# Analysis of Slow Frequency Hopping Networks

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# Analysis of Slow Frequency Hopping Networks

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*Abstract*— **We derive an approximation of the throughput of strongly interfering frequency-hopping radio systems, where packet collisions always result in lost data. A system is defined to consist of a certain number of radio networks, each network with an arbitrary number of communicating units coordinated to communicate without interference.**

## I. Introduction

The development of small and mobile laptops and terminals has created a demand for fast and convenient Wireless Local Area Network (WLAN) access. A possibility for these systems is to use the unlicensed ISM-band which is open to the public and to all kinds of radio systems, as for example Bluetooth[1], a short range system for wireless connectivity. As the ISM-band is shared between many users and systems, interference and reduced performance must be accepted.

For communication in the ISM-band regulations require that some type of spread spectrum technique is used. In this paper we focus on slow frequency-hopping (FH) spread spectrum systems, where the transmitters of packets hop between several available frequency channels. More specifically, we consider packet based FH systems that interfere strongly, which means that if there is a packet collision we assume that all data is lost. This assumption is not always realistic, but it will make it possible to derive a useful analytical expression for the data throughput of the systems. The novelty of our approach lies in the fact that we consider packets of varying lengths.

This paper consists of preliminary results performed within the PCC project "Mobile Wireless Access to Fixed Networks." These results have also been submitted to VTC 2001 Spring in Tel Aviv.

## II. System Model and Analysis

Generally, in radio communication, there is one sender transmitting and one or more receivers listening. We will define a network to consist of such an arbitrary number of units that communicate without interference. The transmissions within a network could e.g. be coordinated by some master unit, which is the case in a Bluetooth piconet. Furthermore let the networks transmit packets continuously in the following way. Firstly, a network selects a packet from a set of packet types. Each packet type is selected with a specified probability and consists of a header, a payload and a guard interval. After transmitting the header and payload the network holds transmission during the guard interval, and selects both a new channel and a new packet type before next transmission (see figure 1).

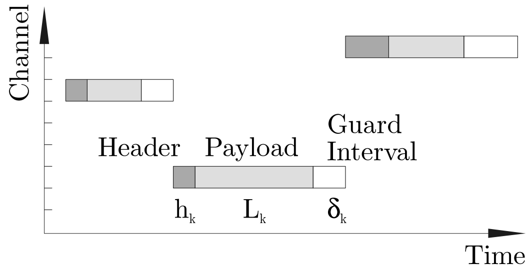


Fig. 1. Three packets and their components transmitted by a frequency-hopping network.

Using the system model above we derive an analytical expression for the data throughput of the networks, given the following assumptions: collisions always result in lost packets; if no collisions occur, packets are successfully received; frequency hops are perfectly random; all networks transmit packet types drawn from the same probability distribution upon transmission; networks always have packets to transmit; there is no time synchronization between networks.

These assumptions enable us to express the probability, $P_s(T)$, of successful transmission of a packet of a certain length $T$. When analyzing the problem we calculate $P_s(T)$ for one reference network and regard all other networks as jammers. In order to perform this calculation we note that, with $q$ available channels, the probability of successful transmission is $(1-1/q)^{H(T)}$, given that the $N-1$ jammers in total have $H(T)$ packets overlapping the packet of length $T$ in time. Averaging over the probability of $h$ overlapping packets, the general expression for $P_s(T)$ becomes

$$P_s(T) = \sum_{h=0}^{\infty} p_{H(T)}(h) \left(1 - \frac{1}{q}\right)^h, \tag{1}$$

where $p_{H(T)}(h)$ is the PDF of $H(T)$. If an expression for $P_s(T)$ is derived, the throughput for a network can easily be calculated.

Clearly, an expression for $p_{H(T)}(h)$ will be difficult to obtain in closed form. Therefore we would like to have an approximation of $p_{H(T)}(h)$. To this end, we use the central limit theorem which states that the number of overlapping packets $H(T)$ is asymptotically Gaussian as the number of jammers goes towards infinity[2]. Since $H(T)$ is a discrete stochastic variable we hence assume that $p_{H(T)}(h)$ has an approximately Gaussian *shape* as the number of jammers increases. Denoting the approximate expression of $P_s(T)$

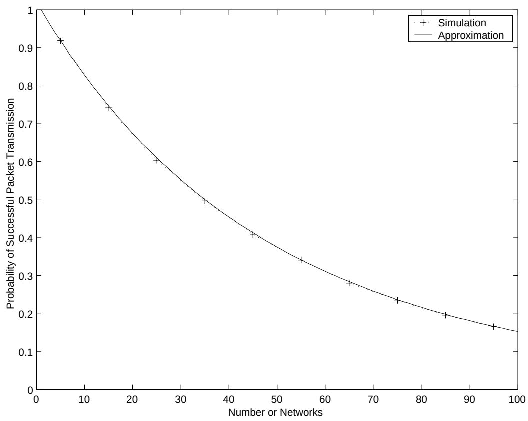


Fig. 2. Simulation and approximation for $q = 79$, $r_1 = 1/2$, $r_2 = 1/6$, $r_3 = 1/3$, $l_1 = 405$, $l_2 = 1655$, $l_3 = 2905$, and $\delta_1 = \delta_2 = \delta_3 = 220$.

as $\tilde{P}_s(T)$ we obtain

$$\tilde{P}_s(T) \simeq \int_{-\infty}^{\infty} f_{H(T)}(h) \left(1 - \frac{1}{q}\right)^h dh, \tag{2}$$

where $f_{H(T)}(h)$ is a Gaussian distribution. To further simplify the expression above, we approximate $f_{H(T)}(h)$ with a Dirac function located at the mean of $H(T)$. This yields

$$\tilde{P}_s(T) \simeq \int_{-\infty}^{\infty} \delta(h - E[H(T)]) \left(1 - \frac{1}{q}\right)^h dh \tag{3}$$

$$= \left(1 - \frac{1}{q}\right)^{E[H(T)]}. \tag{4}$$

Although we give no bounds for the approximation here, simulations show that the approximation is very accurate. The approximate probability of successful transmission and results from a simulation are showed in figure 2.

