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## Digital Simulation of Spatial Xenon Oscillations

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## DIGITAL SIMULATION OF SPATIAL XENON OSCILLATIONS

Gustaf Olsson

### ABSTRACT

A nonlinear model of xenon spatial oscillations in one dimension based on one group diffusion theory and finite differences is presented. The process has been simulated on a digital computer.

In the report is discussed the domain of linearity of the model. The influence of different core parameters on reactor stability and on amplitude of the oscillations is presented. The results are compared to other models.

Influence of nonlinear terms, such as temperature feedback and control rod, indicate that periodic solutions can appear. This has been predicted earlier with a simple two point model. The rod movement has a very big influence on amplitude and character of the oscillations. It can be explained from a very simple model of the core.

The simulations have indicated suitable approximations to get a space independent nonlinear model of the process. Physical interpretation and drawbacks of this model are discussed.

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    Flux shapes and distributions.
2. Numerical methods.
3. Short description of the TRAXEN program. Program listings.
4. Derivation of a transfer function for a two point xenon model.
5. Description and proof of the rod movement for a simplified flux model.

## 1. INTRODUCTION

Xenon spatial instability is a problem in large power reactors. It depends on the fission product xenon, which has a tremendously large neutron cross section. The reactivity feedback from xenon can cause oscillations in the power spatial distribution, which have to be avoided.

The fundamental equations are rather complicated and it is impossible to treat them analytically as they stand. In an earlier report [5] is described different mathematical models, and an extensive study has been made of a simple two point model. This model has given physical insight in the problem.

In this report the equations are simulated on a digital computer. The study includes the xenon instability problem along the core axis. The importance of axial oscillations is discussed in [5].

The simulations were started at Swedish State Power Board, Stockholm, where the main part of the program TRAXEN was written. It was necessary to know the xenon stability for the Marviken reactor.

The purpose of the report is moreover:

- to study the influence of nonlinear terms and compare the result to previous linear model studies,
- to examine the influence from a control rod on stability,
- to calculate the amplitudes for the transients of different disturbances.

In the next section the results are summed up.

## SUMMARY OF THE RESULTS

The report deals with the axial problem of xenon spatial instability. In [5] is discussed the reasons for choosing the axial direction.

The dynamical behaviour is governed by a nonlinear one group diffusion equation for the neutron flux, which is coupled to the ordinary nonlinear differential equations for xenon and iodine. The equations are presented in 2.1. The solution method is described in 2.2. Due to the nonlinear character it is necessary to know time and space distribution of the disturbances. The most probable ones are presented in 2.3. In 2.4 is the choice of core parameters reported.

The simulations have been divided into two main fields, small and large disturbances.

In chapter 3 is discussed the region for linearity. Inside this domain the results from the simulations can be compared to those of linear models. In 3.2 is discussed how stability can be determined from the trajectories. Critical height as function of different core parameters is presented in 3.3. The results are compared to linear finite difference models and modal expansion models with good agreement.

Of technological reasons the amplitude of the transients is important to know. It depends strongly on core height. The maximum flux deviation as function of core height for step disturbances is presented in 3.4. It is compared to a two point model from [5]. The agreement is rather good.

For large disturbances there are mainly two nonlinear terms, which will affect the stability of the solutions, the absorption term and the temperature coefficient. Depending on rod insertion length in equilibrium rod absorption, amplitude and direction of the disturbance the effect of the rod is very different. In 4.1 is discussed the rod movement from a very simple model. The results from the simulations are then compared to those of the simple model and the agreement is surprisingly good.

The trajectories can be damped or amplified by the rod. Due to this fact an unstable limit cycle occurs in some cases. For rod control is shown in 4.2 that there are stable trajectories for small disturbances but unstable ones for large disturbances.

In 4.2 is also shown that stable periodic solutions can appear. This type of performance was predicted with a simple two point model in [5]. In the nonlinear case the amplitude of the transients are no longer proportional to the disturbances.

In 4.3 is shown that the rod, the temperature coefficient as well as the direction of the disturbance are very important for the amplitude.

In chapter 5 is derived a space independent nonlinear xenon model. Similar models have been derived by other authors. It is possible to give a nice physical interpretation of the equations. However, it is shown that the model describes badly what happens for big disturbances.

## 2. MATHEMATICAL MODEL

The xenon process is described by the coupling between the neutron distribution equation and the radioactive decay differential equations of xenon and iodine.

Since the xenon oscillations appear only in large thermal reactors the neutron distribution is treated by one group diffusion theory. Due to the long period of the oscillations the neutron flux can be regarded stationary, why the flux distribution is completely determined by the time dependent material buckling.

The control rod is simplified to have space independent absorption.

In 2.2 is briefly described the solution method.

The type of disturbances which are relevant are presented in 2.3, while the choice of core parameters are discussed in 2.4.

## 2.1 FUNDAMENTAL EQUATIONS OF THE XENON PROCESS

### 2.1.1 NEUTRON DISTRIBUTION

The fundamental equations and the conditions for the xenon process are described in [5] but are repeated here by convenience. The motivation to study the axial oscillations is also found there, why we directly describe the one group diffusion equation in one dimension:

$$\frac{\partial}{\partial z} \left[ D(z,t) \frac{\partial}{\partial z} \phi(z,t) \right] + (\nu \Sigma_f - \Sigma_a) \phi(z,t) = 0 \quad (1)$$

where  $\phi(z,t)$  is the neutron thermal flux and

$D(z,t)$  is the time and space dependent diffusion.

After division with a suitable mean value of  $D$ , called  $D^0$ , equation (1) is transformed to:

$$\frac{\partial}{\partial z} \left[ E(z,t) \frac{\partial}{\partial z} \phi(z,t) \right] + B^2(z,t) \phi(z,t) = 0 \quad (2)$$

where  $B^2(z,t)$  is the material buckling and

$E(z,t)$  is a normalized diffusion parameter.

The boundary conditions are:

$$\begin{aligned} \phi(z,0) &= \phi^0(z) \\ \phi(0,t) &= \phi(H,t) = 0 \end{aligned} \quad (3)$$

$$\int_z K(z) \phi(z,t) dz = P(t) \quad (4)$$

We approximate the space derivatives by finite differences and get from (2):

$$\left[ (E_{k+1} - E_k)(\phi_{k+1} - \phi_k) + E_k(\phi_{k+1} - 2\phi_k + \phi_{k-1}) \right] + h^2 B_k^2 \phi_k = 0$$

$$k = 1, \dots, N$$

where the subscript means space point



or

$$\phi_{k+1} \cdot E_{k+1} - \phi_k (E_{k+1} + E_k) + \phi_{k-1} E_k + h^2 B_k^2 \phi_k = 0 \quad (5)$$

$$k = 1, \dots, N$$

where we have defined:

$$h = \frac{H}{N+1}$$

The boundary conditions are described in discrete form as:

$$\begin{aligned} \phi_0(t) = \phi_{N+1}(t) &= 0 \\ \phi_k(0) = \phi_k^0 & \quad k = 1, \dots, N \end{aligned} \quad (6)$$

$$\sum_{v=1}^N K_v \cdot \phi_v(t) = P(t)$$

The buckling can be expanded into two parts, one equilibrium part,  $B_k^{2*}$ , and one perturbed part,

$$\begin{aligned} B_k^2(t) = B_k^{2*} + \alpha_k \cdot \left( \phi_k(t) - \phi_k^0 \right) + \beta \cdot \left( X_k(t) - X_k^0 \right) + \\ + c_k(t) + u_k(t) \end{aligned} \quad (7)$$

The coefficients  $\alpha_k$  and  $\beta$  express the dependence of buckling on changes in flux (and temperature) and xenon respectively. The term  $c_k$  is the influence on buckling from control rod movement and  $u_k$  is a general control term, available for the operator.

### 2.1.2 XENON AND IODINE EQUATIONS

Xenon concentration is built up mainly by radioactive decay of iodine and a smaller part by the fission. It is destroyed by capture of neutrons and by radioactive decay. Iodine is also got by fission and is destroyed by radioactive decay to xenon.

The xenon and iodine differential equations thus read:

$$\frac{dX_k}{dt} = -\lambda_x X_k(t) + \lambda_i I_k(t) + \gamma_x \sigma_x \phi_k(t) - \sigma_x X_k(t) \phi_k(t) \quad (8)$$

$$\frac{dI_k}{dt} = -\lambda_i I_k(t) + \gamma_i \sigma_x \phi_k(t) \quad k = 1, \dots, N \quad (9)$$

The boundary conditions are:

$$\begin{aligned} X_0(t) &= X_{N+1}(t) = 0 \\ I_0(t) &= I_{N+1}(t) = 0 \\ X_k(0) &= X_k^0 \quad k = 1, \dots, N \\ I_k(0) &= I_k^0 \quad k = 1, \dots, N \end{aligned} \quad (10)$$

### 2.1.3 ABSORPTION TERM IN BUCKLING

The absorption or control term  $c$  in the buckling (7) represents an absorption which must be added or subtracted in order to maintain criticality of the reactor.

As shown in [5], chapter 2.7, only one parameter is necessary to describe uniquely the absorption distribution  $c_k$  ( $k = 1, \dots, N$ ). Physically we have discussed three alternatives, which we call:

- rod control with variable insertion length,
- rod control with variable absorption,
- homogeneous control.

## (i) ROD CONTROL WITH VARIABLE INSERTION LENGTH

We assume the absorption constant along the rod, called  $c^1$ . The insertion length is called  $\lambda$ , where  $0 \leq \lambda \leq 1$ , and  $\lambda$  determines uniquely the absorption. Of computational reasons  $\lambda$  can exceed these limits in order to maintain criticality. Then we have the two cases:

$$A. c(z,t) = \lambda(t) \cdot c^1 \text{ if } \begin{cases} \lambda < 0 & 0 \leq z \leq H \\ \lambda > 1 \end{cases}$$

or in discrete form

$$c_k(t) = \lambda(t) \cdot c^1 \quad \begin{cases} \lambda < 0 \\ \lambda > 1 \end{cases} \quad k = 1, \dots, N \quad (11)$$

and

$$B. c(z,t) = \begin{cases} c^1 & 0 \leq z \leq \lambda H \\ 0 & \lambda H < z \leq H \end{cases} \text{ if } 0 \leq \lambda(t) \leq 1$$

or in discrete form

$$c_k(t) = \begin{cases} c^1 & \text{for } k < [\lambda N] + 1 \\ \{\lambda N - [\lambda N]\} \cdot c^1 & \text{for } k = [\lambda N] + 1 \\ 0 & \text{else} \end{cases} \quad (12)$$

$$k = 1, \dots, N \quad 0 \leq \lambda(t) \leq 1$$

where  $[y]$  assigns integer part of  $y$ .

## (ii) ROD CONTROL WITH VARIABLE ABSORPTION

Now the insertion is assumed to be constant and the absorption is variable. Physically this case can be interpreted as if many fine control rods are inserted from top to this insertion length. We regard all fine rods as one big rod with variable absorption. The absorption along this rod cluster is assumed to increase or decrease if some fine rod is moved in or out. We determine a point  $K^*$  inside the core and get:

$$c_k(t) = \lambda(t) \cdot c^1 \quad \text{if } k \leq K^* \leq N \quad (13)$$

$$c_k = 0 \quad \text{else}$$

The parameter  $\lambda$  now determines the absorption along the rod.

## (iii) HOMOGENEOUS CONTROL

The absorption is constant in the whole core, and we get this case by setting  $K^* = N$  in (ii)

$$c_k = \lambda(t) \cdot c^1 \quad k = 1, \dots, N \quad (14)$$

Physically this control may be regarded as an absorption control by a liquid or gaseous absorber.

## 2.2 SOLUTION METHOD

A Fortran program TRAXEN (TRAnsients of XENon) is written to solve the system equations (5), (8), (9), for all types of disturbances. The program is described in more detail in appendix 3.

As input data we must give geometrical data, mean flux, nuclear constants, control configuration, time and spatial distribution of disturbances, core parameters, such as spatial distribution of buckling, temperature coefficient and diffusion, power condition and desired accuracy. The program calculates both equilibrium flux distribution and transients of the flux, xenon and iodine distributions.

As the neutron diffusion equation (5) is always stationary, the program calculates iteratively the new flux distribution in every time step from the known value of the state in the previous time step.

The equations are integrated with a Runge - Kutta method, corrected with Richardson extrapolation. The numerical methods are described in appendix 2.

In introductory simulations have been tried different time step lengths. It was found that one hour was suitable, and regarding the period of about 24 hours it is accurately enough.

The program is made for maximum 50 node points. As the computing time increases as about  $N^2$ , it is necessary to compromise between accuracy and computing time. In [5] chapter 4.2.1 we found that the stability limit could be accurately determined with 20 space points, and for the following simulations we have chosen  $N = 20$  throughout.

## 2.3 TYPE OF DISTURBANCES

For the nonlinear analysis it is very important to know the type of disturbances which are relevant. As we want to know the conditions for xenon instability, in order to build the reactor inherently stable, we want to know the most serious disturbance in order to be able to predict, if the actual reactor is stable or not for all possible disturbances.

### 2.3.1 SPATIAL DISTRIBUTION OF THE DISTURBANCE

Due to the complexity of the problem we cannot analytically derive the most serious disturbance. However, we can regard the xenon problem as mainly first overtone oscillation. Therefore it seems natural to give the flux a disturbance mainly in the first overtone by moving reactivity from one half of the core to the other.

The disturbance  $u(z,t)$  (7) is assumed to be separable in space and time:

$$u(z,t) = r(t) \cdot R(z) \quad (15)$$

We have standardized the distribution  $R(z)$  to be constant in every half, as in figure 1.

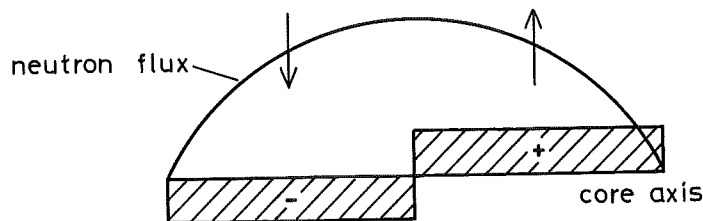


Fig. 1: Space distribution of the reactivity disturbance of the neutron flux.

The shattered areas mark the movement of reactivity. We call the variable  $r(t)$  the amplitude of the disturbance. It can, of course, be both positive and negative.

As an example of a disturbance with a distribution of this shape we will regard a refuelling process.

We assume the total power to be constant. The refuelling process takes place during the operation of the reactor. Before the refuelling the rod (rods) may be in a position like figures 2a or 2b.

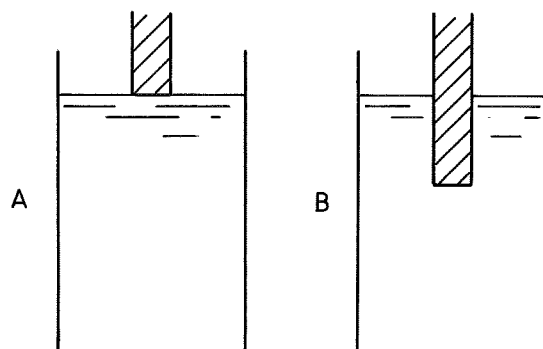


Fig. 2: Examples of rod positions before refuelling

When new fuel elements are inserted, the reactivity in the core increases, why the rods must be inserted in order to hold the power constant (see fig. 3: a, b)

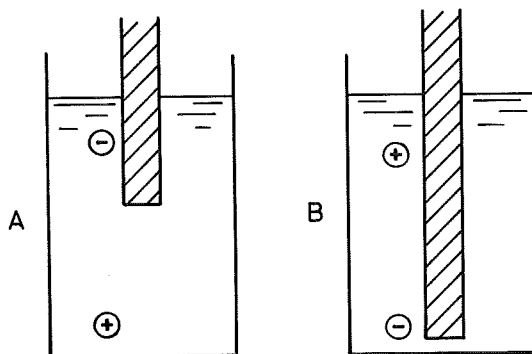


Fig. 3: Examples of position of control rod after refuelling.

The plus and minus signs stand for changes in reactivity.

Figure 3 shows the sign of the disturbances in the two halves of the core for these two standard cases.

### 2.3.2 TIME DISTRIBUTION OF THE DISTURBANCE

In the linear and nonlinear stability analysis the zero solution is disturbed during a finite time. We have chosen the time distribution  $r(t)$  to be a rectangular pulse of one or two hours duration. Except the stability analysis we want to know the amplitude of the transient, which is caused by a reactivity disturbance.

The time function  $r(t)$  has been chosen to a step in some cases. This disturbance may be relevant e.g. for a refuelling process.

The amplitude of the disturbances has been chosen between 10 and 1000 pcm in the simulations. The biggest disturbance may occur in a reactor when a fuel element accidentally falls down in the reactor. During a refuelling process the movement of reactivity is likewise considerable.

### 2.4 CHOICE OF REACTOR CORE PARAMETERS

All parameters and their values are found in appendix 1. The equations are valid for both heavy water and light pressurized water reactors. In these simulations is used only heavy water reactor datas. They are much standardized but several core parameters are taken from the Marviken reactor, [4], which was the first object for this study.

The most interesting parameters affecting the stability are:

- core height  $H$
- temperature (or flux) coefficient  $\alpha$
- mean flux level  $\bar{\phi}$
- flux shape  $\bar{\psi}$ ,  $B^{2*}$
- absorbtion configuration  $c$
- type of disturbance

The rod absorbtion  $c^1$  (12) is very important, as the insertion of the rod is dependent of  $c^1$ . Likewise the rod movement during an oscillation is dependent of  $c^1$ , which will have big influence on stability, as we will discuss in section 4.2.



In Marviken a shim rod will hold 1000 pcm. As our rod may represent several fine rods we have chosen 500 pcm as a representative value of the rod absorption, when it is inserted in the whole core. In this study we are only interested in the general behaviour for a standard reactor, but in a case study this absorption should be carefully taken into account.

Even if the TRAXEN program is prepared for hydrodynamic studies with a variable diffusion constant we do not take the void into account in this study. Thus:

$$E_k = \text{constant} = 1 \quad \text{for all } k.$$

Likewise, we have incomplete information about the space distribution of  $\alpha$  (eq. (7)) and have treated  $\alpha$  as space independent.

The mean value of the flux is calculated as:

$$\bar{\phi} = \frac{1}{N+1} \sum_{k=1}^N \phi_k \quad (16)$$

When the numerical values of the parameters are not specially mentioned, we have used the values from appendix 1.

### 3. SMALL DISTURBANCES

Several previous reports on the xenon problem have treated the linearized equations, just in order to find the stability boundaries. An extensive reference list is found in [5].

By studying the nonlinear equations for different disturbances from equilibrium it is possible to get a good feeling for the domain where the linear approximation is valid with reasonable accuracy. This is discussed in 3.1.

In 3.2 is defined stability criteria and in 3.3 is shown how critical height depends on different core parameters.

The amplitude of the transient is an important measure of performance of a reactor. We measure the maximum flux deviation from equilibrium in the core. The amplitude depends on several core parameters, mainly on core geometry. This relationship is discussed in 3.4. It is possible to get an estimation of the amplitude as function of core height from a two point model. A comparison is made between simulation results and an analytical derivation of the amplitude.

#### 3.1 THE DOMAIN OF LINEARITY

One purpose of this study was to find the importance of the nonlinear terms in different situations. It is impossible to give a general criterion of the boundaries of the linear region. We will, however, give some hints where reasonable accuracy of the superposition principle is to be found.

An easy criterion to check is the rod movement. From (2:7) we find that this is a small term. In [5] chapter 2 was found that the control term had no influence on linear stability for a symmetric two space point model or for a linear finite difference multipoint model with flat flux. It is reasonable to assume that the rod movement at small disturbances has small influence on stability even for other flux forms.

In the simulations we use the rod movement as the primary indicator to determine whether the disturbance can be called small or large. Further, the amplitude of the transient shall be linearly related to the amplitude of the disturbance.

The permitted domain for small disturbances depends strongly on the core parameters, and it is, of course, larger for smaller core sizes and more stable reactors (stability is defined in 3.3).

We show some numerical examples to illustrate the linear domain. We have measured the first maximum of the amplitude of some transients for different disturbances and have checked the linearity by the superposition principle. As a measure of transient amplitude we use the maximum flux deviation from equilibrium in the core. The result is described in table 1.

Table 1: Maximum amplitude of neutron flux deviation transients related to different disturbances. The flux form is found in fig. 1:E. The control is homogeneous.

$$H_{\text{crit}} = 7.5 \text{ m.}$$

Core height	Step disturbance r(t) pcm	Normalized values	
		Disturbance	Ampl. of transient
7.5 m	10	1	1
	20	2	2.00
	50	5	4.98
	100	10	9.64
7.0 m	20	2	2
	50	5	5.00
	100	10	9.80
6.0 m	20	2	2
	100	10	10.04

As table 1 shows we can regard 50 pcm as a small disturbance (within 1% accuracy) for a reactor near the stability limit, while 100 pcm gives a rather good linear relationship for a 6 m reactor for this special flux, representative for the Marviken reactor (see fig. 1:D). When studying the linear stability we have used 10 - 50 pcm as disturbances.

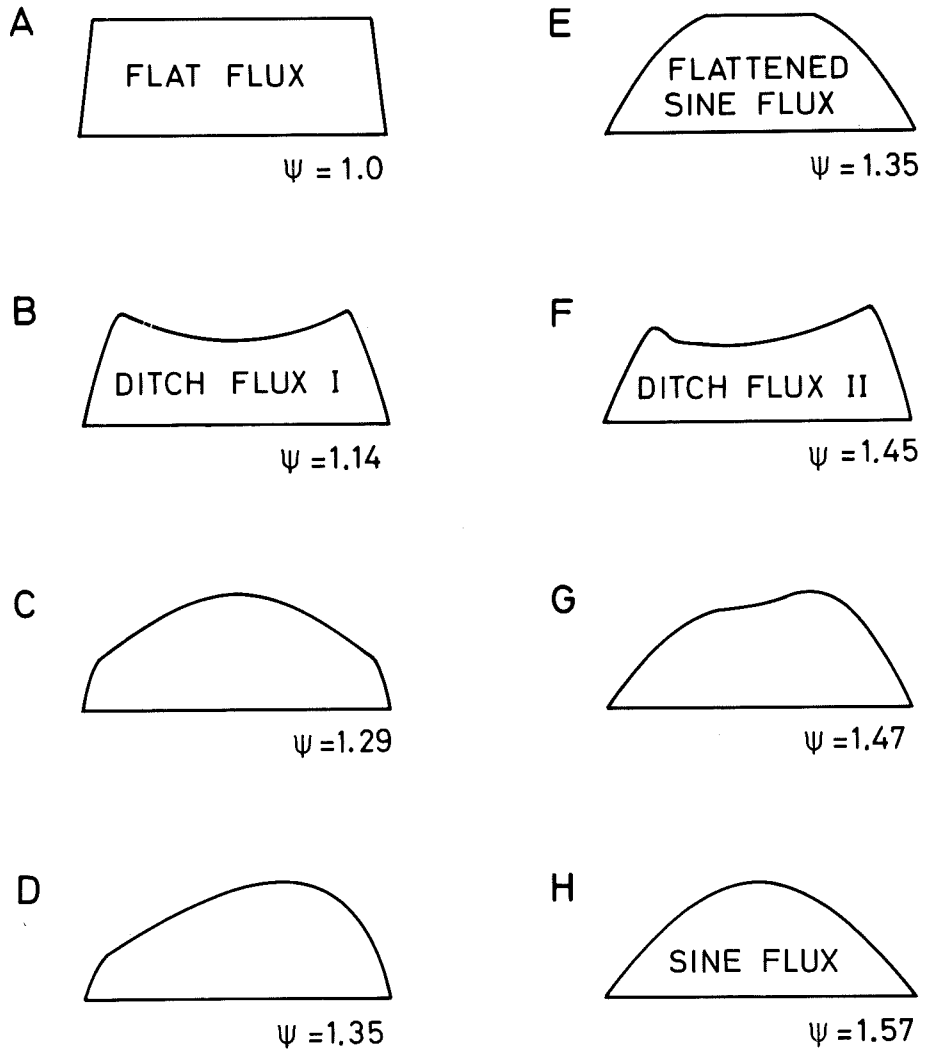


Fig. 1 - Different neutron flux axial distributions. Fluxes A,B,C,E,H, are symmetric.

### 3.2 STABILITY IN CASE OF SMALL DISTURBANCES

We use the definition of stability in the sense of Lyapunov, which means that a trajectory is stable (asymptotically) when the deviation of the state after a disturbance of the zero solution converges to zero. Otherwise it is unstable.

The stability can be measured or calculated in several ways. In [5] we regarded the eigenvalues of the system matrix. Here we will study the convergence of the trajectories of every space point.

It is sufficient to check the stability in one point. This is realized by studying (2:5), which couples all the state variables strongly together. The system can be described by:

$$\dot{x} = A \cdot x$$

where  $x$  is the state vector and  $A$  is a  $2N \times 2N$  matrix.

Eq. (2:5) causes every state variable to be coupled to its neighbours in  $A$ . Thus if one state variable is unstable, all the state variables must be unstable, due to the strong coupling in the system matrix.

It is even sufficient to decide convergence or divergence near the stability limit by studying only two consecutive extremum values of the transient. This is important, as the computation takes much time.

If a transient seems to be at the stability limit when looking at two peaks, one must be sure that no slowly varying unstable oscillation is added to the dominating oscillation of 24-hours period. In [5] sect. 4.2.2 is calculated the eigenvalues for two standard fluxes at the stability limits, and this shows that no unstable oscillation is added to the dominating oscillation. Only fast decaying oscillations appear beside the undamped oscillation.

A number of simulations of about twenty periods ( $> 500$  h) for different fluxes have confirmed this statement.

One observation may be done. We have neglected influence from the fission product samarium, which has a cross section about a hundred of that of xenon and may cause small oscillations of a periodicity of several days.

We will often express the stability in a significant core parameter and define the critical height as the core height where the stability limit is reached.

### 3.3 CRITICAL HEIGHT AS FUNCTION OF DIFFERENT CORE PARAMETERS

As pointed out in 3.1 the rod movement can be neglected in all the linear studies. Thus we need not distinguish between rod control and homogeneous control in the stability analysis or at amplitude calculations. Simulations have confirmed the statement.

Generally the stability is decreasing when core size, temperature coefficient or mean flux is increasing. For very high flux levels, however, the stability is increasing again. A slight asymmetry in the flux shape can also make stability better compared to a symmetric flux shape. Moreover, a flat flux has a lower critical height than a sinusoidal flux (see also [5], chapter 4).

The flux shape may be described by the form factor  $\Psi$ , the symmetry or it can be characterized by a ditch form (flux B and F in fig. 1). Generally the critical height is bigger for a bigger form factor. In table 2 are compared the critical heights as function of the form factors for some common flux shapes.

