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ADAPTIVE CONTROL OF SYSTEMS SUBJECT TO LARGE PARAMETER CHANGES

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Abstract. A method to handle large parameter changes in adaptive control is described. A fault detection procedure is introduced, and the gain in the estimator is increased whenever a fault occurs. A new fault detection procedure is presented, which fulfils the requirements of this special application. One of these requirements is, that the noise variance is not assumed to be constant. The fault detection method can also be applied to ordinary fault detection problems.

Keywords: Adaptive control, Fault detection, Parameter estimation, Time-varying systems.

INTRODUCTION

The ability to track time-variations in the process model is one of the main advantages of an adaptive controller, compared with fixed parameter controllers. Estimation of time-varying parameters is therefore a key issue in adaptive control. To be able to track time-varying parameters, the measurements entering the estimator must be weighted properly. If the plant to be identified is time-varying, old input-output pairs may not be relevant for the actual model. Their influence on the estimates should therefore be reduced.

The set of admissible time-variations in the adaptive control concept can be separated into two categories: Slow parameter changes, and large parameter changes which occur infrequently.

Slow parameter changes are handled by preventing the gain in the estimator from becoming too small. This is mostly done by introducing a forgetting factor, which causes an exponentially decreasing weight of the measurements. This method is known to cause problems in situations of poor excitation. Another way of handling slow parameter changes is to restart the estimation repeatedly. In Hägglund (1983), a method is given which makes the estimator retain a constant amount of information.

Large parameter changes may be treated in a special way, since these changes can be detected. This paper is concerned with parameter estimation in the case of large parameter changes. The material is picked from Hägglund (1983), where more details are to be found.

The problem can be divided into two parts: Detection of parameter changes and modification of the estimation algorithm. The first part is related to fault detection. The paper therefore begins with a short discussion of earlier fault detection methods, and requirements on a fault detection procedure which is suitable for adaptive control are set up. A new fault detection approach is then presented, and the problem of modifying the estimation procedure when a fault is detected is treated. Finally, the new fault detection procedure is illustrated by a simulation example.

FAULT DETECTION METHODS

Throughout the paper, it is assumed that the process can be described by the model

$$y(t) = \theta(t-1)^T\varphi(t) + e(t) \qquad (1)$$

where $y(t)$ is the measured output from the process, $\varphi(t)$ is a vector containing old inputs and outputs of the process, $\{e(t)\}$ is a disturbance sequence of independent random variables and $\theta(t)$ is a parameter vector. Furthermore, it will be assumed that the disturbances $\{e(t)\}$ have a symmetrical probability distribution. The restriction to white noise disturbances is made just for convenience. Coloured noise can also be treated, as is discussed below.

It should first of all be mentioned that the notation "fault" in this paper means a change in the process model, more precisely in the parameter vector $\theta(t)$, which does not necessarily originate from a physical fault in the process. It can e.g. just as well be a parameter change due to a shift of the operating point in a nonlinear system.

A great variety of methods for fault detection has appeared in recent years. Some of them are general, while others are devoted to special applications or concerned with voting between some known models. Surveys of fault detection methods and references to applications are given in Basseville (1982) and in Willsky (1976). The adaptive control problem requires a general method. The following discussion is therefore restricted to such approaches.

A fault detection procedure consists in forming a test sequence which is sensitive to faults, i.e. which has significantly different properties before and after a fault. This sequence is then analysed and decision theory is applied to decide if and when a fault occurs.

The residual sequence $\{\varepsilon(t)\}$, i.e. the differences between the true output signals and the expected output signals of the system, is the predominantly used test sequence. The expected output signals are mostly derived from a Kalman filter or a parameter estimation algorithm. When the statistics of $\{\varepsilon(t)\}$ differs considerably from the measurement noise sequence $\{e(t)\}$, a fault is concluded.

There are two great disadvantages with such tests. First, the statistics of the noise sequence $\{e(t)\}$ must be known to enable any decision about faults. This is easily seen in Equation (1), where a registered change of the statistical properties of $\{\varepsilon(t)\}$ obviously can originate from either a fault or a change in the noise sequence $\{e(t)\}$. The assumption of known disturbance statistics is further discussed in the next section.

