



# LUND UNIVERSITY

## Simple Self-Tuners 1

Åström, Karl Johan

1979

*Document Version:*

Publisher's PDF, also known as Version of record

[Link to publication](#)

*Citation for published version (APA):*

Åström, K. J. (1979). *Simple Self-Tuners 1*. (Technical Reports TFRT-7184). Department of Automatic Control, Lund Institute of Technology (LTH).

*Total number of authors:*

1

### General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

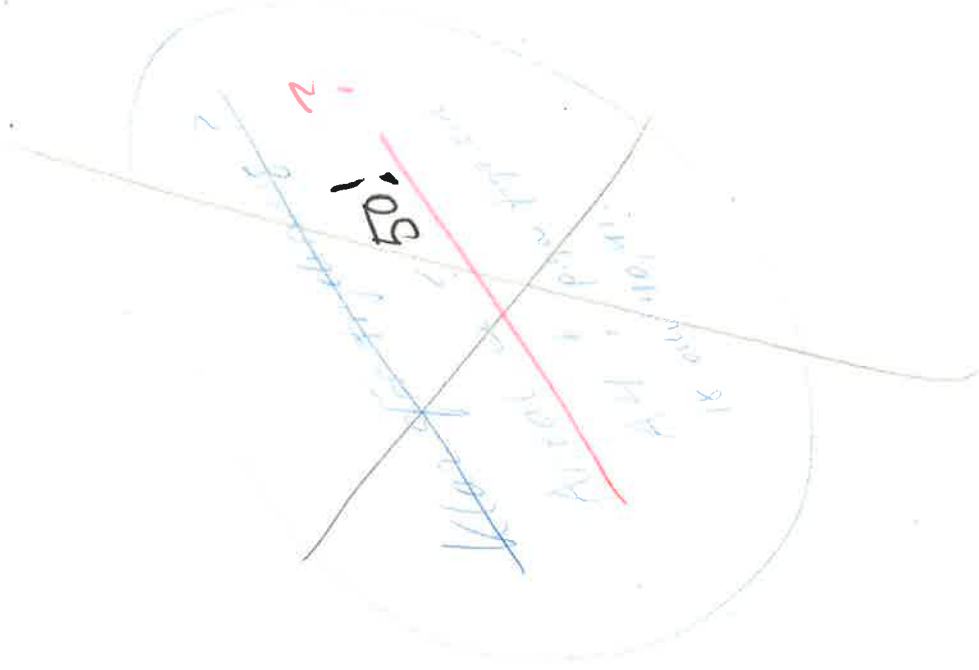
LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00

original kopier

SIMPLE SELF-TUNERS I

K. J. ASTROM



DEPARTMENT OF AUTOMATIC CONTROL  
LUND INSTITUTE OF TECHNOLOGY  
DECEMBER 1979

Organization <b>LUND INSTITUTE OF TECHNOLOGY</b> Department of Automatic Control Box 725 S-220 07 LUND 7 SWEDEN		Document name Internal report	
Author(s) K J Åström		Date of issue December 1979	
Title and subtitle Simple self-tuners I		CODEN: LUTFD2/(TFRT-7184)/1-63/(1979)	
Abstract Design of a self-tuning servo-controller based on explicit identification of a second order model and pole-zero placement design is discussed.  This report is the first in a series dealing with simple self-tuners. The idea is to design self-tuners based on simple models which admits analytical solutions to the design equation. The self-tuning servo discussed in this report is based on explicit identification of a second order model and a pole-zero placement design. The design trade-offs are discussed and the properties of the algorithm are illustrated using simulations.		Sponsoring organization	
Key words		A4	
Classification system and/or index terms (if any)			
Supplementary bibliographical information		Language English	
ISSN and key title		ISBN	
Recipient's notes		Number of pages 66	
		Security classification	
Distribution by (name and address)			

Organization LUND INSTITUTE OF TECHNOLOGY Department of Automatic Control Box 725 S-220 07 LUND 7 SWEDEN	Document name Internal report Date of issue December 1979 CODEN: LUTFD2/(TFRT-7184)/1-63/(1979)
Author(s) K J Aström	Sponsoring organization
Title and subtitle Simple self-tuners I	A4 A5
<p>Abstract          Design of          a self-tuning servo-controller based on explicit          identification of a second order model and pole-zero          placement design. is discussed.</p> <p><i>This report is the first in a series dealing with          simple self-tuners. The idea is to design self-tuners          based on simple models which admit analytical solutions          to the design equations. The self-tuning servo          discussed in this report is based on explicit identification of a          second order model and a pole-zero placement          design. The design trade-offs and <del>the</del> <del>same</del>  <del>paper</del> the robustness are discussed together  <del>with</del> and the properties of the algorithms are illustrated          using simulations.</i></p>	
Key words	A4 A5
Classification system and/or index terms (if any)	
Supplementary bibliographical information Language English	
ISSN and key title ISBN	
Recipient's notes Number of pages 66 Price Security classification	
Distribution by (name and address)	

Contents

1. INTRODUCTION	1
2. ALGORITHM DESIGN	3
Problem Formulation	4
Control Design for Known Parameters	5
Common Factors in the Process Model	7
Initial Values of Parameter Estimates	13
The Sampling Period	17
Reset	21
3. THE ALGORITHM	28
Parameter Estimation	28
Regulator Design	29
Parameters	30
Application and Tuning Rules	31
4. SIMULATIONS	33
Process Model	33
Bandwidth Changes	34
Load Changes	36
Changes in Process Gain	37
Changes in Process Time Constants	39
High Order Process Dynamics	42
5. CONCLUSIONS	47
6. ACKNOWLEDGEMENT	48
7. REFERENCES	49
APPENDIX A - SIMNON PROGRAMS STRE3, PROC1	50
APPENDIX B - EXAMPLE 2.2	53
APPENDIX C - EXAMPLE 2.3	57
APPENDIX D - SIMNON PROGRAMS STSE2, PROC2	61

## 1. INTRODUCTION

A PI-regulator has only two tuning parameters, which are easy to adjust using simple tuning rules. The PID-regulator, which has three tuning parameters, is more difficult to tune. A consequence of this is that the derivative action is often switched off in industrial applications. One motivation for introducing a self-tuning regulator is to simplify the tuning problem. The self-tuning regulator has, however, also tuning parameters. When using a self-tuner it can thus be said that one set of tuning parameters is replaced by another set. Hopefully the new parameters are easier to choose. In the early applications (Åström et al, 1977) the self-tuners were applied to special problems. Good rules for choosing the parameters could then be found (Wittenmark, 1973). Some parameters are, however, critical. For self-tuners based on minimum variance control and least squares parameter estimation it is crucial to have an upper bound on the time delay of the process.

It has also been proposed to design universal self-tuners which do not have any adjustable parameters. A moments reflection shows that this is an impossible task. It is at least necessary to provide the self-tuner with information about the desired specification. Once this is realized there appear to be many possibilities to design practical regulators for special classes of problems based on self-tuning. These regulators will all have tuning parameters which have to be selected by the user. The tuning parameters may, however, be directly performance-related and consequently also easy to choose.

This report is the first in a series which describes simple self-tuners. The particular self-tuner is intended for simple servo applications where the process can be described by a low order model. The regulator is based on recursive least squares parameter estimation and pole-placement design. See Åström and Wittenmark (1979). The regulator has only *one* tuning parameter, namely the

desired bandwidth of the closed loop system. Apart from being of interest in themselves the results also give interesting insights into the properties of self-tuning regulators. The **regulator discussed** is related to the self-tuning PID regulator discussed in Wittenmark (1979). The main difference is that a PID structure is not imposed on the regulator.

The report is organized as follows. The algorithm is presented in Section 2 together with a discussion of the major trade-offs done in the algorithm. Section 3 presents a few simulations which show the behaviour of the algorithm in typical cases. Problems which appear appropriate for using the algorithm and those which do not are discussed in Section 4. The tuning rules for the algorithm are also given in that section.

## 2. ALGORITHM DESIGN

The algorithm is intended to solve simple servo problems for systems which can be described by low order models. It is natural to characterize the performance of a simple servo by the bandwidth of the closed loop system. The basic idea is thus to design a self-tuning servo which literally speaking only has one dial, marked bandwidth, on the front panel. A servo problem is conveniently formulated in terms of pole-placement. It is then natural to use the formulation of self-tuning servos discussed in Åström and Wittenmark (1979). A low order process model is thus estimated recursively. Using the pole-placement design method a regulator is then designed which gives a closed loop system with the desired bandwidth. Having specified the desired bandwidth the other system parameters like sampling period are then determined from the desired closed loop bandwidth. Since the low frequency behaviour of many systems can be approximated by low order models it can be expected that the self-tuning regulator will work satisfactorily, provided that the chosen bandwidth is sufficiently small. An indication of this is e.g. that the overshoot of the closed loop system is that specified in the design. This is supported by the sensitivity analysis presented in Åström (1979a), which guarantees that the pole-placement design based on a simplified model will always give a stable closed loop system provided that the process is stable and the desired bandwidth sufficiently small. As the desired bandwidth is increased it can be expected that the closed loop system will respond faster. As the bandwidth of the system is increased it may, however, happen that a low order model is not sufficient. The performance of the self-tuner will then deteriorate. The overshoot may deviate from the specifications and the closed loop system may be unstable. The tuning rule for the regulator is thus very simple. Start with a small bandwidth. Establish the possible range of bandwidths for which the regulator will work for the process by increasing the specified bandwidth until the overshoot deviates from the specified value. If the desired bandwidth is outside the range found it is



necessary to use a more complex regulator. Since the tuning involves only one parameter it should be fairly easy to do.

### Problem Formulation

Consider a process described by the model

$$y(t) + a_1 y(t-h) + a_2 y(t-2h) = b_1 u(t-h) + b_2 u(t-2h) + b_3 \quad (2.1)$$

where  $h$  is the sampling period and  $b_3$  is a bias term. Find a feedback such that the closed loop system has poles in

$$z = e^{-\zeta\omega h} (\cos \omega h \sqrt{1-\zeta^2} \pm i \sin \omega h \sqrt{1-\zeta^2}). \quad (2.2)$$

This means that the closed loop poles correspond to a continuous time system with poles in

$$s = -\zeta\omega \pm i\omega\sqrt{1-\zeta^2}.$$

The characteristic polynomial for the closed loop system is thus given by

$$P(z) = z^2 + p_1 z + p_2$$

where

$$\begin{aligned} p_1 &= -2 e^{-\zeta\omega h} \cos \omega h \sqrt{1-\zeta^2} \\ p_2 &= e^{-2\zeta\omega h}. \end{aligned} \quad (2.3)$$

The process model has a zero at

$$z = -b_2/b_1.$$

If this zero corresponds to a well-damped mode then the factor  $b_1 z + b_2$  can be cancelled by the regulator and the desired closed loop response is characterized by the pulse transfer function

$$G_d(z) = \frac{z(1 + p_1 + p_2)}{z^2 + p_1 z + p_2}. \quad (2.4)$$

The scaling factor is necessary to ensure that the desired closed

loop transfer function has unit gain.

If the process zero corresponds to an unstable ( $|b_2| > |b_1|$ ) or poorly damped mode the factor  $b_1z + b_2$  can not be cancelled by the regulator and the desired closed loop transfer function is instead given by

$$G_d(z) = \frac{1 + p_1 + p_2}{b_1 + b_2} \cdot \frac{b_1z + b_2}{z^2 + p_1z + p_2} \quad (2.5)$$

To complete the problem statement it is also necessary to specify the observer. To simplify the problem and to avoid the necessity of introducing extra parameters it is assumed that a Luenberger observer is used and that the observer polynomial is

$$T_1(z) = z. \quad (2.6)$$

This means that the observer settles in one sampling period. It also means that the characteristics of the observer will critically depend on the chosen sampling period.

### Control Design for Known Parameters

The calculation of the control law when the process model (2.1) is known is straightforward. See e.g. Åström (1979b). The feedback law is given by

$$Ru = Tu_c - Sy - u_b \quad (2.7)$$

where  $u_c$  is the command signal.

When the process zero is cancelled the polynomials  $R$ ,  $S$ , and  $T$  are given by

$$\begin{cases} r_1 = -b_2/b_1 \\ s_0 = (p_1 - a_1)/b_1 \\ s_1 = (p_2 - a_2)/b_1 \\ t_0 = (1 + p_1 + p_2)/b_1 \\ u_b = b_3/b_1 \end{cases} \quad (2.8)$$

When the process zero is not cancelled the polynomials  $R$ ,  $S$  satisfy the algebraic equation

$$AR + BS = PT_1 \quad (2.9)$$

where

$$T = t_0 T_1.$$

The unique solution with  $\deg R = \deg S = 1$  is chosen. The equation (2.9) can then be written as

$$(z^2 + a_1 z + a_2)(z + r_1) + (b_1 z + b_2)(s_0 z + s_1) = z^3 + p_1 z^2 + p_2 z.$$

The following equations are then obtained:

$$a_1 + r_1 + b_1 s_0 = p_1$$

$$a_2 + a_1 r_1 + b_2 s_0 + b_1 s_1 = p_2$$

$$a_2 r_1 + b_2 s_1 = 0.$$

The solution is

$$\begin{cases} r_1 = [(p_1 - a_1) b_2^2 - (p_2 - a_2) b_1 b_2] / N \\ s_0 = [(p_1 - a_1)(a_2 b_1 - a_1 b_2) + (p_2 - a_2) b_2] / N \\ s_1 = -a_2 r_1 / b_2 \\ t_0 = (1 + p_1 + p_2) / (b_1 + b_2) \\ N = b_2^2 - a_1 b_1 b_2 + a_2 b_1^2 \end{cases} \quad (2.10)$$

The polynomial  $T$  is given by

$$T = z(1 + p_1 + p_2) / (b_1 + b_2). \quad (2.11)$$

To find the control law when there is a bias  $b_3$  in the process model, use the design identity (2.9) i.e.

$$PTy = ARy + BSy.$$

Substitute Ay using the process model (2.1) i.e.

$$PTy = RBu + Rb_3 + BSy = B(Ru + Sy) + Rb_3 \equiv BTu_c.$$

Hence

$$Ru = Tu_c - Sy - \frac{R}{B} b_3.$$

Since B is not always a stable operator the operator R/B is replaced by its static gain. Hence

$$u_b = \frac{(1+r_1) b_3}{b_1 + b_2}. \quad (2.12)$$

#### Common Factors in the Process Model

The linear polynomial equation (2.9) is singular when the polynomials A and B have common factors. In the particular case this happens when  $z = -b_2/b_1$  is a zero of the polynomial A i.e.

$$N = b_2^2 - a_1 b_1 b_2 + a_1 b_1^2 = 0.$$

When N is small the equations are also poorly conditioned. This means that the parameters  $r_1$ ,  $s_0$ , and  $s_1$  which define the feedback law may become very large. This is a potential danger since the high feedback gains may result in very large control signals. It is easy to avoid the difficulty simply by canceling the common factor in the estimated transfer function and design the appropriate feedback for the reduced model. The process model (2.1) has a zero at  $z = -b_2/b_1$ . If there is a corresponding pole we get

$$\frac{b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{b_1 (z + b_2/b_1)}{(z + b_2/b_1)(z + a)} = \frac{b}{z + a}.$$

Hence

$$\begin{cases} b = b_1 \\ a = a_2 b_1 / b_2 = a_1 - b_2 / b_1 \end{cases} \quad (2.13)$$

To design a feedback (2.7) for the process model

$$y(t) + ay(t-1) = bu(t-1) \quad (2.14)$$

which gives the closed loop transfer function (2.4) the equation (2.9) has to be solved. Since the model is of first order we can choose  $T_1 = 1$ . A solution to (2.9) is then given by

$$\left\{ \begin{array}{l} r_1 = 0 \\ s_0 = \frac{p_1 - a_1}{b} \\ s_1 = \frac{p_2}{b} \\ t_0 = \frac{1 + p_1 + p_2}{b} \\ u_b = \frac{b_3}{b_1} \end{array} \right. \quad (2.15)$$

The variable  $N$  is used as a test quantity to decide when the estimated process model has common factors. To obtain a dimensionfree test quantity the following test is used

$$N \leq \epsilon [\max(b_1^2, b_2^2)] \quad (2.16)$$

where the number  $\epsilon$  determines the maximum size of the feedback gain. An example is used to determine a suitable size of  $\epsilon$ .

#### EXAMPLE 2.1

Consider the continuous time process

$$\frac{dy}{dt} = -y + u.$$

Let this process be controlled by an adaptive controller where it is desired to have the closed loop poles given by (2.2) with

$\omega = 1.5$  and  $\zeta = 0.707$ . The adaptive regulator is initialized with a second order process model with

$$\begin{aligned}\hat{a}_1 &= -1.5 & \hat{b}_1 &= 0 \\ \hat{a}_2 &= 0.7 & \hat{b}_2 &= 0\end{aligned}$$

The simulation programs are given in Appendix A. Figure 2.1 shows what happens when the process zero is not cancelled when computing the control law.

Notice that the controller performs very poorly although the estimates of the process parameters converge very quickly. The reason for the poor performance is the common factor in the process transfer function. The presence of this common factor implies that the linear equations for the regulator gain are poorly conditioned. The regulator gains will therefore occasionally be very large. See for example the parameter  $s_0$  in Fig. 2.1. Notice the scale! These very large gains imply that small numerical errors will give very large control signals and very erratic behaviour. In the particular case the common factor is  $z - 0.736$ . The particular value of the common zero, which is obtained, depends on the initial estimates. Figure 2.2 shows what happens when it is attempted to cancel the common factor before calculating the feedback gains. The test quantity used to decide if there is a common factor is given by (2.16).

The value  $\epsilon = 0.0001$  is used in Fig. 2.2. Notice that the closed loop system behaves quite well in this case but that the feedback gains are still quite large during the transient. The gains are almost two orders of magnitude larger in the transient compared to the steady state case.

To ensure that the feedback gains have reasonable values also during the transients it is necessary to use smaller values of  $\epsilon$ . Figure 2.3 shows a simulation with  $\epsilon = 0.01$ .  $\square$

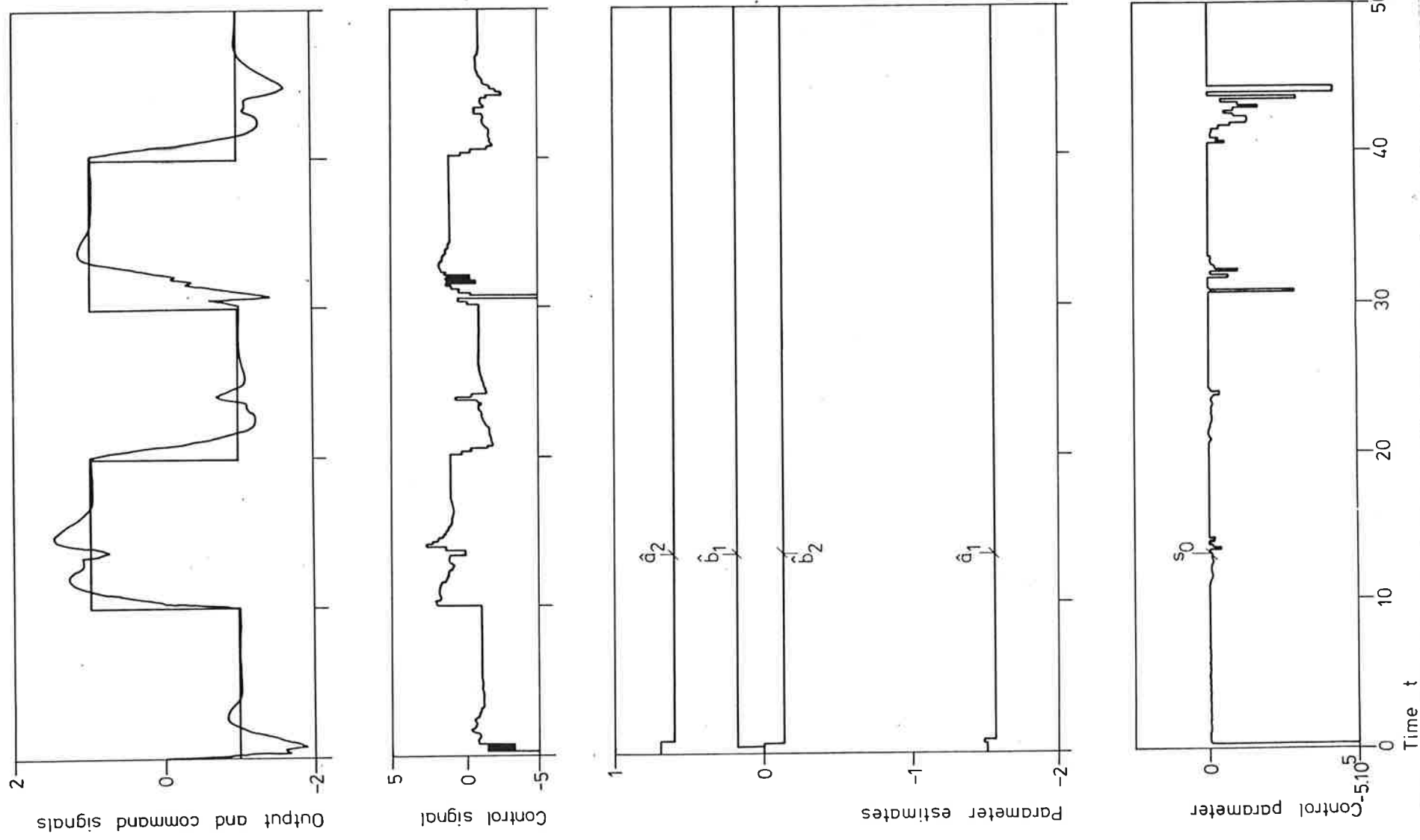


Fig. 2.1 - Simulation of a self-tuning controller based on a second order process model on a first order process.

