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This report describes simulation of an STR applied to a fixed bed chemical reactor system. The purpose is to illustrate how a self-tuning regulator (STR) based on least squares parameter estimation and pole-placement design behaves when applied to a realistic system. The control problem is not trivial because the reactor has non-minimum phase characteristicts. It is shown that the reactor can be controlled well even if the model used in the STR is much simpler than the controlled process. The results of the simulations are supported by analysis.

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1. INTRODUCTION

The purpose of this report is to show how a self-tuning regulator (STR) based on least squares parameter estimation and pole placement design works when applied to a realistic system of moderate complexity. In particular it will be shown what happens when the estimated model in the STR is considerably simpler than the controlled system. The process considered is a mathematical model of a fixed bed chemical reactor system. The reactor is a pilot plant which has been in operation at the Department of Chemical Engineering at University of California, Berkeley, for several years, Foss (1978 a). Several different control designs have been applied to the reactor. Control design based on state-feedback and Kalman filtering is discussed in Silva (1978) and Wallman (1977). Multivariable frequency domain techniques have also been applied, Foss and Edmunds (1978).

The model used in the simulations is presented in Section 2. The model is a linear system of 7th order. The reactor is a multivariable system. In this report we will, however, only discuss control of two simple loops. The formulation of the control problem is discussed in Section 3. The structure of the STR used is also described in this section. The SISO control loops, where the outputs are compositions, are easy to control because the dynamics is stable and minimum phase. The results obtained when controlling such loops by the STR are given in Section 4. It is investigated systematically what happens when the model used in the STR is simpler than the simulated model. The simple loops, where the outputs are temperatures, are more difficult to control because they exhibit nonminimum phase behaviour. The performance obtained by applying the STR to such a loop is explored in Section 5. In the simulated examples in Sections 4 and 5 it is shown that very good behaviour can be obtained with STR:s whose models are considerably simpler than the simulated process. If the model used in the STR is too simple the closed loop will, however, be unstable. Preliminary analysis which supports this observation is given in Section 6.

2. THE REACTOR MODEL

A full description of the experimental reactor is given in Silva (1978). A schematic diagram of the reactor is given in Fig. 2.1.

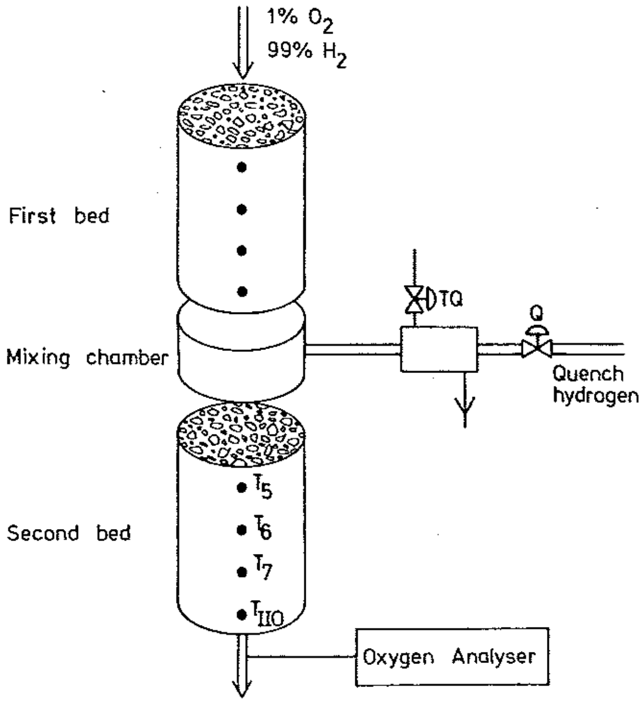

Figure 2.1. Schematic diagram of the reactor.

Oxygen and hydrogen react over a platinum catalyst on silica gel particles. The reactor has two beds as is seen in Fig. 2.1. The beds are separated by a mixing chamber where hydrogen is added. Only the second bed is controlled. The control variables are the temperature and the flow of the added hydrogen, called the "quench flow". The gas temperature and concentration at the outlet of the first bed are regarded as disturbances. The concentration of oxygen at the outlet of the second bed is the major control variable but temperatures are also measured at several positions in the second bed. A linearized model, which described fluctuations around an operating point, was derived by Silva (1978). A model of 14th order was obtained by discretizing the partial differential equations describing the reactor by the collocation method. This model was then reduced to a 7th order model by making an eigenvalue expansion and retaining the dominant terms. The model has the form

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du,$$

where

u_1 = Outlet gas temperature of first bed (TIO)

u_2 = Oxygen concentration at outlet of first bed (CIO)

u_3 = Quench flow (Q)

u_4 = Temperature of quench flow (TQ)

and

$$\left.\begin{array}{l} y_1 = T5 \\ y_2 = T6 \\ y_3 = T7 \end{array}\right\} \text{Intermediate temperatures in second bed}$$

y_4 = Outlet temperature of second bed (TIIO)

y_5 = Oxygen concentration at outlet of second bed (CIIO).

The numbers used in this report were obtained by taking the model of Silva (1978) and rounding all numbers to three significant digits. The numbers used are given in Appendix A. All numbers are scaled. The time unit is 87.5 s, the temperature unit is $167^{o}C$, the concentration unit is 1 mole %, and the flow unit is 13.5 l/min.

The model obtained was tested by calculating step responses and comparing with the responses of the more complete model. Typical step responses are shown in Figs. 2.2 - 2.5. The step responses were generated using the simulation language SIMNON, Elmqvist (1975).

It is seen from Figs. 2.4 and 2.5 that the dynamics relating outlet concentration to quench flow or quench flow temperature can be described by a typical process control dynamics which exhibits time-delay and monotone step response. It is also seen from Fig. 2.4 and Fig. 2.5 that the response of outlet temperature to quench flow and quench flow temperature has typical nonminimum phase behaviour.

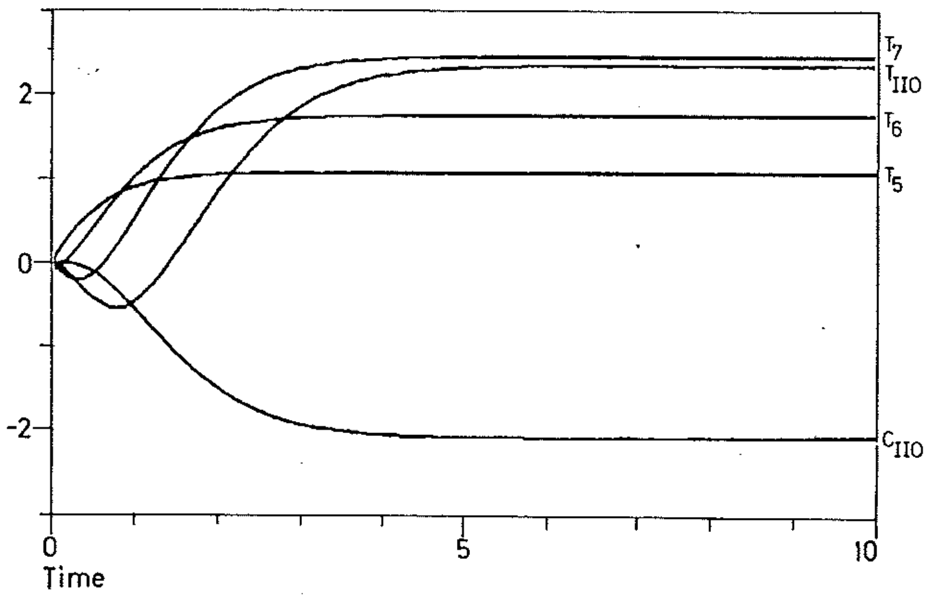

Figure 2.2. Response of the system to a unit step in inlet temperature (TIO).

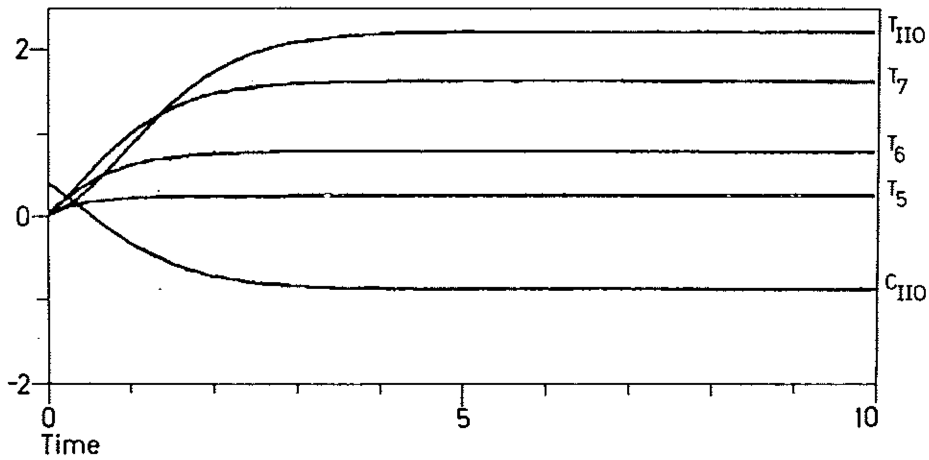

Figure 2.3. Response of the system to a unit step in inlet oxygen concentration (CIO).

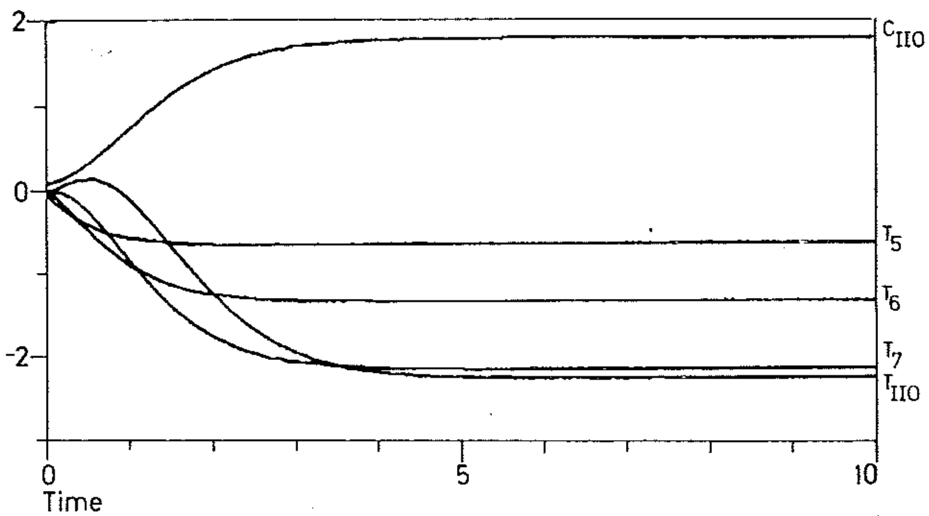

Figure 2.4. Response of the system to a step in quench flow (Q).

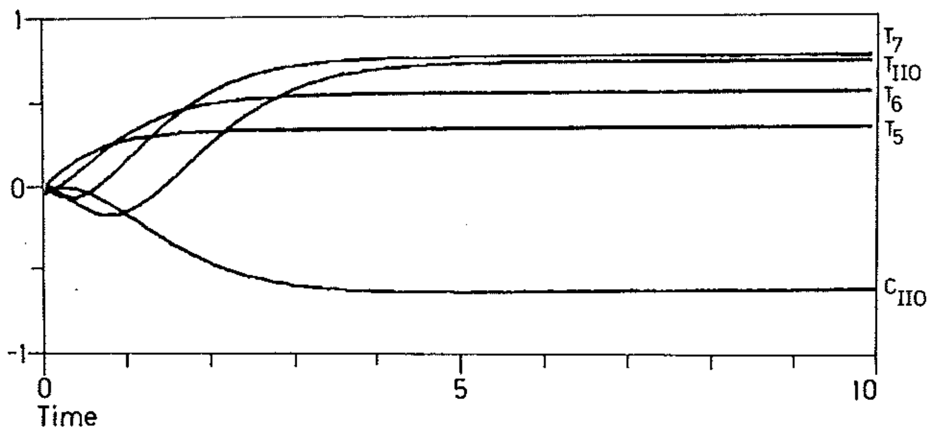

Figure 2.5. Response of the system to a unit step in the quench flow temperature (TQ).

Also notice that there are significant direct terms in the model. These terms were introduced in the model reduction procedure when rapid process dynamics was approximated by a direct signal transmission. The terms do not have physical significance. Actuator dynamics were introduced to avoid the direct terms. The actuators were modeled simply as first order systems with time constants of 0.1 time unit (≈ 9 s).

The seventh order model has the following poles:

$- 2.75 \pm i\ 0.77$

$- 2.14 \pm i\ 0.71$

$- 1.93 \pm i\ 0.30$

$- 1.93$

The transfer function relating outlet temperature to quench flow temperature has the following zeros:

44.3

1.09

$- 2.80$

$- 3.00 \pm i\ 0.40$

$- 3.02$

$- 98.9$

The transfer function thus has two zeros in the right half plane.

3. THE CONTROL PROBLEM

When controlling the reactor the major disturbances enter in the form of variations of the properties of the feed and the catalyst. In this particular case the fluctuations in the feed appear as disturbances in temperature and concentration at the outlet of the first bed. It has been shown by Foss (1978 b) that there is, surprisingly enough, little incentive in using feedforward from these signals. To achieve a desired concentration of the oxygen at the outlet of the second reactor it is necessary to measure oxygen concentration at the outlet of the second bed. It is therefore of interest to see what can be achieved by feedback from the outlet concentration to quench flow or temperature of the quench flow. Since the concentration measurement is expensive, it is also of interest to find out what can be achieved by feedback from the outlet temperature too.

There is no obvious choice of criteria for control. Roughly speaking it is desired to have the reactor return to its steady-state condition in reasonable time after a disturbance, but there is no obvious optimization criterion. Such features can, for example, be captured in a pole placement design formulation.

It is thus assumed that the control problem can be formulated as a deterministic pole placement problem. The purpose is thus to find a control law such that the transfer function from the reference value y_r to the controlled output y is given by Q/P. The desired poles are chosen in such a way that there is a reasonable compromise between the speed of return to the steady state and the magnitudes of the control signals. It is furthermore assumed that the observer polynomial is specified. This type of self-tuning regulators are described in detail in Åström, Westerberg, and Wittenmark (1978). It is not claimed that this is the best formulation of the problem of controlling the chemical reactor. It is, however, one possibility and a study of this type will certainly give insight into the properties of the self-tuning regulator based on pole placement when applied to a typical chemical process.

It is assumed that the reader is familiar with the self-tuning regulator based on pole-placement design which was described in Åström, Westerberg, and Wittenmark (1978). The parameters of a process model described by

$$y(t) + a_1 y(t-1) + \ldots + a_{n_A} y(t-n_A) =$$

$$= b_1 u(t-k-1) + \ldots + b_{n_B} u(t-k-n_B) \tag{3.1}$$

are estimated recursively. A regulator of the form

$$r_0 u(t) + r_1 u(t-1) + \ldots + r_{n_R} u(t-n_R) =$$

$$= t_1 y_r(t) + \ldots + t_{n_T} y(t-n_T+1) - s_1 y(t) - \ldots - s_{n_S} y(t-n_S+1) \tag{3.2}$$

is determined in such a way that the transfer function from the command signal y_r to the output y is Q/P. The observer polynomial is also specified. The structure of the STR is thus given by the integers n_A, n_B, and k, which specifies the model (3.1) and the degrees of the polynomials P, Q, and T or equivalently the integers n_R, n_S, and n_T. Two specific structures are discussed in more detail below.

Minimum-phase Systems

When controlling minimum-phase systems all process zeros can be cancelled and it can thus be required that the desired closed loop transfer function is 1/P. The regulator polynomials are then determined by

$$R = BR_1,$$

where

$$AR_1 + S = PT$$

and

$$\deg S < \deg A$$

Nonminimum-phase Systems

For nonminimum-phase systems the unstable process zeros can not be cancelled. To obtain a simple design algorithm it is assumed that all process zeros are retained as zeros of the desired closed loop. It is thus assumed that the desired closed loop transfer function is given by B/P, where B is the estimated numerator of the process transfer function.

Simulations

In the simulations the response of the system to step changes in the command signal was explored. The interactive simulation language SIMNON, developed by Elmqvist (1978), was used. A particular subsystem REG, written by Gustavsson (1978), was used to implement the adaptive regulators. The SIMNON systems used are listed in Appendix B.

4. STR CONTROL OF A MINIMUM-PHASE LOOP

It will first be attempted to control the oxygen concentration at the reactor outlet. The control variable is chosen as the quench flow. This control loop is fairly easy to control because the system is minimum-phase and the step response is monotone. Both explicit and implicit control algorithms have been investigated.

Implicit Self-tuning Algorithms

Since the system dynamics is minimum-phase it is possible to cancel all process zeros. The pole-placement problem can then be specified by requiring that the desired closed loop transfer function is given by

$$G_d = \frac{z^{k+1}}{P} \ . \qquad (4.1)$$

The polynomial P is chosen as

$$P(z) = K \cdot z^k (z^2 - 0.96\, z + 0.323) \qquad (4.2)$$

where K is such that P(1) = 1. The system thus has poles at the origin and at

$$z = 0.48 \pm 0.30\ i.$$

The complex poles correspond to the poles obtained when sampling a continuous time system with relative damping $\zeta = 0.707$. The specifications were determined from the desire to have a reasonable damping. The value of the sampling period was chosen so that the control signal had reasonable magnitude for a typical step change using the nominal model. The process model has zeros on the negative real axis. If a shorter sampling period was chosen the cancelled modes were well noticable in the control signal. The observer is chosen to have all its poles at the origin i.e. $T = z^{k+1}$. The adaptive STR algorithm II described in Åström, Wittenmark, and Westerberg (1978) was used to control the process. The initial values of the parameter estimates were chosen as:

$$s_0 = s_1 = \dots = s_{n_S} = 0$$

$$r_0 = 1$$

$$r_1 = r_2 = \dots = r_{n_R} = 0.$$

The initial "covariance" was chosen as 10 times the unit matrix. The forgetting factor was 0.98. Regulators having different complexity were investigated by simulation. The response of the system to step changes in the command signal was investigated.

