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Throughput Analysis of Strongly Interfering Slow Frequency-Hopping Wireless Networks

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Abstract

We derive an approximation for the throughput of strongly interfering frequency-hopping wireless networks, where packet collisions always result in lost data. A system is defined to consist of a certain number of radio networks, each with an arbitrary number of communicating units, coordinated to communicate without interference. Using the approximation, we estimate upper and lower bounds on system throughput, as well as the number of networks which gives maximum system throughput.

1 Introduction

The development of small and mobile laptops and terminals has created a demand for fast and convenient Wireless Local Area Network (WLAN) access. Some systems providing this kind of access use the unlicensed ISM-band at 2.4 GHz. As the ISM-band is shared between many users and systems, robustness against interference is required and reduced performance due to interference must be accepted. ISM-band regulations therefore require that some type of spread spectrum technique is used for communication.

In this paper we focus on slow frequency-hopping (FH) spread spectrum networks, where the transmitters and receivers of packets hop between several available frequency channels. More specifically, we consider packet based FH networks that interfere strongly. By strongly we mean that in a packet collision all data is lost in the colliding packets. This assumption is not all realistic but makes it possible to derive a rather simple analytical approximation of the throughput. The approximation is, in fact, a linearization of the exact expression for the throughput, but showing this is beyond the scope of this paper.

Different approaches have been used earlier by other authors in the analysis of FH networks. In [1, 2, 3] the performance of FH networks on fading channels has been examined through simulations. The performance of multiple access protocols and automatic repeat request (ARQ) schemes in FH networks is treated in [4] and [5], respectively. The novelty of our approach lies in that we consider packets of varying lengths.

We start by introducing the system model. Using the model, we derive an approximation of the probability of successful packet transmission, which is then used for approximating throughput. The results are also verified against simulations, showing the accuracy of the approximations. Finally, we apply these results on a specific system consisting of networks roughly corresponding to Bluetooth piconets [6].

2 System Model

Generally, in radio communication, there is one sender transmitting and one or more receivers listening. We will define a network to consist of an arbitrary number of such units that communicate without interference. The transmissions within a network could for example be coordinated by some master unit, which is the case in a Bluetooth piconet. Furthermore, let the networks transmit packets continuously in the following way. Firstly, a network selects a packet from a set of packet types. Packet type i will be selected with a specified probability r_i and consists of a header of length h_i, a payload of length l_i, and a guard interval of length

δ_i. This is explicitly shown in Fig. 1, where we have also introduced L_i as a short notation for $h_i + l_i$. After

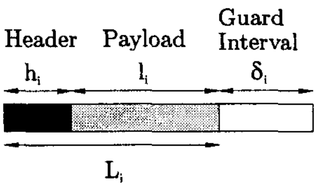

Figure 1: The components of a type i packet.

transmitting the header and payload the network holds transmission during the guard interval, and selects both a new frequency channel and a new packet type before next transmission.

Using the system model above we derive an analytical approximation of throughput, given the following assumptions: collisions always result in lost packets; if no collisions occur, packets are successfully received; frequency hops are perfectly random and each channel is selected with the same probability; all networks use the same packet type distribution; networks always have packets to transmit and there is no time synchronization between the networks.

These assumptions enable us to approximate the probability of successful transmission of a reference packet of a certain length.

3 Analysis

In this section we state the expression for the probability of successful packet transmission. This expression is then approximated and finally we derive the throughput.

3.1 Successful Packet Transmission Probability

Under the assumptions stated in section II, an expression for the probability of successful packet transmission (that is, no packet collision), Pr{no coll.}, can be derived. To do so, we introduce a reference packet of length T (without guard interval). For the transmission of this reference packet to be successful, no other network may transmit a packet at the same time on the same frequency channel.

