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At the Ronneby Symposium on Superheavy elements it was suggested by Bohr and by Swiatecki [1] that one should consider quasi-stable superheavy nuclei on the neutron deficient side of the beta-stability line. For these nuclei the value of the fissility parameter, x, will be larger than one, which means that the liquid-drop energy has no minimum in deformation space but instead has a *saddle point* for oblate shapes. One might exploit shell structure in N and Z coincident with the occurrence of a relatively flat region of the liquid-drop energy surface around this oblate saddle point.

Our investigation is based on the macroscopic-microscopic method developed by Strutinsky [2]. Its ingredients are a liquid-drop macroscopic-energy term, in our case based on the Myers-Swiatecki 1966 Lysekil parameters [3], furthermore a shell-energy term, in our case obtained from the modified-oscillator model (M.O.) [4] or alternatively the scaled variant (S.M.O.) suggested by Bohr and Mottelson [5]. (For the details and parameters of S.M.O., see the work by Bengtsson [6].) The shell-energy term also includes the pairing energy calculated according to conventional prescriptions [4].

To obtain an overview over a large region of nuclei we employ a plot of the liquid-drop and shell-energy properties as shown in fig. 1. This fairly complicated plot, which represents a further development of a plot by Tsang [7], exhibits first of all the liquid-drop saddle-point deformation as a function of N and Z. The lower part of fig. 1 maps prolate saddle-point distortions while the upper part of fig. 1 covers values of x in excess of 1, in which case the saddle point appears in oblate deformation space. Thus e.g. all nuclides with a prolate saddle point of, say, $\epsilon = 0.3$ are connected in the N–Z plane by a contour line. In the calculation of the saddle points for each ϵ-value a minimization in ϵ_4 has been performed. The two coordinates ϵ, ϵ_4 are sufficient to reproduce liquid-drop saddle points out to $\epsilon = 0.7$–0.8. Beyond $\epsilon = 0.8$ a larger deformation space is needed.

For each combination of N and Z a liquid-drop saddle-point distortion ϵ is thus assigned. For this deformation the corresponding neutron and proton shell energies have been calculated and reproduced in the plot. The areas in N and Z where neutron shell energies are smaller than 0.0 MeV are shaded by vertical lines while those areas where the neutron shell energy is smaller than minus 1.5 MeV have been marked by a denser vertical shading. The proton shell energy areas corresponding to E_{shell} smaller than 0.0 MeV and smaller than minus 1.5 MeV, respectively, have analogous horizontal shadings.

Areas favourable for well-developed potential-energy minima obviously correspond to maximally negative neutron and proton shell energies occurring for ϵ-values equal to the liquid-drop saddle points. They can then easily be identified on the basis of these plots. As the liquid-drop energy surface is rather flat in the vicinity of the saddle point (at least for x-values not too different from 1.0), the minimum of the potential-energy surface may occur somewhat removed from the liquid-drop saddle. Some minima may then develop in excess of those indicated in fig. 1.

From fig. 1 one can identify first of all the well-known and rather unique second minimum (at $\epsilon =$

