$$\theta_u = -\lambda\theta_u + u$$
$$\theta_y = -\lambda\theta_y + y,$$

filtered sensitivity function $\phi = [\phi_u, \phi_y, \theta_y]'$, where

$$\phi_u = -\lambda\phi_u + \theta_u$$
$$\phi_y = -\lambda\phi_y + \theta_y$$

parameter adjustment law

$$k = \phi y,$$

Gain $N(x) = x\cos(x)$, where $x = k'k$, and feedback law

$$
\begin{aligned}
u &= N(x)k'\theta + \left(\frac{\partial N(x)}{\partial x}\right)(k'\phi)^2 y + N(x)\phi'\phi y \\
&= x\cos(x)(k'\theta + \phi'\phi y) + (\cos(x) - x\sin(x))(k'\phi)^2 y \\
&= N(x)k'\theta + \phi'\frac{d}{dt}(N(x)k)
\end{aligned}
$$

stabilizes any relative degree 2, minimum phase system of any dimension. What's surprising is that this control can also stabilize any relative degree 1 minimum phase system of any dimension. In other words, to achieve stability with the above control, it is only necessary to know that the minimum phase system to be controlled has relative degree not exceeding 2; i.e., it is not necessary to know relative degree exactly only an upper bound is required. It is natural to speculate that this should be true in general. In other words, with apriori knowledge of an upper bound $\bar{n}$, it should be possible to construct a smooth control system not incorporating a probing signal, which can stabilize any minimum phase system of any dimension, provided the system's relative degree does not exceed $\bar{n}$.

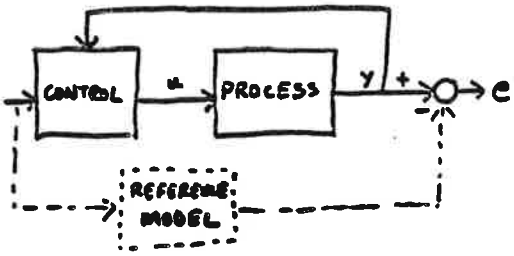
/

GENERAL ISSUES

Stability is of central importance – why?

What should be the role of a probing signal?

Should we insist on "smooth algorithms"




OBJECTIVES:

Tracking: $e \to 0$ as $t \to \infty$
Internal Stabilization – all "state" bounded on $[0,$

Control Class

1. Smooth finite dimensional dynamical system
2. Arbitrary bounded reference input – "sufficiently rich" probing not incorporated for stabilization!

Process Model Assumptions – Linear system with transfer function $g_p(\alpha_p(s)/\beta_p(s))$

1. $\alpha_p(s)$ stable – minimum phase
2. bound $n \geq$ degree $\beta_p(s)$ known
3. rel. degree $n^* =$ degree $\beta_p(s)$ – degree $\alpha_p(s)$ known
4. sign $g_p$ known

2

3

Base Problem: Given $\dot{y} = ay + gu$, with $a$ and $g$ unknown and $g \neq 0$, does there exist an integer $m \geq 0$ and smooth functions $f: \mathbb{R}^{m+1} \to \mathbb{R}^m$, $h: \mathbb{R}^{m+1} \to \mathbb{R}$ such that the closed loop system

$$\dot{y} = ay + g\,h(x,y)$$
$$\dot{x} = f(x,y)$$

is stable in the sense that for any initial state $(x_0, y_0)$, there exists a solution $(x(t), y(t))$ bounded on $[0,\infty)$ and $y(t) \to 0$ as $t \to \infty$?

If sign $g$ known, the classical adaptive control with $n=1$, $f = y^2$, $h = -(\text{sign}(g))xy$ will stabilize:

$$\dot{y} = (a - |g|x)y$$
$$\dot{x} = y^2$$

The control $u = y^2 \sin(y)$ [suggested by Wonham] also stabilizes

$$\dot{y} = (a + g y \sin(y))y$$

but $y$ doesn't necessarily go to zero due to multiple equilibria.

NEGATIVE RESULTS

[redacted] If $m=1$ and if $f(x,y)$ and $h(x,y)$ are constrained to be quadratic polynomial in $x$ and $y$, then stabilization of

$$\left.\begin{aligned}\dot{y} &= ay + g\,h(x,y)\\ \dot{x} &= f(x,y)\end{aligned}\right\} (1)$$

is impossible.

Nussbaum: If $m=1$ and if $f(x,y)$ and $h(x,y)$ are constrained to be rational functions in $x$ and $y$, then stabilization of (1) is impossible.

Open Problem: Generalize the above by proving that Nussbaum's assertion is true for rational controllers of any dimension $m \geq 0$.

**Nussbaum's Positive Result**: It is possible to stabilize $\dot y = ay + gu$ with a smooth 1-dimensional controller.

**Similar control** $\quad u = N(x)y \quad \dot x = y^2 \qquad N(x) = x^2\cos(x)$

$$\dot y = (a + gx^2\cos(x))y \qquad (1)$$
$$\dot x = y^2 \qquad (2)$$

Analysis (Nussbaum)

$$\dot{y^2} = 2(a + gx^2\cos(x))y^2$$

$$\frac{dy^2}{dx} = 2(a + gx^2\cos(x))$$

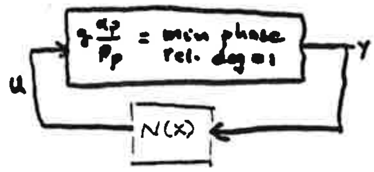
$$y^2(t) - y^2(0) = \int_{x(0)}^{x(t)} 2(a + g\lambda^2\cos(\lambda))d\lambda$$

$$y^2(t) = \pi(x(t)) - \pi(x_0) + y^2(0)$$

$$\pi(\mu) = 2a\mu + 2g(\underbrace{\mu^2\sin(\mu)}_{\text{dominates}} + 2\mu\cos(\mu) - \sin(\mu))$$



**Willems - Byrnes**




$N(x) = x^2\cos(x)$
$\dot x = y^2$

SAME control just used to stabilize $\dot y = ay + gu$

parameterization: $\quad \beta_p = (s-a)\alpha_p + \rho$

$$\beta_p y = g\alpha_p u$$

$$((s-a)\alpha_p + \rho)y = g\alpha_p u$$

$$(s-a)y + \frac{\rho}{\alpha_p}y = gu$$

$$\dot y = ay + gu + L(y) \qquad L(y) = -\frac{\rho}{\alpha_p}y$$

Analysis: $\quad \dot y = (a + gx^2\cos(x))y + L(y)$
$\dot x = y^2$

$$\dot{y^2} = 2(a + gx^2\cos(x))y^2 + 2yL(y)$$

$$y^2(t) = \pi(x(t)) - \pi(x_0) + y^2(0) + 2\int yL(y)dt$$

$$\le c_1 + c_2\int y^2 dt \le c_1 + c_2 x$$

$$y^2(t) \le \pi(x(t)) - \pi(x_0) + y^2(0) + c_1 + c_2 x$$

$$\pi(\lambda) = 2a\lambda + 2g(\lambda^2\sin(\lambda)) + 2\lambda\cos(\lambda) - \sin(\lambda))$$



GENERAL PARAMETERIZATIONS
- Observer Based (MRAC, STR)
- Dynamic Compensator Based ~~(.......)~~

Let $(A, b)$ be any $(n-1)$-dimensional $\{(\bar n - 1)$-dimensional$\}$, controllable pair with $A$ stable. Define sensitivity function $\theta = [\theta_u', \theta_y', y]'$ where

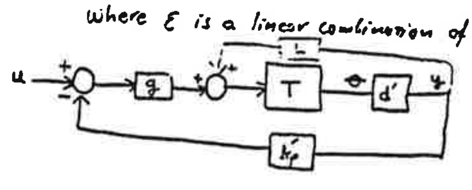
$$\dot\theta_u = A\theta_u + bu \qquad \theta \in \mathbb{R}^{2n-1} \quad \theta \in \mathbb{R}^{2\bar n - 1}$$
$$\dot\theta_y = A\theta_y + by$$
$$y = d'\theta$$

Lemma: For each $n_j \le n$ dimensional, minimum phase system of relative degree $n^* \le \bar n$ and each monic, stable polynomial $\alpha^*$ of degree $n^*$ there exist a nonzero constant $g$, a strictly proper, stable transfer matrix $T$, a strictly proper, stable transfer function $L$ and a constant vector $k_p \in \mathbb{R}^{2n-1}$ $[k_p \in \mathbb{R}^{2\bar n - 1}]$ such that

$$y = \frac{1}{\alpha^*}(g(u - k_p'\theta)) + \varepsilon + L(y)$$

$$\theta = T(g(u - k_p'\theta)) + \varepsilon + L(y)$$

where $\varepsilon$ is a linear combination of decaying exponentials.




DISCRETE - TIME ADAPTIVE CONTROL

FUNDAMENTAL PROBLEM

For the one-dimensional, discrete time system

$$y(t+1) = ay(t) + gu(t)$$

with $a$ and $g$ unknown and $g \ne 0$, does there exist an integer $m \ge 0$ and smooth functions

$$f: \mathbb{R}^{m+1} \to \mathbb{R}^m$$
$$h: \mathbb{R}^{m+1} \to \mathbb{R}$$

such that the system

$$y(t+1) = ay(t) + gh(x(t), y(t))$$
$$x(t+1) = f(x(t), y(t))$$

is stable in the sense that $(x(t), y(t))$ is bounded for $t \ge 0$ and $y(t) \to 0$ as $t \to \infty$.

# Adaptive stabilization of linear multivariable systems

Christopher I. Byrnes

Division of Applied Sciences
Harvard University
Cambridge, Mass. 02138, USA

Framework:
We have a system known only imprecisely, and we want to find a "universal stabilizing controller" which makes the output $y(t)$ of the system $\rightarrow 0$ as $t \rightarrow \infty$, while the parameters in the controller stay bounded.

I. The SISO case
Theorem. Consider the SISO system $\dot{x} = Ax + bu$; $y = cx$. Assume it is minimum phase and $cb > 0$. Define the controller $\dot{k} = y^2$, $u = -ky$. Then, for all $(x_0, k_0)$ it is true that $k_t \rightarrow k_\infty < \infty$ and $x_t \rightarrow 0$ as $t \rightarrow \infty$.

II. The MIMO case
Theorem. Consider the m x m system $\dot{x} = Ax + Bu$; $y = Cx$ and suppose that $\det G(s) = 0 \Rightarrow \mathrm{Re}(s) < 0$ and that $\mathrm{spec}(CB) \subset C^+$. Then, the controller $\dot{k} = \|y\|^2$, $u = -ky$ satisfies $x_t \rightarrow 0$ and $k_t \rightarrow k_\infty$ as $t \rightarrow \infty$.

The proof consists of two ideas: First multivariable root-locus methods are employed to show that the eigenvalues will go into the left half plane, then "frozen analysis" as below shows that this is sufficient to deduce stability. An example of multivariable root locus was analyzed.

III. Frozen Eigenvalue Analysis
Theorem. Consider the system $\dot{x} = Ax - k(t)Bx$, where (i) for $k \gg 0$ $A - kB$ is stable, and, (ii) $k(t) \uparrow +\infty$ as $t \rightarrow \infty$. Then each solution $x(t)$ tends to 0 exponentially.

IV. Modifications and Extensions
The standard assumptions of necessary a priori knowledge for adaptive stabilization of an unknown SISO system are:

1) minimum phase
2) $n \leq N$, i. e. we have an upper bound on the order of the system
3) $n^*$, the relative degree of the system, is known
4) $\mathrm{sign}(cA^{n^*-1}b)$ is known

We have shown that 2) is not needed, actually under weak condition the results can be generalized to a Hilbert space context. By a variant of Nussbaum's result, it was demonstrated that 4) is not needed either.

## V. Necessary Conditions for Adaptive Stabilization

Metaprinciple. "Whatever can be done adaptively can be done if we know (A,b,c)."

Framework: Let the adaptive regulator be a $n_z$-dimensional linear system with internal state z, and dependent on a parameter k, which is updated according to $\dot{k} = f(k,y)$. By convergence we mean that $(x_t,z_t,k_t) \to (0,0,k_\infty)$ as $t \to \infty$.

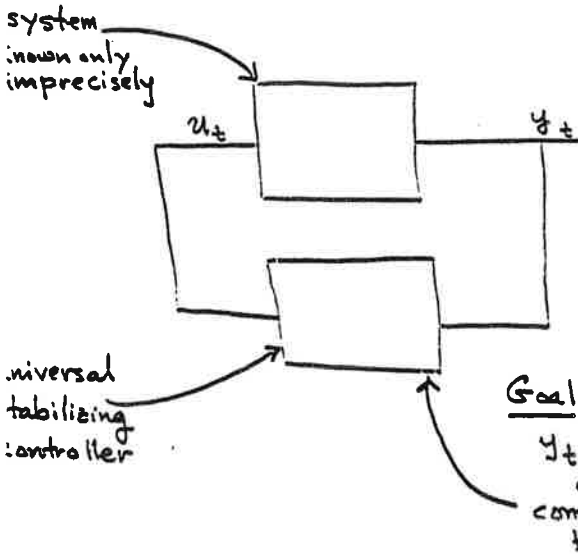
Theorem. If we have a convergent parameter adaptive stabilization scheme, then $n_z \geq n^* - 1$.

Theorem. Suppose $g(s) = c(sI - A)^{-1}b$ is minimum phase and of relative degree $\bar{n} \leq n^*$. Then, if p(s) is minimum phase of degree $n^*$, the regulator

$$k(s) = \frac{k\beta^{n^*} p(s)}{(s + \beta)^{n^*}} \quad , \quad \dot{k} = y^2 \; , \; \beta = e^k$$

will stabilize g(s).

# Adaptive Stabilization of Linear Multivariable Systems




**Goal:**
$y_t \to 0$
as $t \to \infty$,

14

**Hypotheses**: I. $g(s) = c(sI-A)^{-1}b = 0$

$\Rightarrow \operatorname{Re}(s) < 0$

minimum phase

II. $g'(\infty) = cb > 0$

**Theorem**: Suppose

$$\dot{x} = Ax + bu$$
$$y = cx \qquad (1)$$

is minimum phase with $cb > 0$. Define the controller

$$\dot{k} = y^2 \quad , u = -ky \qquad (2)$$

Then (1)-(2) satisfies, $\forall (x_0, k_0)$:

(i) $k_t \to k_\infty$ $\quad t \to \infty$

(ii) $x_t \to 0$

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**Proof**: $cb \neq 0 \Rightarrow b \notin \ker c$

$\therefore$ $\ker c + \operatorname{span}\{b\} = \mathbb{R}^n$

$(x_1, y) = x \in \mathbb{R}^n$

(1)-(2):

$$\dot{x}_1 = A_{11} x_1 + A_{12} y$$
$$\dot{y} = A_{21} x_1 + A_{22} y - \beta k y$$
$$\dot{k} = y^2$$

**1.3.** $\operatorname{sign}(\beta) = \operatorname{sign}(cb) > 0$, $\operatorname{spec}(A_{11}) = Z(g(s)) \subset \mathbb{C}^-$

**Pf.** $\beta = cb$.

**Proof 1**: $g(s_0) = 0 \Rightarrow \hat{y}(s_0) = 0$ where $x(0) = 0$

Therefore, at $s = s_0$, we have

$$s_0 \hat{x}_1(s_0) = A_{11} \hat{x}_1(s_0)$$

and $s_0 \in \operatorname{spec}(A_{11})$.

**Proof 2**. $A_{11} = \overline{A + bf} : \mathcal{V}^*/\mathcal{R}^* \to \mathcal{V}^*/\mathcal{R}^*$

where $\mathcal{V}^* = \ker c$

$\mathcal{R}^* = (0)$

claim: It suffices to prove $|k_t| < M$, because

1) $k_t$ bounded $\Rightarrow \lim k_t = k_\infty$

B) A) $\Rightarrow y_t \in L^2(0,\infty)$

C) B) $\Rightarrow (x_1)_t \in L^2(0,\infty)$

D) A, B, C $\Rightarrow \dot{y} \in L^2(0,\infty) \Rightarrow y_t \to 0$

E) A, B $\Rightarrow \dot{x}_1 \in L^2(0,\infty) \Rightarrow (x_1)_t \to 0$

D, E $\Rightarrow x_t = \begin{pmatrix} x_1 \\ y \end{pmatrix}_t \to 0$.

**Lemma** $k_t$ is bounded

**Proof.** Set $V(x_1, y, k) = \frac{1}{2} y^2$.

$$\dot{V} = y A_{21} x_1 + \int A_{22} y^2 - \beta k y^2$$

$$V \Big|_0^T = \int_0^T y A_{21} x_1 \, dt + A_{22} k \Big|_0^T - \beta k^2 \Big|_0^T$$

claim $\left| \int_0^T y A_{21} x_1 \, dt \right| \le c_1 + c_2 \int_0^T y^2 dt$

$\therefore \; -\frac{1}{2} y^2(0) \le \alpha k \Big|_0^T - \beta k^2 \Big|_0^T + c_1$

$\therefore \; \gamma \le \alpha k(T) - \beta k^2(T)$

$\therefore \; |k_t| \le M$.

claim: $\left| \int_0^T y A_{21} x_1 \, dt \right| \le c_1 + c_2 \int_0^T y^2 dt$

set $z = A_{21} x_1$, $z = A_{21} e^{A_{11} t} x_1(0) + L_2 y$
$= L_1 x_1(0) + L_2 y$

$L_1, L_2$ bounded operators

$$\therefore \; |\langle y, z \rangle| \le |\langle L_1 x_1(0), y \rangle| + |\langle y, L_2 y \rangle|$$
$$\le d_1 \|x_1(0)\|^2 \|y\|^2 + d_2 \|y\|^2$$
$$\le d_1' \|x_1(0)\|^2 + d_2' \|y\|^2$$
$$= c_1 + c_2 \int_0^T y^2 dt$$

QED

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## II. Multivariable Linear Systems

$$(1) \quad \dot{x} = Ax + Bu, \quad y = Cx$$
$$G(s) = C(sI - A)^{-1} B \qquad m \times m$$

**Theorem** (B-W) Suppose $\det G(s) = 0 \Rightarrow Re(s) < 0$ and $spec(CB) \subset \mathbb{C}^+$. Then, the controller

$$(2) \quad \dot{k} = \|y\|^2, \quad u = -ky$$

satisfies

$$x_t \to 0, \quad k_t \to k_\infty.$$

Remarks 1. Multivariable Root-Locus $\Rightarrow$ "Frozen Analy
"Frozen Analysis" $\Rightarrow$ Stability

2. spec (CB) $\subset$ [illegible]

Multivariable Root-Loci d'apres Postlethwait - MacFarlane

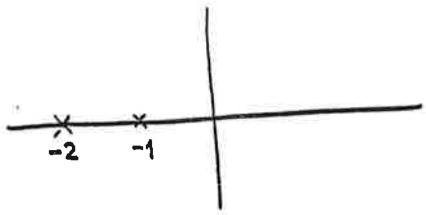
Frozen Eigenvalue Analysis d'apres B & Martensson

Thus we ask 3 questions
1) Where do the root-loci start ?
2) Where do the root-loci end ?
3) How do they get there ?

**Example**: $G(s) = \frac{1}{(1.25)(s+1)(s+2)} \begin{bmatrix} s-1 & s \\ -6 & s-2 \end{bmatrix}$

$$K_k = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}, \quad u = -ky$$

1) $s_i(0) \in \text{Poles } G(s) = \{-1, -2\}$
2) $\lim_{k \to \infty} s_i(k) = \infty$ , $\text{Zeroes } G = \{\infty\}$
3) How do they get there ?





N.B. 1. $0 \le k < 1.25$ stable
2. $1.25 \le k \le 2.5$ unstable
3. $k > 2.5$ stable



In particular $k \to s_i(k)$ is not 1-1!!

$m = p = 1$ $\quad 1 + kg(s) = 0 \Rightarrow k = -1/g(s)$

Indeed $\boxed{\det(I + kG(s)) = 0}$

Set $g = -1/k$

(*) $\boxed{\det(gI - G(s)) = 0}$

In our example

$$0 = f(s,g) = 1.25[s^2g^2 + 3sg^2 + 2g^2] - 2sg + 3g + \frac{4}{5}$$

$$\therefore \quad g_{\pm}(s) = \frac{(2s-3) \pm \sqrt{1-24s}}{(2.5)(s+1)(s+2)}$$

The algebraic functions have a branch point at

$$s = 1/24$$

Similarly, one computes

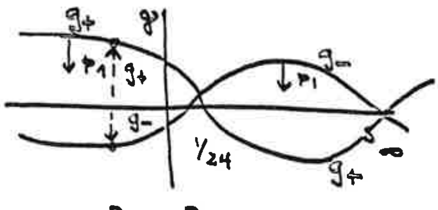
$$s_{\pm}(g) = \frac{(2-3g) \pm \sqrt{9g - g^2}}{(2.5g)}$$

$g_-(-2) = \infty$, $g_-(-1) = \infty$ $\quad g_+(-2) = \frac{-7 + \sqrt{49}}{\quad}$

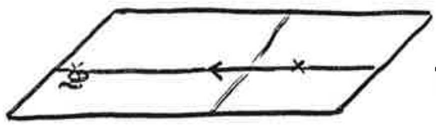


Consider $f(s,g) = 0$ . This defines the algebraic curve / Riemann surface $X$

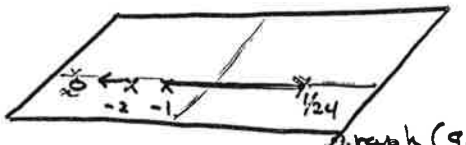
X




$X \subset S^2 \times S^2$, $p = \text{proj}_1(s,g) = s$



graph$(g_+)$




graph$(g_-)$

Newton (1680)

$$(1+x)^{\alpha} = \sum a_i x^{(p/q)^i}$$

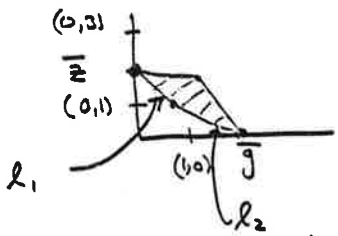
$$y = (1+x)^{\alpha} : f(x,y) = y(1+x)^{-\alpha} - 1 = 0$$

$$f(s,g) = c\left[s^2 g^2 + 3sg^2 + 2g^2\right] - 2sg + 3g + d = 0$$

set $z = 1/s$ obtaining

$$f(z,g) = 0$$

how does $z_i(g) \to 0$ as $g \to 0$




slope $\ell_1 = -q_1/p_1$, height $= r$

Newton: $\exists$ $r$ branches of $z_i(g)$

$$z_i(g) = \alpha_i g^{p_1/q_1} + \ldots$$

Scalar I/O $\ell_1$ has slope $-n^*$

$$z_i(g) = \alpha_i g^{1/n^*} \leftarrow \text{Butterworth pattern}$$

Newton's Theorem $f(s,g) = 0$ has $n$ branches

$$f(s,g) = \prod_{i=1}^{n} (s - s_i(g))$$

At any branch point, each $s_i(g)$ can be developed into a Newton-Puiseux expansion

$$s_i(g) = \alpha_i g^{p_i/q_i} + \ldots$$

The sum of the orders $= n$.

---

Applications:

1) Root Locus Plots (Postlethwaite-MacFarlane)

2) General Position Lemmas:

Lemma Given $G(s)$, $\delta(G(s)) = n$. There exists $u = -ky$ such that $G_K(s)$ has distinct poles.

3) Adaptive Control: $G(s)$ $m\times m$ $\delta(G)$ a. Suppose $\det G(s) = 0 \Rightarrow s \in \mathbb{C}^+$, $\mathrm{spec}(CB) \subset \mathbb{C}$. Then $u = -ky$, $\dot{k} = \|y\|^2$ is self-tuning. That is

(i) $k_t \to k_\infty$ ; (ii) $x_t \to 0$.

## Frozen Eigenvalue Analysis

L8

Theorem Consider the system

$$\dot{x} = Ax - k(t)Bx \qquad (*)$$

where

(i) for $k \gg 0$ $A - kB$ is stable

(ii) $k(t) \uparrow +\infty$

Each solution to $(*)$ tends to zero exponentially.

Proof: $x \in \mathbb{R}^n$, $A^{n\times n}$, $B^{n\times n}$ ..

$$\mathrm{rank}\, B = m \le n$$

$\therefore$ $B = B_1 C_1$, $B_1$ $n\times m$, $C_1$ $m\times n$

$$\underset{x_1}{\ker C_1} + \underset{y}{\mathrm{span}\, B_1} = \mathbb{R}^n$$

Consider $(x_1, y) = x \in \mathbb{R}^n$ and $(*)$

$$\dot{x}_1 = A_{11}x_1 + A_{12}y$$

$$\dot{y} = A_{21}x_1 + A_{22}y - k(t)\beta y$$

I. $\mathrm{spec}(\beta) = \mathrm{spec}(C_1 B_1)$ ✓

II. $s \in \mathrm{spec}(A_{11}) \iff \det C_1(sI-A)^{-1}B_1 = 0$.

Frozen eigenvalue hypotheses imply

I. $\mathrm{spec}(\beta) \subset \mathbb{C}^+$

II. $\mathrm{spec}(A_{11}) \subset \overline{\mathbb{C}^-}$

Choose coordinates $z = Ty$ so that $\beta$ is block

$$\beta = \begin{bmatrix} \alpha_1 & \beta_1 & & \\ -\beta_1 & \alpha_1 & & \\ & & \beta_2 & \\ & & & \ddots \\ & & & & \beta_r \end{bmatrix}$$

Then,

$$\frac{d}{dt}\left(\tfrac{1}{2}\|z\|^2\right) = \langle z, A_{21}x_1\rangle + \langle z, A_{22}z\rangle - k(t)\langle \beta z, z\rangle$$

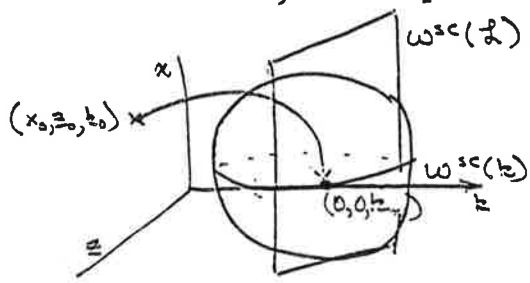
$$\le x^T(AC_1^TC_1 + C_1^TC_1A^T)x - \beta_{min}k(t)\|z\|$$

$$\le -c\|z\|^2 \qquad t \gg 0.$$

or $\|z_t\|^2 \le e^{-ct}\|z_0\|^2$ $t \gg 0$

**Theorem** If we have a convergent parameter adaptive stabilization scheme, then $n_z \geq n^* - 1$.

**Proof**




**Lemma 1** $u = \dim W^u_{(0,0,k_\infty)} = 0$.

**Pf** Follows from Reduction Theorem of Shoshitaishvili / Palis–Takens:

$$\dot{x} = f(x), \quad \dot{y} = y, \quad \dot{z} = -z \qquad \text{local canonical form}$$

$x \in W^c$, $z \in W^s$, $y \in W^u$

$$Jf_{(0,0,k_\infty)} = \begin{bmatrix} A + bJ_\infty c & bH_\infty & 0 \\ G_\infty c & F_\infty & 0 \\ k_x & 0 & k_z \end{bmatrix}$$

**Lemma 2** $\operatorname{spec}\begin{pmatrix} A + bJ_\infty c & bH_\infty \\ G_\infty c & F_\infty \end{pmatrix} \subset$ [illegible]

**Lemma 3** Consider $g(s) = \dfrac{1}{s^n - s^{n-1} \dots - 1}$

suppose $(F_\infty, G_\infty, H_\infty, J_\infty)$ stabilizes (internally) $g(s)$ and has degree $n_z$. Then,

$$n_z \geq n - 1.$$

Q.E.D.

---

**Theorem** (B & B) Suppose $g(s)$:

$$\dot{x} = Ax + bu, \quad y = cx$$

is minimum phase and has relative degree

$$\bar{n} \leq n^*$$

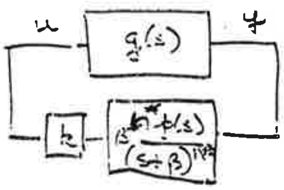
If $p(s)$ is minimum phase of degree $n^*$

$$h(s) = \frac{k\beta^{n^*} p(s)}{(s+\beta)^{n^*}}$$

where

$$\dot{k} = y^2, \quad \dot{\beta} = e^{k}$$

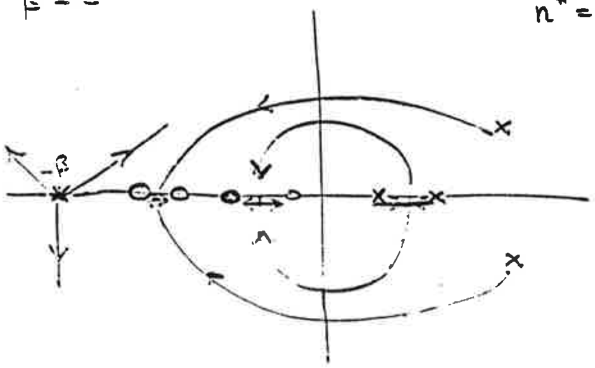
stabilizes $g(s)$.




$\dot{k} = y^2$

$\beta = e^{k}$

$n^* = 3$




$$(s+\beta)^{n^*} d(s) + k\beta^{n^*} p(s)\, n(s) = 0$$

Frozen eigenvalue analysis: stable for $k >> 0$
- algebraic function theory but:

**Sketch** Assume $cA^{n^*-1}b > 0$.

$$c(s) = \frac{k\beta^{n^*-1} p(s)}{(s+\beta)^{n^*-1}}$$

$p(s)$ Hurwitz of degree $n^* - 1$

$$g(s) = \frac{n(s)}{d(s)}$$

**N.B.** In closed loop, this is the same as

$$c'(s) = \frac{k\beta^{n^*-1}}{(s+\beta)^{n^*-1}}$$

compensating

$$g'(s) = \frac{p(s)\, n(s)}{d(s)}$$

$g'(s)$ has relative degree 1 !!

$$\underset{x_1}{\ker c} + \underset{y}{\operatorname{span}(b)} = \mathbb{R}^n$$

## Modifications and Extensions

$$g(s) = c(Is-A)^{-1}b = \frac{p(s)}{q(s)}$$

1. minimum phase
2. $n \le N$ upper bound on order of system
3. $n^* = \deg q - \deg p$ is known
4. $\mathrm{sign}(cA^{n^*-1}b)$ is known

2) is not needed, in fact $n = \infty$,

$$\dot{x} = Ax + bu, \quad x \in \mathcal{H}$$
$$y = cx \quad u, y \in \mathbb{R}$$

$A: \mathcal{D}(A) \to \mathcal{H}$ densely defined + pure point spectrum.

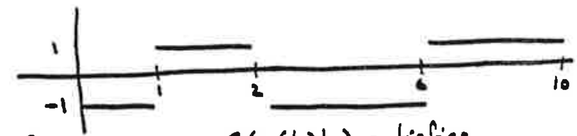
Lemmas 1. $\overline{A+bf}$ pure point spectrum – Weyl von-Neumann Theorem

2. $\mathrm{spec}\, \overline{A+bf} = Z(g(s))$
$\Rightarrow \overline{A+bf}$ stable : Hille-Yosida Thm.

Examples delay or neutral systems, $u_{tt} + u_{xxxx} = 0$
Flexible structures

Ex. $\dot{x} = ax + bu$, $\dot{k} = x^2$, $u = -\mathrm{sign}(b) kx$

Instead, set $u = s(k)ky$ R. Nussbaum



so that Césaro mean $C(s(k)k)$ satisfies

$$\overline{\lim_{k \ge 1}}\, C(s(k)k) = +\infty \qquad \underline{\lim_{k \ge 1}}\, C(s(k)k) = -\infty$$

$$\int_0^T \dot{V}\, dt = \frac{x^2}{2}\Big|_0^T = a\int_0^T x^2 + b\int_1^T s(k)kx^2\, dt$$
$$= ak\Big|_0^T + b\, kC(k)\Big|_0^T$$

$\therefore ak + bkC(k)$ is bounded below
$\therefore |k_t| \le M$ Q.E.D.



## Necessary Conditions in Adaptive Stabilization.

"Whatever can be done adaptively can be done if we know $(A, b, c)$"

or,

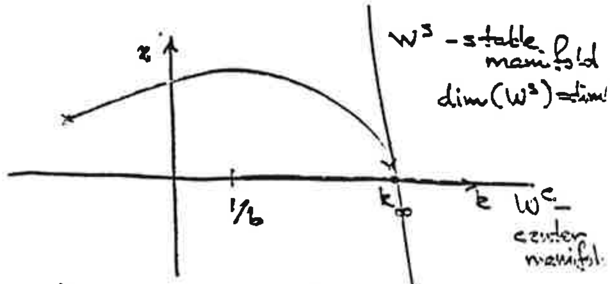
Adaptive stabilization with smooth nonlinear controllers of dim $\le N$ implies (classical) stabilization with linear compensators of dim $\le N$. (B.-Morse)

Proposition If $C$ is a collection of minimum phase systems which can be stabilized by some linear system of order $\le N$, then for all $g \in C$

$$n^*(g) \le N+1.$$

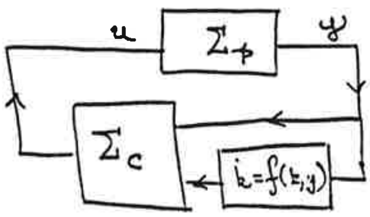
Corollary For minimum phase systems, an upper bound on $n^*$ is necessary for adaptive stabilization.