Let there be $N-1$ interfering networks. If the total number of packets overlapping the reference packet in time is denoted n_{tot}, then

$$\begin{aligned} P(T) &= \Pr\{\text{no coll.}\} \\ &= \sum_{n_{\text{tot}}=0}^{\infty} \Pr\{n_{\text{tot}}\} \Pr\{\text{no coll.}|n_{\text{tot}}\}, \end{aligned} \quad (1)$$

where $\Pr\{n_{\text{tot}}\}$ is the probability of all interfering networks transmitting n_{tot} packets overlapping the reference packet in time and $\Pr\{\text{no coll.}|n_{\text{tot}}\}$ is the probability of successful transmission, given n_{tot} overlaps. The first of these two is rather difficult to derive, but with q channels available and networks selecting each channel with the same probability $1/q$, the second one can be expressed

$$\Pr\{\text{no coll.}|n_{\text{tot}}\} = (1 - \frac{1}{q})^{n_{\text{tot}}}. \quad (2)$$

Considering that the average number of overlapping packets $\bar{n}_{\text{tot}}$ is easier to find than the actual distribution of overlapping packets, we have chosen to approximate (1) as

$$P(T) \approx \tilde{P}(T) = (1 - \frac{1}{q})^{\bar{n}_{\text{tot}}}. \quad (3)$$

When applying this approximation in Section 4 we also address the accuracy.

Given the approximation in (3) we have reduced the problem to finding an expression for $\bar{n}_{\text{tot}}$. Using the average packet transmission dwell time $\sum_k r_k(L_k + \delta_k)$, we can calculate the average number of packets transmitted by a single interfering network during the interval T as

$$\bar{n}(T) = \frac{T}{\sum_k r_k(L_k + \delta_k)}, \quad (4)$$

where T is the duration of an arbitrary packet. Per definition, we know that a fraction r_i of the interfering packets are of type i. Hence, the average number of type i packets is

$$\bar{n}_i(T) = r_i \bar{n}(T). \quad (5)$$

Consulting Fig. 2, we realize that for type i packets we need to take the time interval $T + L_i$ into account when counting the number of overlapping packets, resulting in an average number of $\bar{n}_i(T + L_i)$ (possibly) interfering packets of type i. Since all $N-1$ interfering networks transmit independently, the average number of type i packet transmissions by all networks during that time interval is expressed

$$\bar{n}_{i,\text{tot}}(T + L_i) = (N-1)\bar{n}_i(T + L_i). \quad (6)$$

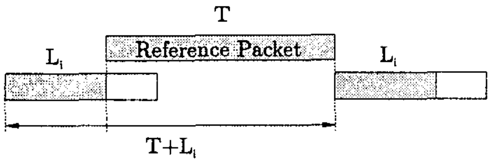

Figure 2: A reference packet transmission of length T and the time interval $T + L_i$ during which a type i packet may overlap in time.

Given the average number of overlapping packets of each type, the average number of overlapping packets $\bar{n}_{\text{tot}}$ is given by the sum

$$\bar{n}_{\text{tot}} = \sum_i \bar{n}_{i,\text{tot}}(T + L_i) \tag{7}$$

$$= (N-1)\frac{T + \sum_i r_i L_i}{\sum_k r_k(L_k + \delta_k)}, \tag{8}$$

where we made use of $\sum_i r_i = 1$.

Substituting (8) in (3) we reach a closed form approximation of the probability of successful transmission

$$\tilde{P}(T) = (1 - \frac{1}{q})^{(N-1)\frac{T+\sum_i r_i L_i}{\sum_k r_k(L_k+\delta_k)}} \tag{9}$$

of a packet of length T.

3.2 Network and System Throughput

From the probability of successful packet transmission $P(T)$, the throughput can be derived. The definition of throughput, R, adopted here is the fraction of time spent on successful payload transmission. The type i packets of length L_i consist of both payload of length l_i and header of length h_i (see Fig. 1). With an average dwell time of $\sum_k r_k(h_k + l_k + \delta_k)$, the part related to transmission of type i packets is $r_i(h_i + l_i + \delta_i)$. However, only $r_i l_i$ of these time units are spent on payload transmission. We also know that only a fraction $P(h_i + l_i)$ of these are successfully received. Hence, we can conclude that the contribution to the throughput by type i packets is

$$R_i = \frac{r_i l_i P(h_i + l_i)}{\sum_k r_k(h_k + l_k + \delta_k)}. \tag{10}$$

For a single network transmitting all packet types, the throughput becomes

$$R = \sum_i R_i = \frac{\sum_i r_i l_i P(h_i + l_i)}{\sum_k r_k(h_k + l_k + \delta_k)}. \tag{11}$$

