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SUPERHEAVY ELEMENTS AND VARIATIONS IN THE NUCLEAR SKIN THICKNESS

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Although the interpretation of the data is uncertain, the recently claimed empirical evidence of element $Z = 126$ and some of its neighbours, provides the impetus for a reexamination of the nuclear one-body potentials, on the basis of which extrapolations have been made up to now. In particular the skin thickness parameters of the neutrons and protons are of interest and the authors presently consider the possibility that the nucleus may adjust its skin thickness to achieve a maximal shell energy.

Most of the presently available predictions concerning the regions of existence of superheavy elements are based on the assumed existence of some constant or slowly varying parameters, whose properties are known over the entire periodic table. These parameters then hopefully lend themselves to an extrapolation to higher mass regions. The nuclear one-body potential is characterized by a few such parameters, as the nuclear root mean square radius, assumed to vary strictly as $A^{1/3}$ and the nuclear diffuseness or skin thickness, usually assumed constant and roughly equal for neutrons and protons. There enters in addition the spin orbit strength, which is usually taken to vary as $A^{-1/3}$ or $A^{-2/3}$. This holds true for the calculations performed on the basis of the modified oscillator [1] (M.O.), the Woods–Saxon potential [2] or the Folded–Yukawa potential [3]. In the latter two of these the entering diffuseness parameter a is assumed to be simply constant. A corresponding parameter in the M.O. potential is the strength of the l^2 term, $\mu' = \kappa\mu$. This term is generally assumed to vary smoothly with A.

Other nuclear shape parameters, as the angular shape coordinates: ϵ, γ, ϵ_4 etc. in the M.O. case, are assumed to be what can be termed "Strutinsky parameters" and are left to vary freely, their equilibrium values determined from a minimisation of the total energy. The latter is assumed to be built up from a macroscopic part and a shell correction part. One might now raise the question whether or not also the skin thickness could be such a Strutinsky parameter whose detailed value varies with N and Z and dependent on shell structure. In the Strutinsky's formula-

tion, the nucleus may be assumed to vary its radial shape so as to maximize the relevant shell gaps. An even more radical concept appearing in the extension of this line of thinking is the nuclear bubble as proposed by Wong [4].

A calculation treating the skin thickness as a Strutinsky parameter was undertaken in 1972 by a group in Berkeley–Los Alamos [5] and is also described in ref. [6]. A more detailed study along the same lines by a Heidelberg group [7] appeared recently (see also Pauli [8]). Both of these calculations suffered from the fact that no reliable macroscopic restoring force relating to the nuclear skin thickness is available, stabilizing the skin thickness for neutrons as well as protons around the same average values. The straightforwardly interpreted empirical evidence particularly from the $A = 20–60$ region is that of a skin thickness varying with N and Z (see fig. 1). Thus from its spectrum $^{40}_{20}\text{Ca}_{20}$ appears to be associated with a large skin thickness ("harmonic-oscillator-like", corresponding to a small μ') while $^{56}_{28}\text{Ni}_{28}$ appears to be a relatively thin-skinned nucleus ("square-well-like" or corresponding to a large μ'). The calculations of ref. [5] brought out strong shell effects corresponding to a "driving force" in the expected directions. However, the macroscopic term providing the restoring force of the Berkeley calculations was found not to permit anything but very small fluctuations in skin thickness (never in excess of 5%) from the average value [5].

The obvious inference is now that such fluctuations in μ' with A in addition to the smooth variation apparent in fig. 1 might be expected also in the extra-

Fig. 1. Spherical single-particle levels of the M.O. potential as a function of μ' ($= \mu\kappa$) with $\kappa = 0.06$. Filled circles correspond to optimal fits to signle-particle data. Circles in parentheses refer to the deformed regions. Note the extrapolation to $Z = 114$ and $Z = 126$, the latter corresponding to an increased skin thickness of the proton potential.

polated regions of nuclei. On the other hand, permitting no or only very small fluctuations in the diffuseness, the nuclear potentials studied so far in almost all cases extrapolate in such a way that the next proton shell is $Z = 114$ and thereupon $Z = 164$ but hardly $Z = 126$ (see fig. 1). For neutrons the next predicted shells are at $N = 184$ and 228 for most straightforward extrapolations. Remarkably enough the gaps