The second disadvantage is that only faults that have a large influence on the output signal can be expected to be detected. In processes with reasonable noise levels, large faults may often occur without any immediate large effects on the output signals. It should be possible to detect such successive effects in the output signal by a suitable nonlinear dynamic manipulation of the measurement sequence.

The parameter estimator used in an adaptive controller is such a filter, and it produces estimates of the parameter vector $\theta(t)$. Since the problem of fault detection is concerned with changes in this vector, it seems natural to use the estimate sequence $\{\hat{\theta}(t)\}$ as a starting point for detection.

In spite of the drawbacks of using the residuals $\varepsilon(t)$ as a test sequence, this use is seldom questioned in the literature. Far more interest is payed to the choice of decision method. All variants from the Sequential Probability Ratio Test, see Wald (1947), to simple cumulative sum tests have been suggested. It would lead too far from the theme of this paper to discuss these methods in detail, but the reader is refered to the references, Basseville (1982) and Willsky (1976), which give extensive reviews with many references.

REQUIREMENTS ON THE FAULT DETECTION

To facilitate the choice of fault detection method, to be used in an adaptive controller, some natural requirements for this special application will be stated here.

(R1) The times when the faults occur are not known.

(R2) The nature of the faults is not known.

Since the transformation between the physical parameters in the process and the parameters in the model (1) is usually quite involved, this is a natural requirement.

(R3) It must be possible to repeat the detection from the new modes of operation.

This means e.g. that there does not exist any "normal mode". As soon as a change in $\theta(t)$ is accepted, the old parameters are forgotten. This requirement is considered to give a general method. In some applications it can be relaxed.

(R4) A change in the noise level must not disturb the detection.

The only assumption made on the noise sequence $\{e(t)\}$ is that it consists of independent symmetrically distributed random variables. Therefore, a change in the noise level does not effect the parameters $\theta(t)$. This is an important requirement, since a change in the noise level is often much more likely than a change in the process parameters.

Requirement (R4) is important, not only for the reason given above. In real processes, disturbances are often entering at several points, and not only additively to the input or output signals. In the process model, the different disturbance sources are represented by one equivalent source entering at one point, see Åström (1970). The characteristics of these equivalent disturbances depend on the process parameters. This means that a change in the parameter vector usually also causes a change in the equivalent output noise level. Under these circumstances, it does not seem very realistic to detect faults under the assumption that the noise level in the output is constant.

According to the previous section, the requirement (R4) unfortunately rules out most of the existing fault detection methods. A new fault detection procedure which satisfies the above requirements will now be presented. It was first described in Hägglund (1982).

A NEW FAULT DETECTION METHOD

A new fault detection method will now be discussed. The least squares parameter estimation method with constant forgetting factor will be used as a starting point. There are two reasons for this. First of all, the new fault detection method will not be restricted to any particular estimation scheme, so the conversion to e.g. the new discounting principle given in Hägglund (1983) is trivial. Secondly, the least squares method with forgetting factor is still the most common estimation scheme in adaptive control.

The equations of the least squares estimator with a constant forgetting factor are

$$\hat{\theta}(t) = \hat{\theta}(t-1) + P(t)\varphi(t)\varepsilon(t) \tag{2a}$$

$$P(t) = \frac{1}{\lambda}\left[P(t-1) - \frac{P(t-1)\varphi(t)\varphi(t)^T P(t-1)}{\lambda + \varphi(t)^T P(t-1)\varphi(t)} \right] \tag{2b}$$

$$\varepsilon(t) = y(t) - \hat{y}(t) = \left[\theta(t-1) - \hat{\theta}(t-1)\right]^T \varphi(t) + e(t) \stackrel{\Delta}{=} \tilde{\theta}(t-1)^T \varphi(t) + e(t) \tag{2c}$$

Here $\hat{\theta}(t)$ is the estimate of $\theta(t)$, $\tilde{\theta}(t)$ is the estimation error at time t and $\hat{y}(t)$ is the prediction of y(t) made at time t-1.