We have used the same packet type distribution and the same packet types for the single network and the interferers, even though this is not necessary in general. By doing this, the throughput of the whole system with N networks is

$$R_{\text{sys}} = NR. \tag{12}$$

Since the absolute values on throughput in (11) and (12) may be difficult to appreciate, it is of interest to study normalized throughput. For the normalization we apply the largest obtainable throughput for a network without interference. For given parameters h_i, l_i and δ_i, we can calculate this maximal throughput as

$$R_{\max} = \max_{r_1 \ldots r_M} \frac{\sum_i r_i l_i}{\sum_k r_k(h_k + l_k + \delta_k)} \tag{13}$$

by setting $P(h_i + l_i) = 1$ in (11) and maximizing over the packet type distribution $[r_1 \ldots r_M]$, where M is the number of packet types. Expression (13) is maximized by only transmitting the packet type with the largest payload to dwell time ratio $l_i/(h_i + l_i + \delta_i)$. Hence, the normalized versions of (11) and (12) are

$$R_{\text{norm}} = \frac{R}{R_{\max}} = \frac{R}{\max_i(\frac{l_i}{h_i+l_i+\delta_i})} \tag{14}$$

$$R_{\text{sys,norm}} = N\frac{R}{R_{\max}} = \frac{R_{\text{sys}}}{\max_i(\frac{l_i}{h_i+l_i+\delta_i})} \tag{15}$$

4 Applications

We will now consider a specific example which demonstrates the usefulness of our analysis. The parameters in the example are chosen to roughly correspond to the ones used in Bluetooth ACL-packet transmission [6]. The throughput of interfering networks will depend on the packet type distribution. Using the fixed parameters in our example in combination with our approximation of the probability of successful packet transmission (9) we can estimate the upper and lower limits on the throughput, obtaining a range in which we can expect a system of interfering networks to operate.

The networks in our example use 79 channels and have three different packet types with different payload lengths l_i, but equal header length $h_i = h$ and guard interval $\delta_i = \delta$. The parameters are:

Number of channels	$q = 79$
Header length	$h = 160\,\mu s$
Guard interval	$\delta = 220\,\mu s$
Payload lengths	$l_1 = 250\,\mu s$
	$l_2 = 1500\,\mu s$
	$l_3 = 3000\,\mu s$

With the above parameters fixed, the only free variables are the packet type probabilities $[r_1, r_2, r_3]$. To obtain estimates on upper and lower limits on throughput, we maximize and minimize the approximated throughput

$$\tilde{R} = \frac{\sum_{i=1}^{3} r_i l_i (1-\frac{1}{q})^{(N-1)\frac{2h+l_i+\sum_{k=1}^{3} r_k l_k}{h+\delta+\sum_{k=1}^{3} r_k l_k}}}{h+\delta+\sum_{k=1}^{3} r_k l_k}, \quad (16)$$

where we have used the approximation $\tilde{P}(\cdot)$ instead of the exact expression $P(\cdot)$ in (11). For an evaluation of the approximation error, we refer to Appendix A. The maximization and minimization of $\tilde{R}$ is done with respect to r_k for all k. The optimizations are subject to the constraints

$$\sum_{k=1}^{M} r_k = 1, \quad (17)$$

$$0 \leq r_k \leq 1, \; k = 1, \ldots, M, \quad (18)$$

and need therefore only be performed in $M-1$ dimensions. Furthermore, since (16) is a non-linear function of r_k it is convenient to employ a numerical method for the optimization. Fig. 3 shows the result of the optimizations in terms of the normalized network throughput $\tilde{R}_{\mathrm{norm}}$, following (14), when the number of interfering networks is between 1 and 150. In the figure we can see that the throughput, as expected, decreases with the number of interfering networks. Further, the obtained throughput differs by about a factor of two between a good and a bad packet type distribution over the displayed range of interfering networks.

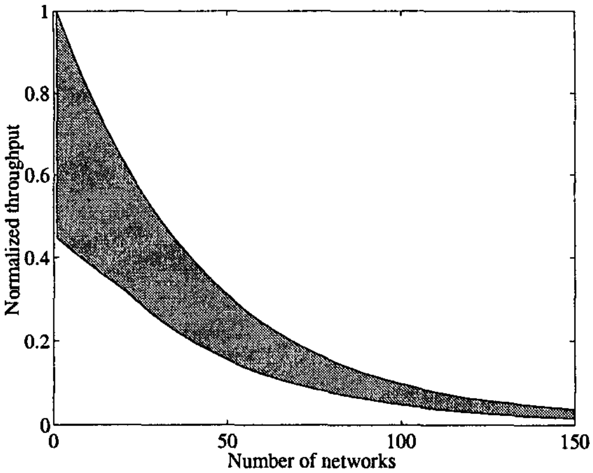

Figure 3: Obtainable values of normalized network throughput as a function of number of interfering networks.