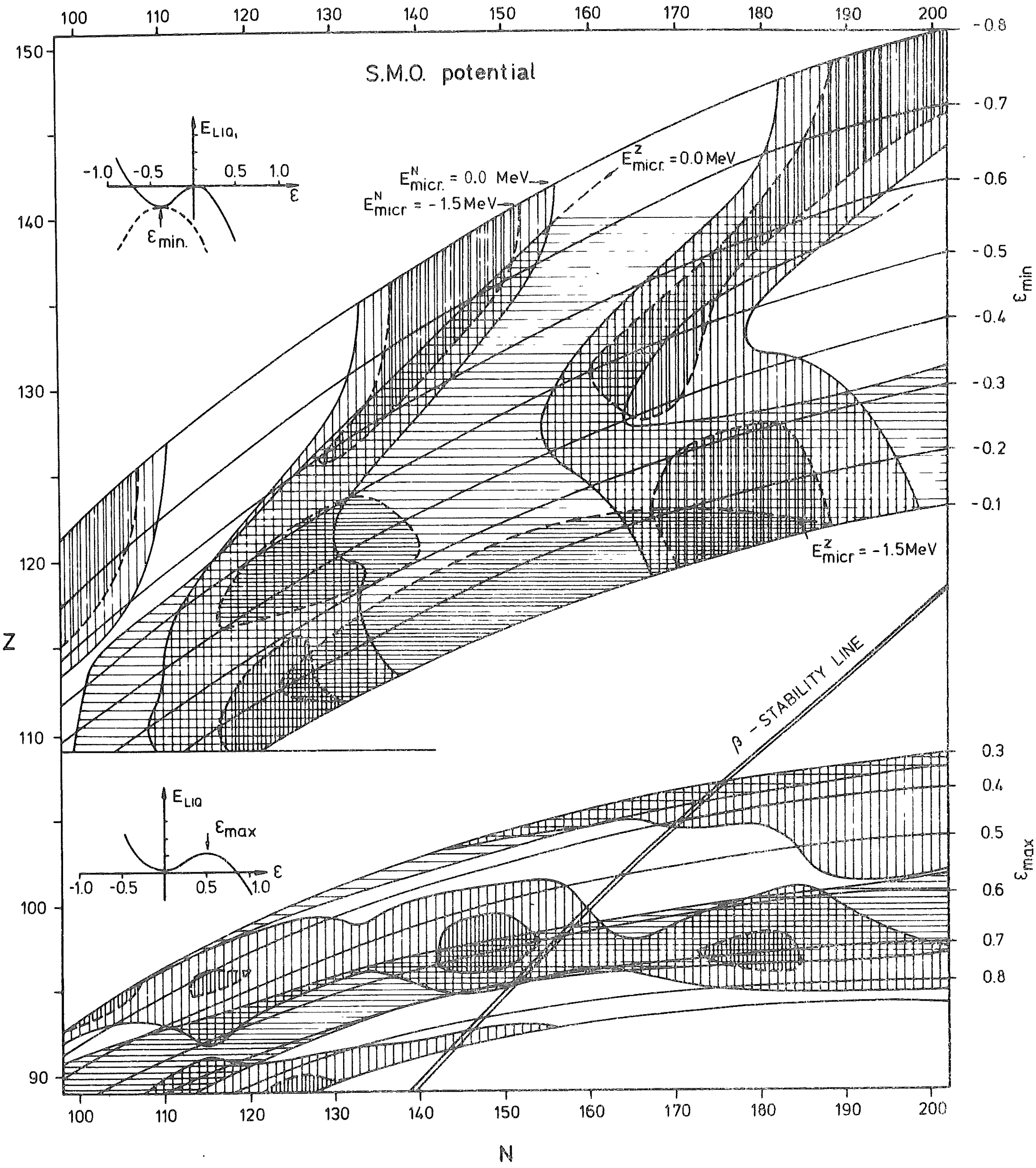

Fig. 1. Plot exhibiting the liquid-drop saddle-point distortion as a function of N and Z and in addition the neutron and proton shell energies for the deformations in question. Areas with both horizontal and vertical shading represent the most favourable regions for the occurrence of well-developed potential-energy minima. Note that the liquid-drop oblate "minimum" is converted into a saddle point.

0.6) in the potential-energy surface in the $N \approx 146$, $Z \approx 94$ region of nuclides (here indicated as $Z \approx 96$). Of the other favourable prolate regions, two have alrady been discussed in the literature. One corresponds to the region around $N \approx 110$, $Z \approx 90$ which seems favourable in the search for second minima. Parts of this region were studied by Tsang and Nilsson [8] and by Pashkevich [9]. Another favourable prolate region corresponds to $N \approx 180$, $Z \approx 96$ and $\epsilon \approx 0.7$. The occurrence of the second shell minimum leads to a drastic erosion of the fission barrier in this region of nuclei. For its probable connection with the termination of the multiple-neutron-capture path see refs. [10, 11].

On the oblate part of fig. 1 (corresponding to $x > 1$) there is more new information to be obtained. Thus reasonably favourable cases are indicated in the vicinity of $N \approx 176$, $Z \approx 128$ extending down to $N = 184$, $Z = 114$ and another region from $N = 160$, $Z = 130$ extending to the end of the figure at $N = 200$, $Z = 150$. In these regions we have in particular studied $N = 176$, $Z = 124$ and $N = 172$, $Z = 136$. Relative to the liquid-drop background there is thus a comfortable shell energy of -6 MeV for $N = 176$, $Z = 124$ at $\gamma = 124$ at $\gamma = 60°$, $\epsilon = 0.2$ and -7 MeV for $N = 172$, $Z = 136$ at $\gamma = 60°$, $\epsilon = 0.6$. The corresponding liquid-drop energy is, however, highly unstable to distortions in the gamma direction. The problem is how much stabilization the shell energy provides.