Table 2: Critical height H(m) for different form factors  $\psi$ ,  
calculated with TRAXEN for 20 space points.

( $\bar{\phi} = 1$ ;  $\alpha = -0.0514$ )

The flux forms are shown in figure 1.

The critical height is the same for rod control and  
homogeneous control.

$\psi$	Flux	$H_{\text{crit}}$ (m)
1.0	A	5.36
1.14	B	5.15
1.29	C	7.25
1.35	D	7.3
1.35	E	7.5
1.45	F	5.05
1.47	G	7.7
1.57	H	8.89

The table also shows that the form factor is no unique measure  
of flux shape.

Fluxes B and F have a lower critical height than A, depending  
on the ditch shape along the axis. As the xenon oscillations are  
mainly affecting the first overtone, the loose coupling between  
the two halves of the core for a ditch flux shape causes a less  
stable flux. These results are compared to other models in [5],  
chapter 4.3 and the accuracy is satisfactory.

The dependence of the temperature coefficient  $\alpha$  has been calcu-  
lated. For a flat flux (fig 1:A) the following values of critical  
height H(m) have been computed ( $\bar{\phi} = 1$ , homogeneous control).

$\alpha$	$H_{\text{crit}}$ (m)
-0.0514	5.36
0	5.02

In [5] is calculated the quotient

$$K = \frac{\Delta H}{H_0} \cdot \frac{\alpha}{\Delta \alpha}$$

for a two point model, where we found:

$$K = 0.062.$$

Here we find:

$$K = 0.063.$$

In [5] was proved for a two point model that the critical height is lower for a symmetric than for an asymmetric flux shape. This is verified by computation with the TRAXEN model. In table 2 we see that flux D has a higher critical height than flux C (see fig. 1). The fuel distributions in the fluxes C and D are the same. Flux D is got from flux C only by moving absorption from one half to the other in order to get an asymmetric equilibrium flux.

We conclude, that the control rod movement does not affect the value of the critical height. On the other hand we see, that the rod insertion length in equilibrium may affect the stability. It will influence the flux distribution, which in turn influences stability.



### 3.4 AMPLITUDE OF THE TRANSIENTS

For small disturbances the type of the input signal is irrelevant for stability tests. We have therefore used steps as disturbances, as they are rather common in real reactors (see 2.3).

Besides stability boundaries there are bounds on the amplitude of transients in a power reactor. The flux deviation must not deviate more than some 5 or 10% from equilibrium. As the amplitude of the transient is related to stability, we use even the maximum amplitude of the transient as a stability measure. In order to be able to compare different reactors we choose a step in reactivity as a standard disturbance. The disturbance consists of a stepwise movement of 100 pcm from one half to the other half of the core.

We treat only stable reactors, which means that the maximum of a step response appears in the first overshoot.

#### 3.4.1 SIMULATION RESULTS

Due to the nature of the oscillations the most serious point of the core is situated around the center of one of the two core halves, i.e. at the coordinates  $z = 0.75 H$  or  $z = 0.25 H$ . The simulations show, that for the standard flux shape of figure 1:C the most serious point is  $z = 0.810 H$  or  $z = 0.190 H$ .

After a step disturbance we get a transient in every space point like figure 2. We are now interested in the amplitude of the first overshoot.

Table 3 shows the result of a number of simulations. Flux shape is that of figure 1:C, the disturbance is 100 pcm and the control is homogeneous.

Table 3: Amplitude of first overshoot as function of core height for the flux C of figure 1.

$$H_{\text{crit}} = 7.25 \text{ m}; \quad \alpha = -0.0514; \quad \bar{\phi} = 1;$$

Control is homogeneous. Disturbance 100 pcm.

Core height (m)	Amplitude $\varphi(z = 0.81 H)(\%)$
5.0	5.47
6.0	10.54
6.5	13.7
7.0	19.29

Figure 2 shows the biggest flux transients for two of the fluxes after a 100 pcm disturbance, the 6.0 and 6.5 m cores.

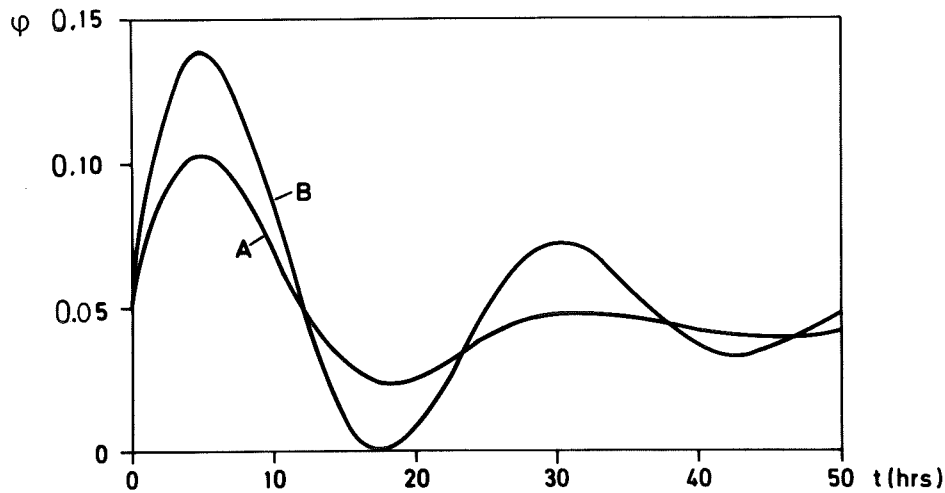


Fig. 2 - Neutron flux transients after a 100 pcm step disturbance in the most serious space point  $z = 0.81 H$ . The flux form is shown in figure 1:C

$$\phi = 1$$

$$\alpha = -0.0514$$

Rod control

Core height: A) 6.0 m

B) 6.5 m

The relationship between core height and amplitude is presented in figure 3. We can see, that the amplitude is increasing very fast with core height.

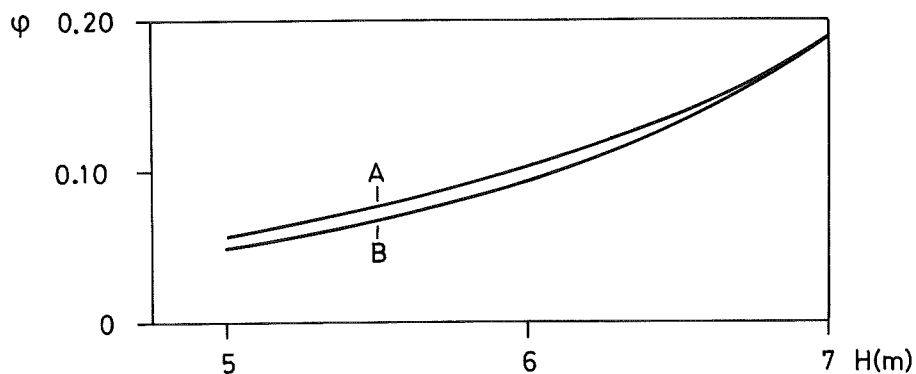


Fig. 3 - The maximum amplitude of the flux deviation after a 100 pcm step disturbance as function of core height. Comparison between simulations with TRAXEN (A) and analytical results with a two point model (B). The flux form is shown in figure 1:C

$$\phi = 1$$

$$\alpha = -0.0514$$

Rod control (same result as homogeneous control)

### 3.4.2 COMPARISON WITH A SIMPLE TWO POINT MODEL

In [5] is presented a xenon model when the diffusion equation is given in only two core points, called the two point model. From this model is derived in appendix 4 a linear two point model of the form:

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

where  $y = \varphi_1$  = flux deviation  
 $u$  = absorption input.

The transfer function  $G(s)$  is of second order. It is easy to derive the step response and its maximum values analytically.

For the case:

$$\alpha = -0.0514$$

$$\bar{\phi} = 1$$

$$\text{giving } H_{\text{crit}} = 6.93 \text{ m}$$

the values of maximum flux deviation are compared to the simulations in figure 3. The values are also shown in table 4.

Table 4: Amplitude of the first overshoot as function of core height for a two point reactor model.

$$\bar{\phi} = 1; \alpha = -0.0514; H_{\text{crit}} = 6.93 \text{ m}$$

Disturbance 100 pcm.

Core height (m)	Amplitude $\varphi(\%)$
5.0	4.97
5.5	6.78
6.0	9.32
6.5	13.03
7.0	18.79

The comparison can only be qualitative as the flux shapes and critical heights are quite different, but the agreement between the simulations and the two point model is all the time within 10%. Near the stability limit the difference is only 2.6%.

#### 4. LARGE DISTURBANCES. INFLUENCE OF NONLINEAR TERMS.

In this chapter is discussed the influence of different nonlinear terms. The most important ones are the absorption term (e.g. from the rod) in the buckling, the temperature coefficient and the quadratic term in the xenon equation.

If a rod is used for control, the influence may be very strong of this nonlinearity. In 4.1 is discussed qualitatively the influence from a very simplified model. It is shown, that this model can explain all the different types of rod movement, that have appeared in the simulations.

The next important nonlinear term is the temperature coefficient  $\alpha$ . It has influence both on the linear stability and on the nonlinear character of the solutions, and a more negative  $\alpha$  has a strong stabilizing effect on the oscillations.

In 4.2 is discussed appearance of periodic solutions, limit cycles, both with rod control and with homogeneous control. Soft self excitation and hard self excitation have been shown. The periodic solutions were discovered with a simple two point model, [5], chapter 5, and have been verified by simulations here.

In 4.3 is discussed influence of nonlinear terms on the amplitude of the transients. As a criterion of nonlinear influence we calculate the accuracy of the superposition principle in different cases.

#### 4.1 INFLUENCE OF ROD MOVEMENT ON STABILITY

As the absorption  $c(z,t)$  is completely determined by one parameter, one of the constants in (1:11) or (1:12) can be arbitrarily chosen, e.g. the "thickness" of the rod. The rod insertion length and the movement are related to the thickness, and, as we will show, the stability in turn is dependent of rod movement.

Now, we will discuss, what causes the absorption to increase or decrease, or alternatively what causes the rod to move up and down. From some very simple examples we will demonstrate the rod movements for different phases of the oscillations.

##### 4.1.1 GENERAL DISCUSSION

###### Case I

Regard a flux during an oscillation. We will study the special condition when the oscillation passes a symmetric shape as in fig. 1. For simplicity we assume the buckling to be space independent in the two halves of the core. The buckling is called  $B_1^2$  and  $B_2^2$ .

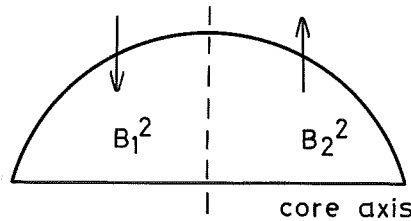


Figure 1: Symmetric flux distribution.

Assume, that the flux deviation will be as in figure 1. The buckling  $B_1^2$  decreases and the  $B_2^2$  increases.

In appendix 5 is shown, that, in order to satisfy all the boundary conditions, we must have:

$$\left| \Delta B_1^2 \right| > \left| \Delta B_2^2 \right| \quad (1)$$

where  $\Delta B^2$  is the total finite change in buckling.

Now the change of buckling is caused by four different terms (2:7). We assume, that the mean value of xenon and flux deviations are equal but of opposite sign in the two core halves, as the total power is constant. Thus the contribution to  $\Delta B^2$  from the xenon and temperature coupling are approximately equal. Further we assume the disturbance  $u$  is of the same amplitude but opposite sign in the two halves.

In order to satisfy (1) we must add absorption to the core, which can be done in a couple of ways:

- in the case homogeneous control the same absorption is added to both halves which directly makes (1) to be satisfied,
- in the case rod control we must either decrease  $B_1^2$  by inserting the rod in the left part; or we can decrease  $B_2^2$  by inserting it into the right part, or a combination of these movements.

An analogous discussion is valid for the opposite flux movement to that in fig. 1.

To sum up, absorption must always be added when the flux deviates from its symmetric shape. The deviation may be caused by disturbance of an equilibrium flux or a free oscillation or both these changes.

### Case II

We regard the flux during an oscillation or in equilibrium when it is asymmetric and we assume

$$B_1^2 < B_2^2$$

where  $B_1^2$  are assumed to be space independent bucklings.

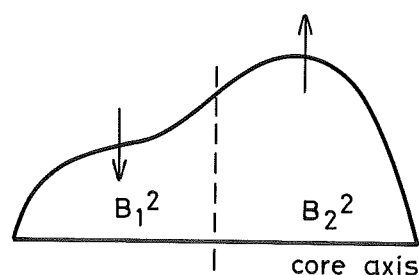


Figure 2: Asymmetric flux

If the deviation has a direction as in fig. 2, we get deviations of the buckling called  $\Delta B_1^2$  and  $\Delta B_2^2$ . In appendix 5 is proved that for the simple case, that  $B_1^2$  and  $B_2^2$  are space independent

$$\left| \Delta B_1^2 \right| > \left| \Delta B_2^2 \right|$$

If we further assume as in case I the xenon deviations to be almost equal in the two parts we must even here add absorption to the core, e.g. insert a rod.

### Case III

For the flux movement in figure 3, where  $B_1^2 < B_2^2$  we can distinguish between two cases. If the deviation of the flux is so small, that it does not reach the symmetric shape we have (see appendix 5)

$$\left| \Delta B_1^2 \right| > \left| \Delta B_2^2 \right|$$

where  $\Delta B_1^2$  are the changes of the bucklings and absorption must be subtracted, or the rod is moved out.

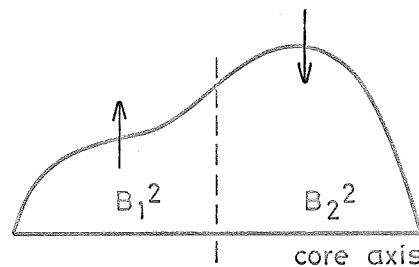


Figure 3: Asymmetric flux

If the deviation is so big, that the flux shape passes the symmetric shape (see fig. 4) the movement can be divided into two parts. For the first part, up to the symmetric shape, fig. 4, we have to subtract absorption (to draw out the rod). For the second part of the movement, we have exactly case I and absorption must be added again (the rod is inserted again).

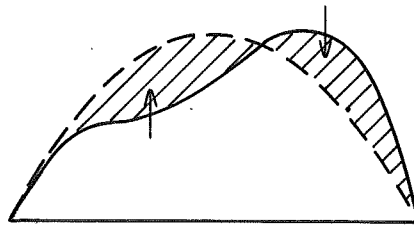


Figure 4: Deviation of flux from asymmetric to symmetric shape.

Simulations have shown that this reasoning is valid for a great number of different flux forms. A contradiction has not been shown for neither rod control nor homogeneous control during a single simulation.

#### 4.1.2 SIMULATION RESULTS

In a couple of numerical examples is demonstrated the variation of the absorption. In the first example we have homogeneous control, and in the second and third examples we use rod control. For the two latter cases is demonstrated the effect of the rod on stability, depending on direction of the disturbance.

##### Example 1: H o m o g e n e o u s   c o n t r o l

Figure 5 shows a flat flux oscillation (fig. 3:1:A) during 30 hours. Core height is 5.40 m, 0.04 m above the critical height. The disturbance is a 1000 pcm pulse, moved from "left" to "right" in the flux during 2 hours. After this time the oscillation is free. The pulse causes the flux to deviate directly from equilibrium and absorption is added ( $\Delta c < 0$ ). We observe the flux deviation after the pulse has finished at  $t=2$  (B). When this flux form is oscillating to the symmetric shape (C), absorption is taken away ( $\Delta c$  is growing in the upper curve). In the next phase, from C to D, we add absorption at the same time as the almost symmetric flux shape C oscillates to D, and then a similar sequence takes place in the continuation of the transient.



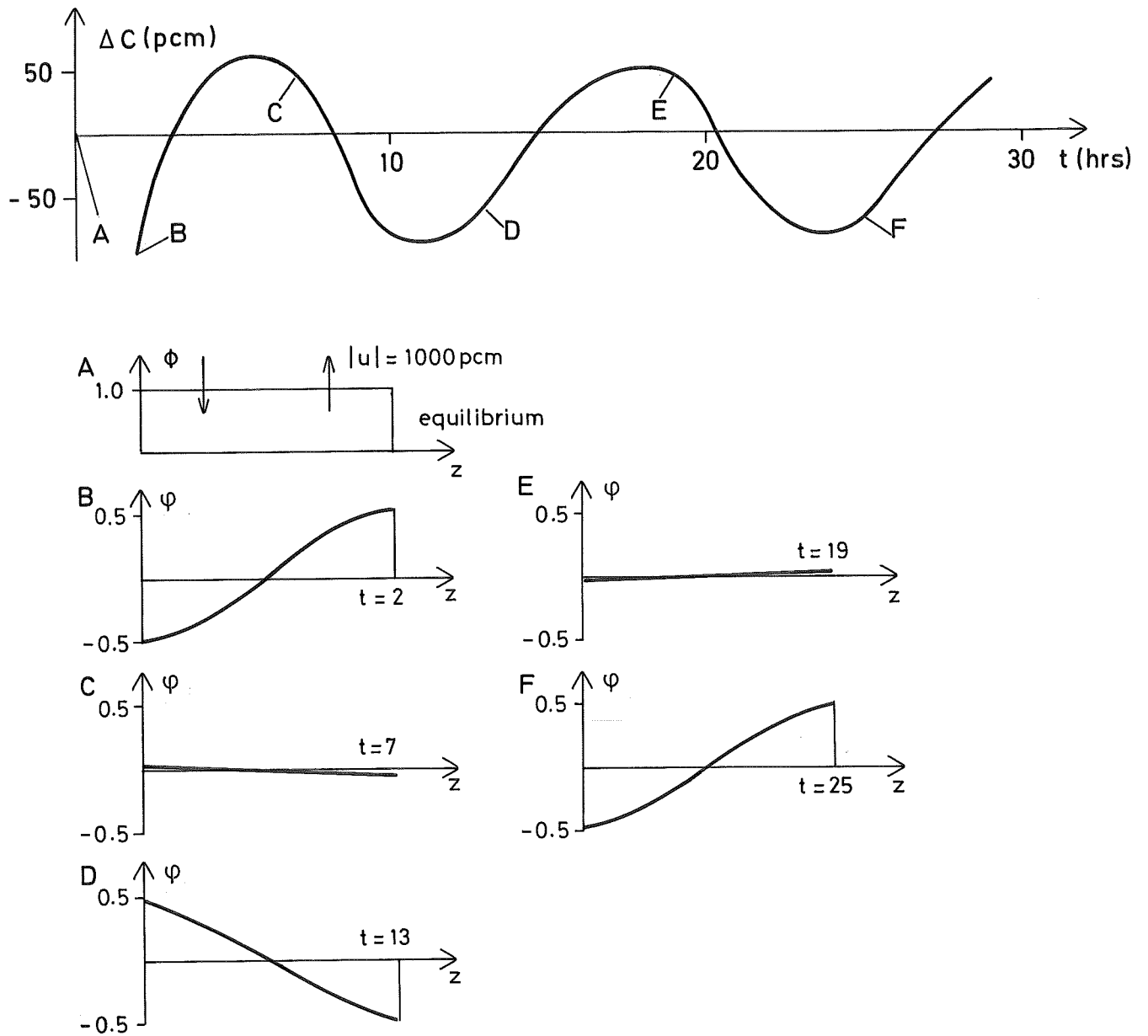


Fig. 5 - Variation of control term  $C$  during an oscillation for a flat flux with homogeneous control

$$H = 5.40 \text{ m} \quad \phi = 1.0$$

$$\alpha = -0.0514$$

Disturbance 1000 pcm during 2 hours with direction shown in A. The curves B-F show the spatial distribution of flux deviation. Coincident points of time (B-F) are marked.

Thus we observe that the absorbtion must oscillate with double frequency compared to the flux or xenon oscillations.

### Example 2 - 3: R o d c o n t r o l

A similar oscillation of the absorbtion as in example 1 is observed in the rod control case. During an oscillation the rod has to move to and fro in order to maintain constant power and this will have a strong influence on stability and on the amplitude of the transients. We will study two different disturbances on a flat flux, the same flux as in example 1.

Figures 6 and 7 show the result of two different signs of a disturbance on the same flux and rod configuration. The total absorbtion varies in the same way but with different amplitudes and is also similar to that of homogeneous control, fig. 5. It can be explained from the general discussion of the preceding section exactly as example 1.

Thus the rod movement must have different influence on the transients in fig. 6 and 7. Compare the curves B in fig. 6 and 7. In the former case the amplitude of the flux deviation is smaller than in the latter case depending on the rod. In both cases the rod has moved to right, into the right core half, and has caused a damping of the first flux (fig. 6) and an amplification of the second flux (fig. 7).

We can see from fig. 6 and 7 that every second time the rod moves in, it causes amplification and every second time it causes damping (curves B, D, F).

The transient in figure 7, however, are all the time bigger than those of figure 6. This depends on what has happened during the first two hours, when the disturbance was acting. In both cases it was inserted, but in figure 7 it caused already here an amplification. The flux is more sensitive to the disturbance of figure 7 than that of figure 6 for this rod configuration.

We realize immediately, that if the rod from the beginning is inserted only in a small part of the core, we get the same type of variation as in figure 6 and 7, but the mean value of the insertion is now  $\lambda_0 < 0.5$ . Thus the disturbance like figure 6

can be amplified instead of damped by the rod as the absorption increases in the left part only, where the flux decreases. The opposite effect is got for the opposite disturbance. If the rod has smaller absorption it must be moved a longer way in order to maintain criticality during an oscillation. Then we intuitively realize that the maximum influence of the rod is got for an insertion variation over a whole core half.

We have from this discussion a good explanation to the strong asymmetry of the two point model oscillations in ref. [5] chapter 5.1.2. For one half period the transient is amplified (fig. 5:3, 4 in [5]) and for the other half it is strongly damped by the "rod". In the two point model we have all the time a maximum influence of the "rod" as it all the time acts in the whole "core half", namely in one of the two space points.

In 4.2 and 4.3 are shown some more simulations where the rod has influenced the trajectories considerably.

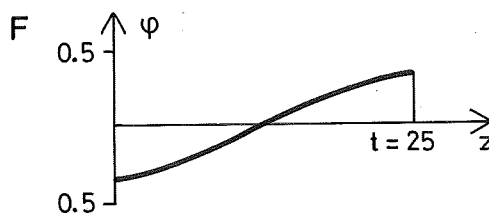
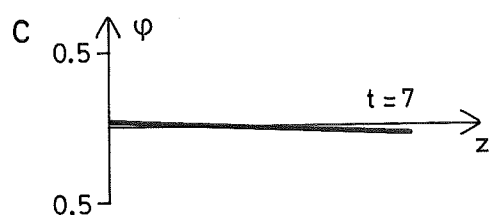
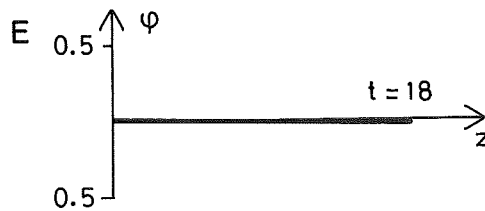
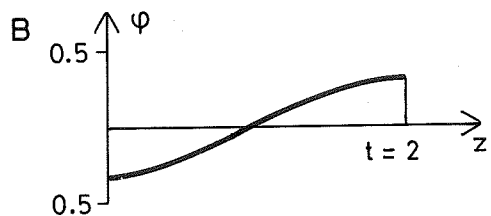
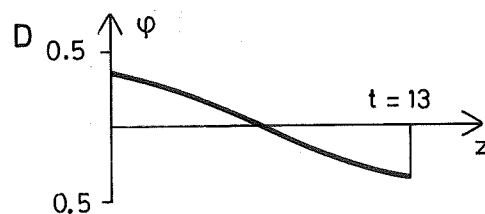
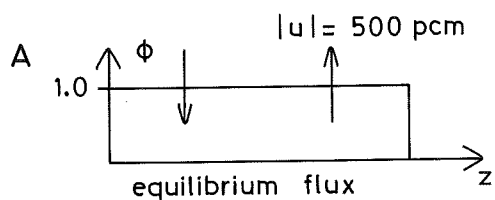
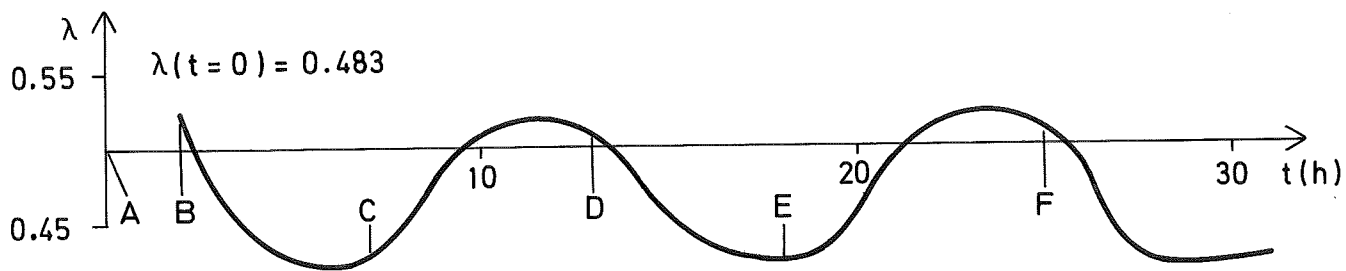


Fig. 6 - Variation of rod insertion  $\lambda$  during a xenon oscillation for a flat flux. Rod absorption = 500 pcm maximum.

$$H = 5.40 \text{ m} \quad \phi = 1.0$$

$$\alpha = -0.0514$$

Disturbance 500 pcm during 2 hours with a direction as in A.

The curves B-F show the spatial distribution of flux deviation.

Coincident points of time (B-F) are marked.