Ex $\dot{x} = x - kbx$
$\dot{k} = x^2$



$(x_t, k_t) \to (0, k_\infty)$

$$\dot{x} = Ax + bu, \quad y = cx$$
$$\dot{k} = f(k, y)$$
$$\dot{z} = F(k)z + G(k)y$$
$$u = H(k)z + J(k)y$$



$(x_t, z_t, k_t) \to (0, 0, k_\infty)$ — Convergence

$$\dot{x}_1 = A_{11} x_1 + A_{12} y$$
$$\dot{y} = A_{21} x_1 + A_{22} y - k e^{rk} A b^{r-1} z_p$$
$$\dot{z} = \left[ e^{k}(-I) + \begin{bmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & & 1 \\ 0 & & & 0 \end{bmatrix} \right] z + \begin{bmatrix} y \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\dot{k} = y^2$$

N. B. $\dot{x}_1 = A_{11} x_1$ is stable.

$$\dot{z} = \begin{pmatrix} -e^{k} & & \\ & \ddots & \\ & & -e^{k} \end{pmatrix} z + \begin{pmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ 0 & & & 0 \end{pmatrix} z$$

are stable: minimum phase

$e^{k} \uparrow +\infty$ + "frozen analysis"

$$\frac{d}{dt} \|y\|^2 = A_{21} x_1 y + A_{22} y^2 - k e^{rk} c b z_1 y$$
$$\sim c_1 + c_2 k + c_2 k - c_1 k^2$$

$\Rightarrow$ $k$ bounded.

$\Rightarrow$ $x_{t,1}, z_t \to 0$ $\quad k_t \to k_\infty$

# Lyapunov functions, cost functions, and adaptive control

Rolf Johansson

Department of Automatic Control
Lund Institute of Technology
Lund, Sweden

Abstract

Lyapunov functions are derived for a class of discrete time adaptive systems. The derivations have been performed under the following assumptions: Reference value = 0, no non-minimum phase zeros of the control object, well damped desired closed loop poles. The parameter estimation is made via a gradient type of algorithm which makes use of the output error.

It is shown that the considered adaptive system have the following properties.

- Global stability in the sense of Lyapunov
- Exponential convergence to zero of the state vector and the output error
- Monotone convergence (not necessarily to zero) of the parameter errors

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# LYAPUNOV FUNCTIONS COST FUNCTIONS AND ADAPTIVE CONTROL

*

ROLF JOHANSSON

LTH

## STABILITY CRITERIA

What conditions for stability prevail in adaptive control theory?

- Strictly positive real transfer functions
- Persistency of excitation
- $\|\varphi\|$ limited

Criticism

- Implicit conditions
- No link between stability and convergence

Remedy?

- Try to stick to "old" concepts like
  - Lyapunov functions
  - Cost criteria
  - Gain margin



- Problem

  Stability of discrete-time direct adaptive control
- Previous work

  Lyapunov functions, SPR – Parks

  BIBO – Goodwin-Ramadge-Caines, Egardt
- My approach

  Deterministic case

  State-space

  Lyapunov function
- New results

  Stability: Lyapunov-stability

  Convergence: Exponential (Global)

## CONTENTS


- Introduction
- Direct adaptive control
- What should a state-space model contain?
- Parameter convergence
- Error and state dynamics
- Lyapunov function
- Stability
- Convergence
- Example
- Conclusions

## DIRECT ADAPTIVE CONTROL

OBJECT, PLANT

$$y(t) = \frac{B_1^*(q^{-1})}{A^*(q^{-1})} u(t)$$

CONTROL LAW

$$R_u^*(q^{-1})\, u(t) = -S_y^*(q^{-1})\, y(t)$$

CLOSED-LOOP POLES ASSIGNMENT

$$R_u^* A^* + S_y^* B^* = T_1^* A_m^* B_2^*$$

PARAMETRIZATION

$$y = \frac{b_0 q^{-k} B_2^*}{T_1^* A_m^* B_2^*}\left[u(t) + \theta^T \varphi(t)\right]$$

CONTROL LAW

$$u(t) = -\hat{\theta}^T(t)\varphi(t)$$

PARAMETER ESTIMATION

$$\hat{\theta}(t) = \hat{\theta}(t-k) + \gamma(\varphi(t-k))\,\varphi(t-k)\, e_f(t)$$

$$e_f(t) = T_1^*(q^{-1}) A_m^*(q^{-1})\, e \quad ; \quad e = y$$

$$\gamma(\varphi(t)) = \frac{1}{\beta_0 \|\varphi(t)\|^2}$$

$$0 < \frac{b_0}{\beta_0} < 2$$

## IDENTIFICATION DYNAMICS

$$\begin{cases} \hat{\theta}(t) = \hat{\theta}(t-k) + \gamma\,\varphi(t-k)\, e_f(t) \\ e_f(t) = b_0 q^{-k}\left[-\tilde{\theta}^T(t)\varphi(t)\right] \end{cases}$$

$$\|\tilde{\theta}(t)\|^2 - \|\tilde{\theta}(t-k)\|^2 =$$

$$= -\varkappa_\theta \cdot \frac{e_f^2(t)}{\|\varphi(t-k)\|^2} =$$

$$= -\varkappa_\theta \|\tilde{\theta}(t-k)\|^2 \cos^2\alpha(t-k)$$

$\alpha$ – Angle between $\varphi$ and $\tilde{\theta}$

! **Notice that $\|\tilde{\theta}(t)\|$ is determined already at time $(t-k)$ !!**

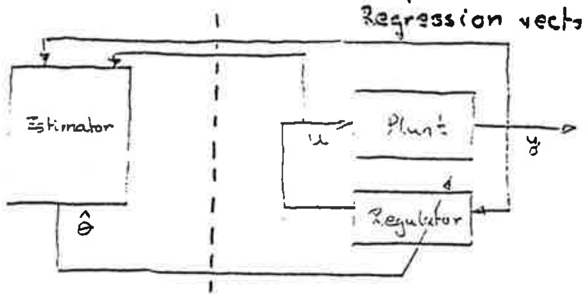
## STATE-SPACE?

Dynamics

| | |
|---|---|
| Control object | $x, y, u$ |
| Regulator | $u, y$ |
| Estimator | $\hat{\theta}, \tilde{\theta}$ |
| | $\varphi, \varphi(t-k)$ |

$\tilde{\theta}, \hat{\theta}$ ⟷ $x, y, u, \varphi, \varphi(t-k)$

Parameters — Input, Outputs, Regression vectors




**State at time $t$ includes**

$$\tilde{\theta}(t+k-1), \ldots, \tilde{\theta}(t)$$

**Introduce**

$$z(t) \triangleq \left[\tilde{\theta}^T(t+k-1) \ldots \tilde{\theta}^T(t)\right]^T$$

$$V_\theta(z(t)) \triangleq \|z(t)\|^2 = z^T(t) z(t)$$

$\Rightarrow$

$$V_\theta(z(t+1)) - V_\theta(z(t)) =$$

$$= -\varkappa_\beta \|\tilde{\theta}(t)\|^2 \cos^2\alpha(t)$$

$\alpha$ – angle between $\varphi, \tilde{\theta}$

Standard proof of parameter convergence

## ERROR DYNAMICS

More difficult?

| | |
|---|---|
| Control object | $u, y, x$ |
| Regulator | $u, y$ |
| Regression vectors | $\varphi$ |

**IDEA #1**

Relate all of them to $\xi(t)$?

$$u(t) = A^*(q^{-1})\,\xi(t)$$
$$y(t) = B_1^*(q^{-1})\,\xi(t)$$

$$x(t) = [x_1(t) \; \dots \; x_n(t)]^T$$

where $x_i(t) = \xi(t-i)$

$$\varphi(t) = [u(t-1) \dots u(t-k+1) \; y(t) \dots \;]^T$$
$$= M\,x(t)$$

State-space representation

$$x_i(t) \triangleq \xi(t-i)$$

$$\begin{bmatrix} x_1(t+1) \\ \vdots \\ x_{n_A}(t+1) \end{bmatrix} = \begin{bmatrix} -a_1 & \dots & \dots & -a_{n_A} \\ 1 & \ddots & & \\ 0 & & \ddots & \\ 0 & \dots & 0 \;\; 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_{n_A}(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u(t)$$

$$y_1(t) = [0 \dots 0 \; b_0 \; b_0 b_1 \; \dots \;]\, x(t)$$

Controllable canonical form

How does $\xi(t)$ develop?

$$\xi(t) = \dots = \frac{1}{T_1^* A_m^* B_2^*}\left[-\tilde{\theta}^T(t)\varphi(t)\right]$$

State-space formulation

$$x(t+1) = F x(t) + G\left(-\tilde{\theta}^T(t)\varphi(t)\right)$$

Remember that

$$\varphi = M x$$

Direct adaptive control reparametrization of $\xi(t)$?

$$\xi = \frac{R^* A^* + S^* B^*}{T_1^* A_m^* B_2^*}\,\xi =$$
$$= \frac{1}{T_1^* A_m^* B_2^*}\left[u + \theta^T \varphi\right]$$
$$= \frac{1}{T_1^* A_m^* B_2^*}\left[-\tilde{\theta}^T(t)\varphi(t)\right]$$

State-space formulation

$$\begin{bmatrix} x_1(t+1) \\ \vdots \\ x_n(t+1) \end{bmatrix} = \begin{bmatrix} -p_1 & \dots & -p_{n_p} & 0 & \dots & 0 \\ 1 & \ddots & & & 0 & \\ & & \ddots & & & \\ & 0 & & & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \left[-\tilde{\theta}^T(t)\varphi(t)\right]$$

$$T_1^* A_m^* B_2^* = 1 + p_1 q^{-1} + \dots + p_{n_p} q^{-n_p}$$

$$x(t+1) = F x(t) + G\left(-\tilde{\theta}^T(t)\varphi(t)\right)$$

## A STATE VECTOR

Parameter errors

$$z(t) = [\tilde{\theta}^T(t+k-1) \dots \tilde{\theta}^T(t)]^T$$

Error dynamics, regulator etc

$$x(t) = [\xi(t-1) \dots \xi(t-n)]^T$$

$$\mathbf{X}(t) = [z^T(t) \;\; x^T(t)]^T$$

$$\mathbf{X}(t+1) = \begin{bmatrix} 0 & \cdots & 0 & I & 0 \\ I & 0 & \cdots & & 0 \\ 0 & \ddots & \ddots & & \vdots \\ \vdots & & I & 0 & 0 \\ 0 & \cdots & 0 & 0 & F \end{bmatrix} \mathbf{X}(t) + \begin{bmatrix} L(\mathbf{X}(t)) \\ 0 \\ \vdots \\ 0 \\ G \end{bmatrix} \nu[\mathbf{X}(t)]$$

$$L[\mathbf{X}(t)] = \gamma(\varphi(t))\, b_0\, \varphi(t)$$

$$\nu[\mathbf{X}(t)] = -\tilde{\theta}^T(t)\varphi(t)$$

We have a linear system with

$$\nu[\mathbf{X}(t)] = -\tilde{\theta}^T(t)\varphi(t)$$

as a formal input ?

**$\nu$ is bilinear in state vector components !**

## IDEA # 2

A logarithmic function would dissolve the bilinear product

$$V_x(x(t)) \triangleq \ln[1 + x^T(t) Q^T Q x(t)]$$

**Q - weighting matrix**

$$V_x(x(t)) = \ln[1 + \|Qx(t)\|^2]$$

$$V_x(x(t+1)) - V_x(x(t)) \le$$

$$\le \varkappa_x \|\tilde{\theta}(t)\|^2 \cos^2 \alpha(t)$$

$\alpha$ - angle between $\tilde{\theta}$ and $\varphi$

**Matching with**

$$V_\theta(z(t)) = \sum_{i=0}^{k-1} \|\tilde{\theta}(t+i)\|^2$$

gives

$$V[\mathbf{X}(t)] = \sum_{i=0}^{k-1} \|\tilde{\theta}(t+i)\|^2 + K \cdot \ln[1 + \|Qx\|^2]$$

Lyapunov function

$$V[\mathbf{X}(t)] = \sum_{i=0}^{k-1} \|\tilde{\theta}(t+i)\|^2 + \ln[1 + \|Qx\|^2]$$

$$V[\mathbf{X}(t+1)] - V[\mathbf{X}(t)] \le$$

$$\le -\varkappa_V \|\tilde{\theta}(t)\|^2 \cos^2 \alpha(t)$$

## Conclusions - Stability

- Stability in the sense of Lyapunov
- Hard bound on output error from $\|Qx(t)\|^2 < e^{V[\mathbf{X}(0)]}$
- **No assumptions on persistent excitation**

# CONVERGENCE

- $\hat{\theta}(t), \hat{\theta}(t-1), \dots, \hat{\theta}(t-k+1)$

  do not necessarily converge to $\theta$

- $\|x\| \to 0$

Assume

$$\hat{\theta}(t) = \hat{\theta}(t-k) + \frac{1}{\beta_0 \|\varphi(t-k)\|^2} \varphi(t-k)\, e_f(t)$$

for all $\|\varphi\| \neq 0$. Then

Theorem 2

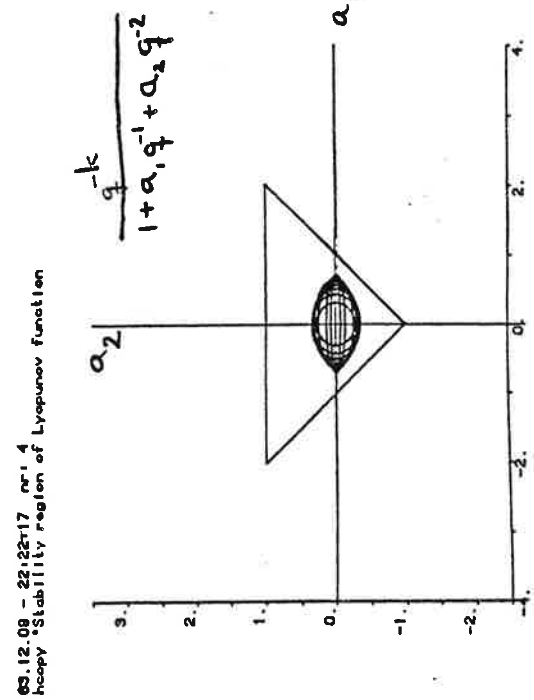
$$\sup \|Qx(t)\|^2 \leq C(t_0)\, e^{-\delta(t-t_0)}$$

$$V[X(t_0)] < C_0$$

with

$$C(t_0) = \exp[C_0/k]$$

$$\delta \in (0, 1)$$




# RESTRICTIONS

- ☐ Regulators only
- ☐ Only well damped closed-loop poles

$$T_1 A_M^* B_R^* = 1 + p_1 q^{-1} + \dots + p_{n_p} q^{-n_p}$$

$$\|Q^{-1} p\| < \tfrac{1}{2}[1-\rho]$$

- ☐ $\hat{\theta}(t) = \hat{\theta}(t-k) + \frac{1}{\beta_0 \|\varphi(t-k)\|^2} \varphi(t-k)\, e_f(t)$

  ↑ may become large

- ☐ Prior knowledge of

  $k$ - time delay

  $\beta_0$ - estimation of gain

$$0 < \frac{b_0}{\beta_0} < 2$$

[Ex.:

# Example

$$u \longrightarrow \boxed{\frac{b_0 q^{-3}}{1+\alpha q^{-1}+\beta q^{-2}}} \longrightarrow y$$

Develop an adaptive dead-beat control strategy

$$R^* A^* + S^* B^* = 1$$

$$\begin{cases} A^*(q^{-1}) = 1 + \alpha q^{-1} + \beta q^{-2} \\ B^*(q^{-1}) = b_0 q^{-3} \end{cases}$$

$$\begin{cases} R^*(q^{-1}) = 1 + r_1 q^{-1} + r_2 q^{-2} \\ S^*(q^{-1}) = s_0 + s_1 q^{-1} \end{cases}$$

$$\Rightarrow y = b_0 q^{-3}[u + \theta^T \varphi]$$

$$\begin{cases} \theta = [r_1 \; r_2 \; s_0 \; s_1]^T \\ \varphi(t) = [u(t-1) \; u(t-2) \; y(t) \; y(t-1)] \end{cases}$$

Ex. 2

$$\begin{cases} u(t) = A^x(q^{-1})\xi(t) = [1+\alpha q^{-1}+\beta q^{-2}]\xi(t) \\ y(t) = B^x(q^{-1})\xi(t) = b_0 q^{-3}\xi(t) \end{cases}$$

$\Rightarrow$

$$\begin{bmatrix} u(t-1) \\ u(t-2) \\ y(t) \\ y(t-1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \alpha & \beta & 0 \\ 0 & 1 & \alpha & \beta \\ 0 & 0 & b_0 & 0 \\ 0 & 0 & 0 & b_0 \end{bmatrix}}_{M_1} \begin{bmatrix} \xi(t-1) \\ \xi(t-2) \\ \xi(t-3) \\ \xi(t-4) \end{bmatrix}$$

$$\lambda_{max}(M_1^T M_1) \le 2(1+\alpha^2+\beta^2+b_0^2)$$

IDENTIFICATION

$$\hat\theta(t) = \hat\theta(t-k) + \frac{1}{\beta_3 \|\varphi(t-k)\|^2}\varphi(t-k)\, e_f(t)$$

$$e_f(t) = y(t) = b_0 q^{-k}(u+\theta^T\varphi)$$

$$= b_0 q^{-k}(-\tilde\theta^T(t)\varphi(t))$$

STATE VECTOR

$$[\tilde\theta^T(t+2)\ \tilde\theta^T(t+1)\ \tilde\theta^T(t)]^T = z(t)$$

$$V_\theta(z(t)) = z^T(t)z(t) = \sum_{i=0}^{2}\|\tilde\theta(t+i)\|^2$$

Ex. 2

$$x(t) = [\xi(t-1)\ldots\ldots\xi(t-7)]^T$$

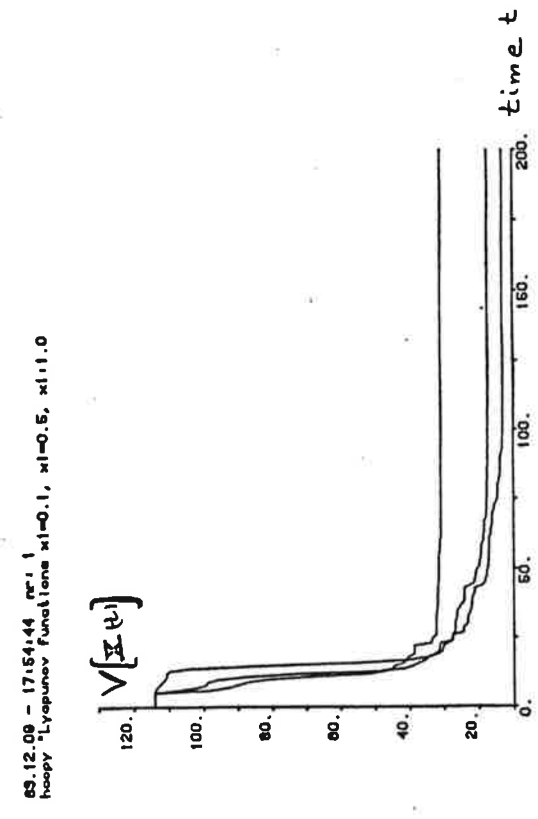
$$V_x(x(t)) = \ln[1+\|Qx(t)\|^2] =$$

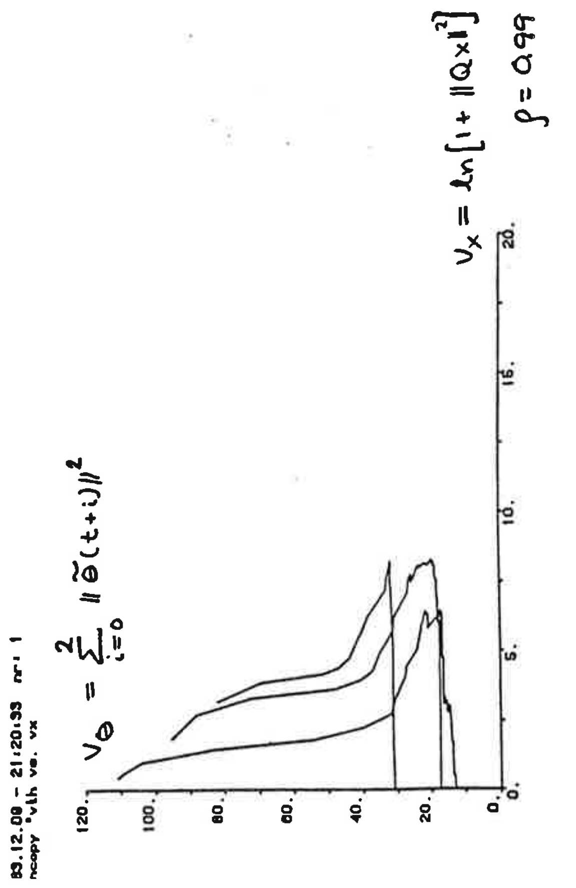
$$= \ln\left[1+\sum_{i=1}^{7}\xi^2(t-i)\rho^{2(i-1)}\right]$$

$$X(t) = [\tilde\theta^T(t+2)\ldots\tilde\theta^T(t)\ x^T(t)]^T$$

$$V(X(t)) = \sum_{i=0}^{2}\|\tilde\theta(t+i)\|^2 +$$

$$+\ln[1+\|Qx(t)\|^2]$$



# CONCLUSIONS

- Global stability in the sense of Lyapunov
- Exponential convergence of error (globally)
- **What remains?**

  Include reference value

  LS - identification

  Include noise and disturbances

# Instrumental variable methods for systems operating in closed loop with application to adaptive control

Torsten Söderström

Automatic Control and Systems Analysis Group
Department of Technology, Uppsala University

Instrumental variable methods are normally designed for systems operating in open loop. The reason is that the instruments are typically computed by filtering the input signal. Such an approach does not work if the system operates under feedback control, since then the instruments and disturbances becomes correlated. However, if there is a measurable setpoint (which anyhow is a modest identifiability condition) then this signal can be used for constructions of instruments. Some ways to achieve this are described.

Recent theory of instrumental variable methods show that the estimates are asymptotically Gaussian distributed. The instruments and data prefilters influence the covariance matrix of the estimates. They can in particular be chosen so that an optimal accuracy is obtained. This result is extended to the case of closed loop operation. A key part is then to use the "noisefree" part of the input and the output, i.e. the part that depends of the setpoint but not on the disturbances. The optimal instruments are easily computed once these noisefree signals (or consistent estimates thereof) and the noise correlation properties are known.

An optimal IV method can also be designed on a minimax basis. Minimization is then with respect to the instruments and maximization with respect to the disturbance properties. Also this IV variant is closely related to the noisefree input and output data.

It is further discussed how the above results for a time invariant feedback can be used for design of adaptive control system. Some few numerical simulated examples are also presented.

References

B. Bauer and H. Unbehauen (1978), On-line Identification of a load- dependent heat exchanger in closed loop using a modified instrumental variable method. Proc. IFAC 7th World Congress, Helsinki.

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P. Stoica and T. Söderström (1983), Optimal Instrumental Variable Estimation and Approximate Implementation. IEEE Transactions on Automatic Control, Vol AC-28, pp 757-772.

E. Trulsson (1983), Adaptive Control Based on Explicit Criterion Minimization. Doctoral Thesis, Department of Electrical Engineering, Linköping University, Linköping, Sweden.

## INSTRUMENTAL VARIABLE METHODS FOR CLOSED LOOP SYSTEMS

- MOTIVATION
- IV METHOD
  - DEFINITION
  - CLOSED LOOP SYSTEMS
  - CONSISTENCY
  - EXAMPLES
- OPTIMAL IV METHODS
  - OPTIMAL COV MATRIX
  - MIN MAX OPTIMALITY
  - EXAMPLES
- EXTENSION TO MULTIVARIABLE SYSTEMS
- APPLICATION TO ADAPTIVE CONTROL
  - APPROACHES
  - EXAMPLES
- CONCLUSIONS

## MOTIVATIONS

- INSTRUMENTAL VARIABLE METHODS GIVE CONSISTENCY IN PRESENCE OF "ARBITRARY" DISTURBANCES
- POTENTIALLY USEFUL FOR INDIRECT ADAPTIVE CONTROL
  - RECURSIVE IMPLEMENTATION EASY
  - QUICK CONVERGENCE
  - FEW PARAMETERS

## BASIC NOTATIONS

SYSTEM (S) $A_o(q^{-1})y(t) = B_o(q^{-1})u(t)+v(t)$

$v(t) = H(q^{-1})e(t) \quad Ee(t)e(s) = \lambda^2\delta_{t,s}$

$y(t) = \varphi^T(t)\theta_o+v(t)$

MODEL (M) $A(q^{-1})y(t) = B(q^{-1})u(t)$

$y(t) = \varphi^T(t)\theta$

EXPERIMENTAL CONDITION (X)

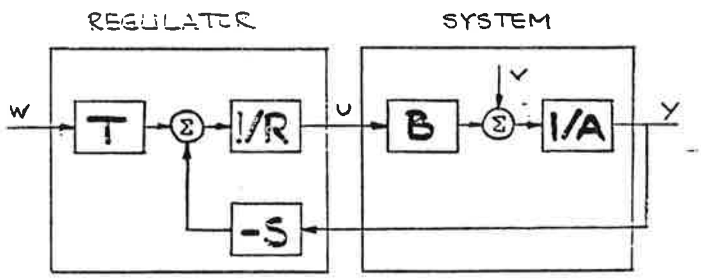
$R(q^{-1})u(t) = -S(q^{-1})y(t)+T(q^{-1})w(t)$

w(t) SETPOINT

$A_o$, $B_o$, A, B, R, S, T POLYNOMIALS IN $q^{-1}$

H RATIONAL FUNCTION IN $q^{-1}$

$\varphi(t) = [-y(t-1)\ldots-y(t-na)\ u(t-1)\ldots u(t-nb)]^T$



## IV METHOD

$$\hat{\theta} = [\frac{1}{N}\sum_{t=1}^{N} z(t)F(q^{-1})\varphi^T(t)]^{-1}[\frac{1}{N}\sum_{t=1}^{N} z(t)F(q^{-1})y(t)]$$

- $z(t)$ VECTOR OF INSTRUMENTS
- $F(q^{-1})$ (SCALAR) PREFILTER

## CONSISTENCY ANALYSIS

$$\hat{\theta}-\theta_o = [\frac{1}{N}\sum_{t=1}^{N} z(t)F(q^{-1})\varphi^T(t)]^{-1}[\frac{1}{N}\sum_{t=1}^{N} z(t)F(q^{-1})v(t)]$$

CONDITIONS

- i) $R \triangleq Ez(t)F(q^{-1})\varphi^T(t)$

  NONSINGULAR

- ii) $0 = Ez(t)F(q^{-1})v(t)$

ASSUME

- i) $z(t)$, $v(s)$ INDEPENDENT

  FOR ALL $t$ AND $s$

- ii) $w(t)$ PERSISTENTLY EXCITING

## EXAMPLES

1) $z(t) = [w(t-1)\dots w(t-na-nb)]^T$

   $F(q^{-1}) = 1$

2) $z(t) = [K_1(q^{-1})w(t)\dots K_{na+nb}(q^{-1})w(t)]^T$

## ACCURACY

$$\sqrt{N}(\hat{\theta}-\theta_o) \xrightarrow{DIST} N(0, P_{IV})$$

$$P_{IV} = \lambda^2 R^{-1} S R^{-T}$$

$$S = E[F(q^{-1})H(q^{-1})z(t)][F(q^{-1})H(q^{-1})z(t)]^T$$

## OPTIMAL COVARIANCE MATRIX

$$\boxed{P_{IV} \geq P_{IV}^{OPT}} = [E H^{-1}(q^{-1})\tilde{\varphi}(t)][H^{-1}(q^{-1})\tilde{\varphi}(t)]^T$$

$$\tilde{\varphi}(t) = [-\tilde{y}(t-1)\ldots-\tilde{y}(t-na)\ \tilde{u}(t-1)\ldots\tilde{u}(t-nb)]^T$$

$\tilde{\varphi}(t)$ DISTURBANCE FREE PART OF $\varphi(t)$

$$\tilde{\varphi}(t) = E[\varphi(t)|w(t-1), w(t-2)\ldots]$$

$$y(t) = \underbrace{\frac{B(q^{-1})T(q^{-1})}{A(q^{-1})R(q^{-1})+B(q^{-1})S(q^{-1})} w(t)}_{\tilde{y}(t)} + \frac{R(q^{-1})}{A(q^{-1})R(q^{-1})+B(q^{-1})S(q^{-1})} v(t)$$

## OPTIMAL IV METHODS

- CASE 1: MINIMAL COVARIANCE MATRIX

$$z(t) = H^{-1}(q^{-1})\tilde{\varphi}(t)$$

$$F(q^{-1}) = H^{-1}(q^{-1})$$

- CASE 2: MINIMAX IV METHOD

$$z(t) = \tilde{\varphi}(t) \qquad F(q^{-1}) = 1$$

SOLUTION TO

| MIN | MAX | $P_{IV}$ |
|---|---|---|
| $z(t)$ | $H(q^{-1})$ | |
| $R$ nonsing | $\lvert H(e^{i\omega})\rvert \leq \alpha$ | |

## EXAMPLE OF OPTIMALITY

SYSTEM $\quad y(t)+ay(t-1) = bu(t-1)+e(t)+ae(t-1)$

REGULATOR (DEADBEAT)

$$u(t) = \frac{a}{b}y(t)+\frac{1}{b}w(t)$$

$w(t)$ WHITE NOISE $\quad Ew^2(t) = \sigma^2$

- CASE 1

$$z(t) = \begin{bmatrix} w(t-1) \\ w(t-2) \end{bmatrix} \qquad P_{IV} = \frac{\lambda^2}{\sigma^2}\begin{bmatrix} 1+a^4 & a^3 b \\ a^3 b & (1+a^2)b^2 \end{bmatrix}$$

- CASE 2

OPTIMAL IV

$$P_{IV} = \frac{\lambda^2}{\sigma^2}\begin{bmatrix} 1-a^2 & 0 \\ 0 & b^2 \end{bmatrix}$$

## EXTENSIONS

- MULTIVARIABLE SYSTEMS
- $nz \geq n\theta$ (OVERDETERMINED IV EQUATIONS)

SYSTEM $\quad y(t) = \phi^T(t)\theta_0+v(t)$

$$v(t) = H(q^{-1})e(t) \quad Ee(t)e^T(s) = \Lambda\delta_{t,s}$$

$$\sqrt{N}(\hat{\theta}-\theta_0) \xrightarrow{DIST} N(0, P_{IV})$$

$$\boxed{P_{IV} \geq P_{IV}^{OPT}} = (E[H^{-1}(q^{-1})\tilde{\phi}^T(t)]^T\Lambda^{-1}[H^{-1}(q^{-1})\tilde{\phi}^T(t)])^{-1}$$

EQUALITY FOR

$nz=n\theta \qquad z(t) = [\Lambda^{-1}H^{-1}(q^{-1})\tilde{\phi}^T(t)]^T$

$$F(q^{-1}) = H^{-T}(q^{-1})$$

## APPLICATION TO ADAPTIVE CONTROL

ASSUMPTIONS

1. POLE ASSIGNMENT DESIGN
   CLOSED LOOP CHAR POL $P_o(q^{-1})$

2. COVARIANCE OPTIMAL IV
   WITH $H(q^{-1})$ DESIGN VARIABLE

## APPLICATION TO ADAPTIVE CONTROL

$$z(t) = H^{-1}[-\tilde{y}(t-1)\ldots-\tilde{y}(t-na)\ \tilde{u}(t-1)\ldots\tilde{u}(t-nb)]^T$$

$$\tilde{y}(t) = \frac{B_oT}{A_oR+B_oS} w(t)$$

$$\tilde{u}(t) = \frac{A_oT}{A_oR+B_oS} w(t)$$

GOAL FOR DESIGN

$$A_oR+B_oS \equiv P_o$$

$$B_o(1)T(1) = P_o(1)$$

## APPLICATION TO ADAPTIVE CONTROL

ALGORITHM

1. ESTIMATE A AND B BY RECURSIVE IV
2. SOLVE $\hat{A}_tR_t + \hat{B}_tS_t \equiv P_o$

   $\hat{B}_t(1)T_t = P_o(1)$

   FOR $R_t$ $S_t$ $T_t$
3. COMPUTE u(t) FROM

   $$R_t(q^{-1})u(t) = -S_t(q^{-1})y(t)+T_tw(t)$$
4. COMPUTE $\hat{\tilde{y}}(t)$, $\hat{\tilde{u}}(t)$ FROM

   $$\hat{\tilde{y}}(t) = \frac{\hat{B}_t(q^{-1})T_t}{P_o(q^{-1})} w(t)$$

   $$\hat{\tilde{u}}(t) = \frac{\hat{A}_t(q^{-1})T_t}{P_o(q^{-1})} w(t)$$

   (ONLY NUMERATORS ARE TIME VARYING!)
5. COMPUTE z(t) as

   $$z(t) = (1-q^{-1})[-\hat{\tilde{y}}(t-1)\ldots-\hat{\tilde{y}}(t-na)\ \hat{\tilde{u}}(t-1)\ldots\hat{\tilde{u}}(t-nb)]^T$$

   NOTE $H(q^{-1}) = \dfrac{1}{1-q^{-1}}$

   (DRIFT, ACTION FOR RESET WINDUP)

## APPLICATION TO ADAPTIVE CONTROL EXAMPLES

CASE 1

$\mathcal{S}$ $y(t)-1.9y(t-1)+0.9y(t-2) = 1.0u(t-1)+0.5u(t-2)$

$\mathcal{M}$ $y(t)+a_1y(t-1)+a_2y(t-2) = b_1u(t-1)+b_2u(t-2)$

CLOSED LOOP POLES: 0.5, 0, 0

CASE 2

$\mathcal{S}$ $y(t)-0.8y(t-1) = 1.0u(t-1) + \dfrac{1}{1-q^{-1}} e(t)$

$\lambda_e = 0.05$

$\mathcal{M}$ $y(t)+ay(t-1) = bu(t-1)$

CLOSED LOOP POLE: 0

METHOD

- $I_1$ $z(t) = [w(t-1)\ldots w(t-na-nb)]^T$ — $F(q^{-1}) = 1$ SIMPLE IV
- $I_2$ $z(t) = (1-q^{-1})\hat{\tilde{\varphi}}(t)$ — $F(q^{-1}) = 1-q^{-1}$ [illegible]
- $I_3$ $z(t) = \hat{\tilde{\varphi}}(t)$ — $F(q^{-1}) = 1$ MINMAX IV