If we model a jammer as a semi-Markov process, we can calculate the mean number of frequency hops a jammer performs during the transmitter's packet transmission time by solving the stationary state probabilities. This gives us the probability of successful transmission of a packet in the presence of a single jammer. The generalization to $N - 1$ jammers is then straightforward since they are all independent of each other. Using the probability of successful transmission the expression for the normalized system throughput is derived.

We will now calculate the probability that a packet transmitted by the transmitter does not collide with one or more packets transmitted by the jammer. The jammer transmits a packet of length $\mu_k$ with probability $r_k$. The packet length includes both header and payload, and hence $\mu_k = h_k + L_k$. After the transmission of the packet, a time $\delta_k$ is spent in a guard interval during which the jammer is idle, and then the process starts over again. This process is depicted in figure 3, where the upper row of states are transmission states and the lower row represents guard interval states.

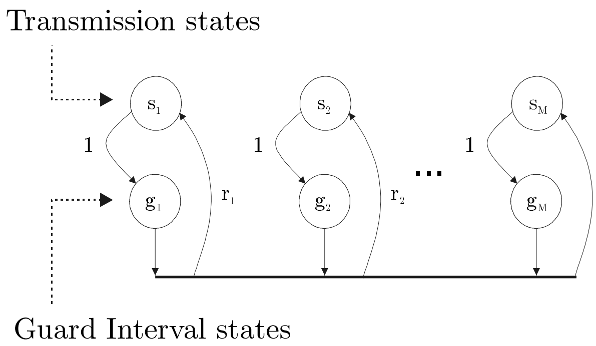


Fig. 3. The jammer modeled as a semi-Markov process.

The transmission states are labeled $s_k$ for $k = 1, 2, \ldots, M$ and their respective guard interval states are labeled $g_k$ for $k = 1, 2, \ldots, M$. After visiting one of the transmission states, the jammer jumps to the guard interval state belonging to that particular transmission state with probability one. The jammer then selects transmission state $s_k$ with probability $r_k$. For convenience, a "bus" has been drawn in figure 3 instead of drawing an arrow from each of the guard interval states to all of the transmission states.

We denote the duration of the transmission of the transmitter's packet by $T$, and the mean number of jammer transitions out of all states $g_k$ during $T$ by $F(T)$. With standard techniques[2] $F(T)$ can be shown to be

$$F(T) = 1 + \frac{T - \sum_{k=1}^{M} r_k \delta_k}{\sum_{n=1}^{M} r_n (\mu_n + \delta_n)}. \tag{5}$$

In order for the transmitter to avoid collision with the jammer, the transmitter and the jammer must not transmit on the same frequency at the same time. The probability of the transmitter not selecting the same frequency as the jammer is, among $q$ frequencies, $1 - 1/q$ [3]. Thus, according to the approximation, if the jammer makes $F(T)$ frequency hops during $T$ this probability instead becomes $(1 - 1/q)^{F(T)}$. Since all networks have the same packet length distribution, we realize that it is sufficient to calculate the mean number of hops for one jammer, and then multiplying with the number of jammers, in order to obtain $E[H(T)]$. We therefore have $E[H(T)] = (N-1)F(T)$ if here are $N - 1$ jammers. The probability of successful transmission of a packet of length $T$ then becomes [4]

$$P_s(T) = \left(1 - \frac{1}{q}\right)^{E[H(T)]} = \left(1 - \frac{1}{q}\right)^{(N-1)F(T)}$$

$$= \left(1 - \frac{1}{q}\right)^{(N-1)\left(1 + \frac{T - \sum_{k=1}^{M} r_k \delta_k}{\sum_{k=1}^{M} r_k (\mu_k + \delta_k)}\right)}. \tag{6}$$

If the transmitter is modeled in the same manner as the jammers, the average probability of successful transmission over all packet types for the transmitter is

$$P_A = \sum_{k=1}^{M} r_k P_s(\mu_k). \quad (7)$$

Even if the average probability of successful transmission is an interesting quantity, the throughput is of even more significance. The throughput is simply the ratio between the mean length of the successfully transmitted payloads and the mean packet length, including the guard interval. The normalized information data rate, or normalized throughput, becomes

$$R = \frac{\sum\limits_{n=1}^{M} r_n L_n \left(1-\frac{1}{q}\right)^{(N-1)\left(1+\frac{L_n+h_n-\sum_{k=1}^{M} r_k\delta_k}{\sum_{k=1}^{M} r_k(h_k+L_k+\delta_k)}\right)}}{\sum\limits_{k=1}^{M} r_k\left(h_k+L_k+\delta_k\right)}. \quad (8)$$

## III. Conclusions

In this paper we have presented a model of a FH system and an analytical approximate expression for the capacity of such a system. It has been assumed that collisions result in a total loss of all packets involved in the collision, since the model does not consider the distances between networks. The throughput will depend on the number of networks in the system, the length of the available packets, their respective guard intervals, and the probabilities of a network transmitting a certain type of packet. By varying the parameters mentioned above the maximum throughput for a network can be found.

Even though the analyzed model is somewhat simplified, as compared to a real system, the new analytical evaluation of throughput, following from the assumptions and approximations, readily compensates for the simplifications. More realistic models usually lead to extensive simulations, from which general conclusions are more difficult to draw.

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