Detailed information on this point is supplied by a potential-energy calculation covering the (ϵ, γ) plane as shown in figs. 2 and 3 (no ϵ_4 is included). (The calculations are based on the M.O. model while the graph of fig. 1 is based on the S.M.O. model. At the small deformations in question the difference is negligible.) As seen from the plots there is obviously a trough for both nuclei leading from the oblate minimum in the gamma direction. For $^{300}124$ the saddle point is situated at $\gamma \approx 30°$. The resulting barrier height is only about 2.5 MeV. As pointed out above, the exploited minimum of fig. 2 is directly connected to the $N = 184$, $Z = 114$ spherical shell energy minimum. Thus, while $^{300}124$ has the minimum at $\epsilon = -0.20$, $^{296}120$ has two near-lying minima one at $\epsilon = -0.1$ and one at $\epsilon = -0.2$ and a barrier E_A of about 3 MeV; furthermore $^{292}116$ has $\epsilon = -0.08$ and $E_A \approx 4$ MeV. Obviously by approaching $N = 184$, $Z = 114$ a higher and broader barrier is achieved although the saddle point remains rather fixed in the (ϵ, γ) plane over this entire region of nuclei. The nucleus in turn becomes more difficult to produce experimentally as a compound nucleus in heavy-ion fusion.

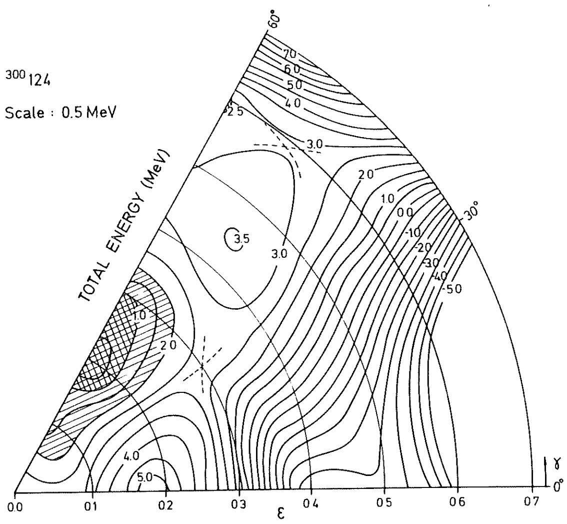

Fig. 2. Potential-energy surface for the nucleus $^{300}124$ as a function of ϵ and γ based on the M.O. potential. Note the axially asymmetric saddle point at $\gamma \approx 30°$.

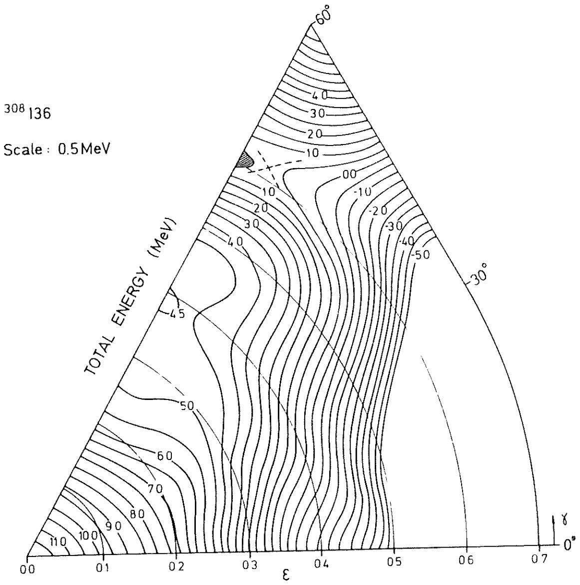

Fig. 3. Potential-energy surface for $^{308}136$. Note the very weak stability towards γ-deformation for the minimum located at $\epsilon \approx 0.6$, $\gamma = 60°$.

A few words should be added concerning fig. 3. Here the minimum is situated along the oblate axis, i.e., $\gamma = 60^\circ$ and $\epsilon = 0.60$. As seen from fig. 1, the corresponding neutron shell energy minimum is by a valley connected with the oscillator oblate 1 : 2 gap ($\epsilon = -0.75$) occurring for $N = 190$. However, the gamma instability of the liquid-drop energy is so large at this ϵ-value that the resulting barrier in the gamma direction is too weak to make the minimum experimentally observable. This is the case for all nuclei in the region starting at $N = 160$, $Z = 130$ and extending to the upper right corner of fig. 1 ($N = 200$, $Z = 150$).

The particular nucleus $^{300}124$ is actually possible to reach by a variety of heavy-ion reactions, in contrast to the superheavy elements situated near the beta-stability line.

Half-life estimates. To estimate the half-life towards fission of the minimum we need in addition the inertia tensor in the (ϵ, γ) plane. Although a cranking-model mass tensor would be interesting, a semiempirically corrected irrotational-flow value is quite sufficient for an order of magnitude estimate. In the latter approximation we have [12]

$$B_{\beta\beta} = B_0,\ B_{\gamma\gamma} = B_0\beta^2,\ B_{\beta\gamma} = 0, \tag{1}$$

where $B_0 = (3/8\pi)MAR^2$, which corresponds to $B_0/\hbar^2 = 55\ \mathrm{MeV}^{-1}$ for $A = 300$. (We shall below replace B_0 by $k \cdot B_0$.) As the circular path length on the triangular plot is $\beta d\gamma$, the circle radius being β, the geometry is thus isotropic in this type of plot.

