# The Identification of Linear Ship Steering Dynamics Using Maximum Likelihood Parameter Estimation

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# THE IDENTIFICATION OF LINEAR SHIP STEERING DYNAMICS USING MAXIMUM LIKELIHOOD PARAMETER ESTIMATION

by

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## 1. SYNOPSIS

The project design for steering qualities of a new ship as well as the prediction and control of its behaviour when at sea may all be based on records of the motions of scale models or previous ships, or even on the observation of the particular ship in the immediate past. In recent years new techniques of stochastic system identification have appeared as alternatives to the deterministic methods hitherto applied to the analysis of free-sailing experiments.

The present report covers the initial results of co-operation on ship steering problems between the Swedish State Shipbuilding Experimental Tank (SSPA) and the Lund Institute of Technology Department of Automatic Control (LTH), where the theory of the maximum likelihood parameter estimation has earlier been developed for a variety of dynamic processes.

The role of system identification within ship steering analysis is discussed, the kinematics of ships are high-lighted and the mathematical models for the ship dynamics are reformulated. The system identification complex is introduced with special emphasis on parameter identifiability and estimation, and on the requirements to be placed on input-output models.

A general program package developed for the computation of the maximum likelihood estimate of the parameters of either a continuous or a discrete model is described. A special subroutine containing three different types of ship models is included.

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A program for calculating the sway velocity from heading and Decca coordinates is put forth to use the special information from manoeuvring tests.

Finally several examples of estimations of the hydrodynamical coefficients and the components of the second order transfer function, relating the yaw rate to the rudder angle, are given.

# 2. SHIP KINEMATICS AND SHIP DYNAMICS

This chapter will discuss the steering and manoeuvring characteristics of ships and the way "models" are used as an aid to understanding.

## 2.1 Mathematical Models, Scale Models, and Ships

Ship model basins routinely test "scale models" prior to the building of a ship of new design, mostly for propulsive performance only; less frequently these models are used to support the prediction of manoeuvring qualities. Whenever available the results from scale model manoeuvring tests should therefore be analyzed not only with a view to predict the behaviour of the particular ship prototype but to add to the collected and much-needed knowledge of manoeuvring characteristics related to hull geometry.

Shipyards run a number of delivery or acceptance trials with every new ship; at least one ship of each class will be subject to some manoeuvring trials. There are several recommendations and codes for manoeuvring trial programmes. The Manoeuvrability Committee of the International Towing Tank Conference is at present reviewing a new ITTC code, which will pay proper attention to the needs of ship officers who will handle the ship as well as to those of the analyst, who will correlate the results with pre-trial predictions and, again, extract as much as possible of general information from the data.

Ship trials are too expensive and time is often too short for adequate information to be obtained. Steering tests that can be performed in the course of a routine passage are strongly desired. Such tests may also be used to acquire supplementary information on the characteristics in shallow water areas, etc.

The prediction, analysis and correlation of manoeuvring performance, and the transformation and storage of information on manoeuvring characteristics are possible only be reference to the equations of motion for or the "mathematical model" of the ship-and-screw-and-rudder system. Together with telemotor system and steering engine this first-mentioned system forms the forward leg of the steered ship loop.

The ultimate information asked for by the naval architect or control engineer is the numeric value of each of the individual coefficients appearing in the mathematical model chosen. The method to find these coefficients from the analysis of experiments is "parameter identification". As will be seen in the next chapter the probabilistic identification theory has the potential of deriving the model structure as well, which motivates the term "system identification".

As far as possible the same model structure should be used in experiment analysis and prediction synthesis. This structure must be soundly based on the dynamics of the ship. Depending on the type of experiment available and on the completeness and accuracy of the data there will still be a need for mathematical models of varying complexity, with and without non-linear elements. In first-order or approximate models the coefficients to be identified are generally of a composite form, built up from combinations of the original coefficients in the "complete" equations of motion. (See below.) Such models have been widely used in the deterministic identification of ship characteristics by "equation error" type methods - Nomoto (1960), Norrbin (1965) - and by the phase-plane analysis technique - cf Bech and Wagner Smitt (1969).

The diagram in Fig 2.1 has been compiled to indicate the role of parameter identification in ship manoeuvring applications. A special note on scale-model-to-ship predictions is pertinent to the subject.

