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Active Distances and Cascaded Convolutional Codes¹

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Abstract — A family of active distances for convolutional codes is introduced. Lower bounds are derived for the ensemble of periodically time-varying convolutional codes.

I. INTRODUCTION

The "extended distances" were introduced by Thommesen and Justesen [1] for unit memory (UM) convolutional codes. We present (non-trivial) extensions to encoder memories $m \geq 1$ and call them *active distances* since they stay "active" in the sense that we consider only those codewords which do not pass two consecutive zero states [2].

II. ACTIVE DISTANCES

Consider the ensemble of binary, rate $R = b/c$, periodically time-varying convolutional codes encoded by a polynomial generator matrix of memory m and period T ,

$$\mathbf{G} = \begin{pmatrix} G_0(t) & \cdots & G_m(t+m) \\ & G_0(t+1) & \cdots & G_m(t+m+1) \\ & & \ddots & \\ & & & \ddots \end{pmatrix} \quad (1)$$

in which each digit in each of the matrices $G_i(t+T)$ for $0 \leq i \leq m$ and $0 \leq t \leq T-1$, is chosen independently and equally likely to be 0 and 1.

Let $\mathcal{U}_{[t_1-m, t_2+m]}^r$ be the set of information sequences $\mathbf{u}_{t_1-m} \dots \mathbf{u}_{t_2+m}$ such that the first m and the last m subblocks are zero and they do not contain $m+1$ consecutive zero subblocks.

Let $\mathcal{U}_{[t_1-m, t_2]}^c$ be the set of information sequences $\mathbf{u}_{t_1-m} \dots \mathbf{u}_{t_2}$ such that the first m subblocks are zero and they do not contain $m+1$ consecutive zero subblocks.

Let $\mathcal{U}_{[t_1-m, t_2]}^s$ be the set of information sequences $\mathbf{u}_{t_1-m} \dots \mathbf{u}_{t_2}$ such that at least one subblock is nonzero and they do not contain $m+1$ consecutive zero subblocks.

Next we introduce the truncated time-varying generator matrix

$$\mathbf{G}_{[t, t+j]} = \begin{pmatrix} G_m(t) & & & \\ \vdots & \ddots & & \\ G_0(t) & & G_m(t+j) & \\ & & \ddots & \\ & & & G_0(t+j) \end{pmatrix}. \quad (2)$$

Definition 1 Let \mathcal{C} be a time-varying convolutional code encoded by a time-varying, polynomial generator matrix. Then the j th order active row distance is

$$a_j^r \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}_{[t-m, t+j+m]}^r} w_H(\mathbf{u}_{[t-m, t+j+m]} \mathbf{G}_{[t, t+j+m]}), \quad (3)$$

the j th order active column distance is

$$a_j^c \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}_{[t-m, t+j]}^c} w_H(\mathbf{u}_{[t-m, t+j]} \mathbf{G}_{[t, t+j]}), \quad (4)$$

and the j th order active segment distance is

$$a_j^s \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}_{[t-m, t+j]}^s} w_H(\mathbf{u}_{[t-m, t+j]} \mathbf{G}_{[t, t+j]}). \quad (5)$$

For a convolutional code encoded by a time-varying, non-catastrophic, polynomial generator matrix we define its free distance as $d_{\text{free}} \stackrel{\text{def}}{=} \min_j a_j^r$.

III. CASCADED CODES

Consider a scheme with two convolutional codes in cascade.

Theorem 1 There exist cascaded convolutional codes in the ensemble of periodically time-varying cascaded convolutional codes whose active distance satisfies

$$\delta_l^r \stackrel{\text{def}}{=} \frac{a_j^r}{mc} \geq (l+1)h^{-1} \left(1 - \frac{l}{l+1}R\right) - O\left(\frac{\log_2 m}{m}\right) \quad (6)$$

for $l \geq l_0^r = O\left(\frac{1}{m}\right)$,

$$\delta_l^c \stackrel{\text{def}}{=} \frac{a_j^c}{mc} \geq lh^{-1}(1-R) - O\left(\frac{\log_2 m}{m}\right) \quad (7)$$

for $l \geq l_0^c = O\left(\frac{\log_2 m}{m}\right)$, and

$$\delta_l^s \stackrel{\text{def}}{=} \frac{a_j^s}{mc} \geq lh^{-1} \left(1 - \frac{l+1}{l}R\right) - O\left(\frac{\log_2 m}{m}\right) \quad (8)$$

for $l \geq l_0^s = \frac{R}{1-R} + O\left(\frac{\log_2 m}{m}\right)$.

By minimizing the lower bound on the active row distance we obtain nothing but the main term in Costello's lower bound on the free distance, viz., $\frac{R}{-\log_2(2^{1-R}-1)}$.

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