The real problem is to detect changes in the parameter vector $\theta(t)$. The vector $\theta(t)$ is not known, and neither is $\tilde{\theta}(t)$. However, the difference between two successive estimation errors $\Delta\tilde{\theta}(t)$ is known for $\theta(t)$ constant, since

$$\Delta\tilde{\theta}(t) \stackrel{\Delta}{=} \tilde{\theta}(t) - \tilde{\theta}(t-1) = \theta(t) - \hat{\theta}(t) - \theta(t-1) + \hat{\theta}(t-1) = -\hat{\theta}(t) + \hat{\theta}(t-1) = -\Delta\hat{\theta}(t) \tag{3}$$

in this case. These differences will give the information needed for the fault detection. To be able to extract this information, the statistics of $\{\Delta\hat{\theta}(t)\}$ will first be investigated.

From Equation (2) the differences between two successive estimates are given by

$$\Delta\hat{\theta}(t) = P(t)\varphi(t)\left[\varphi(t)^T\tilde{\theta}(t-1) + e(t)\right] \tag{4}$$

At time t, the estimates are thus updated in the direction of the vector $P(t)\varphi(t)$. The probabilities of positive and negative direction are almost the same in normal operation when no fault has occurred, i.e. when the estimated parameters are close to the true ones. This is intuitively seen from the following arguments.

When $\lambda = 1$, the estimation procedure is the ordinary recursive least squares algorithm without any discounting of past data. It is known to be the best linear unbiased estimator, see e.g. Goodwin and Payne (1977). This implies that there is no correlation between the increments of the parameter estimates in normal operation. If there were a correlation, it would be possible to modify the algorithm so that a smaller variance of the estimates were obtained. Hence, when $\lambda = 1$ the probabilities for the estimate increments to have positive and negative $P(t)\varphi(t)$ direction are the same, 0.5.

When $\lambda < 1$, a negative correlation between two successive estimate increments is expected. If a forgetting factor less than one is used, the gain in the parameter estimator is greater than it should be for $\lambda = 1$. Intuitively this means that the algorithm in each updating of the estimates has to compensate for the large step taken previously. Hence the expected correlation is negative. However, from continuity arguments this correlation is small when λ is close to one, and the probabilities of positive and negative $P(t)\varphi(t)$ direction of the estimate increments are approximately the same. This is illustrated in Example 1 below.

The arguments above imply that under normal operation

$$P\left[\Delta\hat{\theta}(t)^T\Delta\hat{\theta}(t-1)>0\right] \approx P\left[\Delta\hat{\theta}(t)^T\Delta\hat{\theta}(t-1)<0\right] \quad (5)$$

where P denotes the probability measure.

When $\tilde{\theta}(t)$ is not close to its true value, i.e. when a fault has occurred, the approximations used in the heuristic arguments above are no longer valid. Since the estimated parameters then will be driven towards the new values, the following inequality holds

$$P\left[\Delta\hat{\theta}(t)^T\Delta\hat{\theta}(t-1)>0\right] > P\left[\Delta\hat{\theta}(t)^T\Delta\hat{\theta}(t-1)<0\right] \quad (6)$$

The intuitive way of arguing that the correlation between successive estimate increments is small in case of constant parameters may be unappealing to readers familiar with more strict mathematical derivations. In the following example, Equation (5) is verified under fairly hard restrictions.

Example 1. Consider the process model

$$y(t) = \theta\cdot u + e(t) \quad (7)$$

where the input u is constant and $\{e(t)\}$ is a sequence of independent Gaussian random variables. If the estimator defined by Equation (2) is applied, the P-matrix converges to the constant scalar

$$P = \frac{1-\lambda}{u^2} \quad (8)$$

The updating formula of the estimate of θ then becomes

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{1-\lambda}{u^2}\left[\tilde{\theta}(t-1)\cdot u + e(t)\right] \quad (9)$$

The following expression for the probabilities can now be obtained

$$P\left[\Delta\hat{\theta}(t)\cdot\Delta\hat{\theta}(t-1)<0\right] = \frac{1}{2} + \frac{1}{\pi}\arcsin\left[\frac{1-\lambda}{2}\right] \quad (10)$$

For the details, see Hägglund (1983). Table 1 gives some numerical values of the probability of getting a negative scalar product between two successive estimate increments. For reasonable values of λ, Equation (5) is obviously true in this simple example. □

λ	0.9	0.95	0.98	0.99	0.999
Probability	0.5159	0.5080	0.5021	0.5016	0.5002

Table 1 - The probability of getting a negative scalar product between successive estimate increments in Example 1.