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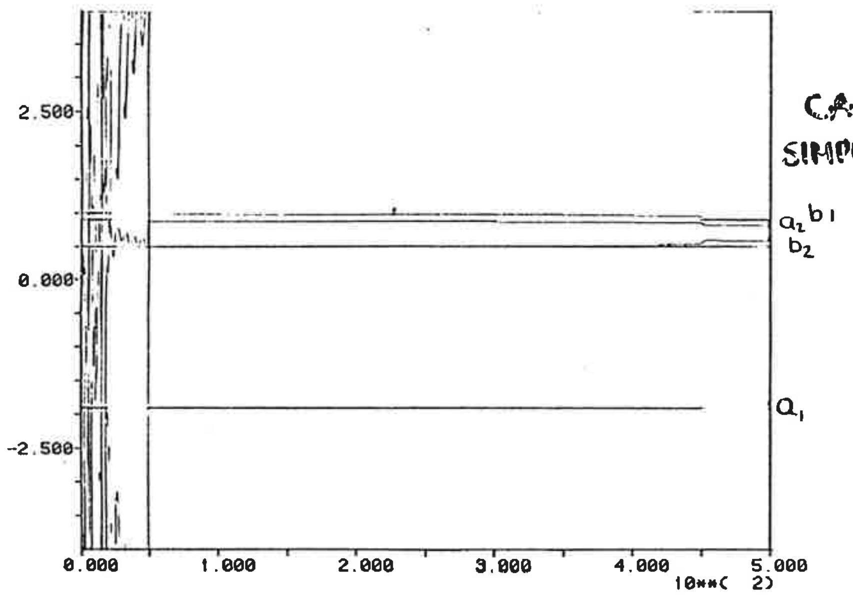


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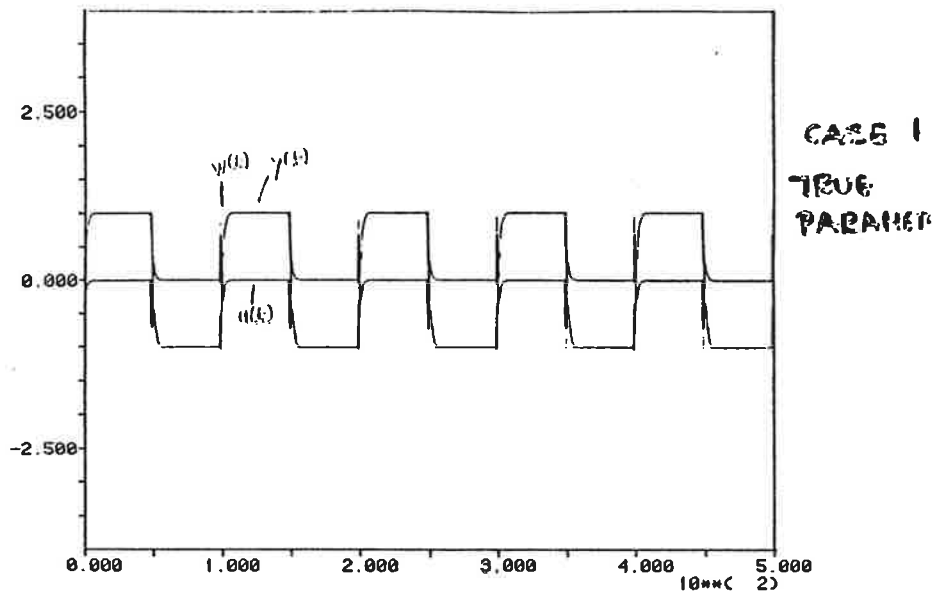


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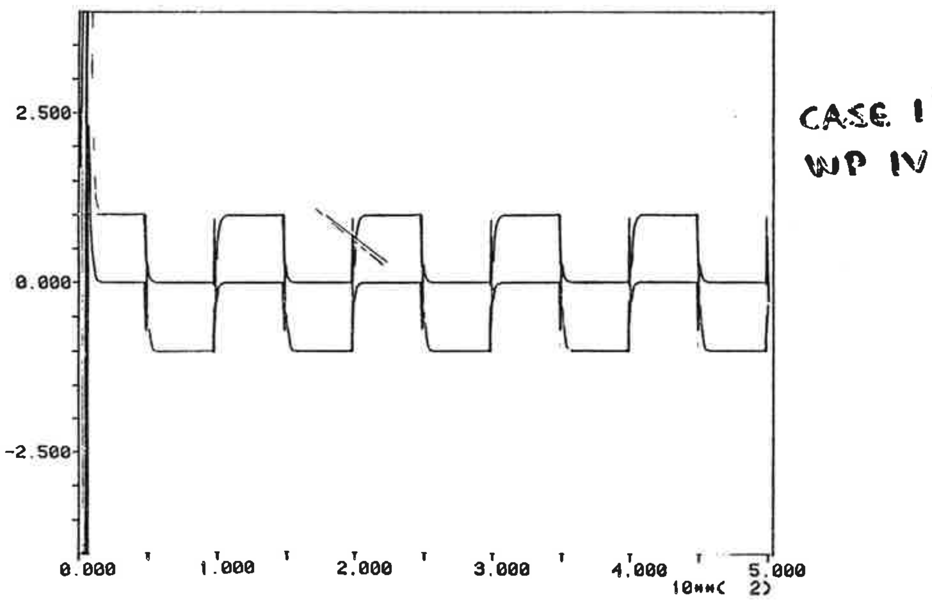


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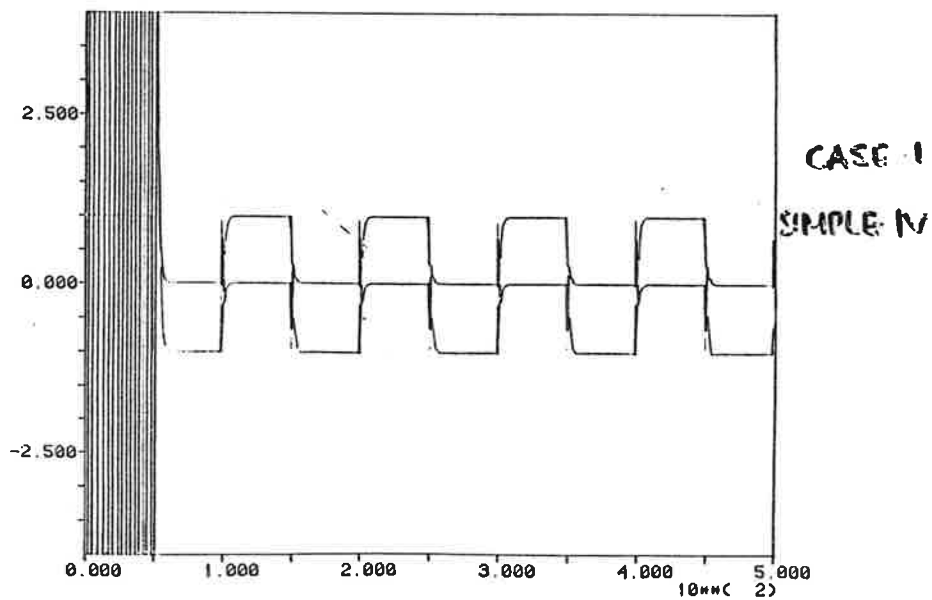

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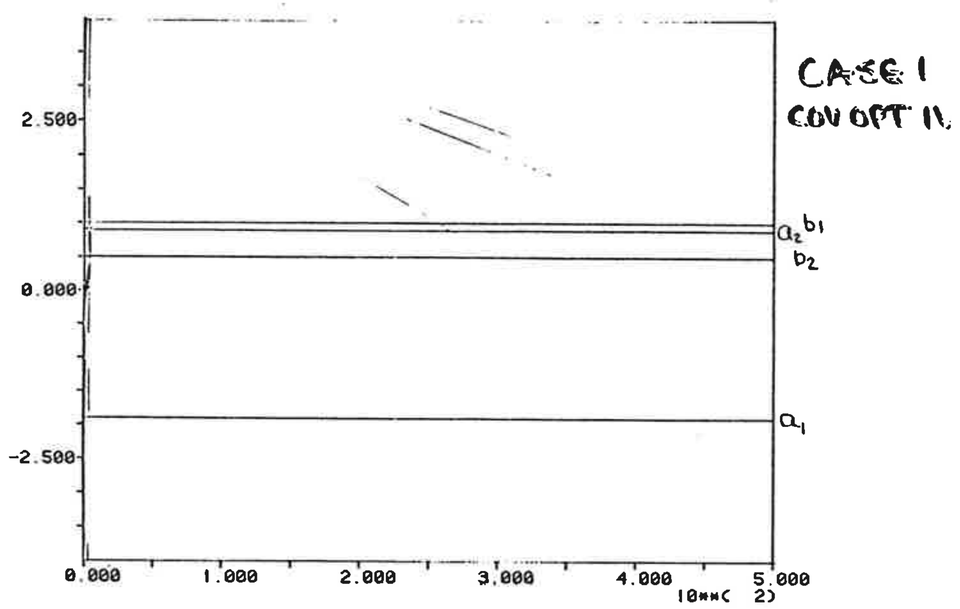


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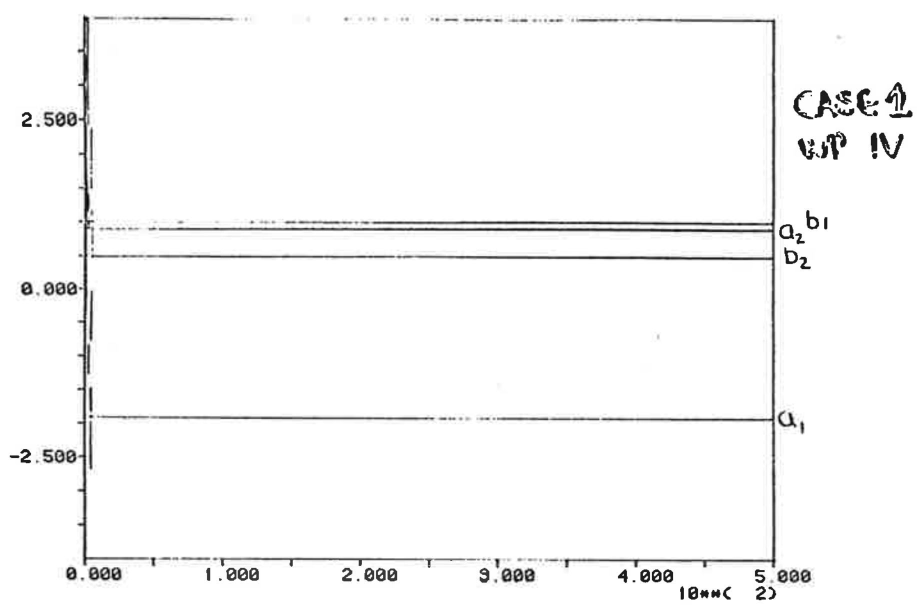


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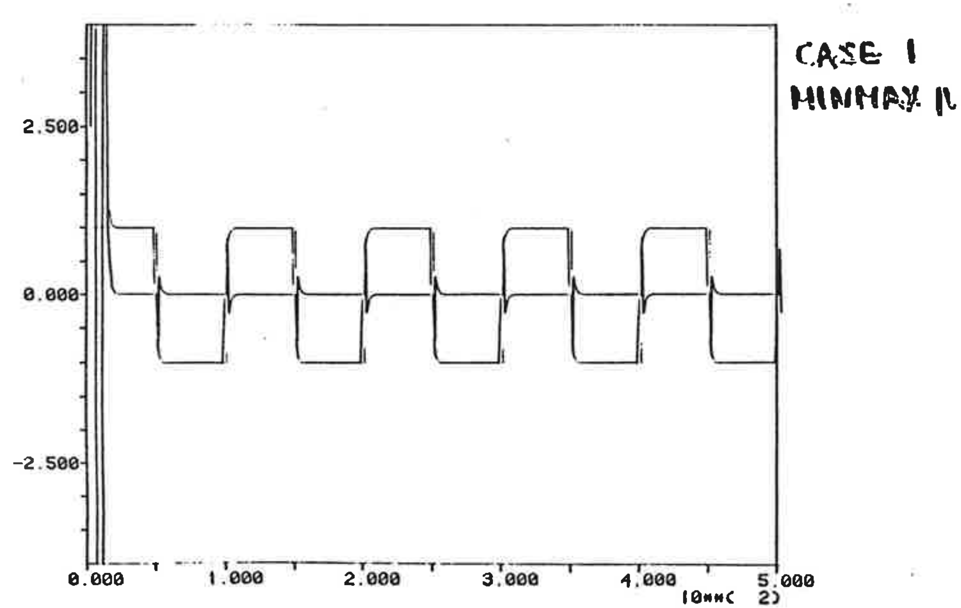


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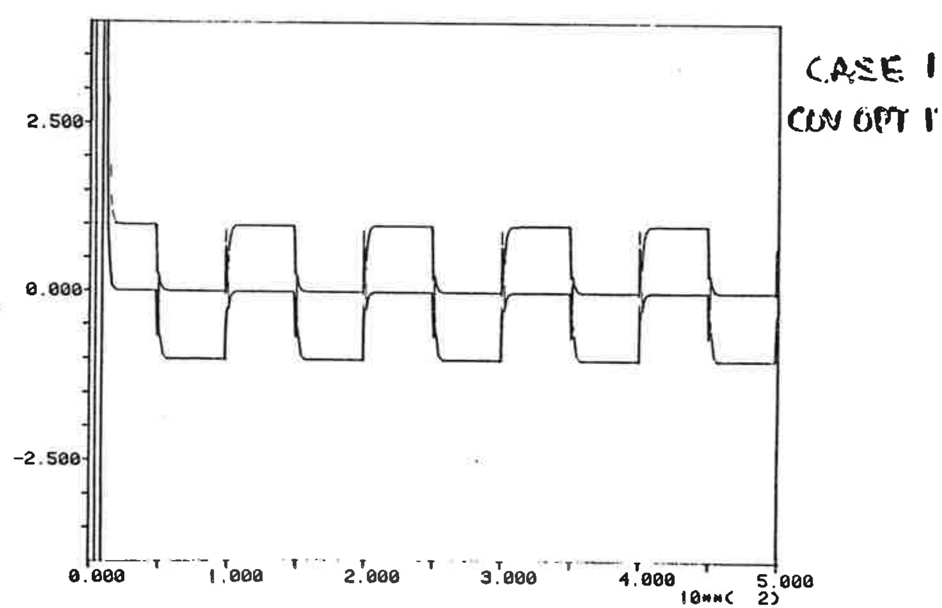

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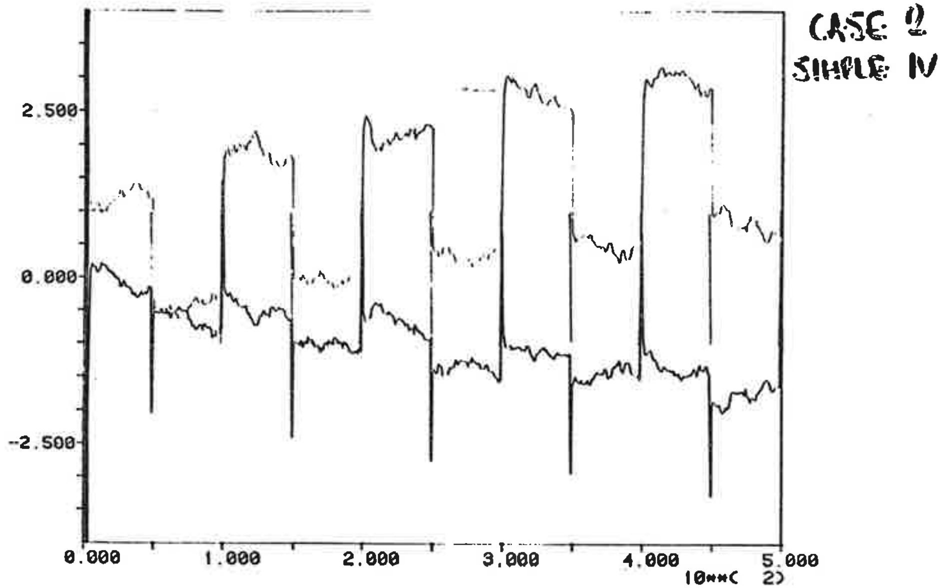


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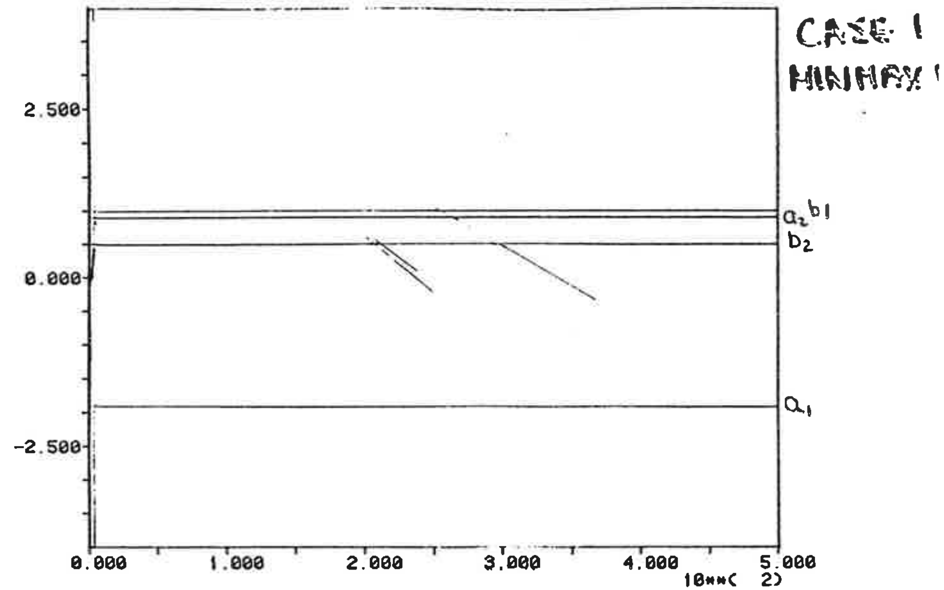


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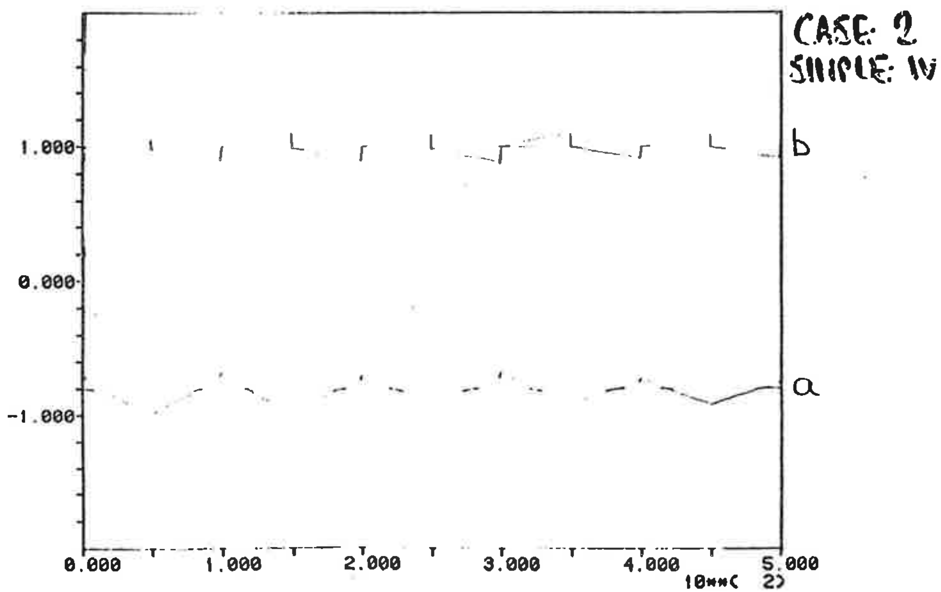


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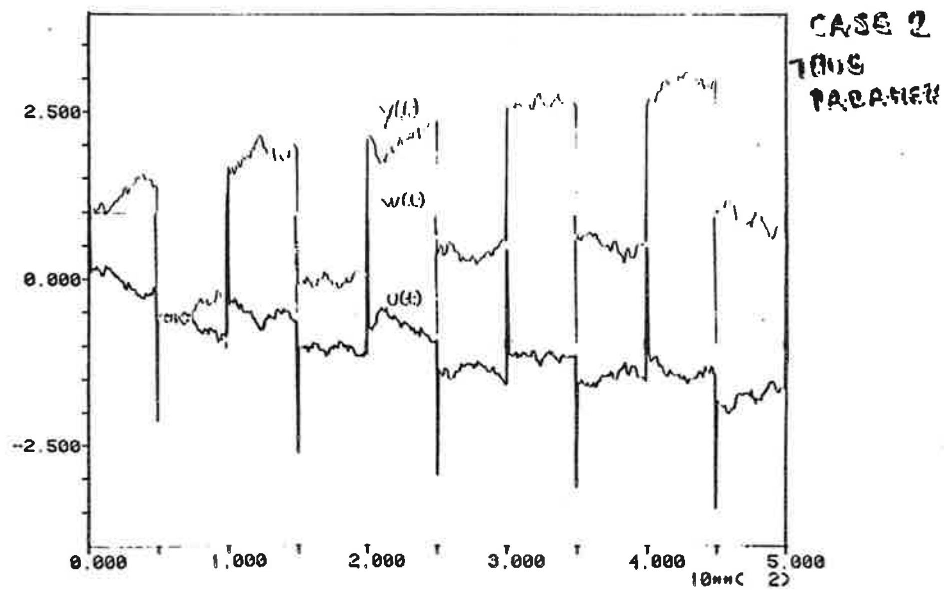

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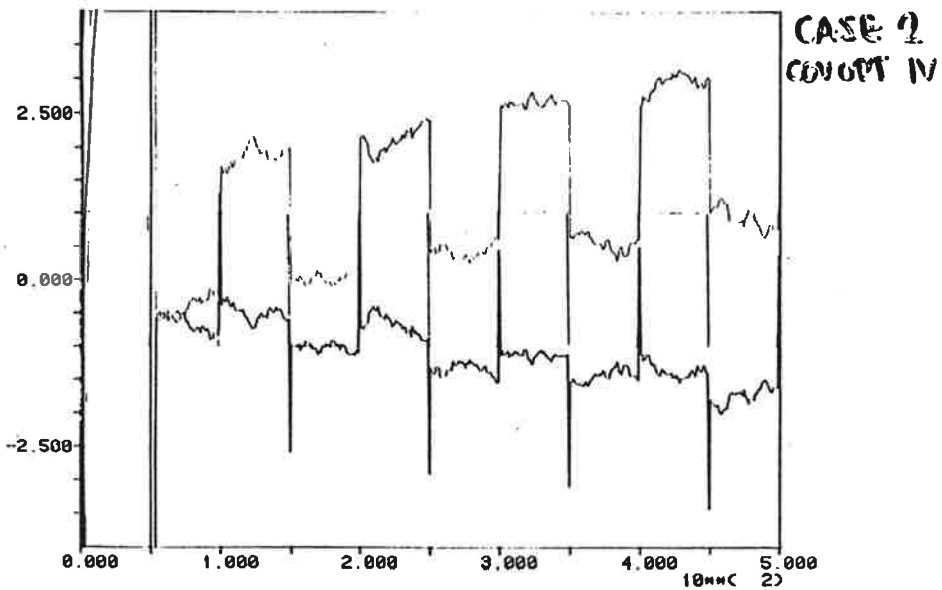


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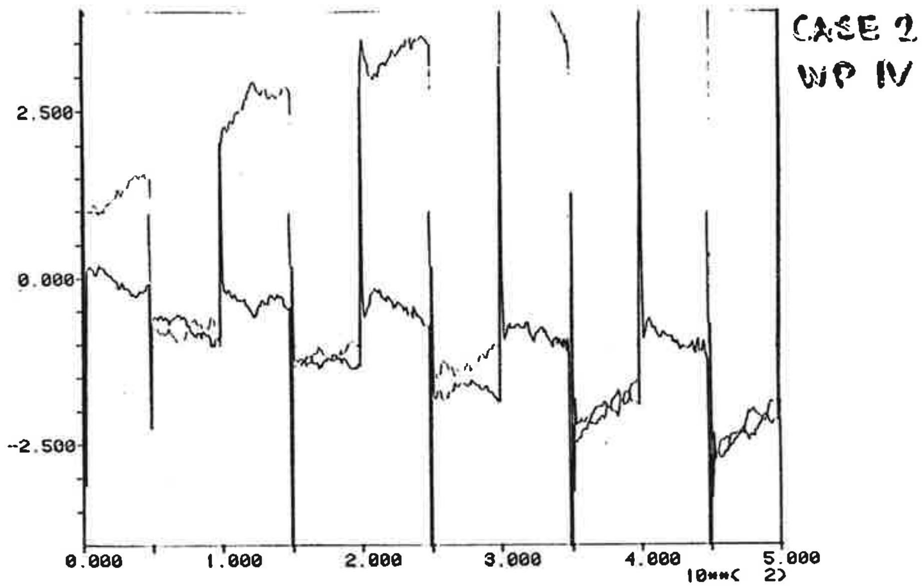


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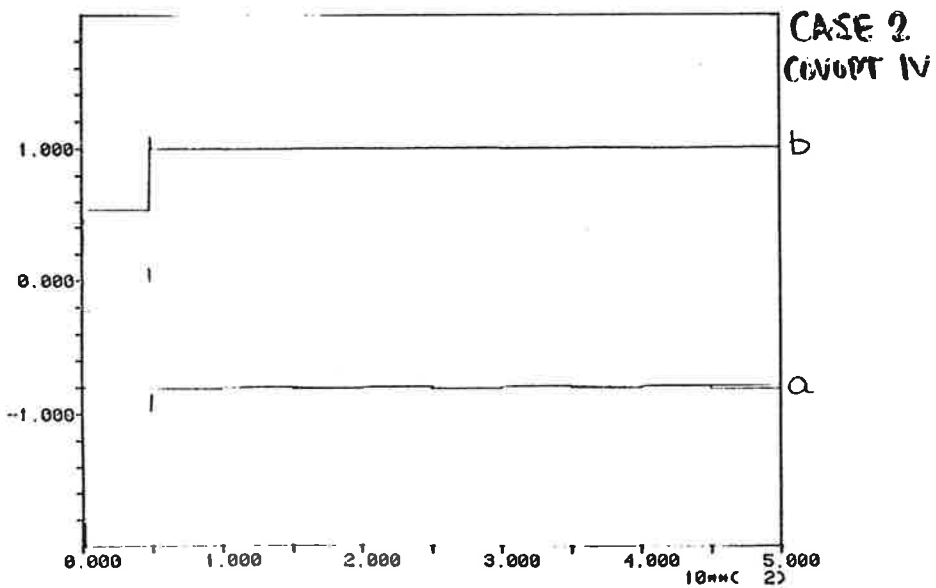


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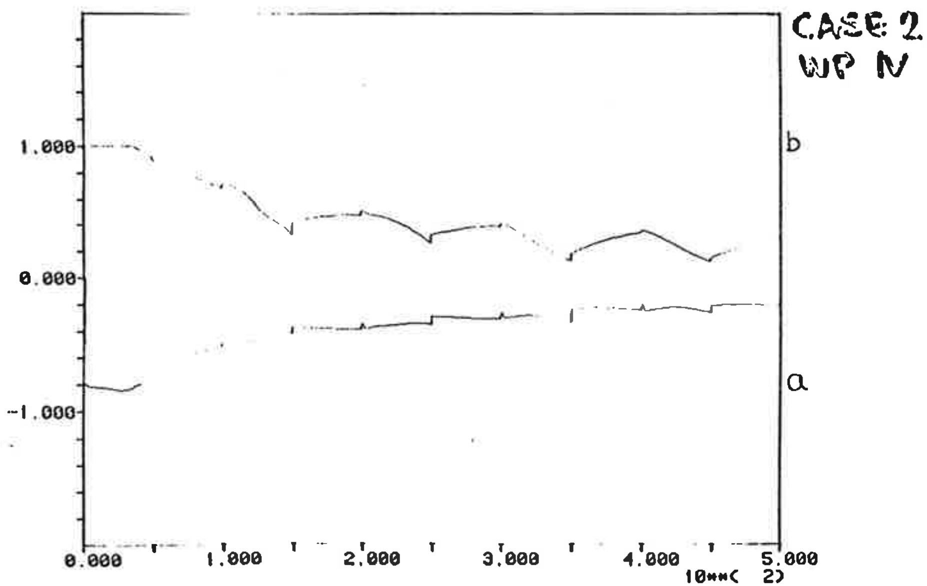

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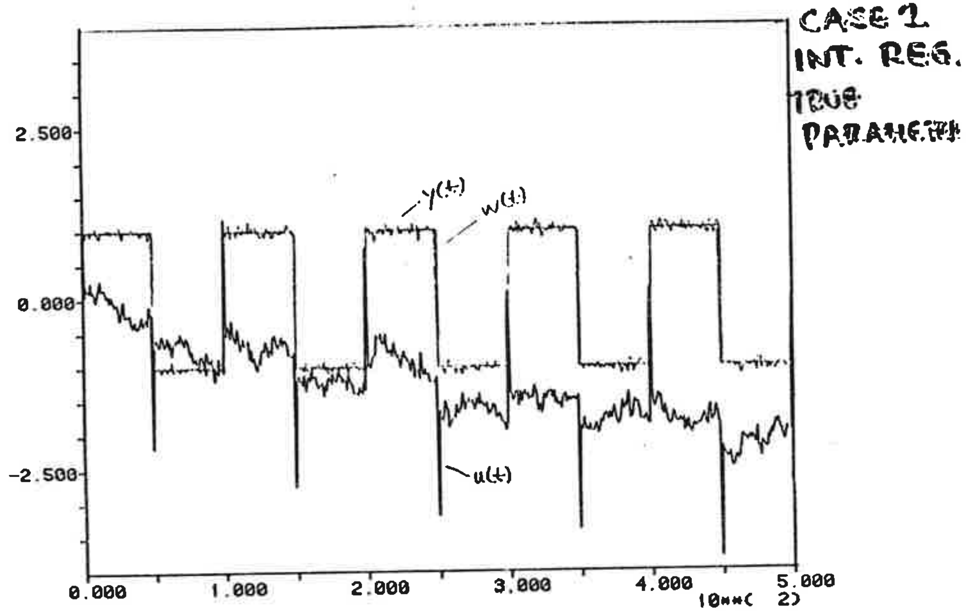


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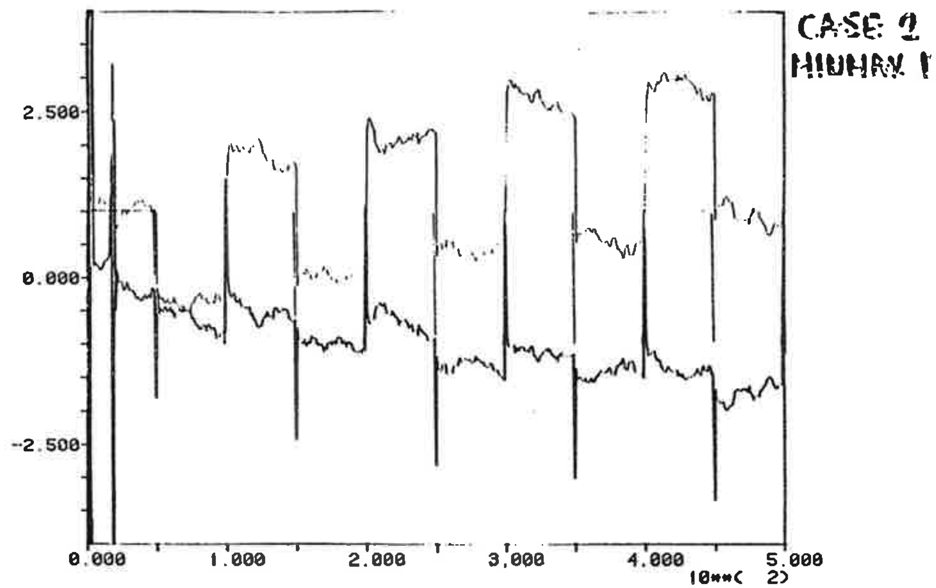


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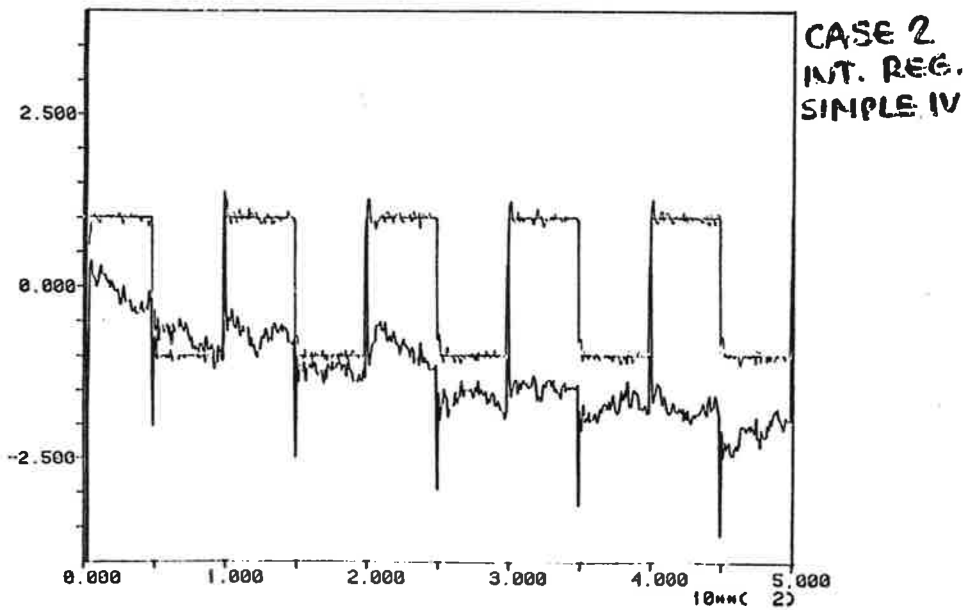


