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Abstract : This paper describes a predictive PI controller with dead-time compensation. The advantage of the new controller compared with other dead time compensating controllers is that it contains only three adjustable parameters; the gain, the integral time and the dead time. The controller is also suited for processes with varying dead times.

1. Introduction

Most of the control problems in the process industry are solved using PID controllers. There are several reasons for this. First of all, the PID controller can be tuned manually by "trial and error" procedures, since it only has three adjustable parameters. The instrumentations of the process plants are also performed to suit the simple structure of the PID controller. This is accomplished by dividing difficult control problems into several loops connected in cascade, ratio, feed-forward etc. There is also a long experience of PID control in the process industry.

When there are long dead times in the process, the control performance obtained with a PID controller is, however, limited. Predictive control is required to control a process with a long dead time efficiently. Predictive control means that the controller contains a mechanism for predicting future process outputs. The derivative part of the PID controller can be interpreted as a prediction mechanism. Unfortunately, prediction through derivation of the measurement signal is not appropriate when the process contains long dead times. Therefore, if a PID controller is applied on this kind of problems, the derivative part is mostly switched off, and only PI control is used. Since no predictive control is used, the control performance deteriorates.

Since there is not enough information in the measurement signal for the purpose of prediction, the prediction has to be based on the control signal, when the process has a long dead time. The prediction can be performed by an internal simulation of the process inside the controller. Such controllers are called dead-time compensating controllers. They require a model of the process, typically consisting of a gain, a time-constant and a dead time. Combined with a PI-control algorithm, this means that there are 5 parameters to tune. This is difficult to do by "trial and error" procedures. A systematic process identification experiment is needed to obtain the process model.

This paper describes a dead-time compensating controller with only three adjustable parameters. It is as easy to tune manually as an ordinary PID controller.

2. Predictive Control

Predictive control means that the controller has the possibility to predict the future changes of the measurement signal, and base the control action on this prediction. This section gives a short summary of two common methods to obtain predictive control, namely prediction by linear extrapolation of the measurement signal and prediction by on-line simulation using a process model.

Prediction by linear extrapolation

The basic structure of the PID controller is

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int e(s)ds + T_d\frac{de(t)}{dt}\right) \tag{1}$$

where u is the control signal, e is the control error signal, K is the controller gain, T_i is the integral time and T_d is the derivative time. In practice, it may look quite different, but that is not important for this discussion. In a PID controller, the prediction is performed by the derivative term. This can be seen more easily if we only look at the P and D parts of the controller:

$$u_{PD}(t) = K\left(e(t) + T_d\frac{de(t)}{dt}\right)$$

If the control error is a linear function of time we have

$$e(t) + T_d\frac{de(t)}{dt} = e(t + T_d)$$

The control law can thus be written as

$$u_{PD}(t) = Ke(t + T_d)$$

The control law is thus proportional to an estimate of the control error e a time T_d ahead, where the estimate is obtained by linear extrapolation.

Prediction by linear interpolation works well in many cases. Since it is sensitive to noise, it requires that the noise level in the measurement signal is not too high. This is the case even though most PID controllers combine a low-pass filter with the derivative part. Prediction by linear interpolation has been proven to be very useful in e.g. temperature control loops, where the need for predictive control is large and the measurement signals normally have a low noise level.

When the process contains a long dead time, prediction through derivation is not possible. The measurement signal does not contain enough information about future changes. Suppose e.g. that we want to change the measurement signal from one set point to another. If the dead time is long, the whole control action will idealy be taken before any change has occured in the measurement signal. Therefore, if a PID controller is used to control a process with a long dead time, it is advisable to switch off the derivative part and only work with PI control. The use of a PI controller means therefore that one has to accept a slow control. On the other hand, processes with long dead time are often difficult to control, and there is a large need for predictive control. This prediction must then be based on the control signal together with a model of the process instead of the measurement signal.