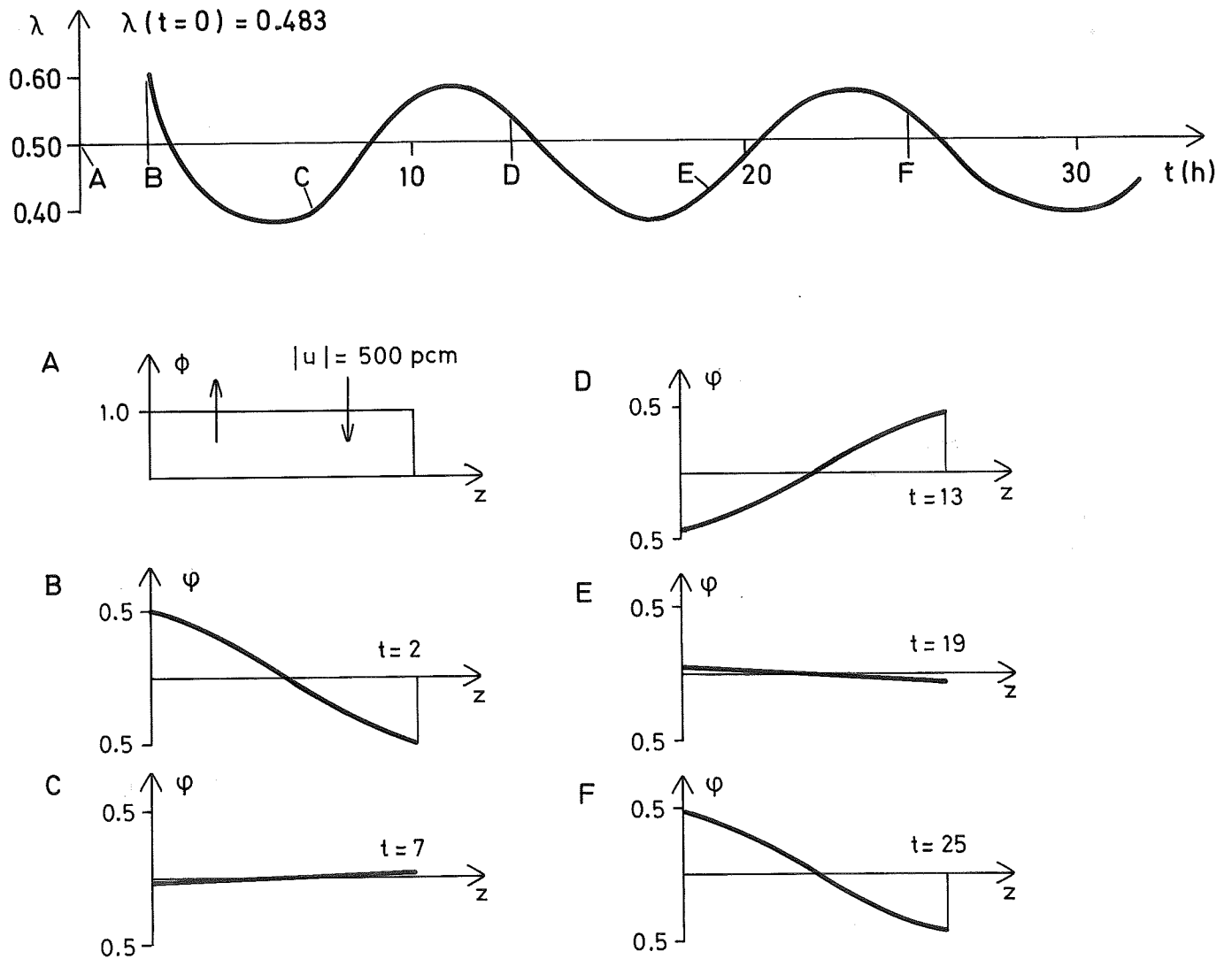


Fig. 7 - Variation of rod insertion  $\lambda$  during a xenon oscillation for a flat flux. Rod absorption = 500 pcm maximum.

$$H = 5.40 \text{ m} \quad \phi = 1.0$$

$$\alpha = -0.0514$$

Disturbance 500 pcm during 2 hours with a direction as in A.

The curves B-F show the spatial distribution of flux deviation.

Coincident points of time (B-F) are marked.

## 4.2 STABILITY.PERIODIC SOLUTIONS

### 4.2.1 CRITERION OF STABILITY IN CASE OF LARGE DISTURBANCES

In 3.2 we defined stability in case of small disturbances. When studying the nonlinear trajectories we must care about every single trajectory.

Let us define the amplitude of a transient to be  $||x||$ , where  $||x||= 0$  in equilibrium.

We call the trajectory stable if:

$$\lim_{t \rightarrow \infty} ||x|| < M$$

where M is a finite number.

If

$$\lim_{t \rightarrow \infty} ||x|| = 0$$

the trajectory is asymptotically stable.

### 4.2.2 ROD CONTROL

In chapter 5.1 in [5] is reported the appearance of periodic solutions for a nonlinear two point model. It is shown that the temperature coefficient  $\alpha$  has a significant influence on the character of the solutions.

With the complex model it is even possible to show a strong influence of the flux shape. In chapter 3.3 was found that a smaller form factor gave a less stable flux or a lower critical height.

Here is shown two cases which confirm what is shown in chapter 5.1 in [5] about periodic solutions. It is possible to get stable periodic solutions for both  $\alpha = 0$  and  $\alpha < 0$  with a core height above the critical height. The small disturbance solutions diverge to the limit cycle, while big disturbance trajectories converge to the same limit cycle.

In [5] was also shown the appearance of unstable limit cycles. It has not been possible to confirm this result for flat fluxes, but for a ditch flux we have got unstable limit cycles.

Example 1: Stable periodic solutions

Figure 8 shows trajectories for a flat flux reactor, bigger than critical height. The solutions converge to a stable periodic solution. The curves B and C show the same oscillations as in fig. 6 and 7. The upper curve, A, shows a small disturbance trajectory of a single space point. The transient in A diverges, while B and C, which are caused by big disturbances converge to a stable periodic solution, whose period time is about 24.1 hours. The trajectories are rather symmetric around the time-axis, because the rod movement is not very big (see fig. 6 and 7). Compare [5] chapter 5.1.

Fig. 8 - Trajectories of flux and xenon deviations in one space point  $z = 0.048 H$  of a flat flux (fig. 3:1:A) reactor with rod control (the same as in figures 6 and 7)

$$H = 5.40 \text{ m} \quad (H_{\text{crit}} = 5.36 \text{ m})$$

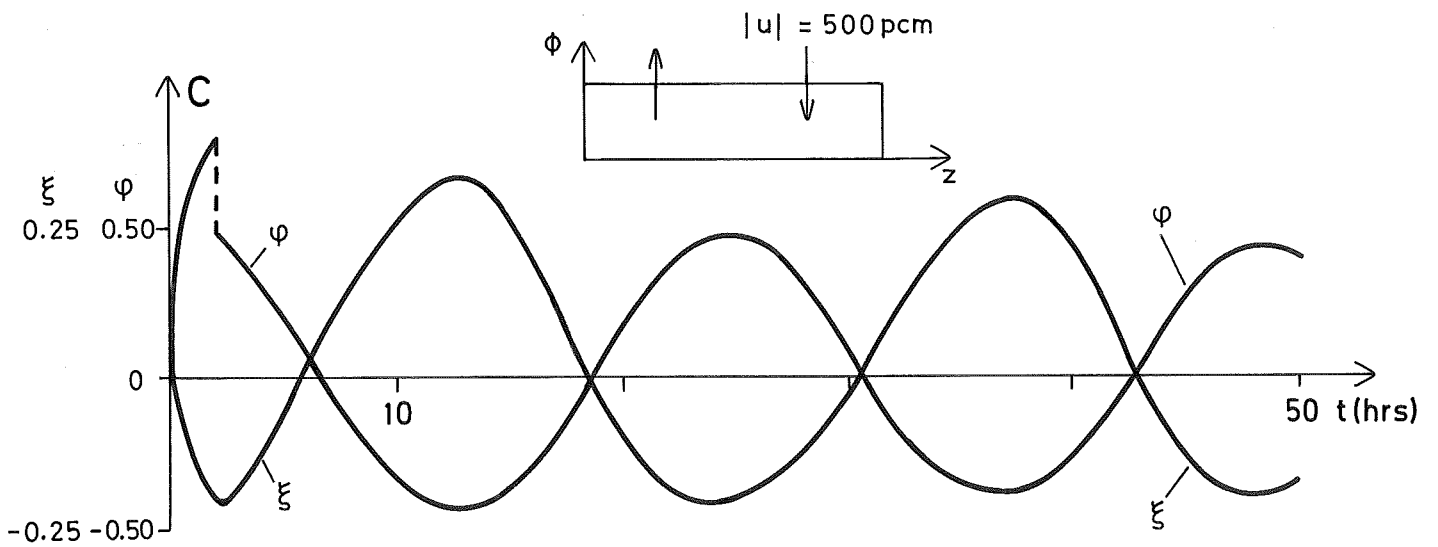
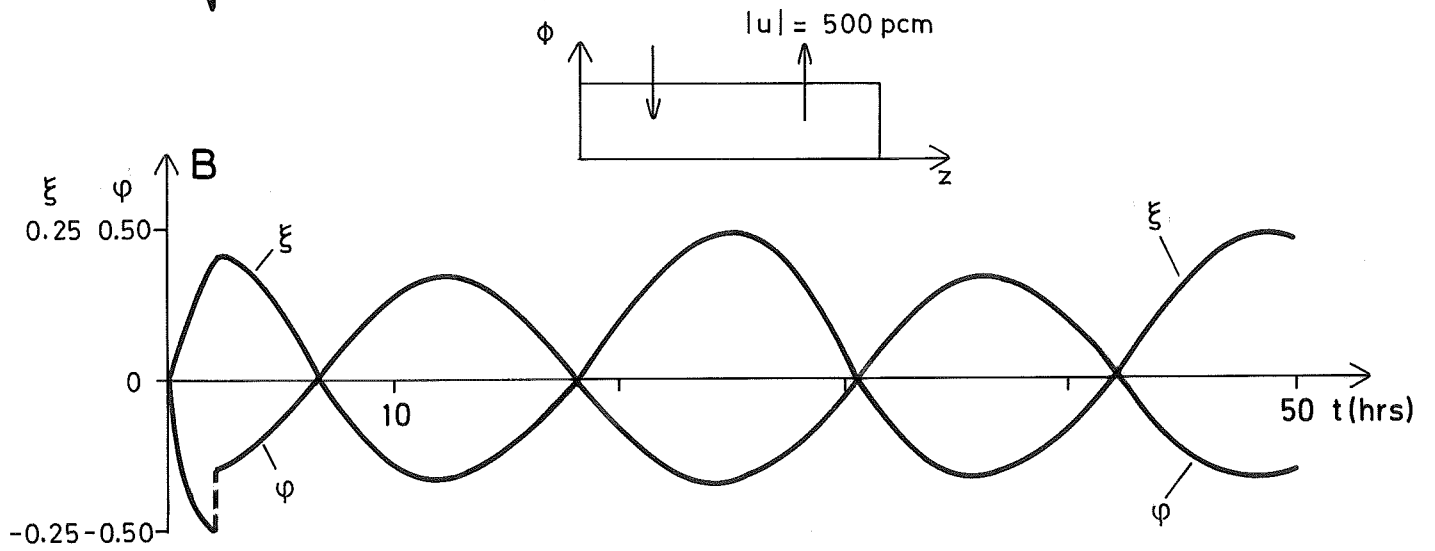
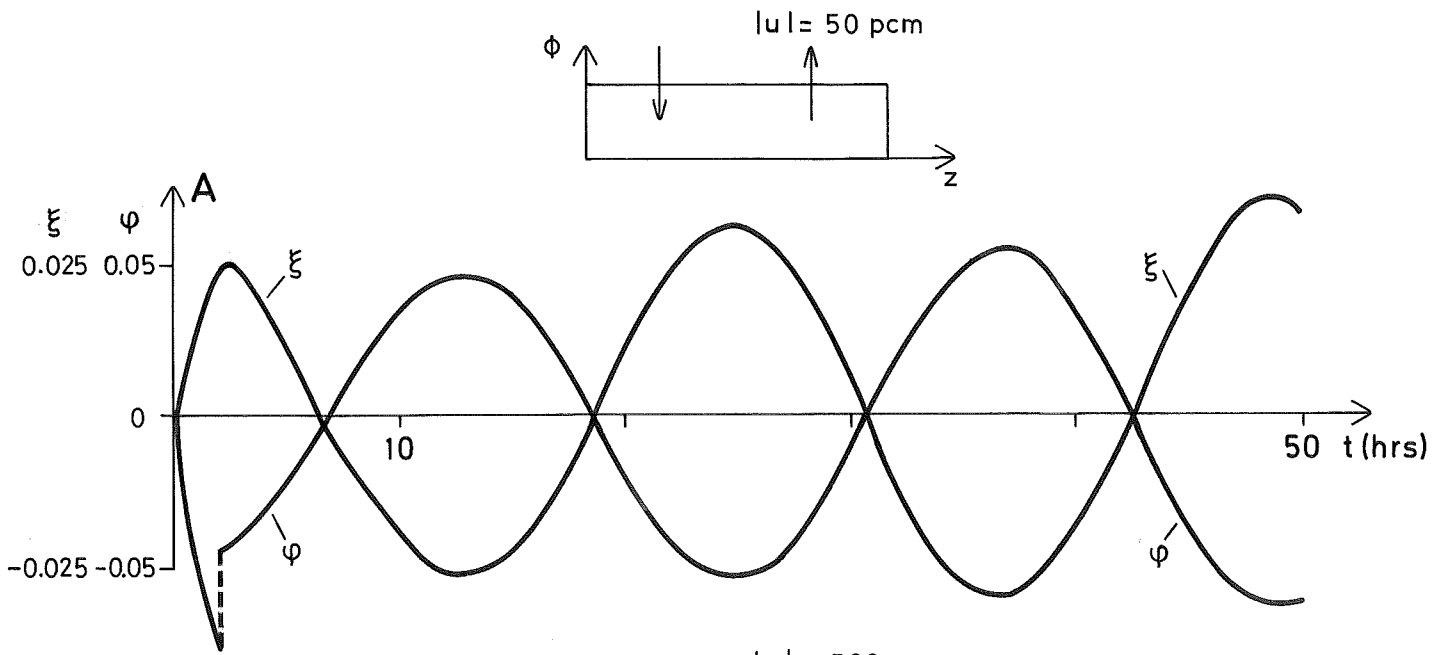
$$\alpha = -0.0514$$

$$\phi = 1.0$$

The direction of the disturbance is shown in the small figures.

The amplitude is

A.	50 pcm	Divergent trajectories
B.	500 "	Convergent "
C.	500 "	" "





Example 2: Unstable periodic solutions

For a ditch flux, fig. 9 (the same as fig. 3:1:F), was found an unstable limit cycle. The core height was 5.0 m, and as the critical height is found to be 5.05 m (section 3.3) it is stable for small disturbances. The flux was disturbed by 100 pcm reactivity moved from one side to the other, as in figure 9. Unstable trajectories were found.

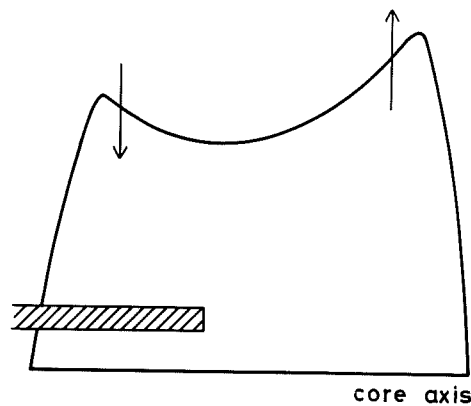


Figure 9: Ditch flux distribution.

$$H = 5.0 \text{ m}$$

The rod at equilibrium is shown.

As mentioned earlier it has been rather difficult to verify unstable periodic solutions. This depends very much on rod configuration. As mentioned before, the "rod" can cause much bigger amplifications in the two point case. Perhaps we have not found the most serious rod configuration in the simulations.

## 4.2.3 HOMOGENEOUS CONTROL

With homogeneous control the nonlinear terms are still damping large trajectories very powerful, why periodic solutions appear even here (compare [5], chapter 5.1). The periodic solutions exist for both positive and negative values of  $\alpha$ . As a negative  $\alpha$  has a damping effect on the trajectories, the amplitude of the periodic solutions are smaller for negative than for positive  $\alpha$ .

We will regard a case when  $\alpha = 0$ . The core height is  $H = 5.02\text{ m}$ , 1 cm over the critical height. Thus the trajectories for small disturbances are unstable, but those for big disturbances are stable, as in figure 10. The period of the periodic solution is about 24.5 hours.

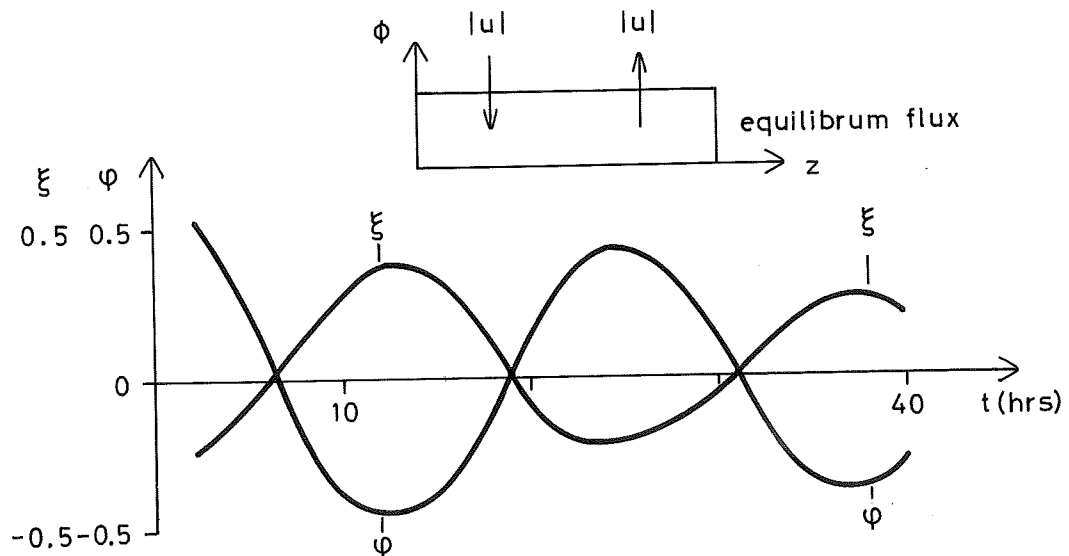


Fig. 10 - Trajectories of flux and xenon deviations in one space point,  $z = 0.952 H$ , of a flat flux (fig 3:1:A) reactor with homogeneous control.

$$H = 5.02 \text{ m} \quad (H_{\text{crit}} = 5.01 \text{ m})$$

$$\alpha = 0$$

$$\phi = 1.0$$

The direction of the disturbance is shown in the small figure. The transient is convergent although the reactor is above the critical height.

### 4.3 AMPLITUDE OF THE TRANSIENTS

#### 4.3.1 INTRODUCTION

As mentioned before, not only the critical height but also the amplitude of the transients are very important of technological reasons. After a disturbance of the reactivity we want to know when the first maximum positive deviation of the flux appears and how big it is. In fig. 8 we can see that it appears at the end of the pulse disturbance at  $t = 2$ . Depending on the direction of the disturbance, the most serious point will be in the upper or lower core half.

In 2.3 we mentioned that step disturbances may also be relevant in order to study the amplitudes.

In next section we will study the influence of the control rod on transient amplitude. In 4.3.3 we use homogeneous control. In this case the nonlinear character is mainly determined by temperature coefficient and xenon feedback in reactivity.

#### 4.3.2 ROD CONTROL INFLUENCE ON AMPLITUDE

In 4.2 is already studied two cases, figures 6 and 7, where the rod has a big influence on the first maximum flux deviation.

##### Example

A further example is studied below. A reactor with 7.0 m core height (figure 11) was disturbed by a step of 500 pcm reactivity. The critical height is 7.3 m. Figure 12 shows the trajectories at the points  $z = 0.81 H$  and  $z = 0.19 H$  respectively, where the flux deviation is largest. In the figure is compared three different cases. Curves A and C are for rod control, where the rod is inserted to about the core center at equilibrium. The direction of the disturbance is opposite in the two cases, curve C as marked in figure 11. Curve B is the result with homogeneous control and is shown for comparison. We can see that the rod has a considerable influence on the amplitude.

The behaviour can be explained as in previous sections.

As shown in 4.2 we have to add absorption during the first part of the oscillation. With a rod we must insert it for both directions of disturbance. Therefore it will damp the movement, caused by a disturbance as in figure 11 (curve C in fig. 12) and will amplify the opposite step answer (curve A). With homogeneous control we have a medium amplitude (curve B).

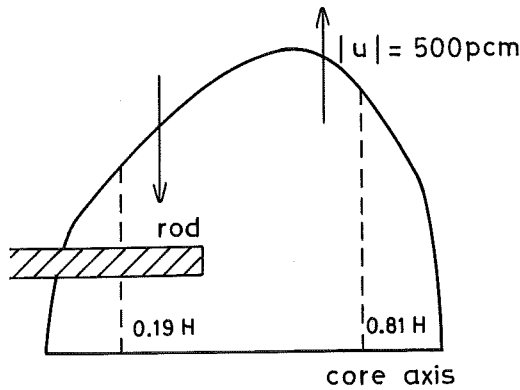


Figure 11: Flux distribution.  
 $H = 7.0 \text{ m}, \psi = 1.33$

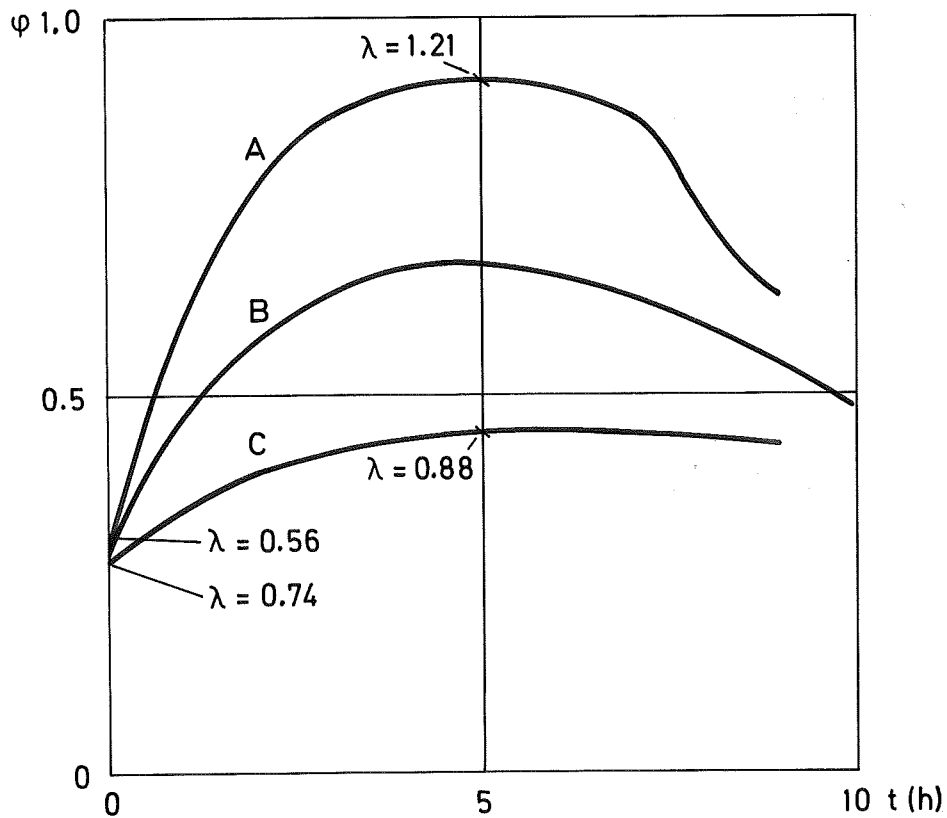


Fig. 12 - Trajectories for maximum flux deviation in one space point of the flux shown in figure 11.

$H = 7.0 \text{ m}$  ( $H_{crit} = 7.3 \text{ m}$ )

$\alpha = -0.0514$

$\phi = 1.0$

The disturbance is a 500 pcm step. The direction is as in fig.11 for B, C and opposite for A.

Rod insertion length  $\lambda$  is shown for its maximum deviation.

	Control	Variable	$\lambda(t=0)$	$\lambda(t=0+)$
A	rod	$\phi(z=0.19H)$	0.54	0.56
B	homogeneous	$\phi(z=0.81H)$	-	-
C	rod	$\phi(z=0.81H)$	0.54	0.74

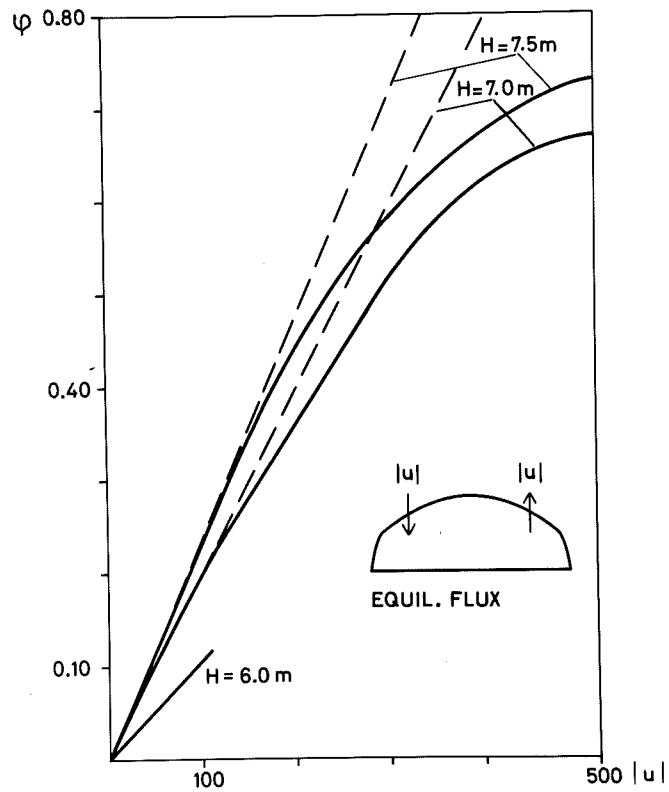


Fig. 13. - Maximum flux deviation at one space point  $z = 0.81H$ . The reactivity disturbance is a step of  $u$  pcm with a direction as in the figure.

Core height 6,7,7.5 m

$$\Phi = 7.0$$

$$\alpha = -0.0514$$

Homogeneous control

A similar result as that in figure 12 is got for a big number of cases. Especially the fluxes D, F, G of fig. 3:1 have been disturbed by a 500 pcm step disturbance of both directions. For all fluxes absorption had to be added during the first 6 - 8 hours of the transient. The rod caused an amplification for one direction and damping for the other.

As mentioned in 3.4 the sensitivity of the fluxes for disturbances increases with core height. Thus a 500 pcm disturbance on an 8 m reactor, 0.65 m over critical height in case D, fig. 3:1, causes a much bigger rod movement than in a 7 m core.

If the rod moves mainly in "left" part during the transient it causes the disturbance shown in figure 11 to be amplified, while it is damped if the rod moves mainly in "right" part. If the rod moves through the whole core it is impossible without simulation to predict the result.

The opposite effect of rod is got for opposite direction of the disturbance.

#### 4.3.3 INFLUENCE ON AMPLITUDE OF OTHER NONLINEAR TERMS.

##### AMPLITUDE OF DISTURBANCE.

In a linear system the superposition principle is valid. For big disturbances in the xenon process, the nonlinear terms, besides the rod, has a damping effect on the amplitude of the trajectories.