Estimating the fission half-life from the WKB-approximation we need the penetrability integral

$$\frac{1}{\hbar}\int_{\alpha_0}^{\alpha_1}\left(\sum_{ij} 2B_{ij}[V(\alpha)-E]\frac{dx_i}{d\alpha}\frac{dx_j}{d\alpha}\right)^{1/2} d\alpha = \frac{1}{\hbar}\sqrt{2kB_0}\int_{s_0}^{s_1}(V(s)-E)^{1/2}\,ds \tag{2}$$

where ds is the line element in the (β, γ)-space and where k is an empirical factor multiplying the irrotational-flow value. The potential-energy plot of figs. 2 and 3 is laid out in ϵ and γ. However, a transformation to β and γ has been carried out and gives in this region a change in path-length of the order of 10%. The penetrability integral can be evaluated numerically and a value of 4.83 is obtained for $^{300}_{124}\mathrm{X}$ under the assumption of only 0.5 MeV total zero-point energy and a minimum-action path (assuming $k = 4$, see below).

The experience from cranking-model calculations in this region of distortions as well as semiempirical estimates from half-lives and other data is that $B_{\epsilon\epsilon}$ is 4–11 times larger than the irrotational-flow value [13]. The upper limit corresponds to the cranking-model result obtained for this region of mass and deformation by Sobiczewski [14, 15]. The two limiting values for $^{300}_{124}\mathrm{X}$ result in fission half-life estimates between 10^{-17} and 10^{-14} sec. The corresponding alpha decay half-life calculated according to ref. [16] is 10^{-9} sec.

In addition some other factors are likely to boost these short half-lives. A somewhat higher (by 1/2–1 MeV) barrier towards gamma distortions might result with the use of an angular-dependent pairing force (as, e.g., a quadrupole pairing term [17]). The odd-A or odd-odd case with the odd-particle(s) placed in high-Ω orbital(s) might also increase the barrier although the gamma distortion already for $\gamma \approx 10$–15° tends to mix Ω-values strongly. (As is well-known, for odd-odd fission isomers in the actinide region hindrance factors of the order of 10^3 are encountered.)

Finally, we have investigated the region of elements associated with spherical doubly closed shells for both protons ($Z = 114$) and neutrons ($N = 126$). The calculated fission half-life for the longest-lived isotope $^{240}114$, with a barrier of 9 MeV, lies between 1 sec and 10^7 years (for the above-mentioned two limits on the inertia, $k = 4$ and 11, respectively). The corresponding alpha decay half-life is 10^{-8} sec. An estimate, based on the liquid-drop formula [3], of the stability with respect to proton emission indicates that nuclei with 126 neutrons become unstable to proton emission by the time $Z = 96$ is reached. It should, however, be mentioned that the liquid-drop formula is not expected to predict the proper position of the proton drip line, since we are far from the beta-stability line. In addition the position of the drip line is influenced by shell effects and odd-even effects.

The synthesis of the nucleus $^{240}114$ obviously appears highly improbable. Of more interest is probably to find some nuclei in the region on the neutron-rich side of $^{240}114$ which still have an appreciable half-life relative to fission due to proximity of the double-shell closure and which on the other hand are more close to the proton drip line (or even inside). The fission barriers are found to be 4–4.5 MeV high for the nuclei $^{236}106$, $^{244}110$ and $^{252}114$ and the fission half-lives between 10^{-6} sec and 10^4 sec (based on the two limits of the *B*-parameter given above). The alpha decay half-lives are estimated around 10^{-11} sec. Furthermore, the latter nuclei are found almost on the proton drip line [3].

These nuclei have thus half-lives long enough for their possible detection provided they can be produced which, however, seems rather difficult but not impossible. The farthest one may reach into this region by the way of a compound nucleus appears to result from the fusion of $^{112}_{50}$Sn with $^{144}_{62}$Sm, which thus leads to $^{256}112$, or alternatively $^{112}_{50}$Sn with $^{124}_{54}$Xe, leading to $^{236}104$.

In total, the possibilities appear sufficiently promising for a search for compound nuclei with half-lives in the nano- to pico-second region with Z between 124 and 120 and N near 176, i.e., in the North-Western part of the usually considered superheavy-element region already considered by, e.g., Nix [18] and possibly also in the South-Western region around the nucleus $^{240}_{114}$X. The alternative region connected with $Z = 134$, $N = 172$ unfortunately appears less promising due to the gamma instability encountered.

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