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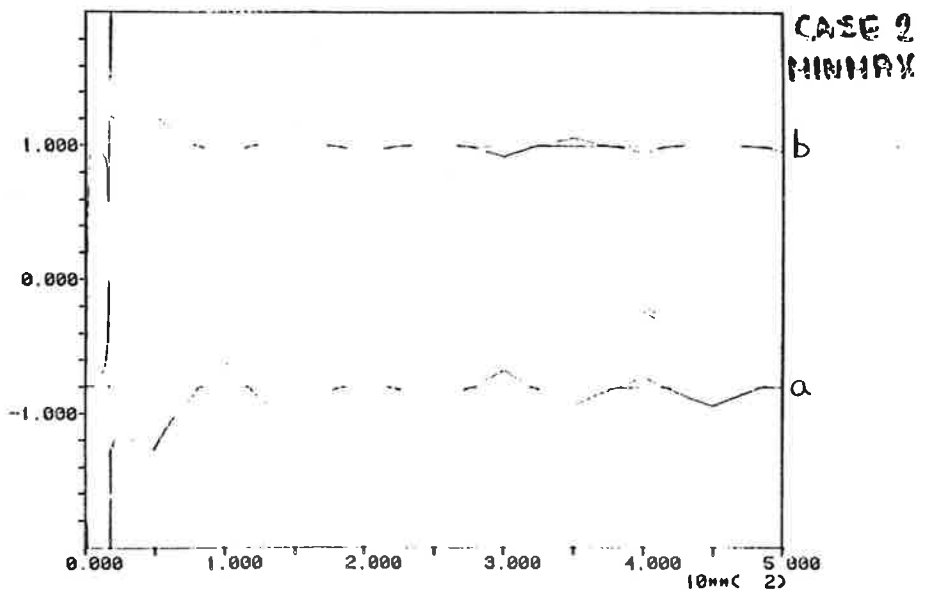

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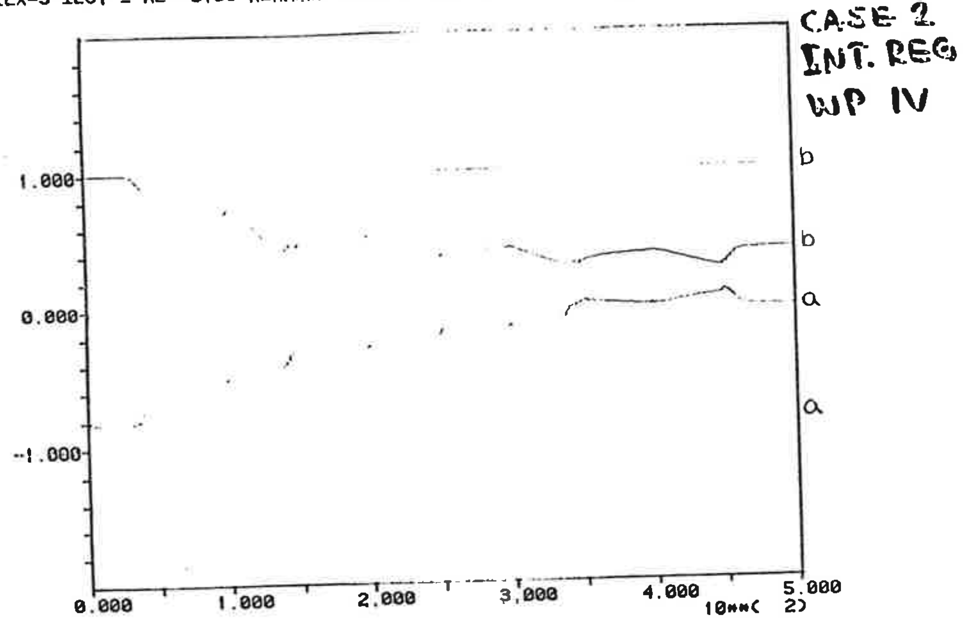


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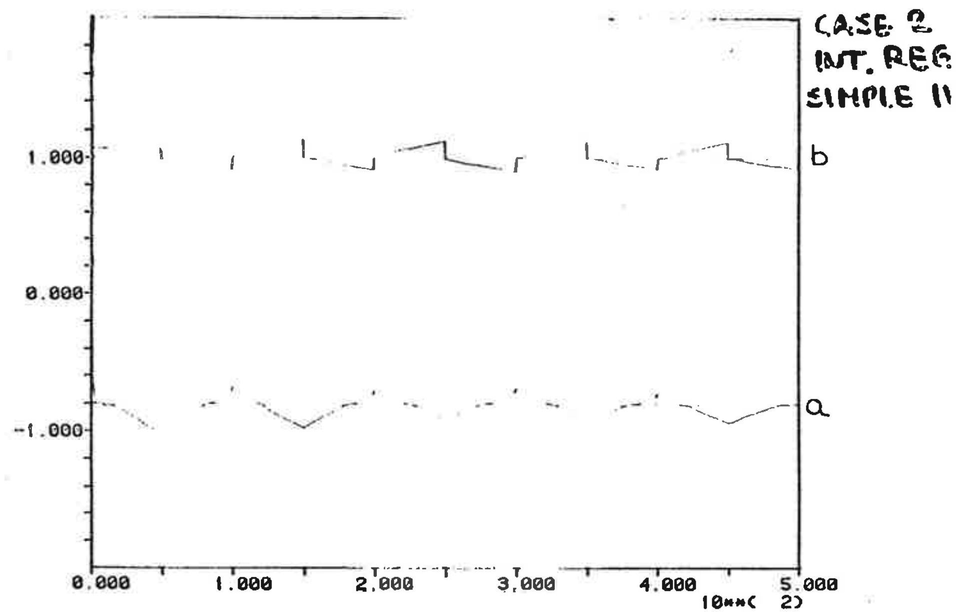


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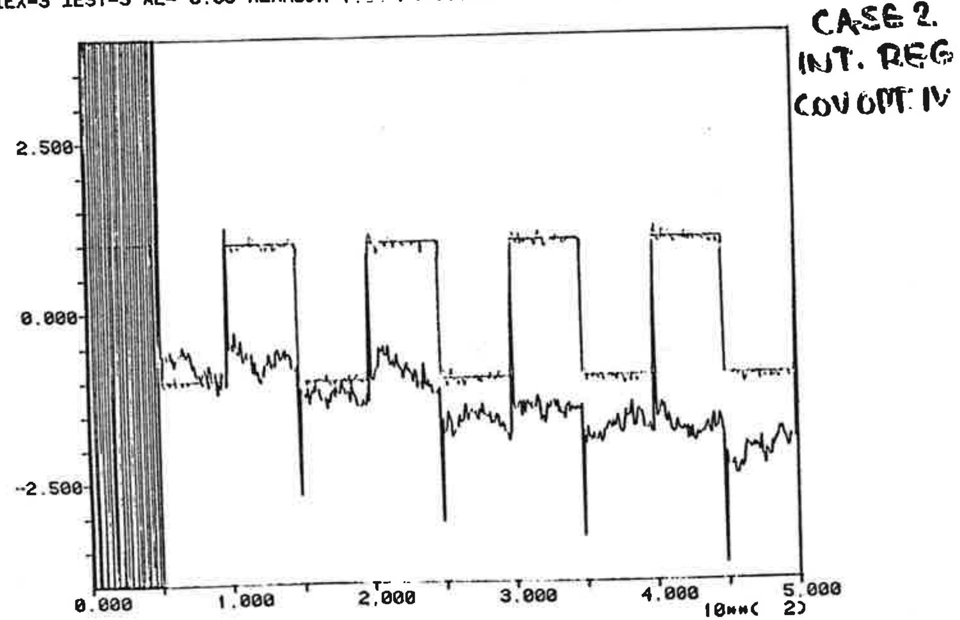


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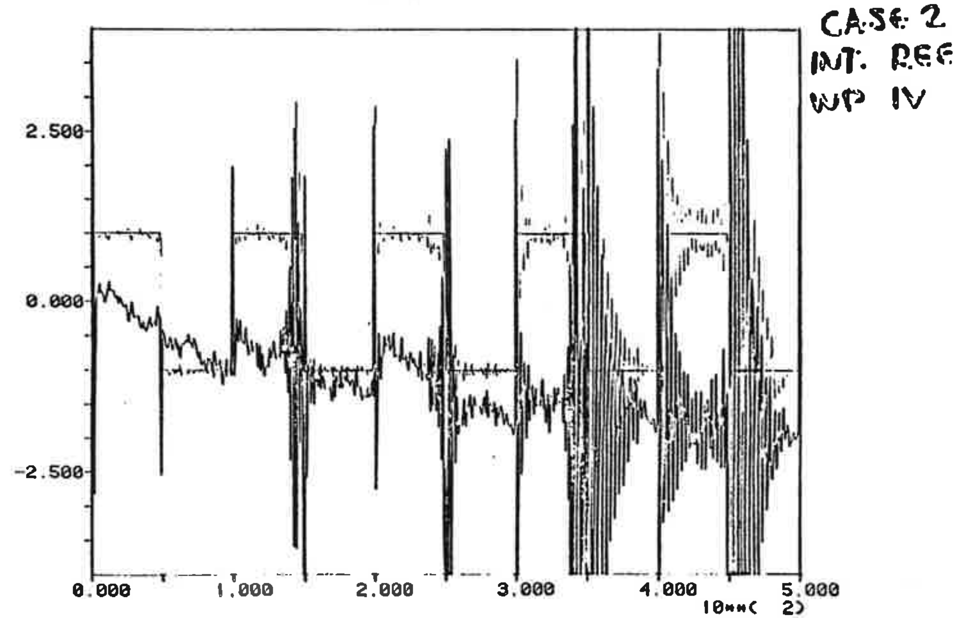

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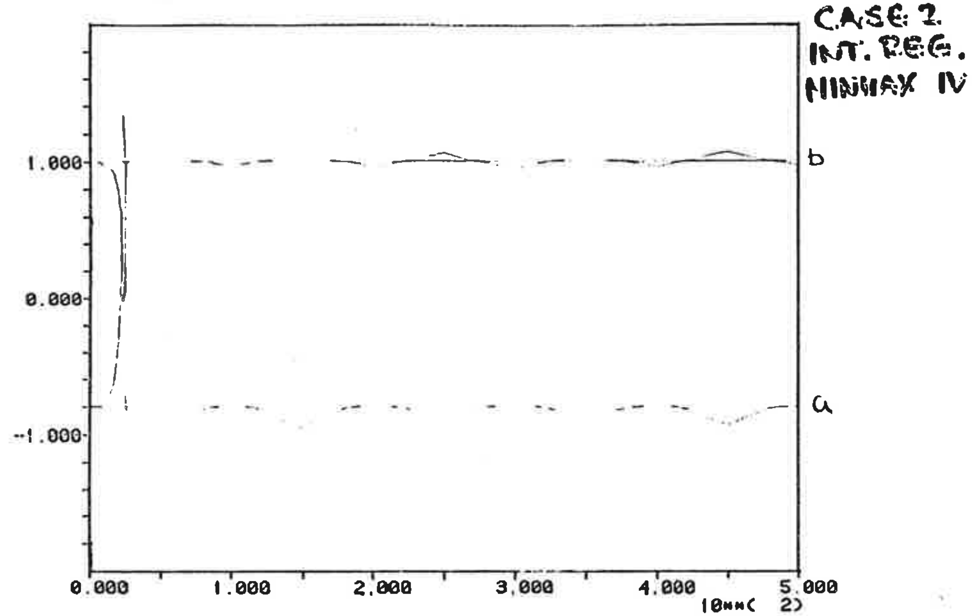


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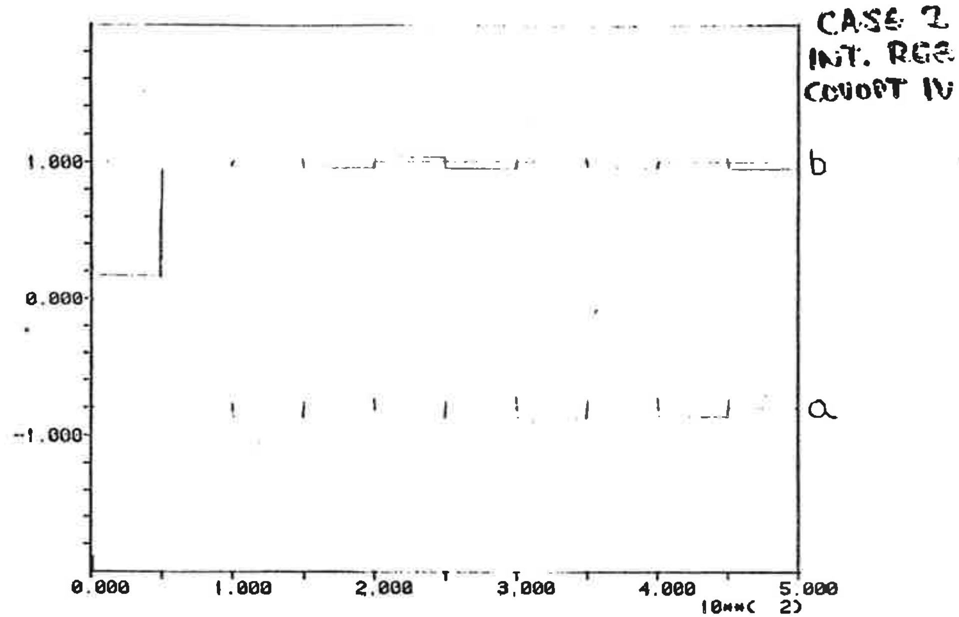


## CONCLUSIONS

- IV METHODS POSSIBLE TO USE IN CLOSED LOOP
- CHOICE OF INSTRUMENTS CAN BE IMPORTANT
- OPTIMAL IV METHODS EXIST
- SOME POTENTIAL FOR DESIGN OF ADAPTIVE CONTROL

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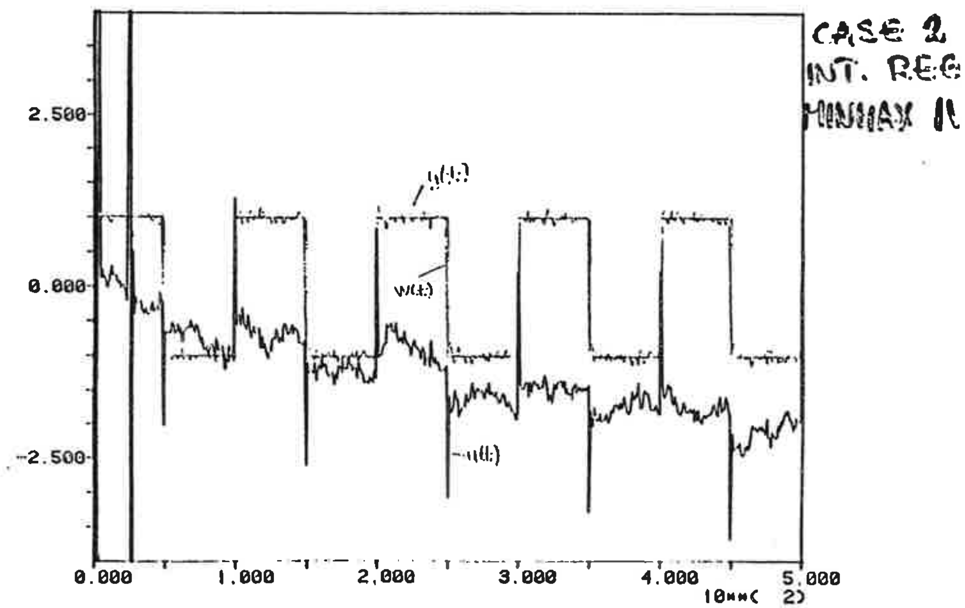

# Adaptive spectral factorization

Victor Solo

Department of Statistics
Harvard University, USA

Despite the great amount of work in the last ten years on the behaviour of various recursive parameter estimation and self-tuning control schemes the situation is disappointing. though a number of global convergence results are available they usually entail restrictions that amount to knowledge of the true system.

Thus, the positive real condition cannot be checked unless the true system parameters are known. Further, monitoring schemes cannot be properly designed unless the true system parameters are known. Also, even for the globally convergent algorithms, various internal filters are not guaranteed to be stable.

In this work some building blocks for algorithm design are suggested. The Levinson, Burg or Lattice algorithm guarantees stability for autoregressive (AR) models. Wilson's (1969) Newton Raphson scheme for spectral factorization guarantees stability of the spectral factor iterate at each iteration. Finally, general regression enjoys a bounded posterior error power property. The use of these building blocks together with the idea of split recursions is illustrated by developing a number of algorithms for ARMA and ARMAX recursive estimation (no iteration is involved).

One of those schemes (called $RF_2$) is shown to be globally convergent. This scheme is then used to develop a convergent self-tuning Kalman filter. The algorithm is free of the criticism mentioned above.

Finally, the three tools above are combined to produce a self-tuning LQG controller. It enjoys some stability properties but no convergence proof is available.

Reference

G.T. Wilson (1969). Factorization of the covariance generating function of a pure moving average process, SIAM J. Numer. Anal., 6, 1-7.

ADAPTIVE

SPECTRAL

FACTORIZATION

Victor Solo

Harvard University

Department of Statistics



• REVIEW

1 MODELS

2 RECURSIVE PARAMETER ESTIMATION

3 STABILITY OF ALGORITHMS

4 SPECTRAL FACTORISATION

• PRESENT

1 FOUR RECURSIVE SCHEMES (SPLIT RECURSIONS)

1 ONE CONVERGES GLOBALLY.

2 A CONVERGENT SELF TUNING KALMAN FILTER

3 USE IN SELF TUNING CONTROL (LQG)

4 A GENERAL MONITORING SCHEME



MODELS ($L \equiv z^{-1} \equiv q^{-1}$)

1) ARMA

$A(L)\, y_k = D(L)\, \epsilon_k$ $\quad A(z^{-1}) = 1 + a(z^{-1}) = 1 + \sum_1^p a_i z^{-i}$

$\Leftrightarrow \quad y_k = \varphi_k' \theta + \epsilon_k \qquad \theta' = (a', d')$

$\varphi_k' = (-y_{k-1}', \epsilon_{k-1}')$

2) ARMAX OR CARMA

$A(L)\, y_k = b(L)\, u_k + D(L)\, \epsilon_k$

3) ARX OR CAR (BUT NOT ARMA)

$A(L)\, y_k = b(L)\, u_k + \epsilon_k$

4) TFARMA

$y_k = \frac{b(L)}{A(L)} u_k + \frac{D(L)}{C(L)} \epsilon_k$

5) FIR OR REGRESSION

$y_k = b(L)\, u_k + \epsilon_k$



RECURSIVE PARAMETER ESTIMATION

• $\theta_{k+1} = F_k \theta_k + P_k \psi_k e_k$

LONG MEMORY i.e. GAIN $= P_k \to 0$

OR RECURSIVE OR SELF-TUNING

SHORT MEMORY i.e. GAIN $= P_k > P > 0$

• WAYS TO GENERATE GAIN & GRADIENT

(1) PER

(2) PLR

(3) EKF

(4) IV

NOTE

SPLIT RECURSIONS

eg YOUNG's

IV-AML

• AIMS FOR AN ALGORITHM

STABILITY e.g. ERROR POWER BOUNDED

CONVERGENCE e.g. ERROR POWER → INNOVATIONS (

EFFICIENCY e.g. CRAMER - RAO BOUND

STABILITY PROPERTIES OF SOME SIGNAL PROCESSING ALGORITHMS

(1) LEVINSON

OR BURG OR LATTICE

ENSURE AR POLYNOMIAL IS STABLE

(2) WILSON'S ALGORITHM FOR SPECTRAL FACTORISATION

SOLVE $|\alpha(z)|^2\sigma^2 = \Sigma_{-p}^{p} \gamma_s z^s$

AT EACH ITERATION $\alpha_j(z)$ IS STABLE

UNTRUE ? OF BAUER

(3) RECRESSION  ERROR POWER $\leq$ DATA POWER

DATA $y_k$  REGRESSORS $x_k$

$\theta_k = \theta_{k-1} + P_k x_k e_k$  $e_k = y_k - x_k'\theta_{k-1}$

$P_k^{-1} = P_{k-1}^{-1} + x_k x_k'$  $\eta_k = y_k - x_k'\theta_k$

$$\Rightarrow \overline{\lim}\ n^{-1} \Sigma_1^n \eta_k^2 \leq \overline{\lim}\ n^{-1} \Sigma_1^n y_k^2$$



PROOF OF STABILITY PROPERTY OF REGRESSION

$\theta_k = \theta_{k-1} + P_k x_k e_k$  $e_k = y_k - x_k'\theta_{k-1}$

$\Rightarrow P_k^{-1}\theta_k = P_{k-1}^{-1}\theta_{k-1} + x_k y_k = P_k^{-1}\theta_{k-1} + x_k e_k$

$\Rightarrow T_k = \theta_k' P_k^{-1} \theta_k = T_{k-1} + \theta_{k-1}' x_k y_k + y_k'\theta_{k-1} e_k + x_k' P_k x_k e_k^2$

$= T_{k-1} + (y_k - e_k)(y_k + e_k) + x_k' P_k x_k e_k^2$

$= T_{k-1} + y_k^2 - e_k^2(1 - x_k' P_k x_k)$

SUM $\Rightarrow$

$T_n + \Sigma_1^n e_k^2(1 - x_k' P_k x_k) = \Sigma_1^n y_k^2$

BUT $\eta_k = y_k - \theta_k' x_k$

$= y_k - \theta_{k-1}' x_k - x_k' P_k x_k e_k = (1 - x_k' P_k x_k) e_k$

$$\Rightarrow T_n + \Sigma_1^n \frac{\eta_k^2}{1 - x_k' P_k x_k} = \Sigma_1^n y_k^2$$

BUT $x_k' P_k x_k \leq 1$



SPECTRAL FACTORISATION

$|\alpha(z)|^2 = \frac{\sigma^2}{\gamma_0} |1 + \Sigma_1^p d_s z^s|^2 = 1 + \Sigma_1^p \rho_s z^s + \Sigma_1^p \rho_s z^{-s}$

EQUIVALENTLY

$V\,|1 + h'(zI - F)^{-1} g|^2 = 1 + m'(zI - F)^{-1} h + m'(z^{-1}I - F)^{-1} h$

$F = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & \\ \vdots & & & \ddots & 1 \\ 0 & 0 & \cdots & & 0 \end{bmatrix}$  $h = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$  $m = \begin{bmatrix} \rho_1 \\ \vdots \\ \rho_p \end{bmatrix}$  $g = \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix}$

STATE SPACE VERSION IS POSITIVE REAL EQUATIONS

$P = F P F' + g V g'$

$g = m - F P h$

$V = 1 - h' P h$

ALGORITHMIC SOLUTIONS

(a) LINEARLY CONVERGENT (BAUER, · )

(b) QUADRATICALLY CONVERGENT (WILSON, HEWER)



OFFLINE SF - LINEAR SCHEME

FREQUENCY DOMAIN - BAUER

$\delta_p^{(j)} = \rho_p$

$\delta_\ell^{(j)} = \rho_\ell - \Sigma_{\ell+1}^{p} \delta_s^{(j)} \delta_{s-\ell}^{(j-1)} / \delta_0^{(j-1)}$  $1 \leq \ell \leq p$

$\delta_0^{(j)} = 1 - \Sigma_1^p (\delta_s^{(j)})^2 / \delta_0^{(j-1)}$

TIME DOMAIN (STATE SPACE)

$\hat{P}_{k+1} = F \hat{P}_k F' + g_k V_k^{-1} g_k'$  ; $\hat{P}_0 = 0$

$g_k = m - F \hat{P}_k h$

$V_k = 1 - h' \hat{P}_k h$

$\delta_k(z) = V_k(1 + h'(zI - F)^{-1} g_k V_k^{-1})$

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OFFLINE SF - QUADRATIC SCHEMES

- FREQUENCY DOMAIN (WILSON, 1969)

SOLVE $\frac{\hat{\delta}_j(z)}{\delta_j(z)} + \frac{\hat{\delta}_j(z^{-1})}{\delta_j(z^{-1})} = \frac{2\sum_p^p \rho_s z^s}{\delta_j(z)\delta_j(z^{-1})}$

$$\delta_{j+1}(z) = \tfrac{1}{2}\delta_j(z) + \tfrac{1}{2}\hat{\delta}_j(z)$$

$$\Leftrightarrow \quad T(\underline{\delta}_j)\,\hat{\underline{\delta}}_j = 2\begin{pmatrix}1\\ \underline{m}\end{pmatrix}$$

$$\underline{\delta}_{j+1} = \tfrac{1}{2}\underline{\delta}_j + \tfrac{1}{2}\hat{\underline{\delta}}_j$$

$$T(\underline{\delta}) = \begin{bmatrix}\delta_0 & \delta_1 & & \delta_p\\ \delta_1 & & \cdot & \\ & \cdot & & \\ \delta_p & & & 0\end{bmatrix} + \begin{bmatrix}\delta_0 & \delta_1 & \cdots & \delta_p\\ & \ddots & & \\ 0 & & & \delta_0\end{bmatrix}$$

- TIME DOMAIN (STATE SPACE) HEWER, 1973

$$\hat{P}_{k+1} = F\hat{P}_{k+1}F' + g_k V_k^{-1} g_k' \quad : \hat{P}_0 = 0$$

$$g_k = m - F\hat{P}_k h$$

$$V_k = 1 - h'\hat{P}_k h$$

$$\delta_k(z) = V_k(1 + h'(zI - F)^{-1} g_k V_k^{-1})$$

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DERIVATION + PROPERTIES OF WILSON'S ALGORITHM

- GIVEN $D_1$ CHOOSE $D_2 = D_1 + dD_1 \Rightarrow$

$$|D_2|^2 = |D_1 + dD_1|^2 \simeq \rho = \sum_p^p \rho_s z^s$$

SO SET $D_1\bar{D}_1 + dD_1\bar{D}_1 + D_1 d\bar{D}_1 = \rho$

$$\Rightarrow \quad dD_1/D_1 + d\bar{D}_1/\bar{D}_1 = \rho/|D_1|^2 - 1$$

- STABILITY

BUT $|D_2|^2 = \rho + |dD_1|^2 \geqslant |dD_1|^2$

$= |D_2 - D_1|^2$ ON $|z| = 1$

ROUCHE $\Rightarrow$ $D_2, D_1$ HAVE SAME NO. OF ZEROES INSIDE $|z| = 1$

- NOTE SET

$$\hat{D}_1/D_1 + \bar{\hat{D}}_1/\bar{D}_1 = 2\rho/|D_1|^2$$

$$\Rightarrow \quad D_2 = \tfrac{1}{2}D_1 + \tfrac{1}{2}\hat{D}_1$$

10

| RHF$_1$ |  $A(z^{-1})y_k = w_k = D(z^{-1})\epsilon_k$

(a) IV (= Hankel) ESTIMATION OF $\underline{a}_k$

(b) GENERATE $\hat{\omega}_k = \hat{A}(z^{-1})y_k$

UPDATE $\hat{\delta}^{(k+1)}_{w}(l) = \hat{\delta}^{(k)}_{w}(l) - \frac{1}{k+1}(\hat{\omega}_{k+1}\hat{\omega}_{k+1-l} - \hat{\delta}^{(k)}_w(l))$

DO ONE STEP OF BAUER

$$\delta_p^{(k+1)} = \hat{\delta}^{(k+1)}_w(p)$$

$$\delta_l^{(k+1)} = \hat{\delta}^{(k+1)}_w(l) - \sum_{l+1}^{p} \delta_s^{(k+1)}\delta_{s-l}^{(k+1)} / \delta_0^{(k+1)}$$

$$\delta_0^{(k+1)} = 1 - \sum_1^p (\delta_s^{(k)})^2 / \delta_0^{(k-1)}$$

OR STATE SPACE

$$\hat{P}_{k+1} = F\hat{P}_k F' + g_k V_k^{-1} g_k$$

$$g_k = (m_k) - F\hat{P}_k h \qquad \underline{m}_k = [\rho_w^{(k)}(1) \ldots \rho_w^{(k)}(p)]'$$

$$V_k = 1 - h'\hat{P}_k h$$

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RHF$_2$  $A(z^{-1})y_k = w_k = D(z^{-1})\epsilon_k$

(a) IV (= Hankel) ESTIMATION OF $\underline{a}$

(b) GENERATE $\hat{\omega}_k = \hat{A}_k(z^{-1})y_k$

UPDATE $\hat{\delta}^{(k+1)}_\omega(l)$

DO ONE STEP OF WILSON

$$\hat{\underline{\delta}}_k = T(\underline{\delta}_k)\begin{bmatrix}1\\ \underline{m}_k\end{bmatrix}$$

$$\underline{\delta}_{k+1} = \tfrac{1}{2}\underline{\delta}_k + \tfrac{1}{2}\hat{\underline{\delta}}_k$$

OR HEWER

$$\hat{P}_{k+1} = F\hat{P}_{k+1}F' + g_k V_k^{-1} g_k$$

$$g_k = (m_k) - F\hat{P}_k h$$

$$V_k = 1 - h'\hat{P}_k h$$

12.

TROUBLE WITH ON-LINE $SF_2$

RECALL $D_2 = |D_2 - D_1|^2 + \rho$

NEED $\rho \geq 0$ TO USE ROUCHE

NO GUARANTEE e.g.

$1 + a\rho_1 \cos\omega \geq 0 \quad \forall\omega$ iff $|\rho_1| \leq \frac{1}{2}$

SO USE A MONITORING SCHEME

1) GET $\hat{\delta}_{j+1}$ AND $\hat{\delta}_{j+1} = \frac{1}{2}\delta_j + \frac{1}{2}\hat{\delta}_j$

2) STABILITY CHECK OF $\hat{\delta}_{j+1}$

3) IF IT FAILS FORM

$$\hat{\hat{\rho}} = |\hat{\delta}_{j+1}|^2$$

AND REDO ONE STEP OF $SF_2$

THEN $\delta_{j+1} = \frac{1}{2}\delta_j + \frac{1}{2}\hat{\delta}_j$

IS GUARANTEED STABLE

- DOES NOT AFFECT CONVERGENCE

13

RLF

(a) GIVEN $a_k(z^{-1})$

FORM $\hat{\omega}_k = (1 + a_k(z^{-1}))y_k$ AND $\hat{\gamma}_\omega^{(k)}(l)$ $l = 0 \ldots p$

$F_1$ DO ONE STEP OF BAUER

$F_2$ DO ONE STEP OF WILSON

GIVES $d_{k+1}(z^{-1})$

(b) WITH $d_{k+1}(z^{-1})$

FORM $\xi_{k+1} = y_{k+1} - d_{k+1}(z^{-1})\xi_k$

USE LATTICE OR BURG TO

UPDATE AR

GIVES $a_{k+1}(z^{-1})$

- GOOD TRANSIENT PROPERTIES
- CONVERGENCE ?

14

| PROPERTY / ALG. | AR STABLE | MA STABLE | TRANSIENTS | CONVERGES |
|---|---|---|---|---|
| $RHF_1$ | | | | |
| $RHF_2$ | | ✓ | | ✓ |
| $RLF_1$ | ✓ | | | |
| $RLF_2$ | ✓ | ✓ | ✓ | |

- PARASITICS $RLF_2$ ✓
- TIME VARYING PARAMETERS

  ALL EASILY MODIFIED

  ONLY $RLF_2$ RETAINS PROPERTIES

- GENERAL MONITORING METHOD

15

SELF TUNING KALMAN FILTER

JUST SLOT RECURSIVE ESTIMATES

OF $F_k$, $V_k$, $K_k$ INTO

$$\hat{x}_{k+1|k} = \hat{F}_k \hat{x}_{k|k-1} + \hat{K}_k e_k / \hat{V}_k$$

$$e_k = y_k - h' \hat{x}_{k|k-1}$$

WELL -IN $SF_1$, $SF_2$

$$\hat{K}_k = g_k - a_k \; ; \; \hat{V}_k \equiv V_k$$

$$\hat{F}_k = \begin{bmatrix} -a_{1k} & 1 & 0 & \cdots \\ \vdots & & \ddots & \\ & & & 1 \\ -a_{pk} & 0 & \cdots & 0 \end{bmatrix}$$

(RECALL COVARIANCE FILTER)

CONVERGENCE $x_{k+1|k} - \hat{x}_{k+1|k} \to 0$

TEDIOUS BUT STRAIGHTFORWARD

SINCE $\hat{K}_k \to g - a = d - a$

## ARMAX

$$(1+a(z^{-1}))y_k = b(z^{-1})u_k + (1+d(z^{-1}))\epsilon_k$$

- USE (IV)H TO GET $a, b$

POOR TRANSIENT BEHAVIOUR ?