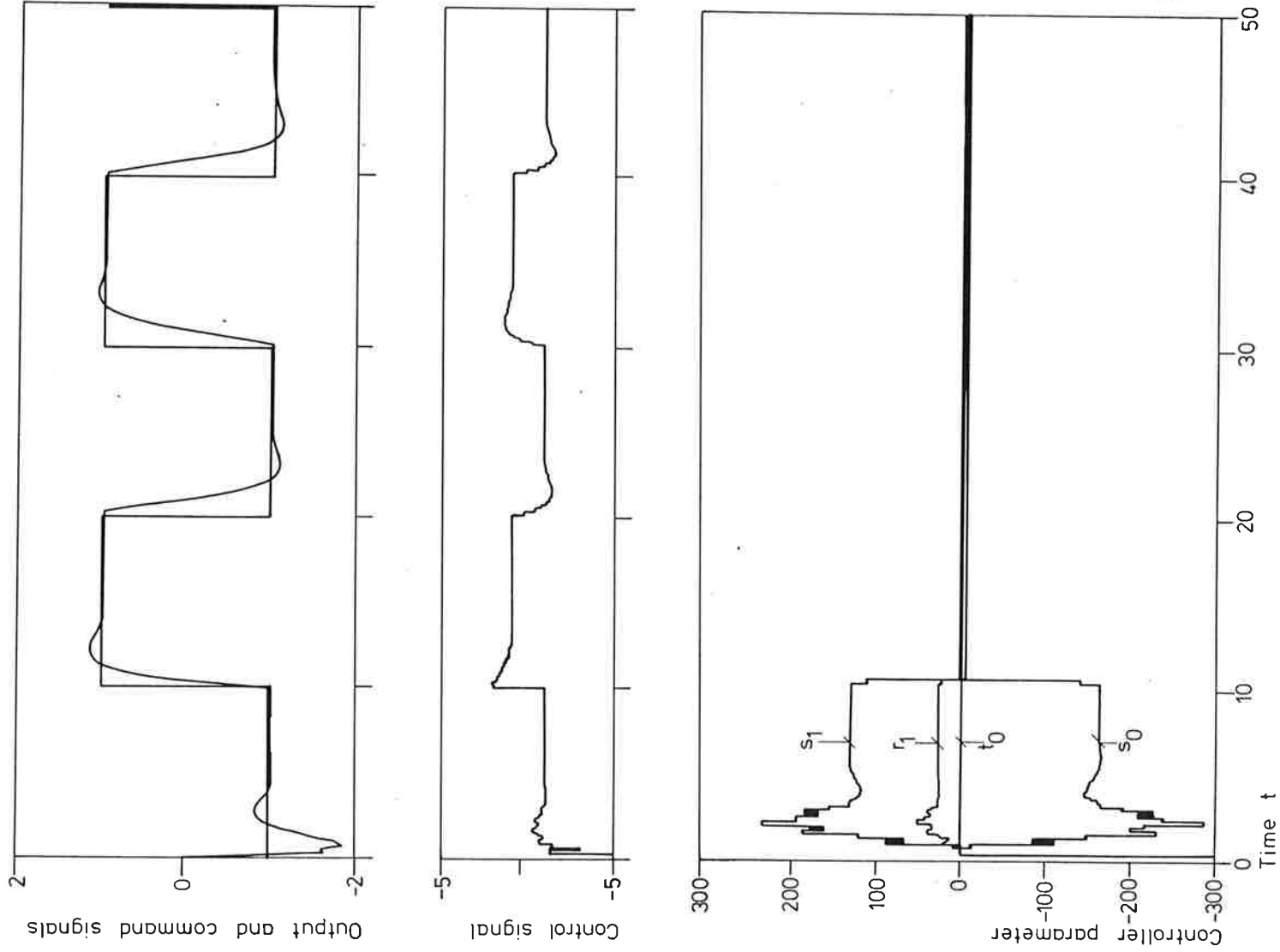


Fig. 2.2 - Simulation of a self-tuning controller based on a second order process model. A possible common factor is cancelled before calculating the control law if the condition (2.16) holds with  $\epsilon = 0.0001$ .



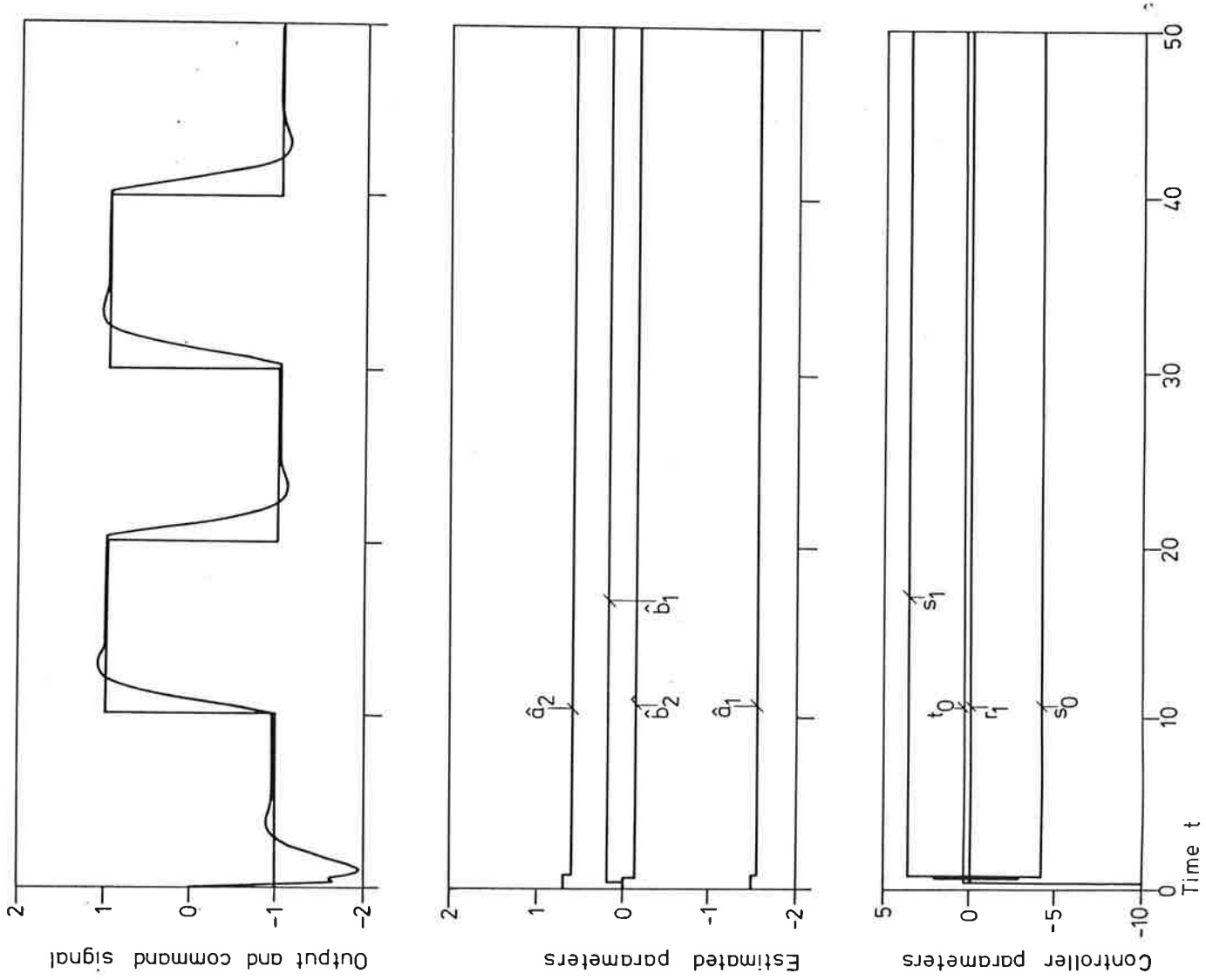


Fig. 2.3 - Simulation of a self-tuning controller based on a second order process model. A possible common factor in the process transfer function is cancelled before calculating the control law if the condition (2.16) holds with  $\varepsilon = 0.01$ .

It is clearly a practical necessity to cancel common factors in the explicit algorithm before attempting the control design. Based on the simulations in Example 2.1 and several other **simulations**, which are not reported, it was decided to choose  $\epsilon = 0.01$  to ensure that the feedback gains have reasonable magnitudes.

#### Initial Values of Parameter Estimates

A self-tuning regulator must be provided with initial values of the parameter estimates. The transient behaviour of the algorithm will depend on the chosen initial values. For an explicit algorithm the parameter estimates are simply the coefficients of a sampled process model. If a priori knowledge is available it is easy to use those. It is also possible to initialize the model with all parameters equal to zero. This means however that the process gain is very small. Consequently the controller gain will be very high and the initial transient may be very large.

In a practical problem it is therefore useful to have reasonable a priori estimates of the parameters. It is easy to find initial estimates of  $a_1$  and  $a_2$ . If the process is sampled very fast we get

$$\hat{a}_1(0) = -2$$

$$\hat{a}_2(0) = 1.$$

For slower sampling a reasonable compromise may be

$$\hat{a}_1(0) = -1.5$$

$$\hat{a}_2(0) = 0.7.$$

It is more difficult to find good universal estimates of the parameters  $b_1$  and  $b_2$ , since any initial estimate will require that the process gain is known approximatively. An example is used to illustrate the consequences of different initial conditions.

## EXAMPLE 2.2

Consider a process with the transfer function

$$G(s) = \frac{1}{(s+1)^2}.$$

Let this process be controlled by an adaptive controller where it is desired to have a closed loop system with  $\omega = 1.5$  and  $\zeta = 0.707$ . The programs for simulating the regulator and the process are the same as those used for Example 2.1. The program listings are found in Appendix A. The parameter values used in the simulation are given in Appendix B.

Figure 2.4 shows the results obtained when all initial values are set to zero. The initial behaviour is far from satisfactory. The controller gain is very large initially. It is only the hard bound on the control signal which keeps the control signal bounded.

Figure 2.5 shows what happens when the initial estimates  $\hat{a}_1(0) = -2$ ,  $\hat{a}_2(0) = 1$ ,  $\hat{b}_1(0) = \hat{b}_2(0) = 0$ . The initial performance is clearly much better. Slightly better results are obtained for the initial values  $\hat{a}_1(0) = -1.5$ ,  $\hat{a}_2(0) = 0.7$ ,  $\hat{b}_1(0) = \hat{b}_2(0) = 0$ . Figure 2.6 finally shows what happens when very good initial values are chosen.  $\square$

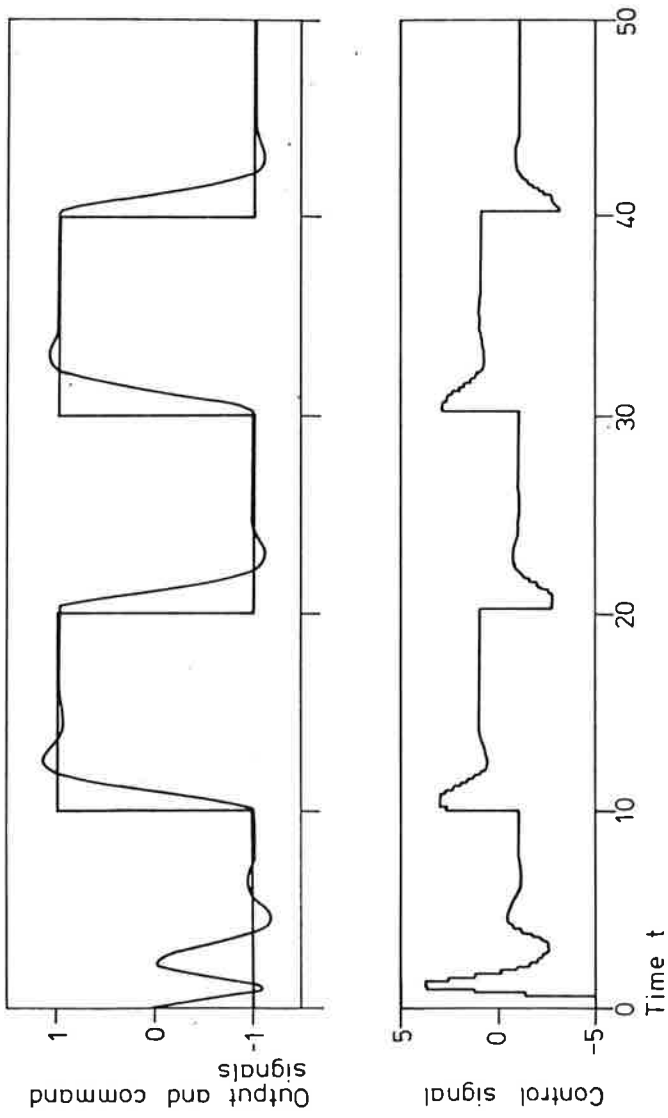


Fig. 2.4 - Simulation which illustrates the transient behaviour of the adaptive controller. The initial values are  $\hat{a}_1(0) = 0$ ,  $\hat{a}_2(0) = 0$ ,  $\hat{b}_1(0) = \hat{b}_2(0) = 0$ .

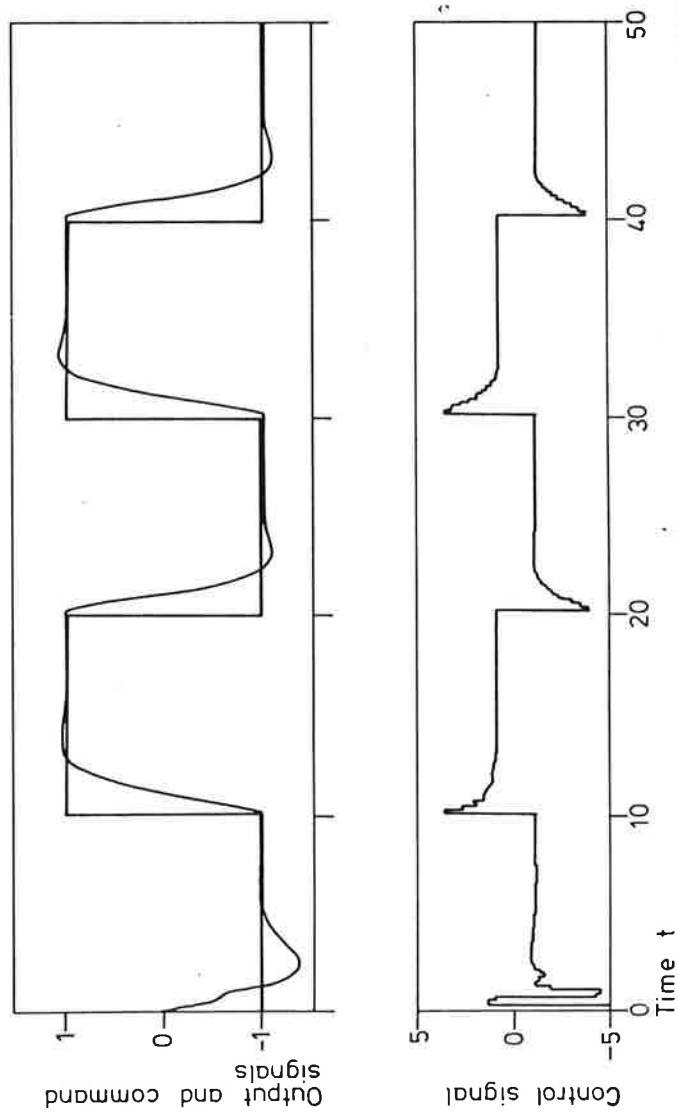


Fig. 2.5 - Simulation which illustrates the transient behaviour of the adaptive controller. The initial values are  $\hat{a}_1(0) = -2$ ,  $\hat{a}_2(0) = 1$ ,  $\hat{b}_1(0) = \hat{b}_2(0) = 0$ .

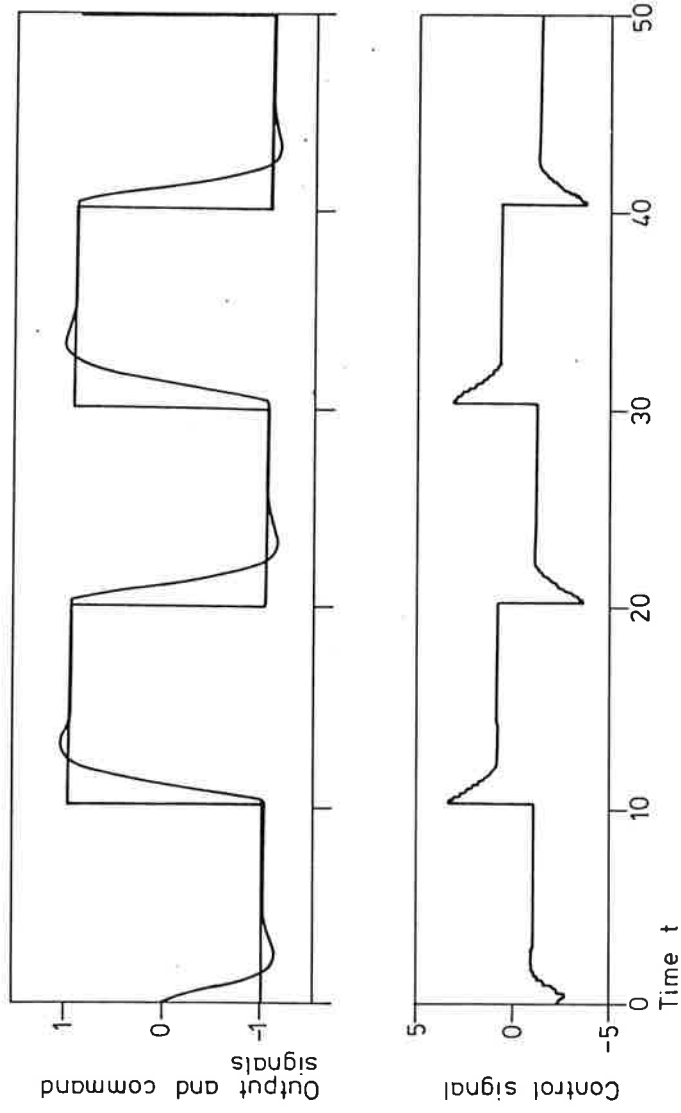


Fig. 2.6 - Simulation which illustrates the transient behaviour of the adaptive controller. The initial values are  $\hat{a}_1(0) = -1.6$ ,  $\hat{a}_2(0) = 0.7$ ,  $\hat{b}_1(0) = 0.02$ ,  $\hat{b}_2(0) = 0.01$ .