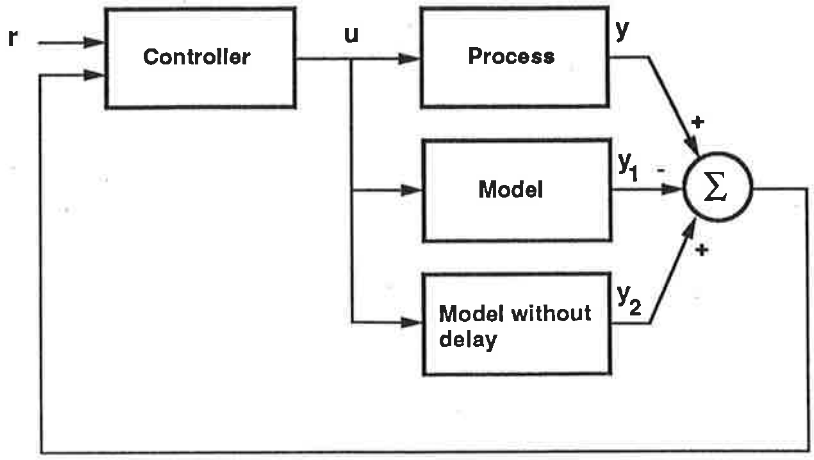


Figure 1. The Smith predictor

Model-based prediction

Prediction of future changes in the measurement signal can of course also be performed if we feed our control signal through a model of the process. This is the principle behind the dead-time compensating controllers. The most common dead-time compensating controller is the Smith predictor. The structure of this controller is shown in Figure 1. The controller output is fed through a model of the process and through the same model without dead time. In this way, the controller acts, in the ideal situation of perfect modeling, on a simulated process which behaves as if there were no dead time in the process. This is accomplished by letting the controller act on the prediction of the measurement signal $y(t+L)$, where L is the dead time. The control algorithm in a Smith predictor is normally a PI controller. The D-part is not needed since the prediction is performed by the dead-time compensation.

A Smith predictor requires a model of the process. The following model structure is commonly used

$$Y(s) = \frac{K_p e^{-Ls}}{1+sT} U(s) \tag{2}$$

i.e. a first order system with static gain K_p, time constant T and dead time L. A Smith predictor using this simple process model combined with a PI controller requires five parameters to be determined, namely K, T_i, K_p, T and L.

A PID controller can be tuned manually. Most process engineers know how to adjust the different parameters of the controller to obtain desired closed loop behaviours. They also know how the three parameters K, T_i, and T_d influence the control. The five parameters of the Smith predictor are very difficult to tune manually without a systematic process identification experiment. Replacing a PID controller with a Smith predictor gives therefore a drastic increase in operation complexity. This is the main reason why most processes with long dead times are still controlled by PI controllers.

3. The PIP Controller

In this section we will describe a model-based predictive PI controller (PIP) with only three adjustable parameters. The aim has been to provide a dead-time compensating controller which can be tuned manually in the same way as a PID controller.

Controller structure

The structure of the PIP controller is the same as the Smith predictor, but with the exception that two of the process model parameters are determined "automatically" based on the PI-parameters. In the PIP controller, the parameters K, T_i and L are determined by the operator. Parameters K_p and T, are calculated as functions of the K, T_i and L, i.e.

$$\begin{aligned} K_p &= f_1(K, T_i, L) \\ T &= f_2(K, T_i, L) \end{aligned} \tag{3}$$

Idealy, the PI controller in a dead-time compensating controller can be set as if no dead time were present. Therefore, it is reasonable to assume that f_1 and f_2 do not depend on L. Furthermore, the controller gain K is independent of the process time constant T, and the integral time T_i is independent of the process gain K_p. Equation (3) can therefore be reduced to

$$\begin{aligned} K_p &= f_1(K) \\ T &= f_2(T_i) \end{aligned} \tag{4}$$

The controller gain is chosen inversely proportional to the process gain, and the integral time is chosen proportional to the process time constant. The following equations are therefore reasonable relations between the controller parameters:

$$\begin{aligned} K_p &= \kappa/K \\ T &= \tau T_i \end{aligned} \tag{5}$$

where κ and τ are constants. Numerical values of these constants will be given later. Hence, instead of having five adjustable parameters, as in the Smith predictor, we have only three adjustable parameters. The process gain K_p is determined from the controller gain and the process time constant T is determined from the controller integral time.

