True equivalent chip thickness for tools with a nose radius

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TRUE EQUIVALENT CHIP THICKNESS FOR TOOLS WITH A NOSE RADIUS

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Abstract: A majority of the established systems for choice and optimization of cutting data are based on Woxén's equivalent chip thickness, h_{eW}. In metal cutting theory and models, the equivalent chip thickness is of vital importance when the depth-of-cut a_p is in the same order or smaller than the nose radius r. Woxén made considerable simplifications in his chip area model, that form the basis for calculations of the equivalent chip thickness. Basic mathematical solutions, e.g. describing the chip area on circular inserts, are lacking. This article describes the geometrical implications when machining with round inserts. The error in Woxén's equivalent chip thickness is largest when the depth-of-cut is less than ¼ of the nose radius. The calculations of the equivalent chip thickness based on the Woxén model are up to 50 % wrong, for some combinations of cutting data in the finishing range. The presented results explain the difficulties in getting a good validity in the models used to calculate tool life in finishing machining. The error leads to an underrating of the tool load in many machining situations.

Keywords: Woxén, equivalent chip thickness, metal cutting, round inserts.

1. INTRODUCTION

When machining with an insert with a nose radius the theoretical chip thickness will vary, from the major cutting edge, along the nose radius, to the minor cutting edge. The theoretical chip thickness h_1 is together with the cutting speed v_c the two most important factors that influences the functionality and productivity of the cutting process.

To produce a cut surface with acceptable properties, the cutting tool must have a curved bridging between the major and the minor cutting edge. In many cases a base geometry in the form of a circular arc with a standardized radius is used. Inserts with a pronounced nose radius are most commonly used in turning operations.

Using a constant approach angle and a constant feed, the major cutting edge will cut a chip with a constant theoretical chip thickness. The tool load will vary along the tool nose due to the variation in theoretical chip thickness. Woxén (Woxén 1932) introduced an equivalent chip thickness, h_{eW}, in 1932, with the purpose to use it as a characteristic parameter describing the mean theoretical chip thickness along the tool nose.

The stresses in a cutting tool are approximately proportional to the theoretical chip thickness h_1. This means that h_1 has a dominant influence on the tool wear and the tool life.

Along the tool nose, h_1 will vary from its maximum value down to 0. When machining with an a_p less than the size of the nose radius, varying cutting conditions will rule along all of the active cutting edge. Models describing tool wear and tool life will be dramatically simplified if a characteristic or equivalent value of h1 can be introduced.

Woxéns equivalent chip thickness describes a kind of theoretical mean chip thickness, based on the active cutting edge length. Another interpretation of the equivalent chip thickness is that it joins together combinations of significant cutting parameters into one single parameter. Woxéns representation gives a value of the equivalent chip thickness for different choices of feed f, depth-of-cut a_p, approach angle κ and nose radius r.

An increased depth-of-cut means that the major cutting edge increases its part of the process energy conversion. This also means that h_{eW} will approach the current value of h_1 for the major cutting edge. For a smaller depth-of-cut, h_{eW} will significantly differ form this value.

Most systems for optimization and choice of cutting data use the equivalent chip thickness. Recommended cutting data from tool and material catalogues are often based on h_{eW}.

2. LIST OF SYMBOLS

A	True chip area	mm^2
A_W	Woxéns chip area	mm^2
a_p	Depth-of-cut	mm
f	Feed	mm/rev
h_1	Theoretical chip thickness	mm
h_e	True equivalent chip thickness	mm
h_{eW}	Woxéns equivalent chip thickness	mm
l_c	Active cutting edge length	mm
l_{ce}	Equivalent active cutting length	mm
l_{cW}	Woxéns active cutting length	mm
r	Nose radius	mm
x, y	Help variables	mm
Δ	Error function	-
δ	Angular variable	°
κ	Approach angle	°

3. PROBLEM DESCRIPTION

Based on experience, it is very hard to predict tool life in finish machining operations. There are several reasons why rough machining easier can be described in tool life models, than finish machining.

One of the reasons is how the theoretical chip thickness behaves, as a constant when rough machining with large values of a_p, compared to the significant variation for $a_p < r$.