Figure 13 shows the sensitivity to different step disturbances and core heights for a flux with homogeneous control. The maximum flux deviation for the most sensitive point is registered, and appears after 5 - 6 hours. For disturbances above 100 pcm the linearity is bad (compare 3.1).

## FLUX SHAPE

An asymmetric flux has different sensitivity depending on the sign of disturbances.

### Example (Homogeneous control)

The flux in fig. 14 ( $H = 8.0$  m,  $0.3$  m over critical height) was disturbed by a  $500$  pcm reactivity step as in figure 14 with maximum flux deviation:

$$\Delta\phi(z = 0.762 H) = 0.689 \quad \text{at } t \approx 6 \text{ hours,}$$

while the opposite direction of the disturbance resulted in:

$$\Delta\phi(z = 0.238 H) = 0.938 \quad \text{at } t \approx 6 \text{ hours.}$$

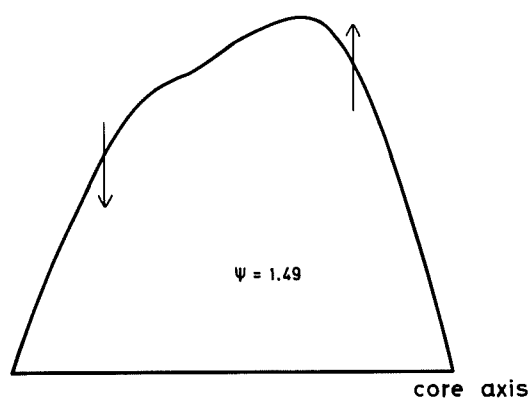


Figure 14 Flux distribution.

$H = 8.0$  m

## 5. A SPACE INDEPENDENT NONLINEAR MODEL

Simulations have shown, that a rather simple, almost linear relationship holds between neutron flux and xenon deviations in every space point. This condition is made use of to get a simpler model. The diffusion equation is replaced by a simple linear condition, which causes the xenon process to be described by a second order nonlinear ordinary differential equation.

This model is compared in 5.2 to a two-point model and is analysed in 5.3. It is shown that the space independent model is only valid for small disturbances.

The result is compared to other space independent models in 5.4.

### 5.1 RELATIONSHIP BETWEEN FLUX AND XENON DEVIATIONS

In the simulations of small disturbances we have observed, that flux and xenon concentration in one core point varies approximately in opposite direction during an oscillation. Margolis [3] showed for a two region core that the transfer function

$$G(s) = \frac{\delta\phi(s)}{\delta X(s)}$$

has a phase which is approximately  $-\pi$ , why  $G(s)$  is approximately real and negative.

In figure 1 is shown the flux and xenon deviations for a flat flux in the point  $z = 0.05 H$ . The variables oscillates to and fro along the line during an oscillation. The oscillation is caused by a small disturbance, 50 pcm, which has been moved between the core halves during 2 hours. In other space points we have similar lines with the same slope but other length (amplitude of the oscillations).

From figure 1 we state the relationship:

$$\varphi = -b \cdot \xi \tag{1}$$

where  $\varphi = \phi - \phi^0$ ,  $\xi = X - X^0$  and  $b = \text{constant}$ .



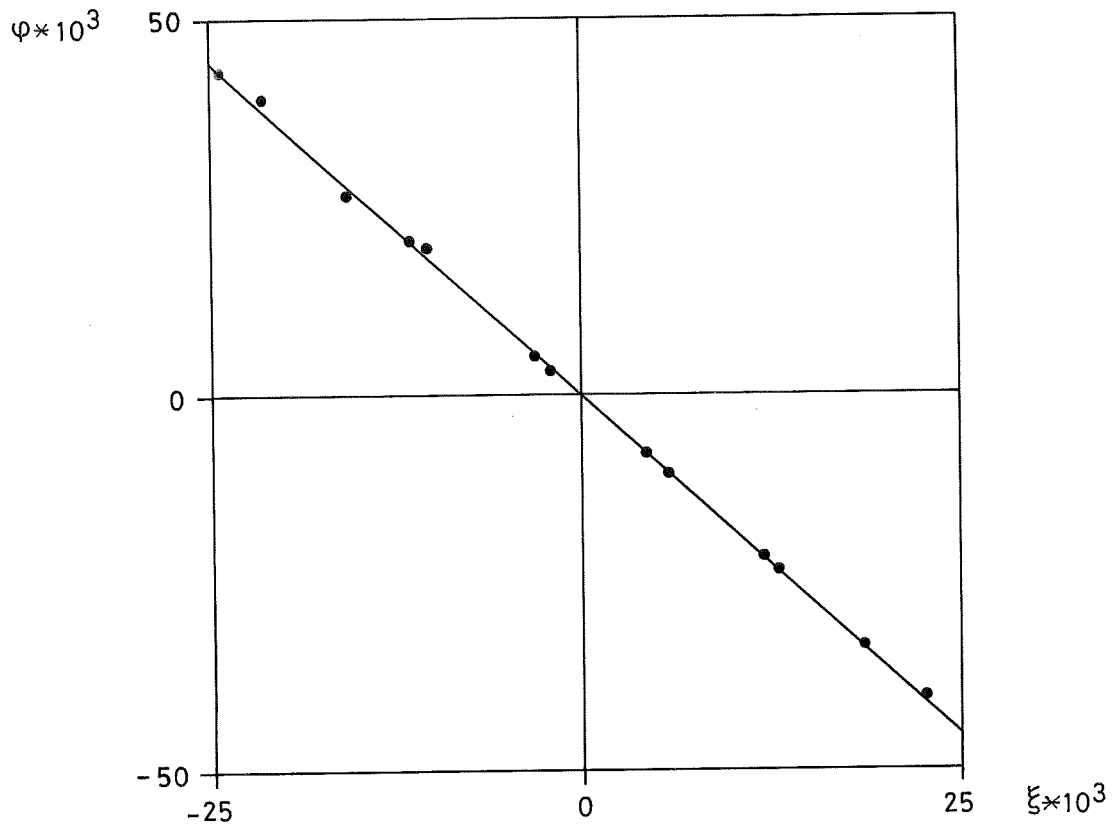


Fig. 1 - Flux and xenon deviation at  $z = 0.05 H$  for a flat flux (fig 3:1A)

$H = 5.40$  (4 cm above crit. height)

Homogeneous control. Disturbance: 50 pcm in 2 hours

$$\underline{\varphi = -1.78 \xi}$$

The same constant  $b$  is valid for every space point.

Now the same good linear relationship is found in several cases at small disturbances ( $< 100$  pcm). A number of fluxes just around the critical heights have been observed for small disturbances, and the linear approximation (1) is valid with good accuracy. In table 1 is summed up the constants  $b$  (eq. (1)) for some cases.

Table 1 Linear relationship between flux and xenon

$\varphi = -b\xi$  The fluxes are shown in figure 3:1.

Case	H(m)	H <sub>crit</sub>	Flux	$\alpha$	b
1	4.95	5.01	A	0	1.73
2	5.02	5.01	A	0	1.76
3	5.40	5.36	A	-0.0514	1.78 <sup>x)</sup>
4	8.90	8.89	H	-0.0514	1.79
5	7.0	7.35	D	-0.0514	1.71
6	7.5	7.35	D	-0.0514	1.73

<sup>x)</sup>(in fig. 1)

We define:

$$\xi = X - X^0$$

$$\eta = I - I^0$$

$$\varphi = \phi - \phi^0$$

and write the xenon and iodine equations in incremental form

from (2:8), (2:9) and neglect the subscript for the space point.  
Thus the following equations are valid:

$$\frac{d\xi}{dt} = -\lambda_X \xi + \lambda_I \eta + \gamma_X \sigma_X \varphi - \sigma_X X^0 \varphi - \sigma_X \phi^0 \xi - \sigma_X \xi \varphi \quad (2)$$

$$\frac{d\eta}{dt} = \gamma_I \sigma_X \varphi - \lambda_I \eta \quad (3)$$

We insert (1) in (2) and (3) and get the dynamic system:

$$\frac{dx}{dt} = \begin{bmatrix} -\lambda_X - \sigma_X \phi^0 + b \cdot \sigma_X (X^0 - \gamma_X) & \lambda_I \\ -\gamma_I \sigma_X b & -\lambda_I \end{bmatrix} x + \begin{bmatrix} b \sigma_X x_1^2 \\ 0 \end{bmatrix}$$

or

$$\frac{dx}{dt} = Ax + \begin{bmatrix} b \sigma_X x_1^2 \\ 0 \end{bmatrix} \quad (4)$$

where

$$x = \begin{bmatrix} \xi \\ \eta \end{bmatrix}$$

## 5.2 COMPARISON WITH A TWO-POINT MODEL

In [5], chapter 2, is derived a model of a two point reactor.

In the linear approximation of the symmetric flux we found

(section 2.7 in [5]) the relationship

$$\varphi_1 = b_1 \cdot (x_1 - x_3) \text{ for rod control (2:48) and}$$

$$\varphi_1 = b_2 \cdot (x_1 - \frac{1}{2} x_3) \text{ for homogeneous control (2:50)}$$

where  $x_1 = \xi_1$ ,  $x_3 = \xi_1 + \xi_2$  and  $b_1, b_2$  are core constants. Now, if we have a symmetric disturbance of the flux, the initial conditions are:

$$x_3 = \xi_1 + \xi_2 = 0$$

$$x_4 = \eta_1 + \eta_2 = 0$$

and (2:49) and (2:52) in [5] show that  $x_3$  and  $x_4$  are identically zero all the time. In this case we get:

$$\varphi_1 = b_1 x_1 = b_2 x_1$$

for both homogeneous and rod control, where

$$b_1 = \frac{\beta \phi^0 h^2}{2 - h^2 \alpha \phi^0} = \frac{\beta \phi^0 H^2}{18 - H^2 \alpha \phi^0} = b_2 \quad (5)$$

For the critical height for  $\phi^0 = 1$ ,  $\alpha = -0.0514$

we find

$$H^0 = 6.93 \text{ m}$$

$$b_1 = -1.71 \text{ or}$$

$$\varphi = -1.71 \xi \quad (6)$$

in the two point symmetric model.

For  $\alpha = 0$  we have the critical height

$$H^0 = 6.498 \text{ m}$$

for the two point model [5] which gives

$$b_1 = -1.71$$

Compare  $b_1$  to the values of  $b$  in table 1 where all the fluxes are situated around the different critical heights. We can also see that  $b$  is very insensitive to variations in  $\alpha$ .

Further in the two point model, the dynamic behaviour of the process for the symmetric initial conditions above is determined from a  $2 \times 2$  submatrix.

$$A = \begin{bmatrix} -\lambda_x - \sigma_x \phi^0 - b_1 \sigma_x (X^0 - \gamma_x) & \lambda_i \\ \gamma_i \sigma_x b_1 & -\lambda_i \end{bmatrix} \quad (7)$$

(see (2:53, 54, 56) in [5])

We see directly, that the linear part of (4) is identical to (7). The matrix  $A$  (7) is got as an approximation of the symmetric two point model when the control term is neglected [5].

### 5.3 ANALYSIS OF THE SPACE INDEPENDENT MODEL

First we find the singular points of (4), which are (0,0) and

$$X_1^0 = \frac{1}{\sigma_X b} \{\lambda_X + \sigma_X \phi^0 + b \sigma_X (1 - X^0)\}, \text{ as } \gamma_X + \gamma_i = 1$$

$$X_2^0 = -\frac{\gamma_i}{\lambda_i} \{\lambda_X + \sigma_X \phi^0 + b \sigma_X (1 - X^0)\} \quad (8)$$

Now we choose  $b = 1.7$ , which causes (0,0) to be a stable focus and the other singular point to be a saddle point. (The choice of  $b$  is not critical. We get one positive real eigenvalue for all values of  $b$  in this singular point.)

Figure 2 shows the phase plane of (4). The saddle point (8) is marked in the figure. The trajectories for large deviations from origo show some odd details.

Trajectories near origo are stable and oscillatory. For bigger disturbances, however, the trajectories will start outside an unstable limit cycle. At first they are diverging in a spiral, but later they are nonoscillatory unstable. For very big disturbances it is possible to get unstable solutions without any oscillation.

These nonperiodic unstable solutions have not been verified by any other space dependent model.

The character of the phase plane is not influenced by the value of the temperature coefficient (see table 1) when  $\alpha$  varies between 0 and -0.05. This contradicts the results from the two point model in [5], chapter 5, as well as the TRAXEN simulations.

To sum up, the space independent model (4) is not sufficient to describe what happens for large amplitudes of the state variables. Then we must include higher order terms in the relationship between flux and xenon.

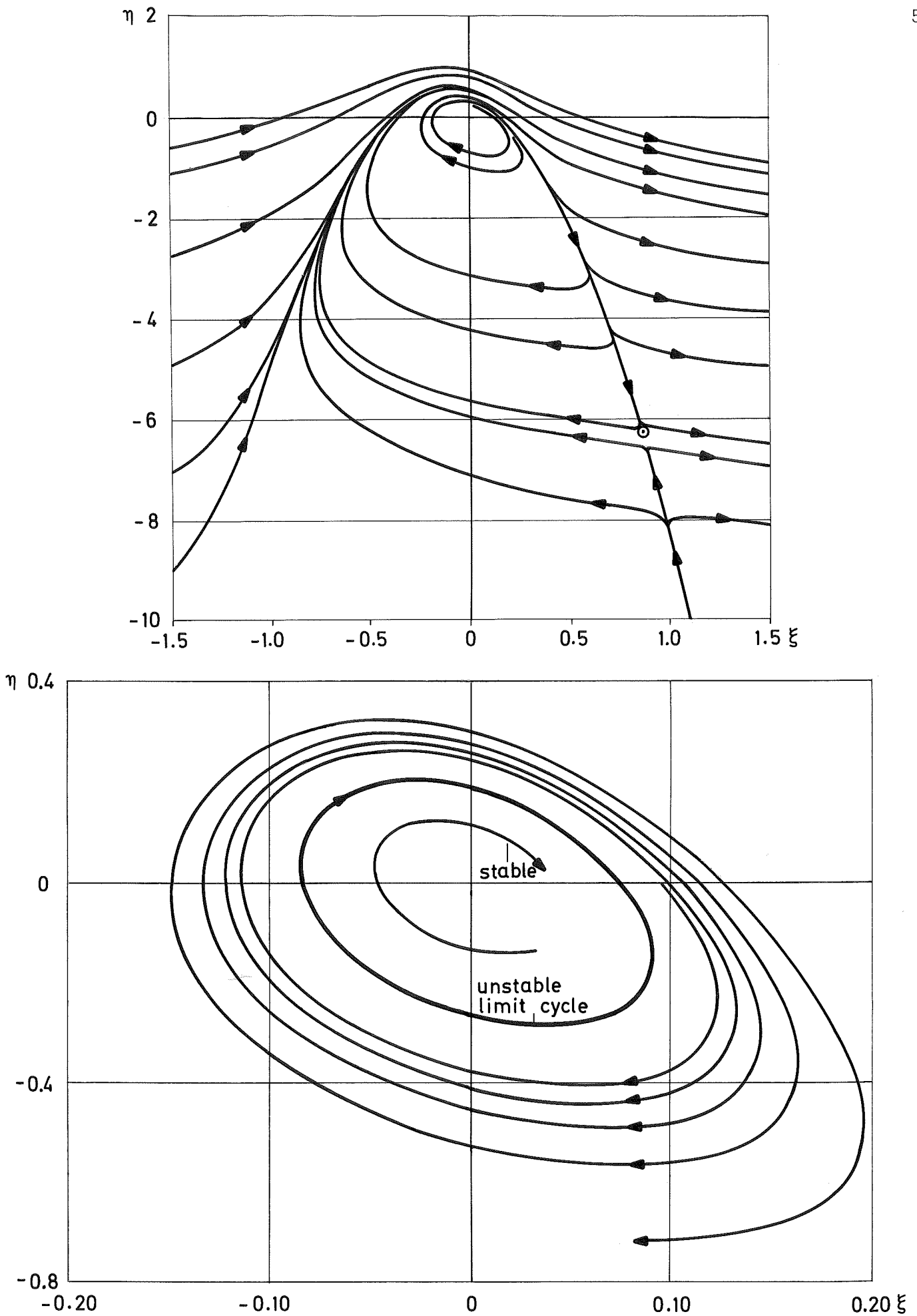


Fig. 2 - The phase plane of the nonlinear space independent xenon model. The lower figure is a detail of the upper one.

#### 5.4 COMPARISON WITH OTHER SPACE INDEPENDENT MODELS

A state independent model of the same structure as presented is proposed by Sha [8]. He assumes the relationship (1) and gets some stability criteria out of Lyapunov theory. However, it is not presented any trajectories.

Chernick et. al [1] have a similar model. Nonoscillatory unstable solutions are got for positive temperature coefficients and this also contradicts the result from the nonlinear two space point model, [5], ch.5, as well as TRAXEN simulations, ch. 4.2.

#### ACKNOWLEDGEMENT

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## Appendix 1

## DEFINITION OF SYMBOLS AND THEIR NUMERICAL VALUES

Symbol	First def. in equation	Explanation	Numerical value
$B^2(z,t)$	2:2	Material buckling	
$(B^2)^*(z)$	2:7	Material buckling, equilibrium value	
$c(z,t)$	2:7	Absorbption term	
$c^1(z,t)$	2:11	Rod absorbption	
$D(z,t)$	2:1	Absolute diffussion	
$E(z,t)$	2:1	Relative diffussion	
H	2:3	Extrapolated core height (m)	
h	2:5	Distance between two node points; $h = \frac{H}{N + 1}$	
$H_{crit}$		Critical core height	
$I(z,t)$	2:8	Iodine concentration, measured with the xenon equilibrium concentration at infinite flux as basis	
$I^0(z)$	2:10	Equilibrium value of iodine concentration	
$n(z,t)$		$I(z,t) - I^0(z)$	
$K(z)$	2:4	Weight function in expression for total power	
$K^*$	2:13	Rod insertion	
N	2:5	Number of node points	
$P(t)$	2:4	Total power	
$r(t), R(z)$	2:15	Time and space distribution of reactivity disturbance	
t	2:1	Time in hours	
$u(z,t)$	2:7	Control term in buckling	

Symbol	First def. in equation	Explanation	Numerical value
$X(z,t)$	2:8	Xenon concentration, measured with the xenon equilibrium concentration at infinite flux as basis	
$X^0(z)$	2:10	Equilibrium value of xenon concentration	
$\xi(z,t)$		$X(z,t) - X^0(z)$	
$z$	2:1	Space coordinate	
$\alpha(z)$	2:7	Temperature coefficient, expressed as reactivity bounded in fuel temperature increase above the moderator at mean flux and infinite gitter.	-0.226%
		Normalization to mean flux $\bar{\phi} = 5.65 \times 10^{13}$ , $M^2 = 440 \text{ cm}^2$ , multiply with $\frac{1}{0.0440}$	$\alpha = -0.0514$
$\beta$	2:7	Xenon influence on changes in buckling (-3.2% on reactivity) at saturation	-0.73
$\gamma_x$	2:8	Fraction of xenon yield (relative to xenon + iodine yield)	0.05
$\gamma_i$	2:9	Fraction of iodine yield (relative to xenon + iodine yield)	0.95
$\phi(z,t)$	2:1	Neutron flux, normalized to $5.65 \times 10^{13}$ neutr/cm <sup>2</sup> *sec.	
$\bar{\phi}$	2:16	Mean flux	
$\phi^0(z)$	2:3	Equilibrium flux	
$\varphi(z,t)$		$\phi(z,t) - \phi^0(z)$	
$\lambda_x$	2:8	Xenon disintegration constant	$0.0756 \text{ h}^{-1}$
$\lambda_i$	2:8	Iodine disintegration constant	$0.1058 \text{ h}^{-1}$
$\lambda_1 \text{ c}^1$	2:11	Rod insertion length, rod absorption	
$\psi$	ch. 3.3	Form factor, $\phi_{\text{max}}/\bar{\phi}$	
$\sigma_x$	2:8	Microscopic xenon cross section normalized to $\bar{\phi} = 5.65 \times 10^{13}$ and time base in hours	$2.29 \times 10^{-18} \text{ cm}^2$ 0.0465
$\Sigma_f$	2:1	Macroscopic fission cross area	
$\Sigma_a$	2:1	Macroscopic absorption cross area	
$\nu$	2:1		

## FLUX SHAPES AND DISTRIBUTIONS

The table shows the distribution in 20 space points of the undisturbed buckling.

The fluxes are shown in figure 3:1.

If only 10 values are written, the flux is symmetric.

## A. Flat flux

$$\psi = 1.05$$

$$h^2 B^2 \quad 1.0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad (\text{symmetric})$$

## B. Ditch flux 1

$$\psi = 1.31 \quad \phi_{\min}/\phi_{\max} = 0.740$$

$$h^2 B^2 \quad \underbrace{0.08155 \quad \dots \quad 0.08155}_{(1, \dots, 7)} \quad \underbrace{-0.07727 \quad \dots \quad -0.07727}_{(8, \dots, 10)} \quad (\text{symm})$$

C.  $\psi = 1.289$ 

$$h^2 B^2 \quad \begin{array}{cccccc} 0.7979 & 0.01015 & 0.00857 & 0.00755 & 0.00666 & \\ 0.00610 & 0.00567 & 0.00533 & 0.00512 & 0.00512 & \end{array}$$

D.  $\psi = 1.35$ 

$$h^2 B^2 \quad \begin{array}{cccccc} 0.8011 & 0.0124 & 0.0107 & 0.00933 & 0.00833 & 0.00755 \\ 0.00688 & 0.00633 & 0.00600 & 0.00567 & 0.00722 & 0.00522 \\ 0.0177 & 0.0178 & 0.0180 & 0.0184 & 0.0190 & 0.0200 \\ 0.0212 & 0.8088 & & & & \end{array}$$

## E. Flattened sine flux

$$\psi = 1.35$$

$$h^2 B^2 \quad \begin{array}{cccccc} 0.0643 & 0.0519 & 0.0467 & 0.0439 & 0.0423 & 0.0415 \\ 0.0411 & 0.0411 & 0.0796 & 0.0796 & - - - - - & (\text{symmetric}) \end{array}$$

## F. Ditch flux 2

$$\psi = 1.45 \quad \phi_{\min}/\phi_{\max} = 0.66$$

$$h^2 B^2 \quad \underbrace{\begin{array}{ccc} 0.0752 & \dots & 0.0752 \\ 0.0816 & \dots & 0.0816 \end{array}}_{(1, \dots, 6)} \quad 0.0790 \quad \underbrace{\begin{array}{cc} -0.0773 & -0.0773 \end{array}}_{(8, \dots, 13)} \quad \underbrace{\hspace{10em}}_{(14, \dots, 20)}$$

$$\begin{array}{rcc}
 \text{G. } \psi = 1.47 & (1, \dots, 7) & (8, \dots, 13) \\
 h^2 B^2 & \underbrace{\begin{array}{ccc} 0.0331 & \dots & 0.0331 \\ 0.0457 & \dots & 0.0457 \end{array}}_{(14, \dots, 20)} & \underbrace{\begin{array}{ccc} -0.0126 & \dots & -0.0126 \end{array}}
 \end{array}$$

H. Sine flux

$$\psi = 1.57$$

$$h^2 B^2 \quad 0.02234 \quad (\text{constant})$$

## Appendix 2

### NUMERICAL METHODS

In order to simulate the xenon process, we have to solve eq. (2:5, 8, 9) with (2:7, 11, 12, 13) and the boundary conditions (2:6, 10) inserted. The solution can be divided into two different parts, here called equilibrium flux and transient calculations.

In the equilibrium state we put the derivatives of xenon and iodine equal to zero in (2:8, 9) and the equilibrium values are found to be:

$$X_k^o = \frac{1}{1 + \frac{\lambda_x}{\sigma_x \phi_k^o}} \quad (1)$$

$$I_k^o = \frac{\gamma_i \sigma_x}{\lambda_i} \phi_k^o \quad k = 1, \dots, N \quad (2)$$

In the xenon process the xenon and iodine concentrations are state variables and are uniquely determined by the differential equations.

## A2.1 CALCULATION OF NEUTRON FLUX

We will solve the diffusion equation in every time step by writing eq. (2:5) in matrix form:

$$E \cdot \phi + h^2(B^{2*} + \Delta B^2)\phi = 0 \quad (3)$$

where

$$E = \begin{bmatrix} -(E_1 + E_2) & E_2 & 0 & - & - & - & - & - & 0 \\ E_2 & -(E_2 + E_3) & E_3 & 0 & - & - & - & - & \\ 0 & E_3 & -(E_3 + E_4) & E_4 & - & - & - & - & \\ | & & & & & & & & \\ | & & & & & & & & \\ | & & & & & & & & \\ | & & & & & & & & E_N \\ 0 & - & - & - & - & 0 & E_N & & -(E_N + E_{N+1}) \end{bmatrix}$$

$$B^{2*} = \text{diag} (B_1^{2*}, B_2^{2*}, \dots, B_N^{2*})$$

$$\Delta B^{2*} = \text{diag} (B_1^2 - B_1^{2*}, \dots, B_N^2 - B_N^{2*})$$

$$\phi = (\phi_1, \phi_2, \phi_3, \dots, \phi_N)^T$$

$$h^2 = (H/N + 1)^2$$

The symbols are explained in appendix 1. See also eq. (2:5).

The flux  $\phi$  is solved from (3)

$$\phi = - E^{-1}(Q + \Delta Q)\phi \quad (4)$$

$$\text{where } Q = h^2 B^{2*}$$

$$\Delta Q = h^2(\Delta B^{2*})$$

and  $\Delta Q$  is a function of  $\phi$ .

In equilibrium we shall solve (4) with (1) inserted in  $Q$ , while  $\Delta Q = 0$ . During the transient, the xenon concentration is found from the differential equations.

We define a new matrix:

$$H = -E^{-1}(Q + \Delta Q) \quad (5)$$

where  $H$  is a function of  $\phi$  and  $X$ .

Our diffusion equation is thus formulated as:

$$\phi = H \cdot \phi \quad (6)$$

In order to solve (6), we use an iterative technique as it is impossible to get an explicit solution of  $\phi$ . We regard an eigenvalue problem:

$$H \phi = \mathcal{H} \phi \quad (7)$$

If we can find a real solution for the eigenvalue  $\mathcal{H} = 1$ , we have also found the solution of (6).

Now we know [2] that the eigenvalue problem for the one group diffusion equation in discrete form:

$$E \phi + \mu h^2 B^2 \phi = 0 \quad (8)$$

has a fundamental solution  $\phi^0$  for the smallest eigenvalue,  $\mu$ , which is proportional to the flux distribution of the poisoned reactor.

