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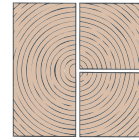
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# RATIONAL MODELLING AND DESIGN IN TIMBER ENGINEERING APPLICATIONS USING FRACTURE MECHANICS

Erik Serrano<sup>1</sup>

**ABSTRACT:** The paper gives a general discussion on methods and theories used for strength analyses of wood and wood-based products. The main aim is to discuss in general terms the benefits gained from using design approaches based on rational and scientifically sound theories. The paper presents mainly fracture mechanics theory but also other advanced modelling techniques and examples from using advanced measurement techniques in testing are briefly discussed. Examples of applications within the current scope of Eurocode 5 that need to be developed are highlighted (dowel type joints and compression perpendicular to the grain). In addition, examples of applications in need of being included in Eurocode 5 are discussed (beams with holes and glued-in rods). The theories presented in the paper are applied to two different applications, namely notched beams and glued-in rods, the latter currently being a candidate for inclusion in Eurocode 5.

**KEYWORDS:** wood, timber, modelling, strength analysis, fracture mechanics, structural design

## 1 INTRODUCTION

### 1.1 BACKGROUND

The word *engineering* stems from ancient words related to the term ingenuity. In some cases, however, this historic coupling to talent, ingenious work and problem solving seems to have been lost [1]. In studying structural failures, irrespective of the main structural material in use, a common explanation for the catastrophic outcome is typically referred to as “human error”. Such a classification covers of course many types of errors, including both ignorance and conscious risk taking. However, there is always the risk of the engineer making mistakes due to the lack of information in or understanding of the structural code as regards the theoretical background of a specific design formula. In such cases, the ingenuity of the engineer might lead to solutions where design formulae are being used outside their intended scopes.

In some rare cases, incorrect and unsafe suggested uses of design formulae were presented in other types of documents such as handbooks, making erroneous references to Eurocode 5 (EC5), [2].

The idea on which this paper is based is that by using rational methods with a clear and physical interpretation such risks can, to a large extent, be avoided.

### 1.2 AIM AND OVERVIEW

The paper presents a number of theoretical approaches that have been used in timber engineering research, but only to a smaller extent have been used in practical design.

The main aim is to highlight the advantages of these approaches in terms of their clear physical interpretation, their ability to model complex behaviour and their suitability as a base for development of design formulae. Another aim of the paper is to discuss some of the sections in the current version of EC5 that are in need of revision, or that are completely lacking.

Following this introductory first section, an overview of a number of theories is given in Section 2, presenting a few basic principles and theories. Detailed information about their use in some applications is given in Section 3. Section 4 discusses briefly a few candidate topics in EC5 that could be up for revision, two of these topics also by use of the theories presented herein. Section 5, finally, presents the conclusions of this paper.

### 1.3 A GENERAL AND RATIONAL APPROACH

By a rational method is meant a method that is based on acknowledged and physically correct theories from solid mechanics. Apart from being based on a sound background, the theories must be combined with the use of model data that have been acquired under well-defined and relevant conditions. In addition, that data must have been evaluated using methods for test result evaluation that are consistent with the intended use of

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the data. Ideally, such a rational approach should then give predictions that are *accurate*, although accuracy is by far the only concern, or perhaps not even the most important concern. The *validity* of the methods used is also extremely important, as is a clear definition and understanding of their limitations. It makes no sense to apply an accurate model outside of its scope.

The use of empirical expressions in design has indeed a long tradition but, generally speaking, such approaches should be avoided, or at least their applicability should be very well defined.

A special class of approaches would be the “empirical” methods based on modelling results (*e.g.* based on parameter studies using finite element models). Such approaches can probably be categorised somewhere in between the fully rational approach and a traditional empirical approach. Their validity is very much dependent on the resolution and range of the parameter studies that were performed in developing the “empirical” expression (curve fitting to FE-results).

Traditional linear elastic fracture mechanics (LEFM) is an example of a more advanced theory (compared to traditional linear elastic stress analysis) that is already included in the current version of EC5 [2], although this fact has been rather well hidden to the user. As an example, the current design formulae regarding beams with notches at the support are based on the work of Gustafsson [3]. For reasons of user-friendliness, Gustafsson’s expressions were reformulated from an energy-based design criterion to a traditional criterion based on shear force capacity. Such a re-formulation inherently includes a risk, since the governing material parameters according to theory have been exchanged for a set of proxy parameters. Any future change of the proxy parameters in other parts of the codes or in standards should therefore also lead to at least a revision of the design formulae that make use of the proxy parameters – is the use of the proxy parameter still valid? As regards notched beams, the introduction of a reduction factor for shear strength in EC5,  $k_{cr}$ , caused problems in terms of interpretation: was it correct to apply  $k_{cr}$  also for formulae referring to notched beams?

## 2 THEORETICAL APPROACHES

Below is given a brief description of some useful theories and modelling approaches that have been suggested for application in timber engineering. The overview is intentionally kept at a simplified level, in order to explain basic concepts. Thus, mainly 2D-applications and uniaxial stress conditions are discussed.