If studied closely, it can be concluded that Woxéns equivalent chip thickness has quantitative imperfections for $a_p < r$. Below, a modified version of Woxéns equivalent chip thickness is derived and presented. This modified equivalent chip thickness h_e is based on Woxéns fundamental conditions according to equation 1, where he is given by the ratio between the chip area A and the active cutting edge length l_c.

$$h_e = \frac{A}{l_c} \tag{1}$$

4. DELIMITATIONS

In the presented work, a simple circular bridging between the major and minor cutting edge is used. This geometry is the most commonly used in turning. The attempt is based on the condition that the bridging between the major and minor cutting edge is represented by a part of a circular arc. The depth-of-cut is limited to the range less than the nose radius r.

5. REALIZATION

The mathematical calculations were performed using the mathematical software Mathcad, versions 11 and 14.

6. THE EQUIVALENT CHIP THICKNESS h_{eW}

Woxén approximates the chip area A using the product between depth-of-cut a_p and feed f. This approximation results in computational errors that are not insignificant in under certain conditions. The equivalent chip thickness according to Woxén can be calculated as:

$$h_{eW} = \frac{A_W}{l_{cW}} = \frac{a_p \cdot f}{\frac{a_p - r(1-\cos\kappa)}{\sin\kappa} + \kappa \cdot r + \frac{f}{2}} \tag{2}$$

where A_W is Woxéns chip area and l_{cW} is the length of the cutting edge that is active in the cutting process. The active cutting length is built by 3 parts, a linear part which is the major cutting edge to the tangation point to the tool nose, the nose part which is equivalent to κ·r, and one final part that is approximated by f/2, according to Figure 1.

The latter approximation is considered to be acceptable from an accuracy viewpoint. In Woxéns model, the tool nose is straightened out, forming a rectangular area that describes the chip area according to Figure 2.

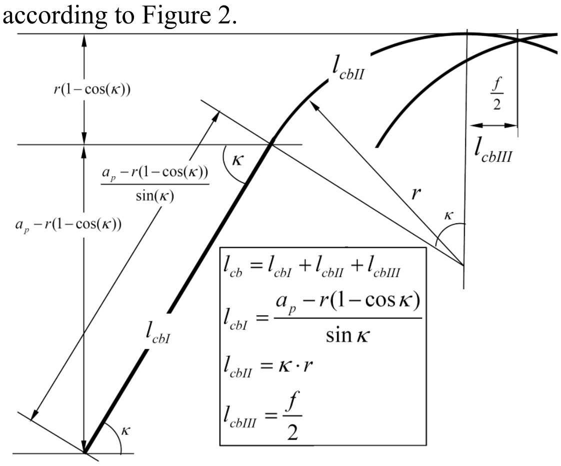


Figure 1 Representation of the active cutting length into 3 parts, $l_{cI}+l_{cII}+l_{cIII}$, *[3]*.

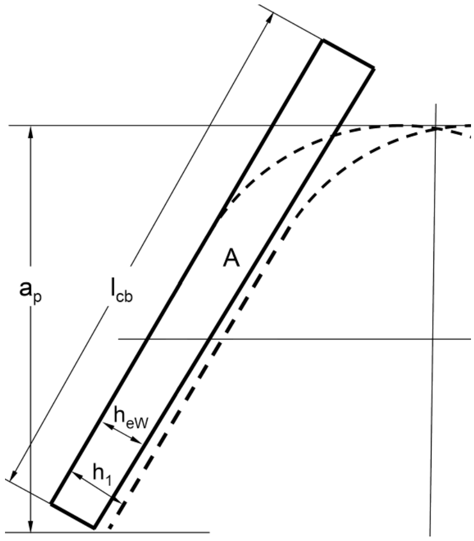


Figure 2 Woxéns chip area A with the equivalent chip thickness h_{eW}.

7. TRUE EQIVALENT CHIP THICKNESS

The design of an insert between the major and the minor cutting edge, is normally achieved through a nose radius r. Along the tool nose the theoretical chip thickness will start at its nominal value and successively decrease to a value of 0 over the minor cutting edge, according to Figure 3.

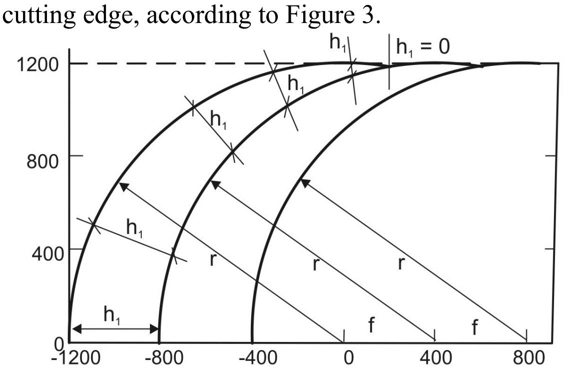


Figure 3. Theoretical chip thickness along the tool nose for r=1.2mm, κ=90° and f=0.4 mm/rev. Scales in µm.