EXAMPLE 4.1 $n_R = 1,\ n_S = 1$

A simple regulator with only two parameters was first investigated. When $n_R = 1$ and $n_S = 1$ the formula (3.2) reduces to

$$r_0\, u(t) = y_r(t) - s_0\, y(t).$$

The results of the simulation are shown in Fig. 4.1 and Fig. 4.2. The response of the closed loop system differs from the desired response specified by (4.1) and (4.2).

It is seen from Fig. 4.2 that the parameter s_0 has a comparatively small value ($s_0 \approx 0.05$). It was therefore also attempted to try a regulator with $s_0 = 0$, i.e. $n_R = 1$ and $n_S = 0$. This regulator corresponds to a pure feedforward

$$r_0\, u(t) = y_r(t).$$

The behaviour of this regulator is shown in the simulation results in Fig. 4.3 and Fig. 4.4 □

The connecting system and the macro used in this simulation together with a typical SIMNON dialog for the simulation is listed in Appendix C.

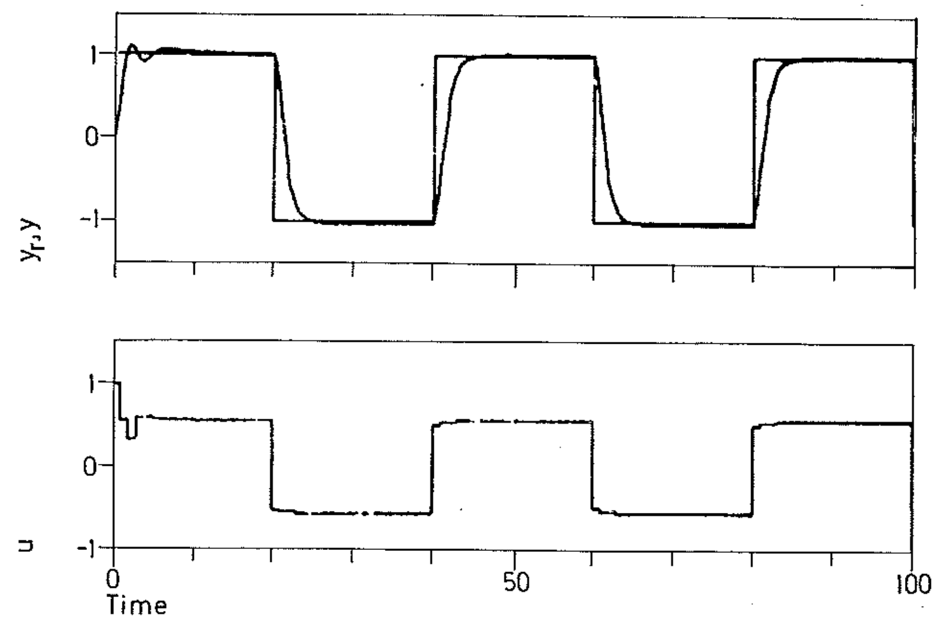

Figure 4.1. Reference y_r, process output y, and control signal u obtained when the implicit algorithm I1 with $n_R = 1$ and $n_S = 1$ is used to control a 7th order reactor model. The control variable is quench flow and the process output is oxygen concentration in the exit gas.

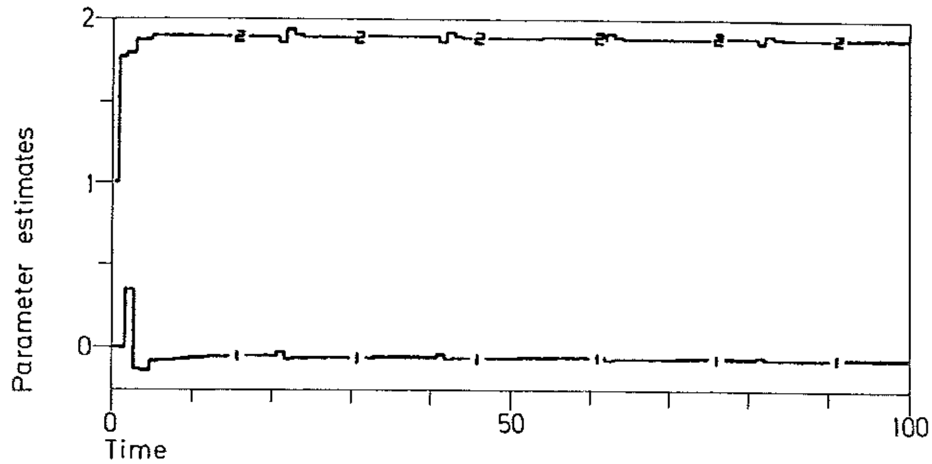

Figure 4.2. Parameter estimates corresponding to Fig. 4.1.

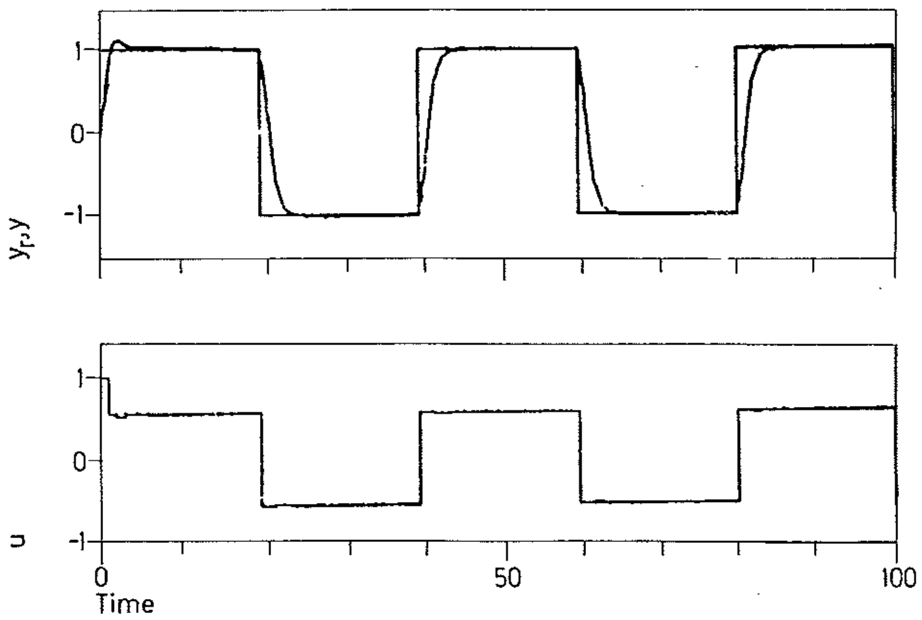

Figure 4.3. Reference y_r, process output y, and control signal u obtained when the implicit algorithm I1 with $n_R = 1$ and $n_S = 0$ is used to control a 7th order reactor model. The control variable is quench flow and the process output is oxygen concentration in the exit gas.

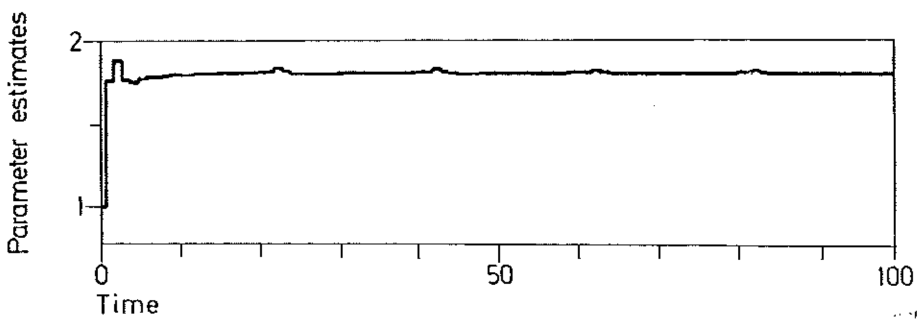

Figure 4.4. Parameter estimates corresponding to Fig. 4.3.

EXAMPLE 4.2. $n_R = 2,\ n_S = 2$

In this case the regulator has the structure

$$r_0\ u(t) + r_1\ u(t-1) = y_r(t) - s_0\ y(t) - s_1\ y(t-1).$$

It is thus characterized by four parameters r_0, r_1, s_0, and s_1. The results of a simulation of the closed loop system are shown in Fig. 4.5 and Fig. 4.6. Notice that the parameter estimates change substantially over the first 100 steps although the closed loop response is almost invariant after a few step changes. The parameter values obtained after a longer simulation period are listed in Table 4.1.

Table 4.1. Parameter estimates obtained at different times for the regulator with $n_R = 2$ and $n_S = 2$.

t	r_0	r_1	s_0	s_1
100	1.841	0.948	-1.080	0.556
200	1.846	1.064	-1.220	0.634
300	1.846	1.077	-1.237	0.642

□

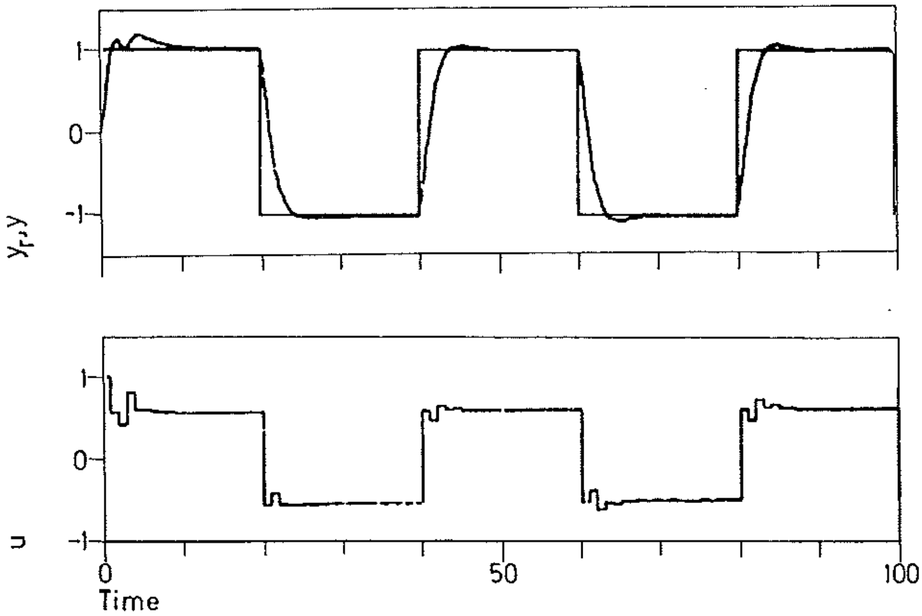

Figure 4.5. Reference y_r, process output y, and control signal u obtained when the implicit algorithm I1 with $n_R = 2$, $n_S = 2$ is used to control a 7th order reactor model. The control variable is quench flow and the process output is oxygen concentration in the exit gas.

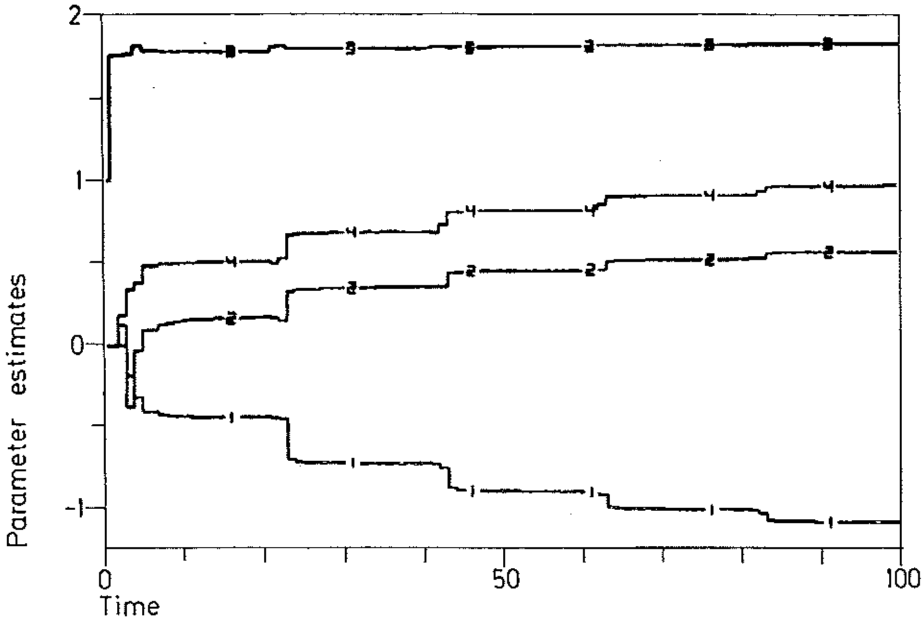

Figure 4.6. Parameter estimates corresponding to Fig. 4.5.

EXAMPLE 4.3. $n_R = 3,\ n_S = 3$

In this case the complexity of the regulator is increased further. The regulator has the form

$$r_0\, u(t) + r_1\, u(t-1) + r_2\, u(t-2) =$$

$$= y_r(t) - s_0\, y(t) - s_1\, y(t-1) - s_2\, y(t-2).$$

The six regulator parameters are estimated directly. The results of a simulation of the closed loop system are shown in Fig. 4.7 and Fig. 4.8. Notice that both the output and the control signal are very close to the signals obtained in Example 4.3.

In this example it was also possible to find several equilibria for the parameter estimates. The simulation results in Fig. 4.9 and Fig. 4.10 shows what happens if the parameters are initialized close to the equilibrium solutions found in Example 4.3. Without analysis it is of course not possible to judge if the solutions shown are true equilibria. It is, however, unquestionable that the estimates remain close to the given values for a substantial time. □

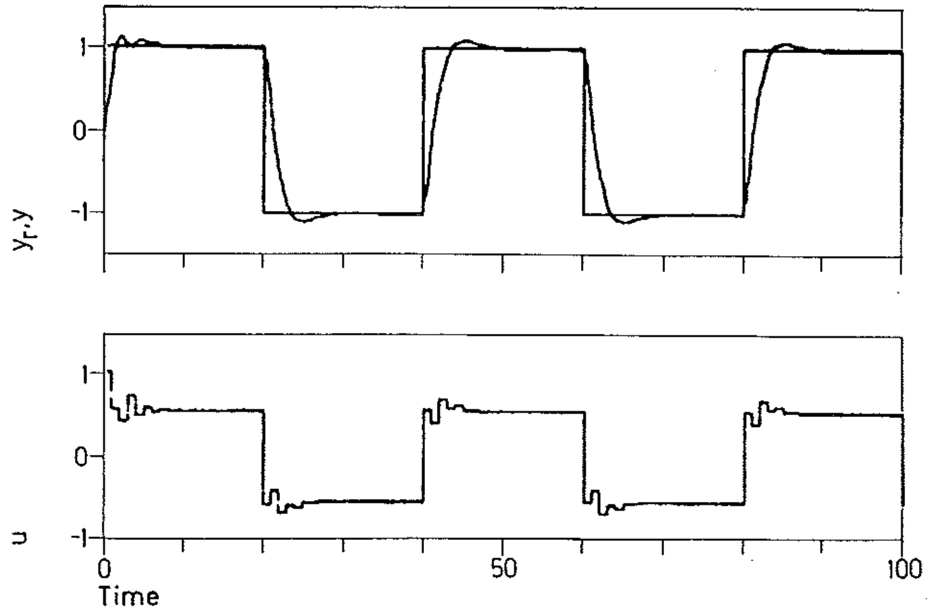

Figure 4.7. Reference y_r, process output y, and control signal u obtained when the implicit algorithm I1 with $n_R = 3$ and $n_S = 3$ is used to control a 7th order reactor model. The control variable is quench flow and the process output is oxygen concentration in the exit gas.

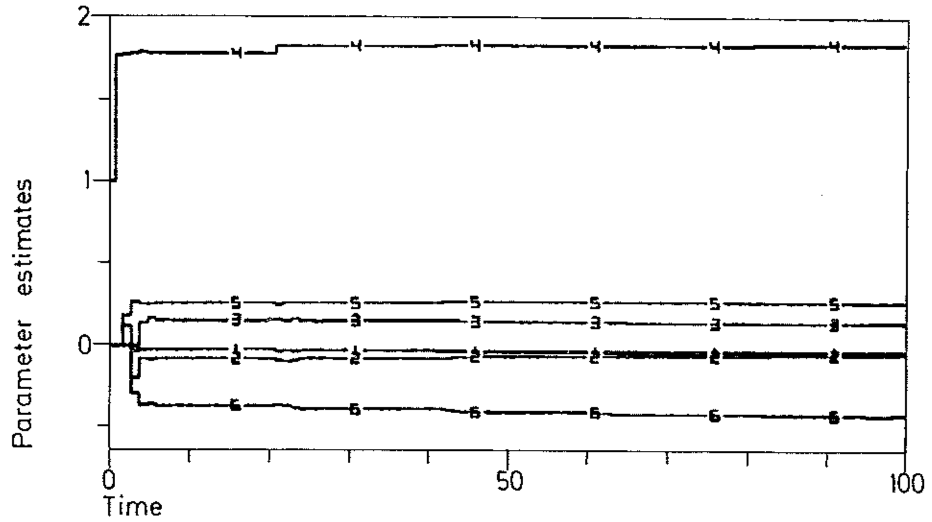

Figure 4.8. Parameter estimates corresponding to Fig. 4.7.