### 2.1 LINEAR ELASTIC FRACTURE MECHANICS

#### 2.1.1 Definition

By LEFM is meant a theory based on the following assumptions:

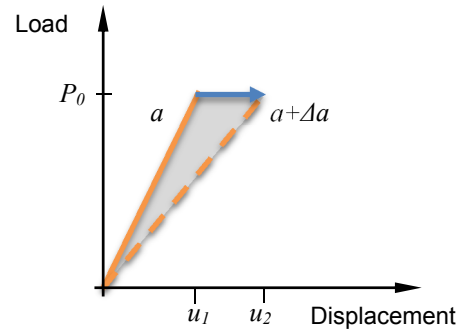
- materials behave linear elastically,
- a sharp crack and an accompanying stress singularity exist (the square root singularity),

- material strength is infinite,
- the size of the fracture process zone (FPZ) in the material is sufficiently small in relation to all other dimensions, including the pre-existing sharp crack, so as to treat it as being of zero size, and
- there exists a material parameter, the fracture energy,  $G_c$ , which, together with the material stiffness, governs the initiation and progressive failure of the material.

A number of different approaches can be used to apply the theory of LEFM. The most commonly used approaches would be the stress intensity approach, J-integral approach or the compliance method [4]. Of special interest here is the compliance method as this lends itself to formulation of hand calculation formulae, which can be used to derive design equations [5].

#### 2.1.2 Compliance method approach

Assuming that a quasi-static (2D)-structure of width  $b$  is loaded by a single constant point load,  $P_0$ , and denoting the length of an existing crack with  $a$ , the load-displacement relations (displacement at loading point) for crack lengths  $a$  and  $a+\Delta a$  are shown in Figure 1, together with the energy released for such crack propagation (grey area).



**Figure 1:** Load-displacement response for crack propagation in a linear elastic structure.

Denoting the structure’s compliance with  $C(a)$ , the following general expression can be derived for the energy,  $W$ , released during crack propagation:

$$W = \frac{1}{2} P_0^2 [C(a + \Delta a) - C(a)]. \quad (1)$$

The energy released per unit area of crack surface is then expressed as

$$G = \frac{W}{b\Delta a} = \frac{1}{2b} P_0^2 \frac{\Delta C}{\Delta a}. \quad (2)$$

From Equation (2), assuming  $\Delta a \rightarrow 0$  and assuming that crack propagation takes place when the released energy per unit area reaches the critical value of the material *i.e.* when  $G=G_c$ , the corresponding critical load  $P_c$  can be calculated by

$$P_c = \sqrt{2bG_c \left( \frac{\partial C}{\partial a} \right)^{-1}}. \quad (3)$$

Since the approach as expressed by Equations (1)-(3) is formulated in a global sense, it is straightforward to apply it for any structural model, be it based on beam

theory and solvable by hand calculation, or be it a full 3D finite element model.

Apart from lending itself to hand calculation, another major advantage is that (at least for 2D-problems) it is straightforward to perform the crack propagation analysis using standard finite element software, since the only demand on the software is that it can perform linear elastic analysis. Equation (3) can of course be evaluated numerically from the results of a series of such linear elastic analyses. Thus, the critical load as a function of crack length is found by performing first a series of analyses, where each analysis is done for a different crack length. Following this, the compliance of the structure (as a function of crack length) is calculated and, finally, Equation (3) is applied in a pure post-processing step.

Another major advantage of the compliance method is that, in general, it is much less sensitive to mesh density as compared to many other LEFM-based methods, such as the stress intensity approach. This is because a) the *global* energy balance is evaluated (and not the local stress/strain state close to the crack tip) and b) the *derivative* of the compliance with respect to crack length rather than the compliance itself is used in Equation (3).

The main limitation of LEFM, and thereby of the compliance method, would be related to the above basic assumptions of the theory. For some applications, the assumption of a negligible size of the FPZ might be questionable [5].

## 2.2 NON-LINEAR FRACTURE MECHANICS

### 2.2.1 Definition

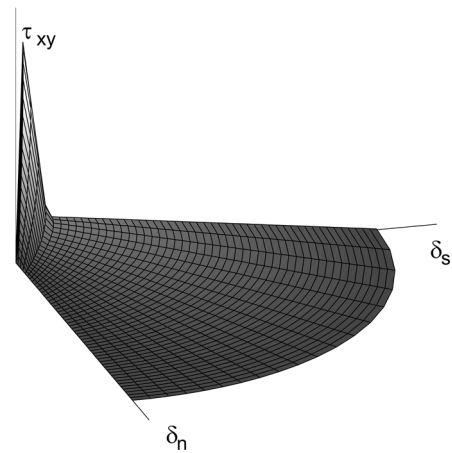
By non-linear fracture mechanics (NLFM) is meant a theory that includes a non-linear behaviour aiming at capturing the strain softening of a material, *i.e.* its gradually diminishing stress transferring capability with increasing deformation. In terms of its implementation several formulations exist, including Hillerborg-type approaches [6] and crack band (smeared crack) approaches [7, 8, 9]. Compared to LEFM, a theory based on NLFM involves additional material parameters that govern the global load bearing capacity of a structure: the strength of the material and the shape of the curve describing the stress versus deformation behaviour. Of central importance for strain softening theories are issues like local and global stability, and mesh dependence issues (in FE-applications) and how these are handled.