Based on the results of the example it seems reasonable to choose the following initial conditions:

$$\hat{a}_1(0) = -1.5$$

$$\hat{a}_2(0) = 0.7$$

$$\hat{b}_1(0) = \hat{b}_2(0) = 0.$$

Since the estimates  $\hat{b}_1$  and  $\hat{b}_2$  are zero the controller gain will be very high initially. It is thus necessary to limit the control signal with this choice.

### The Sampling Period

To obtain a self-tuning controller with only one knob on the front panel it is necessary to fix the sampling period. Since the setting of the knob gives the desired closed loop bandwidth it is natural to choose the sampling period inversely proportional to the bandwidth. A reasonable choice is

$$h = \frac{2\pi}{N_p \omega \sqrt{1-\zeta^2}} \quad (2.17)$$

where  $N$  is the number of samples per period. With  $\zeta = \sqrt{2}/2$  we get

$$\omega h = 2\pi \sqrt{2}/N \approx 9/N.$$

An example is used to illustrate the consequences of choosing different values of  $N$ .

#### EXAMPLE 2.3

Consider the same system as in Example 2.2 i.e. a continuous time system with the transfer function

$$G(s) = \frac{1}{(s+1)^2}.$$

It is desired that the closed loop system has  $\omega = 1.5$  and  $\zeta = 0.707$ . Figure 2.7 shows what happens when different sampling periods are used in the controller. The simulation programs used to generate the curves are listed in Appendix A. The parameter values used are given in Appendix C. In Figure 2.8 are shown the outputs of the regulators with 3, 10, and 30 samples per period at times 30 - 50. It is seen from this Figure that there are no drastic differences between the outputs of the systems. The overshoot is about 5 % higher for the system which has only 3 samples per period.  $\square$

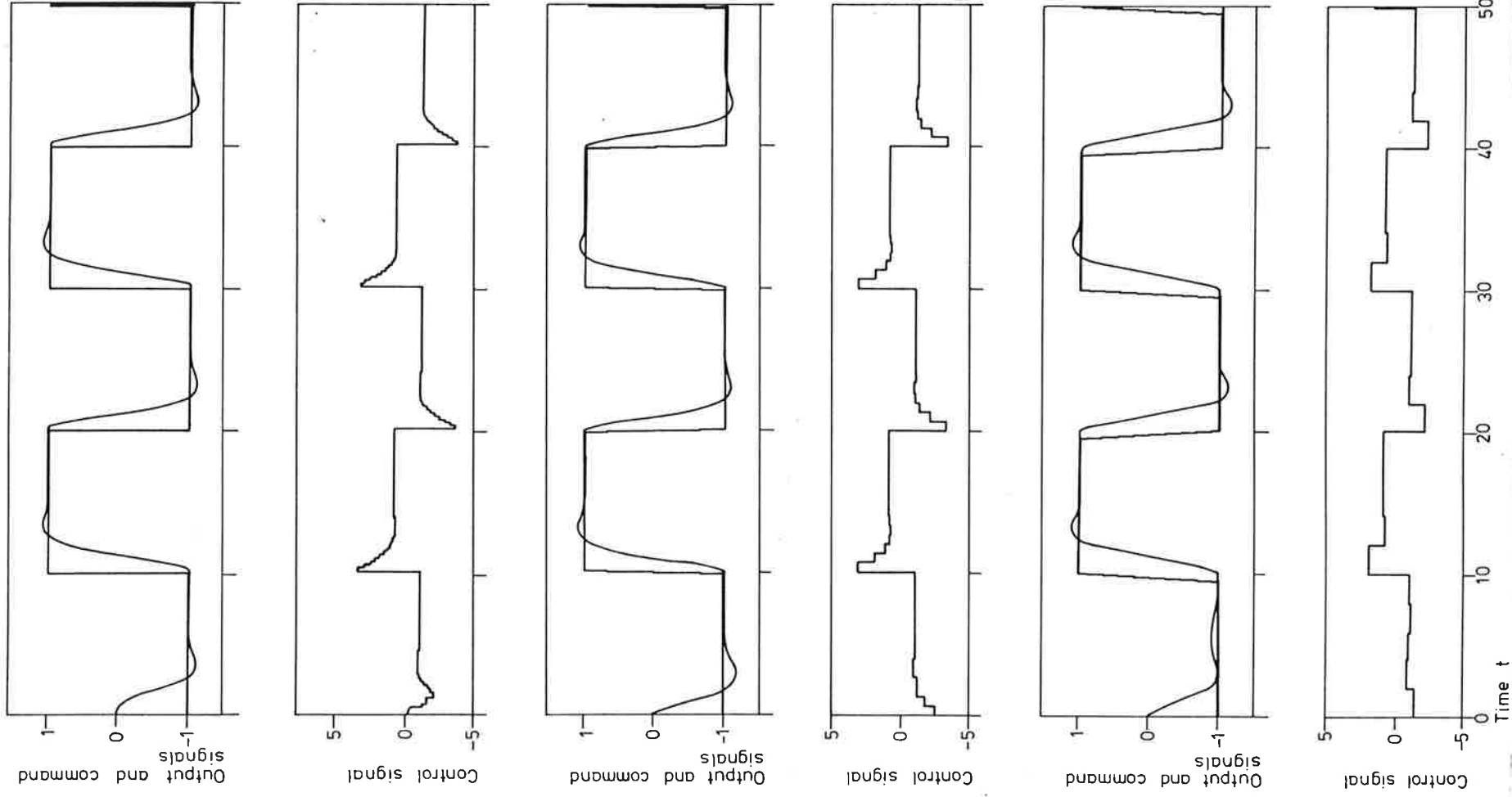


Fig. 2.7 - Illustrates the consequences of different choices of sampling periods in the controller. The curves correspond to 30, 10, and 3 points per period.

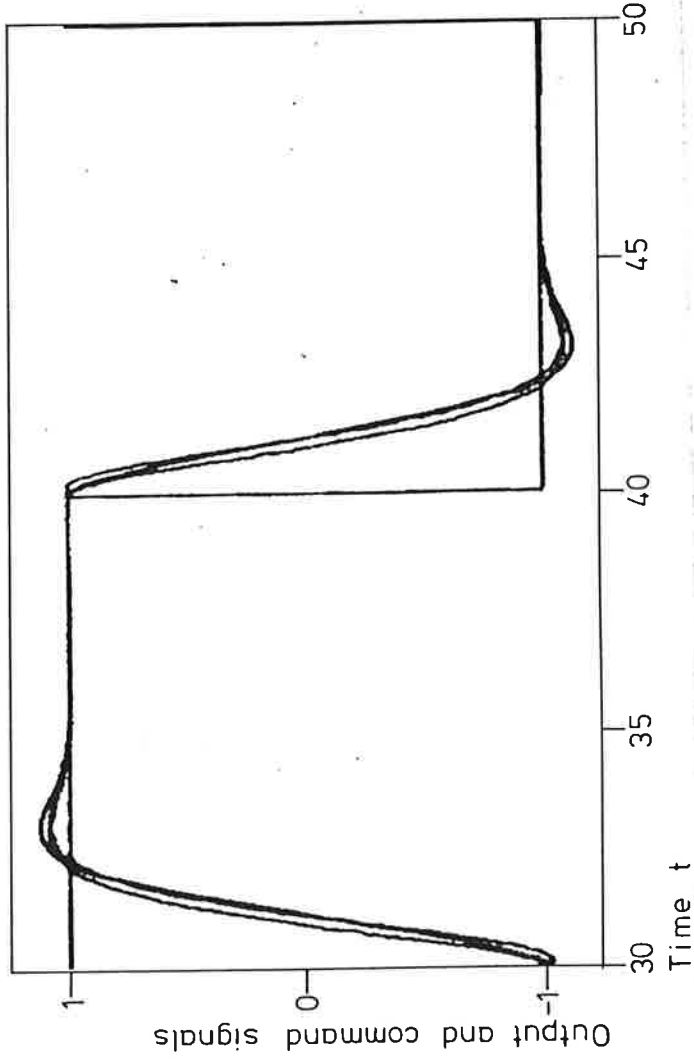


Fig. 2.8 - Outputs at times 30 - 50 of controllers with 3, 10, and 30 samples per period.

The example shows that the sampling period is not critical and that good behaviour is obtained for as slow sampling as 3 samples per period. With a sampled system there will always be an extra delay in response due to the sampling. This delay will on the average be half a sampling period. When  $N = 3$  the delay is an appreciable fraction ( $\approx 15\%$ ) of the settling time. For this reason it appears reasonable to choose a larger value of  $N$ . The sampling period also influences the robustness of the system to time delays and high order process dynamics. This is discussed in Section 3. Based on these considerations the sampling period is chosen so that there are 10 - 20 samples per period. To be specific let us choose

$$\omega h = 0.45.$$

If the sampling period is chosen in this way then it is necessary to change  $h$  whenever  $\omega$  is changed. The parameter estimates will then automatically be modified. It seems, however, useful to speed up the tuning procedure by adjusting the parameter estimates when the sampling period is changed. In principle this is easy to do by



first transforming the sampled data model to a continuous time model which is then sampled with the new sampling period. The calculations are, however, involved and it may be useful to have simplified approximative procedures which involve less calculations. One possibility is to approximate  $\exp(sh)$  with the first term in a Taylor series expansion i.e.

$$z = \exp(sh) = 1 + sh.$$

Simple calculations then show that when the sampling period is changed from  $h$  to  $h_1$  then the  $z$ -transform variable is transformed to

$$z_1 = (h_1/h) z + (1 - h_1/h).$$

The inverse transformation is

$$z = (h/h_1) z_1 + (1 - h/h_1).$$

Under this transfer function the pulse transfer function

$$H(z) = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2} \quad (2.18)$$

is transformed to

$$H_1(z_1) = \frac{b_1' z_1 + b_2'}{z_1^2 + a_1' z_1 + a_2'} \quad (2.19)$$

where

$$\begin{cases} a_1' = (2 + a_1)(h_1/h) - 2 \\ a_2' = (1 + a_1 + a_2)(h_1/h)^2 - (2 + a_1)(h_1/h) + 1 \\ b_1' = b_1(h_1/h) \\ b_2' = (b_1 + b_2)(h_1/h)^2 - b_1(h_1/h). \end{cases} \quad (2.20)$$

When the desired bandwidth and then also the sampling period is changed the process parameters are thus transformed using the above transformation before the updating is started.

### Reset

For all practical regulators it is important to incorporate reset action i.e. the elimination of small static errors. For simple controllers reset action is obtained by integrating the error and making the control signal proportional to the integral of the error. For self-tuning controllers reset can be obtained through several mechanisms. One possibility is to get reset indirectly via the parameter estimation algorithm. This is demonstrated by the following theorem.

#### THEOREM 1

Consider a self-tuning regulator based on estimation of the parameters of the model (2.1) and determination of the control law (2.7) by solving the equation (2.9). Assume that the command signal  $u_c$  is constant. If the closed loop system is in steady state with constant parameter estimates then the control error is zero.

#### *Proof*

Let the constant signals be denoted by the superscript "0". Let  $A^0$  denote  $\hat{A}(1)$  where the parameters of the polynomial have constant values. The closed loop system is then described by the equations

$$A^0 y^0 = B^0 u^0 + b_3^0 \quad (2.21)$$

$$R^0 u^0 = T^0 u_c^0 - s y^0 - (R^0/B^0) b_3^0 \quad (2.22)$$

$$A^0 R^0 + B^0 S^0 = P^0 T_1^0 \quad (2.23)$$

$$T^0 = T_1^0 P^0 / B^0. \quad (2.24)$$

Equations (2.21) and (2.22) give

$$A^0 R^0 y^0 = B^0 R^0 u^0 + R^0 b_3^0 = B^0 T^0 u_c^0 - B^0 S^0 y^0 - R^0 b_3^0 + R^0 b_3^0.$$

Hence

$$(A^0 R^0 + B^0 S^0) y^0 = B^0 T^0 u_c^0 = T_1^0 P^0 u_c^0.$$

It then follows from (2.23) that

$$p^0 T_1^0 y^0 = p^0 T_1^0 u_c$$

which proves the result.

#### *Corollary*

Notice that it follows from the proof that the result holds even if the parameter  $b_3$  is not estimated.  $\square$

The corollary indicates that it is not necessary to introduce integral action in order to eliminate the steady state errors. When using the self-tuning regulator the steady state off-set is instead eliminated indirectly via the parameter estimation. A simulation example will provide further insight.

#### EXAMPLE 2.4

Consider a process with the transfer function

$$G(s) = \frac{1}{(s+1)^2}.$$

Assume that it is desired to have a closed loop system with poles such that  $\omega = 1.5$  and  $\zeta = 0.707$ . A fixed sampled regulator with sampling period  $h$  which achieves this is given by

$$u(t) = t_0 u_c(t) - s_0 y(t) - s_1 y(t-h) - r_1 u(t-h)$$

where

$$\begin{aligned} r_1 &= 0.0198 & s_0 &= 2.060 \\ t_0 &= 2.215 & s_1 &= -0.865 \end{aligned}$$

Figure 2.9 shows what happens with this regulator if the process is subject to a load disturbance.

It is clear from the figure that the constant gain regulator is not capable of dealing with the load disturbance. This is easy to see because the gain is fairly low and there is no integral action.

The corresponding simulation of the self-tuning controller, where the parameter  $b_3$  is kept equal to zero, is shown in Fig. 2.10.

Notice that there is no steady state error as can be expected from the corollary of Theorem 1. Also notice that the step response is unsymmetric and that the parameter estimates change drastically during the transient.  $\square$

Example 2.4 shows clearly that although a regulator, where the bias  $b_3$  is not estimated, will give zero steady state error the regulator will have other undesirable properties. It therefore seems useful to estimate the bias  $b_3$ . The problem is how the bias should be estimated. A simulation example is used to give insight.

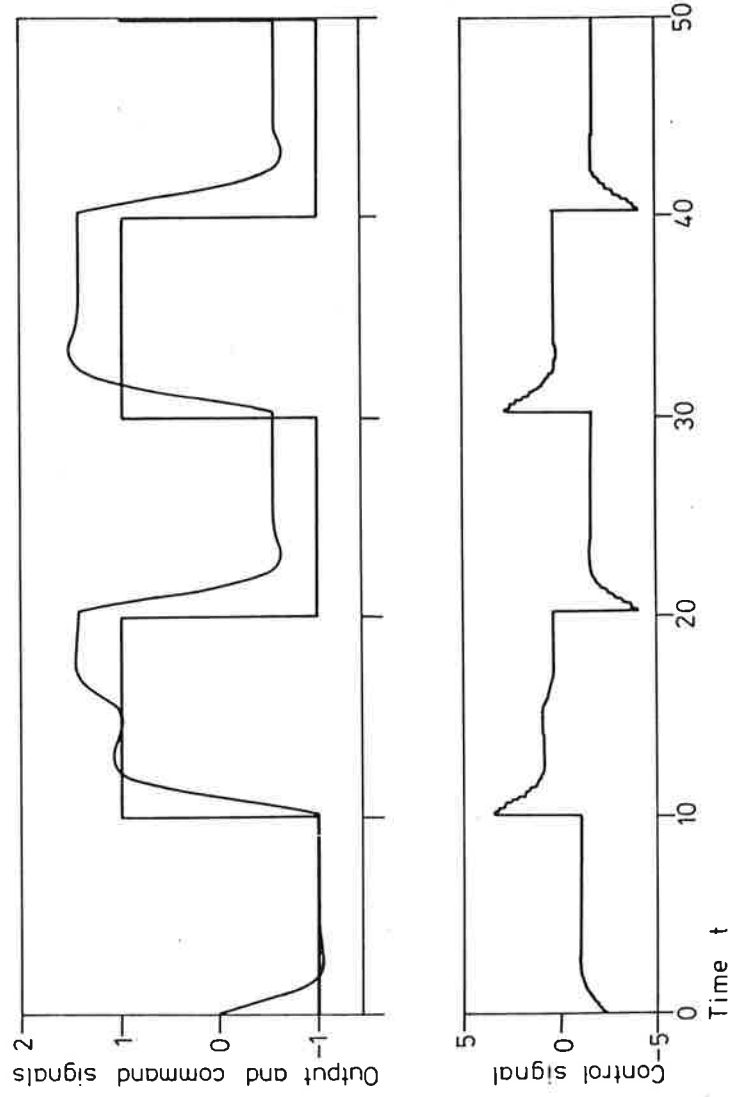


Fig. 2.9 - Simulation of the process  $G(s) = (1+s)^{-2}$  with a constant gain regulator. The process is subject to a unit gain load disturbance at time  $t = 15$  s.