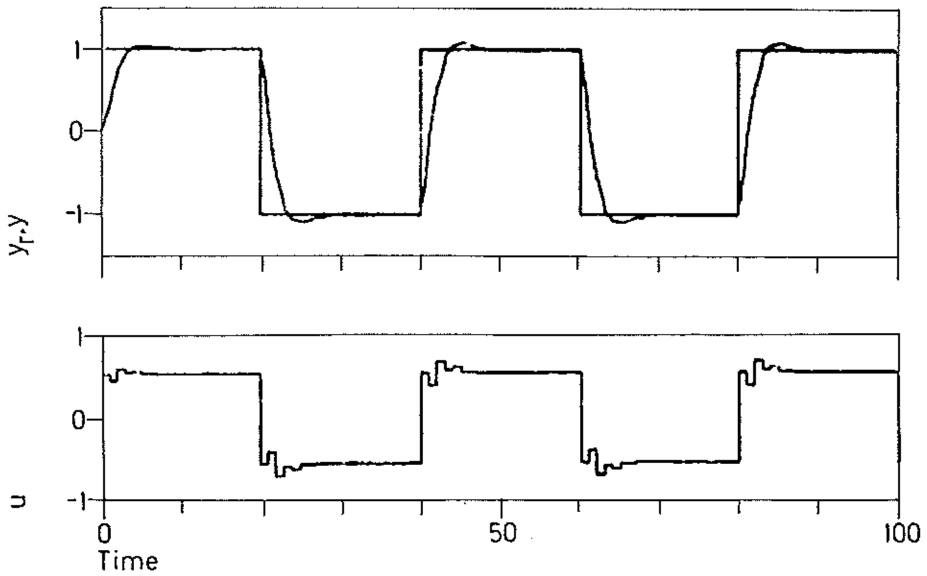

Figure 4.9. Reference y_r, process output y, and control signal u obtained when the implicit algorithm I1 with n_R = 3 and n_S = 3 is used to control a 7th order reactor model. The control variable is quench flow and the process output is oxygen concentration in the exit gas. The simulation is identical to the one shown in Fig. 4.7 except for the initial values of the parameter estimates.

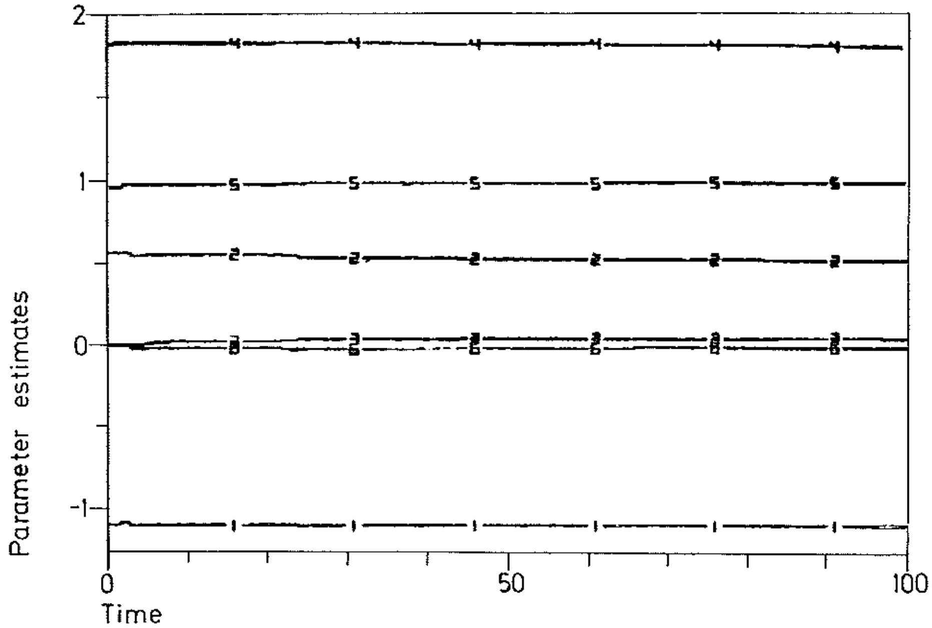

Figure 4.10. Parameter estimates corresponding to Fig. 4.9.

The examples show several interesting features. First it is clear that very good results can be obtained with a regulator having few parameters. From a practical point of view it appears reasonable to choose $n_R = 2$ and $n_S = 2$. This is further illustrated in Fig. 4.11 which is an enlargement of the step responses of the regulators in the different examples. Comparing the curves labeled (2,2) and (3,3) in the figure, it is seen that the regulators in Examples 4.2 and 4.3 give step responses which are similar. The step responses also agree with the specified response at the sampling points. The regulators with fewer than four parameters give responses that deviate from the desired response. In the particular case when all process zeros are cancelled, the complexity of the regulator can thus be determined simply by analysing how much the step response obtained differs from the step response of the desired system.

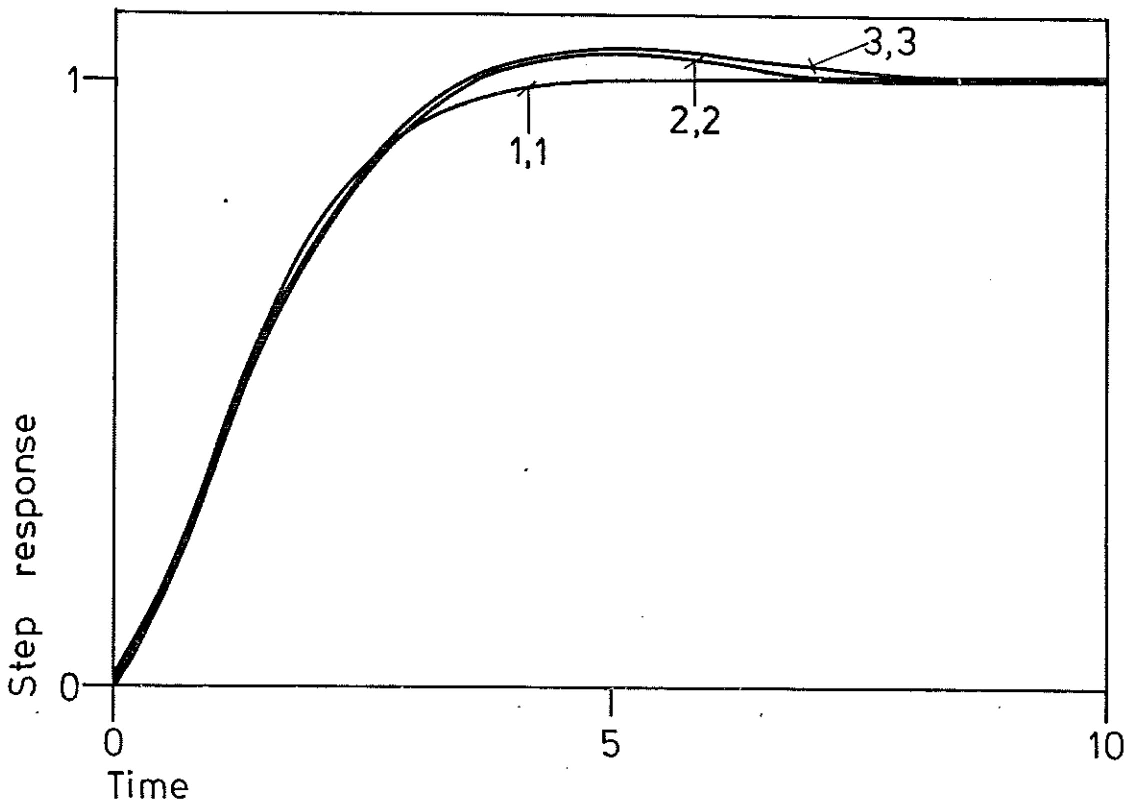

Figure 4.11. Step responses of the systems in Examples 4.1, 4.2, and 4.3.

Notice that the properties of the feedback laws obtained in the different examples have properties which differ substantially. This can be seen from the graphs of the parameter estimates. The limiting values of the parameter estimates are summarized in Table 4.2.

Table 4.2. Limiting estimates of the regulator parameters.

Example	r_0	r_1	r_2	s_0	s_1	s_2	S(1)/T(1)
4.1 a	1.822			0			0
4.1 b	1.900			-0.050			-0.050
4.2	1.846	1.077		-1.237	0.643		-0.594
4.3 a	1.840	0.273	-0.416	-0.028	0.041	0.145	0.158
4.3 b	1.844	0.991	-0.015	-1.100	0.523	0.041	-0.536

In Table 4.2 there is also an entry labeled S(1)/T(1). This number gives the ratio between the low frequency gains of the feedback and feedforward transfer functions. Notice that there are drastic differences between these numbers both in sign and magnitude. The feedforward term dominates in Examples 4.1 a, 4.1 b, and 4.3 a. It is only in examples 4.2 and 4.3 b that there is an appreciable feedback. This means that there are substantial differences in the ability of the regulators to reject disturbances although they have practically the same step responses. Also notice that the feedback gain is positive in all cases except 4.3 a.

Explicit Self-tuning Algorithms

It will now be attempted to control the reactor using the explicit STR algorithm E2 described in Åström, Westerberg, and Wittenmark (1978). A process model of the form (3.1) is thus estimated recursively and the regulator parameters are then calculated using a pole-placement design where all process zeros are retained. The desired closed loop response is given by the pulse transfer function

$$G_d = \frac{B(z)}{P(z)} ,$$

where B is the estimated process numerator and the polynomial P has its zeros at the origin and in

$$z_{1,2} = 0.17 \pm 0.30\ i.$$

These poles correspond to the poles obtained when a continuous time system with relative damping $\zeta = 0.707$ and $\omega = 1.5$ is sampled with a sampling period of one time unit. The observer polynomial was chosen to have all its zeros at the origin.

The initial values of the parameter estimates were chosen as

$$a_1(0) = a_2(0) = \ldots = a_{n_A}(0) = 0$$
$$b_1(0) = b_2(0) = \ldots = b_{n_B-1}(0) = 0$$
$$b_{n_B}(0) = 1.$$

The initial "covariance" was ten times the unit matrix and the forgetting factor was 0.98 in all simulations. The command signal was assumed to be a square wave. Self-tuning regulators having different complexity were tested. Some results are summarized in the following examples.

The connecting system and the macro used in this simulation is listed in Appendix D together with a typical SIMNON dialog.

EXAMPLE 4.4 $n_A = 1,\ n_B = 1$

It was first attempted to estimate a model with only two parameters. The results obtained are shown in Fig. 4.12. The parameter estimates are shown in Fig. 4.13.

It is seen from Fig. 4.12 that the step response converges quickly. The response has, however, too low damping which is due to the fact that the order of the estimated model is too low. The parameter estimates shown in Fig. 4.13 indicate that the solution obtained do not correspond to constant parameter values because the estimates change at each step change. □

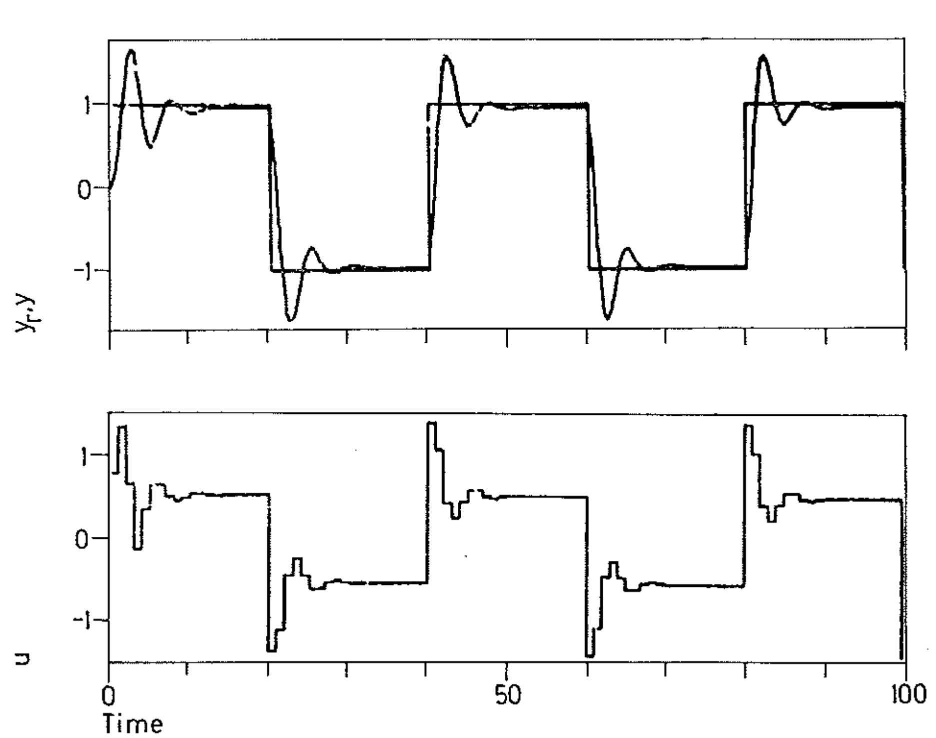

Figure 4.12. Reference value y_r, process output y, and control signal u obtained when the explicit algorithm E2 with $n_A = 1$ and $n_B = 1$ is used to control a 7th order reactor model. The control variable is quench flow and the process output is oxygen concentration in the exit gas.

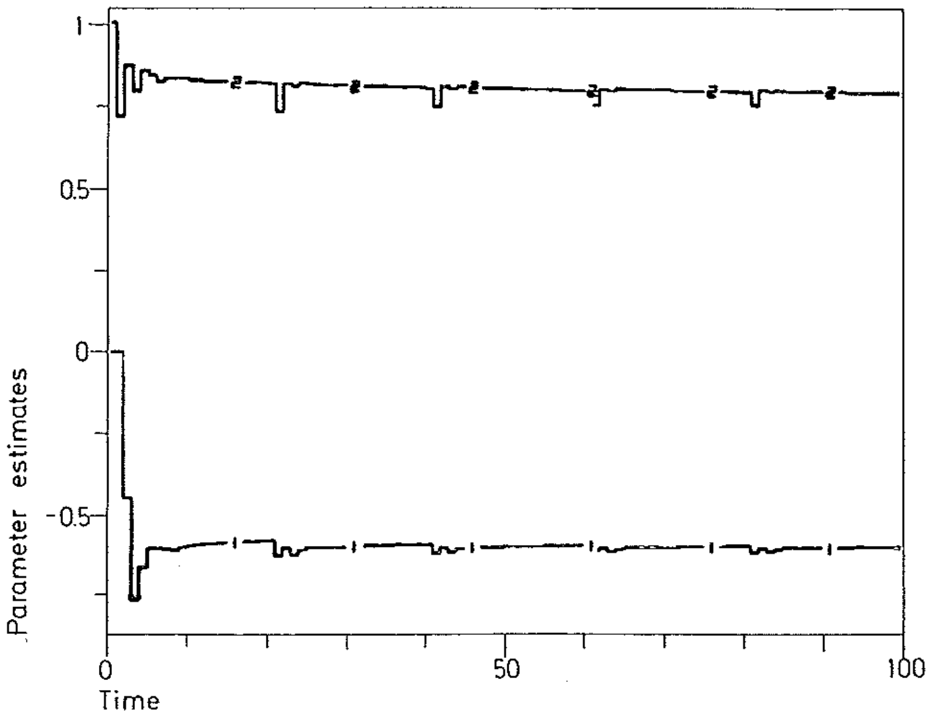

Figure 4.13. Parameter estimates corresponding to Fig. 4.12.

EXAMPLE 4.5 $n_A = 2,\ n_B = 2$

The results obtained when a process model characterized by $n_A = 2$ and $n_B = 2$ are shown in Fig. 4.14 and Fig. 4.15.

Again the step response settles fairly quickly in spite of the fact that the parameter estimates change considerably. It is seen from Fig. 4.15 that the parameter estimates have not converged after 100 steps. The parameter estimates obtained after 100 and 200 steps are listed below.

$a_1(100) = -0.458$ $a_1(200) = -0.433$

$a_2(100) = 0.062$ $a_2(200) = 0.087$

$b_1(100) = 0.662$ $b_1(200) = 0.668$

$b_2(100) = 0.445$ $b_2(200) = 0.399$

A comparison with Fig. 4.12 shows that the damping of the closed loop system is improved considerably when the number of parameters are increased. □

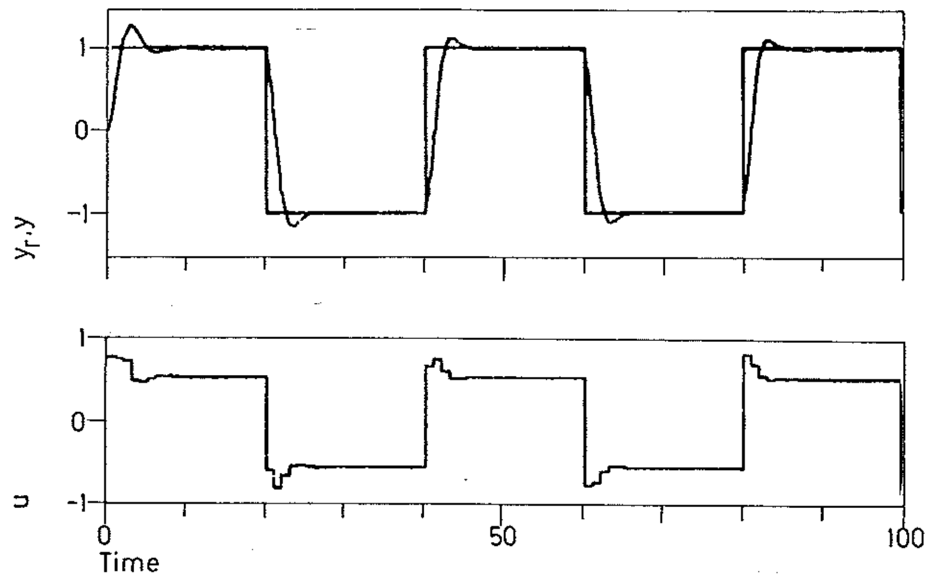

Figure 4.14. Reference value y_r, process output y, and control signal u obtained when the explicit algorithm E2 with $n_A = 2$ and $n_B = 2$ is used to control a 7th order reactor model. The control variable is quench flow and the process output is oxygen concentration in the exit gas.