Implementation

Instead of observing the scalar product between two successive estimate increments, it is often more efficient to study the scalar product between $\Delta\hat{\theta}(t)$ and a sum of the latest estimate increments. To simplify the algorithm, an exponential filtering of the increments of the estimates will be used instead of an ordinary sum. For this purpose, introduce w(t) as

$$w(t) = \gamma_1 w(t-1) + \Delta\hat{\theta}(t) \qquad 0 \le \gamma_1 < 1 \quad (11)$$

In the case when a fault has occurred, w(t) can be viewed as an estimate of the direction of the parameter change. The motivations for the Equations (5) and (6) are valid even when w(t-1) is substituted for $\Delta\hat{\theta}(t-1)$. The test sequence that will be studied is $\{s(t)\}$, where s(t) is defined as

$$s(t) \triangleq \mathrm{sign}\left[\Delta\hat{\theta}(t)^T w(t-1)\right] \quad (12)$$

The sign function makes the test sequence insensitive to the noise variance. It is now clear in principle how to carry out the fault detection:

> "Inspect the latest values of s(t). If s(t) is +1 unlikely many times, conclude that a fault has occurred."

The idea to use the signs of the differences between successive estimates to decide whether the estimates has converged or not has been proposed before. Kesten (1958) proposed a method to accelerate a stochastic approximation method by letting the gain of the estimator depend on the frequency of the changes of these signs.

Testing method

Under normal operation, i.e. when the parameter estimates are close to their true values, s(t) has approximately a symmetric two point distribution with mass 0.5 each at +1 and -1. When a fault has occurred, the distribution is no longer symmetric, but the mass at +1 is larger than the mass at -1. To add the most recent values of s(t), the stochastic variable r(t) defined as

$$r(t) = \gamma_2 r(t-1) + (1-\gamma_2)s(t) \qquad 0 \le \gamma_2 < 1 \quad (13)$$

is introduced. The sum of the most recent values of s(t) is replaced by an exponential smoothing in order to obtain a simple algorithm. When the parameter estimates are close to the true ones, r(t) has a mean value close to zero. When a fault has occurred, a positive mean is expected.

The parameter γ_2 determines, roughly speaking, how many s(t) values that should be included. E.g. $\gamma_2 = 0.95$ corresponds to about 20 values, which is a reasonable choice in many applications. A small γ_2 allows a fast fault detection, although at the price of less security against false alarms. This trade-off is typical for all fault detection methods. When the signal to noise ratio is small, it is not possible to detect the faults as fast as otherwise. It is then necessary to have more information available to decide whether a fault is present. This can be achieved by increasing γ_2.

For values of γ_2 close to one, r(t) will have an approximately Gaussian distribution with variance

$$\sigma^2 = \frac{1-\gamma_2}{1+\gamma_2}. \quad (14)$$

Since γ_2 is generally chosen in this region, it will in the sequel be assumed that r(t) is Gaussian.

If r(t) exceeds a certain threshold r_0, a fault may be concluded with a confidence determined from the value of the threshold. In the present algorithm, the threshold can be computed directly as a function of the rate of false alarms f_f. If a false alarm frequency equal to f_f is acceptable, a fault detection should be given every time r(t) is greater than the threshold r_0, defined by

$$P\{r(t)\geq r_0\} = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{r_0}^{\infty}\exp\left[-\frac{x^2}{2\sigma^2}\right]dx = f_f \quad (15)$$

If a small value of the threshold is chosen to make it possible to detect faults quickly, the false detection rate will be high. This is seen in Equation (15), where there is an inverse relation between r_0 and f_f. As was said before, this compromise between fast detection and security against false alarms must be made in all fault detection methods. The determination of r_0 in this method has the advantage that it is formulated in terms of the expected frequency of false detections, which may be chosen to suit any particular application. In Fig. 1, the error frequency f_f versus the threshold r_0 is presented for some values of γ_2.

