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REFINEMENTS OF THE ZIEGLER-NICHOLS TUNING FORMULA FOR PID AUTO-TUNERS

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Abstract

The accuracy of the Ziegler-Nichols tuning formula is reviewed in the context of PID auto-tuning. It is found that in the case of excessive overshoot in the set-point response, the use of set-point weighting in the proportional term is a superior alternative to the conventional solution of either gain detuning or set-point filtering. In the case of excessive undershoot the tuning formula will need to be modified. The decision to apply set-point weighting and the modification of the tuning formula, as well as the automatic setting of associated parameters can be based simply on the knowledge of normalized dead time. These heuristic refinements to reduce or eliminate manual fine-tuning are substantiated by means of simulation.

1. INTRODUCTION

The Ziegler-Nichols ultimate-cycle tuning procedure (Ziegler and Nichols 1943, Deshpande and Ash 1981) has been widely known as a fairly accurate method to determine good settings of PID controllers for a large range of common industrial processes. However, the procedure is not often applied as it is laborious and time-consuming particularly for a process with large time constants. It also requires close attention of the instrument engineer and the operator as the process has to be operated near instability to measure the ultimate gain and period needed in the tuning formula.

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The advent of microprocessor technology and recent advances in adaptive control strategies have spurred much interests in new methods of automatic controller tuning without the need to operate the process near instability. Several commercial products for automatic tuning or adaptive control (Hoopes, Hawk and Lewis 1983, Krauss and Mayron 1984, Åström and Hägglund 1984, Higham 1985, Hess, Radke and Schumann 1987, Radke and Isermann 1987) have been available since 1981. One simple method of auto-tuning is based on the automatic measurement of the ultimate gain and period, from which the optimal PID controller parameters can be computed using the Ziegler-Nichols tuning formula. The proposed methods of automatic measurement without closed-loop cycling vary from the simple relay-feedback auto-tuning (Åström 1982, Åström and Hägglund 1984), approximate system identification (Hang, Lee and Tay 1985) to correlation technique (Hang, Lim and Soon 1986).

The accuracy of the Ziegler-Nichols tuning formula has been found to be quite adequate when used in the manual procedure as it can be supplemented by fine-tuning based on experience. With the automation of the controller tuning procedure, it becomes attractive to explore the possibility of modifying or augmenting the tuning formula by incorporating heuristic knowledge so that the need for manual fine-tuning by an expert can also be reduced or eliminated. Such a study is presented in this report. It is found that the introduction of set-point weighting (Hägglund and Åström 1985) in the proportional term of the PID control algorithm provides a superior alternative to fine-tuning when the overshoot in set-point response is excessive. The automatic setting of this new weighting factor will be investigated. An extensive simulation study has also indicated that a simple means of developing heuristic rules for the purposes of assessing the controller performance and modifying the controller settings is to characterize process dynamics by the normalized dead time.

The report is organised as follows: A review of the accuracy of the Ziegler-Nichols tuning formula is given in Section 2. The role of fine-tuning and the use of set-point weighting will also be discussed in this section. Process characterization by means of the normalized dead time is then explored in Section 3. The automatic setting of the set-point weighting factor in the case of small dead time is discussed in Section 4 and the modification of the tuning formula in the case of large dead time is discussed in Section 5. Some concluding remarks are given in Section 6.

2. ACCURACY OF THE ZIEGLER-NICHOLS FORMULA

The Ziegler-Nichols tuning formula (Ziegler and Nichols 1943, Deshpande and Ash 1981) is based on the empirical knowledge of ultimate gain k_u and ultimate period t_u as follows:

$$\text{Proportional gain} \quad k_c = 0.6\ k_u \tag{1}$$

$$\text{Integral time} \quad T_i = 0.5\ t_u \tag{2}$$

$$\text{Derivative time} \quad T_d = 0.125\ t_u \tag{3}$$

The PID controller is usually implemented in the following form to avoid 'derivative kick' and to provide noise filtering (Åström and Wittenmark 1984):

$$u_c = k_c \left(e + \frac{1}{T_i} \int e\ dt - T_d \frac{dy_f}{dt} \right) \tag{4}$$

where

$$e = y_r - y \tag{5}$$

$$y_f = \frac{1}{1 + s\ T_d / N}\ y \tag{6}$$

and u_c , y, y_r are the controller output, process output and setpoint respectively. The noise filtering constant N is usually set in the range of 3 to 10. Without loss of generality, N = 10 is used throughout this study. For simplicity the continuous-time formulation will be used as the discretization for digital implementations is straight forward (Deshpande and Ash 1981). The process is also assumed to be linear for the sake of generality; otherwise specific knowledge of the nonlinearity will be required to make additional modifications of the tuning formula.