We see that (8) is transformed to:

$$-E^{-1} h^2 B^2 \phi = \frac{1}{\mu} \cdot \phi \quad \text{or}$$

$$H \phi = \frac{1}{\mu} \cdot \phi \quad (9)$$

Thus, we have to find the largest eigenvalue of (7) and its eigenvector  $\phi$ .

As  $\phi$  is implicitly defined in H, we must iterate. The elements of H are unknown depending on three terms in (2:7) which are included in Q (4) for every space point. In the equilibrium case we have  $\Delta Q = 0$  and it remains to determine xenon equilibrium concentration and absorption c (or rod insertion  $\lambda$ ) in equilibrium.

## A2.2 CALCULATION OF THE EQUILIBRIUM FLUX. Order of operations.

The computation advances as follows:

In brackets are shown the names of Fortran subroutines, described in app. 3:

- {1} Guess a start value of  $\phi$ , e.g. (EGENV)

$$\phi^{(0)} = A \sin\left(\frac{\pi}{H} \cdot z\right)$$

and a rod position  $\lambda^{(0)}$  (FLOW)

(CONT)

- {2} The amplitude A of  $\phi^{(0)}$  is normed by the power condition (2:6) (XNORM)

- {3} The xenon equilibrium first value  $X^{(0)}$  is determined (MATR)  
from (1) and  $\phi^{(0)}$ , and  $X^{(0)}$  and  $\lambda^{(0)}$  are inserted in (RAND)  
Q (eq. (3) - (4)).

- {4} The first iteration (iteration parameter = i) of H = (MATR)  
=  $H^{(i)}$  (i = 0) is then determined. (FFGG)

- {5} With the potense method is determined the largest eigenvalue  $\mathcal{H}_{(i)}$  to the matrix  $H^{(i)}$ , i = 0, 1, 2, ... (iteration variable = v)

$$\mathcal{H}_{(i)}^{(v)} = \frac{\|H^{(i)} \cdot \phi_{(i)}^{(v)}\|}{\|\phi_{(i)}^{(v)}\|} \quad v = 0, 1, 2, \dots \quad (\text{XNORM})$$



where we define

$$\phi_{(i)}^{(v+1)} = H^{(i)} \cdot \phi_{(i)}^{(v)} \quad (\text{GF}_i)$$

The norms are calculated from the power condition (2:6)

$$\|\phi_{(i)}^{(v)}\| = B * \sum_{k=1}^N K_k \cdot \phi_k^{(v)}$$

$$\|H^{(i)} \cdot \phi_{(i)}^{(v)}\| = \|\phi_{(i)}^{(v+1)}\| = B * \sum_{k=1}^N K_k \cdot \phi_{k(i)}^{(v+1)}$$

why

$$\mathcal{H}_{(i)}^{(v)} = \frac{\sum_{k=1}^N K_k \cdot \phi_{k(i)}^{(v+1)}}{\sum_{k=1}^N K_k \cdot \phi_{k(i)}^{(v)}} \quad (\text{XNORM})$$

$\mathcal{H}_{(i)}$  is accepted as an eigenvalue if

$$\left| \frac{\mathcal{H}_{(i)}^{(v+1)}}{\mathcal{H}_{(i)}^{(v)}} - 1 \right| \leq \epsilon_3 \quad (\text{XKAPPA})$$

{6} The new eigenvector  $\phi_{(i)}$  is accepted as the new approximation of  $\phi$  for the  $i$ :th iteration. (EGENV,TEST)

{7} Points 2 - 5 are repeated now for the next iteration. In points 3 and 4 we shall put in the new value  $\phi_{(i+1)}$  to get the matrix  $H_{(i+1)}$ . In order to avoid numerical instability it was necessary to use a relaxation method. Instead of  $\phi_{(i+1)}$  we put in a value  $\phi_{(i+1)}^*$  in the matrix  $H$  in order to get  $H_{(i+1)}$ , where  $\phi_{(i+1)}^*$  is found by the formula:

$$\phi_{(i+1)}^* = \phi_{(i+1)} + \mathfrak{S}(\phi_{(i)} - \phi_{(i+1)}) \quad (\text{TEST})$$

where  $0 < \vartheta < 1$ .

The fastest computations were found for:

$$\vartheta = 0.2 - 0.3$$

- {8} We accept  $\phi_{(i+1)}$  as the right eigenvector for the eigenvalue  $\mathcal{H}_{(i+1)}$  if:

$$\frac{\|\phi_{(i+1)} - \phi_{(i)}\|}{\|\phi_{(i+1)}\|} < \epsilon_2 \quad (\text{TEST})$$

Now we have got a solution of the eigenvalue problem (7) for a certain value  $\lambda^{(0)}$  of the insertion length of the rod. The eigenvalue  $\mathcal{H} \neq 1$  and now  $\lambda$  shall be adjusted  $\lambda^{(i)}$ ,  $i = 0, 1, \dots$ , until we have found the value  $\mathcal{H} = 1$  and corresponding eigenvector  $\phi$  which is the solution of the problem.

- {9} We guess arbitrarily a new value of  $\lambda$ , called  $\lambda^{(1)}$  and get a new solution  $\phi$  and  $\mathcal{H}^{(\mu)}$  by proceeding through points 2 - 8 again (iteration parameter  $\mu$ ). (FLOW)

- {10} The next value of  $\lambda$ ,  $\lambda^{(\mu+1)}$ , is calculated by linear regula falsi from previous values: (FLOW)

$$\lambda^{(\mu+1)} = \lambda^{(\mu)} - \frac{(\mathcal{H}^{(\mu)} - 1)(\lambda^{(\mu)} - \lambda^{(\mu-1)})}{\mathcal{H}^{(\mu)} - \mathcal{H}^{(\mu-1)}}$$

and proceed through 2 - 8 and 10.

- {11} When  $[\mathcal{H}^{(\mu+1)} - 1] < \epsilon_1$  we accept  $\phi$  as the right flux. (FLOW)

- {12} From the beginning we have chosen  $\epsilon_2$  and  $\epsilon_3$  between  $10^{-2}$  and  $10^{-3}$ . Now we make them much smaller,  $10^{-6}$  to  $10^{-8}$  have been found to be acceptable and increase the accuracy of the calculations. (FLOW)

### A2.3 CALCULATION OF THE TRANSIENT

As the flux is assumed to be stationary all the time, we have to resolve the diffusion equation as the xenon concentration varies.

The differential equations (2:8) and (2:9) are integrated with the Runge - Kutta method, where the initial values are found from (1) and (2) after the equilibrium flux calculation. Richardson extrapolation is used in order to increase the accuracy.

The neutron flux is then calculated for every time step as described above. The following changes are made:

- {1} As initial guess of the flux and rod position we use the values of the previous time step.
- {2} The xenon concentration of previous time step is inserted.

Appendix 3

## SHORT DESCRIPTION OF THE TRAXEN PROGRAM

A Fortran program package called TRAXEN (TRANSients of XENon) has been written [6] in order to simulate the xenon process.

It was written initially for IBM 7090, but has later been changed to CD 3300 and CD 3600. The general numerical methods are described in appendix 2.

Figure 1 is a general chart of the subroutines. The inputs and outputs are described briefly and in detail in [6].

The calculation time grows approximately with  $N^2$  where  $N$  is number of meshpoints. Calculation of an equilibrium flux for  $N = 20$  takes about 1 - 4 seconds on a CDC 3600, depending on the buckling and flux shape.

The computing time for a transient depends much on stability or the amplitude of the transients. For bigger deviations for every time step we have either to decrease step length or increase the number of iterations.

The computing time for time step of a transient with 20 space points is 2 - 3 seconds.

Maximum number of meshpoints is 50.

## INPUTS

The MAIN program reads in the following datas in groups  
(see appendix 1 for the terminology).

N, H

$\gamma_i, \sigma_x, \lambda_x, \lambda_i$   
 $\beta(1), \dots, \beta(N)$

$\lambda, c^1, K$  (eq. (2:11-13))

MVOID = 1 if hydraulics shall be calculated (not used here)

STL5 = 1 if a xenon control rod should be used (not used in  
the report)

P(t), r(t) in polygone chains

E(1), ..., E(N)

Q(1), ..., Q(N) (Q = unpoisoned buckling)

$\alpha(1), \dots, \alpha(N)$

R(1), ..., R(N)

stl2 = 1 for power control (eq. (2:6) )

stl2 = 2 for constant flux in one space point

K(1), ..., K(N) (coefficients in eq. (2:6) )

Printing instructions

Accuracy  $\epsilon$  for the iterations

Initial values for flux, xenon and iodine (arbitrarily)

STAT if equilibrium flux is to be calculated

TRAN if transient flux is to be calculated

For a new calculation it is only necessary to read in the  
changed datas.

The input formats are described in detail in [6].

## OUTPUTS

There is possible to print out a number of test values in the subroutines (see [6]).

In subroutine TRY is printed out:

time,  $\lambda$ ,  $\Psi$ ,  $\bar{\phi}$ ,  $P$ ,  $r(t)$ ,  $\mu = \frac{1}{\lambda}$ ,

$\left. \begin{array}{l} \phi \text{ (normalized to } \phi_{\max} = 1), \\ \phi \text{ in physical units,} \\ X, I, \varphi, \frac{d\xi}{dt}, \xi, \end{array} \right\} \text{ for every space point}$

fourier coefficients for fundamental mode and two overtones (sine waves),

## CALCULATION OF EQUILIBRIUM FLUX

1. The MAIN program calls subroutine STAT where a few parameters are set.
2. STAT calls subroutine RAND with the rod insertion  $\lambda$  as a parameter.
3. In RAND is first calculated actual power  $P(t)$  and disturbance  $u(z,t)$ , (zero in this case) with RRPP. If a xenon control rod is used its reactivity value is calculated (ROD). RAND calls FLOW with the rod insertion length as argument.
4. In FLOW is the iteration of rod insertion  $\lambda$  made. First is a flux calculated for the rod position, which was guessed. Then the routine iterates in  $\lambda$  until the eigenvalue  $\lambda$  (eq. (A2:7)) is close to 1 (A2; points 1, 9 - 12).
5. FLOW calls the function EGENV and the subroutine MATR.  
In MATR is defined the matrixes  $E$ ,  $Q$ ,  $\Delta Q$  and  $H$  (eq. (A2:3 - 5)), see A2 points 3 - 4.  
In EIGENV and its subroutine TEST is iteration made of the flux until the flux vector has converged (A2; points 1, 6, 7, 8).
6. EGENV calls the function XKAPPA. In XKAPPA is the biggest eigenvalue  $\lambda$  of the eq. (A2:7) calculated with the potense method (A2, point 5). XKAPPA uses GFI to calculate the product  $H * \phi$

and XNORM to calculate the norm (A2, point 5).  
XKAPPA accepts  $\mathcal{L}$  as an eigenvalue when the iteration has converged.

#### CALCULATION OF THE TRANSIENT FLUX

1. The MAIN program calls subroutine TRAN, where the actual time is defined. If the power is zero TRAN calls subroutine EFFO in order to calculate the transients analytically until power gets positive. If the power is positive it calls RK2.
2. In RK2 is step length of the integration determined. A Richardson extrapolation is also made. The routine calls RK1, which is an ordinary Runge - Kutta subroutine.
3. RK1 calls DERI which defines the right hand side of the equations.
4. DERI in turn calls RAND. The following sequences are the same as in the equilibrium case from point 3. There are some differences, which are pointed out in appendix 2.

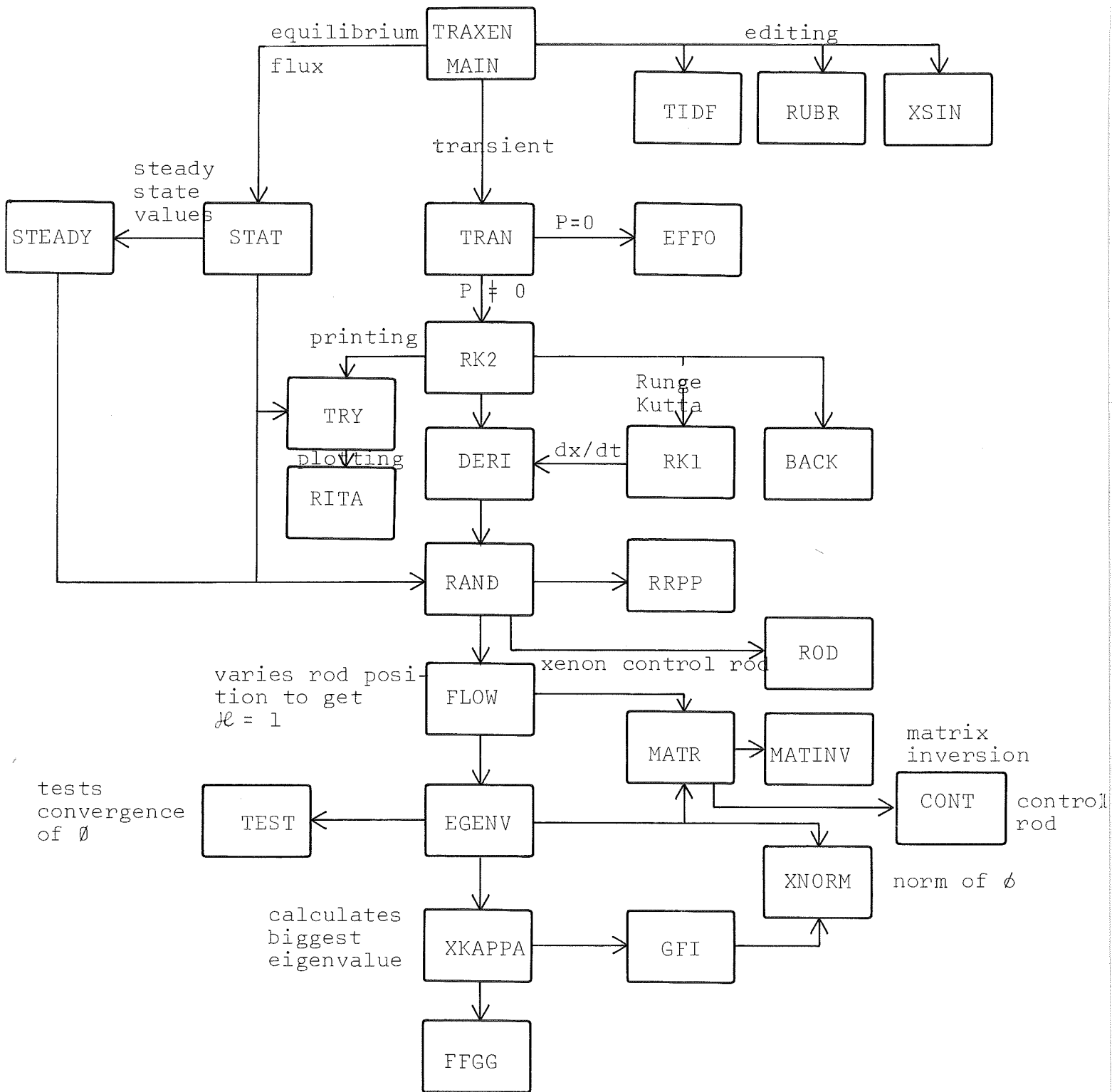


Figure 1. Blockdiagram of the TRAXEN subroutines



## PROGRAM TRAXEN

C AUTHOR GUSTAF OLSSON, DIVISION OF AUTOMATIC CONTROL LUND  
 C THE PROGRAM CALCULATES EQUILIBRIUM NEUTRON FLUX DISTRIBUTION AND XENON  
 C TRANSIENTS FOR A NONLINEAR FINITE DIFFERENCE XENON MODEL

C REF: GUSTAF OLSSON, DIGITAL SIMULATION OF SPATIAL XENON OSCILLATIONS  
 C DIV OF AUTOMATIC CONTROL LUND REPORT 6911

C\*  
 C THE MATHEMATICAL MODEL IS DESCRIBED IN CHAPTER 2  
 C THE NUMERICAL METHODS ARE DESCRIBED IN APP 2

## C\* INPUT VARIABLES

C VARIABLES AND FORMATS ARE DESCRIBED IN  
 C G, OLSSON, DIGITALT PROGRAM TRAXEN FOR TRANSIENTBERAKNINGAR  
 C AV XENONSVANGNINGAR I EN AXIELL REAKTORMODELL  
 C SWEDISH STATE POWER BOARD, STOCKHOLM 1966 REPORT E-53/66

## C\* OUTPUT

C ALL INPUTS ARE REPEATED  
 C TEST VALUES CAN BE PRINTED IF THE VARIABLES NTRY ARE 1  
 C ALL OTHER OUTPUTS ARE PRINTED IN SUBROUTINE TRY

C TIME ROD INSERTION (LAMBDA) FORM FACTOR (PSI)  
 C POWER R = AMPLITUDE OF DISTURBANCE  
 C NY = EIGENVALUE OF FUNDAMENTAL MODE (EQ. A2,8)

C IF STL5 = 1

C AN EXTRA XENON CONTROL ROD IS DEFINED BY ROD, IC, ICU, ICL  
 C THIS ROD CAN BE INSERTED IN ORDER TO DAMP THE OSCILLATIONS  
 C IC = CENTRE OF THE ROD ICU = UPPER BOUNDARY  
 C ICL = LOWER BOUNDARY

C FI = NEUTRON FLUX

C\* FOR EVERY SPACE POINT IS PRINTED

C IF NTRY1 = 2 IS ONLY DELTAFI AND DELTAX IN EVERY SPACE POINT CALCULATED

C FI (NORM) = FI/FIMAX

C FI (ABS) = FI IN PHYSICAL UNITS

C XE = XENON CONC, MEASURED WITH THE XE EQUILIBRIUM CONC,  
 C AT INFINITE FLUX AS BASIS

C JOD = IODINE CONC WITH SAME BASIS AS XE

C DELTAFI(NORM) = (FI(T) - FI(0))/MEAN FLUX

C DXDT = XENON TIME DERIVATIVE

C DELTAX = XE(T) - XE(0)

C FOURIER COEFF, FOR FLUX AND XENON FOR FUNDAMENTAL MODE AND  
 C TWO OVERTONES, THE MODES ARE SINE WAVES

## C SUBROUTINES AND FUNCTIONS

C TRY	STAT	STEADY	TRAN	EFFC
C RK2	RK1	DERI	BACK	XSIN
C RAND	FLOW	MATR	RRPP	EGENV
C CONT	XKAPPA	XNORM	TEST	FFGG
C GFI	MATINY	TIDF	RUBR	RITA
C ROD	HYDRO	VOID	(HYDRO AND VOID ARE NOT USED IN THESE CALCULATIONS, AS THE VOID IS NOT TAKEN ACCOUNT FOR)	

THE MAIN PROGRAM READS IN ALL DATA AND PRINT THEM OUT

IF A TRANSIENT IS TO BE CALCULATED SUBROUTINE TRAN IS CALLED  
IF AN EQUILIBRIUM FLUX SHALL BE CALCULATED IT CALLS SUBROUTINE STAT  
FOR EDITING IT CALLS SUBROUTINES TIDF AND RUBR

C  
C  
C  
C  
C  
C  
C

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COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),          CO      1
1 JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),     CO      2
2 TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA (50),RR,PP, CO      3
3 N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI, CO      4
4 TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50), CO      5
5 Q(50), E(51),R(50),DXDT(50),DIDT (50), HZ,S(150), T(30), W, CO      6
6 AX(20), TETA CO      7
COMMON NRIT, KURV, NSTANS, CO      8
1 IC, ICU, ICL, PROD, DROD, PART , ABSO, STAB(50),KSTYR, CO      9
2 STL1, STL2, STL3, STL4, STL5, STL6, CO     10
3 NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9, CO     11
4 EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3, CO     12
5 DEDA(20),DBDA(19), XALF1 (20),MVOID CO     13
COMMON XSS(50),XD, EPSX, EPSDX, ICONT CO     14
INTEGER AX
REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1 XECO  11
INTEGER STL1, STL2, STL3, STL4, STL5, STL6 XECO  12
DIMENSION P(30), RT(30) MAIN
READ (5,25) AX MAIN0003
WRITE (6,26) AX MAIN0004
1000 STL1 = 0
CALL RUBR ( I ) MAIN0005
GOTO ( 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14), I MAIN0006
1 READ ( 5, 100 ) N, H MAIN0007
HZ = H / FLOAT ( N+1 ) MAIN0008
WRITE ( 6, 101) N, H, HZ MAIN0009
GOTO 1000 MAIN0010
2 READ (5,102) GAMMAI, SIGMAX, LAMDAX, LAMDAI MAIN  11
WRITE (6,103) GAMMAI, SIGMAX, LAMDAX, LAMDAI MAIN  12
WRITE (6,1003) MAIN1012
403 READ (5,1004) I1004, BEX, I1005, BEY, I1006, BEZ MAIN1013
WRITE (6,1005) I1004, BEX, I1005, BEY, I1006, BEZ MAIN1014
IF (I1004.EQ.(-1)) 1000, 31 MAIN
31 IF (I1004.GT,0) 34, 32 MAIN
34 BETA (I1004) = BEX MAIN
32 IF (I1005.GT,0) 35, 33 MAIN
35 BETA (I1005) = BEY MAIN
33 IF (I1006.GT,0) 36, 37 MAIN
36 BETA (I1006) = BEZ MAIN
37 CONTINUE MAIN
GOTO 403 MAIN1019
3 READ(5,104) STL3, LAMBDA, C, K, MVOID MAIN  14
WRITE(6,105)STL3, LAMBDA, C, K, MVOID MAIN  15
IF (MVOID.EQ,1) 83, 84 MAIN
83 CALL VOID
84 CONTINUE MAIN
READ(5,401) STL5, KSTYR, PART,PROD,DROD,ICONT,ABSO,XD,EPSX,EPSDX MAIN
WRITE(6,402) STL5, KSTYR, PART,PROD,DROD,ICONT,ABSO,XD,EPSX,EPSDXMAIN

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	GO TO 1000	MAIN0016
4	READ(5, 108) TMAX, HT	MAIN0017
	WRITE (6, 110) TMAX, HT	MAIN0018
	WRITE (6,111)	MAIN0019
	DO 113 I113 = 1,30	MAIN
50	READ (5,112) TTT, PPP	MAIN0021
	WRITE (6,114) I113, TTT, PPP	MAIN0022
	T(I113) = TTT	MAIN0023
	IF (TTT,LT,0,) 200, 51	MAIN
51	P(I113) = PPP	MAIN
	CONTINUE	MAIN0026
113	CALL TIDF (PI,P)	MAIN0027
200	WRITE (6, 115)	MAIN0028
	DO 116 I116 = 1,30	MAIN
	READ (5, 117) TTX, RRR	MAIN0030
	WRITE (6, 118) I116, TTX, RRR	MAIN0031
	T (I116) = TTX	MAIN0032
	IF (TTX,LT,0,) 201,52	MAIN
52	CONTINUE	MAIN
	RT (I116) = RRR	MAIN0034
116	CONTINUE	MAIN0035
201	CALL TIDF (RI, RT)	MAIN0036
	WRITE (6,160) (I, TI(I,1), TI(I,2), PI(I,1), PI(I,2) , RI(I,1) ,	MAIN0037
C	RI(I,2) , I= 1, M )	MAIN0038
	GOTO 1000	MAIN0039
5	WRITE (6, 121)	MAIN0040
70	READ (5, 120) I116, EX, I117, EY, I118, EZ	MAIN0041
	WRITE (6, 122) I116, EX, I117, EY, I118, EZ	MAIN0042
	IF (I116,EQ,(=1)) 1000,54	MAIN
54	IF (I116,GT,0) 61,55	MAIN
61	E(I116) = EX	MAIN
55	IF (I117,GT,0) 62,56	MAIN
62	E(I117) = EY	MAIN
56	IF (I118,GT,0) 63,57	MAIN
63	E(I118) =EZ	MAIN
57	CONTINUE	MAIN
	GOTO 70	MAIN0047
6	CONTINUE	MAIN0048
	WRITE (6, 126)	MAIN0049
60	READ (5, 128) I128, XQ, XALFA, XR	MAIN0050
	IF (I128,EQ,(=1)) 1000, 65	MAIN
65	Q(I128) =XQ	MAIN
	ALFA (I128) = XALFA	MAIN0053
	R(I128)= XR	MAIN0054
	WRITE (6, 130) I128, Q(I128), ALFA(I128), R(I128)	MAIN0055
	GOTO 60	MAIN0056
7	READ (5, 132) STL2, K1, FIREF, HN	MAIN0057
	WRITE (6,134) STL2, K1, FIREF, HN	MAIN0058
	WRITE (6, 137)	MAIN0059
211	READ (5, 136) I135, BX, I136, BY, I137, BZ	MAIN0060
	WRITE (6, 138) I135, EX, I136, BY, I137, BZ	MAIN0061
	IF(I135,EQ,(=1)) 1000,71	MAIN
71	IF (I135,GT,0) 74,72	MAIN
74	B(I135) = BX	MAIN
72	IF (I136,GT,0) 75,73	MAIN
75	B(I136) = BY	MAIN