Assuming that the controller in Figure 1 is a PI controller, and that the process model is given by Equation (2), the predictive controller can be expressed as

$$\begin{aligned} u(t) &= K\left(1 + \frac{1}{pT_i}\right)\left(e(t) - \frac{K_p}{1+pT}[u(t) - u(t-L)]\right) \\ &= K\left(1 + \frac{1}{pT_i}\right)e(t) - \frac{\kappa(1+pT_i)}{pT_i(1+p\tau T_i)}[u(t) - u(t-L)] \end{aligned} \tag{6}$$

where p is the differential operator $\frac{d}{dt}$. If Equation (6) is compared with the PID controller in Equation (1), we can see that both controllers have three terms. The difference is, that the term performing the prediction now consists of a low-pass filtering of the control signal instead of a high-pass filtering (derivation) of the measurement signal.

Choice of parameters κ and τ

Selection of parameters in the PIP controller will now be discussed. The design goal is to obtain a critically damped closed loop system, which is as fast as possible. Our process model is a first order system with a time delay. The dead time compensation reduces the effects of the time delay. Hence, the choice of the controller parameters K and T_i could ideally be performed as if the process were a pure first order system. For this simple process, it is possible to increase the gain to infinity and the integral time close to zero without instability problems. This would correspond to very large values of κ and τ.

Unmodeled dynamics and high frequency noise will however limit the values of K and T_i. We will therefore restrict the design effort in the following way. The open loop system has a pole at $s = -1/T$. With PI control the closed loop system is of second order. The design criterion is chosen so that the closed loop system has a double pole at $s = -1/T$. The following example gives the details.

EXAMPLE 1
Given a first order process with the transfer function

$$\frac{K_p}{1 + sT} = \frac{b}{s + a}$$

Find a PI controller such that the closed loop system has a double pole at $s = -a$. The characteristic equation and the desired characteristic equation become

$$s^2 + s(a + bK) + \frac{bK}{T_i} = 0$$

$$s^2 + 2as + a^2 = 0$$

The solution is given by the following set of PI parameters

$$K = \frac{a}{b} = \frac{1}{K_p}$$

$$T_i = \frac{bK}{a^2} = \frac{1}{a} = T$$

which corresponds to the choices $\kappa = 1$ and $\tau = 1$. Since the controller zero lies in $s = -1/T_i$, the transfer function between the set point and the measurement signal will be of first order with the time constant equal to the open loop time constant. □

In the simulation examples presented in the next section, we have made the choice

$$\kappa = 1 \qquad \tau = 1 \tag{7}$$

This choice gives a particularly simple form of Equation (6), namely

$$u(t) = K\left(1 + \frac{1}{pT_i}\right)e(t) - \frac{1}{pT_i}[u(t) - u(t - L)] \tag{8}$$

This implies that the open loop time constant of the process T is retained in the closed loop system. Other methods, such as the Ziegler-Nichols and

Cohen-Coon methods, try to obtain a closed loop time constant which is a function of the time delay L. This means that the PIP controller, with the above choices of κ and τ, is supposed to give a faster response when the time delay is long.

4. Properties of the PIP controller

The PIP controller is nothing but a Smith predictor with restrictions on the process model. These limitations are a drawback, in the same way as the simple structure of the PID controller is a drawback compared with other more sophisticated controller structures. On the other hand, the PIP controller has the same advantages as the PID controller, namely that it can be tuned manually.

Compared with a PID controller it has the advantage that it is able to perform predictive control even in the case of long dead times in the process. Furthermore, it performs the prediction without amplifying high frequency noise, as the derivative part of the PID controller does, since the prediction is performed by a low pass filtering of the control signal.

Load disturbances

It is often claimed that the use of a dead-time compensating controller instead of a PI controller is of less importance when the control loop is disturbed by load disturbances. See e.g. Rivera et al (1986). The following analysis shows that, for long time delays, the integrated absolute error (IAE) can be decreased down to a factor of 0.5 compared to what is obtained using a PI controller.