Through a geometrical observation according to Figure 4, presented by (Brammertz 1960) among others, the relations between significant parameters can be identified.

The relations between theoretical chip thickness $h_1(\delta)$, feed f, and nose radius r can be identified in **Figure 4** and calculated according to the equation system according to equation 3.

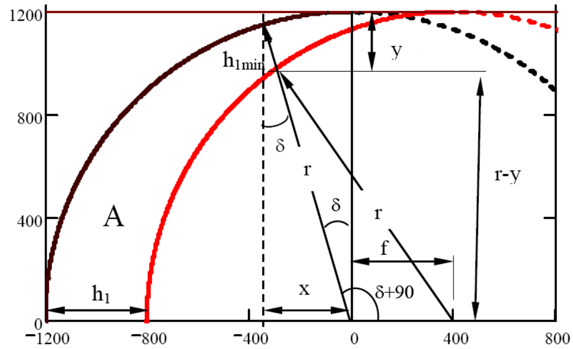


Figure 4. Graphical representation of the chip area end the variation in theoretical chip thickness along the tool nose, for r=1.2 mm, κ=90° and f=0.4 mm/rev. Scales in µm.

The relations can be drawn by studying the two right-angled triangles in the figure, where x and y are help variables and δ the tool nose angular variable. For δ = 90°, $h_1 = f \cdot \sin(\kappa)$. Including the help variables x and y, there are 3 unknowns and 3 equations.

$$r^2 = (f + x)^2 + (r - y)^2$$

$$x = (r - y) \cdot \tan\delta \qquad (3)$$

$$\cos\delta = \frac{r - y}{r - h_1}$$

where h_1 is a function of the angle δ. There exist several solutions to the equitation system. The only valid solution gives the theoretical chip thickness, depending on the angle δ (i.a.) as:

$$h_1(\delta, r, f) = f \cdot \sin\delta + r - \sqrt{f^2 \cdot \sin^2(\delta) + r^2 - f^2} \qquad (4)$$

Figure 5 illustrates equation 4, where the angle δ is variable.

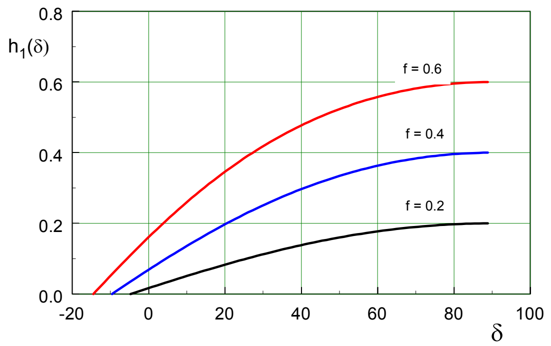


Figure 5 Theoretical chip thickness $h_1(\delta)$ along the tool nose for feeds f=0.2, 0.4 and 0.6 mm/rev, nose radius r = 1.2 mm and approach angle κ =90°.

The angular position for h_1=0 in Figure 5 can be calculated out of equation 4 by inserting $h_1(\delta)$=0, after which the angle δ_0 can be obtained through equitation 5.

$$\delta_0 = -\sin^{-1}\left(\frac{f}{2 \cdot r}\right) \qquad (5)$$

An equation describing the angular function δ_{ap} can be formulated according to equation 6, by studying the large triangle in Figure 6.

$$r^2 = (f + (r - a_p)\cdot \tan\delta_{ap})^2 + (r - a_p)^2 \quad (6)$$

$$\rightarrow$$

$$\delta_{ap} = \tan^{-1}\left(\frac{-f + \sqrt{2\cdot r\cdot a_p - a_p^2}}{r - a_p}\right)$$

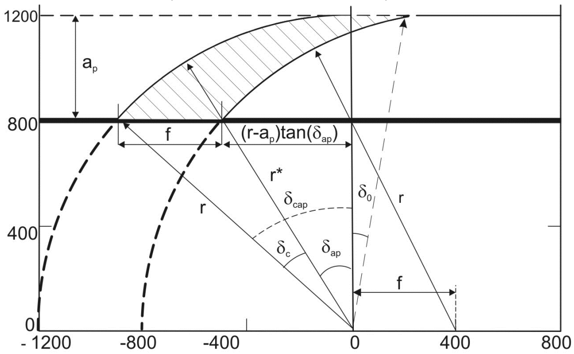


Figure 6. The appearance of the chip area for machining cases where $a_p<r$.