73	IF (I137,GT,0) 76, 77	MAIN
76	B(I137) = BZ	MAIN
77	CONTINUE	MAIN
	GOTO 211	MAIN0066
8	READ (5,140) IT, NRIT, KURV, NTRY1, NTRY2, NTRY3, NTRY4, NTRY5,	MAIN0067
1	NTRY6, NTRY7, NTRY8, NTRY9, NSTANS	MAIN0068
	WRITE (6,142)IT, NRIT, KURV, NTRY1,NTRY2, NTRY3, NTRY4, NTRY5,	MAIN0069
1	NTRY6, NTRY7, NTRY8, NTRY9, NSTANS	MAIN0070
	GOTO 1000	MAIN0071
9	READ (5, 144) EPS1, EPS2, EPS21, EPS3, EPS31, EPS32,	MAIN0072
1	ITE1, ITE2, ITE3, TETA	MAIN0073
	WRITE (6, 146) EPS1, EPS2, EPS21, EPS3, EPS31, EPS32,	MAIN0074
1	ITE1, ITE2, ITE3, TETA	MAIN0075
	GOTO 1000	MAIN0076
10	WRITE (6, 152)	MAIN0077
310	READ (5, 150) I150, XXF, YYX, ZZJ	MAIN0078
	WRITE (6, 154) I150, XXF, YYX, ZZJ	MAIN0079
	IF (I150,EQ,(-1)) 1000, 81	MAIN
81	FI(I150) = XXF	MAIN
	FI1(I150) = FI(I150)	MAIN0082
	XEQ(I150) = YYX	MAIN0083
	JODO(I150)= ZZJ	MAIN0085
	GOTO 310	MAIN0086
11	STL1=1	MAIN0087
	GOTO 1000	MAIN0088
12	STL1 =2	MAIN0089
	GOTO 1000	MAIN0090
13	CONTINUE	MAIN0091
C	THE SUBROUTINE XSIN CALCULATES DIFFERENT SINE FUNCTIONS FOR	
C	THE FOURIER COEFFICIENT CALCULATION IN SUBROUTINE TRY	
C		
	CALL XSIN(N)	MAIN1091
	IF (STL1,EQ,2) 20, 1020	MAIN
1020	CALL STAT	MAIN
	IF (STL1,EQ,1) 21, 1021	MAIN
1021	IF (EPS32,NE,0,) 1023, 20	MAIN
1023	EPS31 = EPS32	MAIN
20	CALL TRAN	MAIN0095
21	CONTINUE	MAIN0096
	STL1 = 0	MAIN *95
	GOTO 1000	MAIN0097
14	STOP	MAIN0098
25	FORMAT (20A4)	MAIN
26	FORMAT (20A4)	MAIN
100	FORMAT ( I10, F10,0 )	MAIN0101
101	FORMAT ( 10X, 1HN, I4, 4X, 1HH, F8,2, 4X, 2HHZ, F8,3 )	MAIN0102
102	FORMAT (4E10,0)	MAIN 103
103	FORMAT (10X, 7HGAMMAI=, F6,2, 4X, 7HSIGMAX=, F10,5, 4X,	MAIN 104
1	7HLAMDAX=, F10,5, 4X, 7HLAMDAI=, F10,5)	MAIN 105
104	FORMAT (I10, 2E10,0, 2I10)	MAIN 106
105	FORMAT (10X, 5HSTL3=,I4, 4X, 7HLAMBDA=, F10,4,4X, 2HC=, F10,4,	MAIN 107
	14X, 2HK=, I4, 4X, 6HMVOID=, I2 )	MAIN 108
108	FORMAT ( 2E10,0)	MAIN0109
110	FORMAT (10X, 7HT(MAX)=, F10,4, 4X, 7HDELTA=, F10,4)	MAIN0110
111	FORMAT (13X,1HI, 4X, 4HT(I), 5X, 6HEFFEKT)	MAIN0111

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112 FORMAT (2E10,0) MAIN0112
114 FORMAT (10X, I4, F10,4, F10,4) MAIN0113
115 FORMAT (13X, 1HI, 3X, 4HT(I), 3X, 10HBUKT,STORN ) MAIN0114
117 FORMAT (2F10,0) MAIN0115
118 FORMAT (10X, I4, F10,4, F10,4 ) MAIN0116
160 FORMAT (/12X, 1HI, 3X, 7HTI(I,1), 3X, 7HTI(I,2), 3X, 7HPI(I,1), MAIN0117
C 3X, 7HPI(I,2), 3X, 7HRI(I,1), 3X, 7HRI(I,2) / (113, 6F10,4)) MAIN0118
121 FORMAT (14X, 1HI, 4X, 4HE(I), 11X, 1HJ, 4X, 4HE(J), 11X, 1HK, 4X, MAIN0119
C 4HE(K) ) MAIN0120
120 FORMAT (I10, E10,0, I10, E10,0, I10, E10,0) MAIN0121
122 FORMAT (10X, I5, F10,4, 5X, I5, F10,4, 5X, I5, F10,4 ) MAIN0122
126 FORMAT (10X, 9X, 4HQ(I), 5X, 7HALFA(I), 3X, 4HR(I)) MAIN0123
128 FORMAT (I10, 3E10,0) MAIN0124
130 FORMAT (10X, I5, 3F10,4 ) MAIN0125
132 FORMAT (I10, 3E10,0) MAIN0126
134 FORMAT (10X, 5HSTL2=, I3, 2X, 3HK1= , F10,4, 3X, 6HFIREF= , F10,5, MAIN0127
C 3X, 3HHN=, F10,4 ) MAIN0128
137 FORMAT (14X, 1HI, 4X, 4HB(I), 11X, 1HJ, 4X, 4HB(J), 11X, 1HK, 4X, MAIN0129
C 4HB(K) ) MAIN0130
136 FORMAT ( I10, E10,0, I10, E10,0, I10, E10,0 ) MAIN0131
138 FORMAT (10X, I5, F10,4, 5X, I5, F10,4, 5X, I5, F10,4 ) MAIN0132
140 FORMAT (13I5 ) MAIN0133
142 FORMAT (10X, 3HIT=, I3, 2X, 5HNRIT=, I2, 2X, 5HKURV=, I2, 2X, MAIN0134
1 5HNTRY=, 9I4, 2X, 7HNSTANS=, I4 ) MAIN1134
144 FORMAT (6E10,0 / 3I10, F10,0) MAIN0135
146 FORMAT (10X, 5HEPS1=, E12,3, 3X, 5HEPS2=, E12,3, 3X, 6HEPS21=, MAIN0136
1 E12,3, 3X, 5HEPS3=, E12,3, 3X, 6HEPS31=, E12,3 MAIN0137
2, 3X, 6HEPS32=, E12,3 / 10X, 5HITE1= MAIN1137
3 I4, 3X, 5HITE2=, I4, 3X, 5HITE3=, I4, 3X, 5HTETA=, F7,2) MAIN0138
152 FORMAT (23X, 2HFI, 10X, 2HXE, 9X, 3HJOD ) MAIN0139
150 FORMAT (I10, 3E10,0) MAIN0140
154 FORMAT (10X, I5, 3X, 3F12,5 ) MAIN0141
401 FORMAT (2I5, 3F5,0, I5, 2F10,0, 2E10,0 ) MAIN
402 FORMAT (10X, 5HSTL5=, I3, 3X, 6HKSTYR=, I3, 3X, 5HPART=, F6,3, 3X, 5HPROD=, MAIN
1 F6,4, 3X, 5HDROD=, F6,4, 3X, 6HICONT=, I4, 3X, 5HABSO=, F7,4, 3X, 3HXD=, MAIN
2 F7,4/ 10X, 5HEPSX=, E12,4, 3X, 6HEPSDX=, E12,4 ) MAIN
1003 FORMAT (14X, 1HI, 4X, 4HBETA, 11X, 1HJ, 4X, 4HBETA, 11X, 1HK, 4X, MAIN 143
1 4HBETA ) MAIN 144
1004 FORMAT (I10, E10,0, I10, E10,0, I10, E10,0) MAIN 145
1005 FORMAT (10X, I5, F10,4, 5X, I5, F10,4, 5X, I5, F10,4) MAIN 146
END MAIN0142

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## SUBROUTINE TRY

TRY 0001

TRY IS A PRINT OUT SUBROUTINE

THE SUBROUTINE IS CALLED BY RK2, STAT, EFF0  
CALLS SUBROUTINE RITA IN ORDER TO PLOT THE SPATIAL DISTRIBUTION  
OF FLUX, XE, AND IODINE

TIDO = ACTUAL TIME                      LAMBDA = ROD POSITION  
FFI = FORM FACTOR                      SUM = MEAN FLUX  
PP = POWER                              RR = AMPLITUDE OF DISTURBANCE  
XNY = INVERTED VALUE OF THE EIGENVALUE YK1  
IC = XENON CONTROL ROD CENTRE  
ICU = UPPER BOUNDARY                  ICL = LOWER BOUNDARY OF THE ROD  
FI2 = FLUX NORMALIZED TO FI(MAX) = 1  
FI = FLUX IN PHYSICAL UNITS          XE0 = XENON CONCENTRATION  
JOD0 = IODINE CONCENTRAION          DELTFI = FLUX DEVIATION  
DXDT = TIME DERIVATIVE OF XE  
A1,A2,A3,X1,X2,X3 = FLUX AND XENON FOURIER COEFF,

COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),	CO	1
1 JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),	CO	2
2 TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA (50),RR,PP,	CO	3
3 N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI,	CO	4
4 TIDO, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),	CO	5
5 Q(50), E(51),R(50),DXDT(50),DIDT (50), HZ,S(150), T(30), W,	CO	6
6 AX(20), TETA	CO	7
COMMON NRIT, KURV, NSTANS,	CO	8
1 IC, ICU, ICL, PROD, DROD, PART , ABSO, STAB(50),KSTYR,	CO	9
2 STL1, STL2, STL3, STL4, STL5, STL6,	CO	10
3 NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,	CO	11
4 EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,	CO	12
5 DEDA(20),DBDA(19), XALF1 (20),MVOID	CO	13
COMMON XSS(50),XD, EPSX, EPSDX, ICONT	CO	14
INTEGER AX		
REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1	XECO	11
INTEGER STL1, STL2, STL3, STL4, STL5, STL6	XECO	12
DIMENSION DELTFI(50)	TRY	
SUM = 0	TRY	0003
FIMAX = 0	TRY	0004
	TRY	0005

THE MAX VALUE OF THE FLUX IS CALCULATED

1 DO 2 I2= 1,N	TRY	0007
SUM = SUM + FI(I2)	TRY	0008
FIMAX = AMAX1 (FIMAX, FI(I2))	TRY	0009
2 CONTINUE	TRY	0010
IF (SUM.EQ.0.) 6,7	TRY	
CONTINUE	TRY	

THE MEAN FLUX IS CALCULATED

SUM = SUM/ FLOAT (N+1 )	TRY	0014
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## EXEMPEL PÅ PRODUKTFUNKTIONSANALYS FÖR DAMMSUGARE

Behovsfunktion: Städa heltäckningsmatta

Huvudfunktion: Avlägsna partiklar

Delfunktioner  
nivå 1:

Frigöra partiklar

Transportera partiklar och luft

Separera partiklar och luft

Magasinera partiklar

Avleda luft

Åstadkomma förflyttning

Stödfunktioner:

Ge beröringsskydd

Ge låg bullernivå

Underlätta hantering

Reducera dammlukt

Ge attraktivitet

Se funktionsträd i figur 34.

FFI = FIMAX / SUM

TRY 0015

FFIX = 1.0/SUM

TRY 0016

XKAPPA ( OR YK1) IS INVERTED (SEE FLOW)

XNY = 1. / YK1

TRY 0018

THE MAXIMUM FLUX IS NORMALIZED TO 1.0

DO 5 I5 = 1, N

TRY 0020

DELTFI (I5) = (FI(I5) - FI1(I5)) \*FFIX

TRY

FI2 (I5) = FI(I5) /FIMAX

TRY 0021

5 CONTINUE

TRY 0022

GOTO 10

TRY 0023

6 FFI = 0

TRY 0024

XNY = 0

TRY 0025

DO 8 I8 = 1, N

TRY 0026

FI2(I8) = 0

TRY 0027

8 CONTINUE

TRY 0028

10 CONTINUE

TRY 0029

WRITE (6,3) TID0, LAMBDA, FFI, SUM, PP, RR, XNY

TRY 0030

STL5 = 1 THE XENON CONTROL ROD IS USED

IF (STL5, EQ, 1) 50, 51

TRY

WRITE (6,103) IC, ICU, ICL

TRY

CONTINUE

TRY

IF (TID0, NE, 0.0, AND, NTRY1, EQ, 2) 202, 201

WRITE (6,4) (I4, FI2(I4), FI(I4), XE0(I4), JOD0(I4), DELTFI(I4),

TRY 34

1 DXDT(I4), I4=1, N)

TRY

GOTO 210

WRITE (6,400) (DELTFI(I4), I4=1, N)

CONTINUE

THE XENON DEVIATION IS CALCULATED

W = 1.0 ASSIGNS THAT TRANSIENT SHALL BE CALCULATED

IF ( STL5, EQ, 1, AND, W, EQ, 1.0 ) 70, 72

TRY

N1

DO 71 I71 = 1, N

TRY

N2

DELTFI( I71 ) = XE0( I71 ) - XSS( I71 )

TRY

N3

WRITE( 6,75 ) ( DELTFI( II ), II = 1, N )

TRY

N4

CONTINUE

TRY

N6

FOURIER COEFFICIENTS ARE CALCULATED IF NTRY1 NOT = 2

IF (NTRY1, EQ, 2) 222, 220

CONTINUE

IF (STL2, EQ, 2) 16, 60

TRY

CONTINUE

TRY

DO 15 I15 = 1, N

TRY 137

FI2 (I15) = FI(I15) /SUM

TRY 237

GOTO 18

TRY 337

DO 17 I17 = 1, N

TRY 437

FI2(I17) = FI(I17) / FIREF

TRY 537

CONTINUE

A1 = 0.

TRY 44



	X1 = 0,	TRY	*44
	A2 = 0,	TRY	45
	X2 = 0,	TRY	*45
	A3 = 0,	TRY	46
	X3 = 0,	TRY	*46
	DO 25 I25 = 1, N	TRY	47
	A1 = A1 * FI2(I25) * S(I25)	TRY	48
	X1 = X1 * XE0 (I25) * S(I25)	TRY	*48
	A2 = A2 * FI2 (I25) * S(2*I25)	TRY	49
	X2 = X2 * XE0 (I25) * S(2*I25)	TRY	*49
	A3 = A3 * FI2 (I25) * S(3*I25)	TRY	50
	X3 = X3 * XE0 (I25) * S(3* I25)	TRY	*50
25	CONTINUE	TRY	51
	GOTO (26,36), STL2		
26	WRITE (6,30) A1, A2, A3		
	WRITE (6,40) X1, X2, X3		
	RETURN	TRY	1055
36	WRITE (6,35) A1, A2, A3		
	WRITE (6,40) X1, X2, X3		
	RETURN	TRY	1355
30	FORMAT (//10X, 29HFOURIERKOEFF FI (FIMED = 1,0) //10X, 3HA1=,	TRY	56
	1E15,5, 5X, 3HA2=, E15,5, 5X, 3HA3=, E15,5 )	TRY	57
35	FORMAT (//10X, 29HFOURIERKOEFF FI (FIREF = 1,0) //10X, 3HA1=,	TRY	58
	C E15,5, 5X, 3HA2=, E15,5, 5X, 3HA3=, E15,5 )	TRY	59
40	FORMAT (//10X, 18HFOURIERKOEFF XENON // 10X, 3HX1=, E15,5,	TRY	159
	1 5X, 3HX2=, E15,5, 5X, 3HX3=, E15,5 )	TRY	259
	3 FORMAT ( /// 10X, 4HTIME , F7,3, 3X, 6HLAMBDA, F12,5, 4X, 3HPSI,	TRY	0031
	C F8,4, 4X, 5HFIMED, F8,3, 4X, 5HPOWER , F8,3, 4X, 1HR, F8,3 ,	TRY	0032
	C 4X, 2HNY , F8,4 )	TRY	0033
103	FORMAT (10X, 3HROD, 3X, 3HIC=, I4, 2X, 4HICU=, I4, 2X, 4HICL=,	TRY	3301
	1 I4 )	TRY	3302
4	FORMAT (// 12X, 1HI, 2X, 8HFI(NORM), 9X, 7HFI(ABS), 13X, 2HXE,	TRY	0035
	1 13X, 3HJOD, 11X, 13HDELTA FI (NORM), 6X, 4HDXDT // (10X, 13, F10,6, E20,6,	TRY	
	2 2F15,6, F20,6, F15,6))	TRY	
75	FORMAT ( 10X, 6HDELTA X, 10F10,6 / (16X, 10F10,6 ) )	TRY	N5
400	FORMAT (/10X, 6HDELTA FI, 10F10,6 / (16X, 10F10,6))		
222	RETURN		
	END	TRY	61



20	WRITE (6,25) TID0	STAT0020
	CALL RITA (FI, XE0, JOD0, KURV, N)	STAT0022
	GOTO 16	
12	FORMAT (10X,20A4)	STAT
10	FORMAT (10X, 16HSTATIONART FLODE )	STAT0016
25	FORMAT (10X, 19HKURVA RITAD, TIME= , F8,3 )	STAT0021
	END	STAT0024

## SUBROUTINE STEADY

C	THE SUBROUTINE IS CALLED BY STAT	
C	CALCULATES THE EQUILIBRIUM VALUES AFTER A STEP DISTURBANCE IN REACTIVITY	
C	CALLS RAND IN ORDER TO CALCULATE EQUILIBRIUM FLUX	
C	THE STEADY STATE VALUES ARE PRINTED OUT BY SUBROUTINE TRY	
C*		
C*		
	COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),	CO 1
1	JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),	CO 2
2	TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA (50),RR,PP,	CO 3
3	N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI,	CO 4
4	TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),	CO 5
5	O(50), E(51),R(50),DXDT(50),DIDT (50), HZ,S(150), T(30), W,	CO 6
6	AX(20), TETA	CO 7
	COMMON NRIT, KURV, NSTANS,	CO 8
1	IC, ICU, ICL, PROD, DROD, PART , ABSO, STAB(50),KSTYR,	CO 9
2	STL1, STL2, STL3, STL4, STL5, STL6,	CO 10
3	NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,	CO 11
4	EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,	CO 12
5	DEDA(20),DBDA(19), XALF1 (20),MVOID	CO 13
	COMMON XSS(50),XD, EPSX, EPSDX, ICONT	CO 14
	INTEGER AX	
	REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1	XECO 11
	INTEGER STL1, STL2, STL3, STL4, STL5, STL6	XECO 12
	W= 1,0	
	STL4 = 1	
	ITID= M	STEAD
	TID0 = TI(ITID,1)	STEAD
	TID = TID0	STEAD
	CALL RAND (LAMBDA)	STEAD
C	REDEFINITION OF XE AND IODINE BEFORE PRINTING	
	DO 2 I2 = 1,N	STEAD
	XSS(I2) = XE0(I2)	
	XE0(I2) = XE(I2)	
	DIDT(I2) = JOD0(I2)	
2	JOD0(I2) = JOD(I2)	
	WRITE (6, 100)	STEAD
100	FORMAT (10X, 12HSTEADY STATE )	STEAD
	CALL TRY	STEAD
	DO 4 I4 = 1,N	
	XE0 (I4) = XSS(I4)	
	JOD0(I4) = DIDT(I4)	
	DIDT(I4) = 0,0	
4	XSS(I4) = XE(I4)	
	RETURN	
	END	

## SUBROUTINE TRAN

REF. GUSTAF OLSSON, DIGITAL SIMULATION OF SPATIAL XENON OSCILLATIONS  
 DIV OF AUTOMATIC CONTROL LUND REPORT 6911

THE SUBROUTINE IS CALLED BY MAIN

TRAN IS THE MAIN SUBROUTINE FOR THE TRANSIENT CALCULATIONS  
 DEFINES THE ACTUAL TIME FOR THE INTEGRATION  
 IF POWER = 0 IT CALLS SUBROUTINE EFF0  
 IF POWER NOT EQUAL 0 IT CALLS RK2  
 W = 1.0 ASSIGNS THAT TRANSIENT SHALL BE CALCULATED

	COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),	CO	1
1	JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),	CO	2
2	TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA(50),RR,PP,	CO	3
3	N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI,	CO	4
4	TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),	CO	5
5	Q(50), E(51),R(50),DXDT(50),DIDT(50), HZ,S(150), T(30), W,	CO	6
6	AX(20), TETA	CO	7
	COMMON NRIT, KURV, NSTANS,	CO	8
1	IC, ICU, ICL, PROD, DR0D, PART, ABS0, STAB(50),KSTYR,	CO	9
2	STL1, STL2, STL3, STL4, STL5, STL6,	CO	10
3	NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,	CO	11
4	EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,	CO	12
5	DEDA(20),DBDA(19), XALF1(20),MVOID	CO	13
	COMMON XSS(50),XD, EPSX, EPSDX, ICONT	CO	14
	INTEGER AX		
	REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1	XECO	11
	INTEGER STL1, STL2, STL3, STL4, STL5, STL6	XECO	12
	STL4 = 2	TRAN0002	
	W = 1.	TRAN0003	
	DO 1 I1 = 2, M	TRAN0004	
	ITID = I1	TRAN0005	
	TID0 = TI(ITID, 1)	TRAN0006	
	IF (PI(ITID,1),EQ,0.,AND, PI(ITID,2),EQ,0.) 5, 3	STAT	
3	CALL RK2	STAT	
	IF(ABS(TMAX - TID0 ),LE, 1,E-4 ) 4,1	STAT	
5	CALL EFF0	TRAN0011	
1	CONTINUE	TRAN0012	
4	RETURN	TRAN0013	
	END	TRAN0014	



## SUBROUTINE RK2

REF. GUSTAF OLSSON. DIGITAL SIMULATION OF SPATIAL XENON OSCILLATIONS  
 DIV OF AUTOMATIC CONTROL LUND REPORT 6911  
 APPENDIX 2

THE SUBROUTINE IS CALLED BY TRAN

RK2 DETERMINES TIME STEP AND MAKES RICHARDSON EXTRAPOLATION (USING BACK)  
 THE ROUTINE DETERMINES WHETHER TO PRINT OR NOT BY CALLING TRY  
 ITITIATES RUNGE KUTTA TIME STEP BY CALLING RK1

RK2 CALLS THE SUBROUTINES RK1, DERI, HYDRO, VOID, TRY, BACK, RITA

```

COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),          CO    1
1 JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),    CO    2
2 TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA(50),RR,PP, CO    3
3 N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI, CO    4
4 TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),  CO    5
5 Q(50), E(51),R(50),DXDT(50),DIDT(50), HZ,S(150), T(30), W,    CO    6
6 AX(20), TETA                                                    CO    7
COMMON NRIT, KURV, NSTANS,                                        CO    8
1 IC, ICU, ICL, PROD, DROD, PART , ABS0, STAB(50),KSTYR,        CO    9
2 STL1, STL2, STL3, STL4, STL5, STL6,                            CO   10
3 NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,      CO   11
4 EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,            CO   12
5 DEDA(20),DBDA(19), XALF1(20),MVOID                            CO   13
COMMON XSS(50),XD, EPSX, EPSDX, ICONT                            CO   14
INTEGER AX
REAL JOD, JOD0,          LAMDAX, LAMDAI, LAMBDA, K1             XECO  11
INTEGER STL1, STL2, STL3, STL4, STL5, STL6                     XECO  12
DIMENSION DXE(50), DJOD(50)                                    RK2
NR0 = NRIT                                                       RK2
CALL DERI                                                         RK2 0003
CALL TRY                                                           RK2 0004
HT1=HT
IF (MVOID,EQ,1) 60, 61                                           RK2
60 CALL HYDRO                                                       RK2
61 CONTINUE                                                       RK2
XXX = AMIN1 (TI (ITID , 2) , TMAX )                               RK2 0005
1 HT = AMIN1 (HT, (XXX= TID0 ))                                    RK2 0006
IF (HT,EQ,0,) 1000, 2                                             RK2
2 X = LAMBDA                                                       RK2
XYZ = RK1(X)                                                       RK2
C                                                                    RK2 0008
C XYZ IS A TRUNCATION ERROR IN RUNGE=KUTTA
C                                                                    RK2 0010
IF (NTRY7,EQ,1) 4,6                                               RK2
4 WRITE (6,100) XYZ                                               RK2
6 DO 10 I10 =1,N                                                  RK2
    DXE (I10) = XE0 (I10)                                         RK2 0013
10    DJOD(I10) = JOD0(I10)                                       RK2 0014
CALL TRY                                                           RK2 0015
IF (MVOID,EQ,1) 62, 63                                           RK2

```

62	CALL HYDRO	RK2
63	CONTINUE	RK2
C		RK2 0016
C	THE SAME INTEGRATION AS BEFORE BUT WITH HALF THE TIME STEP LENGTH	
C		RK2 0018
	CALL BACK	RK2 0019
	HT = 0,5 * HT	RK2 0020
	CALL DERI	RK2 0021
	DO 12 I12 = 1,2	RK2 0022
	XYZ = RK1 (X)	RK2 0023
12	CONTINUE	RK2 0024
	IF (NTRY7,EQ,1) 13,15	RK2
13	WRITE (6,100) XYZ	RK2
C		RK2 0026
C	THE DIFFERENCE BETWEEN THE CALCULATIONS WITH DIFFERENT STEP LENGTHS	
C		RK2 0028
15	DO 14 I14 = 1,N	RK2
	DXE (I14) = (XE0(I14) -DXE(I14))* 1,06667	RK2
	DJOD(I14) = (JOD0(I14)-DJOD (I14))*1,06667	RK2
14	CONTINUE	RK2 0032
	IF (NTRY8,EQ,1) 16,17	RK2
16	WRITE (6,110) (DXE(I) ,I=1,N ), (DJOD(K), K=1,N )	RK2
C		RK2 0035
C	INTEGRATION WITH THE FIRST STEP LENGTH CONTINUES	
C		RK2 0037
17	HT = HT + HT	RK2
	IF (ABS(XXX - TID0),LE,1,E-4) 30,18	RK2
18	DO 22 I22 = 1, IT	RK2 0040
	NRIT = NRIT - 1	RK2 140
	HT = AMIN1 ( HT, (XXX - TID0))	RK2 0041
	XYZ = RK1 (X)	RK2 0042
	IF (NTRY7,EQ,1) 26,27	RK2
26	WRITE (6,100) XYZ	RK2
27	DO 24 K=1,N	RK2
	XE0 (K) = XE0 (K) + DXE (K)	RK2 0045
24	JOD0(K) = JOD0 (K) + DJOD (K)	RK2 0046
	IF (ABS(XXX - TID0),LE,1,E-4) 30, 22	RK2
22	CONTINUE	RK2 0048
	CALL TRY	RK2 0049
	IF (MVOID,EQ,1) 64, 65	RK2
64	CALL HYDRO	RK2
65	CONTINUE	RK2
	IF (NRIT,EQ,0) 50, 18	RK2
30	CALL TRY	RK2 0051
	IF (MVOID,EQ,1) 66, 67	RK2
66	CALL HYDRO	RK2
67	CONTINUE	RK2
100	FORMAT ( 5X, 3HRK2 / 15X, E12,4 )	RK2 0052
110	FORMAT (10X, 3HRK2, 5X, 14HTRUNKATIONSFEL / (10X, 10F11,6))	RK2 0053
	HT=HT1	
1000	RETURN	RK2 54
50	WRITE (6, 55) TID0	RK2 0055
55	FORMAT (10X, 19HKURVA RITAD, TIME= , F8,3)	RK2 0056
	CALL RITA (PI, XE0, JOD0, KURV, N)	RK2 0057
	NRIT = NRC	RK2 0058
	GOTO 18	RK2 0059
	END	RK2 0060