### 2.2.2 Crack band approach

For many wood applications where fracture propagation is of concern, the crack path can be estimated with good accuracy *a priori*. For the analysis of adhesive bond lines, this is especially obvious. In such cases, NLFM can be implemented using a crack band approach where the non-linear softening behaviour is localised to a narrow band representing the bond line. Zero thickness cohesive elements or solid elements of small thickness (one element across in the thickness direction) are common choices. To simplify the description of NLFM

theory, the characterisation of adhesive bonds is taken as an example here.

Following the approach as outlined in [8, 9] a wood adhesive bond line is characterised in stress-deformation space by non-linear relations. One way of visualising such a behaviour (for monotonic loading) is as a surface in 3D, *cf.* Figure 2, showing shear stress ( $\tau_{xy}$ ) versus shear displacement ( $\delta_s$ ) and normal displacement ( $\delta_n$ ) of a bond line. If unloading does not occur, the formulation could be based on non-linear elasticity, see *e.g.* [10], although such a formulation is non-physical. A rather straightforward formulation, fulfilling the basic laws of thermodynamics is achieved by using a damage-based formulation; one such example is given in [11].

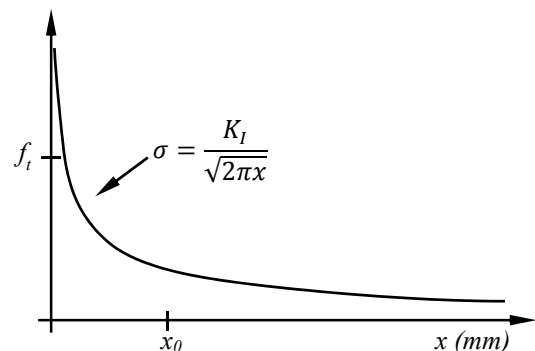


**Figure 2:** Shear stress versus shear displacement and normal displacement of bond line model [11].

## 2.3 GENERALISED METHODS I – MEAN STRESS APPROACH

### 2.3.1 Definition

The starting point for the so-called mean stress approach [4, 5] is LEFM, *i.e.* linear elastic behaviour, infinitesimal size of the FPZ and the existence of a sharp crack. The method has mostly been used for 2D-problems, since its extension into 3D is not so straightforward. The approach (in 2D) is to consider the stress distribution close to a crack tip, and assuming that the strength of the material is limited to a value  $f_t$ , the local (tensile) strength of the material, as depicted in Figure 3. The further discussion here is for simplicity limited to pure Mode I situations.



**Figure 3:** Normal stress distribution ahead of a crack.

Close to the crack tip, the stress field is dominated by the well-known singularity

$$\sigma = \frac{K_I}{\sqrt{2\pi x}} . \quad (4)$$

Assuming now that the above described situation is to be evaluated by a simple stress based failure criterion, the problem of the singularity arises, i.e. since stress is infinite at  $x=0$ , a zero load bearing capacity would be predicted. However, although the single point maximum stress is infinite, the average stress over a certain length is well defined, and finite. Thus, assuming that we instead of the maximum stress in a single point calculate the average stress,  $\sigma_{\text{mean}}$ , along a certain distance,  $x_0$ , from the crack tip, we obtain:

$$\sigma_{\text{mean}} = \frac{1}{x_0} \int_0^{x_0} \frac{K_I}{\sqrt{2\pi x}} dx . \quad (5)$$

Then, by using the average stress  $\sigma_{\text{mean}}$  in a stress based failure criterion, the problem with the singularity has been avoided. Thus, the failure criterion is expressed as

$$\sigma_{\text{mean}} = f_t \quad (6)$$

and the problem is reduced to a stress based evaluation problem. The length  $x_0$  could be determined on the basis of calibration from tests, or from some micro-structural considerations or some other, theoretical, considerations. One such theoretical consideration is that we postulate that, in the case of a sharp crack, the mean stress approach should predict the same load bearing capacity as using a traditional LFM approach, i.e. assuming crack propagation will start if  $K_I = K_{I,c}$ , with  $K_{I,c}$  being the fracture toughness of the material. Making use of Equations (5) and (6) we then obtain

$$\sigma_{\text{mean}} = f_t = \frac{1}{x_0} \int_0^{x_0} \frac{K_I}{\sqrt{2\pi x}} dx = \sqrt{\frac{2K_I^2}{\pi x_0}} \quad (7)$$

From this, we can then express the mean stress length as a material parameter, in terms of fracture toughness and strength:

$$x_0 = \frac{2K_{I,c}^2}{\pi f_t^2} \quad (8)$$

Equation (8) is in terms of fracture energy expressed as (for plane stress)

$$x_0 = \frac{2E_I G_{I,c}}{\pi f_t^2} \quad (9)$$

where  $E_I$  is the Young's modulus of an isotropic linear elastic material. For an orthotropic material, like wood, the corresponding stiffness for tension perpendicular to the grain is obtained from [5]:

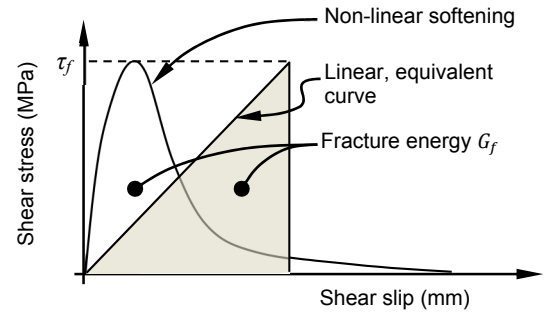
$$\frac{1}{E_I} = \frac{1}{E_0} \sqrt{\frac{E_0}{2E_{90}}} \sqrt{\frac{E_0}{E_{90}} + \frac{E_0}{2G_{0,90}} - \nu_{90,0} \frac{E_0}{E_{90}}} \quad (10)$$

Where indices 0 and 90 denote the parallel and the perpendicular to grain directions, respectively,  $E$  the Young's modulus,  $G$  the shear modulus and  $\nu$  the Poisson's ratio.

## 2.4 GENERALISED METHODS II – QUASI NON-LINEAR FRACTURE MECHANICS

### 2.4.1 Definition

In general, according to the theory NLFM, the shape of the softening curve (stress versus displacement) is a parameter that influences the load bearing capacity. However, it can be shown, see [12], that in some cases of self-similar crack propagation this is not the case, and instead any shape of the softening curve can be used to predict the load bearing capacity. If it is assumed that the shape of the softening curve does not influence the load bearing capacity, then a convenient choice of that shape can be made. One such choice is depicted in Figure 4: a linear elastic behaviour with abrupt “softening”.



**Figure 4:** Shear stress versus shear displacement of a softening material.

The original crack propagation problem is thus treated as a linear elastic one instead, but using a stiffness which is determined by the local strength and the fracture energy of e.g. a bond line. Thus, for the example shown in Figure 3, instead of using the linear elastic shear modulus and the thickness of the bond line, an equivalent shear stiffness,  $k_s$ , based on local strength and fracture energy is used:

$$k_s = \frac{\tau_f^2}{2G_f} \quad [\text{N/mm}^3] \quad (11)$$

Using such a linear elastic equivalent layer approach, simple linear elastic analyses can be performed, and the load bearing capacity is then estimated by a conventional single point maximum stress criterion. Both FE-based approaches and hand calculation approaches have been adopted using this quasi non-linear theory, see e.g. [13, 14] for more recent application of this methodology.

## 2.5 APPLICABILITY AND BENEFITS OF GENERALISED METHODS

The generalised methods described above are generalised in the sense that their applicability is not limited to situations involving stress singularities or involving perfectly brittle materials. Instead, at least formally, any stress field or material brittleness can be handled. Furthermore, the size effects predicted by the methods are in general less pronounced than the extreme size effect predicted by LFM.



### 3 APPLICATION EXAMPLES

Two application examples are given below. One related to the use of the compliance method and one with several of the above-mentioned approaches being used. Some additional details as regards the modelling approaches are included, but the basic concepts are as described in Section 2.

#### 3.1 NOTCHED BEAM

##### 3.1.1 Problem description

The notched beam problem is used here as an example of applying the compliance method from LEFM-theory using a FE-model.

The situation is depicted in Figure 5. At the support, a 90° notch is cut out from a beam. The current study relates to the influence of slope of grain close to the notch, thus a number of grain angles ranging from -10° to +10° (relative the beam axis) were evaluated. A point load was applied at a distance far enough from the support to avoid any disturbance of the stress field. The support, in turn, is modelled as a pinned stiff plate, simulating a roller support.

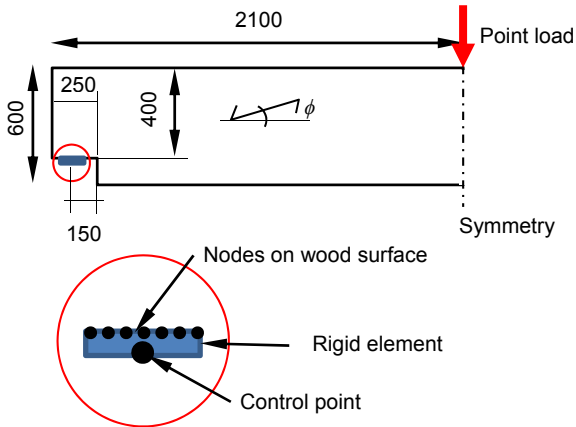


Figure 5: Notched beam problem.

##### 3.1.2 Finite element model and compliance method approach

The finite element model is built up using quadrilateral plane stress elements, element size was varied from 2,5 mm size and up. For all analyses at a given grain angle, the same element mesh was used, but the connectivity of the elements at the crack tip was changed by introducing extra nodes, in order to simulate crack growth. The compliance of the structure was calculated from the FE-results and the estimated critical load was evaluated in terms of Equation (3).