Fig. 2. Shell correction energy as a function of the skin thickness parameter μ' for M.O. for protons or neutrons based on $\kappa = 0.05$. Note the occurrence of fairly deep minima for the magic numbers 114, 126, 184 and 228, here discussed. An increase in κ from 0.05 to 0.06 (see fig. 1) tends to deepen the 114 minimum.
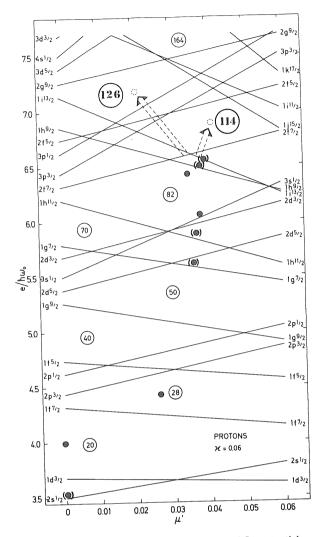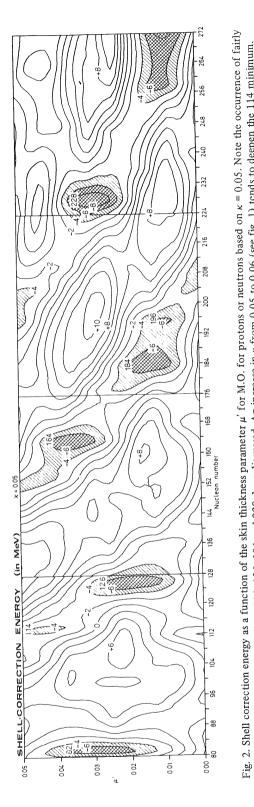
at $Z = 114$ and $N = 184$, 228, but not $Z = 126$, are also brought out in the published Hartree–Fock calculations, based on the Skyrme interaction [9]. The recently claimed empirical evidence of elements in the region of $Z = 126$ (Gentry et al. [10]) provides the opportunity of a reexamination of the different parameters entering in the extrapolations. Especially the nuclear skin thickness appears to be crucial. A more diffuse proton potential than that earlier assumed would indeed give rise to $Z = 126$ appearing as a strong proton gap in the way $N = 126$ is a strong neutron gap in the ^{208}Pb potential. The occurrence of both of the gaps 114 and 126 depend on the value of the skin thickness parameter as can be directly studied in both of figs. 1 and 2. Fig. 1 plots in the M.O. case the proton orbitals as functions of μ', which quantity is roughly inversely linear in the radial diffuseness parameter a of a Woods–Saxon type potential. Table 1 provides the connections between the diffuseness and μ' for protons and neutrons in the $A = 350$ region. The quantity a_p in this table refers to density distributions obtained from actual wave functions. The diffuseness of the density distribution is obtained by comparing the calculated values of $\langle r^4 \rangle$ and $\langle r^2 \rangle$ with those of an assumed Fermi shape density. Finally $\langle a^\rho \rangle$ refers to a Strutinsky smeared density. The *smeared* density distribution should *ideally* correspond to the total (nuclear + Coulomb) potential distribution in r.

Furthermore, in fig. 1 the empirically encountered values of μ' are included in the diagram as filled circles. These refer to fits of single-particle spectra. Circles in

parentheses refer to deformed regions. The totality of circles appear to extrapolate into the μ'-values associated with the $Z = 114$ shell. At least this is true of the M.O. case. For the conventional Woods–Saxon treatment it also seems to hold. The island of nuclei associated with $Z = 114$ in all likelihood also exists and remains the more probable candidate for stability. On the other hand, the $Z = 126$ single-proton gap is reached under the assumption of a radically smaller μ' as in fig. 1. The superheavy nucleus $Z = 126$ is characterized by having of the order of 10% thicker proton skin than that corresponding to the straightforwardly extrapolated value.

In lieu of a reliable macroscopic nuclear skin thickness term we have performed surveys of a possible $Z = 126$ island by the following approach. We have considered the case of such a macroscopic restoring force term being entirely vanishing, thus allowing the nucleus to adjust so as to minimize the shell energy. This corresponds roughly to a maximal $Z = 126$ gap. Pauli and coworkers [7] note that this condition applied to the ^{208}Pb region allowing a variation of both μ' and κ gives a nearly optimal reproduction of the single-particle level order. A similar situation holds according to the cited authors for the corresponding parameters of a Woods–Saxon type spectrum.

In fig. 2 we exhibit a plot of the proton (or neutron) shell energy of the M.O. potential in terms of the surface thickness term μ' and the nucleon number. In this diagram we have assumed $\kappa = 0.05$, and furthermore $\hbar\omega_{N,Z} = 41\,A^{-1/3}$ MeV. One then finds minimum shell energies for $\mu'_p \approx 0.02$ for $Z = 126$ and

Table 1

Skin thickness as a function of M.O. parameter μ' for $Z = 114$, $N = 184$, and for $Z = 126$, $N = 228$. The skin thickness parameter corresponding to a sharp Fermi surface a^ρ is shown for protons and neutrons respectively. Also shown is the quantity $\langle a^\rho \rangle$ corresponding to a Strutinsky smeared level density. The spin-orbit parameter $\kappa = 0.05$.