	DO 5 I5 = 1, N	RK1 0025
	XK2(I5) = HT*0,33333333*DXDT(I5)	RK1 0026
	NPI = N+I5	RK1 0027
	XK2(NPI) = HT*0,33333333*DXDT(I5)	RK1 0028
	XE(I5) = XE0(I5) + 0,5*XK1(I5) + 0,5*XK2(I5)	RK1 0029
	JOD(I5) = JOD0(I5) + 0,5*XK1(NPI) + 0,5*XK2(NPI)	RK1 0030
5	CONTINUE	RK1 0031
	IF (NTRY2,EO,1) 40,41	RK1
40	WRITE(6, 100) TID, (XE(I100), JOD(I100), I100= 1,N )	RK1
41	CONTINUE	RK1
13	CALL DER1	RK1 0034
	DO 6 I6 = 1, N	RK1 0035
	XK2(I6) = HT*0,33333333*DXDT(I6)	RK1 0036
	NPI = N + I6	RK1 0037
	XK2(NPI) = HT*0,33333333*DXDT(I6)	RK1 0038
	XEPS(I6) = XEPS(I6) + 4,5*XK2(I6)	RK1 0039
	XEPS(NPI) = XEPS(NPI) + 4,5*XK2(NPI)	RK1 0040
	XE(I6) = XE0(I6) + 0,375*XK1(I6) + 1,125*XK2(I6)	RK1 0041
	JOD(I6) = JOD0(I6) + 0,375*XK1(NPI) + 1,125*XK2(NPI)	RK1 0042
6	CONTINUE	RK1 0043
14	TID = TID0 + 0,5*HT	RK1 0044
	IF (NTRY2,EO,1) 50,51	RK1
50	WRITE(6, 100) TID, (XE(I100), JOD(I100), I100= 1,N )	RK1
51	CONTINUE	RK1
	CALL DER1	RK1 0047
	DO 7 I7 = 1, N	RK1 0048
	XK4(I7) = HT*0,33333333*DXDT(I7)	RK1 0049
	NPI = N + I7	RK1 0050
	XK4(NPI) = HT*0,33333333*DXDT(I7)	RK1 0051
	XK(I7) = XK(I7) + 4,*XK4(I7)	RK1 0052
	XK(NPI) = XK(NPI) + 4,*XK4(NPI)	RK1 0053
	XEPS(I7) = XEPS(I7) + 4,*XK4(I7)	RK1 0054
	XEPS(NPI) = XEPS(NPI) + 4,*XK4(NPI)	RK1 0055
	XE(I7) = XE0(I7) + 1,5*XK1(I7) + 4,5*XK2(I7) + 6,*XK4(I7)	RK1 0056
	JOD(I7) = JOD0(I7) + 1,5*XK1(NPI) + 4,5*XK2(NPI) + 6,*XK4(NPI)	RK1 0057
7	CONTINUE	RK1 0058
15	TID = TID0 + HT	RK1 0059
	CALL DER1	RK1 0060
	DO 8 I8 = 1, N	RK1 0061
	XK4(I8) = HT*0,33333333*DXDT(I8)	RK1 0062
	NPI = N + I8	RK1 0063
	XK4(NPI) = HT*0,33333333*DXDT(I8)	RK1 0064
	XK(I8) = XK(I8) + XK4(I8)	RK1 0065
	XK(NPI) = XK(NPI) + XK4(NPI)	RK1 0066
	XEPS(I8) = XEPS(I8) + 0,5*XK4(I8)	RK1 0067
	XEPS(NPI) = XEPS(NPI) + 0,5*XK4(NPI)	RK1 0068
	XE0(I8) = XE0(I8) + 0,5*XK(I8)	RK1 0069
	JOD0(I8) = JOD0(I8) + 0,5*XK(NPI)	RK1 0070
8	CONTINUE	RK1 0071
	TID0 = TID	RK1 0072
	EPSMAX = 0	RK1 0073
	N2 = 2*N	RK1 0074
	DO 9 I9 = 1, N2	RK1 0075
	EPSMAX = AMAX1(EPSMAX, ABS( XEPS(I9)))	RK1 0076
9	CONTINUE	RK1 0077
	RK1 = 0,2*EPSMAX	RK1 0078

	TID = TID0	RK1 0079
	DO 16 I16 = 1, N	RK1 0080
	XE(I16) = XE0(I16)	RK1 0081
	J <sup>0</sup> D(I16) = J <sup>0</sup> D0(I16)	RK1 0082
16	CONTINUE	RK1 0083
	IF (NTRY2,E0,1) 60,61	RK1
60	WRITE(6, 100) TID, (XE(I100), JOD(I100), I100= 1,N )	RK1
61	CONTINUE	RK1
	CALL DERI	RK1 0086
	RETURN	RK1 0087
	END	RK1 0088

## SUBROUTINE DERI

DERI0001

REF. GUSTAF OLSSON, DIGITAL SIMULATION OF SPATIAL XENON OSCILLATIONS  
 DIV OF AUTOMATIC CONTROL LUND REPORT 6911  
 APPENDIX 2

THE SUBROUTINE IS CALLED BY RK2 AND RK1

DERI CALCULATES THE RIGHT HAND SIDE OF THE DIFFERENTIAL EQUATIONS (2.8=9)

DERI CALLS SUBROUTINE RAND

	COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),	CO	1
1	JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),	CO	2
2	TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA(50),RR,PP,	CO	3
3	N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI,	CO	4
4	TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),	CO	5
5	O(50), E(51),R(50),DXDT(50),DIDT(50), HZ,S(150), T(30), W,	CO	6
6	AX(20), TETA	CO	7
	COMMON NRIT, KURV, NSTANS,	CO	8
1	IC, ICU, ICL, PROD, DROD, PART, ABSO, STAB(50),KSTYR,	CO	9
2	STL1, STL2, STL3, STL4, STL5, STL6,	CO	10
3	NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,	CO	11
4	EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,	CO	12
5	DEDA(20),DBDA(19), XALF1(20),MVOID	CO	13
	COMMON XSS(50),XD, EPSX, EPSDX, ICONT	CO	14
	INTEGER AX		
	REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1	XECO	11
	INTEGER STL1, STL2, STL3, STL4, STL5, STL6	XECO	12
	GAMMAX = 1, = GAMMAI	DERI0002	
	CALL RAND (LAMBDA)	DERI0003	
	DO 1 I1= 1, N	DERI0004	
	DXDT(I1) = SIGMAX *GAMMAX* FI(I1) + LAMDAI * JOD(I1) -	DERI0005	
C	LAMDAX * XE(I1) - SIGMAX * FI(I1) * XE(I1)	DERI0006	
	DIDT(I1) = GAMMAI * SIGMAX * FI(I1) - LAMDAI * JOD(I1)	DERI0007	
1	CONTINUE	DERI0008	
	IF (NTRY2,E0,1) 10,11	DERI	
10	WRITE(6,100) (DXDT(I2),DIDT(I2), I2=1,N)	DERI	
11	RETURN	DERI	
100	FORMAT(10X, 4HDERI / (10X, 6F10,6))	DERI0010	
	END	DERI0012	

## SUBROUTINE BACK

THE SUBROUTINE IS CALLED BY RK2

BACK ROUTINE BACKS THE PROCESS ONE TIME STEP IN ORDER TO MAKE  
A RICHARDSON EXTRAPOLATION,

```

COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),      CO      1
1 JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100), CO      2
2 TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA (50),RR,PP, CO      3
3 N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI, CO      4
4 TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50), CO      5
5 Q(50), E(51),R(50),DXDT(50),DIDT (50), HZ,S(150), T(30), W,    CO      6
6 AX(20), TETA                                                    CO      7
COMMON NRIT, KURV, NSTANS,                                        CO      8
1 IC, ICU, ICL, PROD, DROD, PART , ABS0, STAB(50),KSTYR,        CO      9
2 STL1, STL2, STL3, STL4, STL5, STL6,                            CO     10
3 NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,      CO     11
4 EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,            CO     12
5 DEDA(20),DBDA(19), XALF1 (20),MVOID                          CO     13
COMMON XSS(50),XD, EPSX, EPSDX, ICONT                          CO     14
INTEGER AX
REAL JOD, JOD0,          LAMDAX, LAMDAI, LAMBDA, K1             XECO   11
INTEGER STL1, STL2, STL3, STL4, STL5, STL6                     XECO   12
1 TID0 = TID0 - HT                                             BACK0002
DO 2 I2 = 1, N                                                 BACK0003
XE0(I2) = XE0(I2) - XK(I2)* 0,5                               BACK0004
NPI = N + I2                                                  BACK0005
JOD0(I2) = JOD0(I2) - XK(NPI) * 0,5                          BACK0006
2 CONTINUE                                                     BACK0007
RETURN                                                         BACK0008
END                                                            BACK0009

```

## SUBROUTINE XSIN(NX)

THE SUBROUTINE IS CALLED BY MAIN

THE SUBROUTINE CALCULATES SINE FUNCTIONS FOR THE FOURIER COEFFICIENT  
CALCULATION IN SUBROUTINE TRY

```

COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),      CO      1
1 JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100), CO      2
2 TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA (50),RR,PP, CO      3
3 N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI, CO      4
4 TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50), CO      5
5 Q(50), E(51),R(50),DXDT(50),DIDT (50), HZ,S(150), T(30), W,    CO      6
6 AX(20), TETA                                                    CO      7
COMMON NRIT, KURV, NSTANS,                                        CO      8
1 IC, ICU, ICL, PROD, DROD, PART , ABS0, STAB(50),KSTYR,        CO      9
2 STL1, STL2, STL3, STL4, STL5, STL6,                            CO     10
3 NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,      CO     11
4 EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,            CO     12
5 DEDA(20),DBDA(19), XALF1 (20),MVOID                          CO     13
COMMON XSS(50),XD, EPSX, EPSDX, ICONT                          CO     14
INTEGER AX
REAL JOD, JOD0,          LAMDAX, LAMDAI, LAMBDA, K1             XECO   11
INTEGER STL1, STL2, STL3, STL4, STL5, STL6                     XECO   12
N3 = 3 * N                                                      XSIN   1
V11= 1./ FLOAT (N+1)                                           XSIN   2
V = 3,1416 * V11                                               XSIN   3
V11= 2.* V11                                                   XSIN   4
DO 20 I20 = 1, N3
S(I20) = SIN (FLOAT(I20) * V ) * V11                          XSIN   6
RETURN
END

```

## SUBROUTINE RAND (X)

REF. GUSTAF OLSSON, DIGITAL SIMULATION OF SPATIAL XENON OSCILLATIONS  
 DIV OF AUTOMATIC CONTROL LUND REPORT 6911  
 APPENDIX 2 POINT 3

THE SUBROUTINE IS CALLED BY STAT, STEADY AND DERI

CALLS RRPP TO GET MOMENT VALUES OF POWER AND DISTURBANCE  
 CALLS ROD TO GET ACTUAL REACTIVITY VALUES FOR THE XENON CONTROL ROD  
 CALLS FLOW FOR CALCULATION OF THE ACTUAL VALUES OF FI, XE AND IODINE

X = GUESSED VALUE OF ROD POSITION, THE VALUE IS READ IN AS DATA FIRST TIME

	COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),	CO	1
1	JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),	CO	2
2	T1(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA(50),RR,PP,	CO	3
3	N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI,	CO	4
4	TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),	CO	5
5	Q(50), E(51),R(50),DXDT(50),DIDT(50), HZ,S(150), T(30), W,	CO	6
6	AX(20), TETA	CO	7
	COMMON NRIT, KURV, NSTANS,	CO	8
1	IC, ICU, ICL, PROD, DROD, PART, ABSO, STAB(50),KSTYR,	CO	9
2	STL1, STL2, STL3, STL4, STL5, STL6,	CO	10
3	NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,	CO	11
4	EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,	CO	12
5	DEDA(20),DBDA(19), XALF1(20),MVOID	CO	13
	COMMON XSS(50),XD, EPSX, EPSDX, ICONT	CO	14
	INTEGER AX		
	REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1	XECO	11
	INTEGER STL1, STL2, STL3, STL4, STL5, STL6	XECO	12
	CALL RRPP	RAND0002	
	IF (STL5,NE,1) 11,10	RAND	
10	CALL ROD	RAND	209
11	CALL FLOW(X)	RAND	3
	IF (STL4,EQ,2) 20,14	RAND	
14	DO 2 I2 = 1,N	RAND	
	JOD(I2) = SIGMAX * FI(I2) * GAMMAI / LAMDAI	RAND0006	
2	CONTINUE	RAND0008	
20	RETURN	RAND	
	END	RAND0010	

## SUBROUTINE FLOW (X)

REF. GUSTAF OLSSON, DIGITAL SIMULATION OF SPATIAL XENON OSCILLATIONS  
 DIV OF AUTOMATIC CONTROL LUND REPORT 6911  
 APPENDIX 2 POINT 1,9 = 12

THE SUBROUTINE IS CALLED BY RAND

FLOW CALLS THE SUBROUTINE MATR AND FUNCTION EGENV

LA0 = THE OLD VALUE OF THE ROD POSITION

LA1 = A NEW VALUE OF THE ROD POSITION

BY CALLING MATR AND EGENV THE ITERATION OF ROD POSITION PROCEEDS  
 UNTIL A VALUE OF THE ROD POSITION CAUSES THE EIGENVALUE FOR THE  
 FLUX TO BE CLOSE TO 1

YK0 = OLD EIGENVALUE

YK1 = NEW EIGENVALUE

	COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),	CO	1
1	JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),	CO	2
2	TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA (50),RR,PP,	CO	3
3	N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI,	CO	4
4	TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),	CO	5
5	Q(50), E(51),R(50),DXDT(50),DIDT (50), HZ,S(150), T(30), W,	CO	6
6	AX(20), TETA	CO	7
	COMMON NRIT, KURV, NSTANS,	CO	8
1	IC, ICU, ICL, PROD, DROD, PART , ABS0, STAB(50),KSTYR,	CO	9
2	STL1, STL2, STL3, STL4, STL5, STL6,	CO	10
3	NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,	CO	11
4	EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,	CO	12
5	DEDA(20),DBDA(19), XALF1 (20),MVOID	CO	13
	COMMON XSS(50),XD, EPSX, EPSDX, ICONT	CO	14
	INTEGER AX		
	REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1	XECO	11
	INTEGER STL1, STL2, STL3, STL4, STL5, STL6	XECO	12
	REAL LA0, LA1, LA2	FLOW0002	
	LA0 = X	FLOW0003	
	IF (STL4,EQ,1) 2, 3	FLOW	
3	CONTINUE	FLOW	
	CALL MATR (LA0)	FLOW0004	
2	YK0 = EGENV (LA0)	FLOW0005	
	IF (ABS(YK0 - 1, ),LE, EPS1) 20,6	FLOW	
6	IF (YK0,LE,0,8,OR,YK0,GE,1,2) 10, 8	FLOW	
8	CONTINUE	FLOW	
	LA1 = LA0 + 0,5 * (YK0 - 1,)	FLOW0008	
	GOTO 15	FLOW0009	
		FLOW0010	
	LAMBDA IS DECREASED OR INCREASED WITH 0,1		
		FLOW0012	
10	LA1 = SIGN (0,1, YK0 - 1, ) + LA0	FLOW0013	
15	DO 1 I1 = 1, ITE1	FLOW0014	
	J1 = I1	FLOW0015	

```

IF(STL4, EQ, 1) 4, 5
5 CALL MATR (LA1)
4 YK1 = EGENV (LA1)
IF (ABS(YK1 = 1, ), LE, EPS1) 22, 9
9 CONTINUE
XYZ = (1, = YK1) / (YK1 = YK0)
IF (ABS(XYZ), GE, 2, ) 12, 16
16 CONTINUE
C
C EXTRAPOLATION TO KAPPA (YK1) = 1.0
C
C LA2 = LA1 + XYZ * (LA1 - LA0 )
C GOTO 14
C
C THE EXTRAPOLATION IS MAXIMIZED TO THE DOUBLE DISTANCE BETWEEN THE POINTS
C
12 LA2 = LA1 + SIGN (2, 0 , XYZ) * (LA1 - LA0) .
14 IF (NTRY2, EQ, 1) 30, 31
30 WRITE(6, 100) I1, LA2, LA1, LA0, YK1, YK0
31 CONTINUE
YK0 = YK1
LA0 = LA1
LA1 = LA2
1 CONTINUE
WRITE (6, 102)
STOP
20 LA1 = LA0
22 LAMBDA = LA1
IF (NTRY9, EQ, 1) 34, 35
34 WRITE (6, 110) J1
35 CONTINUE
C
C AFTER THE ROD POSITION LAMBDA IS FOUND THE VALUE OF EPS2 AND EPS3 IS MADE
C SMALLER IN ORDER TO GET A BETTER ACCURACY
C
EPS03 = EPS3
EPS02 = EPS2
EPS3 = EPS31
EPS2 = EPS21
YK1 = EGENV (LA1)
EPS3 = EPS03
EPS2 = EPS02
RETURN
100 FORMAT (10X, 6H(FLOW), 2X, I3, 2X, 3HLA2, F12,4, 3HLA1, F12,4,
C 3HLA0, F12,4, 3HYK1, F12,4, 3HYK0, F12,4)
102 FORMAT ( 10X, 32HNY KONVERGERAR EJ MOT 1 I FLOW )
110 FORMAT (/ 8X, I4, 1X, 18HITERATIONER I FLOW )
END

```

FLOW  
FLOW  
FLOW0017  
FLOW  
FLOW  
FLOW0019  
FLOW  
FLOW  
FLOW0021  
  
FLOW0023  
FLOW0024  
FLOW0025  
FLOW0026  
  
FLOW0028  
FLOW0029  
FLOW  
FLOW  
FLOW  
FLOW0033  
FLOW0034  
FLOW0035  
FLOW0036  
FLOW0037  
FLOW  
FLOW0040  
FLOW0041  
FLOW  
FLOW  
FLOW  
FLOW0044  
  
FLOW0046  
FLOW0047  
FLOW0048  
FLOW0049  
FLOW0050  
FLOW0051  
FLOW0052  
FLOW0053  
  
FLOW0031  
FLOW0032  
FLOW0038  
FLOW0043  
FLOW0055







## FUNCTION EGENV (X)

REF. GUSTAF OLSSON, DIGITAL SIMULATION OF SPATIAL XENON OSCILLATIONS  
 DIV OF AUTOMATIC CONTROL LUND REPORT 6911  
 APPENDIX 2 POINT 1, 6

THE FUNCTION IS CALLED BY FLOW  
 EGENV CALCULATES THE INITIAL GUESS OF FLUX  
 IF FLUX IN THE FIRST SPACE POINT IS ZERO (WHICH IS THE CASE WHEN  
 THE CALCULATION STARTS FROM EQUILIBRIUM) A SINE DISTRIBUTION IS ASSUMED

IF FLUX IS NOT ZERO THE LAST FLUX DISTRIBUTION IS TAKEN AS THE FIRST  
 ITERATION VALUE  
 SUBROUTINE XNORM IS CALLED IN ORDER TO NORMALIZE TO THE PRESCRIBED POWER

MATR IS CALLED TO INSERT THE VALUES IN THE BUCKLING

FUNCTION XKAPPA IS CALLED TO CALCULATE THE EIGENVALUE FOR THE FLUX  
 FOR THE PRESENT ITERATION OF THE BUCKLING

THE SUBROUTINE CALLS FUNCTION TEST  
 IN TEST IS MADE ITERATION ON FLUX IN THE BUCKLING TERM  
 THE ITERATION IN THE FLUX CONTINUES UNTIL THE FLUX IN THE BUCKLING  
 DIFFERS FROM THE FLUX PREVIOUSLY CALCULATED BY EPS2

```

COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),          CO      1
1 JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),    CO      2
2 TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA(50),RR,PP, CO      3
3 N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI, CO      4
4 TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50), CO      5
5 Q(50), E(51),R(50),DXDT(50),DIDT(50), HZ,S(150), T(30), W,    CO      6
6 AX(20), TETA                                                    CO      7
COMMON NRIT, KURV, NSTANS,                                        CO      8
1 IC, ICU, ICL, PROD, DROD, PART , ABS0, STAB(50),KSTYR,        CO      9
2 STL1, STL2, STL3, STL4, STL5, STL6,                            CO     10
3 NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,       CO     11
4 EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,             CO     12
5 DEDA(20),DBDA(19), XALF1(20),MVOID                            CO     13
COMMON XSS(50),XD, EPSX, EPSDX, ICONT                            CO     14
INTEGER AX
REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1                      XECO    11
INTEGER STL1, STL2, STL3, STL4, STL5, STL6                      XECO    12
IF (FI(1),GT,0.) I2, 1
CONTINUE
DO 2 I2 = 1,N
FI (I2) = SIN ((3,1416 * FLOAT(I2))/FLOAT (N+1))
2 CONTINUE
12 CONTINUE
CALL XNORM (FI ,FI )
DO 4 I4= 1,N
FI2 (I4) = FI (I4)
4 CONTINUE
SIG = SIGMAX / LAMDAX

```

	DO 10 I10 = 1, ITE2	EGEN0011
	IF (STL4,EQ,2) 20,21	EGEN
21	CONTINUE	EGEN
	DO 6 I6 = 1, N	EGEN 211
	FIX = SIG * FI(I6)	EGEN 311
6	XE(I6) = FIX / (FIX+1.)	EGEN 411
	CALL MATR(X)	EGEN 511
20	J1 = I10	EGEN 12
	ZK1 = XKAPPA (FI2, X)	EGEN0013
	IF (TEST(FI,FI2),LE,EPS2) 3, 30	EGEN
30	IF (NTRY7,EQ,1) 31, 10	EGEN
31	WRITE (6,104) I10, (FI2 (I2), I2=1,N)	EGEN
10	CONTINUE	EGEN0019
	WRITE (6, 102)	EGEN0020
	STOP	EGEN
3	CONTINUE	EGEN0023
	IF(NTRY9,EQ,1) 34,35	EGEN
34	WRITE (6,110) J1	EGEN
35	IF (NTRY2,EQ,1) 36,37	EGEN
36	WRITE (6,100) (FI(I2), I2=1,N)	EGEN
37	CONTINUE	EGEN
	EGENV = ZK1	EGEN0033
	RETURN	EGEN0034
100	FORMAT (10X, 2HF1 / (10X, 10F10,6))	EGEN0032
102	FORMAT (10X, 32HINGEN KONVERGENS AV FI I EGENV )	EGEN0021
104	FORMAT (10X, 9HITERATION, I3, 2X, 13HAV FI (EGENV) /	EGEN0017
C	(10X, 10F10,6))	EGEN0018
110	FORMAT ( / 8X, I4, 1X, 10HITER EGENV )	EGEN0030
	END	EGEN0035





FUNCTION XNORM( YYY, ZZZ )

REF, GUSTAF OLSSON, DIGITAL SIMULATION OF SPATIAL XENON OSCILLATIONS  
DIV OF AUTOMATIC CONTROL LUND REPORT 6911  
APPENDIX 2 POINT 2, 5

THE FUNCTION IS CALLED BY THE FUNCTIONS EGENV AND GFI

THE FUNCTION CALCULATES TWO DIFFERENT NORMS  
IF STL 2 = 1 IT NORMALIZE THE FLUX TO THE POWER CONDITION  
IF STL 2 = 2 THE FLUX IN ONE SPACE POINT IS NORMALIZED TO A VALUE FIREF

	COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),	CO	1
1	JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),	CO	2
2	TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA (50),RR,PP,	CO	3
3	N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI,	CO	4
4	TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),	CO	5
5	O(50), E(51),R(50),DXDT(50),DIDT (50), HZ,S(150), T(30), W,	CO	6
6	AX(20), TETA	CO	7
	COMMON NRIT, KURV, NSTANS,	CO	8
1	IC, ICU, ICL, PROD, DRD, PART , ABS0, STAB(50),KSTYR,	CO	9
2	STL1, STL2, STL3, STL4, STL5, STL6,	CO	10
3	NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,	CO	11
4	EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,	CO	12
5	DEDA(20),DBDA(19), XALF1 (20),MVOID	CO	13
	COMMON XSS(50),XD, EPSX, EPSDX, ICONT	CO	14
	INTEGER AX		
	REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1	XECO	11
	INTEGER STL1, STL2, STL3, STL4, STL5, STL6	XECO	12
	DIMENSION YYY(50), ZZZ(50)	XNOR	
	GOTO (1, 4), STL2	XNOR0003	
1	SUM = 0	XNOR0004	
	DO 2 I2 = 1, N	XNOR0005	
	SUM = SUM + B(I2) * YYY(I2)	XNOR0006	
2	CONTINUE	XNOR0007	
	XXX = PP*FLOAT( N+1 ) / ( SUM*K1 )	XNOR0008	
5	DO 3 I3 = 1, N	XNOR0009	
	ZZZ(I3) = XXX*YYY(I3)	XNOR0010	
3	CONTINUE	XNOR0011	
	XNORM = 1./XXX	XNOR0012	
	RETURN	XNOR0013	
4	I4 = HN * FLOAT (N+1) + 0,5	XNOR0014	
	XXX = FIREF/ YYY(I4)	XNOR0015	
	GOTO 5	XNOR0016	
	END	XNOR0017	





L02      FORMAT (10X, 5HD \* F , 6H(FFGG) / (20X, 10F10,5))  
 END

FFGG 22  
 FFGG0024

FUNCTION GFI(YYY)

THE FUNCTION IS CALLED BY THE FUNCTION XKAPPA  
 REF, GUSTAF OLSSON, DIGITAL SIMULATION OF SPATIAL XENON OSCILLATIONS  
 DIV OF AUTOMATIC CONTROL LUND REPORT 6911  
 APPENDIX 2 POINT 5

CALCULATES THE MATRIX=VECTOR PRODUCT G\*FI  
 THE PRODUCT IS USED BY THE FUNCTION XKAPPA IN THE EIGENVALUE  
 CALCULATION WITH THE POTENSE METHOD  
 XNORM IS CALLED IN ORDER TO NORMALIZE THE VECTOR

	COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),	CO	1
1	JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),	CO	2
2	TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA (50),RR,PP,	CO	3
3	N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI,	CO	4
4	TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),	CO	5
5	Q(50), E(51),R(50),DXDT(50),DIDT (50), HZ,S(150), T(30), W,	CO	6
6	AX(20), TETA	CO	7
	COMMON NRIT, KURV, NSTANS,	CO	8
1	IC, ICU, ICL, PROD, DROD, PART , ABS0, STAB(50),KSTYR,	CO	9
2	STL1, STL2, STL3, STL4, STL5, STL6,	CO	10
3	NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,	CO	11
4	EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,	CO	12
5	DEDA(20),DBDA(19), XALF1 (20),MVOID	CO	13
	COMMON XSS(50),XD, EPSX, EPSDX, ICONT	CO	14
	INTEGER AX		
	REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1	XECO	11
	INTEGER STL1, STL2, STL3, STL4, STL5, STL6	XECO	12
	DIMENSION YYY(50), PSI(50)	GFI	
	DO 2 I2= 1,N	GFI	0003
	SUM = 0	GFI	0004
	DO 4 I4 = 1,N	GFI	0005
	SUM = SUM + G (I2,I4) * YYY(I4)	GFI	6
4	CONTINUE	GFI	0007
	PSI (I2) = SUM	GFI	0008
2	CONTINUE	GFI	0009
	IF (NTRY6,EQ,1) 6,8	GFI	
6	WRITE (6,100) (PSI(I2), I2=1,N)	GFI	
8	CONTINUE	GFI	
100	FORMAT (10X, 4HG*FI / (15X, 10F10,6 ))	GFI	0011
	GFI = XNORM (PSI, YYY)	GFI	0012
	RETURN	GFI	0013
	END	GFI	0014