The fault detection method described above fulfils the requirements (R1) - (R4).

MODIFICATION OF THE ESTIMATION ALGORITHM

The first part of a method to handle large parameter changes was given by the fault detection procedure derived in the previous sections. To complete the method, a procedure to increase the gain in the estimator, i.e. the P-matrix in Equation (2b), must also be established. From an information handling point of view, the increase of the gain in the estimator can also be seen as a reduction of the information content in the estimator. The inverse P-matrix denotes the information content. When a fault has occured, P^{-1} indicates a too large information content. By decreasing P^{-1} when a fault is detected, the performance of the estimator can be improved considerably.

The P-matrix can of course be increased in many ways, but there are mainly two methods that have been used previously. The first one is to decrease the forgetting factor λ. The growth of P(t) is then nearly exponential. The second method is to add a constant times the unity matrix to the P(t)-matrix, in which case P(t) is increased instantaneously.

When a large fault has occurred, the most reasonable direction of the parameter updating is along the φ(t) vector, both from stability and rate of convergence

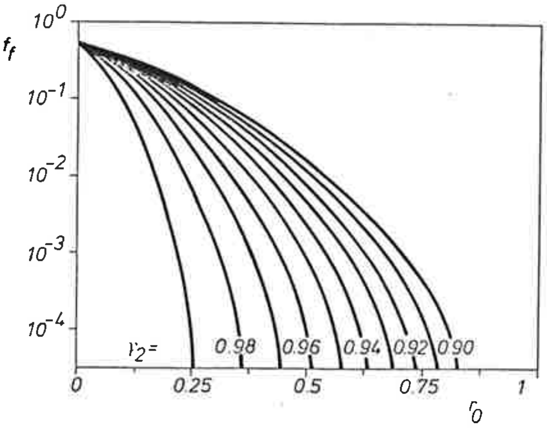

Fig. 1. The error frequency f_f versus the threshold r_0.

point of view. The gain in the estimation algorithm will therefore be increased according to the second method, and Equation (2b) will be substituted by

$$P(t) = \frac{1}{\lambda}\left[P(t-1) - \frac{P(t-1)\varphi(t)\varphi(t)^T P(t-1)}{\lambda + \varphi(t)^T P(t-1)\varphi(t)}\right] + $$
$$+ \beta(t)\cdot I \quad (16)$$

where β(t) is a nonnegative scalar and I is the unity matrix. The variable β(t) is zero except when a fault is detected. When a fault is detected, a positive β(t) has the effect that the P(t)-matrix increases and that the parameter updating is made in a direction closer to φ(t).

The final problem is to choose a suitable β(t). When no fault is detected, β(t) is zero. When a fault is detected, it is reasonable to let β(t) depend on the actual value of P(t) and on how significant the alarm is, i.e. on the value of r(t). This may of course be done in many ways, and the following proposal is just one possibility.

In the noise-free case, the progress of the estimation error, when θ(t) is constant, is given by

$$\tilde{\theta}(t) = \tilde{\theta}(t-1) - P(t)\varphi(t)\varepsilon(t) =$$
$$= \left[I - P(t)\varphi(t)\varphi(t)^T\right]\tilde{\theta}(t-1) \triangleq U(t)\tilde{\theta}(t-1) \quad (17)$$

All eigenvalues of U(t) are one, except the one corresponding to the eigenvector P(t)φ(t). This eigenvalue determines the step length in the algorithm. A small eigenvalue causes large steps, while an eigenvalue close to one means that the step length in the algorithm is small. Using Equation (16), the eigenvalue can be written as

$$1 - \varphi(t)^T P(t)\varphi(t) =$$
$$= \frac{\lambda}{\lambda + \varphi(t)^T P(t-1)\varphi(t)} - \beta(t)\varphi(t)^T\varphi(t) \quad (18)$$

When β(t) = 0, the eigenvalue is thus

$$\nu_0(t) = \frac{\lambda}{\lambda + \varphi(t)^T P(t-1)\varphi(t)} \quad (19)$$

The eigenvalue is obviously between zero and one as long as P > 0. Suppose now, that an eigenvalue equal to ν(t) is desired when a fault is detected. Then β(t) has to be chosen as

$$\beta(t) = \frac{1}{\varphi(t)^T\varphi(t)}\left[\nu_0(t) - \nu(t)\right] \quad (20)$$

The eigenvalue ν(t) should lie in the interval

$$0 < \nu(t) \leq \nu_0(t) \quad (21)$$

in order to keep the P(t)-matrix positive definite. In practice, this choice of β(t) must also be combined with a test for nonsingularity of $\varphi(t)^T\varphi(t)$.