The PI control law is

$$u = Ke + \frac{K}{T_i} \int e(t)dt$$

After a step change in the load, the control signal has changed with the amount

$$\Delta u = \frac{K}{T_i} \int e(t)dt$$

where the integration should be made from the time of the disturbance. For critically damped systems, the integral of the error is equal to the integral of the absolute error when $\Delta u > 0$. Hence, assuming that $\Delta u > 0$, we have

$$IAE_{PI} = \int |e(t)|dt = \frac{T_i}{K}\Delta u \tag{9}$$

The PIP control law is

$$u = Ke + \frac{K}{T_i} \int e(t)dt - \frac{1}{T_i} \int [u(t) - u(t-L)]dt$$

If the dead time is long enough, the control signal u will settle at its new stationary level within the dead time, when the system is disturbed by a step change in the load. The solution to the last integral is then given by

$$\int [u(t) - u(t-L)]dt = L\,\Delta u$$

After a load disturbance, the control signal has therefore been changed according to

$$\Delta u = \frac{K}{T_i} \int e(t)dt - \frac{L}{T_i} \Delta u$$

Assuming critically damped control, the integrated absolute error for the PIP controller is therefore

$$IAE_{PIP} = \int |e(t)|dt = \frac{T_i + L}{K} \Delta u \tag{10}$$

To be able to compare the IAE for the PI and the PIP controllers, typical values for the controller parameters must be obtained. Reasonable values for a PI controller designed for critically damping are

$$K = \frac{1}{4 K_p}$$
$$T_i = \frac{L}{2}$$

This gives an IAE for the PI controller equal to

$$IAE_{PI} = 2 K_p L \Delta u \tag{11}$$

For the PIP controller we have chosen the controller parameters

$$K = \frac{1}{K_p}$$
$$T_i = T$$

This gives an IAE for the PIP controller equal to

$$IAE_{PIP} = K_p(T + L) \Delta u \tag{12}$$

Hence,

$$IAE_{PIP} = \frac{T + L}{2L} IAE_{PI} \tag{13}$$

This means that the PIP controller is superior to the PI controller when $L > T$, and that the improvement is larger for larger L. For very large dead times the IAE of the PIP controller is 50% of the IAE with PI control. (If the Cohen-Coon design methods were used for the PI design, the decrease would have been 67% or 41%, depending on whether the PID or PI design were used.) For shorter dead times, the decrease in IAE is of course less. In the simulation examples presented in this paper, the decrease is about 30%.

Time-varying dead times

Dead times occur typically in processes with mass transportation. It is caused by the time it takes for the material to be transferred from the position of the supply to the position of the sensor. It means that the dead time is inversely proportional to e.g. a flow or a speed of a conveyer. If the flow or the speed vary, so do the dead time. The dead times are therefore often time-varying.

Since the dead time occur explicitly as a parameter in the PIP controller, it is possible to follow variations in the dead time L, by scheduling this parameter to an appropriate flow or speed signal.

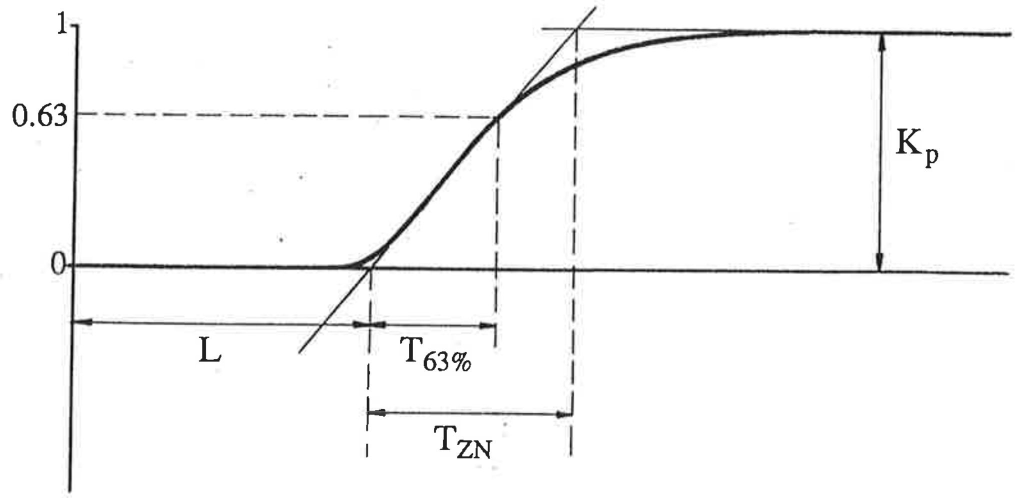


Figure 2. Characterization of a step response with static gain K_p, dead time L, and time constants T_{ZN} and $T_{63\%}$. The curve shows the measurement signal after a unit step change in the control signal at time 0.