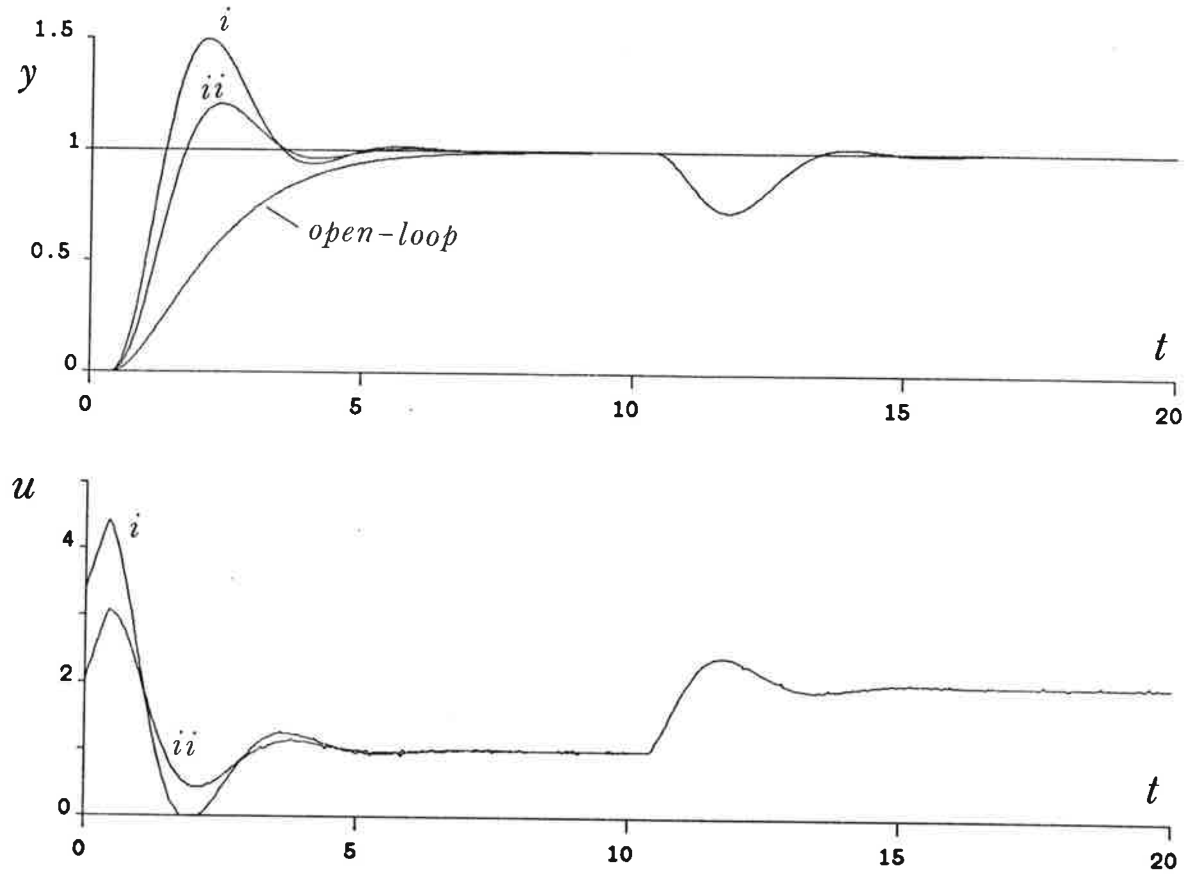

Fig. 1 Excessive overshoot in the set-point response (d = 0.4)
(i) Ziegler-Nichols tuning (k_c = 3.42, T_i = 1.42, T_d = 0.36);
(ii) additional set-point weighting with b = 0.6.

In order to illustrate the accuracy issue of the Ziegler-Nichols tuning formula and to explore appropriate solutions, the performance of the following process tuned by using equations (1)-(3) is first analysed:

$$\text{Process I :} \quad \frac{Y(s)}{U_c(s)} = \frac{e^{-sd}}{(1 + s)^2} \qquad (7)$$

It is found that the performance of the tuned control system varies significantly as the dead time varies. When the dead time d is small, both the set-point and load responses are well tuned in terms of speed of response and damping, as shown in the case of d = 0.4 in Fig. 1. The overshoot in the set-point response is however excessive and it often needs to be reduced to around 20% or 10% depending on applications. The simple solution of de-tuning the gain is not recommended as it will reduce the speed of response in both the set-point and load responses. An acceptable alternative is to filter the set point with a time constant t_f equal to a fraction of the integral time to achieve a reasonable compromise between overshoot and rise time. The merit of this solution is that the load response will not be affected as the PID controller settings are not changed. A recent study on dominant pole designs (Hägglund and Åström 1985) has proposed a new method of reducing overshoot by means of set-point weighting. In this case the controller of equation (4) is modified to include a constant weighting factor 'b' on the set point in the proportional term:

$$u_c = k_c \left[(b\, y_r - y) + \frac{1}{T_i} \int e\, dt - T_d \frac{dy_f}{dt} \right] \qquad (8)$$

It has the same merit that the load response will not be affected. The introduction of 'b' provides a means of adjusting the zero location in the closed-loop process transfer function which in turn affects the overshoot (Franklin and Powell 1980). A heuristic explanation is that the high forward loop gain allowed in the case of small dead time causes the large overshoot; hence a reduction in the initial control signal by means of set-point weighting with $b < 1$ in the proportional term can be effective in a similar sense to the function of feedforward control. This is illustrated in the example shown in Fig. 1. It has been found from extensive simulation studies of the process of equation (7) and other common process models that the use of set-point weighting is a superior solution to set-point filtering in that the speed of response is much less sacrificed for the same reduction in overshoot. The example in Fig. 2(a) shows the effect clearly. The solution by de-tuning through gain reduction is the worst alternative as illustrated in Fig. 2(b). In summary, there is no need to modify the Ziegler-Nichols tuning formula in the case of small dead time as the associated large overshoot can be overcome by the application of set-point weighting. The auto-tuning of the additional weighting factor will be studied in Section 4.

The excessive overshoot in the set-point response discussed above becomes less significant as the dead time increases and the loop gain reduces correspondingly.