## 2.2 A Note on Scale Model Experiments and Data Extrapolation

Scale model experiments may be divided in two principally different groups, one including force measurements on constrained or "captive" models, the other including tests with free-sailing self-propelled models that are manned or remotely controlled, or auto-piloted.

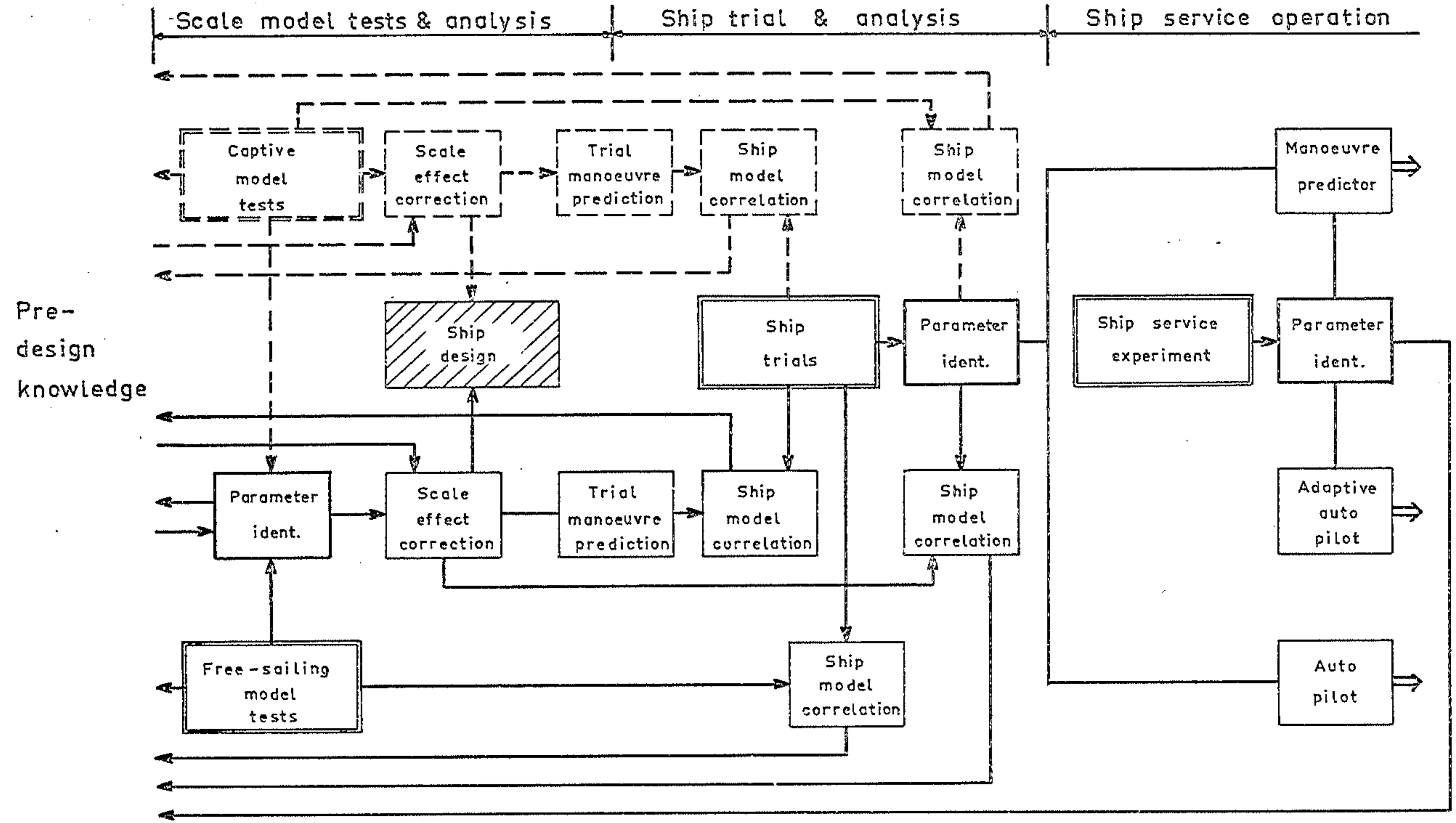



Fig 2.1 Block diagram illustrating the role of steering parameter identification in scale-model and ship experiment analysis, and in ship operation

The principle of the Planar Motion Mechanism (PMM) was introduced by Gertler and Goodman (1960) for force measurements on submerged models, which were oscillated in well-defined motions in the vertical plane. It has since then been widely adopted at tests with surface ship models; cf Strøm-Tejsen & Chislett (1966).