It remains to determine a suitable ν(t). This can be done in many ways. In the example presented in the next section, ν(t) is a piecewise linear function of the significance of the fault alarm, see Fig. 2.

Combining the fault detection procedure with the modification of the estimation algorithm proposed in this section, a method to increase the gain in the estimation algorithm in case of large parameter changes is derived. The method is summarized in a block diagram in Fig. 3

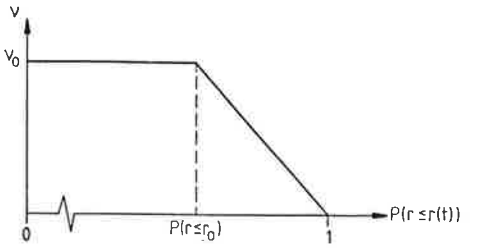

Fig. 2. An example of a choice of $\nu(t)$.

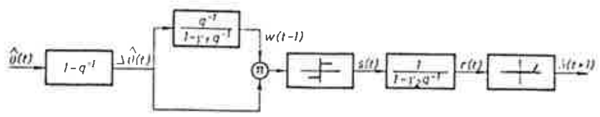

Fig. 3. A block diagram describing the fault detection method.

A SIMULATION EXAMPLE

To illustrate the new fault detection method and the modified estimation algorithm, a simulation study is presented in this section.

The system considered is shown in Fig. 4. The purpose of control is to keep the level in the tank constant. This is done by measuring the tank level and controlling the inlet valve. The dynamics of the tank is described by the equations

$$\frac{dh(t)}{dt} = \frac{1}{10}\left[q_{in}(t)-q_{out}(t)\right]+0.005e(t) \qquad (22a)$$

$$q_{out}(t) = a_{out}\sqrt{2gh(t)} \qquad (22b)$$

where $\{e(t)\}$ is a disturbance sequence and a_{out} is the outlet area. The sequence $\{e(t)\}$ is generated as discrete Gaussian N(0,1) random variables with a sampling period equal to 1/10:th of the controller sampling period. The stochastic part of the equations can be viewed as originating from irregularities in the flow.

The model of the tank used in the estimation algorithm is

$$h(t+1) = a(t)\cdot h(t) + u(t) + \xi(t) \qquad (23)$$

where u(t) is the control signal and $\{\xi(t)\}$ is a sequence of independent random variables. The parameter a(t) is estimated by the recursive least squares method according to equations (2a), (2c) and (16). The equations become

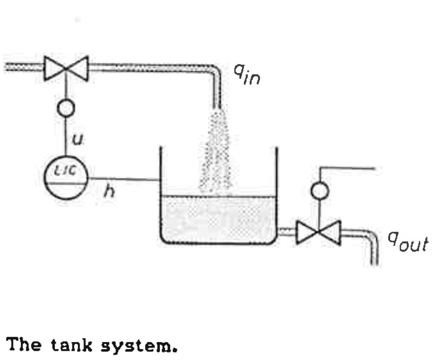

Fig. 4. The tank system.