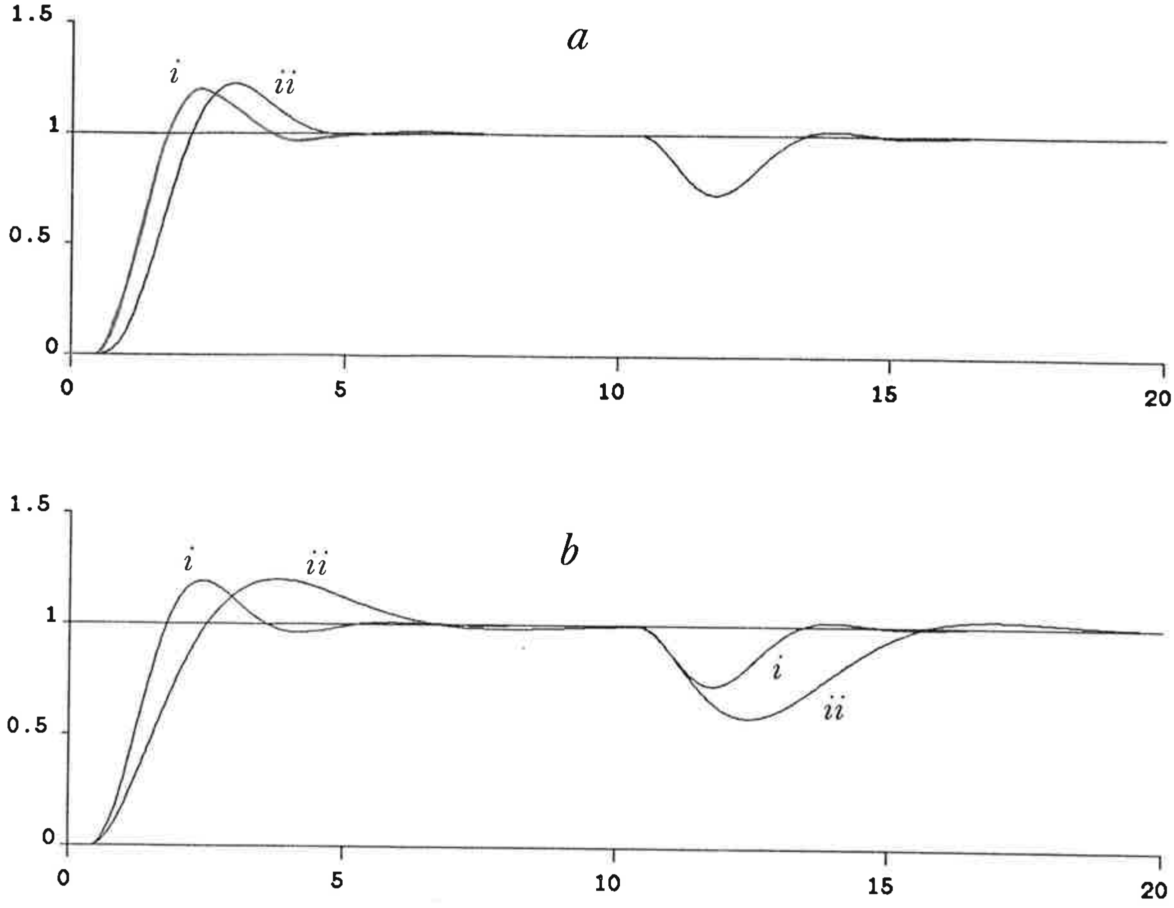

Fig. 2 Comparison of methods for reducing overshoot (d = 0.4)
a) b = 0.6 (i) vs set-point filtering with t_f = 0.8 (ii);
b) b = 0.6 (i) vs detuning gain with k_c = 1.3 (ii).

It has been found in practice and in simulations that a new problem of excessive undershoot would occur in the later part of the transient response when the dead time is large. In an application where very high performance is required, such as a quality control loop, dead-time compensation using a Smith predictor (Deshpande and Ash 1981) or other advanced control technique would be necessary. In less demanding applications, or when the dynamic model of the process is poorly known, PID control may be used and it becomes useful to resolve the excessive undershoot problem. One possible solution is to apply set-point weighting with a 'b' value of more than 1. However this has been found to be effective only if the undershoot is moderate. When the dead time is large and the undershoot is serious, an example being shown in Fig. 3 with d = 2.5, the load response is also poor. Fine-tuning experience has indicated that the problem can be overcome by increasing the integral action, that is by reducing the integral

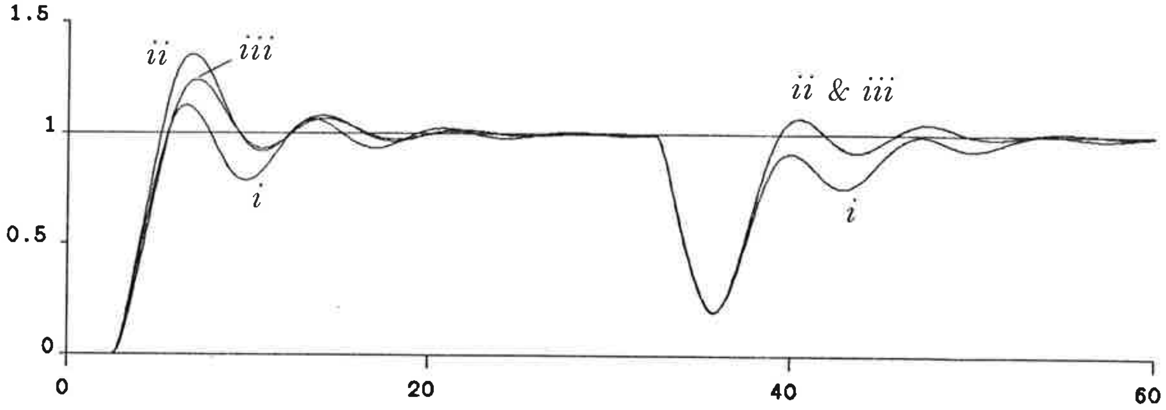

Fig. 3 Excessive undershoot in the set-point response and poor load response: i) Z-N tuning (k_c = 0.92, T_i = 4.17, T_d = 1.04); ii) improvement using T_i = 2.79; iii) T_i = 2.79 and b = 0.8.

time. The large improvement in both the set-point and load responses is shown in Fig. 3. The trade-off then may be a larger overshoot which can be compensated as shown by the use of set-point weighting. The necessary modification of the tuning formula so that manual fine-tuning may be avoided is studied in Section 5.