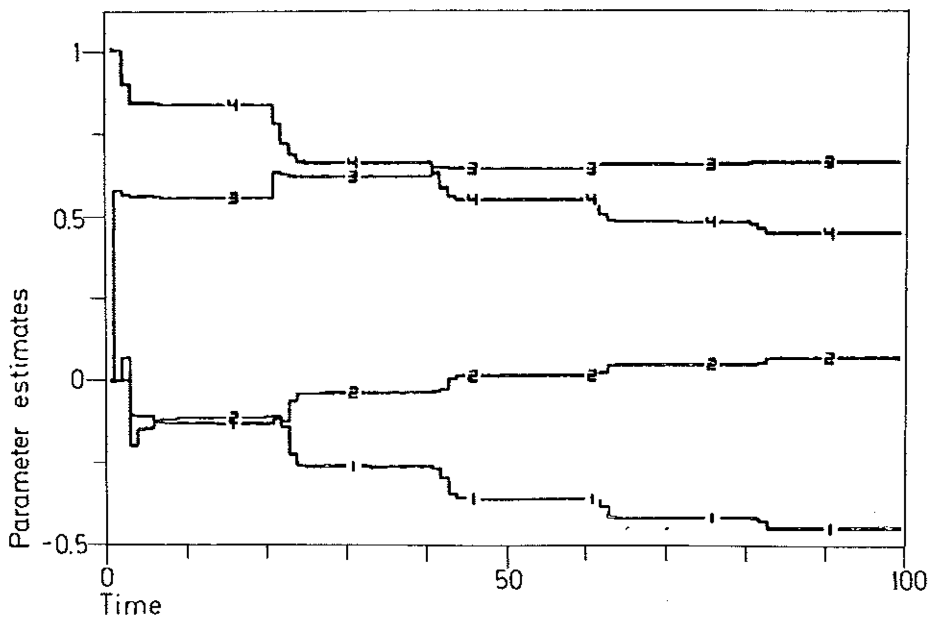

Figure 4.15. Parameter estimates corresponding to Fig. 4.14.

EXAMPLE 4.6 $n_A = 3,\ n_B = 3$

The results obtained when the number of estimated parameters are increased further are shown in Fig. 4.16 and Fig. 4.17.

The step responses again settles fairly quickly although the parameters have not converged after 100 steps. The parameter estimates obtained at times 100 and 200 are listed below.

$a_1(100) = -0.128$	$a_1(200) = -0.142$
$a_2(100) = -0.018$	$a_2(200) = -0.069$
$a_3(100) = -0.011$	$a_3(200) = -0.016$
$b_1(100) = 0.672$	$b_1(200) = 0.670$
$b_2(100) = 0.632$	$b_2(200) = 0.638$
$b_3(100) = 0.242$	$b_3(200) = 0.171$

Notice that the estimates a_2 and a_3 are fairly small and that there are noticable differences between the estimates obtained at $t = 100$ and $t = 200$. Also notice that there are small differences in the step responses, but that the differences in the control signals are more noticable. □

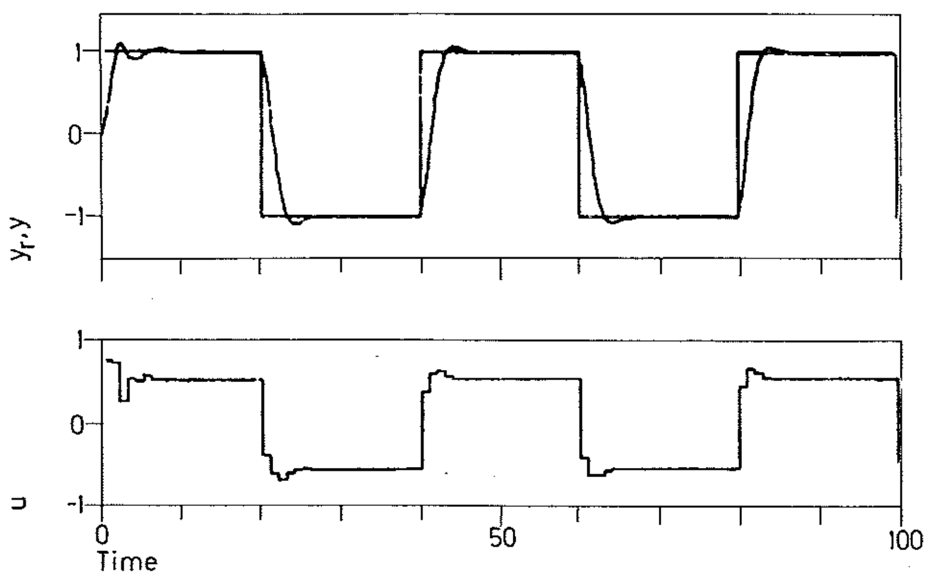

Figure 4.16. Reference value y_r, process output y, and control signal u obtained when the explicit algorithm E2 with $n_A = 3$ and $n_B = 3$ is used to control a 7th order reactor model. The control variable is quench flow and the process output is oxygen concentration in the exit gas.

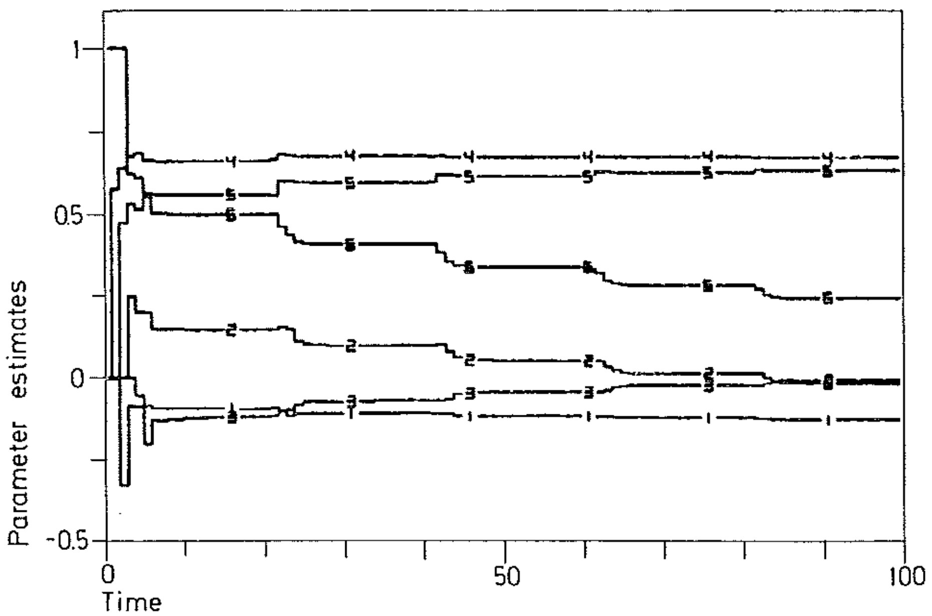

Figure 4.17. Parameter estimates corresponding to Fig. 4.16.

The step responses obtained for systems having different model complexity are compared in Fig. 4.18. It is seen from this figure that the damping is improved by increasing the order of the estimated model but also that the response time increases when more b-parameters are estimated. Several other model structures were also investigated although they are not reported fully here. It was found that systems with $n_A \geq 1$ and $n_B \geq 3$ had almost the same step responses. It was also observed that the convergence rate was slower for systems having many parameters. The parameter estimates could in fact vary substantially over a long time although the step responses were virtually the same.

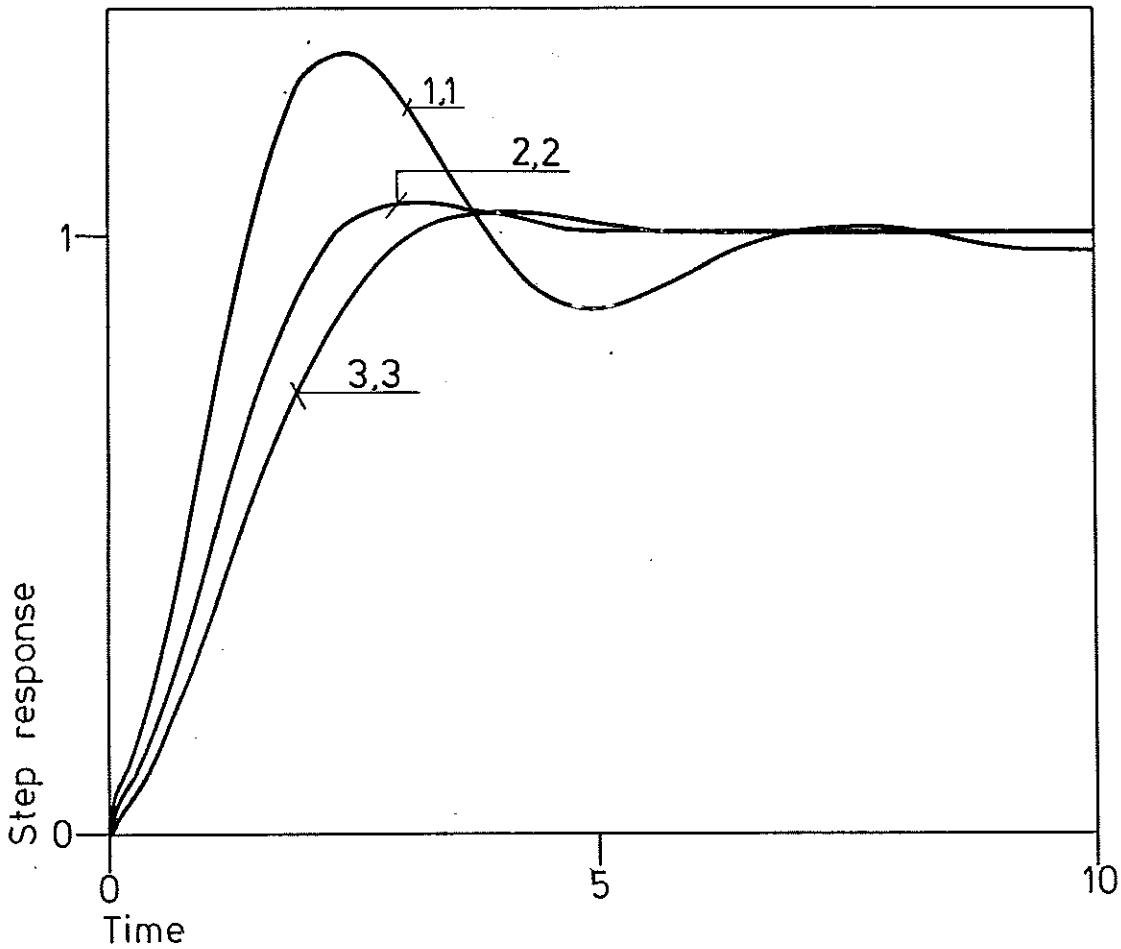

Figure 4.18. Step responses at time 80 for the regulators having different complexity.

The estimated coefficients of the process model (3.1) obtained at times 100 and 200 for regulators having different structures are summarized in Table 4.3. It is seen in this table that the estimates of the parameter b_1 are close in all cases. The estimates of the other coefficients do, however, vary significantly. The estimates of a_2, a_3, and b_4 are also small in all cases. It thus seems reasonable to choose a regulator with $n_A = 1$ and $n_B = 3$. This is also consistent with the investigation of the step responses of regulators having different structures.

Table 4.3. Coefficients of the process pulse transfer function estimated at different times for regulators having different structures.

n_A	n_B	t	a_1	a_2	a_3	b_1	b_2	b_3	b_4
1	1	100	-0.589			0.795			
2	2	100	-0.458	0.0616		0.662	0.445		
2	2	200	-0.505	0.0870		0.668	0.399		
3	3	100	-0.128	-0.0186	-0.0108	0.672	0.632	0.242	
3	3	200	-0.142	-0.0690	0.0166	0.670	0.638	0.171	
1	3	100	-0.230	-	-	0.671	0.568	0.174	
1	3	200	-0.256	-	-	0.670	0.562	0.133	
1	4	100	-0.182	-	-	0.668	0.613	0.177	0.0430
1	4	200	-0.206	-	-	0.669	0.597	0.164	0.0256
2	4	100	-0.297	0.0538	-	0.668	0.536	0.130	0.0550
2	4	200	-0.294	0.0247	-	0.669	0.538	0.117	0.0162

Poles and zeros, DC-gain, and the first error coefficient e_1 for process models of different complexity obtained at different times are listed in Table 4.4. This table shows that the poles and zeros of the different models may vary substantially although the step responses obtained from the regulators designed from the models are very similar. It is also seen in the table that the DC-gain and the first error coefficient of the models are remarkably constant. The choice $n_A = 1$ and $n_B = 3$, which was previously found to be reasonable, gives a model with one real pole and two complex zeros. Also notice in Table 4.4 that there may be substantial differences between the models obtained at different times. Compare for example the models for $n_A = 3$ and $n_B = 3$.

Table 4.4. Poles, zeros, DC-gain, and error coefficients of the process models estimated at different times for adaptive regulators having different structures.

n_A	n_B	t	Poles	Zeros	Gain	e_1
1	1	100	0.589		1.937	-4.72
2	2	100	0.458±i0.096	-0.672	1.835	-3.59
2	2	200	0.200±i0.217	-0.598	1.834	-3.56
3	3	100	-0.088±i0.166, 0.305	-0.470±i0.373	1.836	-3.59
3	3	200	0.216±i0.101, -0.291	-0.476±i0.168	1.835	-3.58
1	3	100	0.230	-0.423±i0.284	1.834	-3.57
1	3	200	0.256	-0.419±i0.153	1.835	-3.58
1	4	100	0.182	-0.126±i0.284, -0.665	1.835	-3.58
1	4	200	0.206		1.835	-3.58
2	4	100	0.148±i0.178	-0.054±i0.340, -0.693	1.833	-3.56
2	4	200	0.146±i0.055	-0.115±i0.170, -0.574	1.835	-3.57

5. STR CONTROL OF A NONMINIMUM-PHASE LOOP

Composition measurements are costly. It is therefore of interest to investigate if the outlet composition of the second bed can be controlled indirectly by feedback from the outlet temperature. To obtain a good system it is of course necessary to have some composition measurement. If indirect control via the temperature measurement works well it may, however, be sufficient to use laboratory measurements for calibration only. The gas temperature at the reactor outlet can be controlled either by the quench flow (Q) or by the temperature (TQ) of the quench flow. It is slightly more difficult to use the quench flow temperature as the control variable. This variable is therefore chosen as the control variable to obtain the worst case.

It was mentioned in Chapter 2 that the control problem is difficult because the linearized process dynamics relating outlet temperature to quench flow temperature is nonminimum-phase. Compare Fig. 2.5. In the pole-placement design formulation it is therefore specified that the desired closed loop transfer function is given by

$$G_d = \frac{B}{P} \tag{5.1}$$

where the numerator polynomial is the B-polynomial in the estimated process model. This means that the specified response will change if the process dynamics changes. There will, however, not be any difficulties to handle nonminimum phase systems. The polynomial P is chosen as

$$P = K \cdot z(z^2 - 0.342 + 0.12).$$

This means that the desired poles are at the origin and at

$$z_{1,2} = 0.17 \pm 0.30\ i.$$

These poles correspond to those obtained when a second order system with damping $\zeta = 0.707$ is samples with a period such that $\omega h = 1.5$. The polynomial P is normalized in such a way that $G_d(1) = 1$, since the B-polynomial is updated at each step. This means that the polynomial P has to be renormalized at each step also. The observer poles are all chosen to be at the origin. The STR algorithm E2 presented in Åström, Westerberg, and Wittenmark (1978) was used in all simulations.

Self-tuning regulators having different structure were investigated in the numerical experiments. Models of the form (3.1) having different numbers of estimated parameters were explored. The purpose was to investigate the influence of model complexity on the performance of the closed loop system.

In the simulations the self-tuning regulator was connected to the process model described in Chapter 2. A square wave command input was applied and the performance of the system was observed. The sampling period was chosen as 1 time unit in all simulations. The following initial values were chosen in all simulations.

$$a_1(0) = a_2(0) = \ldots = a_{n_A}(0) = 0$$
$$b_1(0) = b_2(0) = \ldots = b_{n_B-1}(0) = 0$$
$$b_{n_B}(0) = 1.$$

The initial covariance was chosen as ten times the unit matrix and the forgetting factor was 0.98 in all simulations. The results obtained in some examples are given in the following.