$$\hat{a}(t) = \hat{a}(t-1) + P(t)h(t-1)\varepsilon(t)$$

$$\varepsilon(t) = h(t) - \hat{a}(t-1)\cdot h(t-1) - u(t-1) \qquad (24)$$

$$P(t) = \frac{P(t-1)}{\lambda + P(t-1)h(t-1)^2} + \beta(t)$$

The forgetting factor λ is chosen to 0.995. The equations of the fault detection procedure become

$$w(t) = \gamma_1 w(t-1) + \left[\hat{a}(t) - \hat{a}(t-1)\right]$$

$$s(t) = \text{sign}\left(\left[\hat{a}(t) - \hat{a}(t-1)\right]w(t-1)\right)$$

$$r(t) = \gamma_2 r(t-1) + (1 - \gamma_2)s(t) \qquad (25)$$

$$\nu_0(t) = \frac{\lambda}{\lambda + P(t-1)h(t-1)^2}$$

$$\beta(t) = \begin{cases} 0 & \text{if } r(t-1)<r_0 \\ \frac{1}{h(t-1)^2}\left[\nu_0(t) - \nu(t)\right] & \text{if } r(t-1)\ge r_0 \end{cases}$$

where the two discounting factors γ_1 and γ_2 are 0.85 and 0.95 respectively. The choice of $\nu(t)$ was presented in Fig. 2. The value of the threshold is r_0 = 0.5, which corresponds to an expected false alarm every 1000:th sample instant. The tank is controlled by a minimum variance regulator with set-point

$$u(t) = h_{ref} - \hat{a}(t)\cdot h(t) \qquad (26)$$

For comparison, the problem is first simulated without any fault detection. The result is shown in Fig. 6 and Fig. 7. At t=500, the outlet area is increased from 0.01 to 0.011, corresponding to a sudden increase in the outlet flow or a small leak in the tank. This fault is hard to see directly in the output-, input-, or residual sequences. However, looking at the estimated parameter $\hat{a}(t)$, the fault is obvious. In Fig. 7, the test sequence r(t) is shown. The values of the highest peaks are very unlikely in normal operation, and a fault would have been detected. Note that r(t) has an approximately Gaussian distribution with a standard deviation of 0.16 in case of no fault.

In Fig. 8 and Fig. 9, the result of the simulation is given when the fault detection and the modified estimation algorithm are applied. A detection is made after about 30 samples. The increased convergence rate is obvious. Finally, the loss functions in the two simulations are also compared in Fig. 10. Here the optimal loss function, i.e. the loss function obtained under control with known parameters, is also given.

CONCLUSIONS

The problem of adaptive control of systems subject to large parameter changes has been treated by introducing a fault detection procedure, and increasing the gain in the estimator whenever the faults occur. The new fault detection procedure can be applied to many estimation schemes, since the inputs to the detector are the parameter estimates. In Hägglund (1983), the method is e.g. applied to three different schemes. The situation of coloured noise can therefore also be handled, by using an estimation method suitable for such problems. The method is also able to detect faults that do not influence the magnitude of the residuals much. The new fault detection method fulfils the requirements (R1) - (R4) stated above. Since these requirements are natural, the new method is believed to be useful also in other areas of fault detection.

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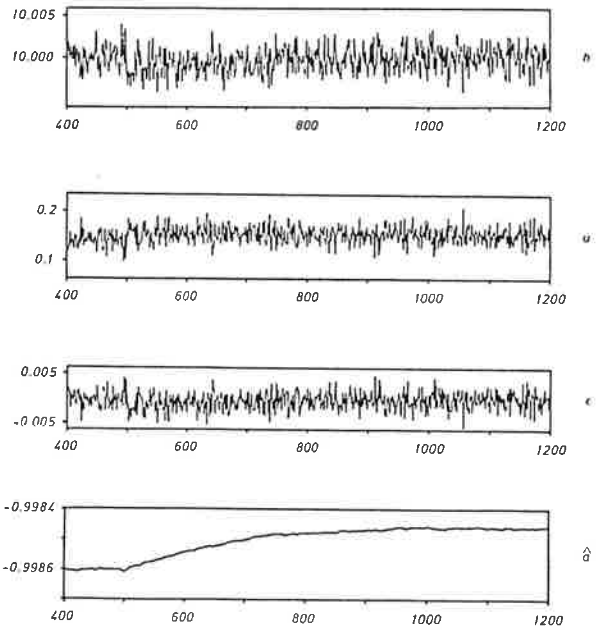

Fig. 6. The result of the simulation without fault detection.