3. PROCESS CHARACTERIZATION

The recent interests in artificial intelligence (Kraus and Mayron 1984, Higham 1985) and expert control (Åström, Anton and Årzen 1986) have motivated the study of heuristics used by expert human operators and designers. One of such heuristics is the broad characterization of process dynamics by means of the normalised dead time (Deshpande and Ash 1981, Åström and Hang 1987), from which an order-of-magnitude prediction of the achievable controller performance can be made. An extensive simulation study has shown that this process characterization factor is strongly correlated with the accuracy issues of the tuning formula discussed in the previous section. The main findings which form the basis of the proposed refinements in the tuning formula in later sections, are summarised in the following.

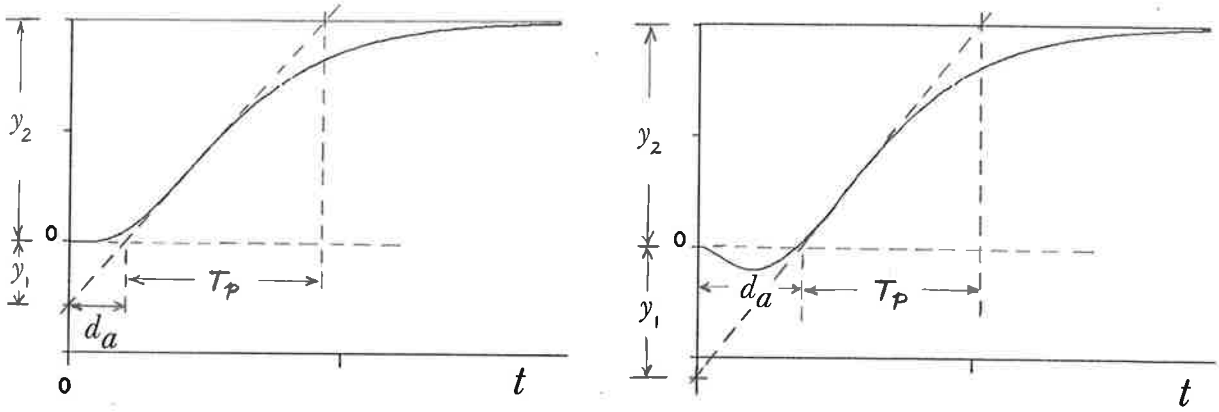

Fig. 4 Step responses of a) a stable and damped process and b) a stable and nonminimum-phase process ($d_n = d_a/T_p = y_1/y_2$)

The normalised dead time, d_n, is defined as the ratio of the dead time or apparent dead time d_a and the major time constant T_p of the open-loop step response of the process. It can be simply determined using a pre-tuning pulse test (Ziegler and Nichols 1943, Kraus and Mayron 1984), as shown in Fig. 4(a) which is well defined for a stable and damped process. The same definition can also be used to characterize a process with inverse dynamics caused by nonminimum-phase zeros, as shown in Fig. 4(b). The physical pulse testing to determine d_n is not needed in the case of the relay-feedback auto-tuner if the available measurements are further employed to estimate a low-order plus dead-time process model (Hägglund and Åström 1987). It can also be avoided in the case of the correlation-based auto-tuner as the impulse response estimates are automatically generated by the correlator from which the step response can be readily computed (Hang, Lim and Soon 1986).

Besides the process given by equation (7), a multiple-lag process and a nonminimum-phase process, which exhibit quite different dynamics, are also used to examine the validity of the correlation results:

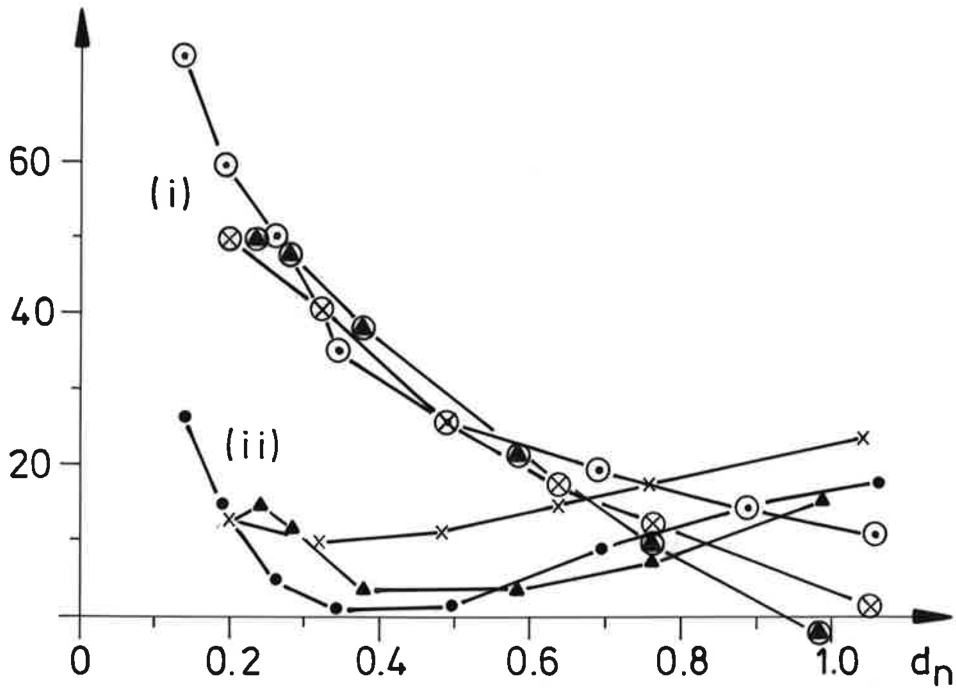

Fig. 5 Characterization of dynamic responses by the normalized dead time: (i) % overshoot; (ii) % undershoot. (... process I ; xxx process II ; ▲▲▲ process III)

$$\text{Process II :} \quad \frac{Y(s)}{U_c(s)} = \frac{1}{(1+s)^n} \tag{9}$$

$$\text{Process III :} \quad \frac{Y(s)}{U_c(s)} = \frac{1 - \alpha s}{(1+s)^3} \tag{10}$$

For convenience the three processes of equations (7), (9) and (10) will be referred to as process I, II and III respectively. In process I, d_n will be varied by changing the dead time d; in process II, d_n will be varied by changing the order n; in process III, d_n will be varied by changing the numerator coefficient α of the nonminimum-phase zero.