SO POWER IN $\hat{v}_k = (1+\hat{a}_k(z^{-1}))y_k - \hat{b}_k(z^{-1})u_k$

TOO LARGE

- SO ADD R STEP ⇒ IVHR

REGRESS $y_k$ ON $\psi_k = (-s'_{1k-1}, u'_{k-1})'$

$$s_{1k} = \hat{a}_k(z^{-1})s_{1k-1} + \hat{b}_k(z^{-1})u_k$$

- THEN $\hat{w}_k = y_k - \frac{\hat{b}_k(z^{-1})}{1+\hat{a}_k(z^{-1})}u_k$

HAS BOUNDED POWER

- FORM $\hat{v}_k = (1+\hat{a}_k(z^{-1}))\hat{w}_k$

AND USE $RF_2$

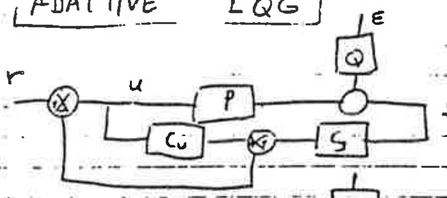
⇒ $R_2HF_2$



## ADAPTIVE LQG

$C_u = \frac{N_u}{\delta}$

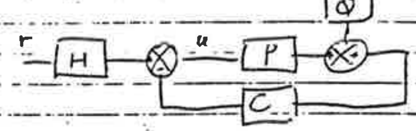
$C_y = \frac{N_y}{\delta}$




$P = \frac{b}{A}$

$Q = \frac{\delta}{A}$

⟺




$$H = \frac{1}{1+C_u} = \frac{\delta}{\delta+N_u} = \frac{\delta}{R}$$

$$C = \frac{C_y}{1+C_u} = \frac{N_y}{\delta+N_u} = \frac{S}{R}$$

$$Ay = bu + \delta\epsilon$$
$$Ru = -Sy + Hr$$

- ESTIMATE $A, b, \delta$ FROM FORWARD LOOP BY $IVHRSF_2$ (⇒ $\delta$ STABLE)

SOLVE $G\delta = AR + bS$ FOR $R, S$

& $\sigma|G|^2 = |b|^2 + \lambda|A|^2$

DO ONE STEP OF WILSON FOR G



$|z| = 1 \Leftrightarrow z^{-1} = \bar{z}$

$$D_{t+1}/D_t + \bar{D}_{t+1}/\bar{D}_t = \rho_{t+1}/|D_t|^2 + 1 \quad (1)$$

$$|D_{t+1}|^2 = |D_{t+1} - D_t|^2 + \rho_{t+1} \geq \rho_{t+1} > 0 \quad (2)$$

$$\Rightarrow 2Re(D_{t+1}/D_t) = 1 + \frac{\rho_{t+1}}{\rho_t}\frac{\rho_t}{|D_t|^2} \geq 1$$
$$\leq 1 + \rho_{t+1}/\rho_t \quad (3)$$

IF $D_t(z)$ HAS NO ZEROES INSIDE $|z| = 1$

ROUCHE → $D_{t+1}$ DOES NOT (4)

⇒ $D_{t+1}/D_t$ ANALYTIC IN $|z| \leq 1$

⇒ $Re(D_{t+1}/D_t)$ IS HARMONIC

⇒ CONSTANT $|z| \leq 1$ OR MAX & MIN ON $|z| = 1$ BUT

SO (3) HOLDS $|z| \leq 1$ i.e. FOR $z = x$ REAL

ALSO (4) ⇒ $D_t$ HAS CONSTANT SIGN (+VE)

⇒ $D_{t+1}(x)/D_t(x) \leq 1 + \alpha_t$

$$\alpha_t = \tfrac{1}{2}(\rho_{t+1}/\rho_t - 1) \to 0$$

⇒ $D_{t+1}/\pi_{t+1} \leq D_t/\pi_t$ : $\pi_t = \pi_1^t(1+\alpha_t) > 0$



$$\pi_t^2 = \pi_{t_0}^t(1+2\alpha_s+\alpha_s^2) = \pi_{t_0}^t\left(\frac{\rho_{s+1}}{\rho_s} + \alpha_s^2\right)$$
$$= \pi_{t_0}^t \frac{\rho_{s+1}}{\rho_s}\left(1 + \frac{\alpha_s^2\rho_s}{\rho_{s+1}}\right) = \frac{\rho_{t+1}}{\rho_{t_0}}\tilde{\pi}_{t_0}^t\left(1+\alpha_s^2\frac{\rho_s}{\rho_{s+1}}\right)$$

$\tilde{\pi}_{t_0}^t$ NONDECREASING

BUT $\rho_t \to \rho$ ⇒ $\pi_t \to \pi$ IF $\tilde{\pi} < \infty$

$$\pi_{t_0}^t(1+\alpha_s^2\rho_s/\rho_{s+1}) < \exp\left(\Sigma_{t_0}^t \alpha_s^2\rho_s/\rho_{s+1}\right)$$

NEED ONLY $\Sigma\alpha_s^2 < \infty$

$$\alpha_s^2 = \frac{1}{4}\left(\frac{\rho_{s+1}}{\rho_s} - 1\right)^2 = \frac{(\rho_{s+1}-\rho_s)^2}{4\rho_s^2}$$

$$\rho_{s+1} = \rho_s + (\xi_{s+1} - \rho_s)/(s+1)$$

$$\Rightarrow \alpha_s^2 \leq \frac{1}{2}\frac{\xi_{s+1}^2}{\rho_s^2 s^2} + \frac{1}{2}\frac{1}{s^2}$$

SO $\Sigma\alpha_s^2 < \infty$ IF $\Sigma E(\xi_s^2/s^2) < \infty$

YES IF $E(\xi_s^2) \to c$

# ADAPTIVE SPECTRAL FACTORIZATION

Victor Solo
Department of Statistics
Harvard University

Despite the great amount of work in the last ten years on the behavior of various recursive parameter estimation and self-tuning control schemes the situation is disappointing. Though a number of global convergence results are available they usually entail restrictions that amount to knowledge of the true system.

Thus, the positive real condition cannot be checked unless the true system parameters are known. Further, monitoring schemes cannot be properly designed unless the true system parameters are known. Also, even for the globally convergent algorithms, various internal filters are not guaranteed to be stable.

In this work some building blocks for algorithm design are suggested. The Levinson, Burg or Lattice algorithm guarantees stability for autoregressive (AR) models. Wilson's (1969) Newton Raphson scheme for spectral factorization guarantees stability of the spectral factor iterate at each iteration. Finally, general regression enjoys a bounded posterior error power property. The use of these building blocks together with the idea of split recursions is illustrated by developing a number of algorithms for ARMA and ARMAX recursive estimation (no iteration is involved).

One of those schemes (called $RF_2$) is shown to be globally convergent. This scheme is then used to develop a convergent self-tuning Kalman filter. The algorithm is free of the criticisms mentioned above.

Finally, the three tools above are combined to produce a self-tuning LQG controller. It enjoys some stability properties but no convergence proof is available.

## Reference

G. T. Wilson (1969). Factorization of the covariance generating function of a pure moving average process, *SIAM J. Numer. Anal.*, 6, 1-7.

# Frequency domain properties of identified transfer function estimates

Lennart Ljung

Department of Automatic Control
Linköping University, Linköping

Consider the problem of identifying the transfer function of the system

$$y(t) = G_o(q)u(t) + v(t)$$

where y is output, u is input and v is a stationary disturbance with spectrum $\Phi_v(\omega)$.

The input spectrum is supposed to be $\Phi_u(\omega)$. From data up to time N an estimate $\hat{G}_N(e^{i\omega})$ is formed. One method for this is a k-step ahead prediction error method in a given model set

$$y(t) = G(q,\theta)u(t) + H(q,\theta)e(t),$$

$$\hat{G}_N(e^{i\omega}) = G(e^{i\omega}, \hat{\theta}_N)$$

$$\hat{\theta}_N = \underset{\theta}{\text{argmin}}\ \theta\ \frac{1}{N}\sum_{t=1}^{N} \varepsilon_F^2(t,\theta)$$

$$\varepsilon_F(t,\theta) = L(q)\ \varepsilon(t,\theta)$$

$$\varepsilon(t,\theta) = W^k(q,\theta)\ [y(t) - G(q,\theta)\ u(t+kT)]$$

$$H(q,\theta) = \bar{H}_k(q,\theta) + q^{-k}\ \tilde{H}_k(q,\theta) \qquad W^k = H_k H^{-1}$$

Several design variables are involved in this method, like k (the prediction horizon), T (the sampling horizon), H (the noise model) and $\Phi_u$.

Then

$$\hat{G}_N(e^{i\omega}) \rightarrow G^*(e^{i\omega}) \quad \text{w.p.1 as } N \rightarrow \infty$$

where $G^*(e^{i\omega})$ is "essentially" determined as the closest function to $G_o(e^{i\omega})$ in the model set, as measured in the weighted $L_2$-norm (over the frequencies $-\pi/T \leq \omega \leq \pi/T$) with weighting function

$$Q(\omega) = \Phi_u(\omega) \cdot |L(e^{i\omega})|^2 \cdot |W^k(e^{i\omega})|^2$$

This can be used for a more or less formal discussion of optimal choices of design variables.

## References


L. Ljung: Estimation of transfer functions. Report LiTH

L. Ljung: Asymptotic variance expressions for identified transfer function estimates. Report LiTH

B. Wahlberg & L. Ljung: Design variables for bias distribution of identified transfer functions. Report LiTH

# FREQUENCY DOMAIN PROPERTIES OF IDENTIFIED TRANSFER FUNCTION ESTIMATES

Lennart Ljung & Bo Wahlberg

PROBLEM:

True system:

$$y(t) = G(q)\, u(t) + v(t)$$

Observed data:

$$Z^N: u(1), y(1), \ldots\ u(N), y(N)$$

Model: $\hat{G}_N(q)$

WHAT CAN WE SAY ABOUT

$$\hat{G}_N(e^{i\omega}) - G_0(e^{i\omega})$$

AS A FUNCTION OF $\omega$?



1. Bias error & random error
2. k-step ahead prediction error methods
3. conventional analysis. Design issues
4. A frequency domain expression for the limiting estimate
5. Choice of $u$, $L$, $k$, $T$
6. Conclusions



## 1 BIAS- & RANDOM ERRORS

Let

$G^*(e^{i\omega}) = E\,\hat{G}_N(e^{i\omega})$ ("$E$" w.r.t. $\{v(t)\}_1^N$)

$G^*(e^{i\omega}) - G_0(e^{i\omega})$ BIAS error

$\hat{G}_N(e^{i\omega}) - G^*(e^{i\omega})$ RANDOM error

$$\hat{G}_N - G_0 = \underbrace{\hat{G}_N - G^*(e^{i\omega})}\ + \underbrace{G^*(e^{i\omega}) - G_0(e^{i\omega})}$$

"Traditional" analysis neglects ↗

Typically: $\sqrt{N}[\hat{G}_N(e^{i\omega}) - G^*(e^{i\omega})] \in AsN(0, P(\omega))$

Expressions for P: .....

Here, we shall concentrate on bias



## 2. PREDICTION ERROR METHODS

Choose:

① $T$ Sampling interval

② $\Phi_u(\omega)$ input spectrum

③ Model set $\mathcal{G} = \{G(q,\theta)\}$ $[y = Gu + He]$

④ Noise model set $\mathcal{H} = \{H(q,\theta)\}$

⑤ Prediction horison $k$

$H = \bar{H}_k + q^{-k}\tilde{H}_k \qquad W^k = \bar{H}_k H^{-1}$

$\hat{y}(t+kT|t,\theta) = W^k(q,\theta)\, G(q,\theta)\, u(t+kT) + (1 - W^k)\, y(t)$

$\varepsilon(t+kT|t,\theta) = W^k(q,\theta)[y(t) - G(q,\theta)\, u(t+kT)]$

$[k=1 \Rightarrow W^k = H^{-1}]$

⑥ Filter $L(q)$

$\varepsilon_F(t+kT|t,\theta) = L(q)\, \varepsilon(t+kT|t,\theta)$

Then

$$\hat{\theta}_N = \arg\min \frac{1}{N}\sum_{t=1}^{N} \varepsilon_F^2(t+kT|t,\theta)$$

## 3. Design issues

$\hat{G}_N(e^{i\omega})$ depends on all the listed 6 choices.

What are 'optimal' choices of

- noise model
- prediction horizon
- prefilter
- input spectrum
- sampling interval

Traditional analysis: "Optimal" = min. var.



## 4. THE LIMITING ESTIMATE

$$\hat{\theta}_N \rightarrow \theta^* = \arg\min E\, \varepsilon^2(t+k, t, \theta) \quad \text{w.p. 1}$$

$$\theta^* = \arg\min \int_{-\pi}^{\pi} \left[ |G_0 - G(\theta)|^2 \Phi_u + \Phi_v \right] \cdot |L|^2 |W_k(\theta)|^2 \, d\omega$$

Fixed noise model $H(\theta) = H^*$ $(W_k(\theta) = W_k^*)$

$$\theta^* = \arg\min \int_{-\pi}^{\pi} |G_0 - G(\theta)|^2 Q^* \, d\omega$$

$$Q(\omega) = \Phi_u(\omega) |L(e^{i\omega})|^2 |W_k^*(\omega)|^2$$

Independently parametrized: $G(\theta) = G(\rho)$ ; $H(\theta) = H(\eta)$

$$\rho^* = \arg\min \int_{-\pi}^{\pi} |G_0 - G(\rho)|^2 Q(\eta^*) \, d\omega$$

$$Q(\eta) = \Phi_u(\omega) |L(e^{i\omega})|^2 |W_k(\eta, \omega)|^2$$

$$\eta^* = \arg\min \int_{-\pi}^{\pi} \left[ |G_0 - G(\rho^*)|^2 \Phi_u + \Phi_v \right] |L(e^{i\omega})|^2 |W_k(\eta, \omega)|^2 d$$

General case:

$$\theta^* = \arg\min \Big[ \int_{-\pi}^{\pi} |G_0 - G(\theta)|^2 Q(\theta^*) \, d\omega + \int_{-\pi}^{\pi} \{ |G_0 - G(\theta^*)|^2 \Phi_u + \Phi_v \} |L(e^{i\omega})|^2 |W_k(\theta)|^2 d\omega \Big]$$



## CONCLUSION:

The weighting function

$$Q(\omega, \theta^*) = \Phi_u(\omega) |L(e^{i\omega})|^2 |W_k(\theta, \omega)|^2$$

determines, entirely or partly, the bias distribution

Formal design problem:

design variables:

$$\mathcal{D} = \{ \Phi_u(\omega), k, L(q), H^*(q) \}$$

$$\theta^* = \theta^*(\mathcal{D})$$

design criterion

$$J(\mathcal{D}) = \int_{-\pi}^{\pi} | G(\theta^*(\mathcal{D})) - G_0 |^2 C(\omega) \, d\omega$$

$$\min_{\mathcal{D} \in \mathscr{D}} J(\mathcal{D})$$

recall:

$$\theta^*(\mathcal{D}) = \arg\min \int_{-\pi}^{\pi} | G(\theta) - G_0 |^2 \underbrace{\Phi_u(\omega) \cdot |L|^2 |W_k}_{\mathcal{D}}$$

SOLUTION: $\mathcal{D}_{opt}$ : $\underline{\Phi_u(\omega) |L(e^{i\omega})|^2 |W_k(e^{i\omega})|^2 \propto C}$

System:

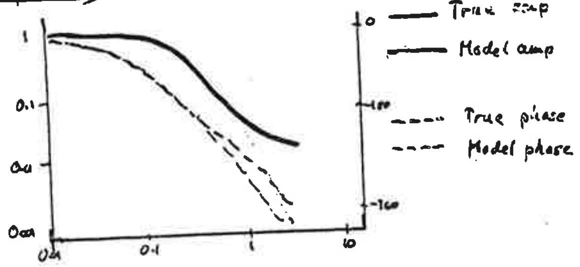
$$y(t) - 2.14\, y(t-1) + .555\, y(t-2) - 0.44\, y(t-3) + 0.042\, y(t-4) =$$
$$= 0.01\, u(t-2) + 0.0074\, u(t-3) + 0.000921\, u(t-4) -$$
$$- 0.0000176\, u(t-5)$$

Model:

$$y(t) + a_1 y(t-1) + a_2 y(t-2) = b_1 u(t-1) + b_2 u(t-2)$$

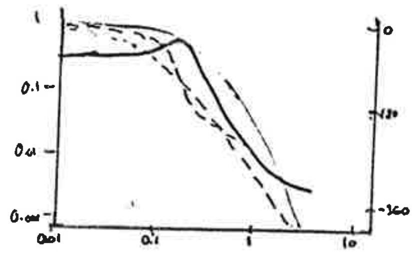
Input: PRBS - white noise

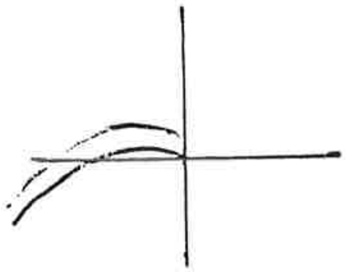
Output error: $H^* = 1$

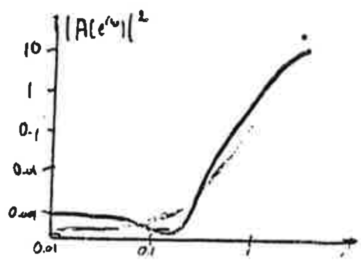




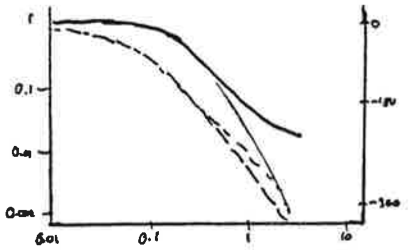

LEAST SQUARES



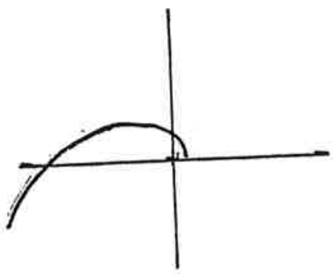




LEAST SQUARES

LP filtered

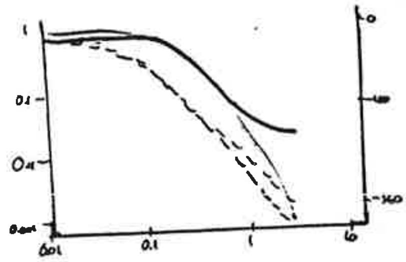
[0, 0.1]

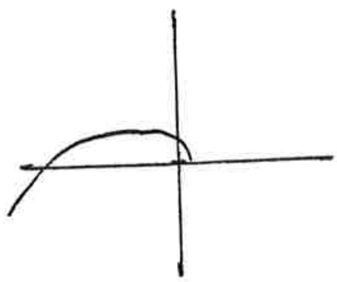


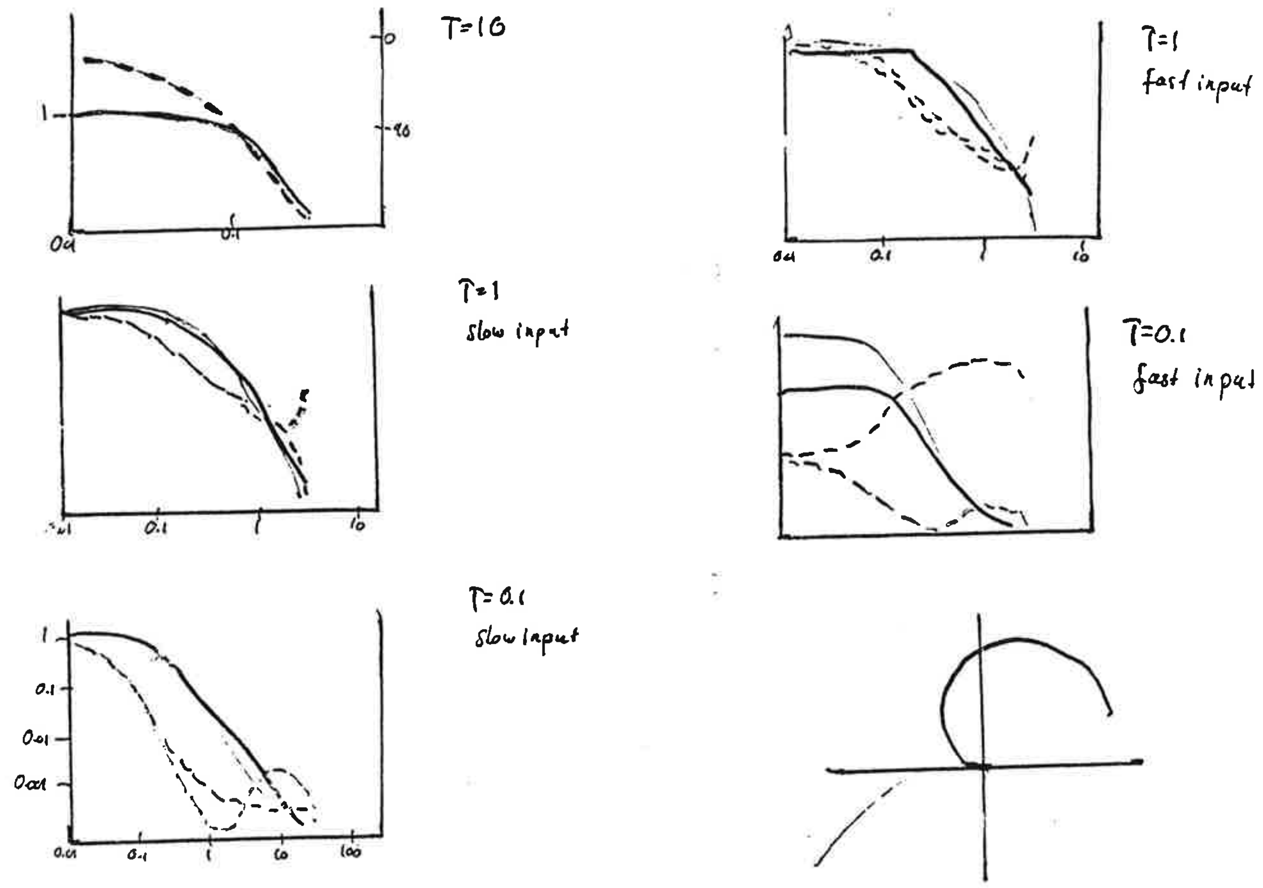
Least squares

BP - filtered

[0.1, 0.5]










## CONCLUSION

The bias distribution is determined by a frequency domain weighting function that is influenced by the noise model, the prediction horizon, the prefilter, the input spectrum and the sampling interval

# Multi-armed bandits

Pravin Varaya

Electrical Engineering
University of California
Berkeley, USA

Contents

1) Bandit process
2) Multiarmed
3) Index
4) Adaption

1. A bandit process is $\{(x(1), F(1)); (x(2), F(2))\ldots\}$, where
$x(s)$ = reward on the sth continuation
$F(s)$ = $\sigma$-field of information after s-1 plays.

<u>Ex</u> Coin toss
$x(s) \in \{0,1\}$. $F(0)$ = prior information about success
$F(s) = F(0) \cup \{x(0)\ldots x(s-1)\}$

2. A multiarmed bandit is defined according to

$$\{x^i(s), F^i(s)\} \quad i = 1,\ldots,n \qquad F^i(\infty) \perp\!\!\!\perp F^j(\infty) \qquad i \neq j$$

$$t - 1 = t^1 + \ldots + t^n \; ; \; F^1(t^1+1) \cup \ldots \cup F^n(t^n+1) = F(t)$$

The control $u(t) \in \{1\ldots n\}$
$u(t)$ is the decision which bandit that shall continue.

$$u(t) = i \longrightarrow \begin{cases} R(t) = x^i(t^i+1) \\ F(t+1) = F(t) \cup F^i(t^i+1) \end{cases}$$

The control $u(t)$ has to maximize the total reward

$$V_\beta(\Pi) = \text{Max } E \sum_{t=1}^{\infty} \beta^t R(t) \qquad 0 < \beta < 1$$

$\beta$ close to zero ---> we want to maximize the reward in the first play(s).

$\beta$ close to unity ---> all rewards are equally valuable

There is a conflict between getting immediate rewards and learning more about the other bandits.

3. Index of a bandit process $\{x(s), F(s)\}$

$$\nu_\beta(s) = \max_{\tau>1} \frac{E\left\{\sum_{t=s}^{\tau-1} \beta^t \, x(t) \mid F(s)\right\}}{E\left\{\sum_{t=s}^{\tau-1} \beta^t \mid F(s)\right\}} = \frac{\text{Acc. reward}}{\text{"Acc. time"}}$$

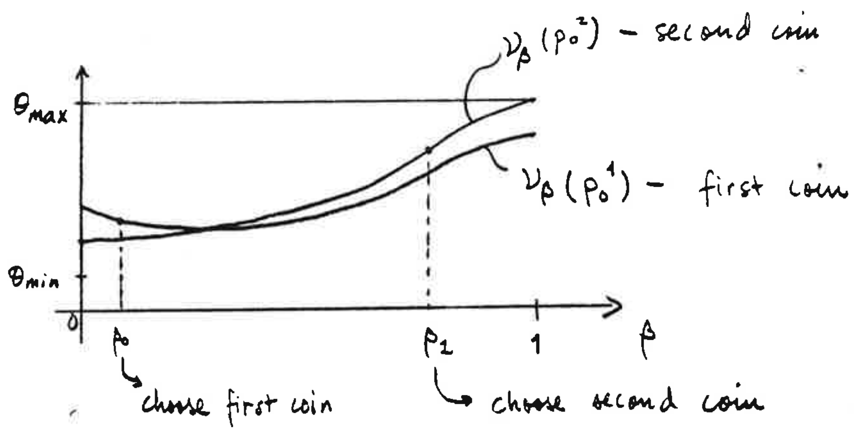
$\tau$ ranges over the stopping times of F(s). The best policy is to continue the bandit with the largest index (multiarmed case).
Notice that the calculation of $\nu_\beta$ for a bandit involves only the bandit itself.

Two coins $\theta_1$ and $\theta_2$
Fact about index $\nu_\beta(s) = \nu_\beta$ (p) (distribution of success probability).

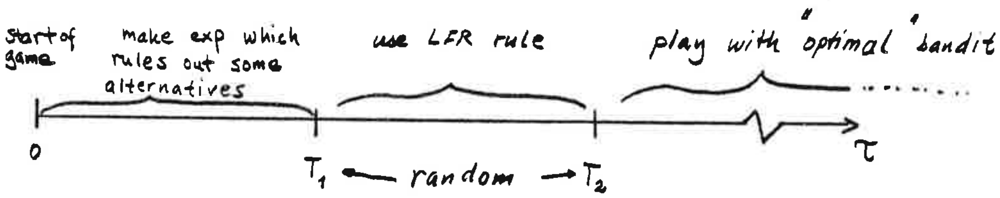
1) Expected reward just now

$\nu_0(p) = \bar{p} =$ exp of $\theta$

2) $\nu_1(p) = \max_{p(\theta) > 0} \theta$ (the most favourable possible event)



Optimal solution for $\beta = 1, \tau \to \infty$
(infinite horizon problem) [Kelly Am. stat. 1981]




LFR = least failure rule: Try the one that currently looks best. As soon as it fails (no reward), choose another one.

4.

The procedure above constitutes a kind of adaptation.
Unsolved problems: Transient behavior
Length of the first two phases.

References

F.P. Kelly: Multi-armed bandits with discount factor near one: the Bernoulli case, Ann. Statist 9, 1981, 987-1001.

P. Varaya, J. Walsand and C Bnynkkoc: Extensions of the multi-armed bandit problem: the discounter case, IEEE AC Trans, 1985, to appear.

J. C. Gittins: Bandit processes and dynamic allocation indices, J. Royal Stat. Soc 41, 1979, 148-177.

# On self-tuning to the optimal controller

P. R. Kumar

Department of Mathematics and Computer Science
U.M.B.C.
Baltimore, Maryland

Summary

The question of whether some adaptive controllers automatically tune themselves to the optimal control law is examined.

First we consider an adaptive control law consisting of a Stochastic Approximation (or Stochastic Gradient) parameter estimator followed by a minimum variance control law. Under some conditions it is shown that, see [1,2].

i) the parameter estimates converge to a random multiple of the true parameter,

ii) and so the adaptive control law converges to the true minimum variance control law,

iii) even though the standard persistency of excitation condition is not satisfied.

Next we consider the same parameter estimator followed by a linear quadratic control law. Ljung's O.D.E.'s are examined to show that though the parameter estimates may converge, they will not generally converge to an optimal control law.

In fact, the result for the case of the minimum variance control law rests on a fortuitous mathematical coincidence, and it is unlikely that self-tuning to the optimal will occur for general cost criteria, see [3].

References

A. Becker, P. R. Kumar and C. Z. Wei: Adaptive Control with the Stochastic Approximation Algorithm: Geometry and Convergence, to appear in IEEE Transactions on Automatic Control, March 1985.

P. R. Kumar: A Survey of Some Results in Stochastic Adaptive Control, March 1985.

W. Lin, P. R. Kumar and T. I. Seidman: Will the Self-tuning Approach Work for General Cost Criteria?

## SELF-TUNING

[illegible]
[illegible]

**Question : Does Self-Tuning Take Place?**

**Will the adaptive controller converge to the optimal controller?**

Yes : [illegible] Controller
Proof of Convergence

**No** : Optimal Controller
General Cost Criterion

**Speculative** : Self-Tuning Result is not possible in general by straightforward schemes.

---

**[S]ystem**

[illegible]

**[M]inimum Variance Control Law**

[illegible]

**[M]ore Conveniently**

[illegible]

[illegible]

[illegible]

**[A]daptive Scheme**

Estimate : $\hat{\theta}_t$

Control : $\phi_t^T \hat{\theta}_t = 0$

[illegible] One degree of freedom
[illegible] enough [illegible] $\hat{\theta}_t \rightarrow$ [illegible]

**Stochastic Approximation or Stochastic Gradient Algorithm**

$$\hat{\theta}_{N+1} = \hat{\theta}_N + \frac{\gamma}{r_N} \phi_N (y_{N+1} - \phi_N^T \hat{\theta}_N)$$

$$r_{N+1} = r_N + \phi_{N+1}^T \phi_{N+1}$$

**More convenient**

$$\hat{\theta}_{N+1} = \hat{\theta}_N + \phi_N \left( \frac{y_{N+1} - \phi_N^T \hat{\theta}_N}{r_N} \right) \gamma$$

$u_N$ chosen such that $\phi_N^T \hat{\theta}_N = 0$

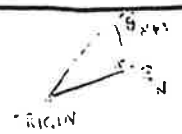
**What does such an algorithm do?**

$\hat{\theta}_{N+1} = \hat{\theta}_N + \phi_N$ (scalar)

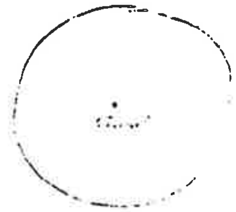
$\hat{\theta}_{N+1} - \hat{\theta}_N$ parallel to $\phi_N$

$\phi_N$ perpendicular to $\hat{\theta}_N$

Hence $\hat{\theta}_{N+1} - \hat{\theta}_N \perp \hat{\theta}_N$



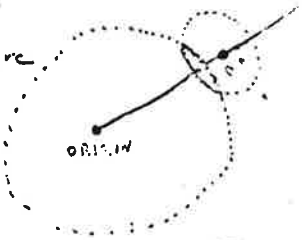
- [illegible] $\|\hat\theta_{N+1}\| \geq \|\hat\theta_N\|$
- Hence $\lim \|\hat\theta_N\|$ exists
- $\hat\theta_N$ converges to random [illegible]



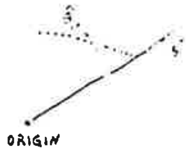
[illegible]

- [illegible] $\|\hat\theta_N - \theta^0\|$ [illegible]
- [illegible] converges
- $\|\hat\theta_N - \theta^0\|$ converges
- $\hat\theta_N$ converges to random [illegible] centred at $\theta^0$


- $\hat\theta_N$ converges to hypersphere
- Line and hypersphere do not intersect
  **except** if hypersphere = point




- [illegible] subsequence $\hat\theta_{N_k} \to$ Line



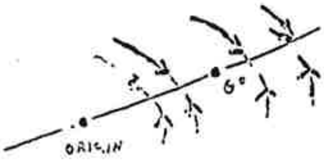

- Proof that $\hat\theta_{N_k} \to$ Line

[illegible]
$$y_{t+1} = a y_t + b_0 u_t + b_1 u_{t-1} + w_{t+1}$$
$$u_t = -\frac{1}{\hat b_0}\left(\hat a y_t + \hat b_1 u_{t-1}\right)$$

- $y_{t+1} = \left(a - b_0 \frac{\hat a}{\hat b_0}\right) y_t + \left(b_1 - b_0 \frac{\hat b_1}{\hat b_0}\right) u_{t-1} + \ldots$
- Optimality: $\lim \frac{1}{N} \sum_1^N y_{t+1}^2 = \lim \frac{1}{N} \sum_1^N$ [illegible]
- Analysis
$$\lim \frac{1}{N} \sum_1^N \left\{\left(a - b_0 \frac{\hat a}{\hat b_0}\right) y_t + \left(b_1 - b_0 \frac{\hat b_1}{\hat b_0}\right) u_{t-1}\right\}^2$$
- Local Convergence Theorem + Analysis
$$\lim \frac{1}{N} \sum_1^N \left(a - b_0 \frac{\hat a}{\hat b_0}\right)^2 + \left(b_1 - b_0 \frac{\hat b_1}{\hat b_0}\right)^2 = 0$$
- $\left(\frac{\hat a}{\hat b_0}\right)_{N_k} \to \frac{a}{b_0}$ and $\left(\frac{\hat b_1}{\hat b_0}\right)_{N_k} \to \frac{b_1}{b_0}$

## Conclusions

- $\hat\theta_N \to k\theta^0$ where $k$ = random scalar.




- $\hat\theta_N \not\to \theta^0$ : Does not converge to true parameter.
- Regulator converges to optimal
- SELF-TUNING

[illegible] $\|\hat\theta_1\| > \|\theta^0\|$

[illegible] $\lim_{N\to\infty} \|\hat\theta_N\| \geq \|\hat\theta_1\| > \|\theta^0\|$

So convergence impossible to $\theta^0$.

## Open Questions

- Proof of self-tuning for
  i) LS scheme ii) Delay iii) Tracking
- Rate of convergence
  Central Limit Theorems
- [illegible]
- [illegible]
- [illegible]
- [illegible]
- More sophisticated control laws.

## Status

- Much progress since 1973
- Much remains to be done.