5. Tuning the PIP controller

This section treats the tuning of the PIP controller, i.e. the problem of finding the parameters K, T_i and L.

There is a fundamental difference between the tuning of a PID controller and the tuning of the PIP controller. In the existing systematic tuning procedures for PID controllers, such as the Ziegler-Nichols and Cohen-Coon methods, the selection of the controller parameters depends essentially on the relation between the apparent dead time L and the dominating time constant T of the process. In the PIP controller, the effects of the dead time are removed from the choice of the parameters K and T_i. Two processes with equal dynamics except for the dead time, should have the same gain K and integral time T_i, but of course with different values of L.

Manual tuning

Two methods to obtain the static process gain, the apparent dead time and the dominating time constant from a step response are given in Figure 2. The methods differ only in the determination of the time constant. In the Ziegler-Nichols method, the time constant T_{ZN} is obtained from the crossings between the tangent with maximum slope and the two stationary levels of the measurement signal. In the second method, the time constant $T_{63\%}$ is obtained as the time when the measurement signal reaches 63% of its final value. The last method has been shown to be the best for our purpose. Both methods give the exact solution in the case of a pure first order system with a time delay.

With the parameter choice $\kappa = 1$ and $\tau = 1$, a good starting point for the choice of controller parameters from an open loop step response experiment is therefore

$$\begin{aligned} K &= 1/K_p \\ T_i &= T_{63\%} \\ L &= L \end{aligned} \tag{14}$$

It is of course also possible to tune the controller with a more "trial and error" type of procedure. The old rules of thumb for changing the controller gain and integral time are still valid. This means that an increased gain or decreased integral time mostly gives a faster but less damped control, whereas a decreased gain or an increased integral time gives a slower and more stable control. The fact that these rules are still valid is important even if a more systematic tuning procedure is used. It makes it possible to make fine adjustments of the control in cases where the systematic procedures fail to give a satisfactory control.

If the controller is to be tuned manually, it is advisable to start by giving L a suitable value, and then adjust K and T_i afterwards.

Automatic tuning

The PIP controller requires an estimate of the static process gain K_p, the dominating time constant T, and the dead time L. As seen above, these parameters are easily obtained from a step response experiment. Therefore, automatic tuning procedures for PID controllers which are based on step response analysis can easily be used also for the PIP controller. The only thing that has to be altered is the design calculation.

The relay autotuner procedure, see Åström and Hägglund (1988A) can not be used without modifications. The relay autotuner obtains the process information in terms of the frequency response at one frequency. It corresponds to two process parameters. To obtain the three parameters that we need, one can proceed in two ways. The first is to study the wave-form of the oscillation, and thereby obtain additional information. See Åström and Hägglund (1988B). The second possibility is to determine the static gain by performing a step response after the relay experiment.

6. Simulation Examples

In this section, the properties of the PIP controller are demonstrated by some simulation examples. The following structure of the PIP controller is used.

$$u(t) = K\left(-y(t) + \frac{1}{pT_i}e(t)\right) + \frac{1}{pT_i}[u(t-L) - u(t)] \tag{15}$$

In this controller, the proportional action works only on the measurement signal, instead of on the control error as in the Equation (8). This structure is mostly preferred in process controllers, since it gives a smoother response to set-point changes. The difference does of course not influence control of load disturbances. See Åström and Hägglund (1988A).

The following set of process models have been used in the simulations.