The correlation results which provide a fairly good quantitative assessment of the accuracy issue discussed in the previous section are shown in Fig. 5. When the normalized dead time is small, in the range of $d_n < 0.3$, the overshoot is about four times the undershoot, achieving quarter damping suitable for most

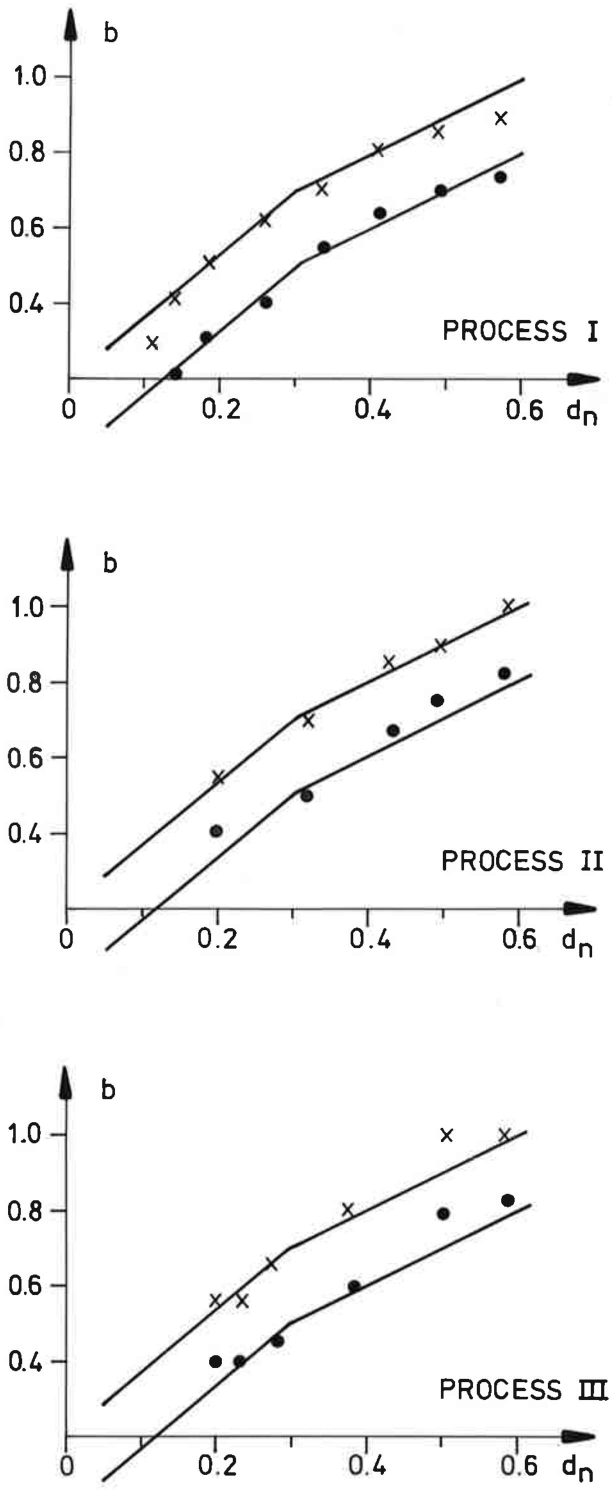

Fig. 6 Optimal set-point weighting factors
(criteria: xxx 20% overshoot ; ... 10% overshoot)

applications (Ziegler and Nichols 1943). The overshoot however is clearly excessive. In the next range of $0.3 < d_n < 0.6$, the quarter damping ratio reduces but the overshoot remains higher than 20%. For $d_n > 0.6$, the undershoot increases and quickly exceeds the overshoot.

Based on the above empirical observations, the following heuristic criterion for refining the tuning formula is recommended : when $d_n < 0.6$, retain the Ziegler-Nichols tuning formula of equations (1)-(3) and apply set-point weighting in the proportional term as in equation (8) to reduce the excessive overshoot; when $d_n > 0.6$, modify the Ziegler-Nichols tuning by a suitable reduction of the integral time to improve both the set-point and load responses. The auto-tuning of the new factors introduced in the above criterion, namly the set-point weighting factor and the reduced integral time will be investigated subsequently.

4. HOW TO CHOOSE THE SET-POINT WEIGHTING FACTOR

In this section we shall explore if a simple correlation formula can be found to choose the required set-point weighting factor 'b' automatically to achieve an acceptable overshoot in the set-point response. Depending on applications, a 10% or 20% overshoot may be a suitable criterion. An extensive simulation study has been carried out to determine the required 'b' factor for processes I, II and III as the normalized dead time d_n changes. The results are summarised in Fig. 6.