The free-sailing scale model technique is often used to supply direct answers to special steering problems. The scale model (moving in obedience of the Froude law) may be said to be a programmed analogue computer, furnishing the model structure of the dynamics - unknown to the experimenter - as well as the computer facility. In common with the ship trial as discussed below this problem-orientated free-model technique has two main drawbacks:

1. The solution of a new manoeuvre problem requires a complete new experiment.

2. New results for a modified configuration or load condition cannot be uniquely interpreted as to their causes.

In addition the scale model may suffer from scaling-law divergences (scale effects), mainly due to screw and rudder loadings being affected by a viscous boundary layer that is "too thick" and a viscous resistance that is "too large". During the tests an auxiliary air screw is sometimes used to supply a friction force allowance, but this artifice will introduce other discrepancies. Also, the dynamics of the propulsion engine will have characteristics which are different from those of the ship.

3. There is no proper way to enter corrections for scale effects into the free-model "computer program", but only to apply empirical correction factors to the over-all or final results.

A typical correction of this kind may be, say, an empirically motivated 7.5 per cent deduction of model steady turning diameter - cf Burcher

(1972); much larger corrections could well be necessary in the transient stage, however.

In contrast to the free-sailing technique the alternative captive scale model technique - with subsequent computer predictions of full scale behaviour - does accept the proper corrections for the scale effects, applied to individual "coefficients" in the equations of motion. These corrections are based on the continuous model/ship correlation as well as on detailed studies of model flow phenomena, which latter add to a special "block" of knowledge.

Obviously the access to a suitable method of parameter identification will greatly enhance the premises of model/ship correlation, and so the free-sailing model technique will gain a new potential as an alternative to the captive model testing. (Again, more or less the same method of identification may eventually be applied to improve the extraction of data from PMM force measurements!)

## 2.3 A Look at the Kinematics

Before discussing different mathematical models it is worth while to look for guidance from the kinematics of typical ships, manoeuvring in a horizontal plane as shown in Fig 2.2. One of these ships is a dynamically unstable full-form tanker, the other a dynamically stable narrow-beam destroyer.

A ship is said to be dynamically stable on straight course or in a turn of constant curvature if, upon a small disturbance from its steady motion, it soon resumes that same motion along a slightly shifted path, without any correcting rudder being applied.

A ship which is dynamically unstable on a straight course will enter a spiral turn, however small the disturbance met, and it will end up moving in a circle of certain radius, in which it will now be stable with zero rudder.

