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An Improved Stochastic Send-on-Delta Scheme for Event-Based State Estimation

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Introduction

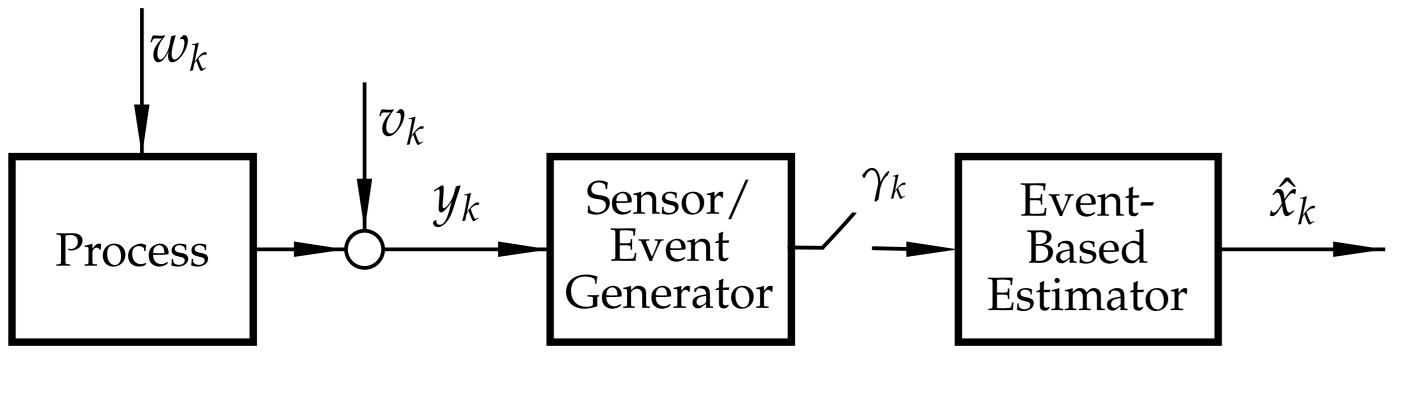
Event-based sensing and communication holds the promise of lower resource utilization and/or better performance for remote state estimation applications in e.g networked control systems (NCS).

However, the problem of designing an optimal event-based state estimator often becomes untractable due to nonlinear measurements. This complexity is avoided with stochastic event-triggering.

In this work [1], we extend the work on stochastic triggering in [2] by proposing a simple predictor in the sensor to further improve the estimation performance.

The Remote Estimation Problem

Compute optimal estimates both with and without transmission:



Process:

$$x_{k+1} = Ax_k + w_k$$

$$y_k = Cx_k + v_k$$

$$w_k \sim \mathcal{N}(0, Q)$$

$$v_k \sim \mathcal{N}(0, R)$$

Two Cases:

$$\gamma_k = \begin{cases} 1 \implies \text{Transmission} \\ 0 \implies \text{No transmission} \end{cases}$$

The MMSE Estimator

Bayes' theorem gives case dependent Kalman filter:

Time Update:

Measurement Update:

 $K_k = P_k^- C^T [CP_k^- C^T + R + (1 - \gamma_k)Y^{-1}]^{-1}$

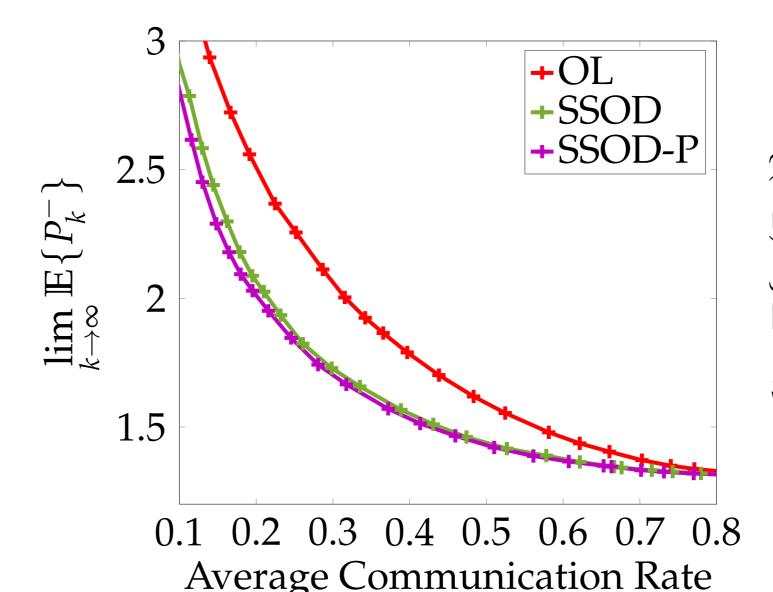
$$\hat{x}_{k}^{-} = A\hat{x}_{k-1} \qquad \hat{x}_{k} = \hat{x}_{k}^{-} + K_{k}[\gamma_{k}y_{k} + (1 - \gamma_{k})S_{l}y_{k-l} - \hat{y}_{k}^{-}]$$

$$\hat{y}_{k}^{-} = C\hat{x}_{k}^{-} \qquad P_{k} = (I - K_{k}C)P_{k}^{-}$$

$$P_{k}^{-} = AP_{k-1}A^{T} + Q \qquad K_{k} = P_{k}^{-}C^{T}[CP_{k}^{-}C^{T} + R + (1 - \gamma_{k})Y^{-1}]^{-1}$$

Numerical Performance Comparison

Performance of SSOD and OL depends on process, while SSOD-P takes the process configuration into account:

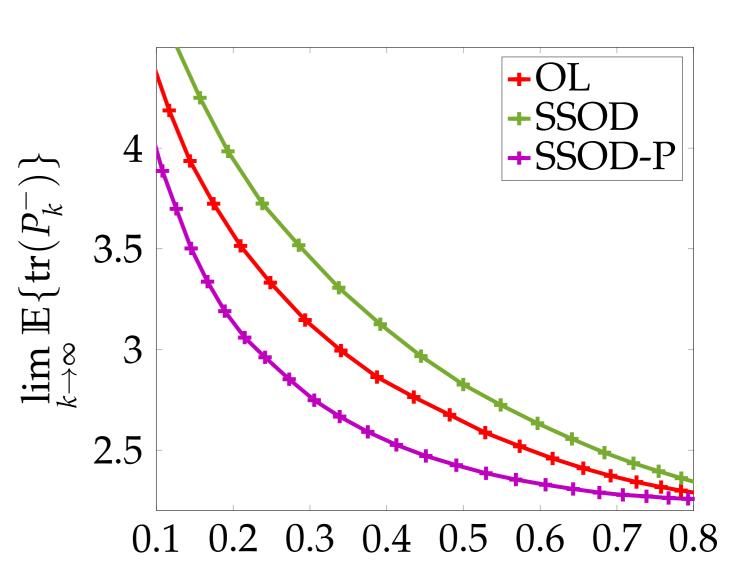


Average Communication Rate

$$A = 0.95, C = 1$$

 $Q = 0.8, R = 1$

Slow 1^{st} -order process:



Highly oscillatory 2^{nd} -order process

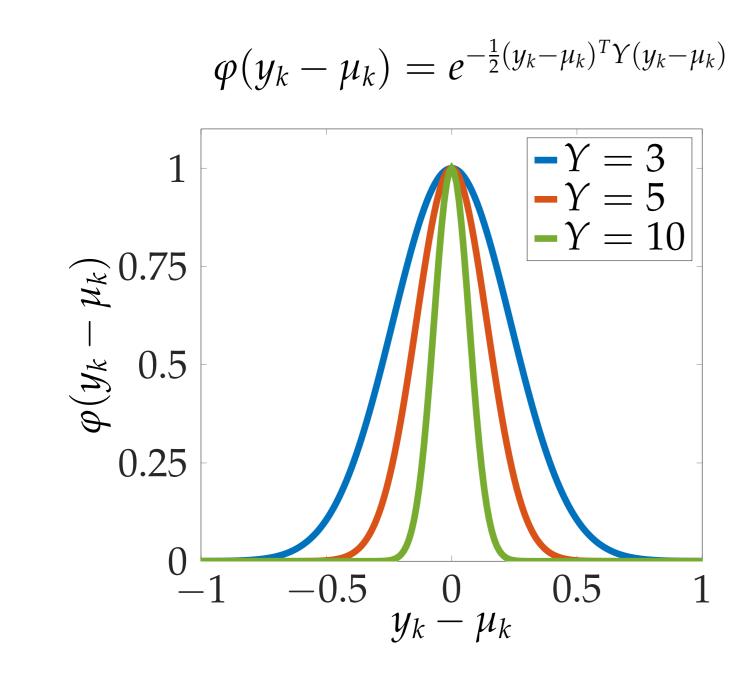
Average Communication Rate

$$A = \begin{bmatrix} -0.85 & -0.35 \\ 0.35 & -0.85 \end{bmatrix}$$
, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$
 $Q = \text{diag}(10^{-3}, 1)$, $R = 0.1$

Stochastic Event-Triggering

Trigger transmission with certain probability:

Decision Function:



Event-Generator:

$$\zeta_k \sim \mathcal{U}(0,1)$$

$$\gamma_k = \begin{cases} 1, & \text{if } \zeta_k > \varphi(y_k - \mu_k) \\ 0, & \text{else} \end{cases}$$

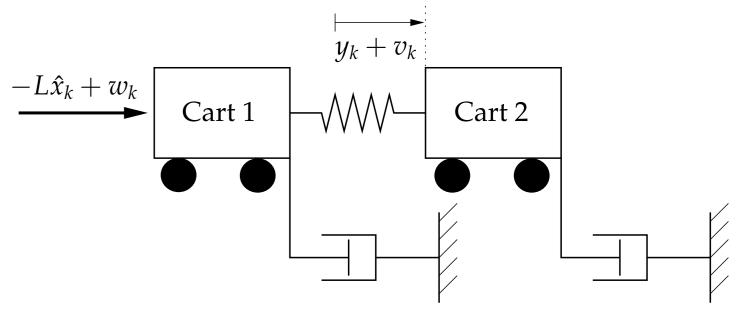
Property of Scheme:

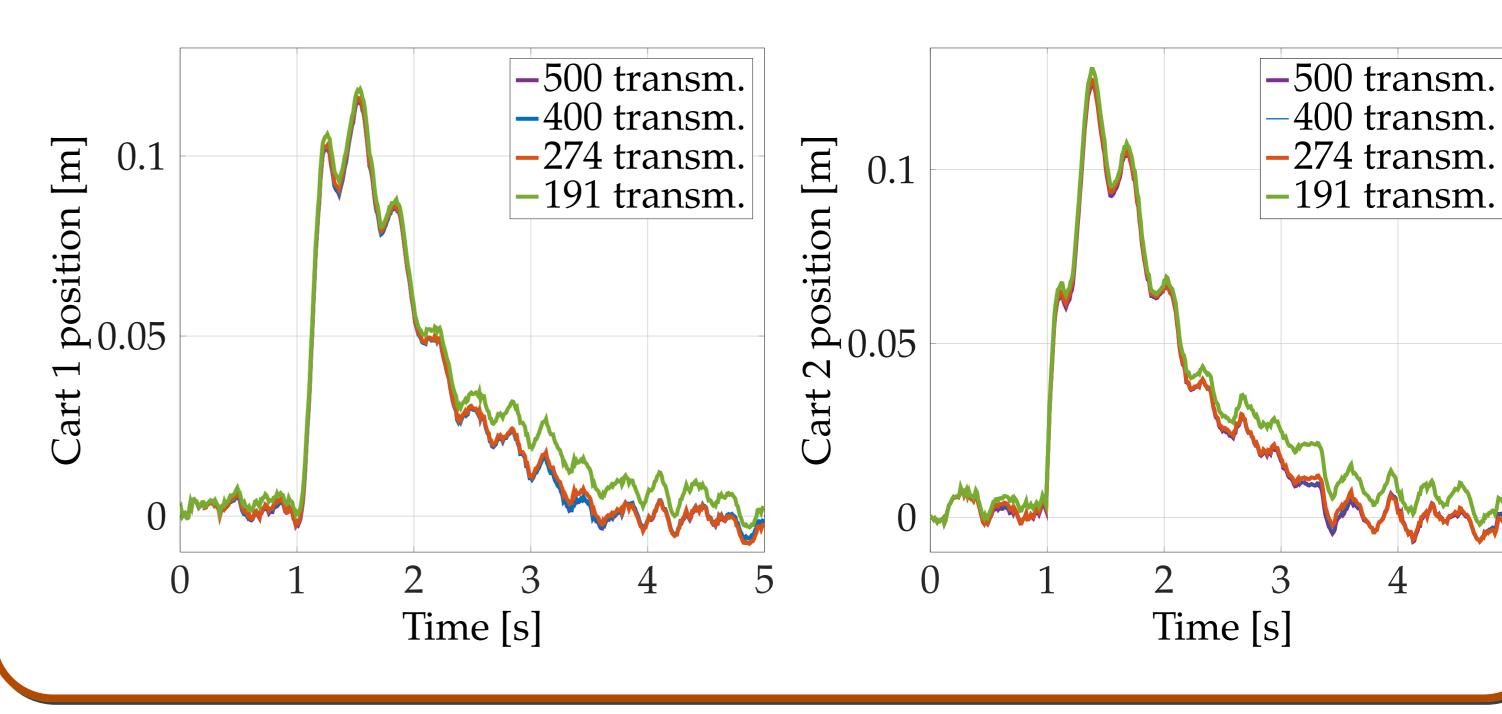
$$\Pr(\gamma_k=0)=\varphi(y_k-\mu_k)$$

Simulation Study

Position control of two carts with state-feedback and SSOD-P:

- System discretized with time step 0.01 s
- Impulse in cart 1 velocity at time 1 s
- Degradation in control performance small even at low communication rates





A Simple Prediction

Proposed μ_k in [2] with no estimator-to-sensor feedback are:

Open-Loop (OL
$$\mu_k = 0$$

Open-Loop (OL): Stoch. Send-on-Delta (SSOD):

 $\mu_k = y_{k-l}$ (Transmit l steps ago)

Based on stationarity, we instead propose:

Stoch. Send-on-Delta with Simple Prediction (SSOD-P):

$$\mu_k = \mathbb{E}\{y_k|y_{k-l}\} = S_l y_{k-l}$$

 $S_l = CA^l\Sigma C^T[C\Sigma C^T + R]^{-1}, \quad \Sigma = \text{Cov}(x_k) \text{ in stationarity}$

Conclusions

- Stochastic Triggering enables simple remote estimator design
- We propose a simple sensor prediction for improved performance
- Prediction implies a scaling of last transmitted value
- Scaling factors can be pre-computed offline
- Proposed scheme compares well in numerical examples

Acknowledgments

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References

[1] M. Thelander Andrén and A. Cervin Event-Based State Estimation Using an Improved Stochastic Send-on-Delta Sampling Scheme In 2nd Int. Conf. on Event-Based Control, Communication and Signal Processing (EBCCSP) (Accepted), Krakow, Poland, June, 2016.

[2] Shi, D., Shi, L. and Tongwen, C. Event-Based State Estimation – A Stochastic Perspective Springer, 2016.