



# LUND UNIVERSITY

## Upper bounds on the probability of the correct path loss for list decoding of fixed convolutional codes

Johannesson, Rolf; Zigangirov, Kamil

*Published in:*  
[Host publication title missing]

*DOI:*  
[10.1109/ISIT.1995.531512](https://doi.org/10.1109/ISIT.1995.531512)

1995

[Link to publication](#)

*Citation for published version (APA):*  
Johannesson, R., & Zigangirov, K. (1995). Upper bounds on the probability of the correct path loss for list decoding of fixed convolutional codes. In *[Host publication title missing]* (pp. 163)  
<https://doi.org/10.1109/ISIT.1995.531512>

*Total number of authors:*  
2

### General rights

Unless other specific re-use rights are stated the following general rights apply:  
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00

# Upper Bounds on the Probability of the Correct Path Loss for List Decoding of Fixed Convolutional Codes <sup>1</sup>

Rolf Johannesson  
Dept. of Information Theory  
Lund University  
P.O. Box 118  
S-221 00 LUND, Sweden

Kamil Sh. Zigangirov  
Dept. of Telecommunication Theory  
Lund University  
P.O. Box 118  
S-221 00 LUND, Sweden

*Abstract* — In list decoding ( $M$ -algorithm) the decoder state space is typically much smaller than the encoder state space. Hence, it can happen that the correct path is lost. This is a serious kind of error event that is typical for list decoding. In this paper two upper bounds on the probability of correct path loss for list decoding are given. For fixed convolutional codes counterparts to Viterbi's upper bounds for maximum-likelihood decoding of fixed convolutional codes are proved. Finally, it is shown that there exists a fixed convolutional code whose probability of correct path loss when decoded by list decoding satisfies a simple expurgated bound.

## I. INTRODUCTION

Viterbi decoding is an example of a non-backtracking decoding method that at each time instant examines the total encoder state space. The error correcting capability of the code is fully exploited.

In *list decoding* ( $M$ -algorithm) we first limit the resources of the decoder, then we choose an encoding matrix with a state space that is larger than the decoder state space. Thus, assuming the same decoder complexity, we use a more powerful code with list decoding than with Viterbi decoding. A list decoder is a very powerful non-backtracking decoding method that does not fully exploit the error correcting capability of the code.

List decoding is a breadth-first search of the code tree. At each depth only the  $L$  most promising subpaths are extended, not all, as is the case with Viterbi decoding. These subpaths form a *list* of size  $L$ .

Since the search is breadth-first, all subpaths on the list are of the same length and finding the  $L$  best extensions reduces to choosing the  $L$  extensions with the largest values of the cumulative Viterbi metric.

## II. THE CORRECT PATH LOSS PROBLEM

Since only the  $L$  best extensions are kept it can happen that the correct path is lost. This is a very severe event that causes many bit errors. If the decoder cannot recover a lost correct path it is of course a "catastrophe", i.e., a situation similar to the catastrophic error propagation that can occur when a catastrophic encoding matrix is used to encode the information sequence.

The list decoder's ability to recover a lost correct path depends heavily on the type of *encoder* that is used. A systematic encoder supports a spontaneous recovery.

<sup>1</sup>This work was supported in part by the Swedish Research Council for Engineering Sciences under Grants 92-661 and 94-83.

## III. UPPER BOUNDS ON THE PROBABILITY OF CORRECT PATH LOSS

The correct path loss on the  $i$ th step of a list decoding algorithm is a random event  $\mathcal{E}_i$  which consists of deleting at the  $i$ th step the correct codeword from the list of the  $L$  most likely codewords.

To upper bound  $P(\mathcal{E}_i)$  we introduce the  *$l$ -list generating function for the path weights*  $T_l(D)$ . Consider the trellis for a rate  $R = b/c$  and memory  $m$  fixed convolutional code. At a given depth consider the set of  $2^{bm}$  paths of least weight leading to the  $2^{bm}$  states. Order these paths according to increasing weights and let  $w_j$  denote the weight of the  $j$ th path ( $w_0 = 0$ ). Introducing

$$T_l(D) = \sum_{j=l}^{2^{bm}-1} D^{w_j},$$

the  $l$ -list generating function of the path weights, we can prove the following

**Theorem 1** For the BSC with crossover probability  $\epsilon$  and fixed convolutional codes with  $l$ -list generating function  $T_l(D)$  the probability of correct path loss is upper bounded by

$$P(\mathcal{E}_i) \leq \min_{1 \leq l \leq L} \frac{T_l(D) |_{D=\sqrt{4\epsilon(1-\epsilon)}}}{L-l+1}.$$

□

For the Gaussian channel we have the corresponding bound:

**Theorem 2** For the channel with additive white Gaussian noise (AWGN) with signal-to-noise ratio  $E_b/N_0$  and fixed convolutional codes of rate  $R$  with  $l$ -list generating function  $T_l(D)$  the probability of correct path loss is upper bounded by

$$P(\mathcal{E}_i) \leq \min_{1 \leq l \leq L} \frac{T_l(D) |_{D=e^{-RE_b/N_0}}}{L-l+1}.$$

□

Furthermore, we can prove

**Theorem 3** There exists a fixed convolutional code satisfying the following expurgated bound:

$$P(\mathcal{E}_i) \leq L^{-\frac{\log_2 \sqrt{4\epsilon(1-\epsilon)}}{\log_2(2^{1-R}-1)}} \cdot O(1).$$

□

## REFERENCES

- [1] Kamil Sh. Zigangirov and Harro Osthoff: "Analysis of Global-list Decoding for Convolutional Codes". European Transaction on Telecommunications, No. 2, 1993.