The most straightforward approach is to use the pure Mode I (opening mode) fracture energy, which is an assumption on the safe side. However, depending on the geometry of the notch (and thereby on the current crack length) the contribution from Mode II will vary. If including Mode II in a mixed-mode fracture criterion, one needs to evaluate the possibly varying critical energy release rate during crack propagation (*i.e.* the current value of  $G_c$  to be used in Equation (3)). This is done in the following manner. First, a relevant stress based

failure criterion in terms of mean stresses  $\sigma_{\text{mean}}$  and  $\tau_{\text{mean}}$  is chosen, the one used here is the Norris criterion

$$\left(\frac{\sigma_{\text{mean}}}{f_t}\right)^2 + \left(\frac{\tau_{\text{mean}}}{f_v}\right)^2 = 1 \quad (12)$$

which in combination with the Wu fracture criterion

$$\frac{K_I}{K_{I,C}} + \left(\frac{K_{II}}{K_{II,C}}\right)^2 = 1 \quad (13)$$

and Equation (4) and its Mode II counterpart can be used to derive an expression for the value of  $x_0$  along which the mean stresses are to be calculated:

$$x_0 = \frac{2 K_I^2}{\pi f_t^2} \left(1 + \frac{(K_{II}/K_I)^2}{(f_v/f_t)^2}\right). \quad (14)$$

In Equation (14)  $K_I$  and  $K_{II}$  represent the magnitudes at the instant of fracture and at the current degree of mixed mode expressed as the ratio  $k=K_{II}/K_I = \tau_{\text{mean}}/\sigma_{\text{mean}}$  (cf. Equation (7)).  $f_t$  and  $f_v$  are the strengths of the material in tension perpendicular to the grain and in shear, respectively.

The expression for  $K_I$  at the instance of fracture is given from Equation (13) and reads, [4]:

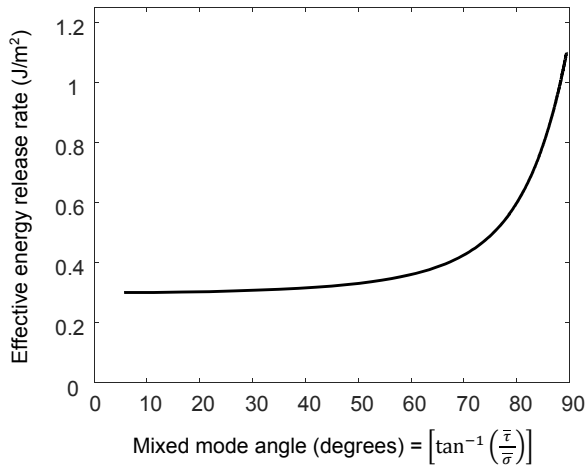
$$K_I = -\frac{K_{II,C}^2}{2K_{I,C}k^2} + \sqrt{\frac{K_{II,C}^4}{4K_{I,C}^2k^4} + \frac{K_{II,C}^2}{k^2}}. \quad (15)$$

By the relations

$$K_I = \sqrt{E_I G_I} \text{ and } K_{II} = \sqrt{E_{II} G_{II}}. \quad (16)$$

the above Equation (14) can be expressed in terms of critical energy release rates  $G_{I,C}$  and  $G_{II,C}$  instead of stress intensities at fracture. That expression is then used to calculate the mean stress length,  $x_0$ . Note that the length  $x_0$  is dependent on the current degree of mixed mode, which in turn is expressed as the ratio of the average stresses along that same length  $x_0$ :  $k = \tau_{\text{mean}} / \sigma_{\text{mean}}$ . Consequently, the value of  $x_0$  has to be calculated in an iterative manner. This iterative procedure is normally fast converging, and typically no more than three iterations are needed to obtain a set of converged values of  $x_0$ ,  $k$  and  $G_c$ .

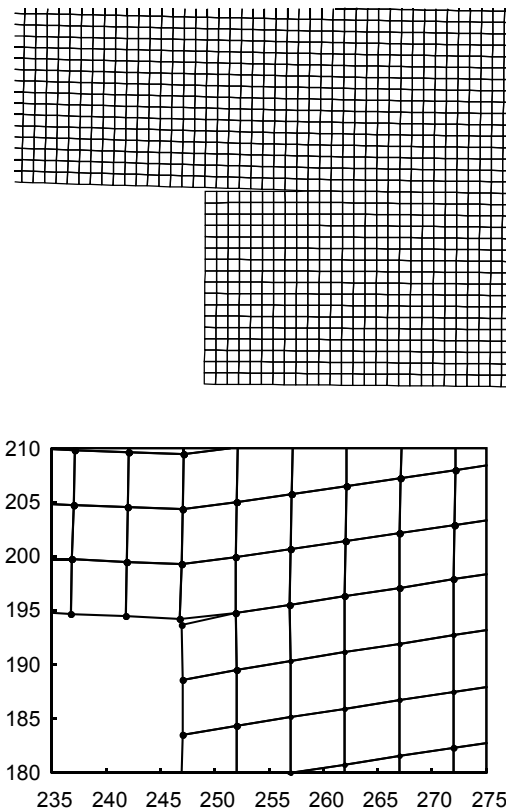
Below is shown a graph indicating the influence of mixed mode on the effective critical energy release rate (assuming 300 J/m<sup>2</sup> and 1150 J/m<sup>2</sup> in mode I and II respectively). As can be seen, the influence on the effective fracture energy is limited and only for stress situations with dominating shear is the value much different from the value corresponding to pure mode I (shear is dominating for small notch depths at constant notch length). An approximation on the safe side would be to use always the value of  $G_{C,I}$ .



