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Upper Bounds on the Probability of the Correct Path Loss for List Decoding of Fixed Convolutional Codes [1]

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Abstract — **In list decoding (M-algorithm) the decoder state space is typically much smaller than the encoder state space. Hence, it can happen that the correct path is lost. This is a serious kind of error event that is typical for list decoding. In this paper two upper bounds on the probability of correct path loss for list decoding are given. For fixed convolutional codes counterparts to Viterbi's upper bounds for maximum-likelihood decoding of fixed convolutional codes are proved. Finally, it is shown that there exists a fixed convolutional code whose probability of correct path loss when decoded by list decoding satisfies a simple expurgated bound.**

I. Introduction

Viterbi decoding is an example of a non-backtracking decoding method that at each time instant examines the total encoder state space. The error correcting capability of the code is fully exploited.

In *list decoding* (M-algorithm) we first limit the resources of the *decoder*, then we choose an encoding matrix with a state space that is larger than the *decoder* state space. Thus, assuming the same decoder complexity, we use a more powerful code with list decoding than with Viterbi decoding. A list decoder is a very powerful non-backtracking decoding method that does not fully exploit the error correcting capability of the code.

List decoding is a breadth-first search of the code tree. At each depth only the L most promising subpaths are extended, not all, as is the case with Viterbi decoding. These subpaths form a *list* of size L.

Since the search is breadth-first, all subpaths on the list are of the same length and finding the L best extensions reduces to choosing the L extensions with the largest values of the cumulative Viterbi metric.

II. The correct path loss problem

Since only the L best extensions are kept it can happen that the correct path is lost. This is a very severe event that causes many bit errors. If the decoder cannot recover a lost correct path it is of course a "catastrophe", i.e., a situation similar to the catastrophic error propagation that can occur when a catastrophic encoding matrix is used to encode the information sequence.

The list decoder's ability to recover a lost correct path depends heavily on the type of *encoder* that is used. A systematic encoder supports a spontaneous recovery.

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III. Upper bounds on the probability of correct path loss

The correct path loss on the ith step of a list decoding algorithm is a random event $\mathcal{E}_i$ which consists of deleting at the ith step the correct codeword from the list of the L most likely codewords.

To upper bound $P(\mathcal{E}_i)$ we introduce the *l-list generating function for the path weights* $T_l(D)$. Consider the trellis for a rate $R = b/c$ and memory m fixed convolutional code. At a given depth consider the set of 2^{bm} paths of least weight leading to the 2^{bm} states. Order these paths according to increasing weights and let w_j denote the weight of the jth path ($w_0 = 0$). Introducing

$$T_l(D) = \sum_{j=l}^{2^{bm}-1} D^{w_j},$$

the l-list generating function of the path weights, we can prove the following

Theorem 1 *For the BSC with crossover probability ε and fixed convolutional codes with l-list generating function $T_l(D)$ the probability of correct path loss is upper bounded by*

$$P(\mathcal{E}_i) \leq \min_{1\leq l\leq L} \frac{T_l(D)\mid_{D=\sqrt{4\varepsilon(1-\varepsilon)}}}{L-l+1}.$$

□

For the Gaussian channel we have the corresponding bound:

Theorem 2 *For the channel with additive white Gaussian noise (AWGN) with signal-to-noise ratio E_b/N_0 and fixed convolutional codes of rate R with l-list generating function $T_l(D)$ the probability of correct path loss is upper bounded by*

$$P(\mathcal{E}_i) \leq \min_{1\leq l\leq L} \frac{T_l(D)\mid_{D=e^{-RE_b/N_0}}}{L-l+1}.$$

□

Furthermore, we can prove

Theorem 3 *There exists a fixed convolutional code satisfying the following expurgated bound:*

$$P(\mathcal{E}_i) \leq L^{-\frac{\log_2\sqrt{4\varepsilon(1-\varepsilon)}}{\log_2(2^{1-R}-1)}} \cdot O(1).$$

□

References

[1] Kamil Sh. Zigangirov and Harro Osthoff: "Analysis of Global-list Decoding for Convolutional Codes". European Transaction on Telecommunications, No. 2, 1993.