As discussed in the previous section, set-point weighting may be applied when $d_n < 0.6$. In this range, the following empirical formula for selecting the 'b' weighting factor has been found to give a close approximation of the experimental results:

$$\begin{aligned} b &= 2x + d_n && \text{for } 0.3 < d_n < 0.6 \\ &= 2(x - 0.1) + \frac{5}{3} d_n && \text{for } d_n < 0.3 \end{aligned} \qquad (11)$$

The available x in the formula is the acceptable % overshoot. The values of b computed from the above formula are shown as solid lines in Fig. 6. An example tuned with this formula is shown in Fig. 7 for process II with n = 3 and a corresponding d_n of 0.2. An example for process III is shown in Fig. 8 with α = 0.5 and a corresponding d_n of 0.38. Another different process given by the following transfer function, which provides a better opportunity to check the

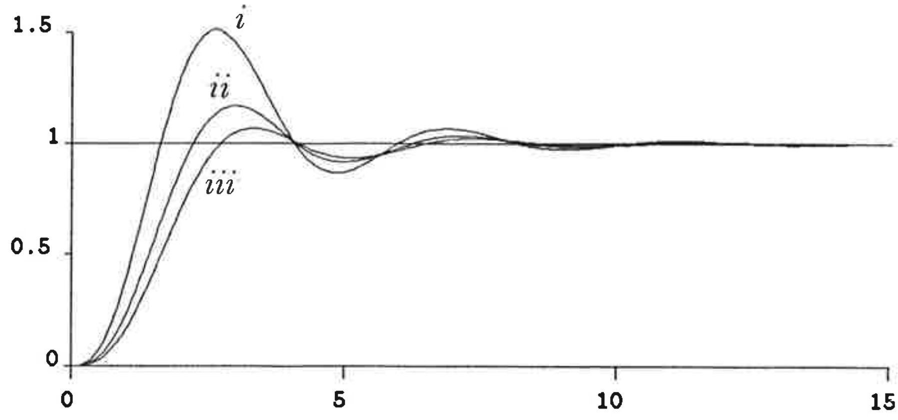

Fig. 7 Performance of formula (11) for process II : (i) b = 1;
(ii) 20% criterion: b = 0.53; (iii) 10% criterion: b = 0.33.

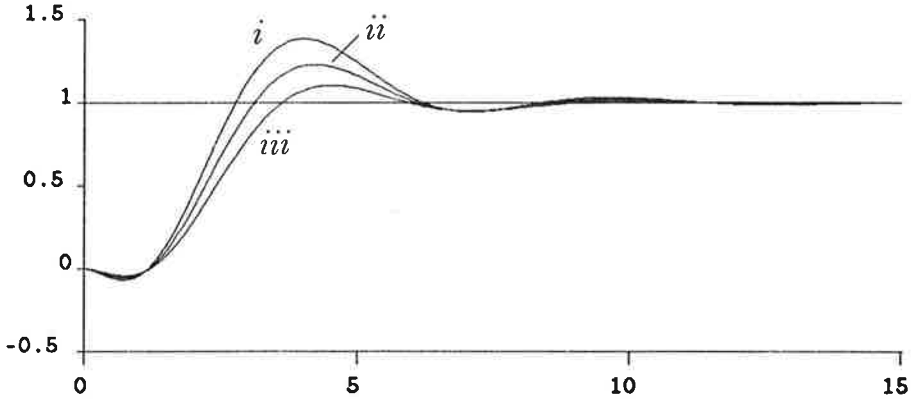

Fig. 8 Performance of formula (11) for process III : (i) b = 1;
(ii) 20% criterion: b = 0.78; (iii) 10% criterion: b = 0.58

accuracy of the formula at small values of d_n, has also been tested and some typical results are shown in Fig. 9:

$$\frac{Y(s)}{U_c(s)} = \frac{1}{(1 + s)(1 + \gamma s)(1 + \gamma^2 s)(1 + \gamma^3 s)} \qquad (12)$$

Notice that when d_n is small, the application of set-point weighting also results in the reduction of undershoots hence achieving faster settling times, in addition to

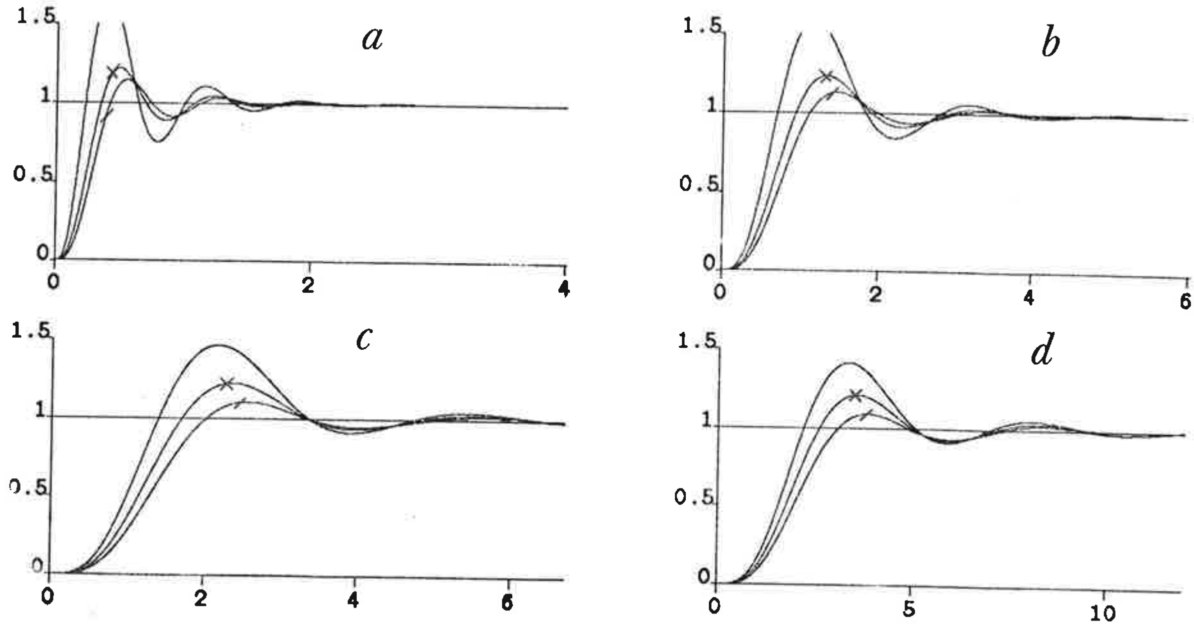

Fig. 9 Performance of formula (11) when applied to the process of eqn. (12): a) $\gamma = 0.2$, $d_n = 0.1$; b) $\gamma = 0.4$, $d_n = 0.19$; c) $\gamma = 0.6$; $d_n = 0.26$; d) $\gamma = 0.8$, $d_n = 0.31$.
(criteria: xxx 20%; /// 10%)