**Figure 6:** Effective energy release rate as a function of mixed mode ratio.

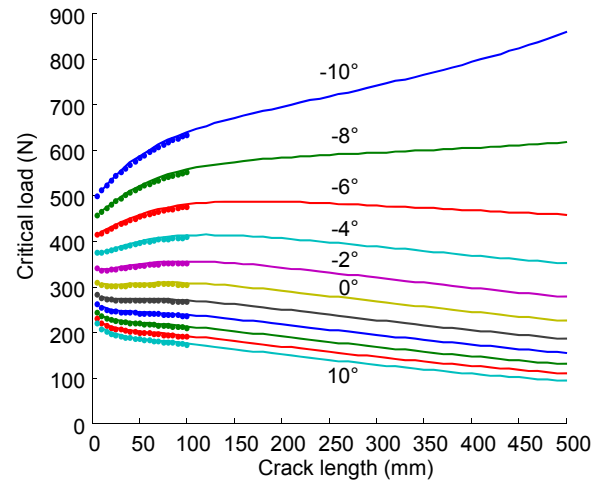
### 3.1.3 FE-analyses and results

The element mesh was constructed such that the element sides were parallel to the grain, and thus, in case of sloping grain the element sides close to the crack tip were not parallel with the beam longitudinal axis. The element mesh used in the analyses is shown in Figure 7, depicting the mesh at the notched corner for a case of straight grain and for a case of sloping grain.



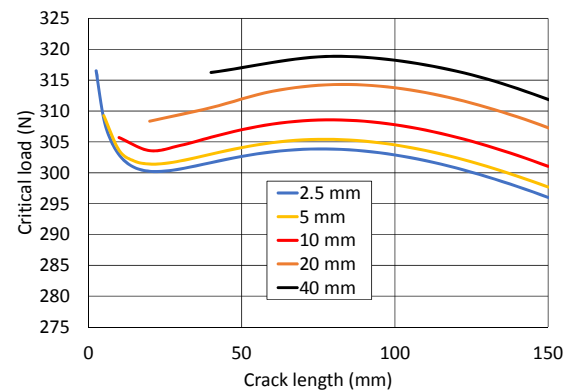
**Figure 7:** Example of 5 mm element meshes in displaced model for straight grain (top, partial zoom) and 10° sloping grain (bottom, partial zoom).

The results from the analyses are shown in Figure 8. In the diagram, results from using both 5 mm element size and from using 10 mm element size are shown.



**Figure 8:** Critical load vs. crack length and grain slope, 5 and 10 mm element size are shown with dots and solid lines, respectively.

An example of the stable performance of the compliance method, with respect to element size, is shown in Figure 9. The influence of element size on critical load bearing capacity is in the range of 5% for a change of element size by more than one order of magnitude (from element size 2,5 mm to element size 40 mm). Using a traditional LEFM approach in terms of evaluating the near crack-tip stress field would probably not be successful for an element size larger than a few millimetres [4].

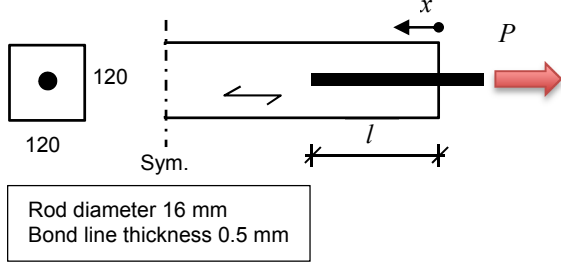


**Figure 9:** Calculated critical load vs. crack length for crack propagation analyses using various element sizes.

## 3.2 GLUED-IN RODS

### 3.2.1 Problem description

The load bearing capacity of a glued-in rod, glued parallel to the grain and loaded in pull-pull according to Figure 10 is used here as an example, see also [9].



**Figure 10:** Glued-in rod problem studied.

The material data used in the comparison was taken similar as those used in [9]. Below the equations for the compliance method approach and for the quasi non-linear fracture mechanics approach using Volkersen theory to determine the stress distribution are presented. Finally, a parameter study involving these approaches and a 3D-FE approach using NLFM is presented.