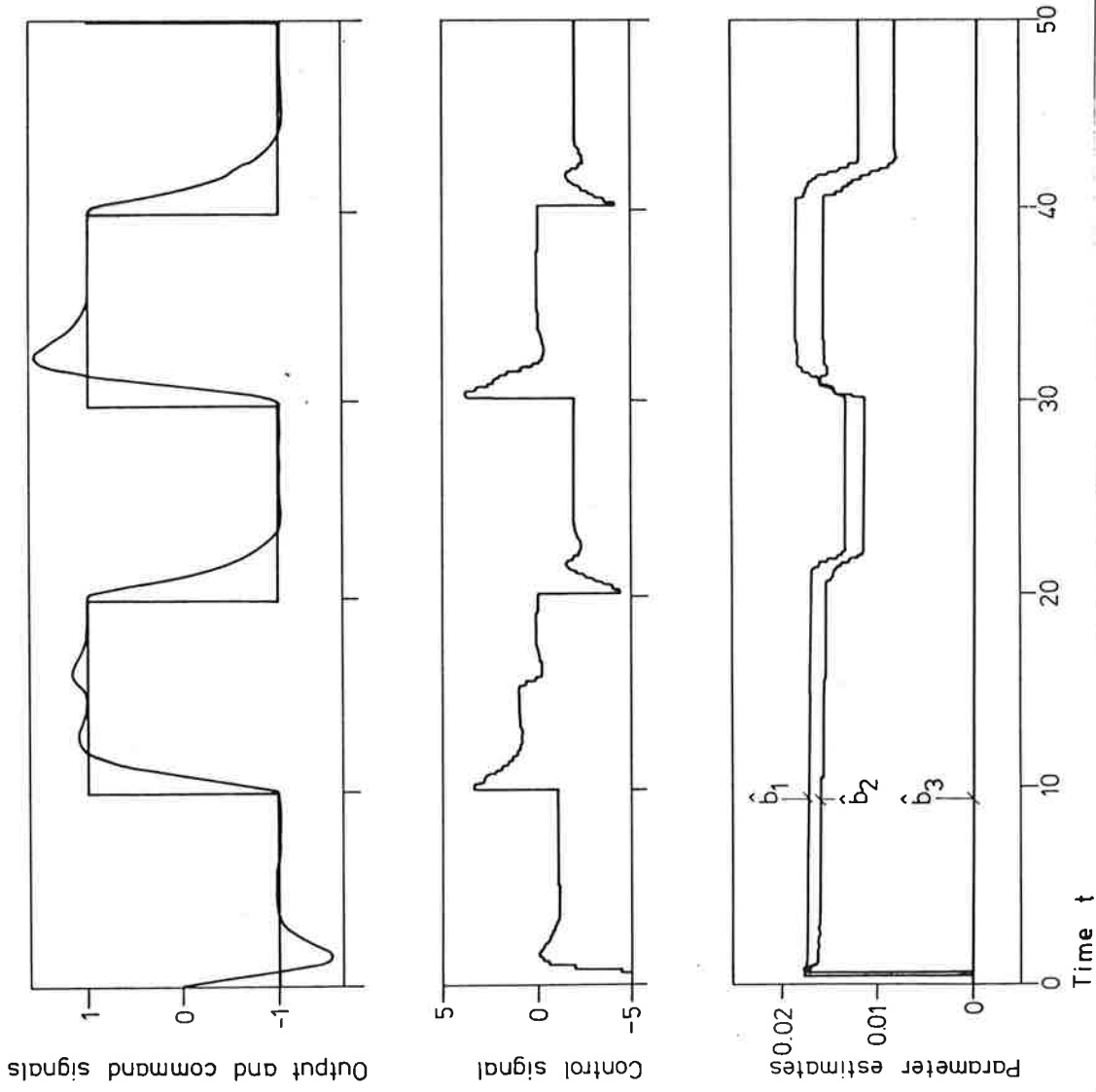
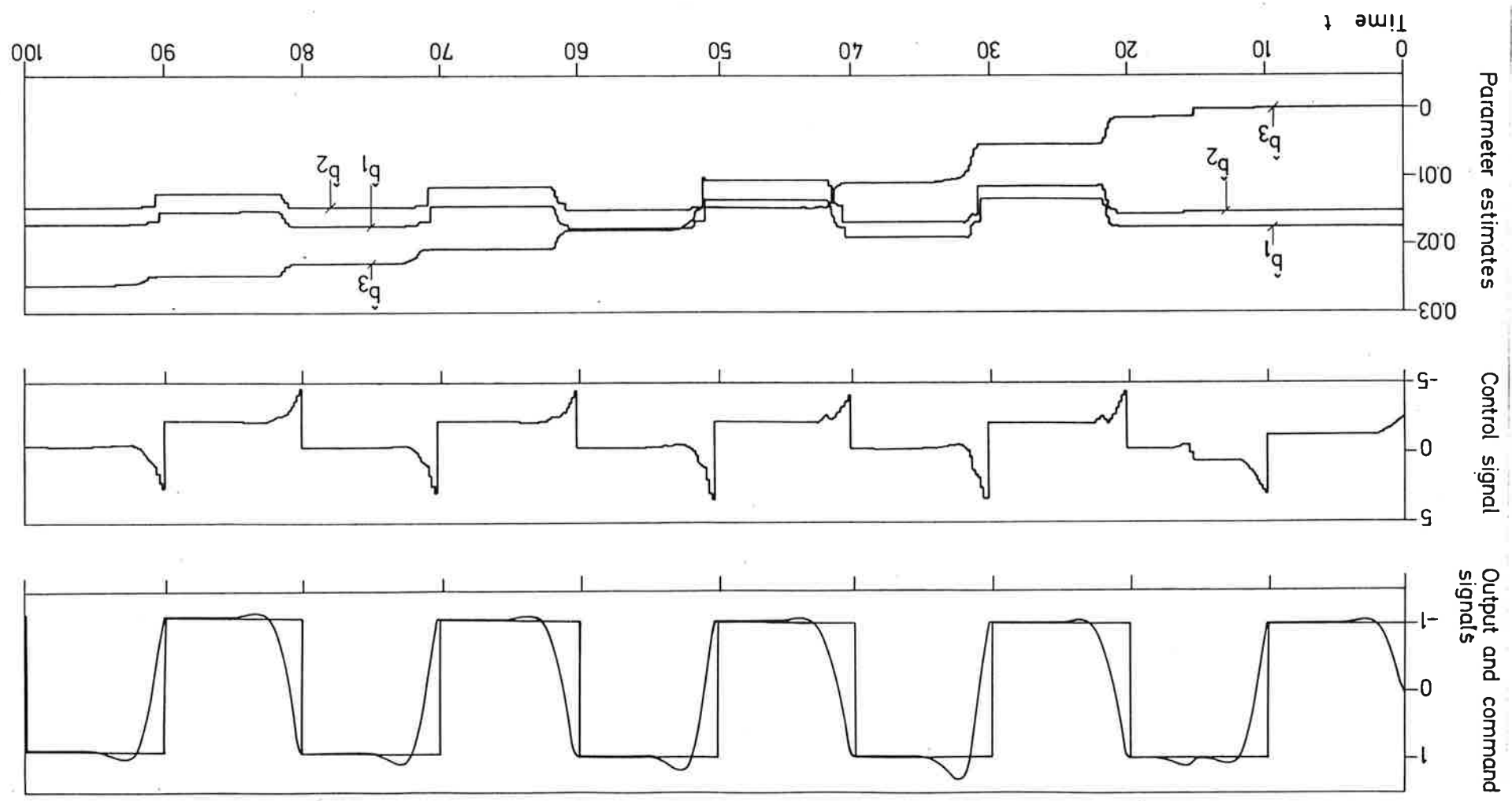


Fig. 2.10 - Simulation of the process  $G(s) = (1+s)^{-2}$  with a self-tuning controller where the bias  $b_3$  is not estimated. The process is subject to a unit step load disturbance at time  $t = 15$  s.

#### EXAMPLE 2.5

Consider the same process and the same regulator as in Example 2.4 but estimate the bias too. Fig. 2.11 shows what happens. There is no steady state error. The response at the system is, however, unsymmetric and the parameters estimates also change significantly during the transients. As time increases the estimate of the bias will, however, improve and the step responses become more and more symmetric.  $\square$

Fig. 2.11 - Simulation of the process  $G(s) = (1+s)^{-2}$  with a self-tuning controller. The process is subject to a unit step load disturbance at time 15 s and the bias  $b_3$  is estimated. The forgetting factor is 0.5 for the bias and 0.98 for the remaining parameters.



The behaviour shown in Example 2.5 indicates that it is advantageous to estimate the bias. The performance shown in Fig.2.11 is, however, far from satisfactory. The convergence is slow and there is too much interaction between the estimates of the bias and of the other parameters. It is possible to improve the convergence rate by choosing a smaller forgetting factor. The estimates will, however, then be more irregular and there will still be a substantial interaction between the estimation of the bias and of the other parameters. The obvious solution is to use different forgetting factors for the bias and for the remaining parameters. It also seems reasonable to simplify the calculations by eliminating the cross covariances between the bias and the remaining parameters. An example illustrates what happens when this is done.

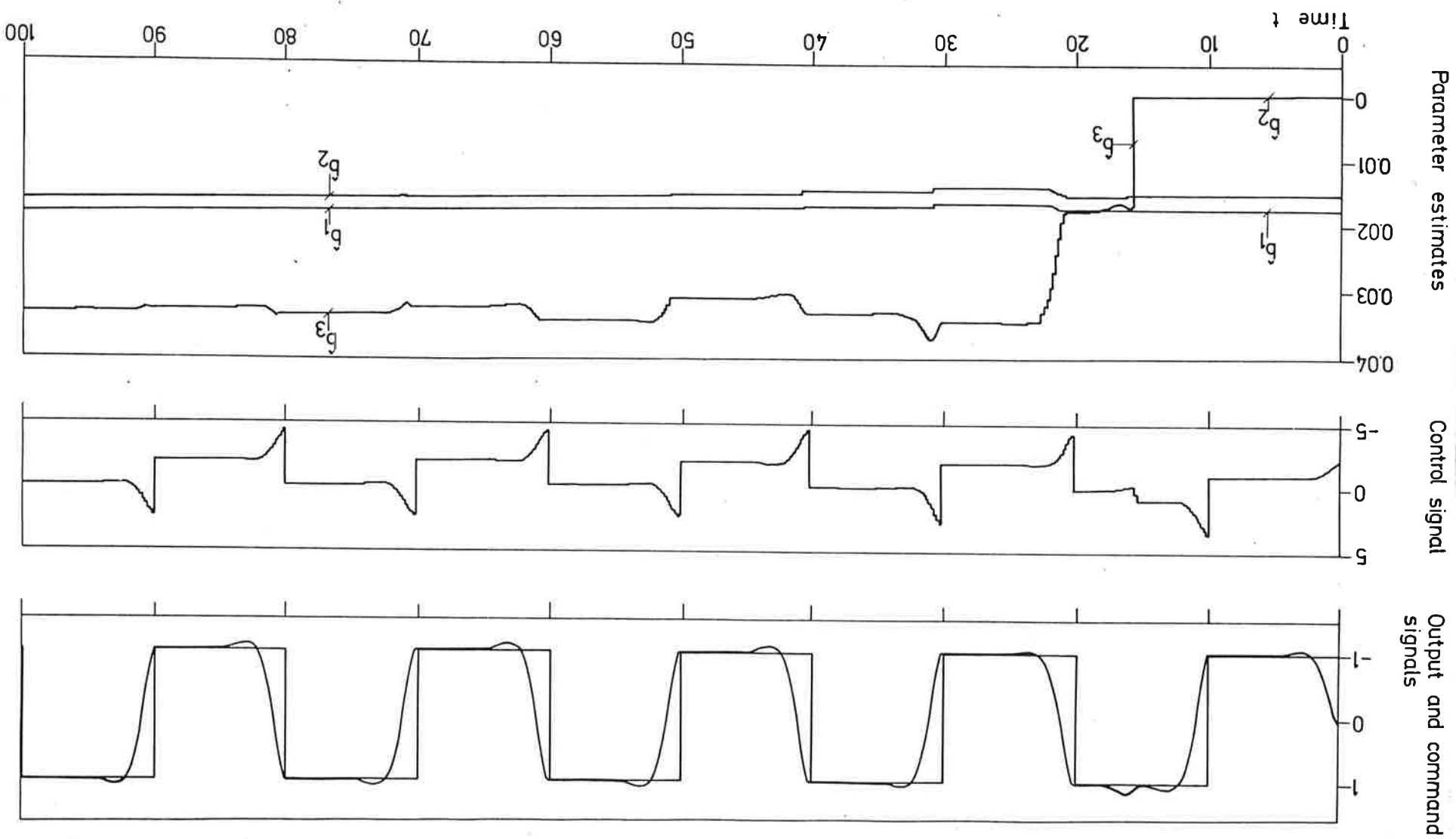
#### EXAMPLE 2.6

Consider the same process as in Example 2.5 but use now a separate forgetting factor for estimating the bias. The forgetting factor  $\lambda = 0.98$  is used for all parameters except for the bias for which  $\lambda = 0.5$  is used. The details of the algorithm are given in the program listing in Appendix A.

Figure 2.12 shows what happens. Notice that the transient behaviour of the adaptation loop is now much faster. Also notice that there is much smaller interaction between the estimation of the bias and the other parameters. The estimate of the bias now changes much more rapidly than before.  $\square$

The conclusions is thus that it is useful to estimate the bias  $b_3$  in the model and that it is advantageous to use a smaller forgetting factor for the bias.

Fig. 2.12 - Simulation of the process  $G(s) = (1+s)^{-2}$  with a self-tuning controller. The process is subject to a unit step load disturbance at time 15 and the bias  $b_3$  is estimated.





### 3. THE ALGORITHM

Based on the discussion in Section 2 it appears possible to design a self-tuning servo which has only one adjustable parameter namely the desired bandwidth of the closed loop system. The basic principles of such a regulator are given below.

#### Parameter Estimation

The parameters  $a_1$ ,  $a_2$ ,  $b_1$ ,  $b_2$ , and  $b_3$  of the process model

$$y(t) + a_1y(t-1) + a_2y(t-2) = b_1u(t-1) + b_2u(t-2) + b_3 \quad (3.1)$$

are estimated using least squares. The forgetting factors are applied only to the diagonal elements of the P-matrix. Separate forgetting factors are used for the bias  $b_3$  and for the remaining parameters. The cross-coupling between the bias and the remaining parameters are neglected. The Simmon code for the parameter estimation is listed below.

```
"PART 1 PARAMETER ESTIMATION
E=Y-TH1*F1-TH2*F2-TH3*F3-TH4*F4-TH5
"ESTIMATOR GAIN
K1=P11*F1+P12*F2+P13*F3+P14*F4
K2=P12*F1+P22*F2+P23*F3+P24*F4
K3=P13*F1+P23*F2+P33*F3+P34*F4
K4=P14*F1+P24*F2+P34*F3+P44*F4
K5=P55*F5
D=1+F1*K1+F2*K2+F3*K3+F4*K4+F5*K5
"UPDATE ESTIMATE
NT1=TH1+K1*E/D
NT2=TH2+K2*E/D
NT3=TH3+K3*E/D
NT4=TH4+K4*E/D
NT5=TH5+K5*E/D
"UPDATE COVARIANCES
N11=(P11-K1*K1/D)/LAM
N12=P12-K1*K2/D
```

N13=P13-K1\*K3/D  
 N14=P14-K1\*K4/D  
 N22=(P22-K2\*K2/D)/LAM  
 N23=P23-K2\*K3/D  
 N24=P24-K2\*K4/D  
 N33=(P33-K3\*K3/D)/LAM  
 N34=P34-K3\*K4/D  
 N44=(P44-K4\*K4/D)/LAM  
 N55=(P55-K5\*K5/D)/LAM1

"UPDATE F-VECTOR

NF1=-Y

NF2=F1

NF3=U

NF4=F3

### Regulator Design

The following control law is used in the algorithm

$$u(t) = t_0 u_c(t) - s_0 y(t) - s_1 y(t-n) - r_1 u(t-n) - u_b. \quad (3.2)$$

The parameters  $t_0$ ,  $s_0$ ,  $s_1$ , and  $r_1$  are chosen in such a way that the closed loop system has the poles obtained when sampling a continuous time system with poles

$$S = -\zeta\omega \pm \omega\sqrt{1-\zeta^2}. \quad (3.3)$$

Common zeros in the process model (3.1) are eliminated before calculating the control law. A normalized test-quantity  $\epsilon$  is used to decide if there are common factors. The process model (3.1) has a zero at

$$z_1 = -b_2/b_1.$$

If this zero corresponds to a well damped mode i.e.

$$z_1 \leq -b_2/b_1 \leq z_2$$

then the process zero is cancelled. Otherwise the process zero is also retained as a zero of the closed loop system. In the design it is also postulated to use a Luenberger observer with a zero at the origin. Since the process model and the desired closed loop system

are of low order the design equations can be solved analytically.  
The Simmon code for the design calculations is listed below.

"PART 2 - REGULATOR DESIGN

```

SQ=SQRT(1-Z*Z)
DT=2*PI/(NP*W*SQ)
P1=-2*EXP(-(2*PI/NP)*Z/SQ)*COS(2*PI/NP)
P2=EXP(-2*(2*PI/NP)*Z/SQ
PS=1+P1+P2
A1=TH1
A2=TH2
B1=TH3
B2=TH4
BS=B1+B2
BMAX=MAX(B1,B2)

N=B2*B2-A1*B1*B2+A2*B1*B1
W1=P1-A1
W2=P2-A2
W3=W1*B2-W2*B1
W4=W1*(A2*B1-A1*B2)+W2*B2
TEST=N/(BMAX*BMAX*EPS)
TEST1=IF -B2/B1<Z1 THEN -1 ELSE IF -B2/B1<Z2 THEN 1 ELSE -1

TO=IF TEST<1 THEN PS/B1 ELSE IF TEST1>0 THEN PS/B1 ELSE PS/BS
R1=IF TEST<1 THEN 0 ELSE B2*W3/N
S0=IF TEST<1 THEN P1/B1-A2/B2 ELSE IF TEST1>0 THEN (P1-A1)/B1 ELSE W4/N
S1=IF TEST<1 THEN P2/B1 ELSE IF TEST1>0 THEN (P2-A2)/B1 ELSE -A2*W3/N

UB=IF TEST<1 THEN TH5/B1 ELSE IF TEST1>0 THEN TH5/B1 ELSE (1+R1)*TH5/BS
U1=TO*UC-S0*Y-S1*(-F1)-R1*F3-UB

U=IF U1<ULOW THEN ULOW ELSE IF U1<UHIGH THEN U1 ELSE UHIGH

```

Parameters

The parameters of the regulator are listed below

$\omega, \zeta$	desired closed loop poles
$N_p$	number of samples per period
$\lambda, \lambda_1$	forgetting factors
$\epsilon$	test quantity for common factors
$z_1, z_2$	bounds on welldamped zero

$u_{\text{high}}, u_{\text{low}}$  hard limit on control signal

$\hat{a}_1(0), \hat{a}_2(0), \hat{b}_1(0), \hat{b}_2(0), \hat{b}_3(0)$  initial estimates

$P_0, P_{0b}$  initial covariances

The parameter  $\omega$  determines the desired closed loop bandwidth. The relative damping of the closed loop poles is normally set to 0.707. The sampling period is specified indirectly by requiring  $N_p$  to be 20 samples per period. There are reasonable universal values of the forgetting factor  $\lambda = 0.98$  and  $\lambda_1 = 0.8$ . The test quantities are chosen as  $\epsilon = 0.01$ ,  $z_1 = -0.1$ ,  $z_2 = 0.99$ . The hard limits of the output are normally set to values which would correspond to the limits of the DA-converters. The initial values are not crucial. They are chosen as follows:  $\hat{a}_1(0) = -1.5$ ,  $\hat{a}_2(0) = 0.7$ ,  $\hat{b}_1(0) = \hat{b}_2(0) = \hat{b}_3(0) = 0$ ,  $P_0 = 100$ ,  $P_{0b} = 0.01$ .

### Application and Tuning Rules

The controller can be expected to give good results for processes which can be modeled by (3.1) and conveniently controlled by a feedback law like (3.2). Besides the self-tuning controller will provide reset action. The controller can thus be expected to give good results when applied to processes which can be modeled by first or second order models. Since many stable high order systems can also be modeled well by (3.1) if the sampling period is sufficiently large, the controller can thus be expected to work well also when applied to high order systems provided that the desired bandwidth is sufficiently small.

The rule for applying the algorithm is thus very simple. Start with a low value of  $\omega$ . Subject the system to a step command and observe if the closed loop behaviour is as can be expected. Increase the bandwidth gradually and observe when the response of the system

starts to deviate from the expected (too high overshoot!) and when the control signal becomes so large that the system saturates. If the desired bandwidth can not be achieved it is **necessary to use a more complex control law.**

## 4. SIMULATIONS

The properties of the self-tuning servo in different situations will now be demonstrated using simulations.

### Process Model

In all simulations it is assumed that the process to be controlled has the transfer function

$$G(s) = \frac{k}{(1+sT)^n}.$$

In the simulations a load disturbance in terms of a step on the process input is also added. The parameters k, T, and N are changed in the simulations. A Simnon program for the process model is listed below.

```

CONTINUOUS SYSTEM PROC2

TIME T
INPUT U V
OUTPUT Y
STATE X1 X2 X3 X4 X5 X6
DER D1 D2 D3 D4 D5 D6

OUTPUT
Y1=IF 0<2 THEN X1 ELSE IF 0<3 THEN X2 ELSE IF 0<4 THEN X3 ELSE X4
Y=IF 0<4 THEN Y1 ELSE IF 0<5 THEN X4 ELSE IF 0<6 THEN X5 ELSE X6
DYNAMICS
PA1=IF T<TPCH THEN 1 ELSE PA11
PA2=IF T<TPCH THEN 1 ELSE PA22
D1=-PA1*X1+PA2*U+V
D2=-PA1*X2+X1
D3=-PA1*X3+X2
D4=-PA1*X4+X3
D5=-PA1*X5+X4
D6=-PA1*X6+X5

PA11:1
PA22:1
TPCH:50
FIRST:-1
0:1
END

```

### Bandwidth Changes

The case  $n = 2$  is first discussed. Since the process model can be described exactly by (3.1) good results can be expected. Figures 4.1, 4.2, and 4.3 shows results obtained for  $\omega = 0.5, 1.5,$  and  $4.5$ . Notice that the parameter estimates, except the bias, settle quickly in all cases. The irregular estimates of  $b_3$  is the prize to be paid for having the possibility to quickly eliminate load disturbances. Also notice that the relative variations in the controller parameters is somewhat larger than in the process parameters. The regulators obtained for different values of  $\omega$  are very different. This is seen from the comparison of the controller parameters in Table 4.1.