the satisfatory reduction of the vastly excessive overshoots. The simple formula of equation (11) is found to be accurate enough for most applications. The effectiveness of set-point weighting may reach its limit when d_n falls below 0.15, which is acceptable as this range of d_n signifies a low-order process where powerful classical or modern analytical tools may be more appropriate than heuristics for a precise control system design. The ultimate loop gain will also become so high that P or PI control may already be adequate even for tight control and hence PID control may not be required (Åström and Hang 1987). Otherwise it is straight forward to use a combination of set-point weighting and set-point filtering to satisfy any specific overshoot criterion.

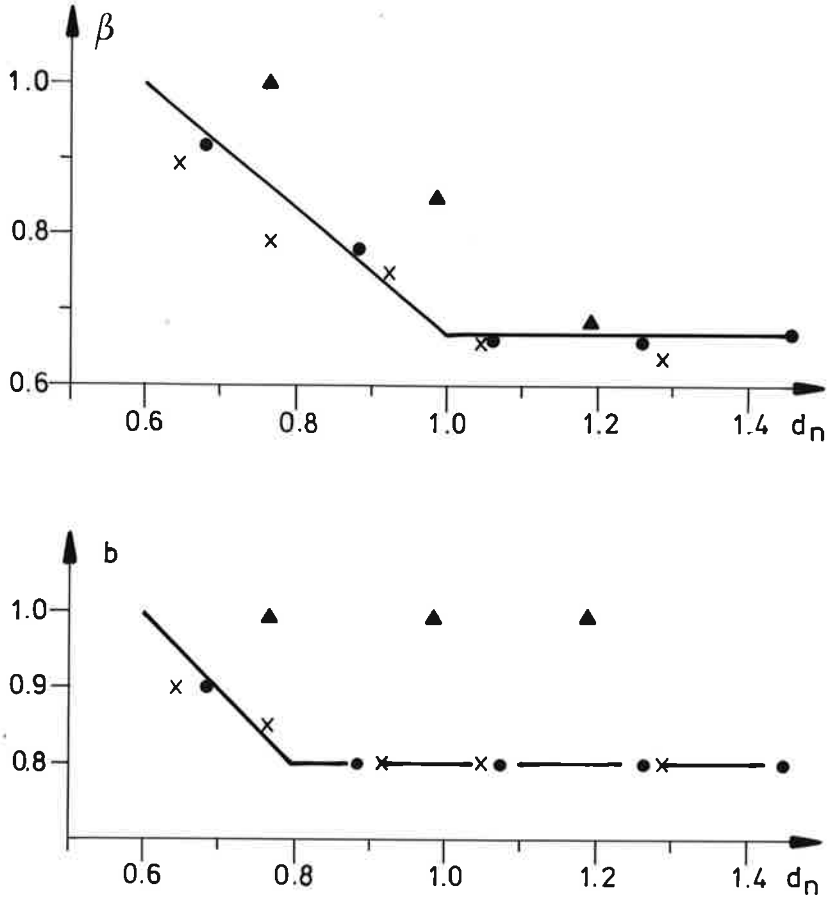

Fig. 10 Optimal values of β and b
(... process I; xxx process II; ▲▲▲ process III)

5. MODIFIED TUNING FORMULA

It has been established in Sections 2 and 3 that as $d_n > 0.6$, the integral term in the Ziegler-Nichols tuning formula needs to be modified in order to reduce the undershoot in the set-point response and to improve the load response. As in the case of set-point weighting, an extensive simulation of processes I, II and III is used to obtain the experimental correlation between the optimal integral time as a function of d_n.

It has been found that a suitable integral time can indeed be determined without the need to change the proportional gain and derivative time. However, the improvement may be accompanied by an excessive overshoot and hence the application of set-point weighting with $b < 1$ may be required. A modification factor β is now defined as the ratio of the optimized integral time and the