Figure 6 and Figure 7 illustrate the chip area when $a_p<r$. The chip area can be identified as the sum of 3 different surface elements, the surface A according to equitation 7 with integration limits (δ_0, δ_{ap}), the triangular surface A_t, and the segmental surface A_c. The segmental surface A_c can be calculated by using the chordal formula, $A=0.5\cdot r^2\cdot(\theta-\sin(\theta))$. The integration limits δ_0 och δ_{ap} are calculated by using equitation 5.

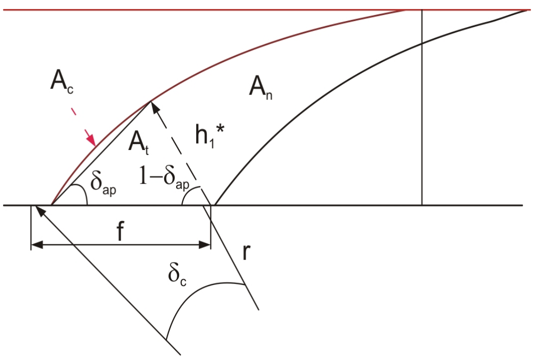


Figure 7. Enlargement of the chip area for the case where $a_p < r$.

$$A = A_n + A_t + A_c$$

$$A_n = \int_{\delta_0}^{\delta_{ap}} f\cdot\sin(\delta) + r - \sqrt{r^2 - f^2\cdot\cos^2(\delta_{ap})}\ \ d\delta \quad (7)$$

$$A_t = \frac{f\cdot h_1^*\cdot\cos(\delta_{ap})}{2}$$

$$A_c = \frac{r^2}{2}\left(\frac{f\cdot\cos^{-1}(\delta_{ap})}{r} - \frac{f\cdot\cos(\delta_{ap})}{r}\right)$$

Each surface can be calculated by using equation 5 and 6, respectively. The surfaces A_n och A_t are dominant in size, the surface A_c is insignificant. Unfortunately, equation 6 is without any analytical solution, due to the fact that the derivate of the chip area as a function of the angular coordinate lacks a primitive function.

For depths-of-cut $a_p>r$ the true equivalent chip thickness he can be calculated by adding the area corresponding to the major cutting edge, as $f\cdot(a_p-r)\cdot\sin(\kappa)$, (Ståhl 2007). Furthermore, h_e can be calculated for any arbitrary value of approach angle κ by setting the upper integration limit to κ in equation 6. Figure 8 illustrates the appearance of the chip area A as a function of a_p.

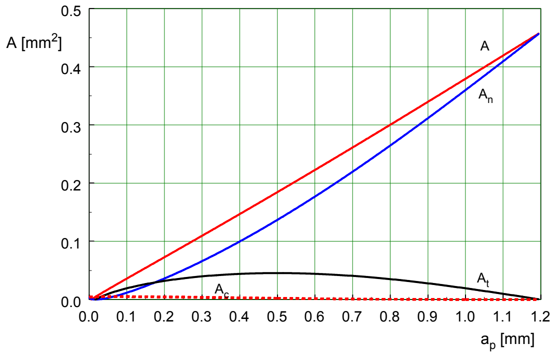


Figure 8. The chip area A and its components, as a function of a_p, f = 0.4 mm/rev.

The length of the active cutting edge l_c is built by 3 parts, according to equation 8, for $a_p < r$.

$$l_c = r\cdot(-\delta_0 + \delta_{ap} + \delta_c) \quad (8)$$

Where δ_c can be calculated as:

$$\delta_c = 2\cdot\sin^{-1}\left(\frac{f\cdot\cos(\delta_{ap})}{2\cdot r}\right) \quad (9)$$