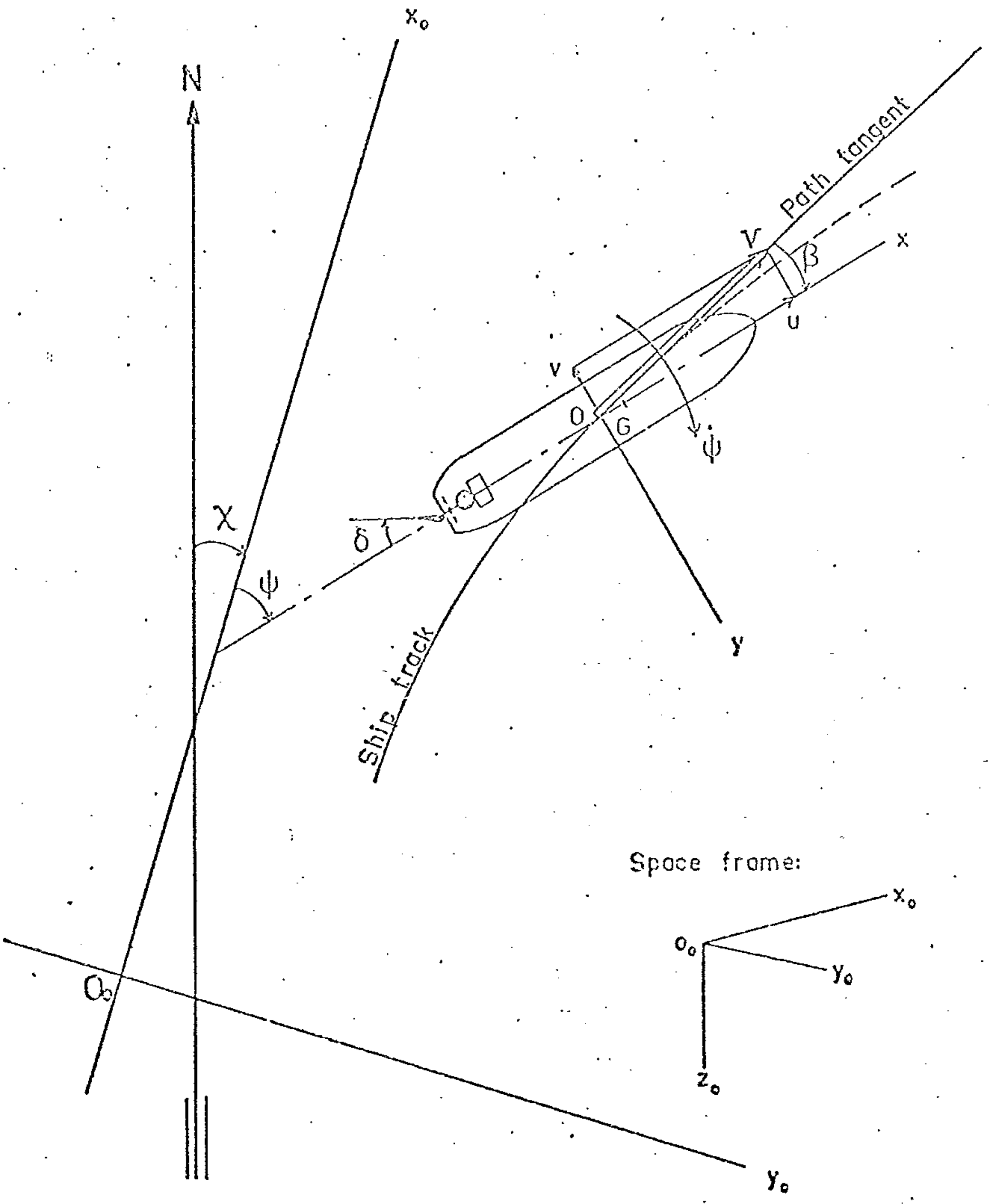



Fig 2.2 Coordinate systems and symbols in surface ship kinematics

The "classical" acceptance trial manoeuvre to be performed with a new ship is the "hard-over turning circle", which is in effect a kind of step response test applied to an over-damped higher-order non-linear element. From the nautical point of view the result is expressed in terms of minimum "reach", "advance" and "tactical diameter" at maximum helm $\delta$, which may be compared with handbook values. The old figures still hold true, the tactical diameter being typically 3.5 ship lengths for the tanker and 7 ship lengths for the destroyer at high speed, as illustrated in Fig 2.3. If, thus, modern technology has not changed these handbook values very much it mainly reflects the fact that ship operators have not specified any new requirements, but also that the turning characteristics are largely inherent in the main proportions of the type of ship considered.

The differences between the two ships become even more obvious if transient be compared to transient and steady turning to steady turning. The time histories of the turning manoeuvres are shown in Fig 2.4, also including a so called "pull-out", in which the rudder is simply returned to midship.

Based on ship lengths travelled the "reach" of the tanker is roughly twice as large as that of the destroyer, indicating a larger effective "time constant". For further reference this time constant of the equivalent first-order system will be denoted by $T'_s$.