## Linear Quadratic Control Law

- $y_{t+1} = a y_t + b u_t + w_{t+1}$
- Cost : $\lim \frac{1}{N} \sum_1^N y_t^2 + \rho u_t^2$
- $u_t = K(a,b)\, y_t$ is optimal

## Adaptive Control Scheme

Estimator: $\begin{pmatrix} \hat a_{N+1} \\ \hat b_{N+1} \end{pmatrix} = \begin{pmatrix} \hat a_N \\ \hat b_N \end{pmatrix} + \frac{1}{N r_N} (y_{N+1} - \hat a_N y_N - \hat b_N u_N)$

Control: $u_N = K(\hat a_N, \hat b_N)\, y_N$

## Rewrite

$$\begin{pmatrix} \hat a_{N+1} \\ \hat b_{N+1} \end{pmatrix} = \begin{pmatrix} \hat a_N \\ \hat b_N \end{pmatrix} + \frac{1}{r_N N} (y_{N+1} - \hat a_N y_N - \hat b_N u_N)\, y_N \begin{pmatrix} 1 \\ K(\hat a_N, \hat b_N) \end{pmatrix}$$

## Ljung's ODE

$$\frac{d}{d\tau} \begin{pmatrix} a(\tau) \\ b(\tau) \end{pmatrix} = c(a,b) \begin{pmatrix} 1 \\ K(a,b) \end{pmatrix}$$

---

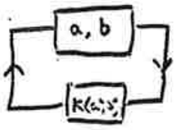
$$\begin{pmatrix} \hat a_{N+1} \\ \hat b_{N+1} \end{pmatrix} = \begin{pmatrix} \hat a_N \\ \hat b_N \end{pmatrix} + \frac{1}{r_N} \underbrace{(y_{N+1} - \hat a_N y_N - \hat b_N u_N)}\, y_N \begin{pmatrix} 1 \\ K(\hat a_N, \hat b_N) \end{pmatrix}$$

$$\{ [a + bK(\hat a_N, \hat b_N)] - [\hat a_N + \hat b_N K(\hat a_N, \hat b_N)] \} y_N + w_{N+1}$$

$$\frac{d}{d\tau} \begin{pmatrix} a(\tau) \\ b(\tau) \end{pmatrix} = \frac{1}{r(\tau)} E(y_t^2) \{ [a + bK(a(\tau), b(\tau))] - [a(\tau) + b(\tau) K(\ldots)] \} \ldots$$

## Equilibrium Points of O.D.E.

$(a^*, b^*)$ : $a + bK(a^*, b^*) = a^* + b^* K(a^*, b^*)$

- Closed Loop Identification Problem
- 

Can only identify $a + bK(a^*, b^*)$

So, $(\hat a_N, \hat b_N)$ MAY converge to such $(a^*, b^*)$

Question: We want self-tuning. S…

Does $a + bK(a^*, b^*) = a^* + b^* K(a^*, b^*) \Rightarrow K(a^*, b^*) = K(a, b)$…

---

$$\frac{db}{da} = K(a,b)$$

- All these Controllers will be [illegible]

[illegible]

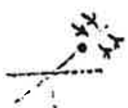
## Why did Min Variance Case Work?

### Mathematical Coincidence

$K(a,b) = -\frac{a}{b}$

$a + bK(a^*, b^*) = a^* + b^* K(a^*, b^*) = 0$

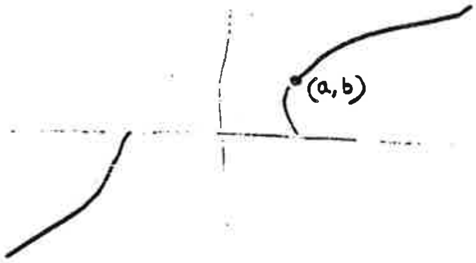
$\Rightarrow K(a^*, b^*) = -\frac{a}{b} = K(a,b)$.



- All equilibrium points are optimal.

---

No: $a + bK(a^*, b^*) = a^* + b^* K(a^*, b^*)$ implies $K(a^*, b^*) = K(a,b)$ only if $(a,b) = (a^*, …$




Hope: If There Are Other Undesired Equilibrium Points, Are They Repellers?

$$\frac{d}{d\tau} \begin{pmatrix} a(\tau) \\ b(\tau) \end{pmatrix} = c(a(\tau), b(\tau)) \begin{pmatrix} 1 \\ K(a(\tau), b(\tau)) \end{pmatrix}$$

[illegible]

## Integral Curves!

$$\frac{db(\tau)}{da(\tau)} = K(a(\tau), b(\tau))$$

Note: Valid No Matter What $K(\cdot, \cdot)$ is

## ENTATIVE CONCLUSIONS

- [illegible] Minimum Variance Control Law [illegible]

Because of Mathematical Coincidence

- [illegible]
- [illegible]

Closed Loop Pole $= a + b\,K(a^*, b^*)$

$= a^* + b^*\,K(a^*, b^*)$

$=$ Desired Pole

- [illegible]

$a^* + b^* K(a^*, b^*)$ stable since $K(a^*, b^*)$ chosen so

$\Downarrow$

$a + b\,K(a^*, b^*) = a^* + b^* K(a^*, b^*)$ also stable.

[illegible] See MP literature.

## FUNDAMENTAL CLOSED LOOP IDENTIFICATI[ON] PROBLEM

- $\hat{\theta}$ [illegible] Estimate (SA, LS, ML)
- $u = K(\hat{\theta}, \ldots$ [illegible]
- Th[illegible]

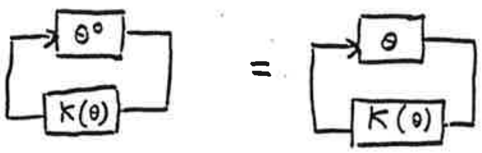
$\hat{\theta}$ [illegible] $A(\cdot) + B(\cdot)K(\cdot)$

$= A(\cdot) + B(\cdot)K(\cdot)$

$= D$

- [illegible] $\in D \not\Rightarrow K(\cdot) = K(\cdot)$
- Question: How do you [illegible] this?

## What properties does $D$ have?




[illegible]

- [illegible]
- [illegible]
- [illegible]

$\hat{\theta}_N$ : minimise $O(N) \log J(\theta) + \sum_{t=1}^{N} \{y(t) - \hat{y}(t|\theta)\}^2$

- [illegible]

## [SUMMARY]

- Proof of Self-Tuning to Minimum Variance Controller
- Self Tuning probably not possible for any other cost criteria [illegible]
- Method to obtain self-tuning sorts of results in general

[illegible]

- [illegible]
- [illegible]
- [illegible]

# A comparison of some control strategies for systems with fast parameter changes

Mille Millnert

Division of Automatic Control
Linköping University, Sweden

This seminar is concerned with adaptive control of systems with abrupt changes in the parameters. First an algorithm for recursive identification of the parameters in a special model-class, suitable for modeling sudden time variations, is presented. Then it is shown how this identification procedur can be used for adaptive control. To illustrate the algorithm and also to discuss some points on adaptive control a comparison study is then referenced.

The basic idea behind the algorithm is to use several parameter sets to model the system. The different parameter vectors correspond to different typical modes of the system. By combining estimation and detection techniques it is possible to estimate the different parameter vectors describing the system.

The purpose of the comparison study was to compare some strategies for adaptive control. Among the "competitors" were for instance a self-tuning regulator (LS with forgetting factor combined with a pole-placement procedure), a time-invariant robust regulator, the regulator mentioned above and an algorithm based on an identification procedur called AFMM (adaptive forgetting through multiple models). A conclusion from the tests was that it can be useful to design regulators which saves information for later use.

References

Millnert, M. (1982): Identification and Control of Systems Subject to Abrupt Changes, Linköping Studies in Science and Technology, Dissertation. No. 82

Millnert, M. (1983): Control Strategies for Systems with Abruptly Changing Parameters, INTERNAL REPORT, LiTH-I-ISY-0634.

Andersson, P. (1983): Adaptive Forgetting in Recursive Identification through Multiple Models, INTERNAL REPORT, LiTH-ISY-I-0638.

A COMPARISON OF SOME CONTROL STRATEGIES
FOR SYSTEMS WITH FAST PARAMETER CHANGES

Mille Millnert
Division of Automatic Control
Linköping University
Sweden



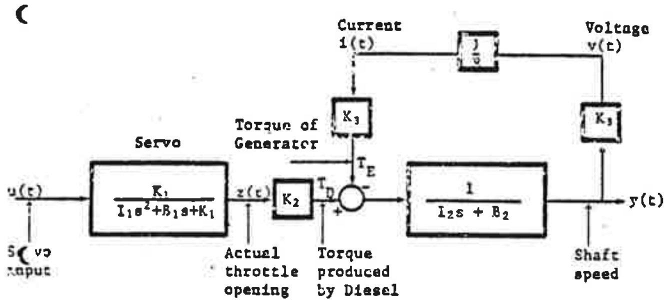
A parameter estimation algorithm

A contest between some strategies for adaptive control

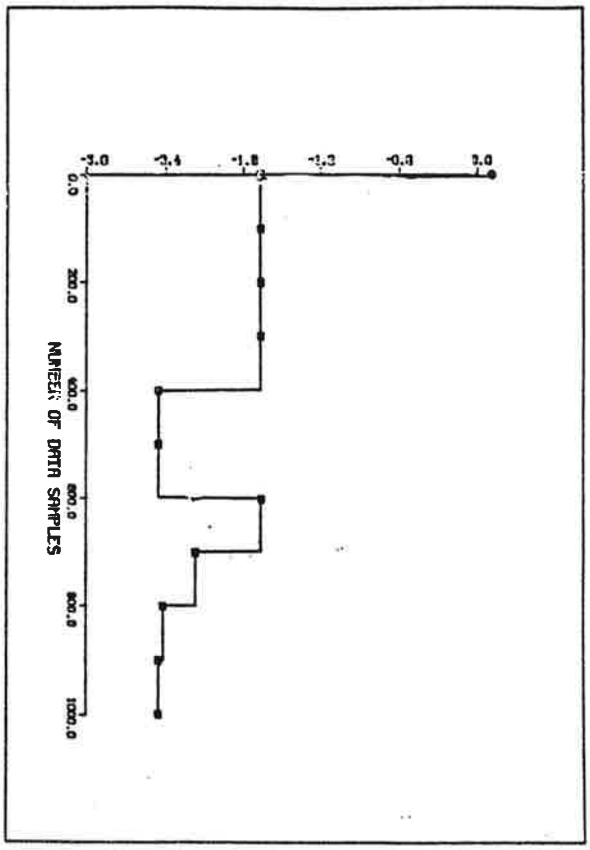


THE CONTEST

the plant





## THE COMPETITORS

1 A self tuner (LS + pole-placement)

2 A constant regulator minimizing a variance-type criterion

3 A regulator minimizing the criterion

$$E\{(y-r)^2+\lambda u^2\}$$

4 The MR algorithm

5 The AFMM algorithm ( forgetting through multiple models)



$$\theta(t+1)=\theta(t)+w(t)$$

$$y(t)=\varphi^T(t)\theta(t)+e(t)$$

$$w(t)=\begin{cases} v(t) & \text{w.p. } q \\ 0 & \text{w.p. } 1-q \end{cases}$$

$$\text{Cov } v(t)=R_1$$

$p(\theta(t)|Y^t)$ = Gaussian sum based on $\bar{\theta}_i(t)$ $i=1\ldots m$

$\bar{\theta}_i \leftarrow$ least squares

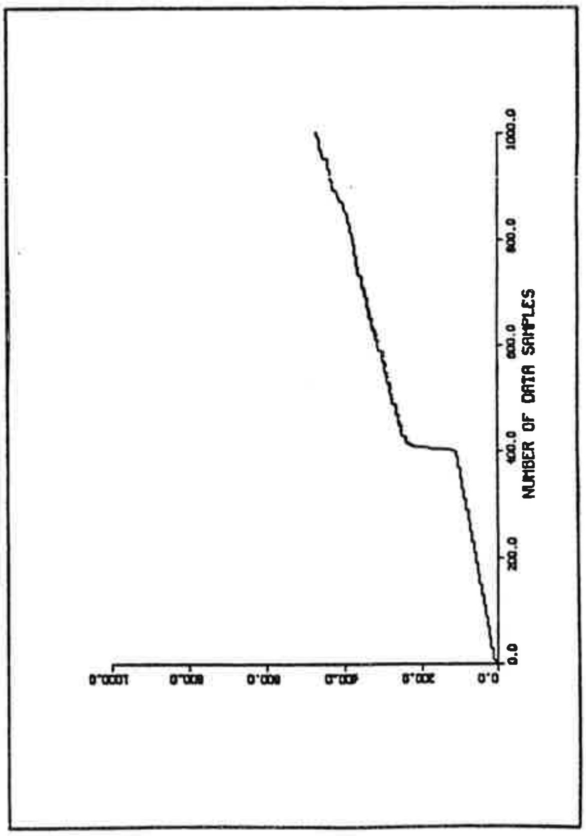
At each time

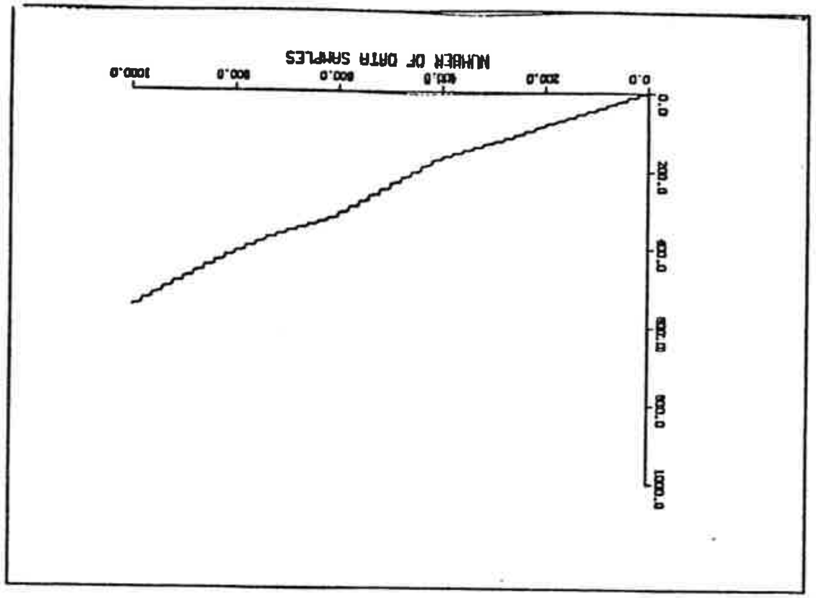
~~$\bar{\theta}_i(t)$~~ with smallest prob.

$\bar{\theta}_i(t)$ with largest prob.

↓

$\bar{\theta}_j(t)$ "Test pilot" alert $P_j=R_1$





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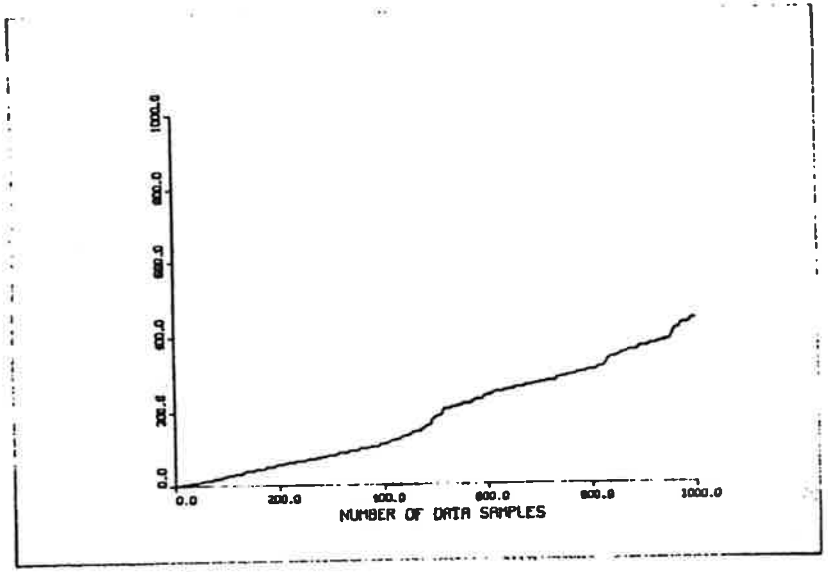

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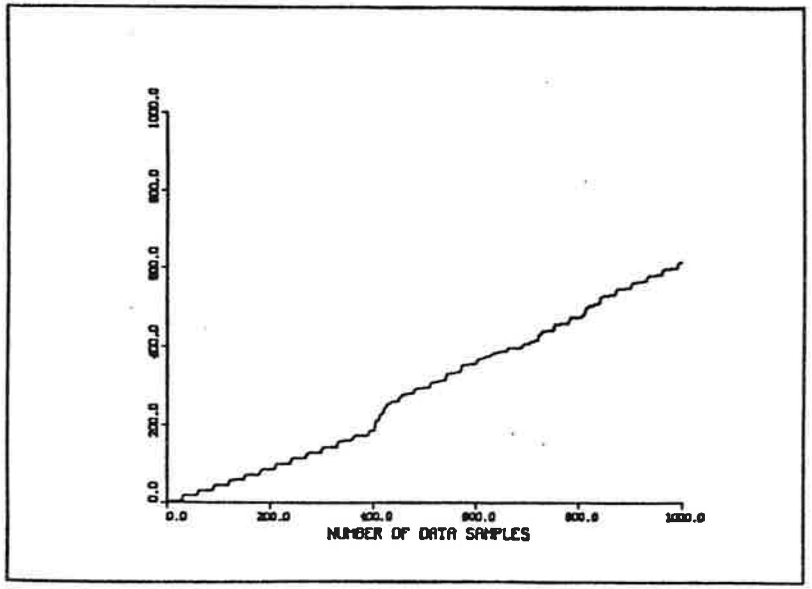

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# Recursive estimation of slowly time-varying parameters

Tore Hägglund

Department of Automatic Control
Lund Institute of Technology
Lund, Sweden

Abstract

The problem to extend traditional algorithms for estimation of constant parameters, such as the least squares algorithm, to capture even the case of time-variable parameters has become important because of their use in adaptive control. Several ad hoc methods have been proposed to handle slowly time-varying parameters. Previous methods, such as the use of a forgetting factor, are here discussed from an information handling point of view. A new method is presented, which is based on the idea to retain a constant amount of information in the estimator. The method is shown to avoid well-known problems associated with other, more heuristic schemes. Analysis as well as simulation experiments are presented.

References

Hägglund, T. The Problem of Forgetting Old Data in Recursive Estimation. Proceedings IFAC Workshop on Adaptive Systems in Control and Signal Processing. San Francisco 1983.

Hägglund, T. New Estimation Techniques For Adaptive Control. PhD-thesis. Report TFRT-1025. Department of Automaic Control, Lund Institute of Technology, Lund Sweden. 1983.

## RECURSIVE ESTIMATION OF SLOWLY TIME-VARYING PARAMETERS

TORE HÄGGLUND
DEPT. OF AUTOMATIC CONTROL
LUND SWEDEN

## TIME-VARYING PARAMETERS

PROCESS MODEL:

$$y(t) = \theta(t-1)^T \varphi(t) + e_n(t)$$

IF THE PARAMETERS TO BE ESTIMATED OR THE NOISE LEVEL VARY, THEY VARY SLOWLY AND/OR SELDOM COMPARED WITH THE TIME CONSTANTS OF THE SYSTEM.

TWO CASES:

1. LARGE PARAMETER CHANGES
2. SLOW PARAMETER CHANGES

## LS ESTIMATION

MINIMIZE $\sum_{i=1}^{t} \frac{1}{\omega(t,i)} \left[ y(i) - \theta(t)^T \varphi(i) \right]^2$

PROBLEM: HOW TO CHOOSE $\omega$

$$y(i) = \theta(i-1)^T \varphi(i) + e_n(i) =$$
$$= \theta(t-1)^T \varphi(i) + \left[\theta(i-1) - \theta(t-1)\right]^T \varphi(i) + e_n(i) =$$
$$= \theta(t-1)^T \varphi(i) + \underbrace{e_m(t,i)}_{\text{model error}} + \underbrace{e_n(i)}_{\text{noise}} \qquad i \le t$$

$$\sigma(t,i)^2 = \sigma_m(t,i)^2 + \sigma_n(i)^2$$

TRY TO CHOOSE $\omega = \sigma^2$

EXAMPLE: CONSTANT PARAMETERS AND CONSTANT NOISE LEVEL

$$e_m(t,i) = \left[\theta(i-1) - \theta(t-1)\right]^T \varphi(i) = 0$$

$\Rightarrow \sigma_m(t,i) = 0$

$\Rightarrow \sigma(t,i)^2 = \sigma_n(t,i)^2 = \sigma^2$

ALL MEASUREMENTS HAVE THE SAME WEIGH

EXAMPLE: EXPONENTIALLY INCREASING MODEL ERROR VARIANCE AND CONSTANT NOISE LEVEL

$$\sigma_m(t,i)^2 = \left[\left(\frac{1}{\lambda}\right)^{t-i} - 1\right]\sigma^2 \qquad \sigma_n(i)^2 = \sigma^2$$

$$\Rightarrow \sigma(t,i)^2 = \left(\frac{1}{\lambda}\right)^{t-i} \sigma^2$$

CONSTANT FORGETTING FACTOR $\lambda$

## SLOW PARAMETER CHANGES

DISCOUNT PAST DATA IN SUCH A WAY THAT A CONSTANT DESIRED AMOUNT OF INFORMATION IS RETAINED, IF THE PARAMETERS ARE CONSTANT.

INFORMATION: $P^{-1}$

Goal: $P \longrightarrow a \cdot I$

$$P_t^{-1} = P_{t-1}^{-1} + \frac{1}{v_t}\varphi_t\varphi_t^T - \alpha_t\varphi_t\varphi_t^T \qquad \alpha_t \ge 0$$

$$P_t = P_{t-1} - \frac{P_{t-1}\varphi_t\varphi_t^T P_{t-1}}{(v_t^{-1}-\alpha_t)^{-1} + \varphi_t^T P_{t-1}\varphi_t}$$

COMPARE:

$$P_t^{-1} = P_{t-1}^{-1} + \frac{1}{v}\varphi_t\varphi_t^T - (1-\lambda)P_{t-1}^{-1}$$

## CHOICE OF $\alpha(t)$

STATIONARITY: $\alpha_t = \frac{1}{v_t}$

$\alpha_t \ge 0$

THEOREM:

IF

$$0 \le \alpha_t \le \frac{1}{\varphi_t^T P_{t-1}\varphi_t} \qquad (*)$$

THEN

* P STAYS POSITIVE DEFINITE
* $\tilde{\theta}_t^T P_t^{-1}\tilde{\theta}_t$ IS DECREASING

□

IN EACH ITERATION, TRY TO OBTAIN

$$\frac{x^T P_t x}{x^T x} = a \qquad x = P_{t-1}\varphi_t$$

BUT FULFIL THE LIMITATION (*)

## CONVERGENCE

IF $a$ NOT TOO LARGE (cf $\lambda < 0$)

THEN

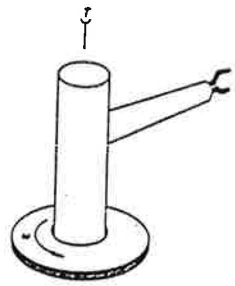
$$W(t) = \sum_1^n [\lambda_i(t) - a]^2 \qquad \lambda_i(t) = eig(P_t)$$

IS DECREASING.

$$W(t) \to 0 \Rightarrow P(t) \to a \cdot I$$

EXCITATION CONDITION $\Rightarrow W(t) \to 0$

## EXAMPLE - INDUSTRIAL ROBOT



$$J\frac{d\omega}{dt} = T_e + T_f + T_D$$

$$\omega(t+1) = \omega(t) + \frac{1}{J}[T_e + T_f + T_D]$$

$$T_f = -k_f \, sign(\omega)$$

$$\theta = [1/J \quad 2k_f/J]^T$$

$$T_e = 0.5\frac{1}{\hat{\theta}_1}(\omega_{ref} - \omega) + \frac{\hat{\theta}_2}{\hat{\theta}_1} sign(\omega)$$

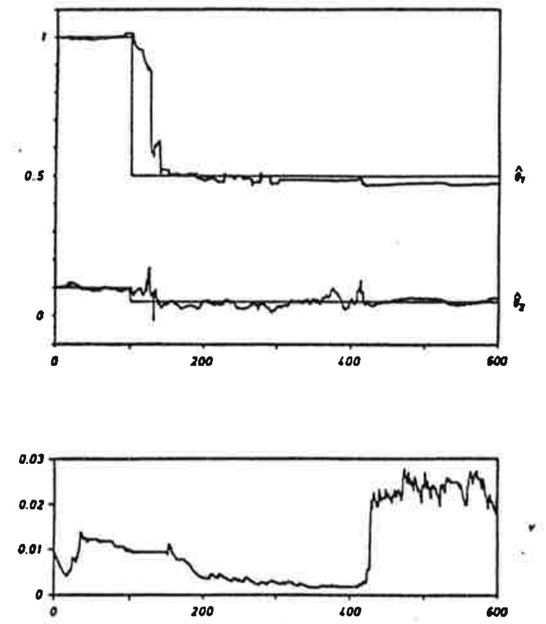



Figure 6.2 - The estimated parameters and the estimated noise variance.



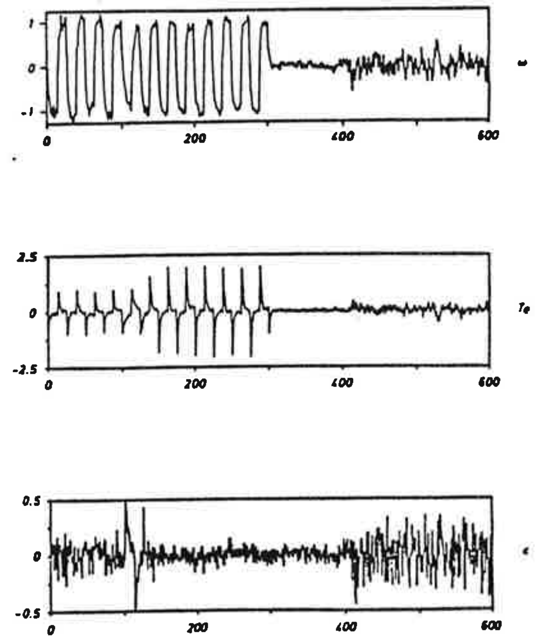



Figure 6.3 - The output- and input-signals of the system, and the residuals ε(t).



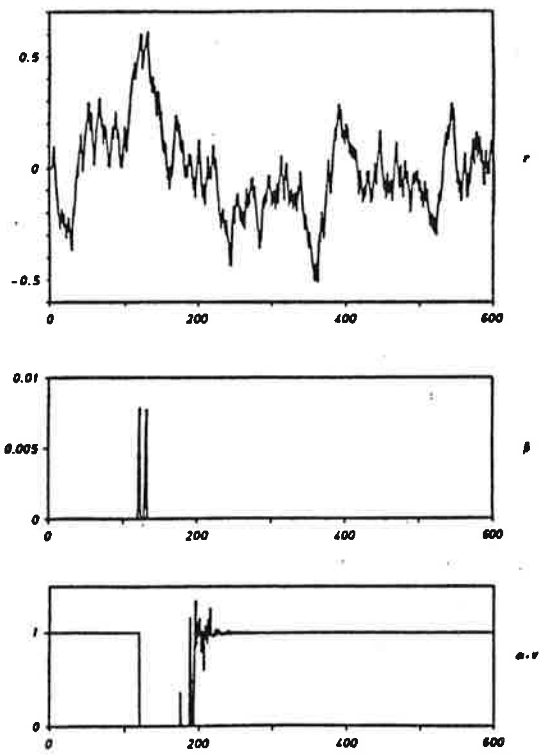



Figure 6.4 - The test signal r(t), the additive gain β(t) and the discounting measure α(t)·ν(t).



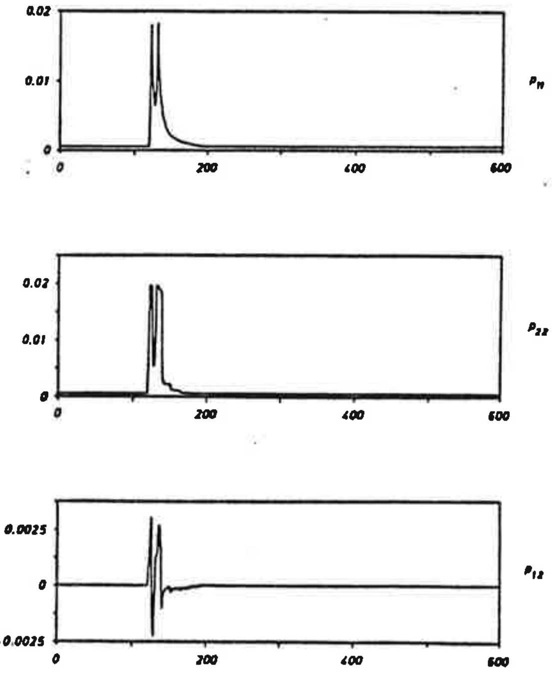



Figure 6.5 - The elements of the P-matrix.

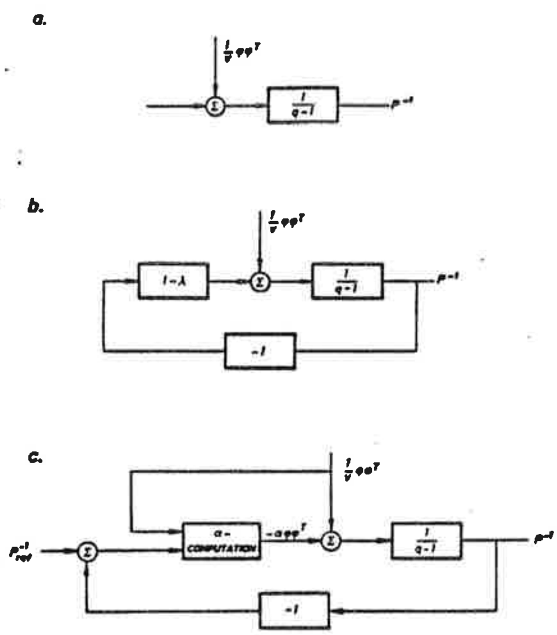



Figure 7.1 - Block diagrams describing the updating of the inverse P-matrix.
a. The original LS procedure.
b. LS with forgetting factor.
c. The new proposed method.

# Robustness of (MRAS) adaptive control

Petar Kokotovic

CSL, University of Illinois
Urbana, USA

The topic of this lecture was robustness of (MRAS) adaptive control subject to high frequency unmodeled dynamics of the process model. This area of stability problems can be classified in three categories: HCG-high controller gain, HAG-high adaptive gain and HF-high frequency input. Here only the adaptation loop is considered i.e. HAG- and HF-instabilities. In this way the resulting system to analys remains linear, time-varying.

The main idea is to approximate the high frequency dynamics with a right half plane zero. (Cf. Padé approximation of a timedelay.) Consider the example below. The approximation is valid for $\mu s << 1$.

$$G(s) = \frac{1}{s+1}\cdot\frac{1}{\mu s+1} = \frac{1}{s+1}\cdot\left(1-\frac{\mu s}{1+\mu s}\right) \approx \frac{1}{s+1}\cdot(1-\mu s) = \frac{1+\mu}{s+1} - \mu \qquad (1)$$

For stability analysis the differentiating effect of the RHP zero is stressed while for synthesis the througput effect gives a clue how to improve robustness.

The MRAS scheme was analysed on the process above (1) using a first order reference model with unknown gain.

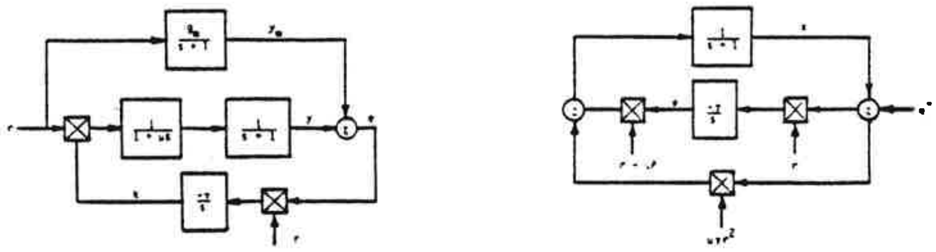


Fig. 1. The MRAS system and the transformed error system (2).

After the approximation (1) of the process and change of coordinates to to an error system, the following equations are obtained:

$$\frac{d}{dt}\begin{bmatrix} x \\ \psi \end{bmatrix} = \begin{bmatrix} -1+\mu\gamma r^2 & r-\mu\dot{r} \\ -\gamma r & 0 \end{bmatrix}\begin{bmatrix} x \\ \psi \end{bmatrix} + \begin{bmatrix} \mu\gamma r^2 \\ -\gamma r \end{bmatrix} e \qquad (2)$$

where

$x$ = difference in output between the actual and tuned system.
$\psi$ = difference in controller gain in reference to the tuned system.

HAG instability is shown in the following way. Assume $r=R$ const. and $e\equiv0$. The characteristic equation gives the stability condition on the adaptive gain $\gamma R^2 < \mu^{-1}$.

To reveal HF-input instability, choose $r=R\cdot\sin(\omega t)$ and assume $\mu\gamma r^2 \ll 1$, $e\equiv0$. Simulation shows a slow drift in $\psi$ and a limit cycle in $x$. Approximation of the equations (2) for slow adaptation, i.e. $\gamma R^2/\sqrt{(1+\omega^2)}$ is sufficiently small, gives the stability condition $\omega^2 < \mu^{-1}$. One interpretation derived from the approximated equations is that instability is reached when the noise-to-signal ratio is larger than unity. Another interpretation is that the positive real condition for the plant is violated at high frequencies due to the RHP zero.

For the combined problem of HAG and HF stability, a detailed analysis shows that the joint stability condition is somewhat conservative.

In the approximation (1) it is seen that the unmodeled high frequency dynamics can be seen as a negative throughput. A simple device to increase the stability properties of the MRAS scheme could therefore be to introduce a positive bypass $\beta$. Analysis by means of a Lyapunov function shows that stability is achieved for $\beta \geq \mu$. The positive real condition for the plant with bypass is also satisfied for these $\beta$.