By plotting the normalized system throughput $\tilde{R}_{\mathrm{sys,norm}}$, following (15), for the same set of parameters, we obtain a system perspective in Fig. 4. Here we can observe that the system throughput increases up to about 45 interfering networks. Beyond that point the system throughput decreases with additional networks. We can also conclude that we can never achieve a throughput greater than that of about 16 non-interfering networks. In the figure we have also indicated the packet distribution that yields the maximum system throughput. For our parameters, the maximum is obtained when only transmitting one packet type. Up to a quite large number of interfering networks, about 80, we should transmit only the longest packets.

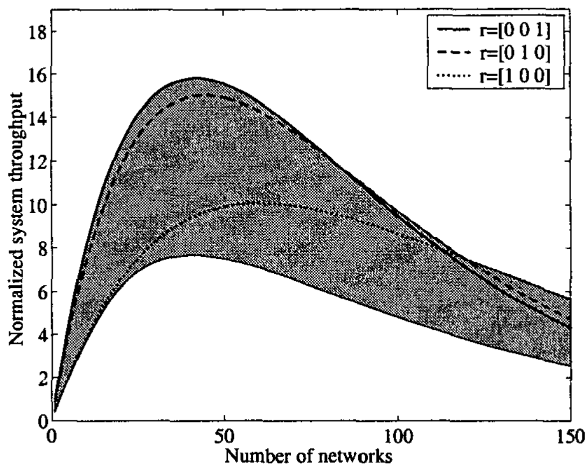

Figure 4: Obtainable values (shaded) of normalized system throughput and throughput for the three single packet type cases.

5 Conclusions

In this paper we have presented a model of a frequency-hopping system and a simple analytical approximation of the throughput. The expression has been applied to a specific system and bounds for the throughput have been found.

By using an example, roughly corresponding to a Bluetooth scenario, we have illustrated how to locate the maximum throughput for a system of interfering networks. The location of this point tells us when adding more networks to the system will decrease the obtainable system throughput.

It has been assumed that collisions result in a total loss of all packets involved. The model used does not account for attenuation and fading of the useful and interfering signals, adjacent channel interference, increased robustness due to error-correcting coding or traffic and load aspects. However, even though the analyzed model is somewhat simplified as compared to a real system, the new analytical evaluation of throughput, following from the assumptions and approximations, readily compensates for the simplifications. More realistic models usually lead to extensive simulations from which general conclusions are more difficult to draw.

A Approximation accuracy

Since this paper does not contain any analysis of the impact of approximation errors in (9), we provide a comparison between simulated and calculated values on throughput. We have used the parameters from Section 4 and calculated the throughput by simulating 5 seconds of transmission, using four different packet type distributions.

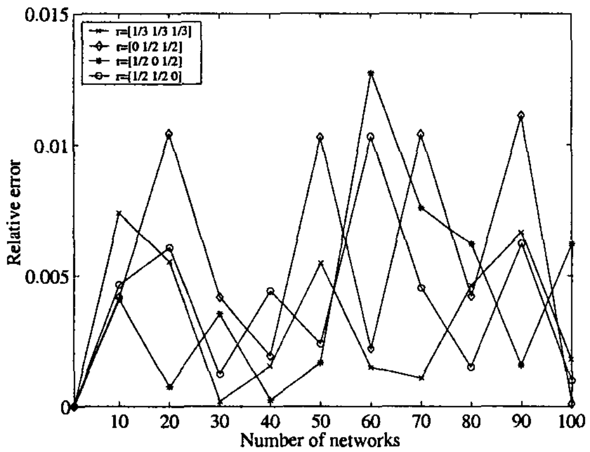

Figure 5: The relative error between simulated and approximated values on throughput.

The relative errors between simulations and the approximation formula are shown in Fig. 5, for different numbers of interfering networks. As can be seen in the figure, the relative error does not exceed 1.5%.

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