### 3.2.2 Compliance method equations

For a compliance method approach, it is necessary to assume the existence of an initial crack causing pull-out of the rod from the timber. We here assume that this crack exists at the loaded end of the joint. Assuming a pure axial action in the wood and steel parts, the compliance of the glued-in rod joint can be expressed in terms of the axial stiffness ( $EA$ ) of the different parts of the joint. Using now the approach described in Equations (1)-(3) we can write for the ultimate load,  $P_u$ :

$$P_u = \sqrt{2\overline{EA}G_c\pi\phi} \quad (17)$$

$$\overline{EA} = \left( \frac{1}{(EA)_{rod}} - \frac{1}{(EA)_{rod} + (EA)_{wood}} \right)^{-1} \quad (18)$$

where  $EA$  denotes the axial stiffness of the rod and the net wood cross section,  $G_c$  the fracture energy (here assumed in pure mode II), and  $\phi$  is the diameter of the cylindrically shaped assumed fracture surface along the rod. From Equation (18) it is obvious that a simplified equation can be obtained by assuming that the axial stiffness of the wood part is much larger than the axial stiffness of the steel part, in which case  $\overline{EA} = (EA)_{rod}$ .

### 3.2.3 Quasi non-linear fracture mechanics equation

Using the classical Volkersen approach, the stress distribution along the rod-timber interface is determined. This is done under the assumption of a linear elastic behaviour, and assuming that the adhesive layer is acting in pure shear. By such shear lag analysis, the shear stress along the rod for an applied load,  $P$ , is described by [15]:

$$\tau = \frac{PG}{t\omega(EA)_r} \left\{ \left[ \cosh(\omega l) + \frac{(EA)_r}{(EA)_w} \frac{\cosh(\omega x)}{\sinh(\omega l)} \right] - \sinh(\omega x) \right\} \quad (19)$$

where  $G/t$  is the shear stiffness of the adhesive layer with thickness  $t$ , and shear modulus  $G$  and where the parameter  $\omega$  expresses the ratio of shear stiffness of the

adhesive layer to axial stiffness of the adherends through:

$$\omega^2 = \frac{G}{t} \pi \phi \left( \frac{1}{(EA)_r} + \frac{1}{(EA)_w} \right). \quad (20)$$

Assuming now that  $(EA)_w/(EA)_r > 1.0$  maximum shear stress is found at  $x=0$ . According to the above described theory of quasi-nonlinear fracture mechanics, the shear stiffness  $G/t$  is exchanged for the equivalent stiffness based on fracture energy,  $G_f$ , and shear strength,  $\tau_f$ , of the bond between rod and timber. Thus, it is assumed that

$$\frac{G}{t} = \frac{\tau_f^2}{2G_f} \quad [N/m^3]. \quad (21)$$

Inserting this into Equation (19) and making use of Equation (20) we finally obtain an expression for the load bearing capacity,  $P_u$ , of the glued-in rod:

$$P_u = \frac{2G_f\omega(EA)_r}{\tau_f} \left( \frac{\sinh(\omega l)}{\cosh \omega l + \frac{(EA)_r}{(EA)_w}} \right) \quad (22)$$

with

$$\omega^2 = \frac{\tau_f^2}{2G_f} \pi \phi \left( \frac{1}{(EA)_r} + \frac{1}{(EA)_w} \right). \quad (23)$$

### 3.2.4 3D FEM approach

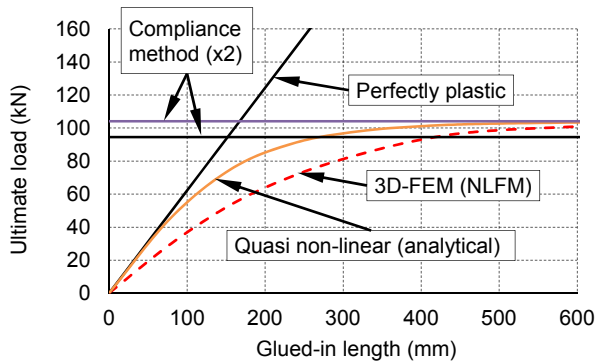
A 3D cohesive element based model was used to calculate the load bearing capacity of a glued-in rod. The model consisted of linear elastic material for the wood and the steel, and the cohesive element behaviour was based on a damage formulation as described in [11], resulting in stress versus displacement responses similar to what is shown in Figure 2. The bond line behaviour includes two shear stress components and one peel stress component. All other stress components in the bond line are assumed zero (this being consistent with an assumption of a thin bond line). It is beyond the scope of this paper to give a detailed description of the model, for such details, please see [11].

### 3.2.5 Results – Influence of glued-in length

In Figure 11 is shown the results from a parameter study of the influence of glued-in length on the ultimate load bearing capacity, as predicted by the three above-described approaches. The prediction “perfectly plastic” is simply the load bearing capacity obtained from assuming a uniform shear stress distribution along the rod. Two curves derived from Equations (17) and (18) are shown, the lower one corresponding to  $\overline{EA} = (EA)_{rod}$ .