μ'	$Z = 114$		$N = 184$		$Z = 126$		$N = 228$	
	a_p^ρ (fm)	$\langle a_p^\rho \rangle$ (fm)	a_n^ρ (fm)	$\langle a_n^\rho \rangle$ (fm)	a_p^ρ (fm)	$\langle a_p^\rho \rangle$ (fm)	a_n^ρ (fm)	$\langle a_n^\rho \rangle$ (fm)
0.00	0.78	0.79	0.73	0.77	0.78	0.83	0.84	0.81
0.01	0.78	0.76	0.73	0.74	0.78	0.80	0.77	0.77
0.02	0.76	0.74	0.73	0.71	0.78	0.78	0.70	0.74
0.03	0.69	0.72	0.69	0.68	0.78	0.75	0.70	0.70
0.04	0.69	0.70	0.65	0.65	0.76	0.73	0.70	0.67
0.05	0.69	0.68	0.64	0.62	0.72	0.71	0.64	0.63
0.06	0.69	0.66	0.59	0.58	0.71	0.68	0.62	0.62

$\mu'_n \approx 0.03$ for $N = 228$. (The combination of $Z = 126$ and $N = 184$ was investigated but found less interesting because of the short alpha half-lives.) In order to simplify the calculations we choose $\mu'_n = \mu'_p = 0.025$. The corresponding proton and neutron shell energies are -6 MeV for protons and -11 MeV for neutrons. These large values appear highly promising for the formation of an island of stability. (This is somewhat of an upper limit as the inclusion of a surface diffuseness stabilisation term of a macroscopic type should lead to less negative shell energies.)

These values of μ'_p and μ'_n correspond to Strutinsky smeared diffuseness constants $\langle a_p^\rho \rangle$ and $\langle a_n^\rho \rangle$ of 0.76 and 0.72 fm respectively, as seen from table 1. For the actual unsmeared proton and neutron densities one obtains $a_p^\rho = 0.78$ and $a_n^\rho = 0.70$ fm. In this case we are thus rather near to self-consistency. Incidentally we have verified in the calculations that self-consistency occurs for the same μ'-values that give shell energy minimum. One can thus note a strong excess of protons in the skin and a large skin thickness.

In conventional extrapolation of the Woods–Saxon potential parameters, generally leading to the prediction that the proton number $Z = 114$ shows shell closure, the diffuseness parameter a of the potential, as distinguished from that of the density, is usually uncritically taken to be the same for neutrons and protons and for all nuclei throughout the periodic table. With such a parametrization, however, the surface diffuseness of the effective proton potential including nuclear and Coulomb field is markedly reduced by the large Coulomb contribution in heavy nuclei. Following a suggestion in the paper by Yariv et al. [7] one may instead examine the consequences of a postulate that the neutrons and protons should have the same *density* skin thickness *on the average*. As we shall see, this leads to a quite different extrapolation into the superheavy region.

For the numerical calculations we have employed a set of parameters from ref. [11]. In the nucleus ^{208}Pb the ratio of the potential parameters a_n/a_p has to be chosen about 0.7 to give $\langle a_p^\rho \rangle \approx \langle a_n^\rho \rangle \approx 0.65$ fm, which is close to the value determined in ref. [7] from the criterion of optimal shell structure. Using the above condition on the densities for superheavy nuclei one finds that one has to employ $a_n = 0.65$, $a_p = 1.0$ fm to obtain $\langle a_n^\rho \rangle = \langle a_p^\rho \rangle = 0.65$ fm. This has a drastic effect on the single-particle level scheme. The proton

shell correction energy E_p for the nucleus ^{298}114 becomes very nearly zero, requiring instead a skin thickness readjustment in a of 10% for the magicity of $Z = 114$ to be retained. Instead a gap appears at $Z = 126$. In the superheavy nucleus ^{354}126 the value of E_p is -6 MeV for the standard value [11] $a_n = 0.63$ fm, and dE_p/da_n is -15 MeV/fm. The neutron shell correction energy E_n is -6.5 MeV and dE_n/da_n is about $+50$ MeV/fm. Thus a total shell correction energy of -12.5 MeV is predicted. Still lower values are possible if the surface diffuseness degree of freedom can be exploited along the lines discussed above. The driving force towards a larger skin thickness for $Z = 126$ can also be seen from the fact that the non-smeared skin thickness of the matter distribution is about 10% larger than the Strutinsky smeared one, which thus appears to require a larger a_p in the proton potential to obtain self-consistency.