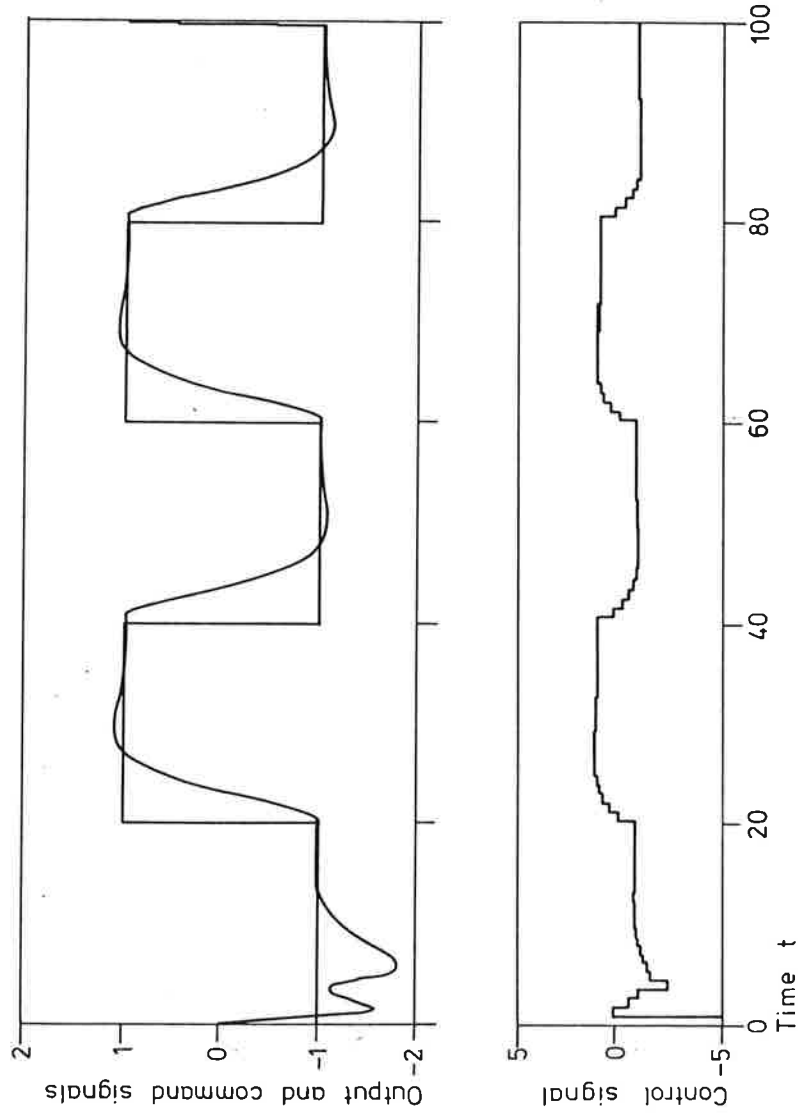


Fig. 4.1 - Simulation of the self-tuning controller applied to the system  $(1+s)^{-2}$  with specifications  $\omega = 0.5$ .

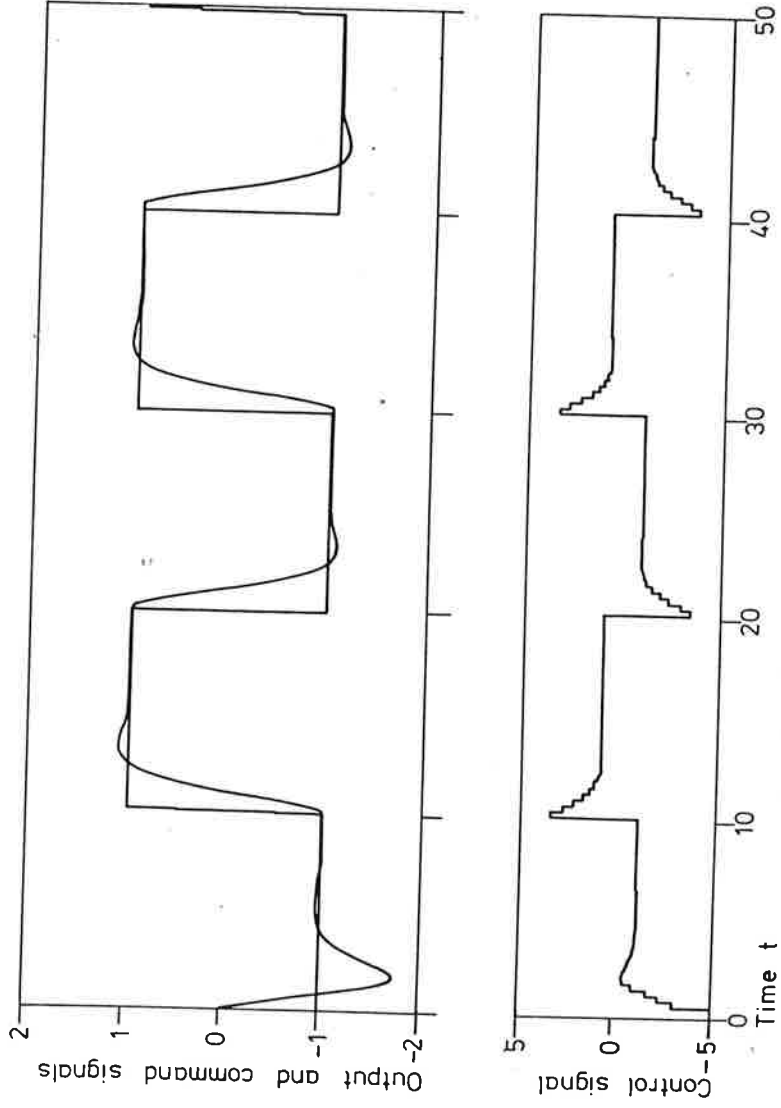


Fig. 4.2 - Simulation of the self-tuning controller applied to the system  $(1+s)^{-2}$  with specifications  $\omega = 1.5$

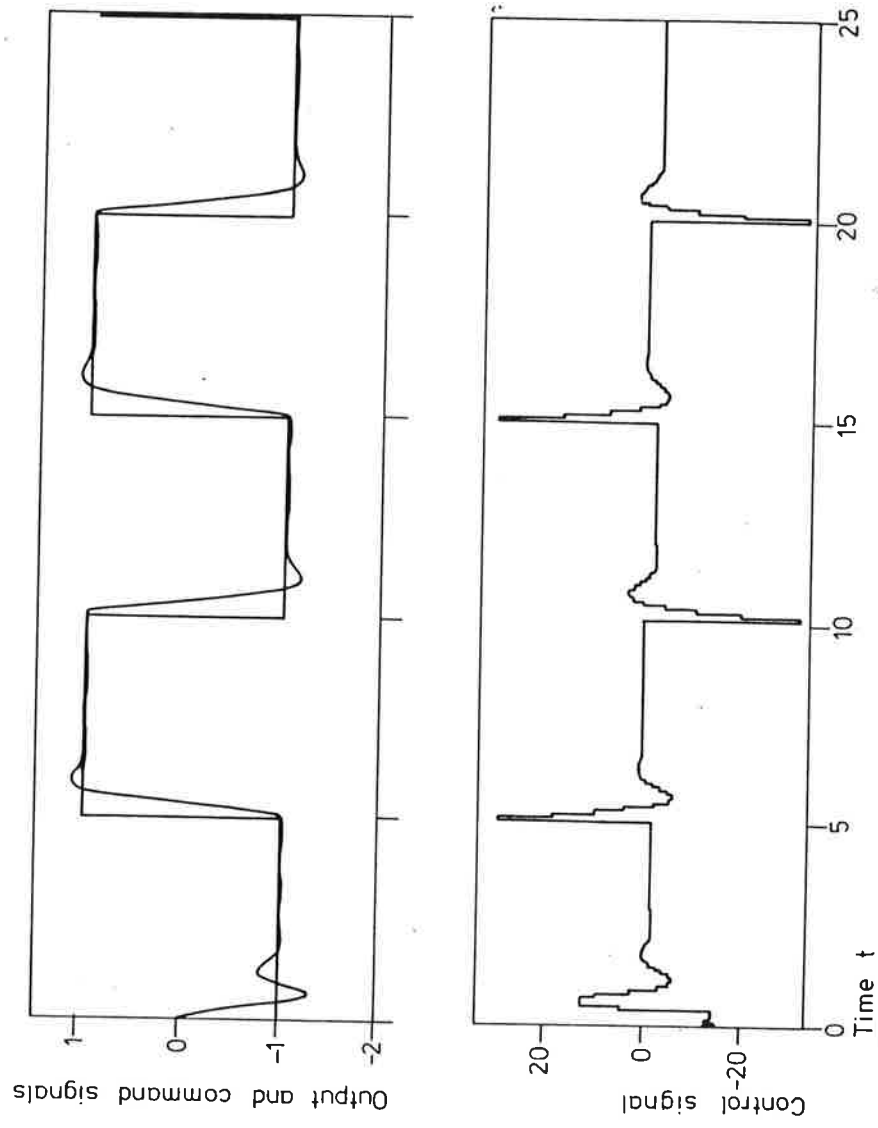


Fig. 4.3 - Simulation of the self-tuning controller applied to the system  $(1+s)^{-2}$  with specifications  $\omega = 4.5$ .



Table 4.1 - Limiting values of the controller parameters.

$\omega$	$r_1$	$s_0$	$s_1$	$t_0$
0.5	-0.332	-0.849	0.616	0.436
1.5	0.035	1.835	-0.663	2.208
4.5	0.189	51.26	-36.18	16.27

For  $\omega = 0.5$  the natural process response is faster than the desired response. The regulator gain is therefore negative. The control signal in Fig. 4.1 also shows a gradual buildup. For  $\omega = 1.5$  the controller gain is positive. In Fig. 4.2 it is seen that the initial value of the control signal at a step change is larger than the steady state value. For  $\omega = 4.5$  the controller gain is quite large. It is also seen from Fig. 4.3 that the regulator generates very large control signals during the transients to achieve the desired performance. Notice that the scale is different in Fig. 4.3.

#### Load Changes

The behaviour of the algorithm subject to load changes were discussed extensively in Section 2. See for example Fig. 2.9. It should be remembered, however, that the properties of the system in this respect depend drastically on the chosen value of  $\omega$ . In Fig. 4.4 are shown the response to load changes of a system with  $\omega = 4.5$

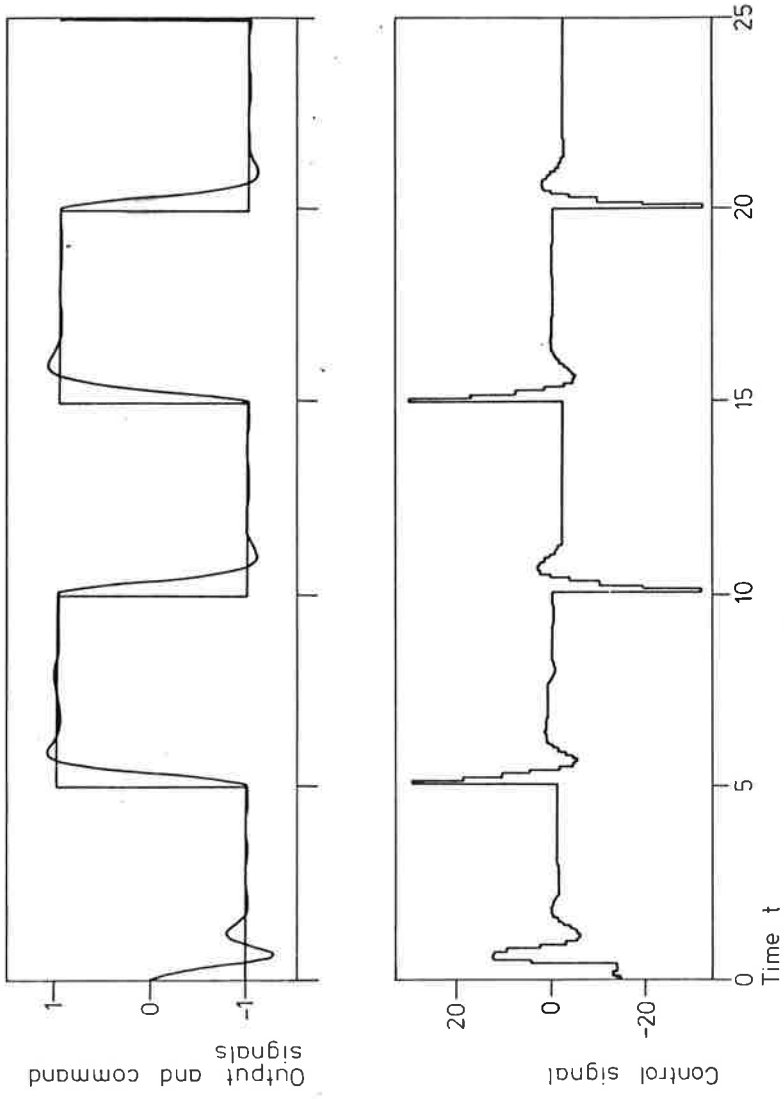


Fig. 4.4 - Simulation of the process  $G(s) = (1+s)^{-2}$  with a self-tuning controller. The process is subject to a unit step load disturbance at  $t = 15$  s. Compare with Figure 2.9.

#### Changes in Process Gain

The simulation in Fig. 4.5 shows what happens when the process gain is suddenly increased from  $k = 1$  to  $k = 4$  at time  $t = 15$  s. The increased gain makes the system momentarily unstable. The regulator recovers quickly and the step response is back to normal again after two steps. Also notice that the regulator parameters quickly change to the new values. The speed of response will of course depend on the chosen forgetting factors. The value  $\lambda = 0.95$  was used in the simulations. With  $\lambda = 0.98$  the convergence is somewhat slower.

The values of the controller gains that are obtained when the process gain is increased or decreased are shown in Table 4.2.

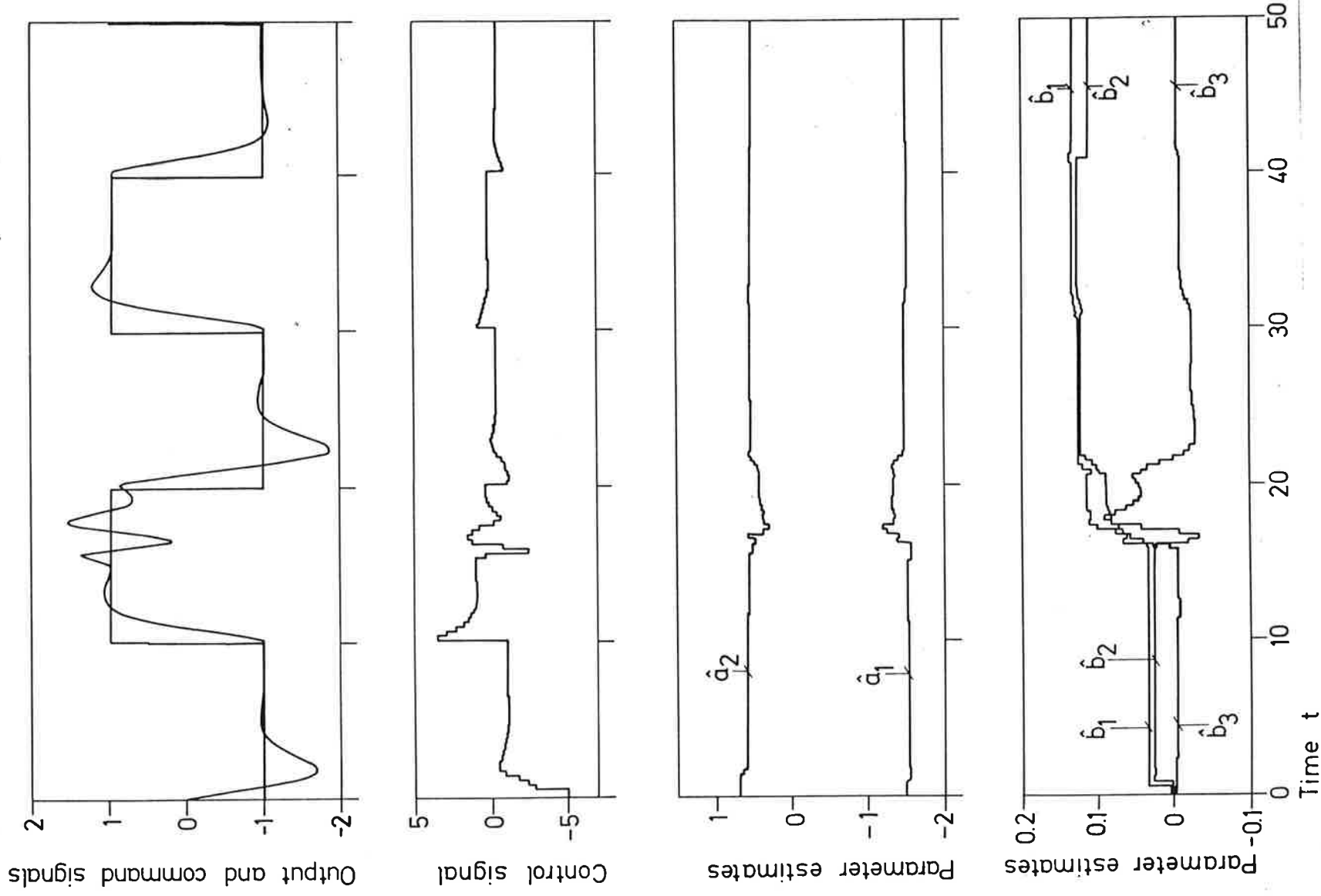


Fig. 4.5 - Simulation of the process  $G(s) = (1+s)^{-2}$  with a self-tuning controller. The process gain is increased from 1 to 4 at time  $t = 15$  s.

Table 4.2 - Limiting controller gains obtained for different values of process gains.

$k$	$r_1$	$s_0$	$s_1$	$t_0$
0.25				
1	0.035	1.835	-0.663	2.208
4	0.004	0.482	-0.179	0.558

The simulations indicate clearly that the controller copes well with changes in the process gain.

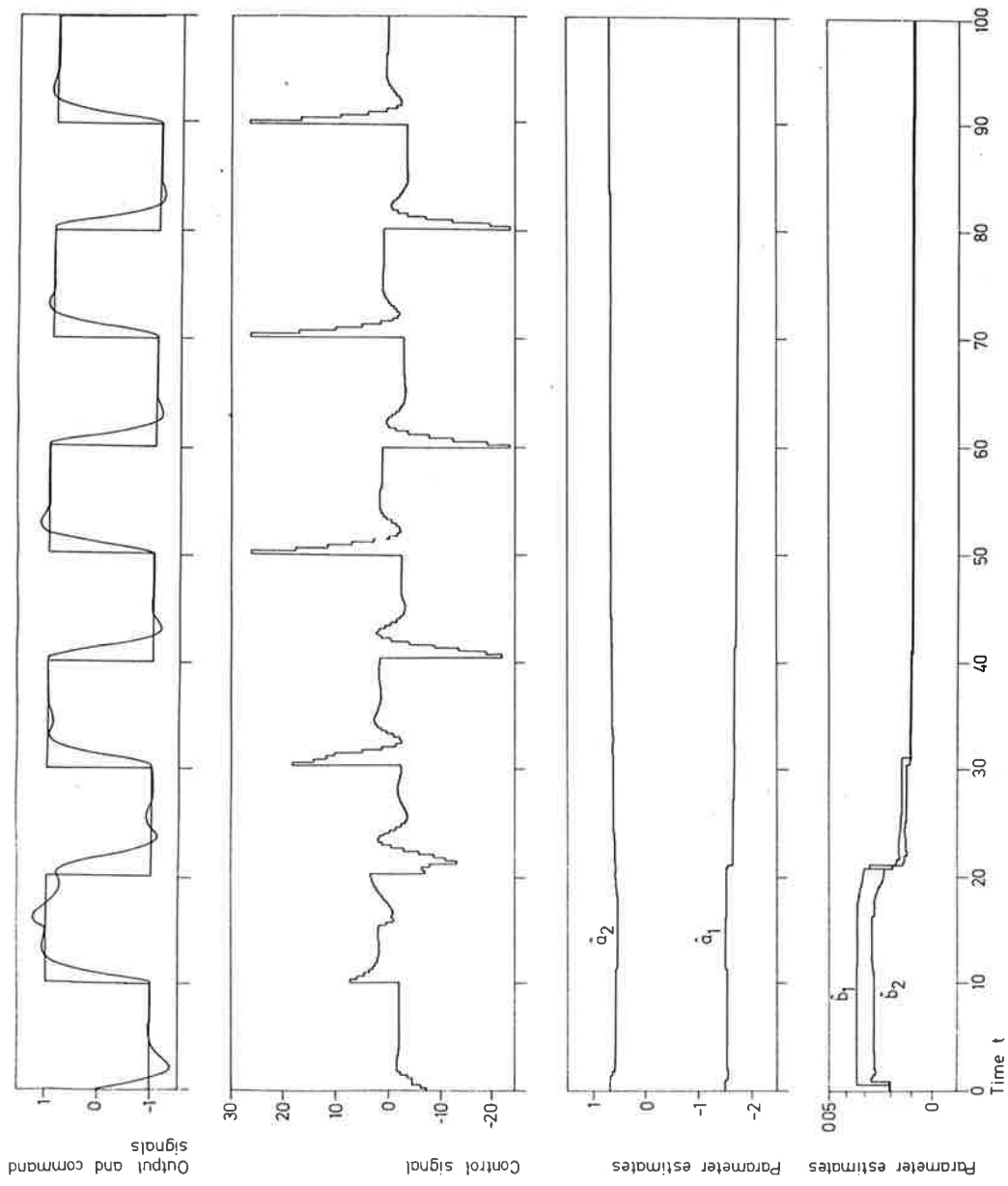
#### Changes in Process Time Constants

Figure 4.6 shows what happens when the process time constant  $T$  is increased from  $T = 1$  s to  $T = 2$  s at time  $t = 15$  s. Since the process model given by (2.1) is of second order this means that the effective process time constant is four times as large. Fig. 4.7 shows similar results when the process time constant is decreased from  $T = 1$  s to  $T = 0.5$  s. Notice that the limiting regulators obtained in the two cases are quite different. This is seen from Table 4.3.