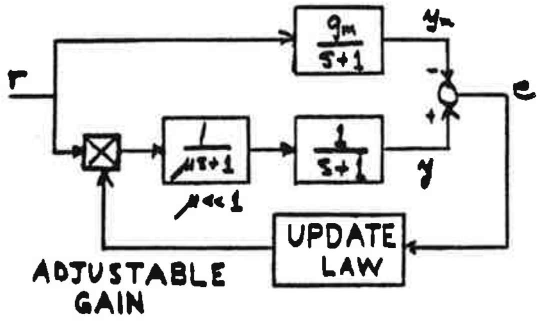
The analysis shows that a bypass increases stability properties against unmodeled high frequency dynamics. However, a tracking error is introduced, which is difficult to predict when $\beta \neq \mu$. For instance, when $\beta$ increases toward $\mu$, the tracking error can decrease or increase depending on the frequency of the reference signal.

<u>Reference</u>

Kokotovic, P., Riedle, B.: Instabilities and stabilization of an adaptive system. Prec. American Control Conference, San Diego, California, June 6-8, 1984.

Riedle + Kokotovic * Lund, July 84 w NSF-STU

# CAN THE SIMPLEST ADAPTIVE LOOP




## BE UNSTABLE ?

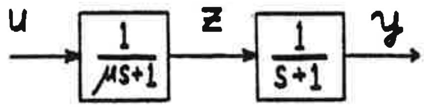
YES, IN TWO WAYS:

1. FAST (HAG)
2. SLOW (HF)

(Discussions with Åström, Bitmead, Rohrs et al.)

· A METHOD TO STABILIZE THE LOOP

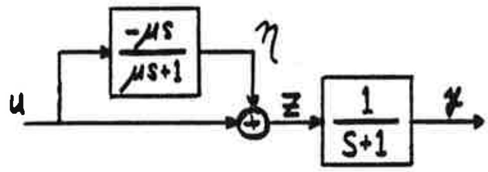
# RIGHT-HALF-PLANE ZERO INSTEAD OF HF PARASITICS



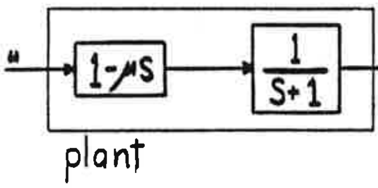

$$\dot y = -y + z \qquad \mu \dot z = -z + u$$

$$\eta = z - u \iff$$

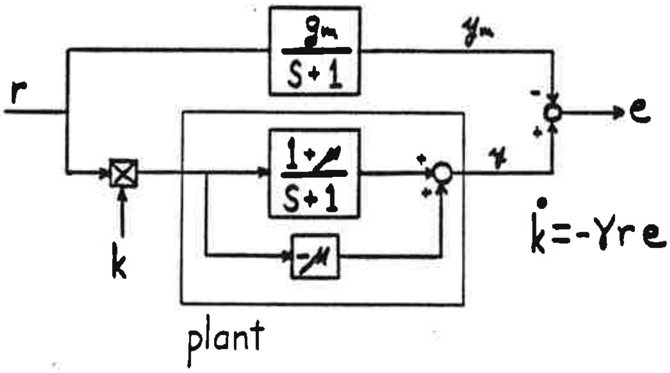
$$\dot y = -y + u + \eta \qquad \mu \dot \eta = -\eta - \mu \dot u$$



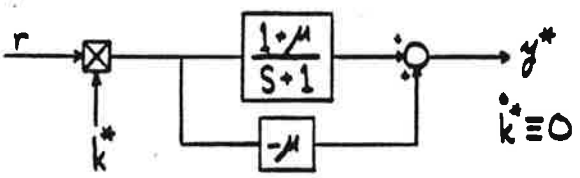

AT LOW FREQUENCIES, LET $\eta = -\mu$




## ADAPTIVE SYSTEM




## TUNED SYSTEM




## TUNED ERROR

$$e^* \triangleq y^* - y_m$$

## FACTS FROM STABILITY THEORY

### FACT 1

$$\lim_{T\to\infty} \int_0^T \text{trace } A(t)\, dt = +\infty$$

IS SUFFICIENT FOR INSTABILITY OF

$$\dot x = A(t)\, x \qquad \square$$

### FACT 2

IF $\dot x = A(t)\, x$ IS EXPONENTIALLY STABLE, THEN

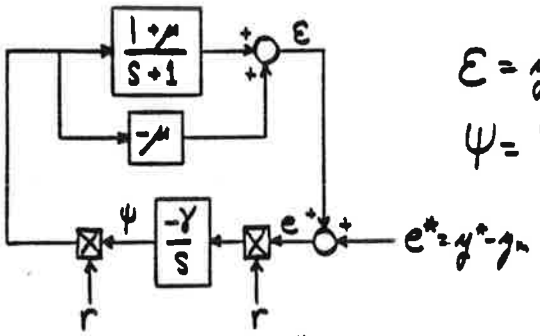
$$\dot x = (A(t) + B(t))\, x$$

IS EXPONENTIALLY STABLE WHEN

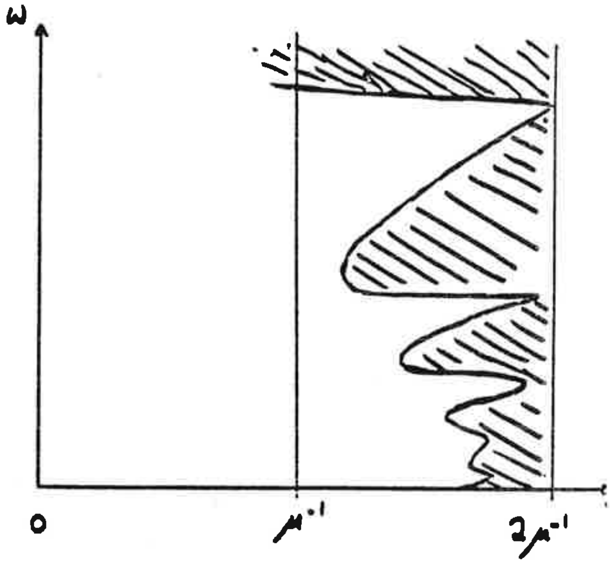
$$\int_0^t \|B(\tau)\| d\tau < c_1 t + c_2 \quad , \forall t \geq 0$$

FOR SOME CONSTANTS $c_1$ and $c_2$.

# FAST ADAPTATION ($\mu \dot{r}$ SMALL)




$$\varepsilon = y - y^*$$
$$\psi = k - k^*$$




$r = \sin \omega t$ $\quad$ $\mu\gamma < 1$ sufficient ($\mu\omega$ small) $\quad$ $\frac{\mu\gamma}{2} < 1$ necessa

LINEAR, LET $e^* = 0$

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\psi} \end{bmatrix} = \left( \begin{bmatrix} -1+\mu\gamma r^2 & r \\ -\gamma r & 0 \end{bmatrix} + \begin{bmatrix} 0 & -\mu\dot{r} \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} \varepsilon \\ \psi \end{bmatrix}$$

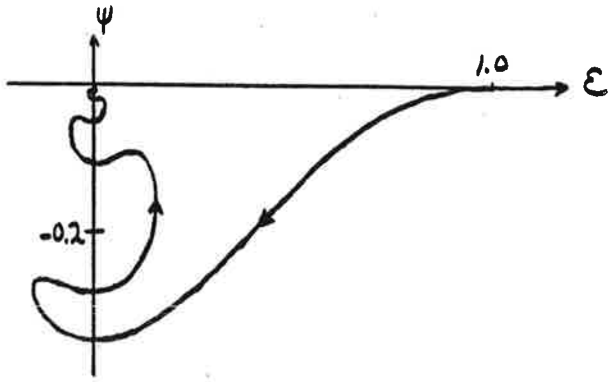
NECESSARY FOR STABILITY

$$\mu\gamma\lambda_1 < 1 \quad , \quad \lambda_1 \triangleq \lim_{T\to\infty} \frac{1}{T} \int_0^T r^2(t)\,dt \quad (= \text{average of } r^2(t))$$

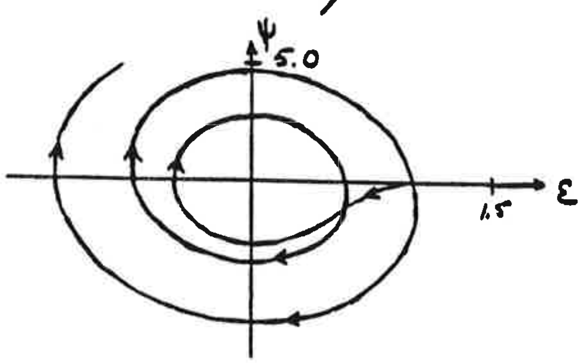
SUFFICIENT FOR STABILITY ($\mu \dot{r}$ SMALL)

$$\mu\gamma r^2(t) < 1 \quad \forall t$$

$$V_1 = \frac{1}{2}\varepsilon^2 + \frac{1}{2\gamma}\psi^2 \quad , \quad \dot{V}_1 = (-1+\mu\gamma r^2(t))\,\varepsilon^2$$

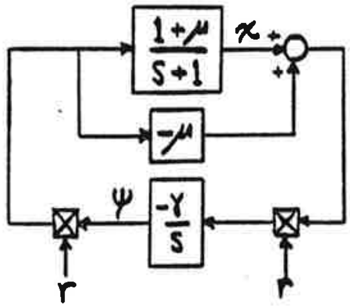




$r = \sin t$ $\quad$ $\mu = 0.1$ $\quad$ $\gamma = 1$

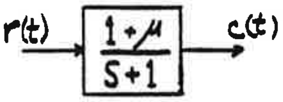



$r = \sin .5t$ $\quad$ $\mu = 0.1$ $\quad$ $\gamma = 24$

## SLOW ADAPTATION ($\gamma$ and $\mu\gamma r^2$ SMALL)




$$\dot{x} = -x + \psi(1+\mu)r$$
$$\dot{\psi} = \gamma(-rx + \mu r^2)$$




$$c(t) \triangleq \int_0^t (1+\mu)e^{-(t-\tau)} r(\tau)\, d\tau$$

WHEN $\gamma = 0$

$$x(t) = c(t)\,\psi(t)\ ,\quad \psi(t) = \text{const.}$$

DEFINE $\delta(t)$ SUCH THAT

$$x(t) = c(t)\,\psi(t) + \gamma\,\delta(t)$$

## SLOW ADAPTATION ($\gamma$ and $\mu\gamma r^2$ SMALL)

$$\begin{bmatrix}\dot{\delta}\\ \dot{\psi}\end{bmatrix} = \left( \begin{bmatrix} -1+\gamma rc & (c-\mu r)rc \\ 0 & \gamma(\mu r^2 - rc) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -\gamma r^2 & 0 \end{bmatrix} \right) \begin{bmatrix}\delta\\ \psi\end{bmatrix}$$

$$\dot{\delta} = (-1 + \gamma r(t)c(t))\,\delta \qquad \dot{\psi} = \gamma(\mu r^2(t) - r(t)c(t))\,\psi$$

STAB. COND. $\quad -1 + \gamma\lambda_2 < 0 \qquad \gamma(\mu\lambda_1 - \lambda_2) < 0$

$$\lambda_1 = \lim_{T\to\infty} \frac{1}{T}\int_0^T r^2(t)\,dt = (\text{average of } r^2) \qquad \lambda_2 = \lim_{T\to\infty} \frac{1}{T}\int_0^T r(t)c(t)\,dt = (\text{average of } rc)$$

NECESSARY AND SUFFICIENT CONDITION FOR SLOW ADAPTATION STABILITY

$$\mu < \frac{\lambda_2}{\lambda_1} = \frac{\text{average of } rc}{\text{average of } r^2}$$

$$\boxed{\mu < \frac{\lambda_2}{\lambda_1}}$$

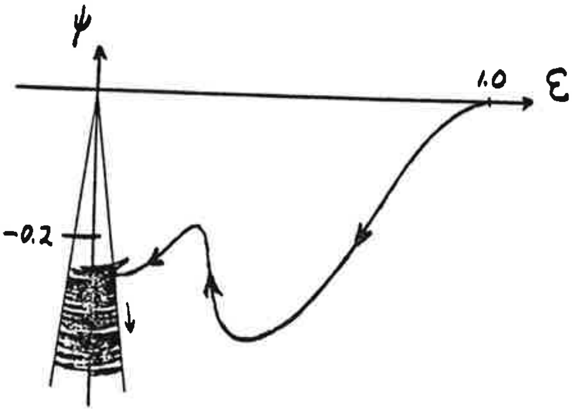
REQUIRES LOW FREQUENCY REFERENCE INPUTS

$$r(t) = \sum_{i=1}^{N} R_i \sin(\omega_i t + \theta_i)$$

$$\mu < \frac{\lambda_2}{\lambda_1} = \frac{\sum_{i=1}^{N} \frac{1+\mu}{1+\omega_i^2} R_i^2}{\sum_{i=1}^{N} R_i^2}$$

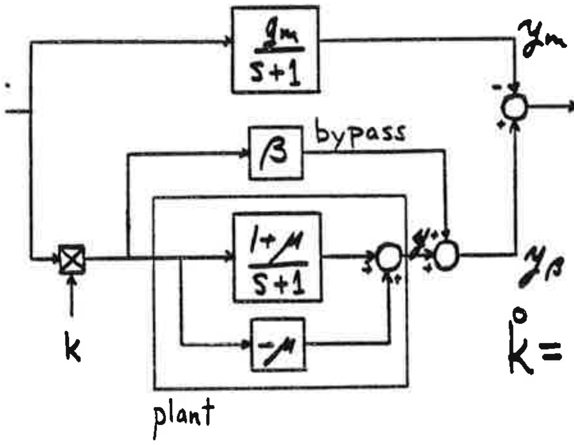
$$\boxed{\sum_{i=1}^{N} \frac{1}{1+\frac{1}{\mu}} R_i^2 < \sum_{i=1}^{N} \frac{1}{1+\omega_i^2} R_i^2}$$

Sufficient $\quad \frac{1}{\mu} > \omega_i^2\ ,\quad \boxed{\omega_i < \frac{1}{\sqrt{\mu}} \quad \forall i \in [1,N]}$



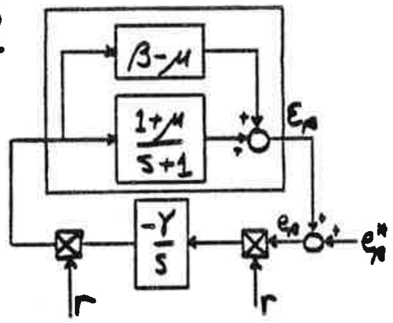

$r = \sin 4t \qquad \mu = 0.1 \qquad \gamma = 1.0$

# STABILIZATION WITH BYPASS




$\mathring{k} = -\gamma r e_\beta$

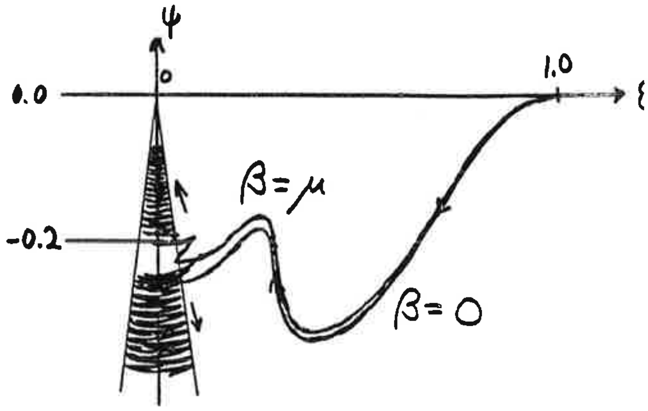
SPR for $\beta \geq \mu$




$\varepsilon_\beta = y_\beta - y_\beta^*$

$\psi = k - k^*$

$e_\beta^* = y_\beta^* - y_m$




$r = \sin 4t \quad \gamma = 1.0 \quad \mu = 0.1$

# CONCLUDING REMARKS

* SIMPLEST ADAPTIVE SYSTEM EXHIBITS TWO TYPES OF INSTABILITY CAUSED BY ADAPTATI

* THESE TWO MECHANISMS PLUS LINEAR HIGH CONTROLLER GAIN CAN EXPLAIN ALL PHENOMENA OBSERVED IN MORE COMPLEX ADAPTIVE SYSTEMS

* BOUNDS DERIVED HERE WILL GENERALIZE

$$\mu\gamma r^2 < 1 \longrightarrow \mu\omega^T\Gamma\omega < 1$$

$$\text{ave}(rc - \mu r^2) > 0 \longrightarrow \text{ave}(\omega c^T - \mu\omega\omega^T) > 0$$

# Parameter convergence issues in MRAC

S. Shankar Sastry
Dept. of Elect. Eng. and Comp. Sciences
University of California
Berkeley, USA

This contribution addresses parameter convergence in continuous time model reference adaptive control. The scheme of Narendra et al. is considered. The reported result can be summarized as follows:

- The technique of generalised harmonic analysis is employed to translate the persistent excitation condition on the signal vector into an equivalent condition on the exogenous reference signal only for parameter convergence.

- Partial convergence results for the case where the reference signal is not sufficiently rich are derived.

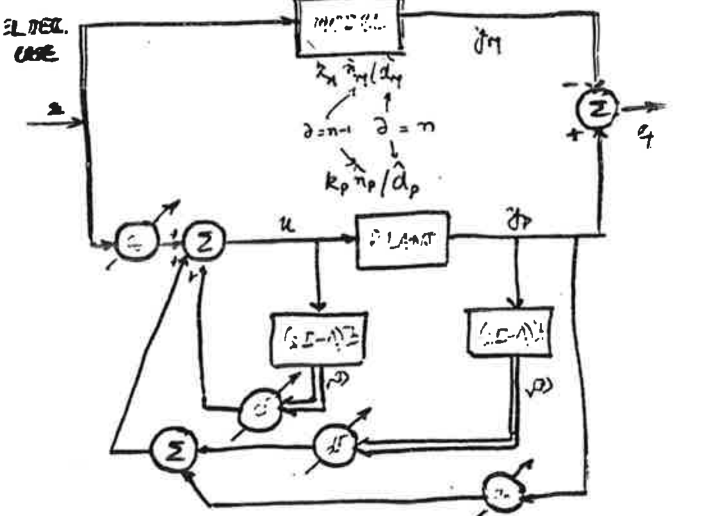
- Connections with robustness are illustrated by a first order example with output plant disturbance and plant parameter variation.

PARAMETER CONVERGENCE ISSUES IN MODEL REFERENCE ADAPTIVE CONTROL

S. S. SASTRY — Joint work with S. Boyd, M. Bodson

UNIVERSITY OF CALIFORNIA BERKELEY

NEC. & SUFF. CONDITIONS FOR PARAMETER CONVERG. NOMINAL




PARAMETER VECTOR $\theta^T = [c_0, c^T, d_0, d^T] \in \mathbb{R}^{2n}$

SIGNAL VECTOR $w = [r, v^{(1)T}, y_p, v^{(2)T}] \in \mathbb{R}^{2n}$

CONTROL LAW $u = \theta^T w$. UPDATE LAW $\dot{\theta} = -e_1 w$

NOTHING CAN BE SAID ABOUT CONVERGENCE OF $\theta$.

$\exists!\ \theta^{*T} = [c_0^*, c^{*T}, d_0^*, d^{*T}] \in \mathbb{R}^{2n}$ such that

closed-loop plant transf. fn = model transfer function.

$\phi = \theta - \theta^*$ PARAMETER ERROR

THEOREM (ANDERSON, MORGAN-NARENDRA, ....)

$\phi, e_1 \to 0$ exponentially $\iff \exists\, \alpha, \delta > 0 \ni$

$$\int_s^{s+\delta} w w^T dt \geq \alpha I \quad \forall s.$$ PERSISTENT EXCITATION

$w^T = [r, v^{(1)T}, y_p, v^{(2)T}]$ ).

Remarks: (1) Condition not explicit since it is on signals inside time-varying adaptive loop.

Contributions: (1) Conditions on exogenous ref signal for parameter convergence

(2) PARTIAL CONVERGENCE RESULTS

(3) CONNECTIONS WITH ROBUSTNESS

Technique: GENERALIZED HARMONIC ANALYSIS. A LÀ WIENER

Generalized Harmonic Analysis

$u: \mathbb{R}_+ \to \mathbb{R}^n$ is said to have autocovariance $R_u(\tau) \in \mathbb{R}^{n \times n}$ iff

$$\lim_{T \to \infty} \frac{1}{T} \int_s^{s+T} u(t)\, u(t+\tau)^T dt = R_u(\tau)$$

uniformly in $s$ (!).

Normal: $w$ is persistently exciting $\iff R_w(0) > 0$.

Fact: $R_u(\tau)$ has the Bochner (spectral) representation

$$R_u(\tau) = \int e^{j\nu\tau} S_u(d\nu)$$ SPECTRAL MEASURE

Note $R_w(0) > 0 \iff \int S_w(d\nu) > 0$

Lemma 3 LINEAR FILTER LEMMA

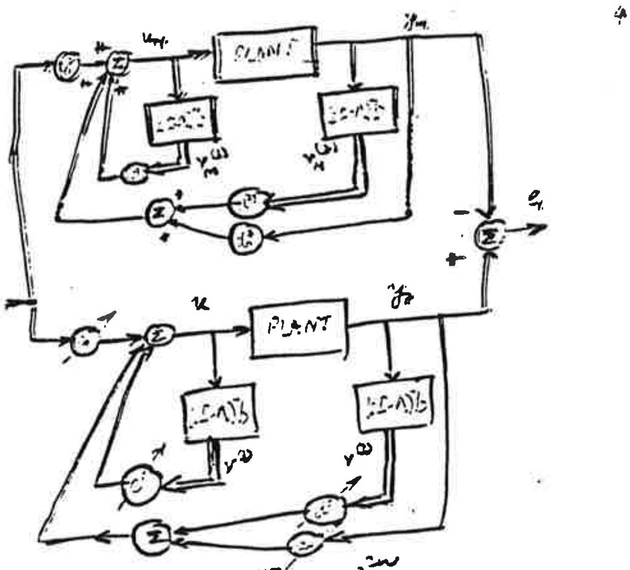
Let $u: \mathbb{R}_+ \to \mathbb{R}^n$ have spectral measure $S_u(d\nu)$ and $h$ be an $m \times n$ matrix of bounded measures. Then $y = h * u$ has spectral measure

$$S_y(d\nu) = \hat{h}(j\nu)\, S_u(d\nu)\, \hat{h}(j\nu)^*$$

Lemma 4 TRANSIENT LEMMA

$u - v \in L^2 \implies R_u(\tau) = R_v(\tau)$

4




$v_M^T = [r, v_M^{(1)T}, y_M, v_M^{(2)T}] \in \mathbb{R}^{2n}$

$v^T = [r, v^{(1)T}, y_p, v^{(2)T}] \in \mathbb{R}^{2n}$

Idea $\Rightarrow$ $w - w_M \in L^2 \Rightarrow R_w(\tau) = R_{w_M}(\tau)$.

i.e. $R_w(0) > 0 \iff R_{w_M}(0) > 0 \iff \int S_{w_M}(d\nu) > 0$.

[illegible] transfer function from r to $w_M$ is

$[1, \frac{\hat{M}}{\hat{P}}(sI-\Lambda)^{-1}b, \hat{M}, \hat{M}(sI-\Lambda)^{-1}b] = \hat{Q}^T(s)$

$\therefore S_{w_M}(d\nu) = \hat{Q}(j\nu)\hat{Q}^*(j\nu) S_r(d\nu)$.

5

Theorem: $w_M$ is Persistently Exciting $\iff$ Spectral measure of r is not concentrated on $k < 2n$ points.

Pf: $\Rightarrow$ Assume that $S_r$ is conc. on $\nu_1, \dots, \nu_k$ where $k < 2n$.

Then

$$R_{w_M}(0) = \int \hat{Q}(j\nu) S_r(d\nu) \hat{Q}^*(j\nu) = \sum_{m=1}^{k} \hat{Q}(j\nu_m)\hat{Q}^*(j\nu_m) S_r(\{\nu_m\})$$

Since it is the sum of $k < 2n$ dyads,

$R_{w_M}(0)$ is singular.

$\Leftarrow$ Assume that w is not P.E. Then, $\exists c \in \mathbb{R}^{2n} \ni$

$$0 = c^T R_w(0) c = \int |\hat{Q}(j\nu)^* c|^2 S_r(d\nu)$$

i.e. $\hat{Q}^*(j\nu) c = 0 \quad \forall \nu \in \text{Supt}(S_r)$.

Now $\exists$ constant matrix $M \ni$

$$\hat{Q}^T(s) M = \frac{1}{\hat{n}_p d_M}[\hat{d}_p, \dots, \hat{d}_p s^{n-2}, \hat{n}_p, \dots, \hat{n}_p s^n].$$

Then, we have

$[\hat{Q}^*(j\nu) M]\tilde{c} = 0 \quad \forall \nu \in \text{Supt}(S_r)$

i.e. $\exists$ poly. $\hat{a}(s)$ & $\hat{b}(s)$ of $\partial$ $n-2$, and $n$ resp $\ni$

$$\underbrace{\hat{a}(j\nu)}_{\partial = n-2}\underbrace{\hat{d}_p(j\nu)}_{\partial = n} + \underbrace{\hat{b}(j\nu)}_{\partial = n}\underbrace{\hat{n}_p(j\nu)}_{\partial = n-1} = 0 \quad \forall \nu \in \text{Supt}\, S_r.$$

6

$(\text{Supt}\, S_r) \geq 2n$ points.

$$\hat{a}(s)\hat{d}_p(s) + \hat{b}(s)\hat{n}_p(s) \equiv 0$$

$$\frac{\hat{n}_p}{\hat{d}_p} = \frac{-\hat{a} \leftarrow n-2}{\hat{b} \leftarrow n}$$

$\not\leftarrow$ Coprimeness of $\hat{n}_p$, $\hat{d}_p$ $\square$

Remarks: 1. We have shown that when reference signal is sufficiently rich

Supt $S_r(d\nu) \geq 2n$ points then we have parameter convergence.

2. Only assumption: r have autocovariance

Rate of Convergence: let $\alpha', \delta'$ be such that

$$\int_s^{s+\delta'} w_M(\hat{M} w^T)\,dt \geq \alpha' I \quad \forall s \in \mathbb{R}_+$$

← Connection with location of spectrum $S_r(d\nu)$??

Then, Lyapunov exponent of $\phi$. PARAMETER ERROR

is

$$\frac{\log(1+\alpha')}{2\delta'} \approx \frac{\alpha'}{2\delta'} \quad \text{for small } \alpha'$$

7

PARTIAL CONVERGENCE

If $S_r$ is concentrated on $k < 2n$ points $\nu_1, \dots, \nu_k$. Then let $\Theta$ be the set of $\theta$'s for which c.l. plant matches model i.e.

$$k \updownarrow \underbrace{\begin{bmatrix} \hat{P}(j\nu_1)\hat{Q}(j\nu_1)^T \\ \vdots \\ \hat{P}(j\nu_k)\hat{Q}(j\nu_k)^T \end{bmatrix}}_{2n} \theta = \begin{bmatrix} \hat{M}(j\nu_1) \\ \vdots \\ \hat{M}(j\nu_k) \end{bmatrix}$$

$$\Theta = \theta^* + \text{Nullspace}\begin{bmatrix} \hat{P}(j\nu_1)\hat{Q}(j\nu_1)^T \\ \hat{P}(j\nu_k)\hat{Q}(j\nu_k)^T \end{bmatrix}$$

$$= \theta^* + \text{Null Space}\, R_W(0)$$

$\theta \to \Theta$ as $t \to \infty$, $\theta$ itself need not converge but $\lim_{t\to\infty} \text{dist}(\theta(t), \Theta) = 0$.

Rate of convergence may not be exponential.

HIGHER REL. DEGREE CASES 8

Rel. degree 2 same statement – parameter convergence iff $\text{Supp}\, S_\nu(d\nu) \geq 2n$ points.

Rel. degree $\geq 3$ new parameter $\theta_{2n+1}$ associated with gain parameter for augmented error signal. Using partial convergence theorem, all but $\theta_{2n+1}$ converge iff $\text{Supp}\, S(d\omega) \geq 2n$ points. (!!). WATCH OUT FOR $\theta_{2n+1}$ in the augmented error scheme.

CONNECTIONS WITH ROBUSTNESS 9

(VIDYASAGAR – VANNELLI)

Consider $\dot{x} = f(x,u,t)$, locally Lipschitz in $x, u$.

Then, if $\dot{x} = f(x,0,t)$ is exponentially stable (uniformly)

$\Rightarrow$ $\dot{x} = f(x,u,t)$, $y = h(x,t)$ is small signal $L_\infty$ BIBO stable with $x(0) = 0$.

i.e. $\exists\, c \ni \|u\|_\infty \leq c \Rightarrow \|y\|_\infty \leq k\|u\|_\infty$

EXTENSION: If $x(0) \neq 0$, $\forall \varepsilon > 0\ \exists T_\varepsilon \ni$

$\|y\|_\infty \leq k(1+\varepsilon)\|u\|_\infty$ for $t \geq T_\varepsilon$.

REMARKS: (1) EXPONENTIAL STABILITY IS ABSOLUTELY NECESSARY FOR THIS THEOREM TO WORK – NUMEROUS COUNTEREXAMPLES EXIST. – (How about Rel. degree $\geq 3$, with augmented error)

2) $L_\infty$ gain $k$ depends on rate of exponential convergence (inversely); and so does $T_\varepsilon$.

Application 10

| | | |
|---|---|---|
| 1ST ORDER PLANT | $\dot{y}_p = -a_p^{*} y_p + b u$ (UNKNOWN) | $e_1 = y_p - y_m$ O/P ERROR |
| MODEL | $\dot{y}_m = -a_m y_m + b r$ | |
| ADAPTIVE LAW CONTROL | $u = r + d\, y_p$ | $\phi = d - d^{*}$ PARAMETER ERROR |
| | $\dot{d} = -\frac{e_1 y_p}{b}$ | |

$$\begin{pmatrix}\dot{e}_1\\ \dot{\phi}\end{pmatrix} = \begin{pmatrix}-a_m & y_p(t)\\ -y_p(t) & 0\end{pmatrix}\begin{pmatrix}e_1\\ \phi\end{pmatrix} = \begin{matrix}-a_m e_1 + \phi y_m + \phi e_1\\ -e_1^2 - e_1 y_m\end{matrix}$$

$\dot{x} = f(x,t)$ EXP. CONVERG. IFF $\text{Supp}\, S_\nu \geq 1$ pt.

O/P DISTURBANCE

$\dot{e}_1 = -a_m e_1 + \phi y_m + \phi e_1 + \phi p + d^{*} p$

$\dot{\phi} = -e_1^2 - e_1 y_m - 2p e_1 - p y_m - p^2$

$\dot{x} = f(x,t) + g(x,t)\begin{pmatrix}p\\ p^2\end{pmatrix}$

SLOW PARAMETER VARIATION

$\dot{e}_1 = -a_m e_1 + \phi y_m + \phi e_1$

$\dot{\phi} = -e_1^2 - e_1 y_m - \dot{a}_p$

$\dot{x} = f(x,t) + g(x,t)\begin{pmatrix}0\\ \dot{a}_p\end{pmatrix}$

UNMODELLED DYNAMICS – CHOICE OF MODEL CRUCIAL.
TECHNIQUE – CONVERT TO O/P DISTURBANCE.
AMT THAT CAN BE HANDLED DEPENDS ON EXTENT OF ... CONVERG...

# On adaptive control with prescribed robustness properties

Eva Trulsson

Department of Electrical Engineering
University of Linköping
Linköping, Sweden

We will describe a frequency domain method for regulator design. The resulting regulator has a stability margin in terms of the Nyquist curve which is independent of the system. This is a useful property in an adaptive control application, where the controller at every step is designed for a recursively identified model of the system, for the following reason. Since the Nyquist curve is known it is known at which frequences it is important to have a good model of the system in order to have a good control result. Then the input-output data can be filtered through filters which emphasizes these frequences and this will lead to a model of the system which is best there. (See B. Wahlberg and L. Ljung (1984)).

The main idea of the method is to obtain a given Nyquist curve by a cancellation of the system poles and zeroes. Therefore the basic version can only be applied to stable minimum phase systems. By a slight modification it is however possible to handle also pure integrators and discrete time real unstable zeroes.

Reference

B. Wahlberg and L. Ljung (1984): Design variables for bias distribution in transfer function estimation. Internal report, Department of Electrical Engineering, Linköping University, S-581 83 Linköping, Sweden.

# On an adaptive control algorithm with prescribed robustness properties.

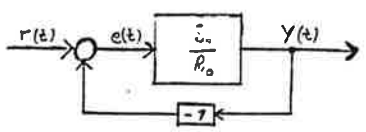
A frequency domain method.

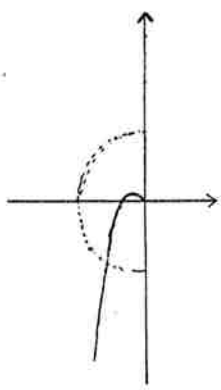
- There will always be unmodeled dynamics.
- Most important to have a good model of the system around the cross-over frequency.
- Filtering can be used to distribute the bias

Poleplecement: The Nyquist curve depends almost as much on the system as on the desired closed loop poles.
That may be a problem in an adaptive control application.

## An other design method.

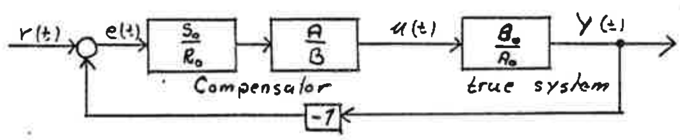
Choose some 'ideal' open loop reference system $S_0/R_0$.



## Use the control:

$$u(t) = -\frac{S_0}{R_0} \cdot \frac{A}{B} \left( y(t) - r(t) \right) \qquad (*)$$




## Observations

- Same robustness against multiplicative errors in $\frac{B}{A}$ for all systems. (Given by $\frac{S_0}{R_0}$)
- In order to have (*) realizable $\frac{S_0}{R_0}$ and $\frac{B}{A}$ must be of the same type. (Same pole-excess or same number of delays
- Only stable minimum phase systems can be handled. (With some modifications also integrators and real discrete time zeroes can be treated.)