$$G_1(s) = \frac{e^{-5s}}{1+s} \qquad G_2(s) = \frac{e^{-10s}}{1+s}$$

$$G_3(s) = \frac{e^{-5s}}{(1+s)^3} \qquad G_4(s) = \frac{e^{-10s}}{(1+s)^3}$$

$$G_5(s) = \frac{e^{-5s}}{(1+s)(1+0.5s)(1+0.25s)(1+0.125s)}$$

$$G_6(s) = \frac{e^{-10s}}{(1+s)(1+0.5s)(1+0.25s)(1+0.125s)}$$

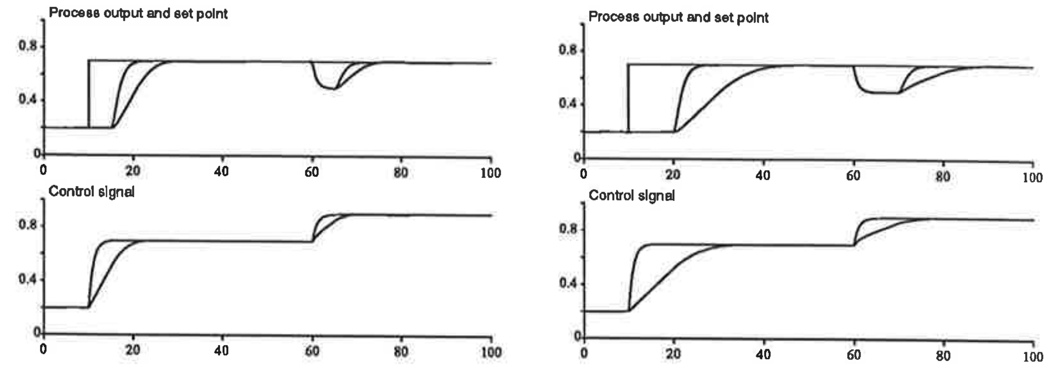
These process models capture typical dynamics encountered in the process industry, of course with the exception that the ratios between the apparent dead time and the dominating time constant may take other values. In Åström et al (1989), rules for the applicability of PID controllers with the Ziegler-Nichols tuning rules were given. There it was found that PID controllers with Ziegler-Nichols tuning rules could be used for processes were the ratio between the apparent dead time L and the dominating time constant T was less than 1. Dead-time compensation was recommended for systems with longer dead times. The process models used in these simulations have ratios of L/T ranging from 2 to 10.

The results of the simulations are presented in Figure 3, and in Table 1. In the simulations, the PI and the PIP controller are compared with respect to set-point changes and load disturbances. The design goal for both controllers has been to obtain fast robust control without any overshoot. The design rules given in Section 4 have not been used. Instead, both controllers have been tuned manually to obtain the design goal. In Table 1, the controller parameters, the ratio between the IAE for the two controllers, and the estimated process model obtained from step response investigations are presented. Figure 3 clearly demonstrates that the PIP controller is superior to the ordinary PI controller for these dead-time dominated processes. The difference is also demonstrated by the decreased IAE (almost 30%).

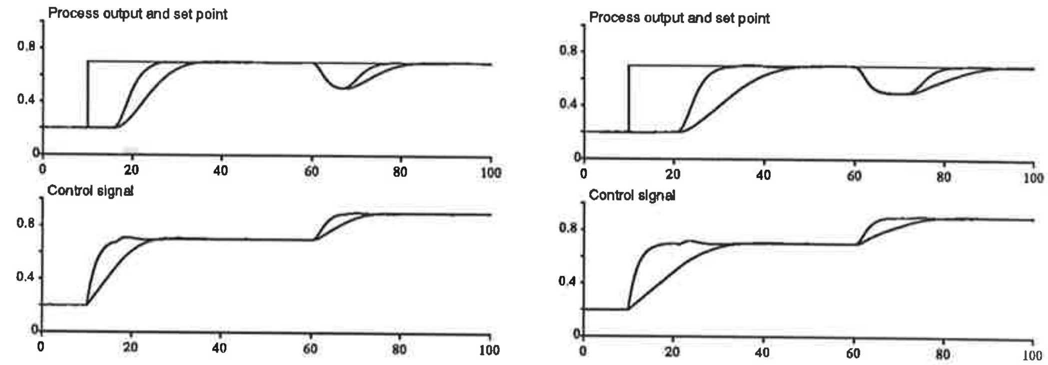
The PIP controller parameters are close to those obtained from the estimated process parameters according to Equation (14), especially K_p and