EXAMPLE 5.1 $n_A = 1,\ n_B = 1$

In this example a model (3.1) of first order was estimated. Such a model has small possibilities to describe the process dynamics. The controlled output did also diverge for many different choices of initial conditions and the system could not be made to function properly. It was therefore attempted to increase the value of k in the model (3.1). When this was done the results in Fig. 5.1 were obtained. The parameter estimates are shown in Fig. 5.2. □

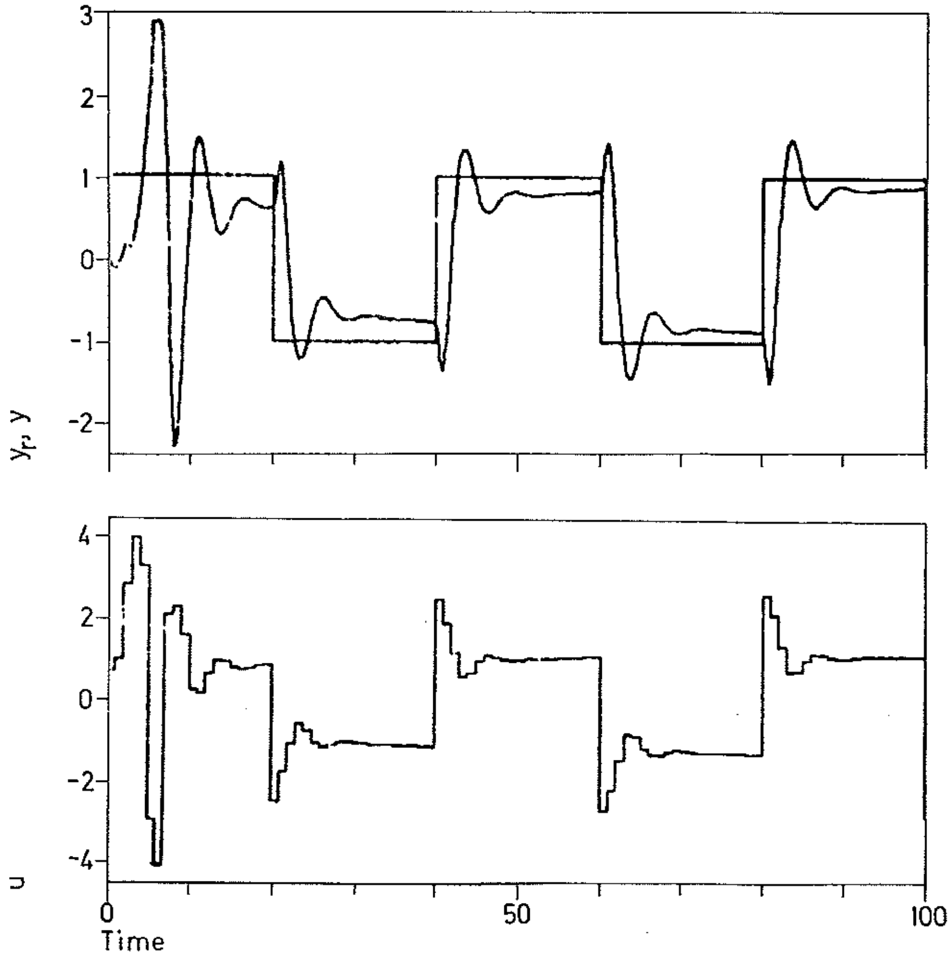

Figure 5.1. Process inputs and outputs when the explicit algorithm E2 with $n_A = 1$, $n_B = 1$, and $k = 1$ is used to control a 7th order model of the reactor. The control variable is quench flow temperature and the process output is the temperature of the exit gas.

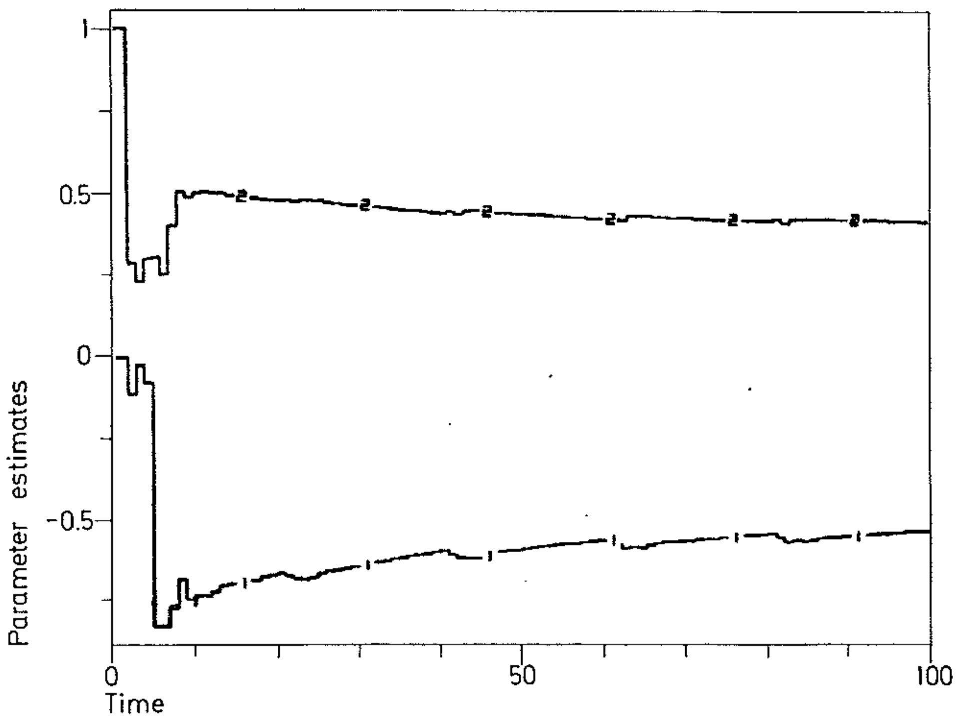

Figure 5.2. Parameter estimates corresponding to Fig. 5.1.

EXAMPLE 5.2 $n_A = 1$, $n_B = 2$

In this case a model (5.3) of second order with $n_A = 1$ and $n_B = 2$ was estimated. Such a model is the simplest model which can describe a sampled nonminimum-phase system. The output and the control variables obtained are shown in Fig. 5.3. Notice that the step response converges quickly. The step response differs, however, substantially from the specified step response. The parameter estimates are shown in Fig. 5.4. Notice that the parameter estimates change immediately after the changes in the command signal. □

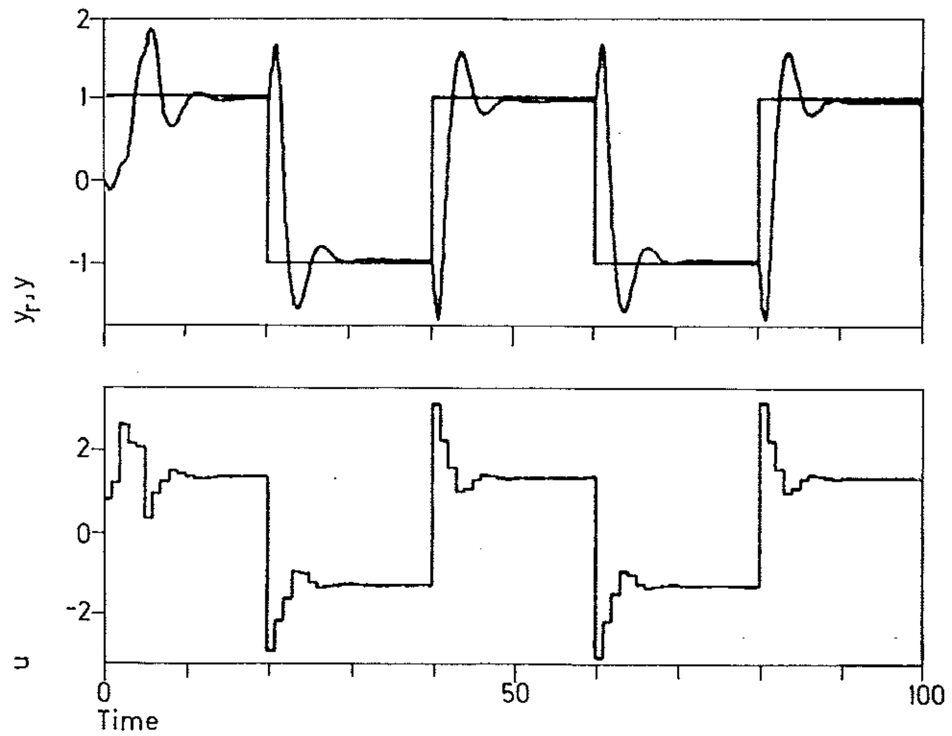

Figure 5.3. Process inputs and outputs when the explicit STR algorithm E2 with $n_A = 1$ and $n_B = 2$ is used to control a 7th order model of the reactor. The control variable is quench flow temperature and the process output is the temperature of the exit gas.

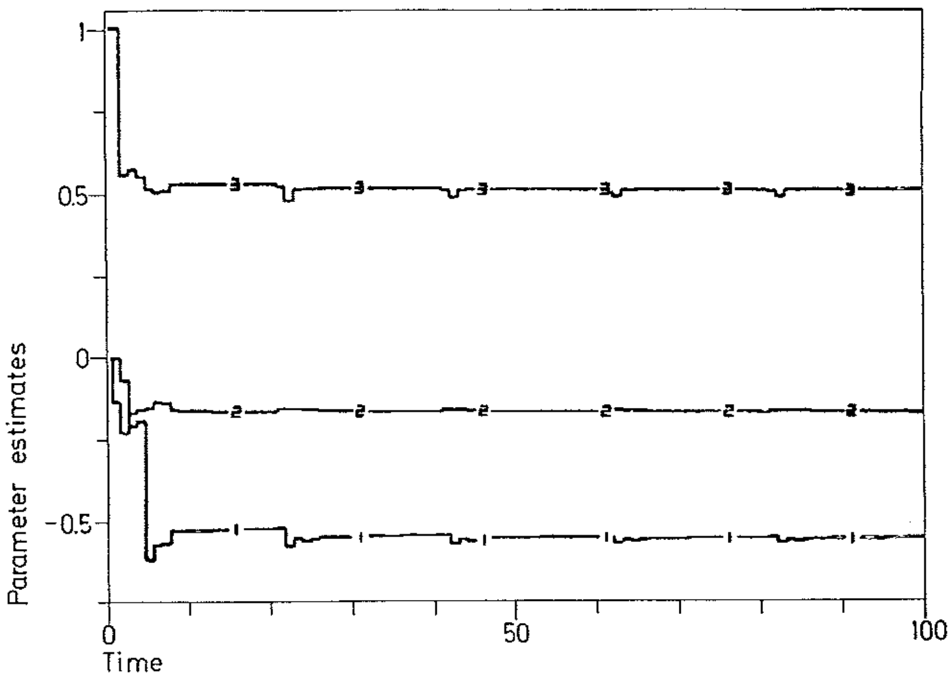

Figure 5.4. Parameter estimates corresponding to Fig. 5.3.

EXAMPLE 6.3 $n_A = 1$, $n_B = 3$

This example is identical to the previous example but the number of b-parameters in the model (3.1) has been increased to $n_B = 3$. The process inputs and outputs are shown in Fig. 5.5 and the parameter estimates in Fig. 5.6. A comparison with Fig. 5.3 shows that the overshoot is reduced drastically. □

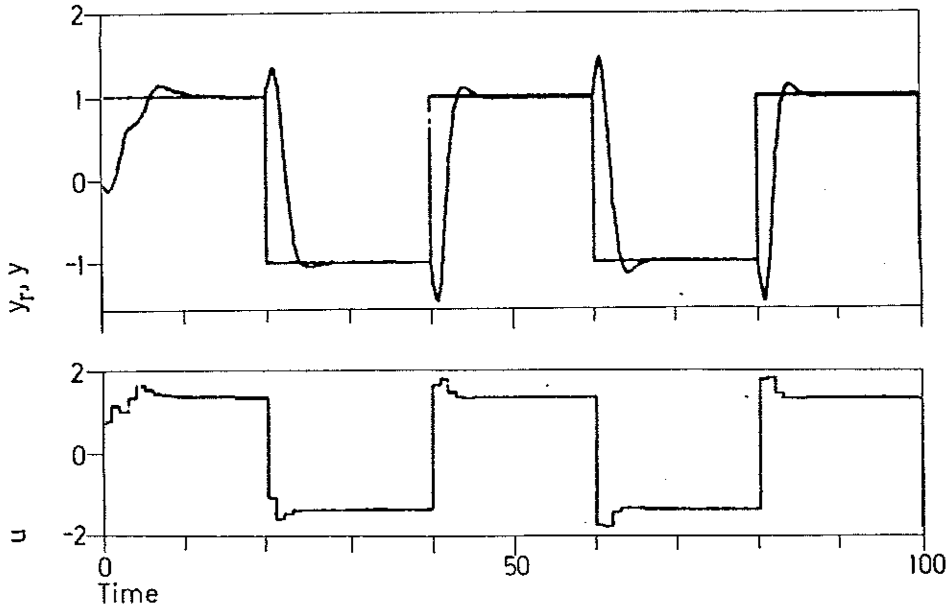

Figure 5.5. Process inputs and outputs when the explicit STR algorithm E2 with $n_A = 1$ and $n_B = 3$ is used to control a 7th order model of the reactor. The control variable is quench flow temperature and the process output is the temperature of the exit gas.

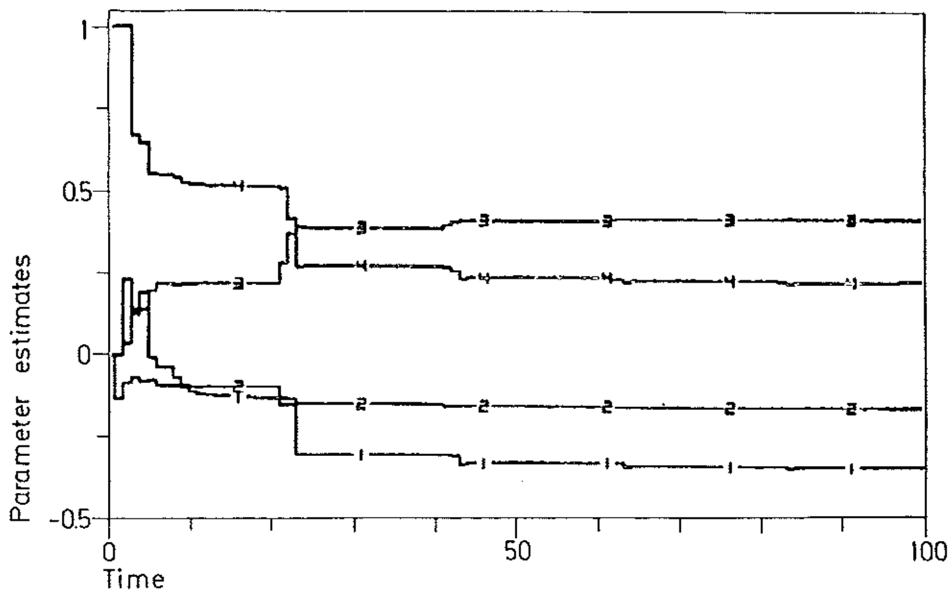

Figure 5.6. Parameter estimates corresponding to Fig. 5.5.

EXAMPLE 5.4 $n_A = 2$, $n_B = 3$

In this example the model complexity is increased by estimating an additional a-parameter. The process inputs and outputs are shown in Fig. 5.7 and the parameter estimates are shown in Fig. 5.8. □

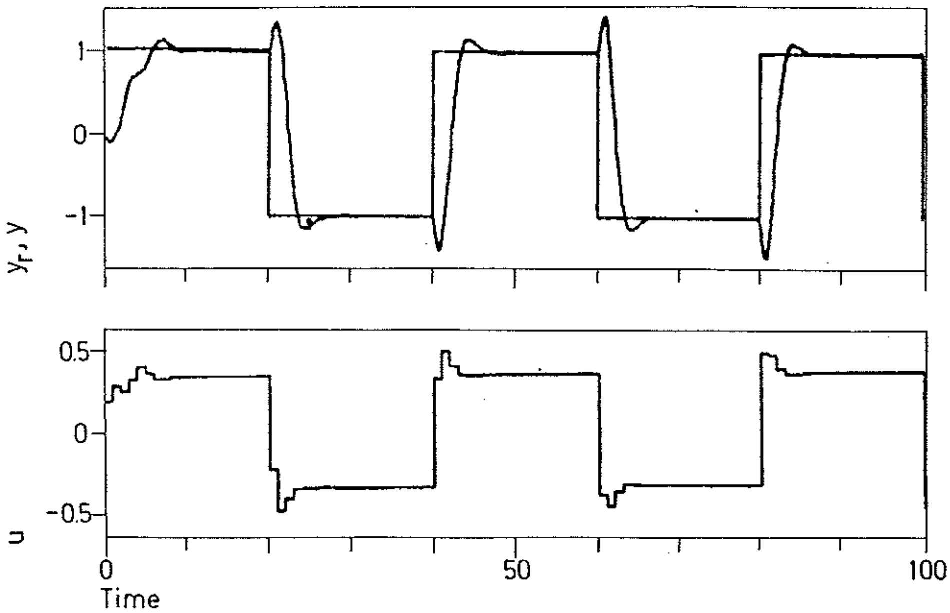

Figure 5.7. Process inputs and outputs when the explicit STR algorithm E2 with $n_A = 2$ and $n_B = 3$ is used to control a 7th order model of the reactor. The control variable is quench flow temperature and the process output is the temperature of the exit gas.