For  $T = 0.5$  the process is faster than the desired response and the controller gain is therefore negative. Also notice that a substantial process gain is required when  $T = 2.0$  s. The simulations show clearly that the controller handles variations in process time constants well.

Table 4.3 - Limiting controller gains obtained for different process time constants.

$T$	$r_1$	$s_0$	$s_1$	$t_0$
0.5	-0.169	-0.800	0.709	0.739
1.0	0.035	1.835	-0.663	2.208
2.0	0.149	18.73	-12.27	7.604



**Fig. 4.6** - Simulation of the process  $G(s) = (1+sT)^{-2}$  with the self-tuning controller. The time constant  $T$  is 1 for  $0 \leq t \leq 15$  s and 2 for  $t \geq 15$  s. The forgetting factor is  $\lambda = 0.98$ .

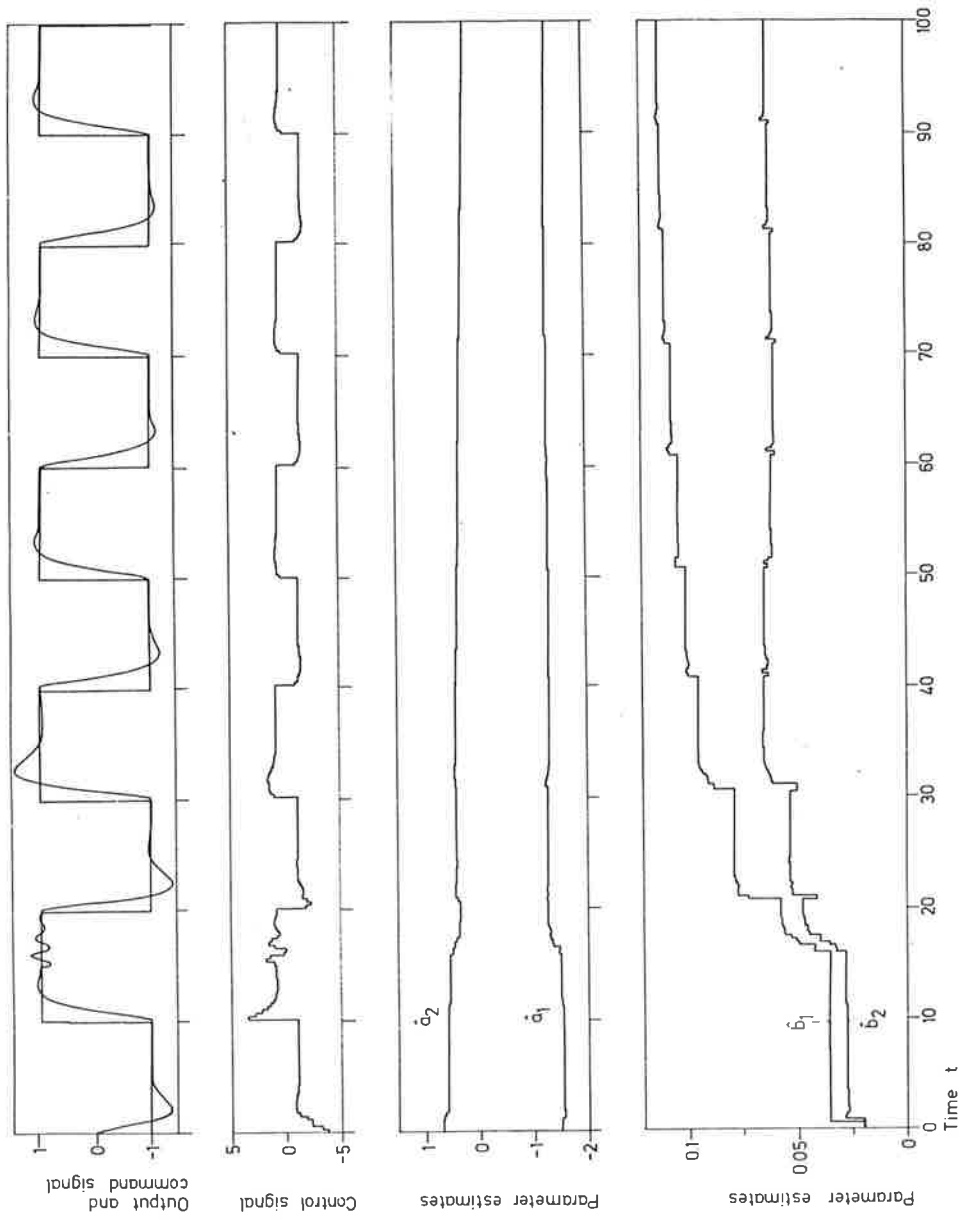


Fig. 4.7 - Simulation of the process  $G(s) = (1+sT)^{-2}$  with the self-tuning controller. The time constant  $T$  is 1 for  $0 \leq t < 15$  s and 0.5 for  $t \geq 15$  s. The forgetting factor is  $\lambda = 0.98$ .

### High Order Process Dynamics

The design of the self-tuning controller was based on the assumption that the process could be described by a second order model. It is of interest to investigate how the regulator behaves when it is applied to a process of higher order. A process with the transfer function

$$G(s) = \frac{1}{(1+s)^4}$$

was simulated. Figure 4.8 shows what happens when it is demanded that the closed loop system has  $\omega = 0.3$ . The result shows that the desired results are obtained. The overshoot and the response time are as expected. When the closed loop bandwidth is increased to  $\omega = 0.6$  the results shown in Fig. 4.9 are obtained. These results are far from satisfactory.

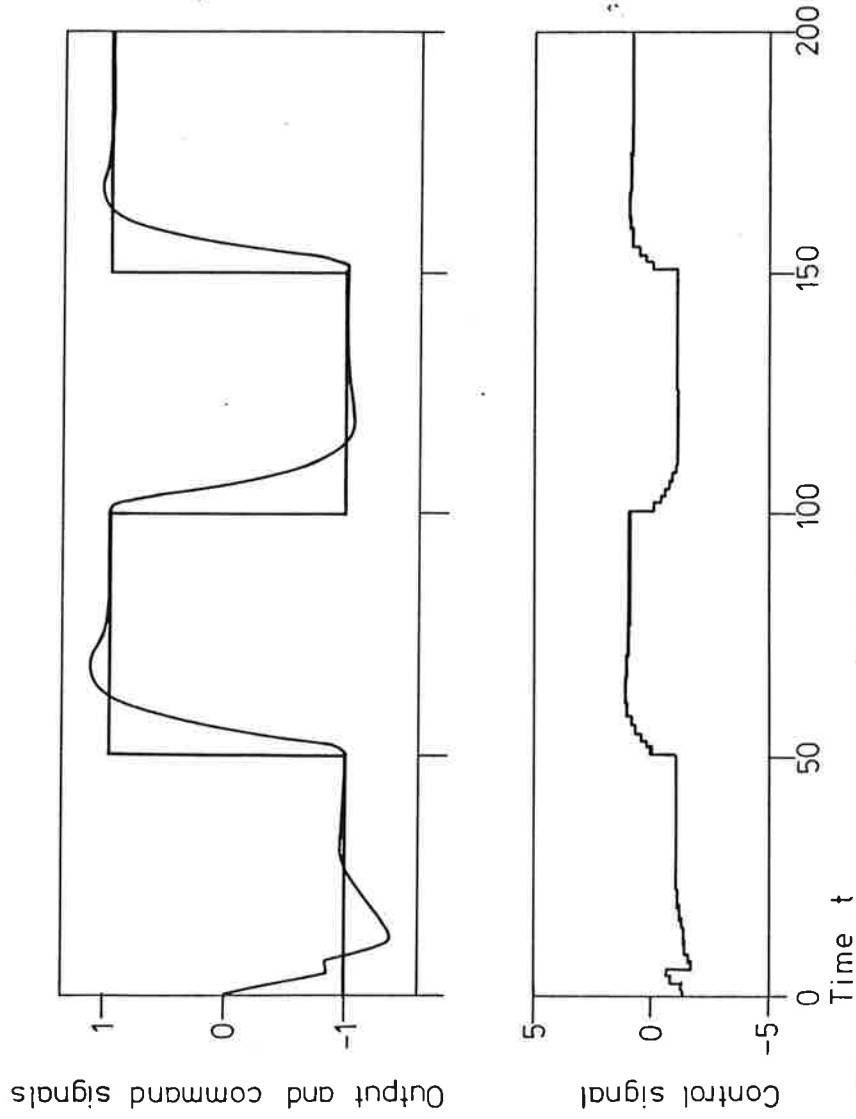


Fig. 4.8 - Simulation of the process  $G(s) = (1+s)^{-4}$  with the self-tuning controller. The specified  $\omega$  is 0.3.

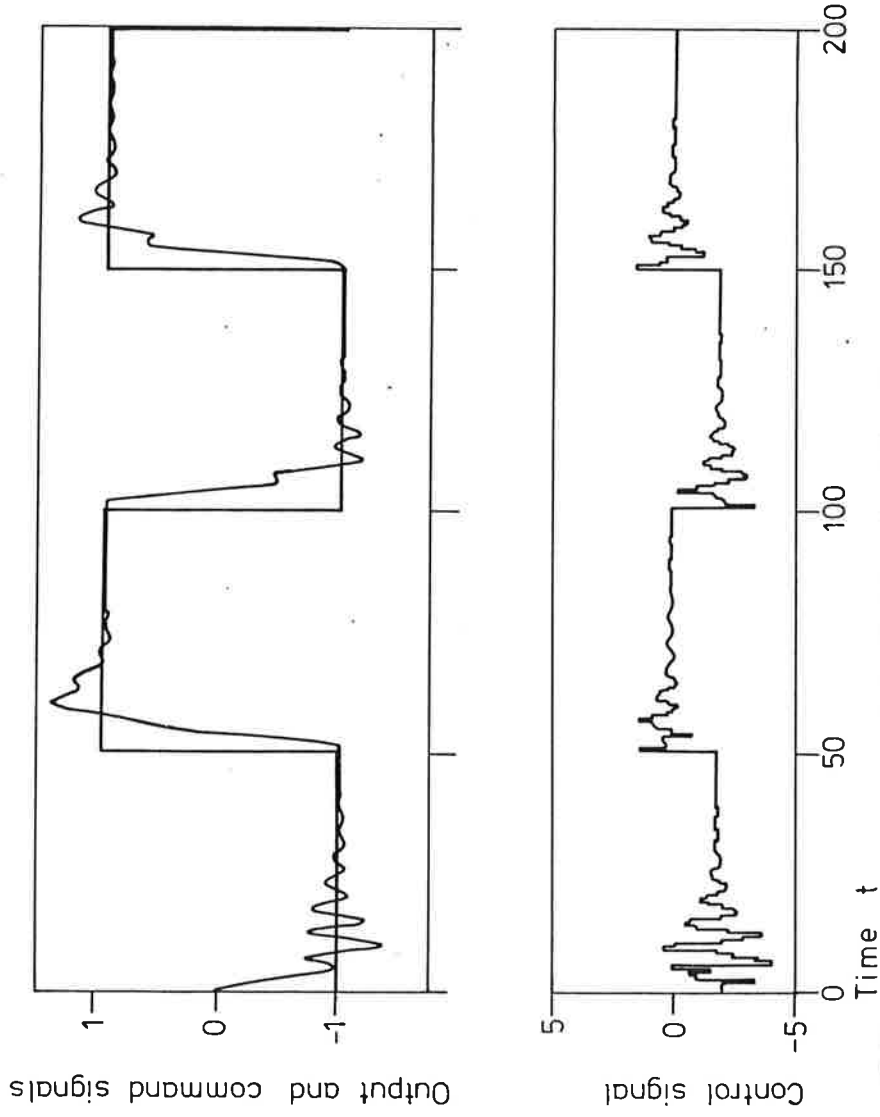


Fig. 4.9 - Simulation of the process  $G(s) = (1+s)^{-4}$  with the self-tuning controller. The specified  $\omega$  is 0.6.

The estimates of the process parameters obtained for some different values of  $\omega$  are shown in Table 4.4. The stepresponses of the estimated process models for  $\omega = 0.3$  and  $\omega = 0.6$  are shown in Fig. 4.10. The Nyquist-diagrams of the systems are shown in Fig. 4.11.

Table 4.4 - Process parameter estimates obtained when applying the self-tuning controller to the process  $(1+s)^{-4}$ .

$\omega$	$h$	$a_1$	$a_2$	$b_1$	$b_2$
0.30	1.48	-1.079	0.351	0.0567	0.2126
0.40	1.11	-1.333	0.496	0.0244	0.1347
0.50	0.89	-1.497	0.609	0.0136	0.0926
0.60	0.74	-1.624	0.707	0.0075	0.0717
0.80	0.56	-1.886	0.944	-0.0014	0.0551



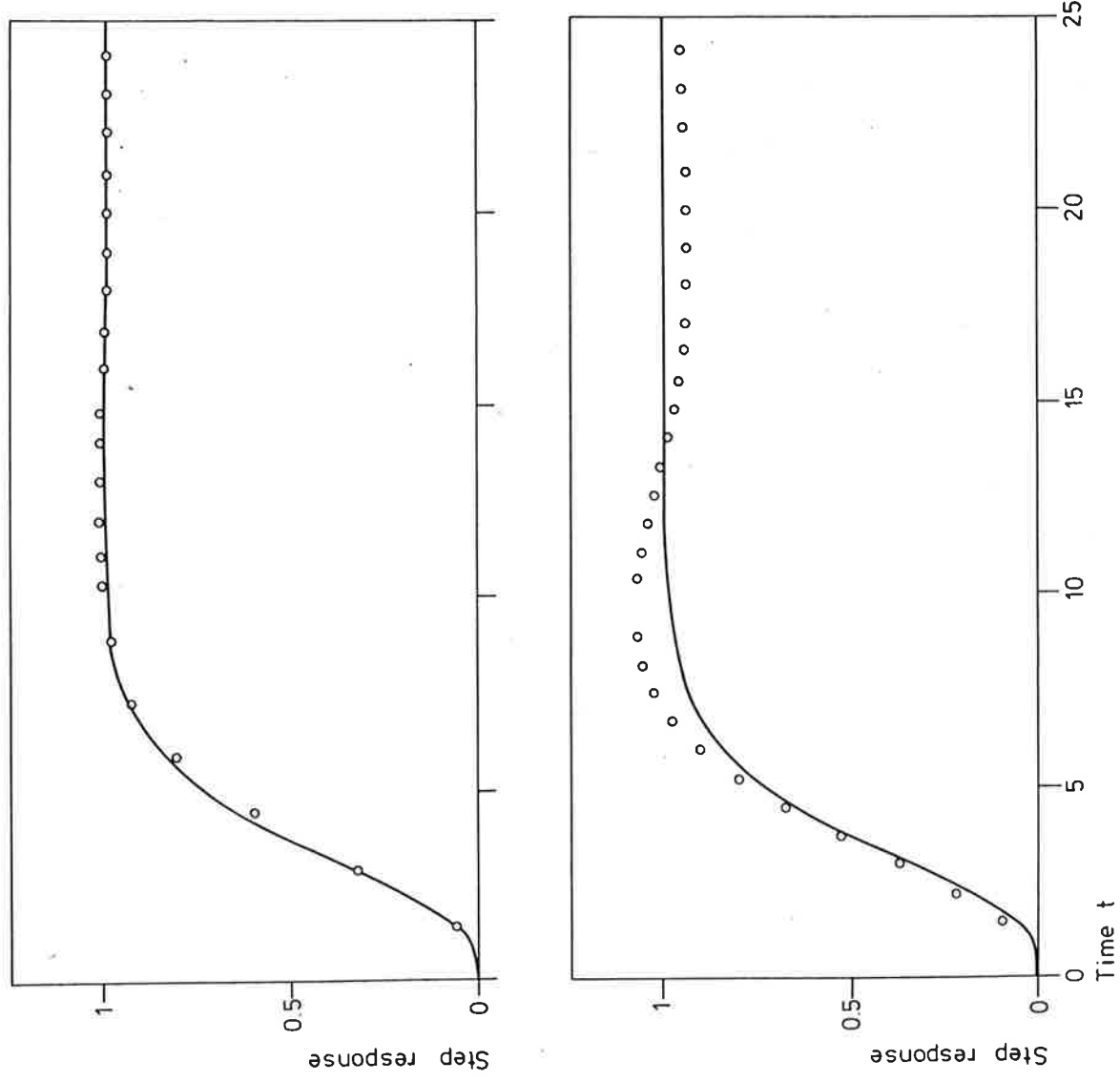


Fig. 4.10 - Step responses of the system  $G(s) = (1+s)^{-4}$  and of the estimated process models for  $\omega = 0.3$  (above) and  $\omega = 0.6$  (below).

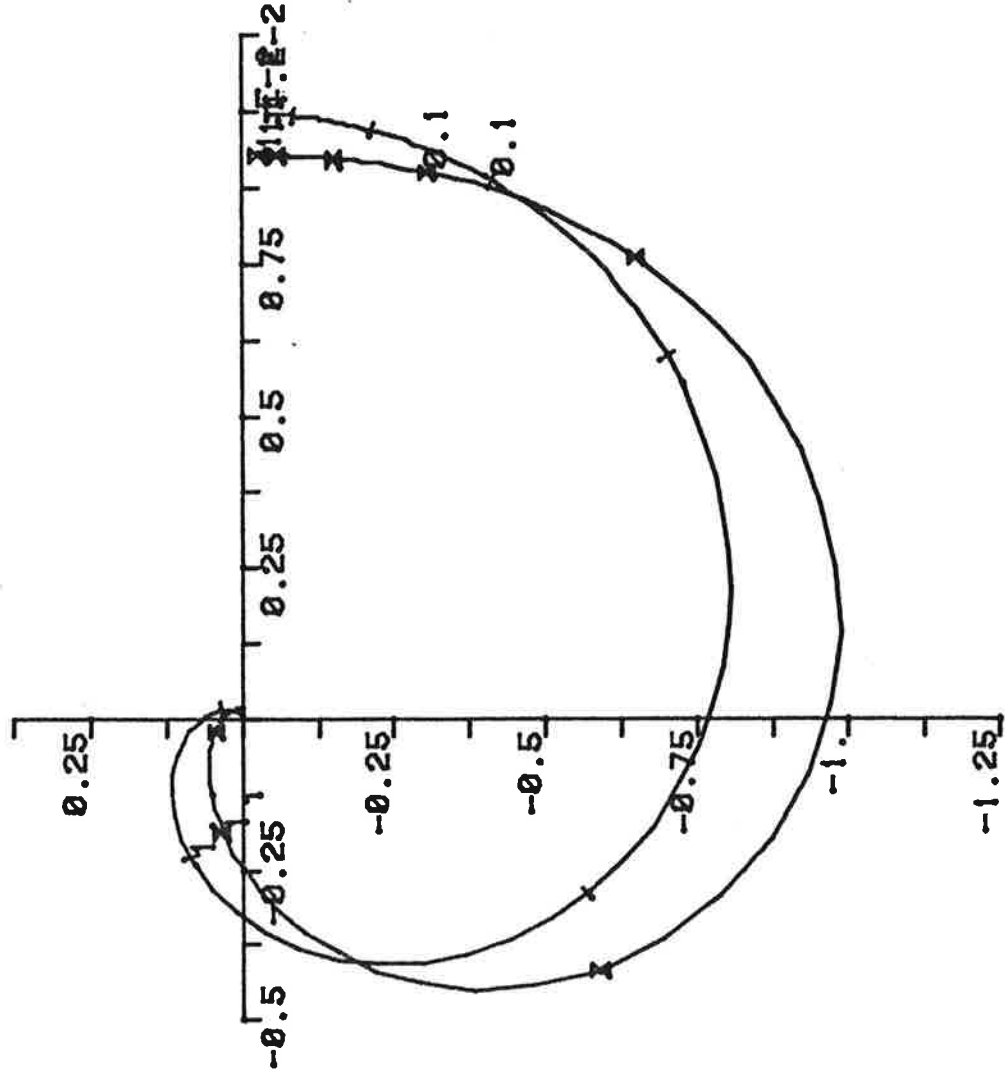


Fig. 4.11 - Nyquist-diagrams for the systems shown in Fig. 4.10.