The equivalent chip thickness he can be calculated by forming the ratio between the chip area A (according to equation 7) and the active cutting length l_c (according to equation 8). By also including the previously presented equations, the equivalent chip thickness he can be calculated as:

$$h_e = \frac{A}{l_c} = \frac{\int_{\delta_0}^{\delta_{ap}} f\sin(\delta) + r - \sqrt{r^2 - f^2\cos^2(\delta_{ap})}\,d\delta}{r\cdot\left(\sin^{-1}\left(\frac{f}{2\cdot r}\right) + \tan^{-1}\left(\frac{-f+\sqrt{2\cdot r\cdot a_p - a_p^2}}{r-a_p}\right) + \sin^{-1}\left(\frac{f\cdot\cos(\delta_{ap})}{r}\right)\right)} + \frac{\frac{f\cdot h_1\cos(\delta_{ap})}{2} + \frac{r^2}{2}\left(\frac{f\cos^{-1}(\delta_{ap})}{r} - \frac{f\cos(\delta_{ap})}{r}\right)}{r\cdot\left(\sin^{-1}\left(\frac{f}{2\cdot r}\right) + \tan^{-1}\left(\frac{-f+\sqrt{2\cdot r\cdot a_p - a_p^2}}{r-a_p}\right) + \sin^{-1}\left(\frac{f\cdot\cos(\delta_{ap})}{r}\right)\right)}$$

In **Figure 9**, h_e for $a_p < r$ and different feeds is presented.

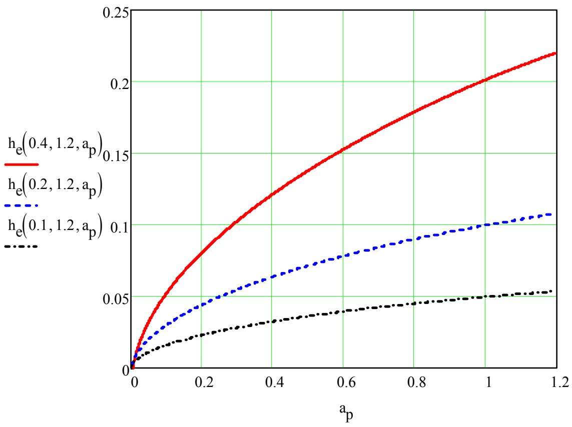


Figure 9. The equivalent chip thickness h_e as a function of depth-of-cut $a_p < r$, f=0.1 mm/rev (lower black curve), f=0.2mm/rev (middle blue curve) and f=0.4 mm/rev (upper red curve) and for nose radius r=1.2 mm.

8. COMPARISON BETWEEN h_{eW} AND h_e

In Figure 10, a comparison between calculated values of the Woxén chip equivalent (equation 2) and the new solution (equation 9) is illustrated. The deviation between the models varies depending on the range of initial conditions.

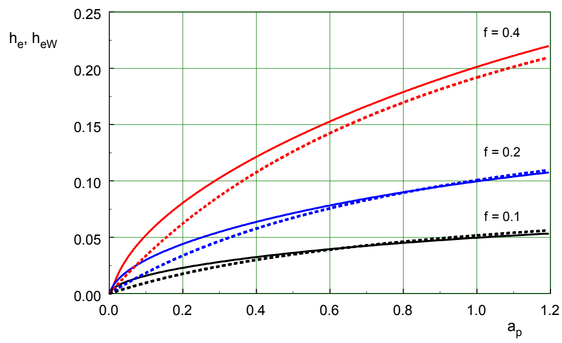


Figure 10. Comparison between true chip equivalent h_e (solid curves) and Woxéns chip equivalent h_{eW} (broken curves), as a function of depth-of-cut a_p, r=1.2 mm, f=0.1, 0.2 and 0.4 mm/rev.

An error analysis of Woxéns chip equivalent h_{eW} can be performed by formulating a function Δ according to equation 10, which describes the relative deviance in % between h_{eW} and h_e.

$$\Delta = \frac{h_e(f,r,a_p) - h_{eW}(f,r,a_p)}{h_e(f,r,a_p)}\cdot 100 \qquad (10)$$

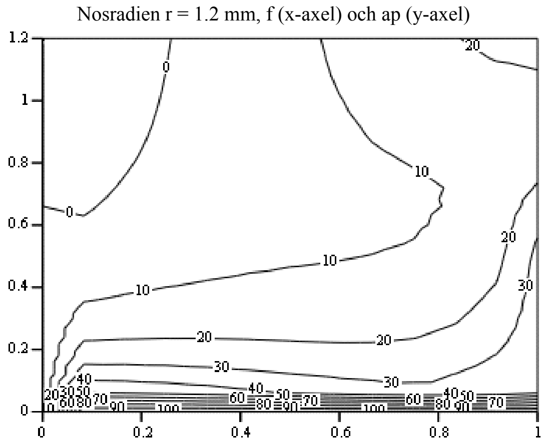
In Figure 11 the error function Δ is presented in the form of contour diagrams, for the nose radii r=1.2mm and r=1.6mm. It is evident that the error can be both positive and negative for the presented cases, where the error lies between -20 to 50 % within the finish machining area. It can also be concluded that combinations of f and a_p generates the same value of the equivalent chip thickness along the 0-line.