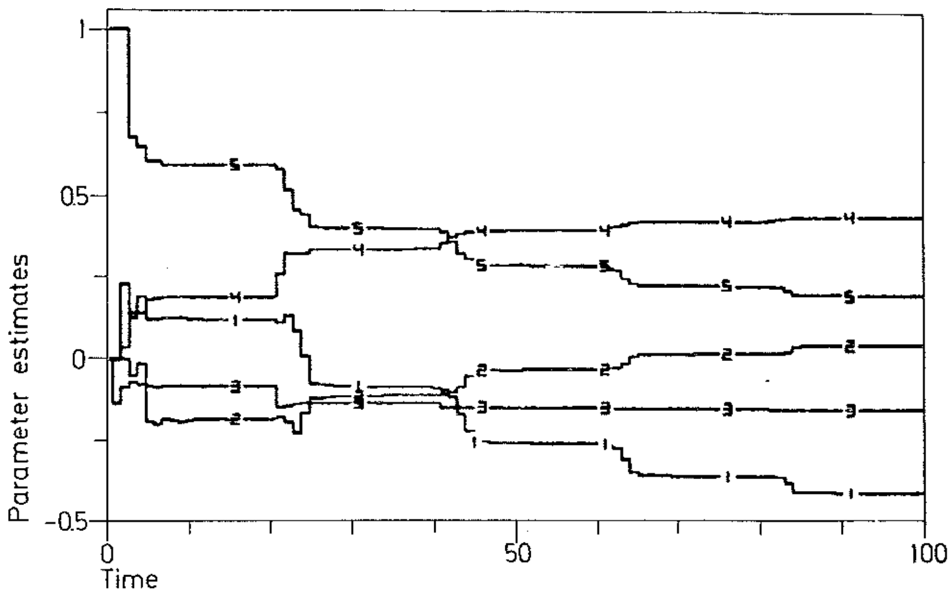

Figure 5.8. Parameter estimates corresponding to Fig. 5.7.

EXAMPLE 5.5 $n_A = 3,\ n_B = 3$

In this case the model complexity is further increased by adding one additional a-parameter. The process inputs and outputs are shown in Fig. 5.9 and the parameter estimates are shown in Fig. 5.10. □

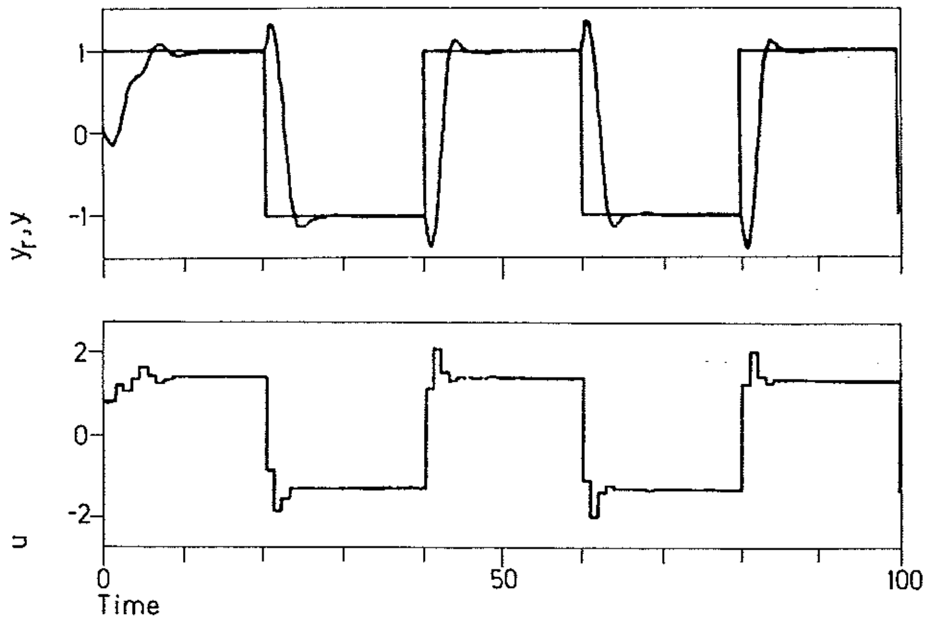

Figure 5.9. Process inputs and outputs when the explicit STR algorithm E2 with $n_A = n_B = 3$ is used to control a 7th order model of the reactor. The control variable is quench flow temperature and the process output is the temperature of the exit gas.

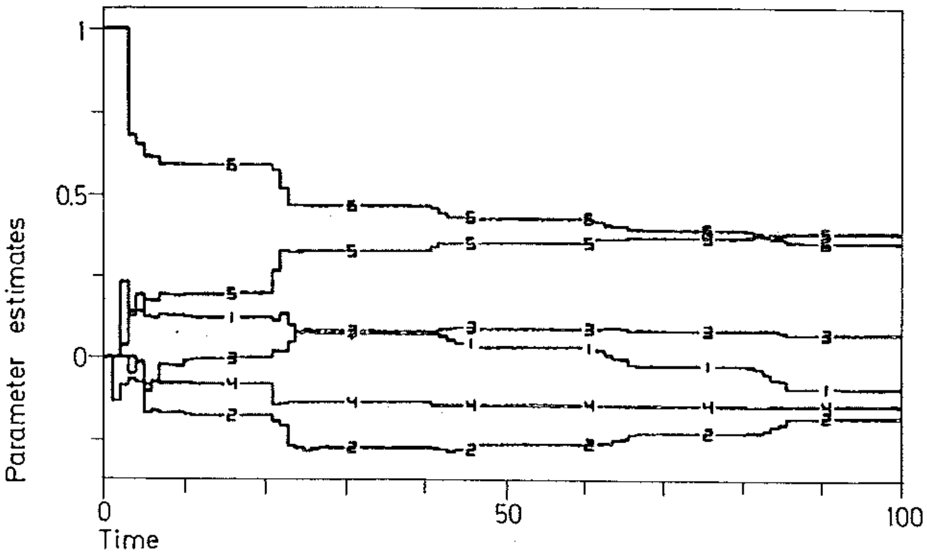

Figure 5.10. Parameter estimates corresponding to Fig. 5.9.

Comparison of Regulators Having Different Complexity

Some empirical observations made in the simulations will now be summarized. If the model used in the STR is too simple the closed loop system does not have the desired performance. The closed loop system may even be unstable as was observed for $n_A = n_B = 1$ and $k = 0$. When the complexity of the model is increased the performance of the closed loop system will approach the specified performance. It was also observed that the convergence was comparatively fast. The step response did not change much after the first two step changes. Compare for example Figures 5.1, 5.3, 5.5, 5.7, and 5.9.

Table 5.1. Coefficients of the process pulse transfer functions estimated at different times for regulators having different structures.

n_A	n_B	t	a_1	a_2	a_3	b_1	b_2	b_3	b_4
1	1	100	-0.541				0.400		
1	2	100	-0.546			-0.164	0.514		
1	3	100	-0.330			-0.152	0.422	0.229	
1	3	400	-0.335			-0.154	0.424	0.224	
2	3	100	-0.402	0.0397		-0.153	0.433	0.193	
2	3	200	-0.457	0.0675		-0.154	0.445	0.162	
3	3	100							
3	3	200	-0.428	0.0458	0.0063	-0.154	0.440	0.177	
3	3	300	-0.488	0.0903	-0.0064	-0.154	0.450	0.147	
3	3	400	-0.494	0.0949	-0.0077	-0.154	0.451	0.144	
2	4	100	0.0253	-0.0831		-0.153	0.372	0.359	0.122
2	4	200	-0.184	-0.0211		-0.154	0.403	0.277	0.0646
2	4	400	-0.378	-0.0420		-0.154	0.433	0.196	0.0190
2	4	600	-0.380	-0.0431		-0.154	0.433	0.194	0.0183

The parameter estimates obtained at different times for regulators having different structures are summarized in Table 5.1. It is seen in the table that the estimates of b_1 are very close in all cases given in the table. The estimates of the parameter b_2 are also fairly close but the other parameter estimates vary substantially. It is also seen in Table 5.1 that the estimates of the parameters a_2, a_3, and b_4 are all small, particularly for large t. It thus seems reasonable to choose a regulator having the structure $n_A = 1$ and $n_B = 3$. Notice that this model is much simpler than the model of the process and the actuators which has $n_A = 8$ and $n_B = 7$. The poles and zeros, the low frequency gain, and the error coefficients of the models having different structures are given in Table 5.2. Notice that the DC-gain, the

Table 5.2. Poles, zeros, DC-gain, and error coefficients of the process models estimated at different times for adaptive regulators having different structures.

n_A	n_B	t	Poles	Zeros	Gain	e_1
1	1	100	0, 0.541		0.872	-2.77
1	2	100	0, 0.546	3.14	0.772	-2.83
1	3	100	0, 0, 0.300	3.24, -0.465	0.744	-2.43
1	3	400	0, 0, 0.335	3.22, -0.454	0.745	-2.43
2	3	100	0, 0.175, 0.227	3.22, -0.392	0.743	-2.40
2	3	200	0, 0.228±i0.124	3.21, -0.328	0.742	-2.40
3	3	100	-0.516, 0.307±i0.190	3.23, -0.717	0.741	-2.38
3	3	200	-0.075, 0.251±i0.142	3.21, -0.358	0.742	-2.39
3	3	300	0.186, 0.151±i0.107	3.21, -0.296	0.742	-2.40
3	3	400	0.223, 0.135±i0.127	3.21, -0.290	0.742	-2.40
2	4	100	0, 0, -0.302, 0.277	3.22, -0.400±i0.291	0.742	-2.40
2	4	200	0, 0, 0.092±i0.112	3.21, -0.300±i0.202	0.742	-2.40
2	4	400	0, 0, 0.189±i0.080	3.21, -0.257, -0.149	0.742	-2.40
2	4	600	0, 0, 0.190±i0.082	3.21, -0.262, -0.140	0.742	-2.40

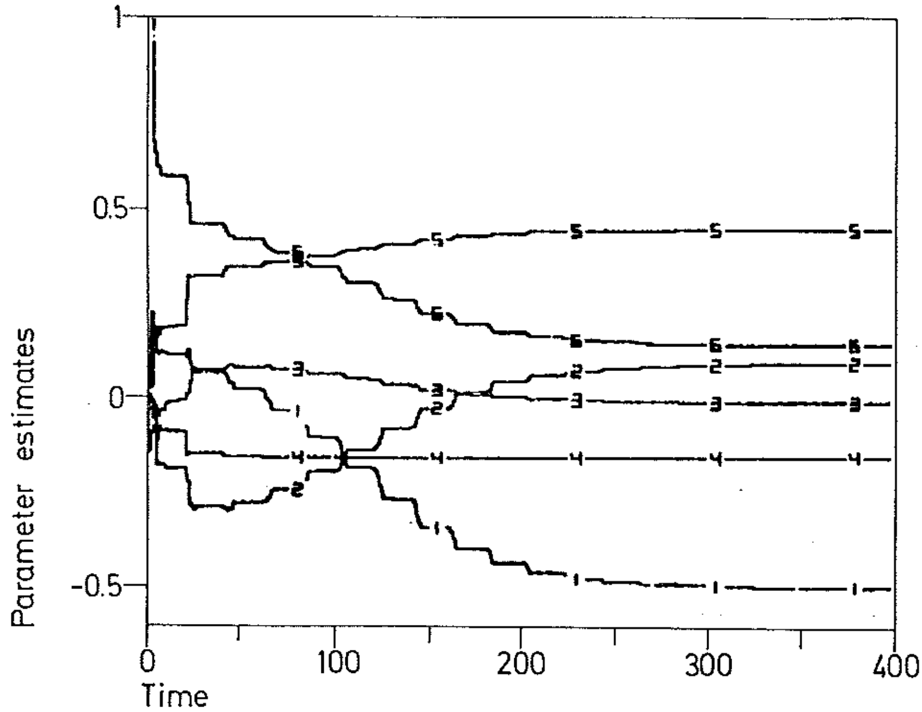

Figure 5.11. Parameter estimates for the regulator with $n_A = 3$ and $n_B = 3$.

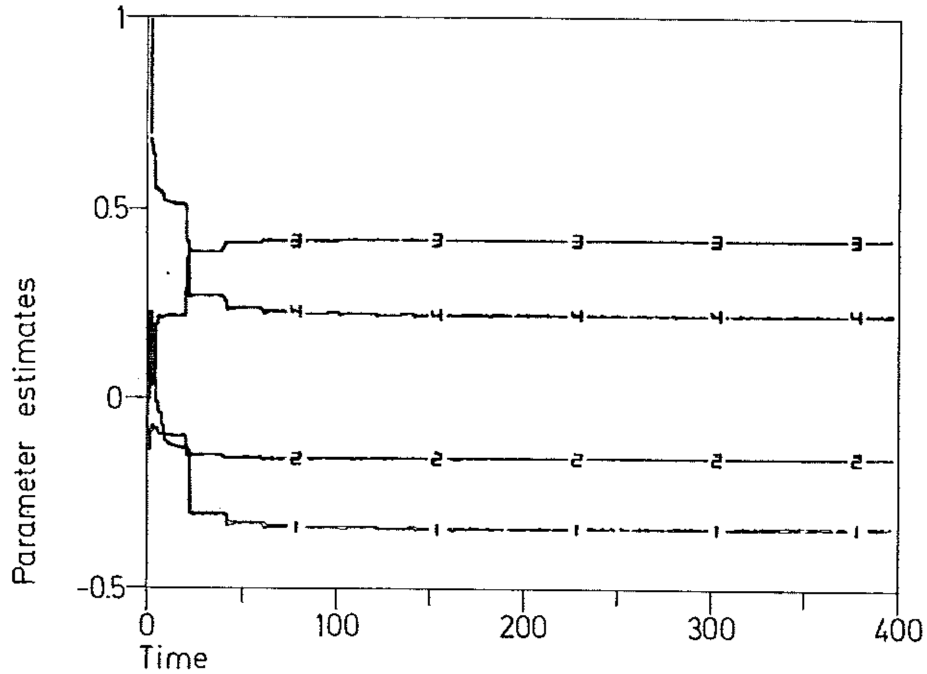

Figure 5.12. Parameter estimates for the regulator with $n_A = 1$ and $n_B = 3$.

error coefficient and the process zero outside the unit disc have almost the same values in all cases even if the other poles and zeros may vary significantly. The fact that the zero outside the unit disc is estimated very well points to the usefulness of self-tuning regulators based on algorithms which cancel all zeros that are sufficiently well damped.

It is also seen in Table 5.1 that the parameter estimates will converge slowly for the regulators which have many parameters, i.e. $n_A = n_B = 3$ and $n_A = 2$ and $n_B = 4$. This is also illustrated in Fig. 5.11, which shows the parameter estimates over a longer period for the system with $n_A = n_B = 3$. Notice that the parameter $\theta_3 = b_1$ converges quickly but the other parameters converge slowly. Compare with Fig. 5.12, which shows the parameter estimates for the regulator with $n_A = 1$ and $n_B = 3$.

It was previously mentioned that $n_A = 1$ and $n_B = 3$ was a reasonable choice of regulator structure. This is further illustrated in Fig. 5.13, which shows the step responses of regulators based on models of different complexity.

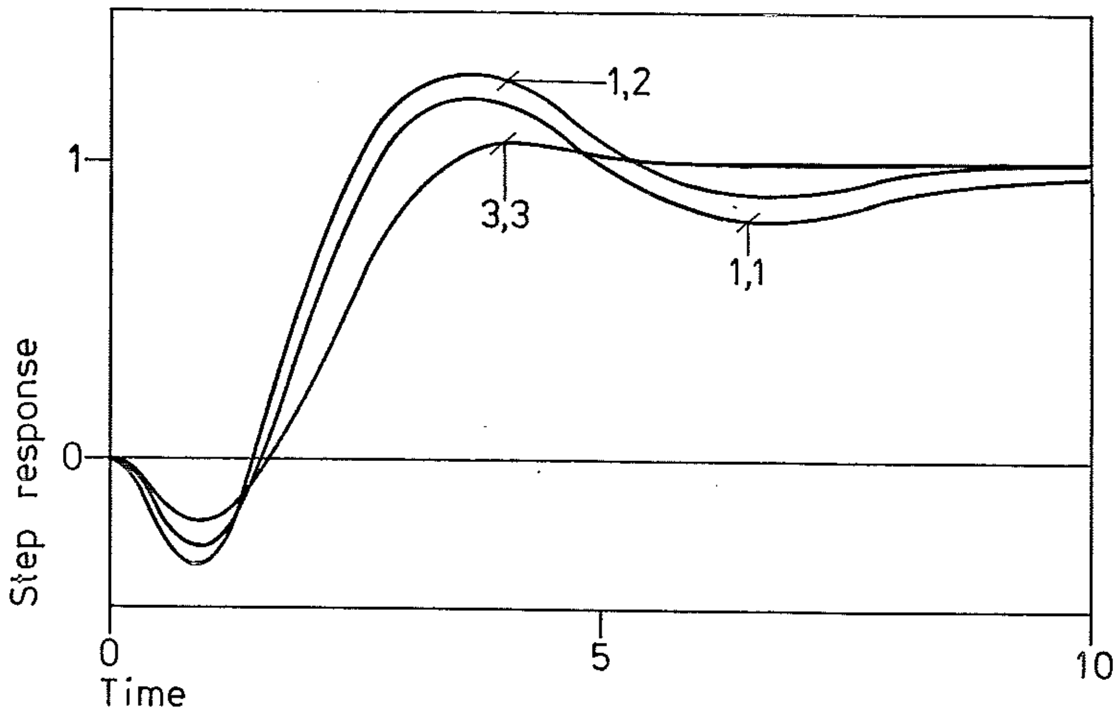

Figure 5.13. Step responses at time 80 for regulators based on models of different complexity. The numbers on the curve refer to the numerator and denominator degrees of the model transfer function.