It is seen from Fig. 4.11 that the estimated model and the sampled system model are closer for  $\omega = 0.3$  than for  $\omega = 0.6$ . According to the sensitivity results given in Aström (1979b) it is shown that the closed loop system designed on the basis of an approximative model  $G$  will be stable if

$$|G - G_0| \leq G_s = \left| \frac{G}{G_M} \right| \cdot \left| \frac{G_{FF}}{G_{FB}} \right|$$

where  $G_0$  is the true transfer function. In this particular case we have

$$G_s = \frac{z(z^2 + p_1z + p_2)}{(s_0z + s_1)(z^2 + a_1z + a_2)}$$

The magnitude of the transfer function  $G_S$  for the two cases are shown in Fig. 4.12. It is seen from this figure that a much more accurate model is given for the design corresponding to  $\omega = 0.6$ .

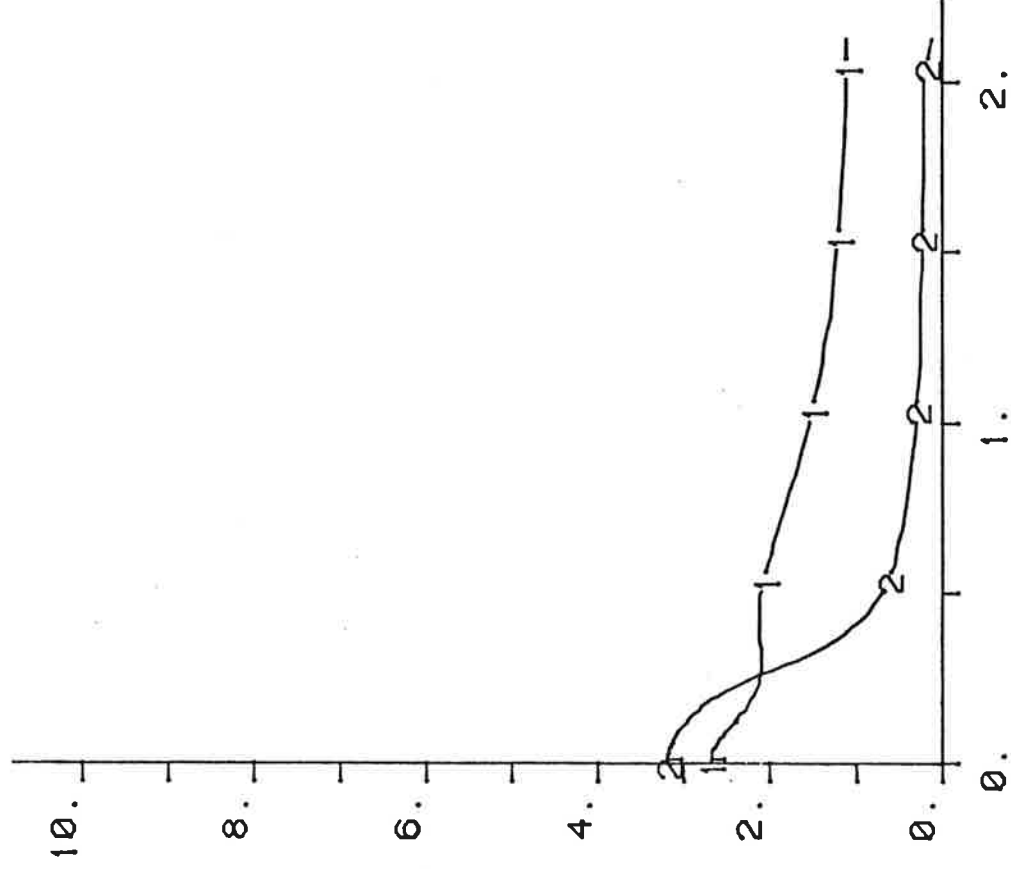


Fig. 4.12 - Magnitude of the transfer function  $G_S$  for  $\omega = 0.3$  and  $\omega = 0.6$ .

## 5. CONCLUSIONS

In modern DDC-packages there are frequently a large number of different versions of the PID-algorithm. It therefore does not seem unreasonable to have a number of different self-tuners for different types of problems. This report presents a self-tuner for typical servo problems. The self-tuner has one major adjustable parameter which is proportional to the desired bandwidth of the closed loop system. All other parameters are fixed or related to the bandwidth. It is shown by simulation that the algorithm will work well in many circumstances. When the estimated parameters are constant the regulator corresponds to a feedback law of the form

$$u(t) = t_0 u_c(t) - s_0 y(t) - s_1 y(t-h) - r_1 u(t-h) - u_D.$$

This feedback law corresponds to a proportional and derivative feedback. There is no explicit integrator in the feedback. In spite of this off sets are eliminated indirectly via the parameter estimation. It is in fact demonstrated that the self-tuning regulator will give zero steady state error.

There are several improvements of the regulator that could be interesting to investigate. The selection of forgetting factors is a compromise. It would thus be worthwhile to consider other ways to eliminate the bias and to choose the forgetting factors.

## 6. ACKNOWLEDGEMENT

The idea of designing a servo with one knob for the desired closed loop bandwidth was triggered during a discussion with my graduate student Carl Fredrik Mannerfelt. When assigned to do a self-tuner which could automatically tune its sampling period Carl Fredrik responded by saying: "Why should you?". I am also grateful to Dr Björn Wittenmark, with whom I have had many discussions on this topic.

This work was partially supported by the Swedish Board of Technical Development (STU) under contract No. 78-3763.

## 7. REFERENCES

- Aström, K J (1979a): Algebraic system theory as a tool for regulator design. Report CODEN: LUTFD2/(TFRT-7164)/1-023/(1979), Department of Automatic Control, Lund Institute of Technology, Lund, Sweden.
- Aström, K J (1979b): Robustness of a design method based on assignment of poles and zeros. Report CODEN: LUTFD2/(TFRT-3153)/1-014/(1979), Department of Automatic Control, Lund Institute of Technology, Lund, Sweden.
- Aström, K J, and B Wittenmark (1979): Self-tuning controllers based on pole-zero placement. Report CODEN: LUTFD2/(TFRT-7180)/1-43/(1979), Department of Automatic Control, Lund Institute of Technology, Lund, Sweden.
- Aström, K J, U Borisson, L Ljung, and B Wittenmark (1977): Theory and applications of self-tuning regulators. *Automatica* 13, 457-476.
- Wittenmark, B (1973): A self-tuning regulator. Thesis TFRT-1003, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden.
- Wittenmark, B (1979): Self-tuning PID-controllers based on pole placement. Report CODEN: LUTFD2/(TFRT-7179)/1-037/(1979), Department of Automatic Control, Lund Institute of Technology, Lund, Sweden.

## APPENDIX A - SIMMON PROGRAMS STRE3, PROC1

DISCRETE SYSTEM STRE3  
 "SELF-TUNING REGULATOR BASED ON POLE AND ZERO PLACEMENT  
 " SPECIAL VERSION FOR SECOND ORDER SYSTEMS  
 " THE PROCFSS ZERO IS RETAINED  
 " EXPLICIT IDENTIFICATION  
 " AUTHOR KJ ASTROM 790R20

INPUT Y UC  
 OUTPUT U  
 STATE F1 F2 F3 F4 TH1 TH2 TH3 TH4 TH5  
 STATE P11 P12 P13 P14 P22 P23 P24 P33 P34 P44 P55  
 NEW NF1 NF2 NF3 NF4 NT1 NT2 NT3 NT4 NT5  
 NEW N11 N12 N13 N14 N22 N23 N24 N33 N34 N44 N55  
 TIME T  
 TSAMP TS  
 INITIAL  
 TH1:-1.5  
 TH2:0.7  
 P11=P0  
 P22=P0  
 P33=P0  
 P44=P0  
 P55=P0

OUTPUT

"PART 2 - REGULATOR DESIGN

P1=-2\*EXP(-Z\*W\*DT)\*COS(W\*DT\*SQRT(1-7\*Z))  
 P2=EXP(-2\*Z\*W\*DT)  
 PS=1+P1+P2  
 A1=TH1  
 A2=TH2  
 B1=TH3  
 R2=TH4  
 BS=B1+R2  
 BMAX=MAX(B1,B2)  
 N=R2\*B2-A1\*B1\*R2+A2\*B1\*B1  
 W1=P1-A1  
 W2=P2-A2  
 W3=W1\*R2-W2\*B1  
 TEST=N/(RMAX\*BMAX\*EPS)  
 T0=IF TEST<1 THEN PS/B1 ELSE PS/RS  
 R1=IF TEST<1 THEN 0 ELSE B2\*W3/N  
 S0=IF TEST<1 THEN P1/B1-A2/B2 ELSE (W1\*(A2\*B1-A1\*B2)+W2\*R2)/N  
 S1=IF TEST<1 THEN P2/B1 ELSE -A2\*W3/N

```

UB=IF TEST<1 THEN TH5/R1 ELSE (1+R1)*TH5/BS
U1=T0*UC-S0*Y-S1*(-F1)-R1*F3-UR
U=IF U1<ULOW THEN ULOW ELSE IF U1<UHIGH THEN U1 ELSE UHIGH

DYNAMICS

"PART 1 PARAMETER ESTIMATION
E=Y-TH1*F1-TH2*F2-TH3*F3-TH4*F4-TH5

"ESTIMATOR GAIN
K1=P11*F1+P12*F2+P13*F3+P14*F4
K2=P12*F1+P22*F2+P23*F3+P24*F4
K3=P13*F1+P23*F2+P33*F3+P34*F4
K4=P14*F1+P24*F2+P34*F3+P44*F4
K5=P55*F5
D=1+F1*K1+F2*K2+F3*K3+F4*K4+F5*K5

"UPDATE ESTIMATE
NT1=TH1+K1*E/D
NT2=TH2+K2*E/D
NT3=TH3+K3*E/D
NT4=TH4+K4*E/D
NT5=TH5+K5*E/D

"UPDATE COVARIANCES
N11=(P11-K1*K1/D)/LAM
N12=P12-K1*K2/D
N13=P13-K1*K3/D
N14=P14-K1*K4/D
N22=(P22-K2*K2/D)/LAM
N23=P23-K2*K3/D
N24=P24-K2*K4/D
N33=(P33-K3*K3/D)/LAM
N34=P34-K3*K4/D
N44=(P44-K4*K4/D)/LAM
N55=(P55-K5*K5/D)/LAM1

"UPDATE F-VECTOR
NF1=-Y
NF2=F1
NF3=U
NF4=F3

"UPDATE SAMPLING TIME
TS=T+DT

PO:100
POB:0
W:1.5
Z:0.707
DT:0.2

```



LAM:0.98  
LAMI:5  
UHIGH:5  
ULOW:-5  
F5:1  
EPS:1.F-5

END

---

CONTINUOUS SYSTEM PROC1

TIME T  
INPUT U V  
OUTPUT Y  
STATE X1 X2 X3  
DER D1 D2 D3  
  
OUTPUT  
Y=IF 0<2 THEN X1 ELSE IF 0<3 THEN X2 ELSE X3  
  
DYNAMICS  
PA1=IF T<TPCH THEN 1 ELSE PA11  
PA2=IF T<TPCH THEN 1 ELSE PA22  
D1=-PA1\*X1+PA2\*U+V  
D2=-PA1\*X2+X1  
D3=-PA1\*X3+X2  
  
PA11:1  
PA22:1  
TPCH:50  
FIRST:-1  
0:1  
END

## APPENDIX B - EXAMPLE 2.2

Data for Figure 2.4

```

DISCRETE SYSTEM STRE3
TIME : T          50.0000
STATE : F1        1.00004
        F4       -0.9999935
        TH3      1.763140E-02
        P11      10.8859
        P14      0.4999973
        P24     -0.345708
        P44      0.199465
INIT  : F1        0.000000
        F4        0.000000
        TH3      0.000000
        P11      0.000000
        P14      0.000000
        P24      0.000000
        P44      0.000000
NEW   : NF1      1.00001
        NF4     -0.9999980
        NT3      1.763140E-02
        N11      10.9928
        N14      0.502027
        N24     -0.343883
        N44      0.203497
INPUT : Y        -1.00001
OUTPUT: U        -1.00001
TSAMP : TS       50.2000
PAR   : PO      100.000
        W       1.50000
        ULOW    -5.00000
        LAM     0.980000
VAR   : P1      -1.58149
        A1      -1.61450
        B2      1.725639E-02
        N       9.908721E-04
        W3      4.829569E-04
        R1      8.410866E-03
        UB      0.000000
        K1      0.461620
        K4     -8.395918E-03
        F2      1.00007
        TH1     -1.61450
        TH4      1.725639E-02
        P12     -9.71358
        P22      9.68449
        P33      0.160839
        P55      0.000000
        F2      0.000000
        TH1      0.000000
        TH4      0.000000
        P12      0.000000
        P22      0.000000
        P33      0.000000
        P55      0.000000
        NF2      1.00004
        NT1     -1.61450
        NT4      1.725639E-02
        N12     -9.81392
        N22      9.79117
        N33      0.164097
        N55      0.000000
        UC       1.00000
        F3     -0.9999980
        TH2      0.649384
        TH5      0.000000
        P13      0.210452
        P23     -9.317899E-02
        P34     -3.679857E-02
        F3      0.000000
        TH2      0.000000
        TH5      0.000000
        P13      0.000000
        P23      0.000000
        P34      0.000000
        NF3     -1.00001
        NT2      0.649384
        NT5      0.000000
        N13      0.212107
        N23     -9.170840E-02
        N34     -3.682867E-02
        Z        0.707000
        EPS      1.000000E-04
        F5        1.00000
        PS       7.279973E-02
        R1       1.763140E-02
        RMAX      1.763140E-02
        W2       4.908860E-03
        T0        2.08668
        S1       -0.316514
        E         2.500601E-07
        K3       -6.765923E-03
        D         1.88697
        POB      0.000000
        DT       0.200000
        UHIGH    5.00000
        LAM1     5.00000
        P2       0.654293
        A2       0.649384
        BS       3.488779E-02
        W1       3.300267E-02
        TEST     31874.5
        S0       1.39477
        U1       -1.00001
        K2       0.410138
        K5       0.000000
        X2       -1.00001
        X2       0.000000
        D2       1.166612E-04
        V        0.000000
        TPCH     50.0000
        FIRST   -1.00000
        PA2      1.00000
        PA11     1.00000
        X3       -0.999794
        X3       0.000000
        D3       -2.128631E-04
        PA11     1.00000

```

## CONTINUOUS SYSTEM PROC1

```

TIME : T          50.0000
STATE : X1       -0.9999891
INIT  : X1       0.000000
DER   : D1      -1.220852E-04
INPUT : U        -1.00001
OUTPUT: Y        -1.00001
PAR   : O        2.00000
        PA22     1.00000
        PA1      1.00000
        X2       -1.00001
        X2       0.000000
        D2       1.166612E-04
        V        0.000000
        TPCH     50.0000
        FIRST   -1.00000
        PA2      1.00000
        X3       -0.999794
        X3       0.000000
        D3       -2.128631E-04
        PA11     1.00000

```

## CONNECTING SYSTEM EX1

TIME : T	50.0000				
PAR : PER	20.0000	U1	-1.00000	U2	1.00000
TV	50.0000	V1	0.00000	V2	0.00000
AP	0.200000	BP	-3.00000		
VAR : UP	-3.20000				

## Data for Figure 2.5

## DISCRETE SYSTEM STRES

TIME : T	50.0000				
STATE : F1	1.00011	F2	1.00012	F3	-0.999890
F4	-0.999870	TH1	-1.64354	TH2	0.675878
TH3	1.748443E-02	TH4	1.485085E-02	TH5	0.000000
P11	8.78099	P12	-7.84062	P13	0.164219
P14	0.375031	P22	7.86464	P23	-7.588292E-02
P24	-0.259255	P33	0.117339	P34	-2.368760E-02
P44	0.146084	P55	0.000000		
INIT : F1	0.000000	F2	0.000000	F3	0.000000
F4	0.000000	TH1	-2.00000	TH2	1.00000
TH3	0.000000	TH4	0.000000	TH5	0.000000
P11	0.000000	P12	0.000000	P13	0.000000
P14	0.000000	P22	0.000000	P23	0.000000
P24	0.000000	P33	0.000000	P34	0.000000
P44	0.000000	P55	0.000000		
NEW : NF1	1.00009	NF2	1.00011	NF3	-0.999911
NF4	-0.999890	NT1	-1.64354	NT2	0.675878
NT3	1.748443E-02	NT4	1.485085E-02	NT5	0.000000
N11	8.86753	N12	-7.92193	N13	0.165418
N14	0.376524	N22	7.95085	N23	-7.480941E-02
N24	-0.257919	N33	0.119717	N34	-2.370731E-02
M44	0.149041	N55	0.000000		
INPUT : Y	-1.00009	UC	1.00000		
OUTPUT: U	-0.999911				
TSAMP : TS	50.2000				
PAR : PO	100.000	P08	0.000000	Z	0.707000
W	1.50000	DT	0.200000	EPS	1.000000E-04
ULOW	-5.00000	UHIGH	5.00000	F5	1.00000
LAM	0.980000	LAM1	5.00000		
VAR : P1	-1.58149	P2	0.654293	PS	7.279973E-02
A1	-1.64354	A2	0.675878	R1	1.748443E-02
R2	1.485085E-02	BS	3.233527E-02	EMAX	1.748443E-02
N	8.539271E-04	W1	6.204939E-02	W2	-2.158515E-02
W3	1.298890E-03	TEST	27933.0	T0	2.25140
R1	2.258930E-02	S0	2.25687	S1	-1.02806
UB	0.000000	U1	-0.999911	E	-1.587905E-07
K1	0.401193	K2	0.359223	K3	-5.296686E-03
K4	-6.595507E-03	K5	0.000000	D	1.77240

## CONTINUOUS SYSTEM PROC1

```

TIME : T          50.0000
STATE : X1        -1.00001
INIT  : X1         0.000000
DER   : D1         9.655952E-05
INPUT : U          -0.999911
OUTPUT: Y         -1.00009
PAR   : O          2.00000
      : PA22       1.00000
VAR   : PA1       1.00000
      : PA11       1.00000
      : X3         -0.999776
      : X3         0.000000
      : D3         -3.178418E-04
      : TPCH       50.0000
      : FIRST     -1.00000
      : PA2       1.00000

```

## CONNECTING SYSTEM EX1

```

TIME : T          50.0000
PAR   : PER       20.0000
      : TV         50.0000
      : AP         0.200000
VAR   : UP        -3.19998
      : U1        -1.00000
      : V1         0.000000
      : BP        -3.00000
      : U2         1.00000
      : V2         0.000000