9. CONCLUSIONS AND DISCUSSION

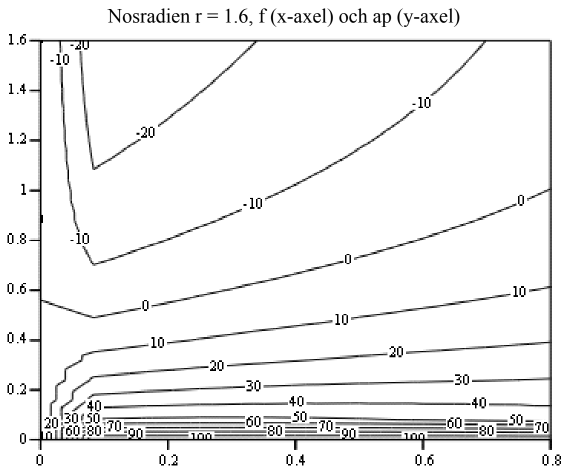
Without any closer examination it can be deduced that tool life models and systems for choice of cutting data based on Woxéns chip equivalent, provides a very limited precision within the finishing area. The largest error in the calculations of Woxéns chip equivalent is obtained with a depth-of-cut less than ¼ of the nose radius, which in the presented cases corresponds to a depth-of-cut a_p between 0.3 och 0.4 mm/rev.

A systematic fault in the determination of model constants and the subsequent application of given cutting data recommendations, can to a certain degree limit the effects of the errors in Woxén's approximation. This is due to the fact that the same cutting data combinations are used both to determine the model constants and in later production and metal cutting.

The basic idea with the chip equivalent is precisely that it is equivalent, meaning all combinations of depth-of-cut and feed that generates the same chip equivalent should also generate the same tool life, under similar conditions. In Figure 12 and Figure 13, contour diagrams illustrate how different combinations of feed and depth-of-cut generate the same chip equivalent. Figure 12 shows the results based on h_{eW}, Figure 13 shows the results based on h_e.

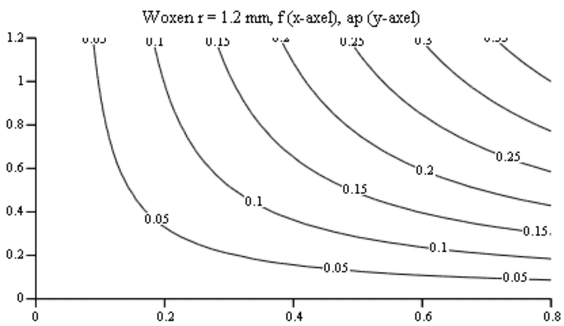


Δ 12



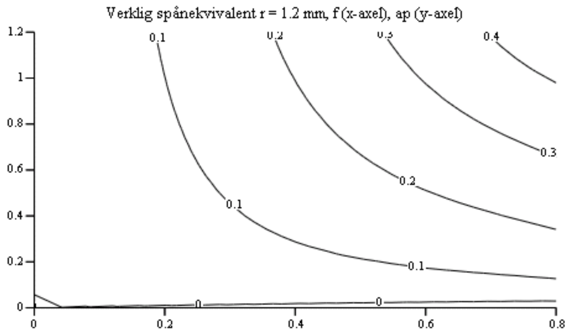
Δ 16

Figure 11. Deviations in % between true chip equivalent h_e and Woxéns chip equivalent h_{eW} as a function of feed (x-axis) and depth-of-cut (y-axis). Upper diagram: nose radius r=1.2mm, lower diagram: r=1.6mm.



h_{W12}

Figure 12. Woxéns chip equivalent h_{eW} for different feed f (x-axis) and depth-of-cut a_p (y-axis). Nose radius r=1.2 mm.



h_{e12}

Figure 13. True chip equivalent h_e for different feed f (x-axis) and depth-of-cut a_p (y-axis). Nose radius r=1.2 mm.

By using the developed models and equations to determine the chip equivalent, better conditions to predict tool life end tool wear in metal cutting are created. The model can be adapted to other types of inserts, not having the circular bridging between major and minor cutting edge.

10. ACKNOWLEDGEMENTS

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