It should be noted that the 3D-FEM approach involves mixed mode behaviour. The same shear strength was used in all the models making use of shear strength (12 MPa), but mixed mode behaviour is only accounted for in the 3D-FE model (which also makes use of the bond line peel strength, here set to 4 MPa). This results in a discrepancy between the 3D-FE model results and the results from the shear lag model based on quasi non-linear fracture mechanics. However, the general shapes of those two curves are similar, which indicates that the analytical quasi non-linear theory is well suited for calibration to test results: the effect of mixed mode could possibly be accounted for in the analytical model by assigning a lower shear strength value.





**Figure 11:** Influence of glued-in length on ultimate pull-out load using various theoretical models.

## 4 RATIONAL APPROACHES FOR EC5

As mentioned in the introduction, one aim of the current paper has been to highlight current candidates for revision in EC5 where new design formulae should be based on rational approaches. Below are given examples of four such candidate applications. Two of the applications could be included using the concepts of fracture mechanics as discussed above, and based on current knowledge, more or less. As regards the remaining two (compression perpendicular to grain and dowel type joints in complex loading), the current status of research has shown that more advanced methods of design could very well be adopted.

### 4.1 FRACTURE MECHANICS BASED DESIGN

#### 4.1.1 Beams with holes

In the current version of EC5, there are design provisions for notched beams, but not for beams with holes. There was, in a draft version of EC5, plans for including this, but the design formulae suggested were based on an erroneous analogy with the formulae for notched beams.

In a recent paper, [16] an approach based on the compliance method and semi-analytical formulae was suggested. Although further verification work is needed, and although the current format of the approach does not lend itself to hand-calculations the concept seems promising.

The approach in [16] is general, with the possibility of including the influence of bending moment, shear force and axial force in the beam. There is also a possibility of using the approach for design of reinforcement of the wood close to the hole in the beam. As stated in the general conclusions of the paper, the approach is appealing since it is based on the same theoretical framework as the currently used end-notched beam design equation of EC5, and it does indeed capture the strong beam size effect as found in experimental work.

#### 4.1.2 Glued-in rods

The inclusion of glued-in rods (bonded-in rods) into EC5 has been up for discussion for more than 20 years. Although a very large number of research projects and investigations have indeed been carried out, there is still no consensus on how to implement this type of joint in the code [17].

Historically, empirical design equations were proposed, but also analytical models based on theories such as the ones described in this paper have been suggested. In the case of glued-in rods, the main problem has probably not been to find consensus on the precise design formulae, but rather that glued-in rods have been treated already in the standardisation and certification system by national approvals, and thus there is no major demand from industry to have them included into EC5. However, from the point of view of promoting the use of glued-in rods, including that joint type in EC5 would certainly be beneficial.

### 4.2 PLASTICITY THEORY BASED DESIGN

#### 4.2.1 Compression perpendicular to grain (CPG)

The design approaches for CPG in previous and current version of EC5 are prime examples of semi-empirical methods. In the work with the EC5 revision, a recently published paper [18] discusses candidate theoretical models for a new approach as regards design for CPG.

Apart from debating how to find an easy-to-use design approach, based on a sound and rational theory, the design of CPG has been debated from a design philosophy point-of-view: In many cases it turns out that the CPG situation is mainly a serviceability limit state problem, with excess deformation rather than an ultimate limit state problem. Even so, a rational approach based on a physically founded model, would be preferable.

#### 4.2.2 Dowel type joints

The use of advanced contact-free deformation measurement technique (digital image correlation, DIC) at testing of dowel type joints has been presented in a number of papers and conference contributions [19, 20, 21]. The applied multi-scale approach (testing materials and components at various scales) in combination with the DIC-technique as presented in [19], has given an in-depth understanding of the kinematics of dowel-type joints under loading conditions involving pure bending moment or involving a combination of bending and normal force. The research has not only provided detailed information about the local deformation of dowels in joints, but has also provided data necessary for implementation of advanced modelling approaches in commercial engineering software.

Thus, it is believed that the basic knowledge is available such that relevant, reliable and accurate analyses can be performed. The main challenge for the scientific and engineering communities is to clarify how such advanced approaches can be included in design work based on FE-analyses and assuring that the provisions of EC5 are met.

A simple starting point for the specific example of dowel type joints would be to give provisions regarding dowel materials such that a ductile behaviour of the dowels can be assured. Today, minimum requirements have to be fulfilled for strength. As regards strength it is however essential that the material can be defined within a narrow *interval* in order to avoid brittle failures due to over strength of the dowels.

Another issue of importance would be to include into EC5 at least a recommended approach for the engineer to follow in order to perform an elasto-plastic design calculation. The current version of EC5 does not provide any detailed information regarding e.g. the load distribution among dowels in a joint, something that would be essential in the design of high deformation capacity joints.

## 5 CONCLUSIONS

The following conclusions are drawn:

- An increased understanding of the applicability of design equations, and of their limitations, is of prime concern as regards minimising the risk for human errors in structural design.
- A rational approach in design, as opposed to a purely empirical approach, will add to such understanding.
- Fracture mechanics is one example of such a rational framework.
- Both traditional LEFM and the so-called quasi non-linear fracture mechanics theory are possible candidates as basis for design formulae.
- The current version of EC5 is in need of revision and additions, and some of the changes currently under discussion could include the theories discussed in this paper.

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