## Non minimum phase zeroes.

Suppose that

$$\frac{B}{A} = \frac{q^{-d} B_I B_s}{A}$$

where

$$B_I = k \cdot (1 + b q^{-1}) \qquad b > 0$$

Then choose $S_0$ with $d+1$ pure delays.

$$\frac{S_0}{R_0} = \frac{q^{-d-1} S_1}{R_0}$$

as reference loop gain.

and use the control

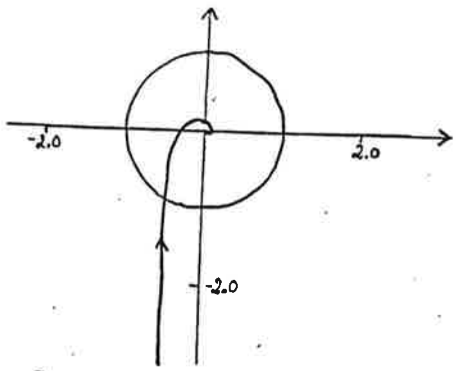
$$u(t) = -\frac{S_1 A}{R_0 B_s}\left(y(t) - r(t)\right)$$

This will lead to the loopgain

$$G_R \cdot G = \frac{S_1 A}{R_0 B_S} \cdot \frac{B_I B_s q^{-d}}{A} = \underbrace{\frac{S_1 q^{-d-1}}{R_0}}_{\text{desired gain}} \cdot \underbrace{q B_I}_{\text{nice}}$$

Reference open loop system:

$$G(q^{-1}) = \frac{0.125\, q^{-2} (1 + q^{-1})}{(1 - q^{-1})}$$



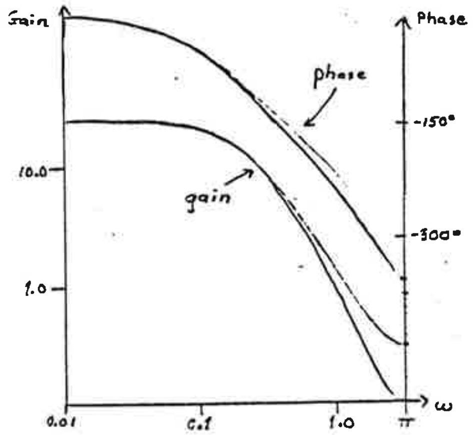

- 60° phase margain
- integral action
- gain decreasing to zero
- poles at -0.27 ; 0.65 ± 0.25i "p"

## An example.

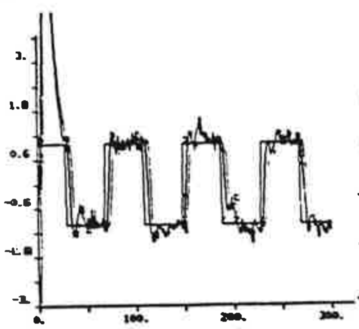
System:

$$y(t) = \frac{q^{-2}\,(1 + 0.25 q^{-1})}{(1 - 0.8 q^{-1})^2 (1 - 0.25 q^{-1})}\, u(t) + e(t)$$

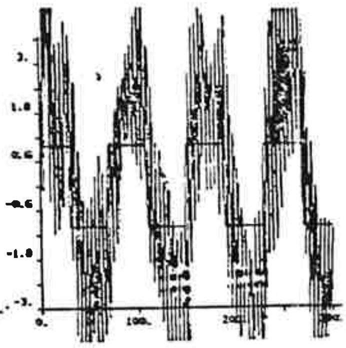
Almost second order



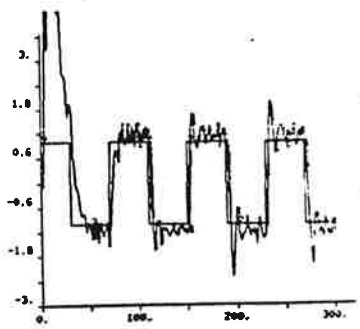

— true system
— a second order system



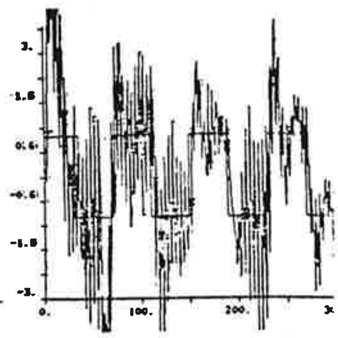

Reference system



poleplacement in the origin

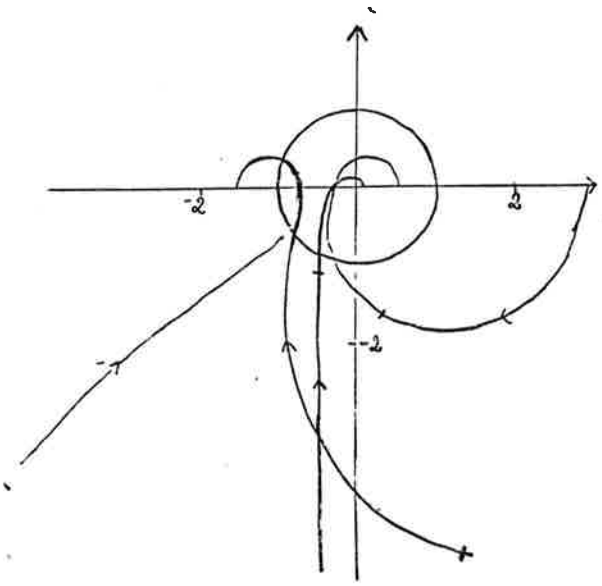



poleplacement at "p"



polplacement at "p" with integral action

p = -0.27 ; 0.65 ± 0.25i




— poleplacement in the origin
— poleplacement at "p"
— reference system
— poleplacement at "p" with integral action.

p = -0,27 ; 0.63 ± 0.25i

## Conclusions

- Frequency domain (robustness) properties which are independent of the system model.
- No problems with common factors in A and B
- Simple.
- The disturbance rejection properties may be bad.
- The choice of reference system may need some initial testing.

# On living with the positive real condition

Charles Rohrs

Deptartment of Electrical Engineering
University of Notre Dame
Notre Dame, Indiana, USA

In this lecture an example is given on a simple model reference adaptive system for which instability is achieved for certain command signals. A modification of the adaptation is suggested to overcome the problem.

In the example the reference model is of 1st order while the process is of 3rd order. The adjustable parameters are the feedforward gain $k_r$ and the feedback gain $k_y$. If $k_y = k_y^*$ is known and the reference signal r(t) is a step $r_0$ then the closed loop system is linear and time invariant. It is then easy to see that for $r_0$ sufficiently large the closed loop system will become unstable unless the process transfer function is positive real. An easy way to eliminate this problem is to use a normed adaptation gain $g_r' = g_r/r^2$ instead of $g_r$. If the reference signal is a sinusoidal $r(t) = \sin\omega t$ then simulations show that for frequencies larger than a certain frequency $\omega_0$ the system will be driven into instability.

The proposed method is based on frequency domain arguments. The idea is to turn off the adaptation for sufficiently high frequencies in the reference input. For these frequencies the adaptation is done with respect to a benign model with positive real transfer function $G_{good}(s)$.

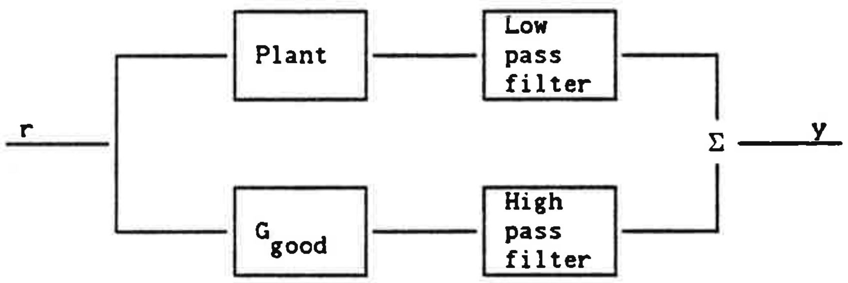



Fig. The "conditioned plant".

The following assumptions are made :

- The sign of the process gain is known.
- The process is minimum phase.
- The relative degree of the process is known.
- The controller uses $2n^*$ parameters, where $n^*$ is an upper bound on the process order.

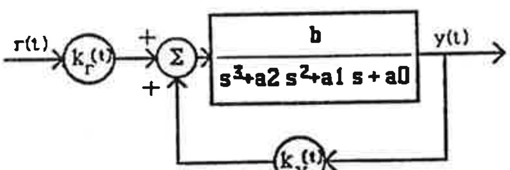
Simulations of the MRAS when applied to the "conditioned plant" show that the parameter drifts due to high frequency command signals are eliminated. There are however no theo-retical results on the robustness of the modified controller.

# Living with Positive Realness

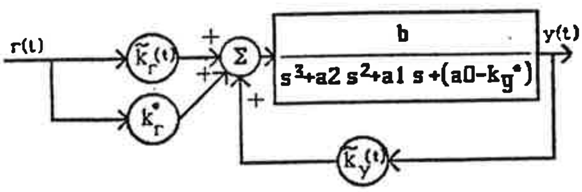
Charles E. Rohrs
Univ. of Notre Dame

1. The positive real condition appears to be necessary as well as sufficient

2. There are measures which can be taken which may be taken to allow us to live with the positive real condition.
   1. Excitation
   2. Plant Conditioning




The adaptive controllers setup




$$P^* = \frac{b}{s^3+a2\,s^2+a1\,s+(a0-k_y^*)}$$

Adaptive controller's error system

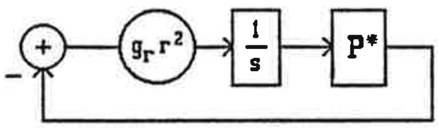
**Case 1**

$k_y^*$ known; $r(t)=r$, a constant

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \\ \ddot{e} \\ \tilde{k}_r \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -(a0-k_y^*) & -a1 & -a2 & br \\ -g_r\,r & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \\ \ddot{e} \\ \tilde{k}_r \end{bmatrix}$$

Stability determined by characteristic equation

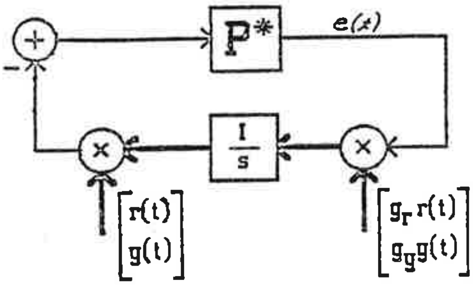
$$s(s^3+a2\,s^2+a1\,s+(a0-k_y^*)) + b g_r r = 0$$




∃ r which will cause instability iff $\arg P^*(j\omega) > 90$ degs for some $\omega$

One should use $g_r = \dfrac{g_r'}{r^2(t)}$

ERRCR SYSTEM CAN BE VIEWED AS FOLLOWS:




If true plant were first order, then there would be feedback parameter $k_y^*$ so that $P^*$ would be positive real or passive.

Then the passivity theorem would say that the loop is stable.

For the case where the plant is properly modeled, the stability result holds despite the fact that the feedback portion of the loop may have large gain.

Example: Assume r(t) and e(t) are sinusoid of the same frequency.

CASE 2

$k_y^*$ known, r(t) sinsoidal
Linear, periodic A(t)

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \\ \ddot{e} \\ \tilde{k}_r \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -(a0-k_y^*) & -a1 & -a2 & br(t) \\ -g_r r(t) & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \\ \ddot{e} \\ \tilde{k}_r \end{bmatrix}$$

$x(t) = P(t)e^{Kt}x(0)$

$P(t) = P(t+T)$

$a0-k_y^* = 2;\ a1 = 5;\ a2 = 4;\ b = V_r = 1$

$r(t) = \sin w_0 t$

$w_0 = 2.3$ $\quad \arg P^*(2.3) = 180°$ $\quad \lambda_{max}(e^{kt}) = 1.08$

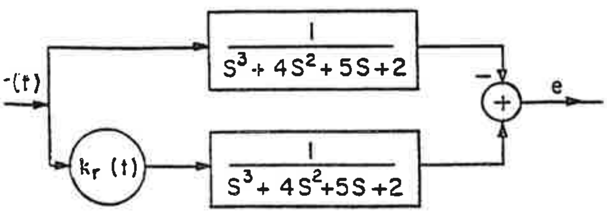
$w_0 = .71$ $\quad \arg P^*(.71) = 90°^+$ $\quad \lambda_{max}(e^{kt}) = 1.007$

$w_0 = .70$ $\quad \arg P^*(.70) = 90°^-$ $\quad \lambda_{max}(e^{kt}) = .984$

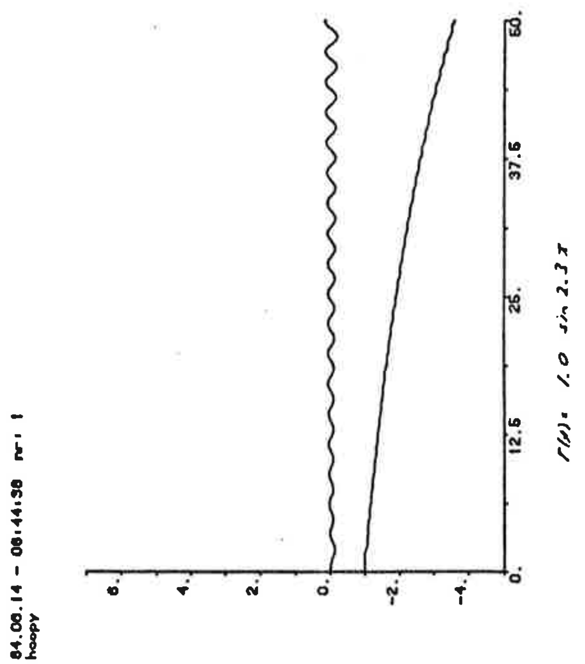
Note that this instability is independent of the reference model used to form the error.

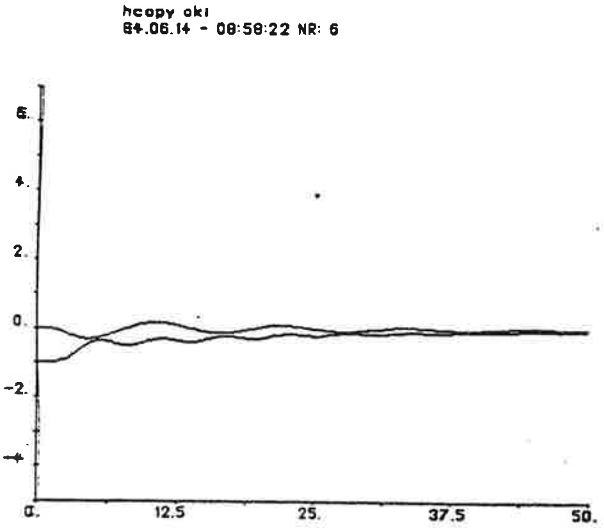
The instability cannot be explained by any inability of the adaptive system to match the model with a well behaved nominal system.

For example, Case 2 could have arisen from the following system.

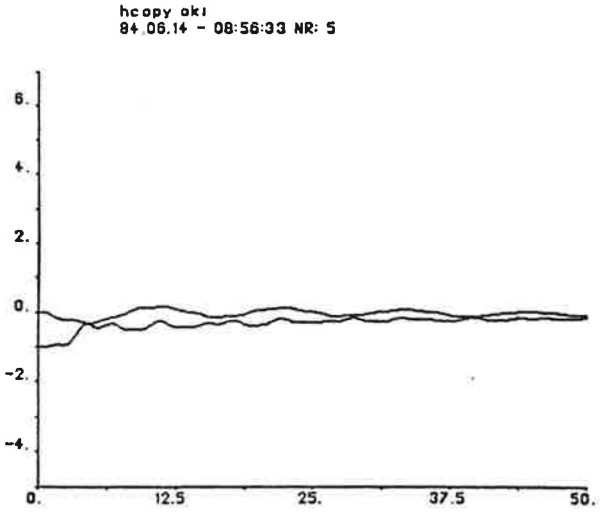



We know from Parks that creating a positive real operator is <u>sufficient</u> for stability. These results indicate that the positive real condition is <u>necessary.</u>

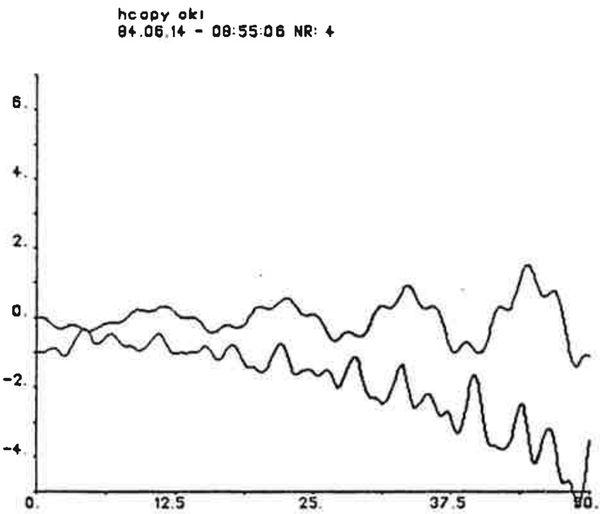





r = 1.0 sin .555t




r = 1.0 sin .555t + 1.0 sin 2.3t




r = 1.0 sin .555t + 1.0 sin 2.3t

Assumptions needed in stability proofs of adaptive algorithms:

1. The sign of $g_p$ is known
2. The zeroes of B(s) are in the left half plane
3. The relative degree of the plant is known exactly
4. The controller uses $2n^*$ parameters where $n^*$ is an upper bound on the plant order.

*<u>THESE ASSUMPTIONS CAN NOT BE MET IN PRACTICAL SITUATIONS</u>*

**INSTABILITY** can result in the presence of unmodeled dynamics due to an infinite gain operator in the feedback loop.

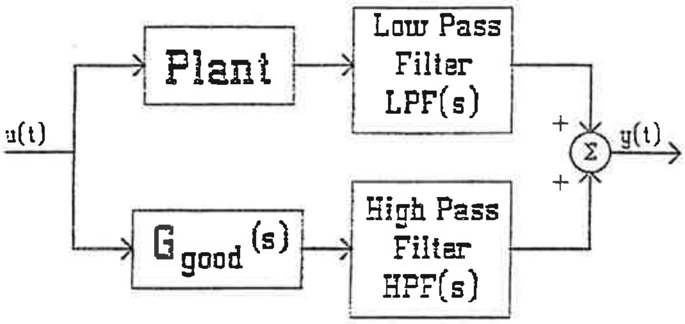
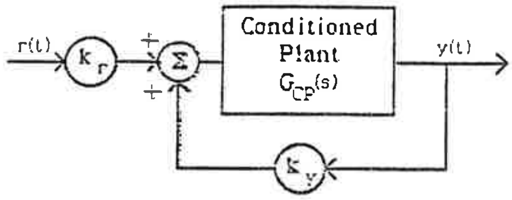



Figure 1. The conditioned plant.

At low frequencies the plant response dominates.

At high frequencies the response of $G_{good}$ dominates.

Frequency selective adaptive control is the result.




The nominally controlled plant.

$$\lim_{s\to\infty} G_{CP}(s) = \lim_{s\to\infty} G_{good}(s) = \frac{g}{s^{n-m}}$$

g is the high frequency gain of $G_{good}(s)$ and $G_{CP}(s)$

n-m is the relative degree of $G_{good}(s)$ and $G_{CP}(s)$

Assumptions 1 and 3 will be met independent of the unmodeled dynamics in the plant

If the right half plane zeroes of the plant are high frequency in nature they will not appear in $G_{CP}$. Assumption 2 will be met for a large class of plants.

## CONCLUSION

Using proper conditioning of the type described here, adaptive control can be made *robust* for any frequency reference input.

## CHALLENGES

1. The inputs used here were exactly sufficiently exciting. What are the problems with under excitation and over excitation?
2. The problems with disturbances are not eliminated by conditioning.

*Simulations*

Model $\frac{3}{s+3}$

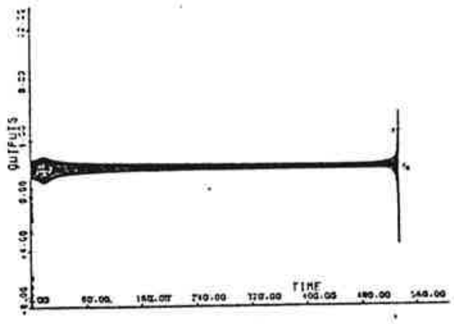
Plant $\frac{2}{s+1} \cdot \frac{229}{s^2+30s+229}$

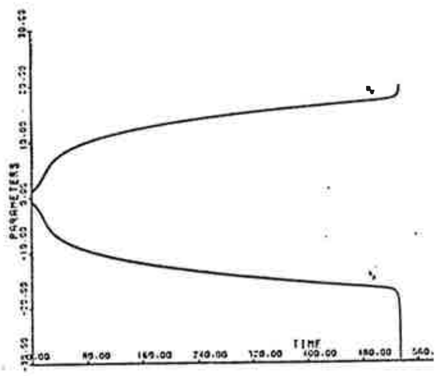
Mechanism I

$r(t) = 2.0 \quad d(t) = 0.5 \sin 16.1t$

Mechanism II

$r(t) = 2.0 \quad d(t) = 0.5 \sin 8t$


OUTPUTS
TIME


PARAMETERS
TIME

# Distributed asynchronous algorithms for deterministic and stochastic optimization

Dimitri P. Bertsekas

Laboratory for Information and Decision Systems
Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

There is presently a great deal of interest in distributed implementations of various iterative algorithms whereby the computational load is shared by several processors while coordination is maintained by information exchange via communication links. In most of the work done in this area the starting point is some iterative algorithm which is guaranteed to converge to the correct solution under the usual circumstances of centralized computation in a single processor. The computational load of the typical iteration is then divided in some way between the available processors, and it is assumed that the processors exchange all necessary information regarding the outcomes of the current iteration can begin.

The mode of operation described above may be termed synchronous in the sense that each processor must complete its assigned portion of an iteration and communicate the results to every other processor before a new iteration can begin. This assumption certainly enhances the orderly operation of the algorithm and greatly simplifies the convergence analysis. On the other hand synchronous distributed algorithms also have some obvious implementation disadvantages such as the need for an algorithm initiation and iteration synchronization protocol. Furthermore the speed of computation is limited to that of the slowest processor. It is thus interesting to consider algorithms that can tolerate a more flexible ordering of computation and communication between processors. Such algorithms have so far found applications in computer communication networks, e.g., ARPANET and other networks designed like it where processor failures are common and it is quite complicated to maintain synchronization between the nodes of the entire network as they execute real-time network functions such as the routing algorithm.

Processor network environments for which weakly coordinated distributed computation seems particularly advantageous typically possess one or more of the following characteristics all of which involve occurrence of some type of unpredictable event.

1/ Computation nodes and communication links are subject to frequent and/or unexpected failures. (For example packet radio networks).

2/ Computation nodes have different and/or time varying speeds of execution. (For example each processor is assigned to a perhaps time varying number of tasks involving computation loads which are not fixed a priori).

3/ Computations at various nodes is event driven. (For example in data collection or sensor networks where the timing, and ordering of measurements may not be predictable.).

It is possible to consider various degrees of coordination in different types of distributed algorithms. An interesting question is to determine the minimum degree of coordination needed in a given algorithm in order to obtain the correct solution. To this end we consider an extreme model of asynchronous distributed algorithms where by computation and communication are performed at each processor completely independently of the progress in the other processors. It is perhaps surprising that even under these chaotic circumstances it is still possible to solve correctly important classes of problems. An account of progress made in this direction is given in a survey jointly written with J. Tsitsiklis and M. Athans (1983). An analysis is given in (Bertsekas, 1982) for broad classes of dynamic programming problems and in (Bertsekas, 1983) for more general fixed point problems involving contractions and monotonicity assumptions. Further related work is (Tsitsiklis, Bertsekas, and Athans, 1983), and (Tsitsiklis, 1983).

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# Expert control

K. J. Åström

Department of Automatic Control
Lund Institute of Technology
Lund, Sweden.

There has been substantial progress in theory and practice of automatic control through application of mathematical analysis and numerics. Nonnumerical data processing has, however, so far only had marginal influence on control systems. The purpose of this paper is to identify possible uses of expert system techniques in implementation of control systems. It is first observed that actual implementation of control laws often involves a substantial amount of heuristic logic. This is true for simple regulators as well as for more sophisticated multivariable control loops. The paper shows that the heuristic logic may be replaced by an expert system. This leads to simplifications in implementation as well as new capabilities in the control system. Selected basic elements of an expert system are presented. Stochastic dynamic programming offers a framework in which the heuristics can be embedded. This points to requirements for a new artificial intelligence approach for heuristic planning under uncertainty. The ideas are illustrated by examples: a smart PID regulator, a self-tuner with safety jackets and a pole-placement adaptive regulator which can by itself determine suitable pole locations. Once the expert system approach is taken it is possible to obtain control systems with new functions. This is illustrated by the smart PID regulator which incorporates learning.

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## EXPERT CONTROL

[> 1. INTRODUCTION
2. CONTROL PRACTICE
3. EXPERT SYSTEMS
4. EXPERIMENTS
5. CONCLUSIONS

## MOTIVATION

SAFE OPERATION OF STR
COMPUTER POWER
SYMBOLIC DATA PROCESSING

## PID CONTROL

manual automatic
parameter changes
switching transients
integral windup

## CONCLUSION I

Even simple regulators
combines algorithms and
heuristics

## ADAPTIVE CONTROL

LOCAL GRADIENT ALGORITHMS
PRIOR KNOWLEDGE
INITIALIZATION
SAFE-GUARDS

## CONCLUSION II

Adaptive controls
contain a lot of
heuristics

## EXPERT SYSTEM (ES)

Data base
Facts, Evidence,
Hypotheses, Goals
Supervisory strategy

## RULE BASED ES

If < > then < >
Forward chaining
Backward chaining
Why

## SUCCESS STORIES

Experts and data available
Problem Scope
Combinatorics
Incremental progress

$$Ay_t = Bu_{t-d} + Ce_t$$
$$Ru = -Sy$$
$$z^{d-1}CB = AR + BS$$
$$y_t = f_o e_t + \ldots + f_{d-1} e_{t-d+1}$$
$$F = R/B$$

## PRIOR KNOWLEDGE

delay
sampling period
regulator complexity
forgetting factor
initial estimates
bounds on control

## OPERATOR CLASSES

Main Monitor
Backup Control
Minimum variance control
Estimation
Tuning
Learning

Main monitor:
- stability-supervisor
- control-quality-supervisor

Back-up control:
- pid-control
- auto-tune

Fixed gain MV control:
- minimum-variance-control
- minimum-variance-supervisor
  - ringing-detector
  - degreesupervisor

Estimation:
- parameter-estimation
  - excitation-supervisor
  - perturbation-signal-generation
  - jump-detector

Self-tuning:
- self-tuning-regulation

Learning:
- get-regulator-parameters
- put-regulator-parameters
- store-regulator-parameters
- test-scheduling-hypothesis
- smooth-table-entries

Main monitoring table

| # | Time | u | $\sigma_u$ | y | $\sigma_y$ | Stable | Reg. type |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

Backup control Table.

| # | Time | $k_c$ | $t_c$ | P | I | D |
|---|---|---|---|---|---|---|
| | | | | | | |

Minimum variance control table

| # | Time | $n_R$ | n | d | h | Parameters |
|---|---|---|---|---|---|---|
| | | | | | | |

## WHY USE ES?

Simple coding
Separate algorithms and logic

## ALGORITHM ORCHESTRATION

Control Algorithms
Diagnosis Algorithms
Logic and sequencing
Tables for learning

## IMPLEMENTATION

Vax 11/780
ES in Lisp
Algorithms in Pascal
Concurrency

## EXPERIMENTS

- o SMART PID
- o INTELLIGENT STR
- o AUTOMATIC $\omega_B$ CHOICE

## JUDGEMENT

Ideas probably much more important for large complex systems. But let us do simple things first.

## CONCLUSIONS

- o New control laws
- o Many algorithms
- o Control and diagnosis
- o Learning

# Experiments with expert control

Karl-Erik Årzén

Department of Automatic Control
Lund Institute of Technology
Lund, Sweden

Abstract:

Expert control refers to a control system where an expert system is used to orchestrate a collection of control and identification algorithms. This is done in real time. Adaptive control algorithms need large safety jackets of logic to work in practice. Existing adaptive algorithms perform well locally but require a priori information about time delay, system order etc.to do so. Simple algorithms exist that can provide some of this information. An expert system is well suited for implementation of logic. Heuristics and rules of thumb are also easily implemented.

A testbench for experiments is presented. The controller is divided in two parts. One algorithm library written in Pascal and one expert system written in OPS4 and Lisp. These parts are implemented as communicating concurrent processes. The communication is done with mailboxes and messages. A typical message is to start or stop an algorithm. OPS4 is a rule based, forward chaining expert system framework.

An experiment is presented where the level of a water tank is controlled by a PID controller with Ziegler-Nichols auto tuning and gain scheduling.

Reference:

Åström K. J. and Anton J. J.: Expert Control, Proceedings IFAC 9th World Congress, Budapest, Hungary. 1984.

# EXPERIMENTS WITH EXPERT CONTROL.

- Introduction
- Implementation
- Experiments
- Demonstration

## MOTIVATION

- "Safety Jacket"
- Many algorithms
- Heuristics & rules of thum
- Make full use of à priori information
- Division : Logic ↔ Algorithm
- Workbench

## Programming Languages

Symbolic Data Processing, AI-tradition ⇒ Lisp, Prolog

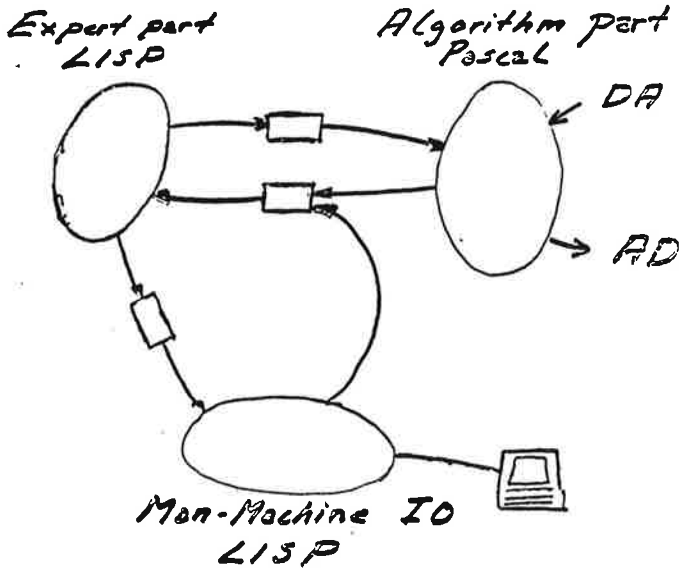
Numerical algorithms ⇒ Pascal, Fortran, ADA

## Concurrency

Expert System in real time.

## IMPLEMENTATION

- VAX 11/780 , VMS
- Pascal , Franz Lisp



## Algorithm part

- Library of control, identification and supervision algorithms with a few well specifed operations upon.

- Control loop

```
while true do
  begin
  if mail_in_box then readmail;
  for all algorithms do
     if algorithm_is_active then
         execute_algorithm;
 wait;
end;
```

## Expert System Part

- Existing Expert System Frame work

OPS 4

Consists of

Working Memory
Production -"-
Control Structure

Forward Chaining
Rule format: Condition ... --> Action ...

Recognize-act cycle

## EXPERIMENTS

1) "Smart" PID that can decide whether
   - The process can be controlled by PI
   - -"- PID
   - The process needs a more complex controller

2) STR that determines the prediction horizon automatically

## DEMONSTRATION

PID with Ziegler-Nichols autotuning and gain scheduling

Algorithms

PID
Relay
Relayguard
Noiseestimator

≈ 50-60 rules.

## Notes from the discussion

A short summary of the discussion on Wednesday afternoon is given here. The conclusions given below should not be taken as declarations everyone agreed on, but rather as a couple of interesting statements brought up during the discussion. Some statements were accepted by most participants, other maybe by very few.

### I. Process control versus aircraft control

Process control and aircraft control are two totally different issues. In aircraft control, lots of time, money and work are spent to obtain a very high performance. In process control, the control work must often be both fast and cheap, and high performance is often not so important. Stability is often enough. Furthermore, these systems have different types of dynamics. These are some differences which has influenced the success of adaptive control in the process industry, and the luck of success in the aircraft control.

### II. Parametrization

Current parametrizations used in adaptive control are not good - we are probably using them only because we know how to solve the identification problem for these parametrizations.

Approximations are best done on an input-output basis, in the frequency domain. Unstructured inaccuracy would e.g. be given as nominal phase and amplitude curves ± ranges. State-space parametrization is not good for approximation. Therefore, the state-space realization part should be left to the end of the design procedure.

State-space is excellent for rigid body dynamics.

You cannot have an adaptive theory unless you have a feedback theory that deals with plant uncertainty. Feedback reduces tolerance bound.

### III. Is identification essential to adaptive control?

Identification is essential to adaptive systems. You have to obtain knowledge of the system from measurements.

From a theoretical point of view, identifiability may not be essential, but for robustness it may. Otherwise we do not catch the whole dynamics. As long as you get the I/O-map, this is enough. The number of parameters may lead to numerical problems. Even if the I/O map is not changing, the internal parameters may change. This is a numerical problem.

### IV. How to take care of apriori knowledge

Prefiltering may be one way to include apriori knowledge to decrease the number of parameters.

We would like to remove as many critical parameters, such as nonminimum phase and time delay, as possible. The relay method can solve the time delay problem, at least for systems with monotone step responses.

A lot of tricks are currently added to take care of non-typical situations. It should be useful to get insight from theory why we need those tricks. Some problems are related to robustness.