Table 1. Results of the simulations with the PIP controller

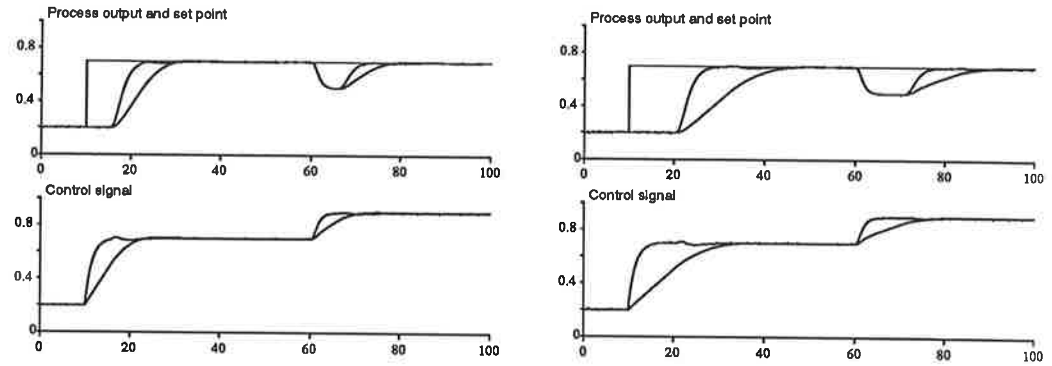
Process	PI		PIP			$\frac{IAE_{PIP}}{IAE_{PI}}$	Estimated Parameters			
	K	T_i	K	T_i	L		K_p	L	$T_{63\%}$	T_{ZN}
G_1	0.28	2.3	1.0	1.0	5.0	0.73	1.0	5.0	1.0	1.0
G_2	0.25	3.9	1.0	1.0	10.0	0.70	1.0	10.0	1.0	1.0
G_3	0.25	2.8	1.0	2.0	6.0	0.71	1.0	5.8	2.7	3.9
G_4	0.27	4.8	1.0	2.0	11.0	0.72	1.0	10.8	2.7	3.9
G_5	0.25	2.4	1.0	1.5	5.5	0.73	1.0	5.5	1.6	2.2
G_6	0.25	4.2	1.0	1.5	10.5	0.71	1.0	10.5	1.6	2.2



PI and PIP control of the process models G_1 (left) and G_2 (right)



PI and PIP control of the process models G_3 (left) and G_4 (right)



PI and PIP control of the process models G_5 (left) and G_6 (right)

Figure 3. Comparisons between the PI and the PIP controller. The graphs show a step response followed by a load disturbance. In all diagrams, the faster response is obtained by the PIP controller, and the slower is obtained by the PI controller.

L. From Table 1 it is also seen that the estimation of the dominating time constant by taking the 63 % value is superior than the Ziegler-Nichols version.

Note that for the PIP controller, the gain K and the integral time T_i take the same values when the process models only differ with respect to the dead time. This property demonstrates the possibility to let L be connected to a signal (e.g. a flow or a speed) and thereby follow time variations in the dead time.

7. Implementation aspects

The PIP controller can of course be implemented in instrument systems as a complement to the PID algorithm, to be used for processes with long dead times. Since the controller structure is relatively simple, it can also easily be incorporated in present single-station PID controllers as an extra feature. The changes that have to be made in the program code are quite small. The derivative part of the PID controller has to be replaced by to the last term in Equation (8). The PIP controller needs only one parameter except for the PI parameters K and T_i, namely the dead time L. The controller requires also the delayed control signal $u(t - L)$. Since most modern controllers already contain the possibility to delay signals, this facility can be used also for this purpose.

If the controller is supposed to handle time-varying dead times, the procedure that performs the delay of the control signal must be able to handle variations in the dead time L. This can be accomplished in two ways. One possibility is to vary the size of the buffer where the control signals are stored. The second method is to let the sampling period of the buffer be proportional to the dead time. The last alternative is preferable in most cases.

8. Conclusions

This paper has presented a predictive PI controller, which is suitable for processes with long dead times. Compared to an ordinary PID controller it has the advantage that it manages to predict the measurement signal even when the process has a long dead time and when the measurement signal is noisy. The benefits of this are demonstrated through simulations and analysis.

The PIP controller has the same structure as a Smith predictor, but with restrictions on the process model. This restriction is a drawback, in the same sense as the simple structure of the PID controller is a drawback compared with more complex controller structures. On the other hand, the PIP controller has the same advantages as the PID controller, namely that it can be tuned manually. This is possible since the PIP controller only contains three adjustable parameters.

9. Acknowledgements

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