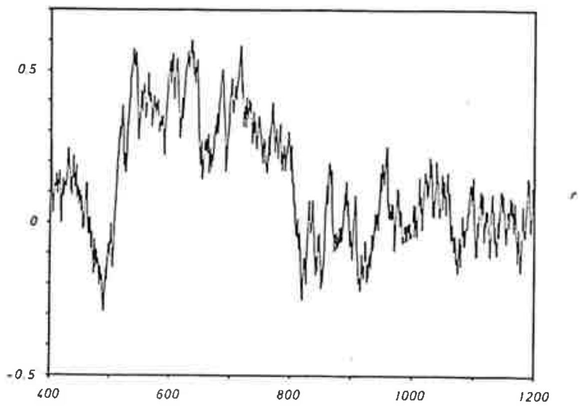

Fig. 7. The r(t) sequence when no modification of the estimation algorithm is done.

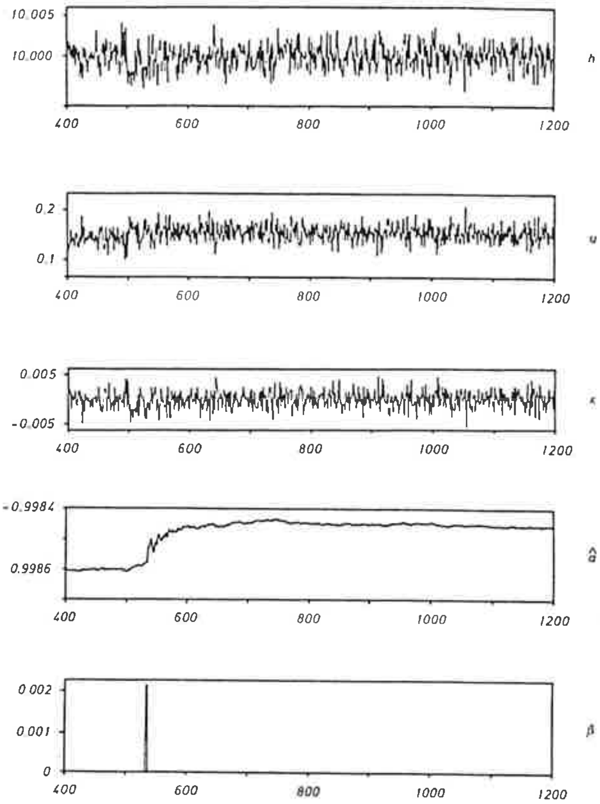

Fig. 8. The result of the simulation when the fault detection and the modified estimation algorithm are applied.

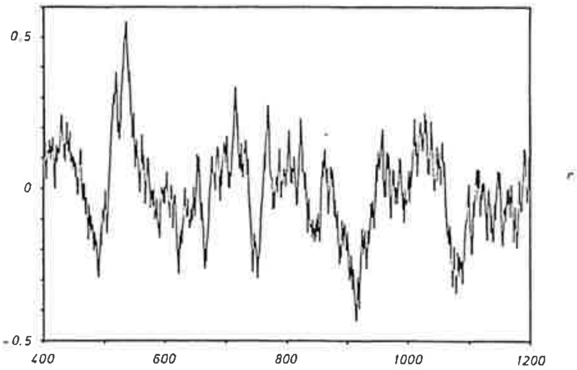

Fig. 9. The r(t) sequence when the fault detection and the modified estimation algorithm are applied.

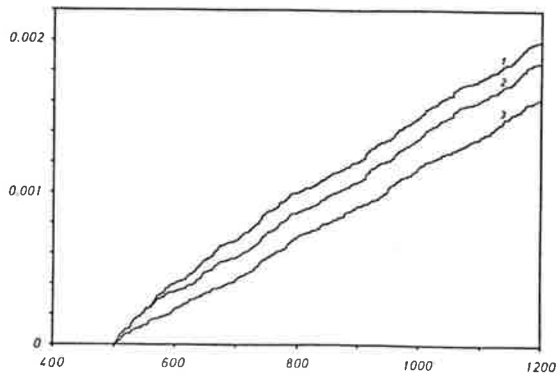

Fig. 10. The loss functions in the simulations without fault detection (1), with fault detection (2) and the optimal loss function (3).