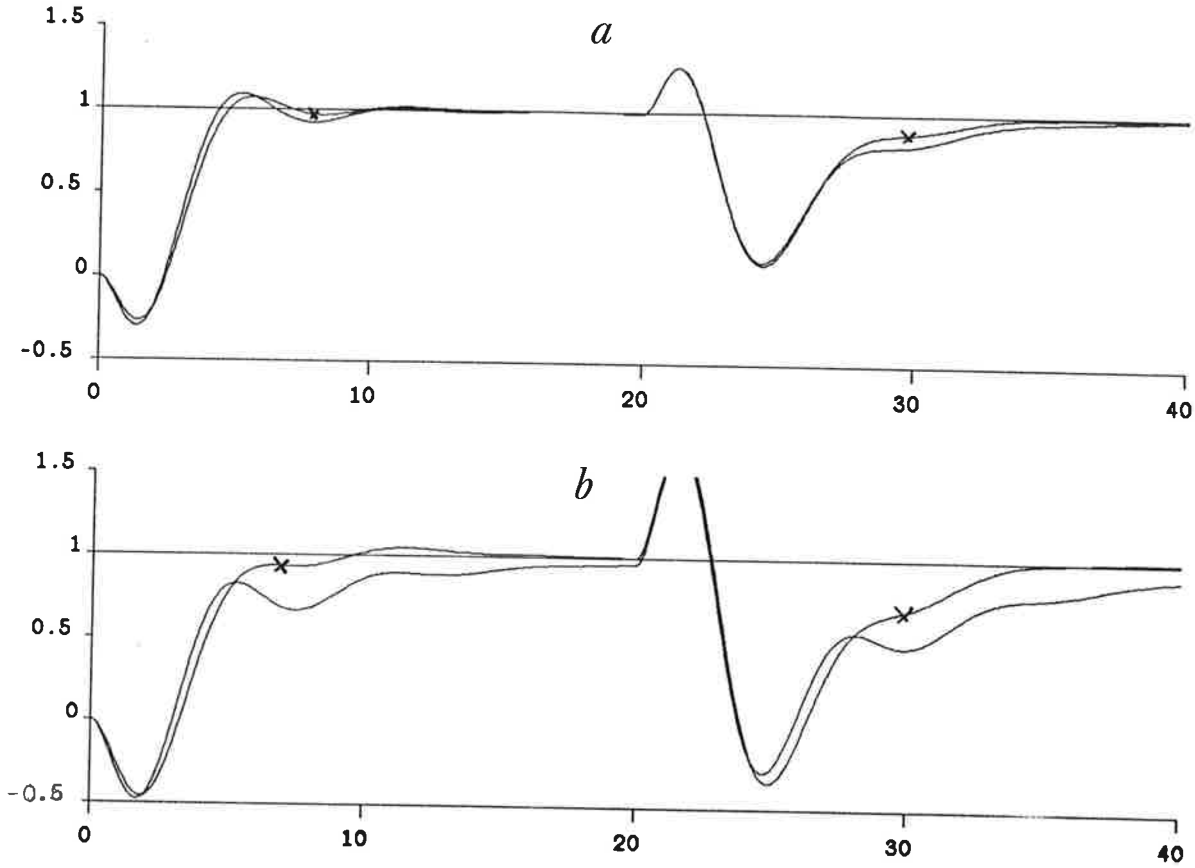

Fig. 11 Moderate accuracy of formulae (14) and (15) for process III
a) $\alpha = -1.5$, $\beta = 0.87$, $b = 0.83$; b) $\alpha = -2.5$. $\beta = 0.67$, $b = 0.8$.
(— ZN tuning; -*- tuning with new formulae)

Ziegler-Nichols integral time. The new tuning formula for the integral term is thus given by:

$$T_i = 0.5\, \beta\, t_u \qquad (13)$$

As the control performance in this region of d_n cannot be expected to be very tight, the optimal criterion for adjusting the integral time and the set-point weighting factor b has been set as an undershoot of 10% and an overshoot of 20%. The empirical results for all the three different processes are shown in Fig. 10. A simple regression exercise then produces the following formulae for computing β and b as a function of d_n:

$$\beta = 1.5 - 0.83\, d_n \qquad \text{for } 0.6 < d_n < 1.0$$

$$= 0.67 \qquad \text{for } d_n > 1.0 \qquad (14)$$

$$= 1 \qquad \text{for } d_n < 0.6$$

and

$$b = 1.6 - d_n \qquad \text{for } 0.6 < d_n < 0.8$$

$$= 0.8 \qquad \text{for } d_n > 0.8 \qquad (15)$$

An example of the accuracy of the above formulae has already been shown in Fig. 3. The computed values of β and b based on the formulae are plotted as the solid lines in Fig. 10. Notice that the results are not very accurate for process III. Fortunately the computed optimal settings are on the safe side, that is the results are better than the set criteria. This is demonstrated in the two extreme cases shown in Fig. 11.

6. CONCLUSIONS

The characterization of process dynamics using the normalized dead time d_n has been found to be an effective means of predicting the performance of the Ziegler-Nichols tuning formula. When $d_n < 0.6$, the overshoot in the set-point response will be excessive. The recommended solution is to apply set-point weighting based on the new formula of equation (11) while retaining the Ziegler-Nichols tuning formula for the PID parameters. When $d_n > 0.6$, the undershoot in the set-point response will become excessive and the load response will be sluggish. The recommended solution is then to modify the integral term of the Ziegler-Nichols tuning formula by a β factor given in equation (13) and formula (14) and using a corresponding set-point weighting factor given by formula (15).

With the proposed refinements of the tuning formulae, PID auto-tuning will be accurate over a much larger range of process dynamics. Consequently the need for manual fine-tuning and the associated human expertise will be largely eliminated. The main merit of the refinements is that it requires only a simple pulse test for the measurement of the normalized dead time. The pulse test can even be avoided in the case of the relay-feedback auto-tuner and the correlation-based auto-tuner as the normalized dead time can be deduced from available measurements. The computation of the tuning factors is also very simple. A more refined formula is generally not warranted as the proposed refinements are found to be effective for a wide range of process dynamics. The extensions of the methodology to the refinements of the Ziegler-Nichols formula for PI controllers and to other tuning formulae are quite straight forward.

It is hoped that the above development will encourage more frequent use of PID auto-tuners for problem loops that require regular re-tuning. When coupled with a suitable on-line parameter estimator, the PID controller may also be used as a robust explicit self-tuning controller. In this more demanding application, the estimated model parameters will be used to compute the ultimate gain, ultimate period and the normalized dead time; the refined tuning formulae will then become the updating rules for the controller parameters in contrast with the usual complex updating rules based on the solution of diophantine type of algebraic equations such as that in the self-tuning pole-placement controller. This new approach to self-tuning PID controller is under development as a module in an intelligent controller.

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