6. ANALYSIS

A striking observation from the numerical experiments, reported in the previous chapters, is that the performance of the self-tuning regulator can be very close to the specifications even if the model used in the regulators is substantially simpler than the controlled process. Specifically it was found that a self-tuning regulator with $n_A = 1$ and $n_B = 3$ gave a closed loop response which deviated from the specifications by at most 1 % even if the model of the reactor with actuator dynamics has $n_A = 8$ and $n_B = 7$. This is particularly surprising since the process has several complex poles. The general observations agree well with observations made when applying system identification methods to a large variety of industrial processes. In such cases it has been observed that simple models are often adequate. See Åström (1976) and (1977). This observation is of substantial practical significance because it supports the use of simple models. It would thus be of interest to have theory which gives insight into properties of self--tuning regulators based on model structures that are simpler than the real process. Unfortunately there are very few results of this type available. Most theory of system identification and adaptive control is based on the assumption that the class of fitted models is sufficiently rich to include the real process. Exceptions are Åström and Wittenmark (1973), Ljung (1976), and Baram and Sandell (1978). Analysis will now be given which gives some insight into the problem. We will start by investigating how sensitive the pole-placement design is to modeling errors.

Consider a process whose dynamics is characterized by the rational pulse transfer function

$$G_0 = \frac{B_0}{A_0} . \tag{6.1}$$

Assume that it is desired to design a combined feedback and feedforward using pole placement such that the closed loop system has the pulse transfer function

$$G_d = \frac{Q}{P} . \tag{6.2}$$

Let the design be based on a simplified model having the pulse transfer function

$$G = \frac{B}{A} . \tag{6.3}$$

The pole-placement design procedure can be described as follows. Find the greatest common divisor B_2 of B and Q. Factor Q and B as

$$Q = Q_1 B_2 \tag{6.4}$$

$$B = B_1 B_2 . \tag{6.5}$$

Solve the equation

$$AR + BS = PB_1 T_1 \tag{6.6}$$

where T_1 is the desired observer polynomial. The controller is then given by

$$Ru = Ty_r - Sy \tag{6.7}$$

where

$$T = T_1 Q_1 . \tag{6.8}$$

Notice that the regulator (6.5) can be interpreted as the combination of a feedforward path with the transfer function

$$G_{FF} = \frac{T}{S} \tag{6.9}$$

and a feedback path with the transfer function

$$G_{FB} = \frac{R}{S} . \tag{6.10}$$

It must be required that the polynomials B_1, P, Q_1, and T_1 have no zeros on the unit circle or outside the unit disc.

Stability of Pole-placement Regulators Designed Using Simplified Models

It will now be investigated what happens when a simplified model (6.3) is used to design the regulator and the regulator then is applied to the system (6.1). An important requirement is that the closed loop system is stable. A sufficient condition is given by

THEOREM 1

Consider a regulator (6.5) obtained by applying pole-placement design to the stable model $G = B/A$ with the specification that the closed loop transfer function should be $G_d = Q/P$. Let the regulator control a stable system with the pulse transfer function $G_0 = B_0/A_0$. The closed loop system is then stable if

$$|G - G_0| < \left| \frac{BPT}{AQS} \right| = \left| \frac{G}{G_d} \right| \cdot \left| \frac{G_{FF}}{G_{FB}} \right| \tag{6.11}$$

on the unit circle and at $z = \infty$. □

Proof

Consider the function

$$F = \frac{A_0R + B_0S}{A_0} = R + \frac{B_0S}{A} . \tag{6.12}$$

This function is regular outside and on the unit circle because the system G was assumed stable. The zeros of the function F are equal to the closed loop poles. Solving (6.6) for R and inserting in (6.12) gives

$$F = PB_1T_1/A - BS/A + B_0S/A_0 = PB_1T_1/A + S(G_0 - G) \tag{6.13}$$

when $G = G_0$ the zeros of F are thus equal to the zeros of the polynomials B_1, P, and T_1. Since both the system and the model were assumed to be stable the functions PB_1T_1/A_1 and $S(G_0 - G)$ are both regular outside the unit disc and on the unit circle. The functions are thus regular in the domain enclosed by the contour C in Fig. 6.1. Notice that

$$\frac{BPT}{AQS} = \frac{B_1B_2PT_1Q_1}{AQ_1B_2S} = \frac{B_1PT_1}{AS} .$$

Condition (6.6) thus implies that

$$|S(G_0 - G)| < |PB_1T_1/A_1|$$

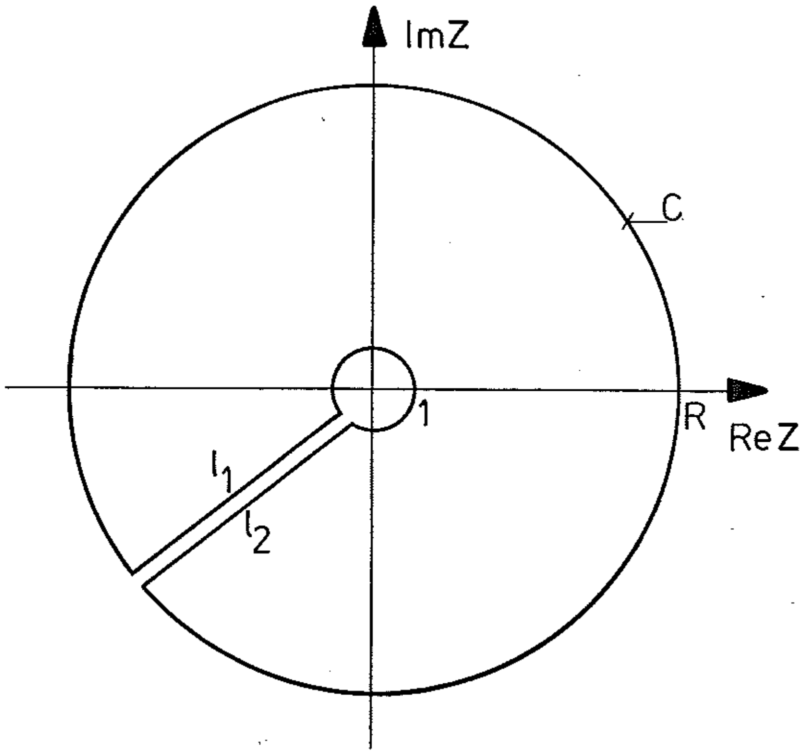

Figure 6.1. The contour C.

on the unit circle and on the circle with radius R provided that R is sufficiently large. The principle of the argument variation (see e.g. Titchmarsh (1932)) then implies that the functions F and PB_1T_1/A have the same number of zeros inside the contour. Notice that the argument variations along the lines ℓ_1 and ℓ_2 cancel each other. Since the function PB_1T_1/A is regular outside the unit disc and on the unit circle it follows that the closed loop system has no poles outside the unit disc. The last equality in (6.6) is obtained from

$$\frac{BPT}{AQS} = \frac{BP\bar{T}R}{AQRS} = \frac{G\ G_{FF}}{G_d\ G_{FB}}$$

and the proof is completed. □

Theorem 1 gives good insight into the sensitivity of the pole-placement design to modeling errors. When a model has been obtained and a regulator has been designed, the right hand side of (6.11) can be determined.

It is then easy to establish bounds on the transfer function G which will result in a stable closed loop system. Notice that the bound is proportional to $|G_{FF}/F_{FB}|$. From the point of view of *stability* the requirements on model precision will thus decrease if the ratio of feedforward to feedback is increased. For single-degree-of-freedom systems $G_{FF} = G_{FB}$ and the bounds are then further simplified. Also notice that the bound is proportional to $|G/G_d|$. From the point of view of stability it is thus advantageous to have a high process gain. The ratio $|G/G_d|$ is normally large for low frequencies because the low frequency gain of the process is typically larger than the desired low frequency gain. Reasonable specifications are also often such that $|G/G_d|$ is constant for high frequencies. Since G_0 and G normally are small for high frequencies, this means that the inequality (6.5) can be satisfied even if G and G_0 deviates substantially at high frequencies. Normally it is only in a fairly narrow frequency range where (6.5) gives critical requirements on the model accuracy. This explains qualitatively why simple models can be useful in the pole-placement design.

Sensitivity of Closed Loop Poles to Model Errors

So far the discussion has been focussed on the stability problem. Having established that a model is sufficiently accurate to guarantee stability it is of course of interest to analyse the problem further and to investigate the requirements on model precision which are necessary to have the dominating poles close to their specified values.

In the proof of Theorem 1 it was shown that the closed loop poles are the zeros of the function F defined by (6.6), i.e.

$$F = PB_1T_1/A + S(G_0 - G) = H + S(G_0 - G).$$

When $G = G_0$ the system has thus poles at the zeros of P, B_1, and T_1. Consider F as a function of z and G. A Taylor series expansion at $z = p_i$ and $G = G_0$ gives

$$F(z) \approx H(p_i) + H'(p_i)(z - p_i) + S(p_i)[G(p_i) - G_0(p_i)].$$

An approximative formula for the change of the pole p_i due to a modeling error is thus

$$z_i = p_i + [H'(p_i)]^{-1} S(p_i)[G_0(p_i) - G(p_i)].$$

If it is required that a pole p_i change by at most $\alpha|p_i|$ due to a modeling error the following inequality is obtained:

$$|G_0(p_i) - G(p_i)| \leq \alpha|H'(p_i)| \; |S^{-1}(p_i)| \cdot |p_i|. \qquad (6.8)$$

A requirement that certain dominant poles do not change too much will thus lead to a requirement that the values of the model pulse transfer function is close to the process pulse transfer function at the poles of interest. Such a requirement can of course also be satisfied by a fairly simple model provided that the number of dominant poles is not too large.

These preliminary results give some insight why simple models are so useful. Much further work is of course necessary to get full insight into the problem.

7. ACKNOWLEDGEMENTS

I am very grateful to professor Alan Foss who let me share some of his knowledge about the reactor control problem. He gave me access to the models and provided many stimulating discussions. This work was partially supported by the Swedish Board of Technical Development (STU) under contract 78-3763.

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APPENDIX A - REACTOR MODEL

The SIMNON program used to simulate the reactor is listed below.

```
CONTINUOUS SYSTEM BEDII
"LINEAR MODEL OF SECOND BED OF FOSS REACTOR"

INPUT TIO CIO Q TQ
OUTPUT T5 T6 T7 TIIO CIIO
STATE X1 X2 X3 X4 X5 X6 X7
DER DX1 DX2 DX3 DX4 DX5 DX6 DX7

OUTPUT
"MODEL OF MIXING CHAMBER"
TM=-0.337*Q+0.237*TQ+0.763*TIO
CM= -0.723*Q+0.763*CIO

S1=-0.110*X1+1.01*X2+0.0706*X3-0.0319*X4+0.0244*X5-0.0289*X6+0.0171*X7
T5=S1+0.048*TM-0.00281*Q

S2=0.190*X1-0.240*X2+0.866*X3+0.382*X4-0.191*X5+0.202*X6-0.117*X7
T6=S2-0.0950*TM-0.00381*Q

S3=-0.119*X1+0.126*X2-0.186*X3+0.632*X4+0.682*X5-0.433*X6+0.235*X7
T7=S3+0.0614*TM-0.00345*Q

S4=-0.00492*X1+0.00488*X2-0.00600*X3+0.00996*X4-0.0263*X5+0.342*X6
TIIO=S4+0.676*X7+0.00259*TM-0.00190*Q

S5=-0.0465*X1-0.114*X2-0.191*X3-0.262*X4-0.263*X5-0.142*X6
CIIO=S5-0.000241*X7+0.243*CM+0.276*Q

DYNAMICS
R1=-2.51*X1-0.128*X2+0.103*X3-0.128*X4+0.158*X5-0.221*X6+0.135*X7
DX1=R1+2.83*TM+0.0679*CM-0.0266*Q

R2=0.768*X1-2.13*X2-0.200*X3+0.200*X4-0.233*X5+0.319*X6-0.194*X7
DX2=R2+2.36*TM+0.281*CM-0.123*Q

R3=0.845*X1+1.80*X2-1.88*X3-0.285*X4+0.269*X5-0.346*X6+0.208*X7
DX3=R3+0.849*TM+0.551*CM-0.203*Q

R4=0.318*X1+1.504*X2+2.41*X3-1.84*X4-0.275*X5+0.282*X6-0.163*X7
DX4=R4-1.46*TM+0.589*CM-0.0781*Q

R5=-0.685*X1-0.218*X2+1.69*X3+2.202*X4-2.052*X5-0.237*X6+0.120*X7
DX5=R5-1.594*TM+0.349*CM+0.219*Q

R6=-1.02*X1-1.46*X2+0.334*X3+1.90*X4+1.43*X5-2.34*X6-0.112*X7
DX6=R6-0.184*TM+0.205*CM+0.381*Q

R7=-1.01*X1-1.66*X2-0.0508*X3+1.72*X4+1.44*X5+0.623*X6-2.83*X7
DX7=R7+0.254*TM+0.187*CM+0.404*Q

END
```

APPENDIX B - THE SYSTEMS ACTUA AND REF

To simulate the closed loop the reactor model BEDII listed in Appendix A was used. The adaptive regulator REG is described in Gustavsson (1978). Systems describing the actuator (ACTUA) and the reference signal generation (REF) were also used. These systems are listed below.

```
CONTINUOUS SYSTEM ACTUA

INPUT U
OUTPUT Y
STATE X
DER DX

OUTPUT
Y=X

DYNAMICS
DX=(U-X)/TA

TA:0.1

END
```

- -

```
DISCRETE SYSTEM REF

TIME T
OUTPUT Y
TSAMP TS

OUTPUT
Y=IF MOD(T,PER)<(0.5*PER-EPS) THEN NIV1 ELSE NIV2

DYNAMICS
TS=T+DT

PER:40
NIV1:1
NIV2:-1
EPS:0.00001

DT:1

END
```

APPENDIX C - CONCENTRATION CONTROL USING IMPLICIT ALGORITHM

The connecting system is listed below.

```
CONNECTING SYSTEM FOSCR
"CONNECTING SYSTEM FOR CONCENTRATION REGULATION

CIO[BEDII]=0
TIO[BEDII]=0
TQ[BEDII]=0
Q[BEDII]=Y[ACTUA]
U1[REG]=CIIO[BEDII]-Y[REF]
U3[REG]=Y[REF]
U[ACTUA]=UR[REG]
U2[REG]=UR[REG]
UPLOT=AS*U[ACTUA]+BS
AS:1
BS:-4
END
```

The following macro was used.

```
MACRO BEDC1
"MACRO FOR CONCENTRATION CONTROL USING STURP1
PAR REG:4                "CHOICE OF REGULATOR ALGORITHM STURP2
PAR N1:2
PAR N2:2
PAR K1:0
PAR K2:0
PAR TH01:0               "INITIAL VALUES OF PARAMETER ESTIMATES
PAR TH02:0
PAR TH03:0
PAR TH04:0
PAR TH05:0
PAR TH06:0
PAR P01:10               "INITIAL COVARIANCES
PAR P02:10
PAR P03:10
PAR P04:10
PAR P05:10
PAR P06:10
PAR WTI:0.98             "FORGETTING FACTOR
PAR REF:1                "AMPLITUDE OF COMMAND SIGNAL
PAR PER:40               "PERIOD OF SQUARE WAVE COMMAND
PAR DT[REF]:1            "SAMPLING PERIOD IN REF
PAR NAM:3                "#COEFFICIENTS IN DESIRED CHAR POLY
PAR NT:2                 "#COEFFICIENTS IN OBSERVER POLYNOMIAL
PAR AM1:2.7548           "CLOSED LOOP CHAR POLY
PAR AM2:-2.6446
PAR AM3:0.8898
PAR T1[REG]:1            "OBSERVER POLYNOMIAL
PAR T2[REG]:0
PAR T3[REG]:0
PAR DT[REG]:1
END
```

A typical SIMNON dialog is listed below.

```
ALGOR RK

LET IVR.=6
LET ISA.=3
LET ISB.=4
LET IPL.=1

SYST REG ACTUA BEOII REF FOSCR
BEDC1
AXES H 0 100 V -5 2
PLOT Y[REF] CIIO UPLOT
STORE TH1 TH2 TH3 TH4 TH5 TH6 Y[REF] CIIO UPLOT
PAR TH03:1
SIMU 0 100
AXES V -0.7  0.7
SHOW TH1 TH2 TH3 TH4
AXES H 78 92  V -3  3
SHOW Y[REF] CIIO
DISP (LP)
```

A copy of the line printer listing obtained after DISP is enclosed.