```

## Data for Figure 2.6

## DISCRETE SYSTEM STRES3

```

TIME : T          50.0000
STATE : F1        1.00011
      : F4        -0.999865
      : TH3       1.752368E-02
      : P11       12.6925
      : P14       0.566340
      : P24       -0.401051
      : P44       0.204404
INIT  : F1        0.000000
      : F4        0.000000
      : TH3       2.000000E-02
      : P11       0.000000
      : P14       0.000000
      : P24       0.000000
      : P44       0.000000
NEW   : NF1       1.00009
      : NF4       -0.999893
      : NT3       1.752368E-02
      : N11       12.8168
      : N14       0.568449
      : N24       -0.399179
      : N44       0.208542
INPUT : Y         -1.00009
OUTPUT: U         -0.999919
TSAMP : TS        50.2000
PAR   : PO       100.000
      : W         1.50000
      : ULOW      -5.00000
      : LAM       0.980000
      : F2        1.00013
      : TH1       -1.63740
      : TH4       1.534014E-02
      : P12       -11.3524
      : P22       11.2686
      : P33       0.163536
      : P55       0.000000
      : F2        0.000000
      : TH1       -1.60000
      : TH4       1.000000E-02
      : P12       0.000000
      : P22       0.000000
      : P33       0.000000
      : P55       0.000000
      : NF2       1.00011
      : NT1       -1.63740
      : NT4       1.534014E-02
      : N12       -11.4696
      : N22       11.3925
      : N33       0.166851
      : N55       0.000000
      : UC        1.00000
      : P0B       0.000000
      : DT        0.200000
      : UHIGH     5.00000
      : LAM1     5.00000
      : F3        -0.999893
      : TH2       0.670267
      : TH5       0.000000
      : P13       0.000000
      : P23       0.000000
      : P34       0.000000
      : NF3       -0.999919
      : NT2       0.670267
      : NT5       0.000000
      : N13       0.264118
      : N23       -0.134870
      : N34       -3.094227E-02
      : Z         0.707000
      : EPS       1.000000E-04
      : F5        1.00000

```

VAR	:	P1	-1.58149	P2	0.654293	PS	7.279973E-02
	:	A1	-1.63740	A2	0.670267	R1	1.752368E-02
	:	B2	1.534014E-02	BS	3.286383E-02	RMAX	1.752368E-02
	:	N	8.813052E-04	W1	5.590987E-02	W2	-1.597397E-02
	:	W3	1.137588E-03	TEST	28699.6	T0	2.21519
	:	R1	1.980105E-02	S0	2.06057	S1	-0.865180
	:	UB	0.000000	U1	-0.999919	F	-1.699664E-08
	:	K1	0.511264	K2	0.453776	K3	-6.532279E-03
	:	K4	-8.166708E-03	K5	0.000000	D	1.97985

-----

CONTINUOUS SYSTEM PROC1

TIME	:	T	50.0000	X2	-1.00009	X3	-0.999790
STATE	:	X1	-0.999977	X2	0.000000	X3	0.000000
INIT	:	X1	0.000000	D2	1.079738E-04	D3	-2.957433E-04
DER	:	D1	5.820394E-05	V	0.000000		
INPUT	:	U	-0.999919				
OUTPUT	:	Y	-1.000009	TPCH	50.0000	PA11	1.00000
PAR	:	O	2.000000	FIRST	-1.00000		
	:	PA22	1.000000	PA2	1.00000		
VAR	:	PA1	1.000000				

-----

CONNECTING SYSTEM EX1

TIME	:	T	50.0000	U1	-1.00000	U2	1.00000
PAR	:	PER	20.0000	V1	0.000000	V2	0.000000
	:	TV	50.0000	BP	-3.00000		
	:	AP	0.200000				
VAR	:	UP	-3.19998				

## APPENDIX C - EXAMPLE 2.3

Data for Figure 2.7 A

```

DISCRETE SYSTEM STRES
TIME : T          50.0000
STATE : F1        1.00011
        F4       -0.9999659
        TH3      2.130907E-02
        P11      9.65710
        P14      0.490416
        P24     -0.351379
        P44      0.175146
INIT  : F1        0.000000
        F4        0.000000
        TH3      0.200000
        P11      0.000000
        P14      0.000000
        P24      0.000000
        P44      0.000000
NEW   : NF1      1.000009
        NF4     -0.9999689
        NT3      2.130907E-02
        N11      9.75132
        N14      0.492226
        N24     -0.349794
        N44      0.178687
INPUT : Y        -1.000009
OUTPUT: U       -0.9999918
TSAMP : TS      50.2222
PAR   : PO      100.000
        W        1.50000
        ULOW     -5.00000
        LAM      0.980000
VAR   : P1      -1.53639
        A1      -1.60198
        R2      1.834659E-02
        N        1.254240E-03
        W3      1.575531E-03
        R1      2.304632E-02
        UB      0.000000
        K1      0.427916
        K4     -7.682636E-03

        F2      1.000013
        TH1     -1.60198
        TH4      1.834659E-02
        P12      -8.53877
        P22      8.46142
        P33      0.133592
        P55      0.000000
        F2      0.000000
        TH1     -1.600000
        TH4      0.200000
        P12      0.000000
        P22      0.000000
        P33      0.000000
        P55      0.000000
        NF2      1.000011
        NT1     -1.60198
        NT4      1.834659E-02
        N12     -8.62708
        N22      8.55517
        N33      0.136299
        N55      0.000000
        UC      1.000000

        F3      0.000000
        TH2      0.700000
        TH5      0.000000
        P13      0.000000
        P23      0.000000
        P34      0.000000
        NF3     -0.9999918
        NT2      0.641634
        NT5      0.000000
        M13      0.201400
        M23     -9.949481E-02
        M34     -2.842231E-02

        Z        0.707000
        EPS      1.000000E-04
        F5        1.000000
        PS       8.778299E-02
        R1       2.130907E-02
        RMAX     2.130907E-02
        K2      -1.746432E-02
        T0       2.21363
        S1      -0.805997
        E       -1.504086E-07
        K3     -5.857613E-03
        D        1.81641

        P0B      0.000000
        DT      0.222222
        UHIGH    5.00000
        LAM1     5.00000
        P2       0.624170
        A2       0.641634
        BS       3.965566E-02
        W1       6.559160E-02
        TEST     27621.8
        S0       1.99658
        U1      -0.999918
        K2       0.374857
        K5       0.000000

        X2       -1.00009
        X2       0.000000
        D2      1.067519E-04
        V        0.000000
        TPCH     50.0000
        FIRST   -1.00000
        PA2      1.00000

CONTINUOUS SYSTEM PROC1
TIME : T          50.0000
STATE : X1       -0.9999982
INIT  : X1       0.000000
DER   : D1      6.382167E-05
INPUT : U       -0.9999918
OUTPUT: Y       -1.000009
PAR   : O        2.00000
        PA22     1.00000
        PA1      1.00000

```

## CONNECTING SYSTEM EX1

TIME : T	50.0000				
PAR : PER	20.0000	U1	-1.00000	U2	1.00000
TV	50.0000	V1	0.00000	V2	0.00000
AP	0.200000	BP	-3.00000		
VAR : UP	-3.19998				

## Data for Figure 2.7 B

## DISCRETE SYSTEM STPE3

TIME : T	50.0000				
STATE : F1	1.00016	F2	0.999913	F3	-0.999902
F4	-0.999842	TH1	-1.02687	TH2	0.263626
TH3	0.144295	TH4	9.246067E-02	TH5	0.000000
P11	1.30879	P12	-0.955527	P13	1.781061E-02
P14	0.226115	P22	0.911915	P23	-2.511634E-03
P24	-0.153217	P33	3.338497E-02	P34	-1.223567E-02
P44	9.913663E-02	P55	0.000000		
INIT : F1	0.000000	F2	0.000000	F3	0.000000
F4	0.000000	TH1	-1.00000	TH2	0.300000
TH3	0.100000	TH4	0.100000	TH5	0.000000
P11	0.000000	P12	0.000000	P13	0.000000
P14	0.000000	P22	0.000000	P23	0.000000
P24	0.000000	P33	0.000000	P34	0.000000
P44	0.000000	P55	0.000000		
NEW : NF1	1.00014	NF2	1.00016	NF3	-0.999890
NF4	-0.999802	NT1	-1.02687	NT2	0.263626
NT3	0.144295	NT4	9.246068E-02	NT5	0.000000
N11	1.32562	N12	-0.965410	N13	1.832678E-02
N14	0.227347	N22	0.920239	N23	-1.985115E-03
N24	-0.151960	N33	3.403823E-02	N34	-1.230129E-02
N44	0.101000	N55	0.000000		
INPUT : Y	-1.00007	UC	1.00000		
OUTPUT: U	-0.999890				
TSAMP : TS	50.0002				
PAR : PO	100.000	P08	0.000000	Z	0.707000
W	1.50000	DT	0.666670	FPS	1.000000E-04
ULOW	-5.00000	UHIGH	5.00000	F5	1.000000
LAM	0.980000	LAM1	5.00000		
P1	-0.749712	P2	0.243167	PS	0.493455
A1	-1.02687	A2	0.263626	B1	0.144295
R2	9.246067E-02	BS	0.236756	RMAX	0.144295
N	2.773812E-02	W1	0.277157	W2	-2.045944E-02
W3	2.857831E-02	TEST	13322.1	T0	2.08423
R1	9.526130E-02	S0	1.26058	S1	-0.271612
UB	0.000000	U1	-0.999890	E	-1.138076E-06
K1	0.109665	K2	0.111863	K3	-5.842642E-03
K4	-1.394033E-02	K5	0.000000	D	1.24131

## CONTINUOUS SYSTEM PROC1

```

TIME : T          50.0000
STATE : X1        -0.999965
INIT  : X1         0.000000
DER   : D1        7.474422E-05
INPUT : U         -0.999890
OUTPUT: Y        -1.000007
PAR   : O          2.000000
      : PA22       1.000000
VAR   : PA1       1.000000
      : TPCB       50.0000
      : FIRST     -1.000000
      : PA2       1.000000
      : PA11      1.000000
      : X2        -1.000007
      : X3         0.000000
      : D2         1.029819E-04
      : V          0.000000
      : X3         0.999844
      : X3         0.000000
      : D3        -2.236664E-04

```

## CONNECTING SYSTEM EX1

```

TIME : T          50.0000
PAR   : PER       20.0000
      : TV         50.0000
      : AP         0.200000
VAR   : UP        -3.19998
      : U1        -1.000000
      : V1         0.000000
      : BP        -3.000000
      : U2         1.000000
      : V2         0.000000

```

## Data for Figure 2.7 C

## DISCRETE SYSTEM STRES3

```

TIME : T          50.0000
STATE : F1        0.999696
      : F4        -1.00161
      : TH3       0.593990
      : P11       7.58123
      : P14       4.33317
      : P24      -1.80747
      : P44       2.85859
INIT  : F1        0.000000
      : F4        0.000000
      : TH3       0.600000
      : P11       0.000000
      : P14       0.000000
      : P24       0.000000
      : P44       0.000000
NEW   : NF1       1.000004
      : NF4      -0.999655
      : NT3       0.593990
      : N11       7.73477
      : N14       4.32302
      : N24      -1.73163
      : N44       2.82552
      : Y         -1.000004
OUTPUT: U         2.04846
TSAMP : TS        52.0000
      : F2        0.996236
      : TH1      -0.270321
      : TH4       0.153863
      : P12      -3.14287
      : P22       1.60430
      : P33       5.971441E-02
      : P55       0.000000
      : F2        0.000000
      : TH1      -0.300000
      : TH4       0.200000
      : P12       0.000000
      : P22       0.000000
      : P33       0.000000
      : P55       0.000000
      : NF2       0.999696
      : NT1      -0.270321
      : NT4       0.153860
      : N12      -3.13428
      : N22       1.57152
      : N33       6.034892F-02
      : N55       0.000000
      : UC        1.000000
      : F3        -0.909655
      : TH2       1.815424E-02
      : TH5       0.000000
      : P13       0.152110
      : P23      -6.453098E-02
      : P34       5.926339E-02
      : F3        0.000000
      : TH2       0.000000
      : TH5       0.000000
      : P13       0.000000
      : P23       0.000000
      : P34       0.000000
      : NF3       2.04846
      : NT2       1.815679E-02
      : NT5       0.000000
      : N13       0.151299
      : N23      -5.846820E-02
      : N34       5.210208E-02

```



PAR	:	PO	100.000	POB	0.000000	Z	0.707000
	:	W	1.50000	DT	2.00000	FPS	1.000000E-04
	:	ULOW	-5.00000	UHIGH	5.00000	F5	1.00000
	:	LAM	0.980000	LAM1	5.00000		
VAR	:	P1	0.125525	P2	1.437881E-02	PS	1.13990
	:	A1	-0.270321	A2	1.815424E-02	B1	0.593990
	:	B2	0.153863	BS	0.747853	RMAX	0.593990
	:	N	5.478432E-02	W1	0.395846	W2	-3.775431E-03
	:	W3	6.314843E-02	TEST	1552.74	T0	1.52424
	:	R1	0.177353	S0	0.367839	S1	-2.092590E-02
	:	UB	0.000000	U1	2.04846	E	1.314655E-05
	:	K1	-4.434407E-02	K2	0.331242	K3	-3.127683E-02
	:	K4	-0.391261	K5	0.000000	D	1.70882

## CONTINUOUS SYSTEM PROC1

TIME	:	T	50.0000	X2	-1.00004	X3	-0.999782
STATE	:	X1	-0.999840	X2	0.000000	X3	0.000000
INIT	:	X1	0.000000	D2	1.955628E-04	D3	-2.538264E-04
DER	:	D1	3.04830	V	0.000000		
INPUT	:	U	2.04846				
OUTPUT	:	Y	-1.00004	TPCH	50.0000	PA11	1.00000
PAR	:	O	2.00000	FIRST	-1.00000		
	:	PA22	1.00000	PA2	1.00000		
VAR	:	PA1	1.00000				

## CONNECTING SYSTEM EX1

TIME	:	T	50.0000	U1	-1.00000	U2	1.00000
PAR	:	PER	20.0000	V1	0.000000	V2	0.000000
	:	TV	50.0000	BP	-3.00000		
	:	AP	0.200000				
VAR	:	UP	-2.59031				

## APPENDIX D - SIMNON PROGRAMS STSE2, PROC2

```

DISCRETE SYSTEM STSE2
"SELF-TUNING REGULATOR BASED ON POLY AND ZERO PLACEMENT
"
" SPECIAL VERSION FOR SECOND ORDER SYSTEMS
" A SECOND ORDER PROCESS MODEL IS ESTIMATED EXPLICITLY
" COMMON FACTORS IN THE ESTIMATED TRANSFER FUNCTION ARE CANCELLED
" THE PROCESS ZERO IS RETAINED IF IT IS POORLY DAMPED
" AUTHOR KJ ASTROM 791012

```

```

INPUT Y UC
OUTPUT U
STATE F1 F2 F3 F4 TH1 TH2 TH3 TH4 TH5
STATE P11 P12 P13 P14 P22 P23 P24 P33 P34 P44 P55
NEW NF1 NF2 NF3 NF4 NT1 NT2 NT3 NT4 NT5
NEW N11 N12 N13 N14 N22 N23 N24 N33 N34 N44 N55
TIME T
TSAMP TS

```

```

INITIAL
TH1=A10
TH2=A20
P11=P0
P22=P0
P33=P0
P44=P0
P55=P0R

```

```
OUTPUT
```

```
"PART 2 - REGULATOR DESIGN
```

```

SQ=SQRT(1-Z*Z)
DT=2*PI/(NP*W*SQ)
P1=-2*EXP(-(2*PI/NP)*Z/SQ)*COS(2*PI/NP)
P2=EXP(-2*(2*PI/NP)*Z/SQ)
PS=1+P1+P2
A1=TH1
A2=TH2
R1=TH3
R2=TH4
BS=B1+R2
BMAX=MAX(B1,R2)

N=R2*B2-A1*B1*B2+A2*R1*R1
W1=P1-A1
W2=P2-A2
W3=W1*R2-W2*R1
W4=W1*(A2*R1-A1*B2)+W2*B2
TEST=N/(BMAX*BMAX*EPS)
TEST1=IF -R2/B1<Z1 THEN -1 ELSE IF -R2/B1<Z2 THEN 1 ELSE -1

```

```

T0=IF TEST<1 THEN PS/B1 ELSE IF TEST1>0 THEN PS/R1 ELSE PS/BS
R1=IF TEST<1 THEN 0 ELSE IF TEST1>0 THEN B2/R1 ELSE P2*W3/N
S0=IF TEST<1 THEN P1/B1-A2/B2 ELSE IF TEST1>0 THEN (P1-A1)/R1 ELSE W4/N
S1=IF TEST<1 THEN P2/B1 ELSE IF TEST1>0 THEN (P2-A2)/B1 ELSE -A2*W3/N

U8=IF TEST<1 THEN TH5/B1 ELSE IF TEST1>0 THEN TH5/R1 ELSE (1+R1)*TH5/BS
U1=T0*UC-S0*Y-S1*(-F1)-R1*F3-UR

U=IF U1<ULOW THEN ULOW ELSE IF U1<UHIGH THEN U1 ELSE UHIGH

DYNAMICS

"PART 1 PARAMETER ESTIMATION
E=Y-TH1*F1-TH2*F2-TH3*F3-TH4*F4-TH5

"ESTIMATOR GAIN
K1=P11*F1+P12*F2+P13*F3+P14*F4
K2=P12*F1+P22*F2+P23*F3+P24*F4
K3=P13*F1+P23*F2+P33*F3+P34*F4
K4=P14*F1+P24*F2+P34*F3+P44*F4
K5=P55*F5
D=1+F1*K1+F2*K2+F3*K3+F4*K4+F5*K5

"UPDATE ESTIMATE
NT1=TH1+K1*E/D
NT2=TH2+K2*E/D
NT3=TH3+K3*E/D
NT4=TH4+K4*E/D
NT5=TH5+K5*E/D

"UPDATE COVARIANCES
N11=(P11-K1*K1/D)/LAM
N12=P12-K1*K2/D
N13=P13-K1*K3/D
N14=P14-K1*K4/D
N22=(P22-K2*K2/D)/LAM
N23=P23-K2*K3/D
N24=P24-K2*K4/D
N33=(P33-K3*K3/D)/LAM
N34=P34-K3*K4/D
N44=(P44-K4*K4/D)/LAM
N55=(P55-K5*K5/D)/LAM1

"UPDATE F-VECTOR
NF1=-Y
NF2=F1
NF3=U
NF4=F3

"UPDATE SAMPLING TIME
TS=T+DT

```

W:1.5  
 Z:0.707  
 NP:20  
 LAM:0.98  
 LAM1:0.8  
 EPS:0.01  
 UHIGH:5  
 ULOW:-5  
  
 A10:-1.5  
 A20:0.7  
 PO:100  
 POB:0.1  
  
 Z1:0.1  
 Z2:0.99  
  
 F5:1  
 PI:3.1415926

END

-----

CONTINUOUS SYSTEM PROC2

TIME T  
 INPUT U V  
 OUTPUT Y  
 STATE X1 X2 X3 X4 X5 X6  
 DER D1 D2 D3 D4 D5 D6  
  
 OUTPUT  
 Y1=IF 0<2 THEN X1 ELSE IF 0<3 THEN X2 ELSE IF 0<4 THEN X3 ELSE X4  
 Y=IF 0<4 THEN Y1 ELSE IF 0<5 THEN X4 ELSE IF 0<6 THEN X5 ELSE X6  
 DYNAMICS  
 PA1=IF T<TPCH THEN 1 ELSE PA11  
 PA2=IF T<TPCH THEN 1 ELSE PA22  
 D1=-PA1\*X1+PA2\*U+V  
 D2=-PA1\*X2+X1  
 D3=-PA1\*X3+X2  
 D4=-PA1\*X4+X3  
 D5=-PA1\*X5+X4  
 D6=-PA1\*X6+X5  
  
 PA11:1  
 PA22:1  
 TPCH:50  
 FIRST:-1  
 O:1  
 END