For $Z = 114$ the non-smeared skin thickness for protons is about 20% smaller than the smeared one. Corresponding to the choice $a_n = 0.65$, $a_p = 1.0$ giving $\langle a_n^\rho \rangle = \langle a_p^\rho \rangle = 0.65$ fm, one finds $a_p^\rho = 0.52$ and $a_n^\rho = 0.71$ fm. This may be taken to imply that a much smaller a_p should be used near $Z = 114$, justifying the assumption of $a_p \approx a_n$ used in earlier calculations, which have predicted the $Z = 114$ gap.

The Woods–Saxon considerations in general support the conclusions based on the M.O. model but the most reasonable extrapolation based on the former potential appears to point in between the two arrows of fig. 1. However, also in the Woods–Saxon case the skin thickness dependence on the shell energy provides a strong driving force towards the realisation of both $Z = 114$ and $Z = 126$ as shell gaps.

Based on the "optimal" value of μ' for the M.O. we have also calculated total-energy maps for a sequence of even nuclei near $Z = 126$ and $N = 228$ in terms of the deformation coordinates ϵ, ϵ_4 and γ. We have employed a one-dimensional path to fission along $\gamma = 0$, solely expressed in terms of ϵ, where a minimisation with respect to ϵ_4 is implied. The effect of γ on the half-lives is found to be negligible at least in the $Z = 126$, $N = 228$ region. For the fission inertial mass we have in this preliminary estimate used the empirical value [12] $B_{\epsilon\epsilon} = 0.054 A^{5/3} \hbar^2$ MeV^{-1}. The resulting fission barriers are similar to those of the $Z = 114$, $N = 184$ region. However, the barriers calculated for $Z = 126$ in the M.O. case under the special

Table 2
Partial half-lives of even-even elements near $Z = 126$, $N = 228$. The top and second line in each box gives the logarithm of fission half-life and the alpha decay half-life in years respectively. Stability to beta decay is indicated by the symbol β.

Proton number	Neutron number							
	218	220	222	224	226	228	230	232
128	−10	−4	3	11	21	32	25	17
	−7	−7	−5	−4	−3	−2	−11	−10
			β	β	β	β		
126		−3	3	10	20	30	38	33	28
		4	5	7	10	12	15	−2	−1
	β	β	β					
124		−8	−2	6	17	27	36	30	25
		8	10	13	16	20	23	2	3

optimistic assumptions of minimal shell energy are found to be higher but somewhat thinner (due to the larger Coulomb repulsion) than those earlier calculated in the $Z = 114$, $N = 184$ region under assumption of a straightforward extrapolation. The collection of half-lives with respect to fission, alpha decay and beta decay stability (indicated with a β) is shown in table 2 for this "shell-optimal" extreme case. An order of magnitude estimate gives the longest total half-life of the order of 10^7 years (the error limits being many powers of ten) and associated with the nucleus $Z = 126$, $N = 222$, based on the M.O. model.

For the results quoted a vanishing skin thickness restoring force has been assumed. A possible way of determining the skin thickness stabilising force would be to fit the spectroscopic data, on the basis of a Woods—Saxon potential, with a as a free parameter, for the odd elements near ^{16}O, ^{40}Ca, ^{48}Ca and ^{56}Ni. A further natural assumption is that the restoring force is proportional to $A^{2/3}$. However, the uncertainties as to the level schemes appear to be so large that no reliable numbers can be given. On the other hand, these data seem off-hand to be compatible with a softer stabilising force than that of the droplet model used in ref. [5]. Thus, it seems conceivable that with a stabilising macroscopic term included one arrives at a considerably smaller than average proton skin thickness for $Z = 114$ and at a larger than average skin thickness for $Z = 126$. A further complication for a quantitative comparison is the coupling between the

proton and neutron skins. Thus the neutrons and protons exhibit opposite trends both for the combination $Z = 114$, $N = 184$ and for $Z = 126$, $N = 228$.

In summary, from this type of consideration it appears possible, though by no means proven, that $Z = 126$ occurs as a shell gap large enough to manifest itself in an island of stability around the proton number $Z = 126$ far beyond the stability penisula. Earlier the prediction was put forth by several authors of another island connected with the proton number $Z = 114$ and neutron number $N = 184$ and associated with rather different relative neutron and proton skin thicknesses. The most straight-forward extrapolation in the nuclear potentials may be taken to give some preference for the $Z = 114$ alternative. For both islands to coexist it appears that one has to assume a strong adjustment in the nuclear skin thickness of a magnitude encountered empirically in the mass region $A = 16$–56, and working in the direction to minimise the shell energy. Should this be the case, it appears somewhat surprising that variations in the skin thickness have not been noted already in Hartree—Fock calculations. The preliminary conclusions here await both the verification of a detailed Hartree—Fock calculation preferably based on a fully realistic two-body interaction, and more importantly, more detailed experimental measurements.

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