DISCRETE SYSTEM REG

INPUT :	U1	2.00274	U2	-0.541717	U3	-1.00000
OUTPUT:	UR	-0.541717	RES	3.234506E-03	TH1	-1.08613
	TH2	0.559511	TH3	1.83779	TH4	0.958587
	TH5	0.000000	TH6	0.000000		
TSAMP :	TS	101.000				
PAR :	ID	1.00000	REG	4.00000	REF	1.00000
	N1	2.00000	N2	2.00000	N3	0.000000
	N4	0.000000	N5	0.000000	KDEL	0.000000
	K1	0.000000	K2	0.000000	K3	0.000000
	K4	0.000000	K5	0.000000	ULIM	-1.00000
	IB0	0.000000	B0	1.00000	DT	1.00000
	WTI	0.980000	WTM	1.00000	RLIM	-1.00000
	IRES	0.000000	ILS	50.0000	DELTA	0.000000
	INCR	0.000000	Q2	0.000000	ITER	10.0000
	R0	1.00000	IOP	0.000000	EPS	0.000000
	FILT	1.00000	MRAS	1.00000	OBS	0.000000
	IWR	0.000000	NWR1	10.0000	NWR2	100.000
	NT	2.00000	T1	1.00000	T2	0.000000
	T3	0.000000	T4	0.000000	T5	0.000000
	NAM	3.00000	AM1	2.75480	AM2	-2.64460
	AM3	0.889800	AM4	0.000000	AM5	0.000000
	NBM	1.00000	BM1	1.00000	BM2	0.000000
	BM3	0.000000	BM4	0.000000	BM5	0.000000
	NQ	1.00000	QP1	1.00000	QP2	0.000000
	QP3	0.000000	QP4	0.000000	QP5	0.000000
	NP1	1.00000	PP11	1.00000	PP12	0.000000
	PP13	0.000000	PP14	0.000000	PP15	0.000000
	NP2	1.00000	PP21	1.00000	PP22	0.000000
	PP23	0.000000	PP24	0.000000	PP25	0.000000
	IPP1	0.000000	IPP2	0.000000	IPP3	0.000000
	IPP4	0.000000	IPP5	0.000000	RPP1	1.00000
	RPP2	1.00000	RPP3	1.00000	RPP4	1.00000
	RPP5	1.00000	TH01	0.000000	TH02	0.000000
	TH03	1.00000	TH04	0.000000	TH05	0.000000
	TH06	0.000000	P01	10.0000	P02	10.0000
	P03	10.0000	P04	10.0000	P05	10.0000
	P06	10.0000	SAP0	100.000	R11	0.000000
	R12	0.000000	R13	0.000000	R14	0.000000
	R15	0.000000	R16	0.000000		
VAR :	V	29.1655	VU	31.7142	WT	0.980000
	AL1	0.000000	AL2	0.000000	AL3	0.000000
	AL4	0.000000	AL5	0.000000	AL6	0.000000

- -

CONTINUOUS SYSTEM ACTUA

STATE :	X	0.546309
INIT :	X	0.000000
DER :	DX	-10.8803
INPUT :	U	-0.541717
OUTPUT:	Y	0.546309
PAR :	TA	0.100000

```
                         CONTINUOUS SYSTEM BEDII
STATE : X1      -0.215342         X2      -0.340615        X3      -0.613175
        X4       -1.01611         X5       -1.27925        X6       -1.26069
        X7       -1.22308
INIT  : X1       0.000000         X2       0.000000        X3       0.000000
        X4       0.000000         X5       0.000000        X6       0.000000
        X7       0.000000
DER   : DX1      1.661736E-05     DX2      4.657730E-05    DX3      1.275931E-04
        DX4      2.613580E-04     DX5      3.104806E-04    DX6      2.451949E-04
        DX7      2.165101E-04
INPUT : TIO      0.000000         CIO      0.000000        Q        0.546309
        TQ       0.000000
OUTPUT: T5      -0.357276         T6      -0.730146        T7       -1.17261
        TIIO     -1.23287         CIIO      1.00274
VAR   : TM      -0.184106         CM      -0.394981        S1      -0.346904
        S2      -0.745554         S3       -1.15942        S4      -0.404556
        S5       0.947643         R1       0.562388        R2       0.612723
        R3       0.484969         R4       6.777178E-03    R5      -0.274948
        R6      -0.160803         R7      -9.986782E-02

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

                          DISCRETE SYSTEM REF
TIME  : T        100.000
OUTPUT: Y       -1.00000
TSAMP : TS       101.000
PAR   : PER      40.0000          EPS      1.000000E-05    NIV1     1.00000
        NIV2    -1.00000          DT        1.00000

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

                          CONNECTING SYSTEM FOSCR
PAR   : AS       1.00000          BS      -4.00000
VAR   : UPLOT   -4.54172
```

APPENDIX D - CONCENTRATION CONTROL USING EXPLICIT ALGORITHM

The connecting system used was FOSCR given in Appendix C. The following macro was used.

```
MACRO BEDC2
"MACRO FOR CONCENTRATION CONTROL USING STURP2
PAR REG:5                      "CHOICE OF REGULATOR ALGORITHM STURP2
PAR N1:2
PAR N2:2
PAR K1:0
PAR K2:0
PAR TH01:0                     "INITIAL VALUES OF PARAMETER ESTIMATES
PAR TH02:0
PAR TH03:0
PAR TH04:0
PAR TH05:0
PAR TH06:0
PAR P01:10                     "INITIAL COVARIANCES
PAR P02:10
PAR P03:10
PAR P04:10
PAR P05:10
PAR P06:10
PAR WTI:0.98                   "FORGETTING FACTOR
PAR REF:1                      "AMPLITUDE OF COMMAND SIGNAL
PAR PER:40                     "PERIOD OF SQUARE WAVE COMMAND
PAR DT[REF]:1                  "SAMPLING PERIOD IN REF
PAR NAM:3                      "¤COEFFICIENTS IN DESIRED CHAR POLY
PAR NT:2                       "¤COEFFICIENTS IN OBSERVER POLYNOMIAL
PAR AM1:1                      "CLOSED LOOP CHAR POLY
PAR AM2:-0,34
PAR AM3:0,12
PAR T1[REG]:1                  "OBSERVER POLYNOMIAL
PAR T2[REG]:0
PAR T3[REG]:0
PAR DT[REG]:1
END
```

A typical SIMNON dialog is listed below.

```
ALGOR RK

LET IVR.=6
LET ISA.=3
LET ISB.=4
LET IPL.=1

SYST REG ACTUA BEDII REF FOSCR
BEDC2
AXES H 0 100  V -5 2
PLOT Y[REF] CIIO UPLOT
STORE TH1 TH2 TH3 TH4 TH5 TH6 Y[REF] CIIO UPLOT
PAR TH04:1
SIMU 0 100
AXES V -0.7  0.7
SHOW TH1 TH2 TH3 TH4
AXES H 78 92 V -3 3
SHOW Y[REF] CIIO
DISP (LP)
```

A copy of the lineprinter listing obtained after DISP is enclosed.

DISCRETE SYSTEM REG

INPUT :	U1	1,99988	U2	-0,863476	U3	-1,00000
OUTPUT:	UR	-0,863476	RES	-7,696977E-05	TH1	-0,458079
	TH2	6,162631E-02	TH3	0,662576	TH4	0,445077
	TH5	0,000000	TH6	0,000000		
TSAMP :	TS	101,000				
PAR :	ID	1,00000	REG	5,00000	REF	1,00000
	N1	2,00000	N2	2,00000	N3	0,000000
	N4	0,000000	N5	0,000000	KDEL	0,000000
	K1	0,000000	K2	0,000000	K3	0,000000
	K4	0,000000	K5	0,000000	ULIM	-1,00000
	IB0	0,000000	B0	1,00000	DT	1,00000
	WTI	0,980000	WTM	1,00000	RLIM	-1,00000
	IRES	0,000000	ILS	50,0000	DELTA	0,000000
	INCR	0,000000	Q2	0,000000	ITER	10,0000
	R0	1,00000	IOP	0,000000	EPS	0,000000
	FILT	1,00000	MRAS	1,00000	OBS	0,000000
	IWR	0,000000	NWR1	10,0000	NWR2	100,000
	NT	2,00000	T1	1,00000	T2	0,000000
	T3	0,000000	T4	0,000000	T5	0,000000
	NAM	3,00000	AM1	1,00000	AM2	-0,340000
	AM3	0,120000	AM4	0,000000	AM5	0,000000
	NBM	1,00000	BM1	1,00000	BM2	0,000000
	BM3	0,000000	BM4	0,000000	BM5	0,000000
	NQ	1,00000	QP1	1,00000	QP2	0,000000
	QP3	0,000000	QP4	0,000000	QP5	0,000000
	NP1	1,00000	PP11	1,00000	PP12	0,000000
	PP13	0,000000	PP14	0,000000	PP15	0,000000
	NP2	1,00000	PP21	1,00000	PP22	0,000000
	PP23	0,000000	PP24	0,000000	PP25	0,000000
	IPP1	0,000000	IPP2	0,000000	IPP3	0,000000
	IPP4	0,000000	IPP5	0,000000	RPP1	1,00000
	RPP2	1,00000	RPP3	1,00000	RPP4	1,00000
	RPP5	1,00000	TH01	0,000000	TH02	0,000000
	TH03	0,000000	TH04	1,00000	TH05	0,000000
	TH06	0,000000	P01	10,0000	P02	10,0000
	P03	10,0000	P04	10,0000	P05	10,0000
	P06	10,0000	SAP0	100,000	R11	0,000000
	R12	0,000000	R13	0,000000	R14	0,000000
	R15	0,000000	R16	0,000000		
VAR :	V	27,0110	VU	33,1612	WT	0,980000
	SAPI	1,000000E-02	YM	0,000000		

- -

CONTINUOUS SYSTEM ACTUA

STATE :	X	0,544851
INIT :	X	0,000000
DER :	DX	-14,0833
INPUT :	U	-0,863476
OUTPUT:	Y	0,544851
PAR :	TA	0,100000

CONTINUOUS SYSTEM BEDII

STATE :	X1	-0,214795	X2	-0,339732	X3	-0,611528
	X4	-1,01326	X5	-1,27550	X6	-1,25688
	X7	-1,21935				
INIT :	X1	0,000000	X2	0,000000	X3	0,000000
	X4	0,000000	X5	0,000000	X6	0,000000
	X7	0,000000				
DER :	DX1	8,840510E-05	DX2	8,119270E-05	DX3	3,189407E-05
	DX4	-6,130803E-05	DX5	-8,294545E-05	DX6	-4,124269E-05
	DX7	-2,692640E-05				
INPUT :	TIO	0,000000	CIO	0,000000	Q	0,544851
	TQ	0,000000				
OUTPUT:	T5	-0,356347	T6	-0,728162	T7	-1,16925
	TIIO	-1,22912	CIIO	0,999876		
VAR :	TM	-0,183615	CM	-0,393927	S1	-0,346002
	S2	-0,743530	S3	-1,15610	S4	-0,403330
	S5	0,944928	R1	0,560959	R2	0,611122
	R3	0,483579	R4	6,437119E-03	R5	-0,274606
	R6	-0,160659	R7	-9,984410E-02		

- -

DISCRETE SYSTEM REF

TIME :	T	100,000				
OUTPUT:	Y	-1,00000				
TSAMP :	TS	101,000				
PAR :	PER	40,0000	EPS	1,000000E-05	NIV1	1,00000
	NIV2	-1,00000	DT	1,00000		

- -

CONNECTING SYSTEM FOSCR

PAR :	AS	1,00000	BS	-4,00000
VAR :	UPLOT	-4,86348		

APPENDIX E - TEMPERATURE CONTROL OF NONMINIMUM PHASE LOOP

The connecting system is listed below.

```
CONNECTING SYSTEM FOSTR
"CONNECTING SYSTEM FOR TEMPERATURE REGULATION

CIO[BEDII]=0
TIO[BEDII]=0
TQ[BEDII]=Y[ACTUA]
Q[BEDII]=0
U1[REG]=TIIO[BEDII]-Y[REF]
U3[REG]=Y[REF]
U[ACTUA]=UR[REG]
U2[REG]=UR[REG]
UPLOT=AS*U[ACTUA]+BS
AS:0.5
BS:-4
END
```

The following macro was used in the simulations.

```
MACRO BEDTR
"MACRO FOR TEMPERATURE CONTROL
PAR REG:5                 "CHOICE OF REGULATOR ALGORITHM STURP2
PAR N1:1
PAR N2:3
PAR TH01:0                "INITIAL VALUES OF PARAMETER ESTIMATES
PAR TH02:0
PAR TH03:0
PAR TH04:0
PAR TH05:0
PAR TH06:0
PAR P01:10                "INITIAL COVARIANCES"
PAR P02:10
PAR P03:10
PAR P04:10
PAR P05:10
PAR P06:10
PAR WTI:0.98              "FORGETTING FACTOR
PAR REF:1                 "AMPLITUDE OF COMMAND SIGNAL
PAR PER:40                "PERIOD OF SQUARE WAVE COMMAND
PAR DT[REF]:1             "SAMPLING PERIOD IN REF
PAR NAM:1                 "¤COEFFICIENTS IN DESIRED CHAR POLY
PAR NT:2                  "¤COEFFICIENTS IN OBSERVER POLYNOMIAL
PAR AM1:1                 "CLOSED LOOP CHAR POLY
PAR AM2:-0.34
PAR AM3:0.12
PAR T1[REG]:1             "OBSERVERPOLYNOMIAL
PAR T2[REG]:0
PAR T3[REG]:0
PAR DT[REG]:1
PAR AS:0.5
PAR BS:-4
END
```

A typical SIMNON dialog is listed below.

```
ALGOR RK

LET IVR.=6
LET ISA.=3
LET ISB.=4
LET IPL.=1

SYST REG ACTUA BEDII REF FOSTR
BEDTR
AXES H 0 100  V -5 2
PLOT Y[REF] TIIO UPLOT
STORE TH1 TH2 TH3 TH4 TH5 TH6 Y[REF] TIIO UPLOT
PAR TH04:1
SIMU 0 100
AXES V -0.7  0.7
SHOW TH1 TH2 TH3 TH4 -MARK
AXES H 78 92 V -3 3
SHOW Y[REF] TIIO
DISP (LP)
```

A copy of the line printer listing obtained after this simulation is enclosed.

```
                        DISCRETE SYSTEM REG
INPUT : U1       1.99700          U2       -1.78684          U3       -1.00000
OUTPUT: UR      -1.78684          RES    -2.178332E-03       TH1      -0.330560
        TH2     -0.152854         TH3       0.421912         TH4       0.229253
        TH5      0.000000
TSAMP : TS       101.000
PAR   : ID       1.00000          REG       5.00000          REF       1.00000
        N1       1.00000          N2        3.00000          N3        0.000000
        N4       0.000000         N5        0.000000         KDEL      0.000000
        K1       0.000000         K2        0.000000         K3        0.000000
        K4       0.000000         K5        0.000000         ULIM      4.00000
        IB0      0.000000         B0        1.00000          DT        1.00000
        WTI      0.980000         WTM       1.00000          RLIM     -1.00000
        IRES     0.000000         ILS       50.0000          DELTA     0.000000
        INCR     0.000000         Q2        0.000000         ITER      10.0000
        R0       1.00000          IOP       0.000000         EPS       0.000000
        FILT     1.00000          MRAS      1.00000          OBS       0.000000
        IWR      0.000000         NWR1      10.0000          NWR2      100.000
        NT       2.00000          T1        1.00000          T2        0.000000
        T3       0.000000         T4        0.000000         T5        0.000000
        NAM      3.00000          AM1       1.00000          AM2      -0.340000
        AM3      0.120000         AM4       0.000000         AM5       0.000000
        NBM      1.00000          BM1       1.00000          BM2       0.000000
        BM3      0.000000         BM4       0.000000         BM5       0.000000
        NQ       1.00000          QP1       1.00000          QP2       0.000000
        QP3      0.000000         QP4       0.000000         QP5       0.000000
        NP1      1.00000          PP11      1.00000          PP12      0.000000
        PP13     0.000000         PP14      0.000000         PP15      0.000000
        NP2      1.00000          PP21      1.00000          PP22      0.000000
        PP23     0.000000         PP24      0.000000         PP25      0.000000
        IPP1     0.000000         IPP2      0.000000         IPP3      0.000000
        IPP4     0.000000         IPP5      0.000000         RPP1      1.00000
        RPP2     1.00000          RPP3      1.00000          RPP4      1.00000
        RPP5     1.00000          TH01      0.000000         TH02      0.000000
        TH03     0.000000         TH04      1.00000          TH05      0.000000
        P01      10.0000          P02       10.0000          P03       10.0000
        P04      10.0000          P05       10.0000          SAP0      100.000
        R11      0.000000         R12       0.000000         R13       0.000000
        R14      0.000000         R15       0.000000
VAR   : V        55.6474          VU        192.658          WT        0.980000
        AL1      0.000000         AL2       0.000000         AL3       0.000000
        AL4      0.000000         AL5       0.000000

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                        CONTINUOUS SYSTEM ACTUA
STATE : X        1.34359
INIT  : X        0.000000
DER   : DX      -31.3043
INPUT : U       -1.78684
OUTPUT: Y        1.34359
PAR   : TA       0.100000
```

```
                         CONTINUOUS SYSTEM BEDII
STATE  : X1     0,346228        X2      0,449164        X3      0,661929
         X4     0,948236        X5       1,09106        X6       1,02767
         X7     0,987347
INIT   : X1     0,000000        X2      0,000000        X3      0,000000
         X4     0,000000        X5      0,000000        X6      0,000000
         X7     0,000000
DER    : DX1    4,619360E-06    DX2     8,732080E-06    DX3     2,211332E-05
         DX4    5,138665E-05    DX5     8,173287E-05    DX6     9,077881E-05
         DX7    9,040348E-05
INPUT  : TIO    0,000000        CIO     0,000000        Q       0,000000
         TQ      1,34359
OUTPUT : T5     0,461145        T6      0,746867        T7       1,04226
         TIIO   0,997001        CIIO   -0,875286
VAR    : TM     0,318432        CM      0,000000        S1      0,445860
         S2     0,777118        S3       1,02271        S4      0,328730
         S5    -0,875048        R1     -0,901157        R2     -0,751490
         R3    -0,270326        R4      0,464962        R5      0,507662
         R6     5,868220E-02    R7     -8,079123E-02

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                         DISCRETE SYSTEM REF
TIME   : T       100,000
OUTPUT : Y      -1,00000
TSAMP  : TS      101,000
PAR    : PER     40,0000        EPS     1,000000E-05    NIV1     1,00000
         NIV2   -1,00000        DT       1,00000

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

                         CONNECTING SYSTEM FOSAD
TIME   : T       100,000
PAR    : T1     0,000000        CIOS    0,000000        T2      0,000000
         TIOS   0,000000        AS      0,500000        BS     -4,00000
VAR    : UPLOT  -4,89342
```