SUBROUTINE MATINV (A, N)

THE SUBROUTINE IS CALLED BY MATR

MATINV IS A MATRIX INVERSION ROUTINE FOR A TRIDIAGONAL MATRIX

C  
C  
C  
C  
C  
C

1

3

2

4

10

20

25

30

40

200

100

```

DIMENSION A(50,50), B(50), C(50)
IF (A(1,1),EQ,0,) 200, 1
CONTINUE
A(1,1) = 1./ A(1,1)
B(1) = A(1,2) * A(1,1)
C(1) = A(2,1)
NM1 = N-1
DO 2 I2 = 2, NM1
C(I2) = A(I2 + 1, I2)
A(I2,I2) = A(I2,I2) - C(I2-1)* B(I2-1)
IF (A(I2,I2),EQ,0,) 200,3
CONTINUE
A(I2,I2) = 1./ A(I2,I2)
B(I2) = A(I2, I2+1) * A(I2,I2)
A(N,N) = A(N,N) - C(N-1) * B(N-1)
IF (A(N,N),EQ,0,) 200,4
CONTINUE
A(N,N)=1./ A(N,N)
I = N
J = N-1
A(I,J) = -A(I,J+1) * C(J) /A(J,J)
J =J- 1
IF (J,EQ,0) 20,10
J= I
I=J-1
A(I,J) = - B(I) * A(I+1,J)
I= I-1
IF (I,EQ,0) 30,25
J=J-1
I= J
A(I,J) = - A(J,J) * (A(I,J+1) * C(J) - 1.)
J= I-1
IF (J,EQ,0) 40,10
RETURN
WRITE (6,100)
FORMAT (// 10X, 15HSINGULAR MATRIS )
STOP
END

```

MATI  
MATI  
MATI  
MATI 4  
MATI 5  
MATI 105  
MATI 6  
MATI 7  
MATI 107  
MATI 8  
MATI  
MATI  
MATI 10  
MATI 11  
MATI 12  
MATI  
MATI 14  
MATI 15  
MATI 16  
MATI 17  
MATI 18  
MATI  
MATI 21  
MATI 22  
MATI 23  
MATI 24  
MATI  
MATI 27  
MATI 28  
MATI 29  
MATI 30  
MATI  
MATI 33  
MATI 34  
MATI 35  
MATI 36  
MATI 37

SUBROUTINE TIDF (ZI, Z)

THE SUBROUTINE IS CALLED BY MAIN

TIDF CALCULATES TIME FUNCTIONS FOR THE DISTURBANCE  
 $U(Z, T) = R(T) * R(Z)$  AND THE TOTAL POWER  $P(T)$  OUT OF A  
 POLYGENE CHAIN,

	COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),	CO	1
1	JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),	CO	2
2	TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA (50),RR,PP,	CO	3
3	N,C,K, LAMDAX, LAMDAI, LAMBDA,SIGMAX, K1, FIREF, IT,HT, GAMMAI,	CO	4
4	TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),	CO	5
5	Q(50), E(51),R(50),DXDT(50),DIDT (50), HZ,S(150), T(30), W,	CO	6
6	AX(20), TETA	CO	7
	COMMON NRIT, KURV, NSTANS,	CO	8
1	IC, ICU, ICL, PROD, DROD, PART , ABS0, STAB(50),KSTYR,	CO	9
2	STL1, STL2, STL3, STL4, STL5, STL6,	CO	10
3	NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,	CO	11
4	EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,	CO	12
5	DEDA(20),DBDA(19), XALF1 (20),NVOID	CO	13
	COMMON XSS(50),XD, EPSX, EPSDX, ICONT	CO	14
	INTEGER AX		
	REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1	XECO	11
	INTEGER STL1, STL2, STL3, STL4, STL5, STL6	XECO	12
	DIMENSION ZI(30,2), Z(30)		
	K= 0		TIDF0004
	M = 0		TIDF0005
	DO 1 I =1, 100		TIDF0006
	IPK = I+K		TIDF0007
	IF (T(IPK +1).LT,0,) 10,5		TIDF
5	M = M+1		TIDF
	IF ( T(IPK),EQ,T(IPK+1),AND, Z(IPK),NE,Z(IPK+1)) 2, 6		TIDF
6	CONTINUE		TIDF
	TI(I,1) = T(IPK)		TIDF0011
	TI(I,2) = T (IPK+1)		TIDF0012
	ZI(I, 1) = Z (IPK)		TIDF0013
	ZI (I,2) = Z (IPK+1)		TIDF0014
	GOTO 4		TIDF0015
2	TI(I,1) = T(IPK+1)		TIDF0016
	ZI(I,1) = Z (IPK +1)		TIDF0017
	TI (I,2) = T(IPK +2)		TIDF0018
	ZI (I,2) = Z (IPK+2)		TIDF0019
	K= K+ 1		TIDF0020
1	CONTINUE		TIDF0021
10	RETURN		TIDF
	END		TIDF0023

SUBROUTINE RUBR ( I )

THE SUBROUTINE IS CALLED BY MAIN  
RUBR IS A EDITING PROGRAM FOR WRITING ALL INPUT DATAS.

C	DIMENSION A(20)	RUBR
C	COMMON /DATA/ B(15)	RUBR
C	INTEGER A,B	
C	DATA ((B(I1), I1=1,14) = 3HGEO, 3HNUK, 3HKON, 3HTID,3HDIF,	RUBR
	1 3HBUK,3HEFF, 3HTRY,3HTOL, 3HBEG, 3HSTA, 3HTRA, 3HEXE, 3HSLU )	RUBR
	READ ( 5, 1) A	RUBR0005
1	FORMAT (A3, 19A4)	RUBR
	WRITE (6,2) A	
2	FORMAT(/10X, 1X, A3, 19A4)	
	DO 3 I3 = 1, 14	RUBR0010
	IF (A(1),NE, B(I3)) 3, 4	RUBR
4	I = I3	RUBR
	RETURN	RUBR0013
3	CONTINUE	RUBR0014
	WRITE (6,10)	RUBR
10	FORMAT (10X, 20HFELAKTIGT RUBRIKKORT )	RUBR
	STOP	RUBR0015
	END	RUBR0016

SUBROUTINE RITA

J = 1  
RETURN  
END

SUBROUTINE HYDRO

J = 1  
RETURN  
END

SUBROUTINE VOID

J = 1  
RETURN  
END

## SUBROUTINE ROD

REF G. OLSSON, DIGITALT PROGRAM TRAXEN FOR TRANSIENTBERAKNINGAR  
 AV XENONSVANGNINGAR I EN AXIELL REAKTORMODELL  
 SWEDISH STATE POWER BOARD, STOCKHOLM 1966 REPORT E-53766

THE SUBROUTINE IS CALLED BY RAND

IT CALCULATES THE REACTIVITY VALUES FOR A XENON CONTROL ROD

THE POSITION OF THE ROD IS DETERMINED BY

IC = ROD CENTRE ICU = UPPER BOUNDARY

ICL = LOWER BOUNDARY

PART = ROD LENGTH (LT 0,5)

COMMON FI(50),FI1(50),FI2(50),XE(50), XE0(50), JOD(50),	CO	1
1 JOD0(50), XK1(100), XK2(100), XK4(100), XK(100), XEPS(100),	CO	2
2 TI(30,2), PI(30,2), RI(30,2), B(50), ALFA(50), BETA(50),RR,PP,	CO	3
3 N,C,K, LAMDAX, LAMDAI, LAMBDA, SIGMAX, K1, FIREF, IT,HT, GAMMAI,	CO	4
4 TID0, TID, ITID, TMAX, M, DELTA, HN, YK1, D(50,50), G(50,50),	CO	5
5 Q(50), E(51),R(50),DXDT(50),DIDT(50), HZ,S(150), T(30), W,	CO	6
6 AX(20), TETA	CO	7
COMMON NRIT, KURY, NSTANS,	CO	8
1 IC, ICU, ICL, PROD, DROD, PART, ABSO, STAB(50),KSTYR,	CO	9
2 STL1, STL2, STL3, STL4, STL5, STL6,	CO	10
3 NTRY1,NTRY2, NTRY3,NTRY4,NTRY5,NTRY6,NTRY7,NTRY8,NTRY9,	CO	11
4 EPS1, EPS2, EPS21, EPS3, EPS31, ITE1, ITE2, ITE3,	CO	12
5 DEDA(20),DBDA(19), XALF1(20),MVOID	CO	13
COMMON XSS(50),XD, EPSX, EPSDX, ICONT	CO	14
INTEGER AX		
REAL JOD, JOD0, LAMDAX, LAMDAI, LAMBDA, K1	XECO	11
INTEGER STL1, STL2, STL3, STL4, STL5, STL6	XECO	12
IF (STL4,EQ,1) 420,1	ROD	
1 GOTO (100,200,400,500)KSTYR		

KSTYR = 1

IFI AND IX ARE CALCULATED (EQ, 2,1 = 2,2)

XFI1 = THE INTEGRATED VALUE OF FLUX DEVIATION OVER THE FIRST HALF OF  
 THE CORE (2,1)

XDX1 = INTEGRATED VALUE OF DXDT (2,2)

XFI2 = THE SAME AS XFI1 BUT OVER SECOND CORE HALF

XDX2 = THE SAME AS XDX1 BUT OVER SECOND CORE HALF

IC IS DEFINED OF XDX AND XFI (2,5)

DROD = CONSTANT PROD = CONSTANT

CONTINUE

N1 = 0,5 \* FLOAT(N) + 0,6

SUM1 = 0

SUM11 = 0,0

DO 2 I2 = 1,N1

SUM11 = SUM11 + FI(I2) - FI1(I2)

SUM1 = SUM1 + DXDT(I2)

XDX1 = SUM1 / FLOAT(N1)

XFI1 = SUM11 / FLOAT(N1)

K = 2 \* N1 - N

IF (K,EQ,0) 11,12

N1 = N1 + 1

CONTINUE

SUM2 = 0,

SUM21 = 0,0

100

2

11

12

```

DO 4 I4 = N1, N
SUM21 = SUM21 + FI(I4) = FI1 (I4)
SUM2 = SUM2 + DXDT (I4)
XX = 1./ FLOAT (N=N1+1)
XDX2 = SUM2 * XX
XFI2 = SUM21 * XX
XDX = 0,5 * (ABS(XDX1) + ABS(XDX2))
XFI = 0,5 * (ABS(XFI1) + ABS(XFI2))
IC = 0,5 * FLOAT (N+1) = FLOAT(N)*(XDX * SIGN (DROD, XDX2) -
1 XFI * SIGN(PROD, XFI2) ) + 0,5
GOTO 300

C
C KSTYR = 2
C IFI = THE COORDINATE WHERE FLUX DEVIATION IS BIGGEST
C IFI = THE COORDINATE WHERE DXDT IS BIGGEST
C IC IS CALCULATED OUT OF IFI AND IDX (2,9)
C
200 CONTINUE
XDX = DXDT(1)
IDX = 1
DO 24 I24 = 2, N
IF (DXDT(I24) = XDX) 22,22,24
22 XDX = DXDT(I24)
IDX = I24
24 CONTINUE
XFI = FI(1) = FI1(1)
IFI = 1
DO 28 I28 = 2, N
DELT = FI(I28) = FI1(I28)
IF (XFI = DELT) 26, 26, 28
26 XFI = DELT
IFI = I28
28 CONTINUE
PROD = 1,0 = DROD
IC = PROD * FLOAT(IFI) + DROD * FLOAT (IDX)
GOTO 300

C
C KSTYR = 3
C IC IS DETERMINED IN DATA INPUT
C XDELX = XE DEVIATION IN POINT IC
C DXDT(IC) = DXDT IN POINT IC
C THE ROD SWITCHES BETWEEN IC AND N + 1 = IC DEPENDING ON DELX AND DXDT
C (2,10=11)
C WHEN XDELX LT EPSX AND DXDT(IC) LT EPSDX THE ROD IS MOVED TO CORE
C CENTRE (2,12=13)
C
400 XDELX = XE(1) = XSS(1)
DO 402 I402 = 2, N
IF (XE(I402) = XSS(I402) = XDELX ) 402,402, 406
406 XDELX = XE(I402) = XSS(I402)
IXE = I402
402 CONTINUE
404 IF (ABS(XDELX), LT, EPSX, AND, ABS(DXDT(IXE)), LT, EPSDX) 420,403
403 IF (XDELX, GT, (=XD * DXDT(IXE)*ABS(DXDT(IXE)))) 408,410
408 IC = N+1 = IXE
GOTO 412

```

ROD  
RODROD  
ROD

ROD

ROD  
ROD

ROD

ROD  
ROD

```

410     IC = IXE
412     CONTINUE
      GOTO 300

C
C     KSTYR = 4
C     THE COORDINATE WHERE FLUX DEVIATION IS BIGGEST IS CALCULATED
C     IF XDELX IN THIS POINT IS BIG ENOUGH (2,15) THE ROD IS MOVED TO DAMP
C     THE FLUX
C     IIC = NUMBER OF SPACE POINTS CORRESPONDING TO THE HALF ROD
C     THE ROD MUST ALL THE TIME BE INSIDE THE BOUNDARIES OF THE CORE
C     STAB = ABSORPTION VALUE ALONG THE ROD
C     ABSO = ABSORPTION CONSTANT
C
500     IXE = ICONT
      XDELX = XE(IXE) = XSS(IXE)
      GOTO 404
420     IC = 0,5 * FLOAT (N+1) + 0,5
300     CONTINUE
      IIC = 0,5*PART * FLOAT (N+1)
      N2 = N/2
      IIC1 = 1 + IIC
      IF (IC,LT,N2) 6, 14
14     CONTINUE
      IIC2 = N - IIC
      IC = MIN0 (IIC2 , IC)
      GOTO 60
6     IC = MAX0(IIC1,IC)
60     CONTINUE
      ICU = IC + IIC
      ICL = IC - IIC
7     DO 8 I8 = 1,N
8     STAB (I8) = 0,0
      DO 10 I10 = ICL, ICU
10    STAB(I10) = ABSO
      STAB (ICU +1) = ABSO * (0,5 * PART *FLOAT (N+1) = FLOAT (IIC))
      STAB (ICL -1) = STAB (ICU +1)
      IF(NTRY9,EQ,1)302,304
302   WRITE(6,306)
304   CONTINUE
306   FORMAT (10X, 8HROD KLAR)
      RETURN
      END

```

ROD

ROD

ROD

ROD

ROD

ROD

ROD

Appendix 4

DERIVATION OF A TRANSFER FUNCTION FOR A TWO POINT XENON MODEL

In order to get a simple estimation of the amplitude of the transients we will derive a transfer function for a two point linear xenon model, defined in [5]. From the transfer function it is possible to analytically derive the maximum amplitude of the output variable.

In [5] eq. (2:24 - 25) is derived an expression for the diffusion equation in two space points:

$$\alpha_1 \varphi_1^2 + \varphi_1 (\beta \xi_1 + c_1 - g_1) + \phi_1^0 (c_1 + \beta \xi_1) = 0 \quad (1)$$

$$\alpha_2 \varphi_1^2 + \varphi_1 (-\beta \xi_2 - c_2 + g_2) + \phi_2^0 (c_2 + \beta \xi_2) = 0 \quad (2)$$

$$\text{where } g_i = \frac{3}{h^2} - (B_i^{2*} + \alpha_i \phi_i^0) \quad i = 1, 2 \quad (3)$$

We add one more control term  $u$  which will make it possible to disturb the flux externally.

$$\alpha_1 \varphi_1^2 + \varphi_1 (\beta \xi_1 + c_1 + u_1 - g_1) + \phi_1^0 (c_1 + u_1 + \beta \xi_1) = 0 \quad (4)$$

$$\alpha_2 \varphi_1^2 + \varphi_1 (-\beta \xi_2 - c_2 - u_2 + g_2) + \phi_2^0 (c_2 + u_2 + \beta \xi_2) = 0 \quad (5)$$

where  $g_i$  is defined in (3).

We linearize eq. (4) and (5) and assume:

$$u_1 = -u_2 = u \quad (6)$$

Further we assume a symmetric equilibrium flux and homogeneous control:

$$g_1 = g_2 = g$$

$$\phi_1^0 = \phi_2^0 = \phi^0$$

$$X_1^0 = X_2^0 = X^0$$

$$c_1 = c_2 = c$$

Then (4) and (5) are simplified to:

$$-\varphi_1 g + \phi^0(c + u + \beta\xi_1) = 0 \quad (7)$$

$$\varphi_1 g + \phi^0(c - u + \beta\xi_2) = 0 \quad (8)$$

For the symmetric flux we have derived in [5], eq. (2:44):

$$g = \frac{2}{h^2} - \alpha \phi^0 \quad (9)$$

We subtract (8) from (7) and eliminate c:

$$-2\varphi_1 g + 2\phi^0 u + \beta\phi^0(\xi_1 - \xi_2) = 0 \quad (10)$$

or

$$\varphi_1 = \frac{\beta\phi^0}{2g} (\xi_1 - \xi_2) + \frac{\phi^0}{g} \cdot u \quad (11)$$

where g is defined in (9).

We introduce the state variables:

$$x_1 = \xi_1 \quad x_2 = \eta_1 \quad x_3 = \xi_1 + \xi_2 \quad x_4 = \eta_1 + \eta_2 \quad (12)$$

and rewrite (11):

$$\boxed{\varphi_1 = \frac{\beta\phi^0}{2g} (2x_1 - x_3) + \frac{\phi^0}{g} u} \quad (13)$$

The xenon and iodine equations are rewritten directly from [5], eqs. (2:37) - (2:40) and we linearize the equations:

$$\frac{dx_1}{dt} = (-\lambda_x - \sigma_x \phi^0)x_1 + \lambda_i x_2 + \sigma_x(\gamma_x - X^0)\varphi_1 \quad (14)$$



$$\frac{dx_2}{dt} = -\lambda_i x_2 + \gamma_i \sigma_x \phi^0 \quad (15)$$

$$\frac{dx_3}{dt} = (-\lambda_x - \sigma_x \phi^0)x_3 + \lambda_i x_4 \quad (16)$$

$$\frac{dx_4}{dt} = -\lambda_i x_4 \quad (17)$$

where the state variables are defined in (12). Eq. (13) is inserted in the system equations (14) - (17).

We get directly:

$$\frac{dx}{dt} = Ax + Bu \quad (18)$$

$$y = Cx + Du$$

where

$$A = \begin{bmatrix} -\lambda_x - \sigma_x \phi^0 \left[ 1 + (X^0 - \gamma_x) \frac{\beta}{g} \right] & \lambda_i & \frac{\sigma_x \beta \phi^0}{2g} (X^0 - \gamma_x) & 0 \\ \gamma_i \sigma_x \frac{\beta \phi^0}{g} & -\lambda_i & -\gamma_i \sigma_x \frac{\beta \phi^0}{2g} & 0 \\ 0 & 0 & -\lambda_x - \sigma_x \phi^0 & \lambda_i \\ 0 & 0 & 0 & -\lambda_i \end{bmatrix}$$

$$B = \frac{\phi^0 \sigma_x}{g} \begin{bmatrix} \gamma_x - X^0 \\ \gamma_i \\ 0 \\ 0 \end{bmatrix}$$

$$C = \frac{\beta\phi^0}{2g} (2 \quad 0 \quad -1 \quad 0)$$

$$D = \frac{\phi^0}{g}$$

The transfer function

$$G(s) = \frac{Y(s)}{u(s)}$$

is easily derived from (18). As only two states are both controllable and observable, the transfer function is of second order

$$G(s) = C(sI - A)^{-1} B + D \quad (19)$$

Simple calculus of (18) inserted into (19) gives:

$$G(s) = \beta\sigma_x \left( \frac{\phi^0}{g} \right)^2 \left\{ \frac{(\gamma_x - X^0)(s - a_{22}) + \gamma_i a_{12}}{(s - a_{11})(s - a_{22}) - a_{12} a_{21}} \right\} + \frac{\phi^0}{g} =$$

$$= \frac{\frac{\phi}{g} s^2 + \left[ \frac{\beta\sigma_x \phi^2}{g^2} (\gamma_x - X^0) - \frac{\phi}{g} (a_{11} + a_{22}) \right] s + \frac{\beta\sigma_x \phi^2 \lambda_i}{g^2} (1 - X^0) + \frac{\phi}{g} (a_{11} a_{22} - a_{12} a_{21})}{s^2 - (a_{11} + a_{22})s + a_{11} a_{22} - a_{12} a_{21}} \quad (20)$$

where the parameters  $a_{ij}$  are elements in A (18).

Appendix 5DESCRIPTION AND PROOF OF THE ROD MOVEMENT FOR A SIMPLIFIED  
FLUX MODEL

The rod movement or absorption variation is included in the buckling term. Here we will show the variation of the buckling of a very simplified core model, when the flux distribution is disturbed. The statement is shown in eg.(4:1). Out of the proof we can conclude how rod movement will take place.

Let us assume a neutron flux which is stationary. Further we assume one group diffusion theory. The core is divided into two parts of equal length, normalized to 1. Buckling is space independent in the two halves of the core in fig. 1. Then the diffusion equation reads:

$$\frac{d^2 \phi_1}{dz^2} + B_1^2 \phi_1 = 0 \quad (1)$$

$$\frac{d^2 \phi_2}{dz^2} + B_2^2 \phi_2 = 0 \quad (2)$$

with solutions:

$$\phi_1 = A_1 \sin (B_1 z + \delta_1) \quad (3)$$

$$\phi_2 = A_2 \sin (B_2 z + \delta_2) \quad (4)$$

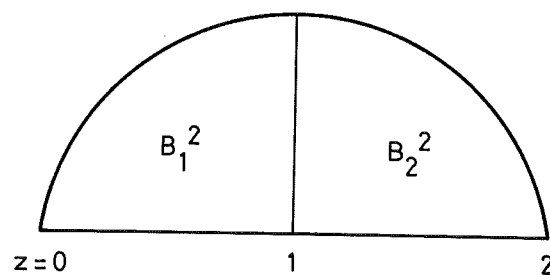


Figure 1: Flux distribution

The unknown parameters  $A_1$ ,  $A_2$ ,  $\delta_1$ ,  $\delta_2$  and one of the terms  $B_1$ ,  $B_2$  are determined out of the boundary conditions (5) - (9).

$$\phi_1(0) = 0 \quad (5)$$

$$\phi_2(2) = 0 \quad (6)$$

$$\left. \frac{d\phi_1}{dz} \right|_{z=1} = \left. \frac{d\phi_2}{dz} \right|_{z=1} \quad (7)$$

$$\phi_1(1) = \phi_2(1) \quad (8)$$

$$\int_0^1 \phi_1(z) dz + \int_1^2 \phi_2(z) dz = C = \text{constant} \quad (9)$$

The last condition means that mean flux is unaltered.

Now, let us fix  $B_1$ . We solve (5) - (9) in order to get  $B_2$  and find:

$$\boxed{\frac{B_2}{B_1} = - \frac{\text{tg } B_2}{\text{tg } B_1}} \quad (10)$$

We assume all the time, that:

$$0 \leq B_1 \leq \pi/2 \quad (11)$$

$$\pi/2 \leq B_2 \leq \pi$$

For the special case:

$$B_1 = \pi/2$$

we have

$$B_2 = \pi/2$$

This is the symmetric sine curve.

We will prove the following statements, and assume all the time (10) is satisfied.

If  $B_1$  decreases (increases) from  $B_{11}$  to  $B_{12}$  we have an increase (decrease) in  $B_2$  from  $B_{21}$  to  $B_{22}$  such as:

$$\left| (B_{11})^2 - (B_{12})^2 \right| > \left| (B_{21})^2 - (B_{22})^2 \right| \quad \text{or}$$

$$\left| \Delta B_1^2 \right| > \left| \Delta B_2^2 \right|$$

Thus if  $B_1$  decreases we have

$$(B_{11})^2 + (B_{21})^2 > (B_{12})^2 + (B_{22})^2$$

The inequality is opposite if  $B_1$  increases.

In order to prove (12) we must take the constraint (10) into account. Due to the cumbersome calculations we prefer to show the statement by numerical calculations.

The function  $f = B_1^2 + B_2^2$  is monotonically increasing with  $B_1$ , when  $B_1$  varies between 0 and  $\pi/2$ . The function is biggest for  $B_1 = \pi/2$  (see figure 2), i.e. the symmetric flux shape, where

$$B_1^2 + B_2^2 = \pi^2/2$$

Out of this discussion we realize that (12) is valid all the time when  $B_1$  varies between two values in the domain

$$0 \leq B_1 \leq \pi/2.$$

To sum up, the variation in the buckling  $B_1^2$  is always bigger than the variation of the buckling  $B_2^2$ , when the bucklings are within the boundaries (11).

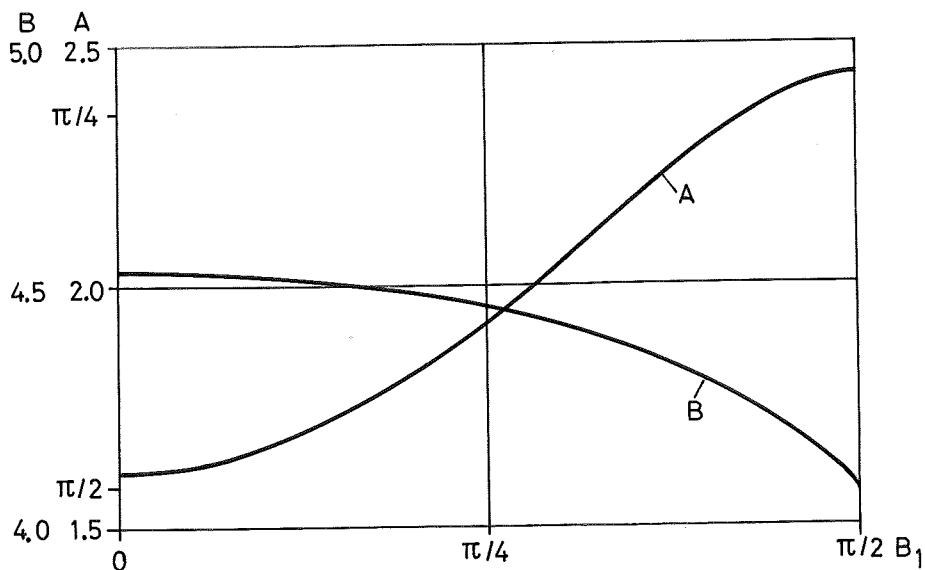


Fig. 2 - The variation of the sum of bucklings  $B_1^2 + B_2^2$  (curve A) and of  $B_2$  (curve B) as function of  $B_1$