Whereas the final turning circles in Fig 2.3 are about the same, the non-dimensional constant turning parth curvature $L/R_c$ (or yaw rate $\dot{\psi}'_c$) is also much larger for the tanker than for the destroyer; this indicates a higher effective gain, $K'_s = \dot{\psi}'_c/\delta_c$, which may be attributed to a smaller effective yaw-damping moment experienced by the tanker hull. In Fig 2.5 the non-dimensional yaw rate obtained from the turning manoeuvre is plotted in a diagram of steady-state characteristics $\dot{\psi}'_c(\delta_c)$. The curves shown are the loci derived from turning circles at several helm angles, and in particular from the special trial manoeuvre known as the "reversed spiral". (Wagner Smitt (1967).) The full drawn

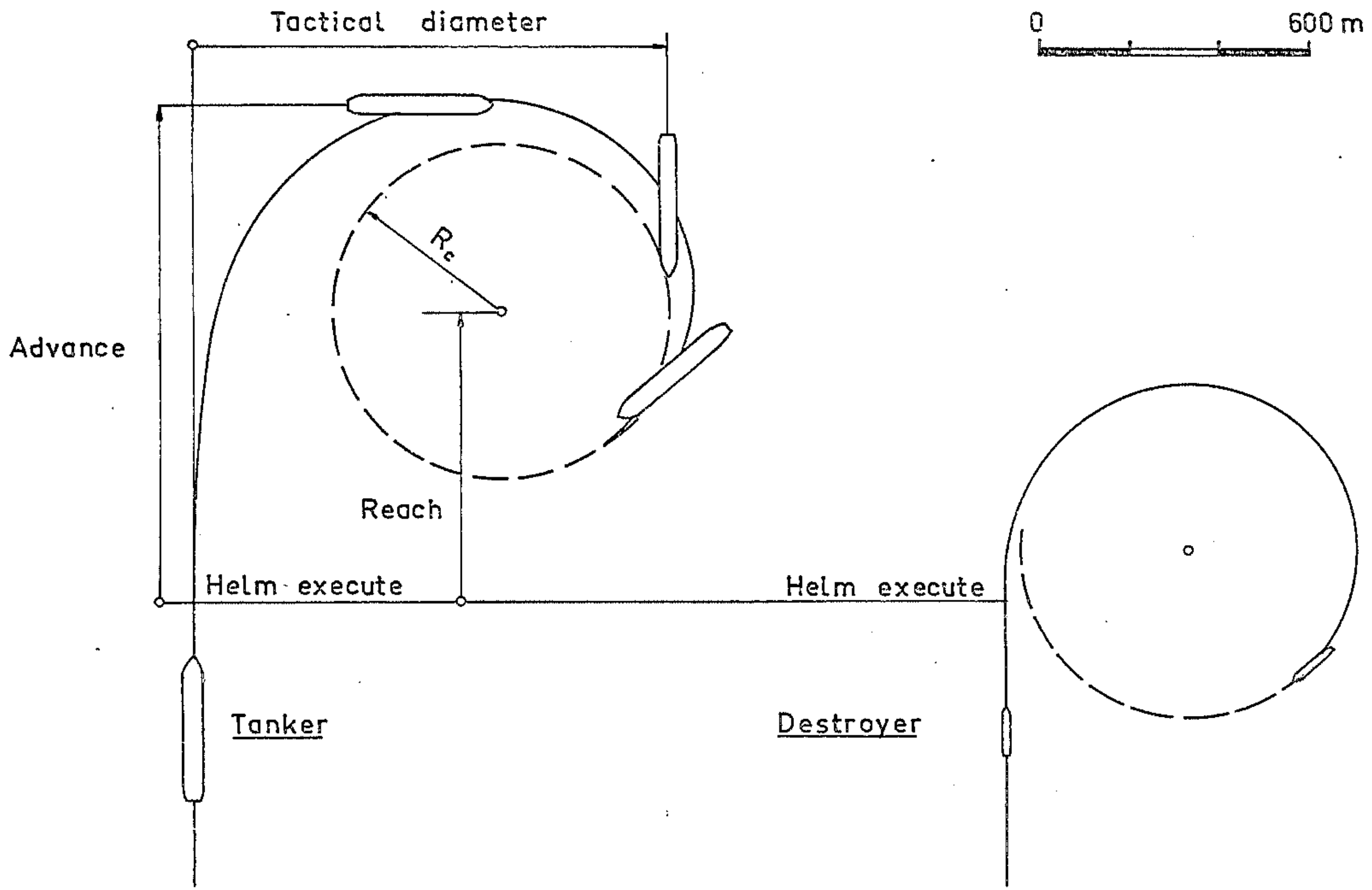



Fig 2.3 Typical turning circle tracks for a 300 m tanker and a 110 m destroyer

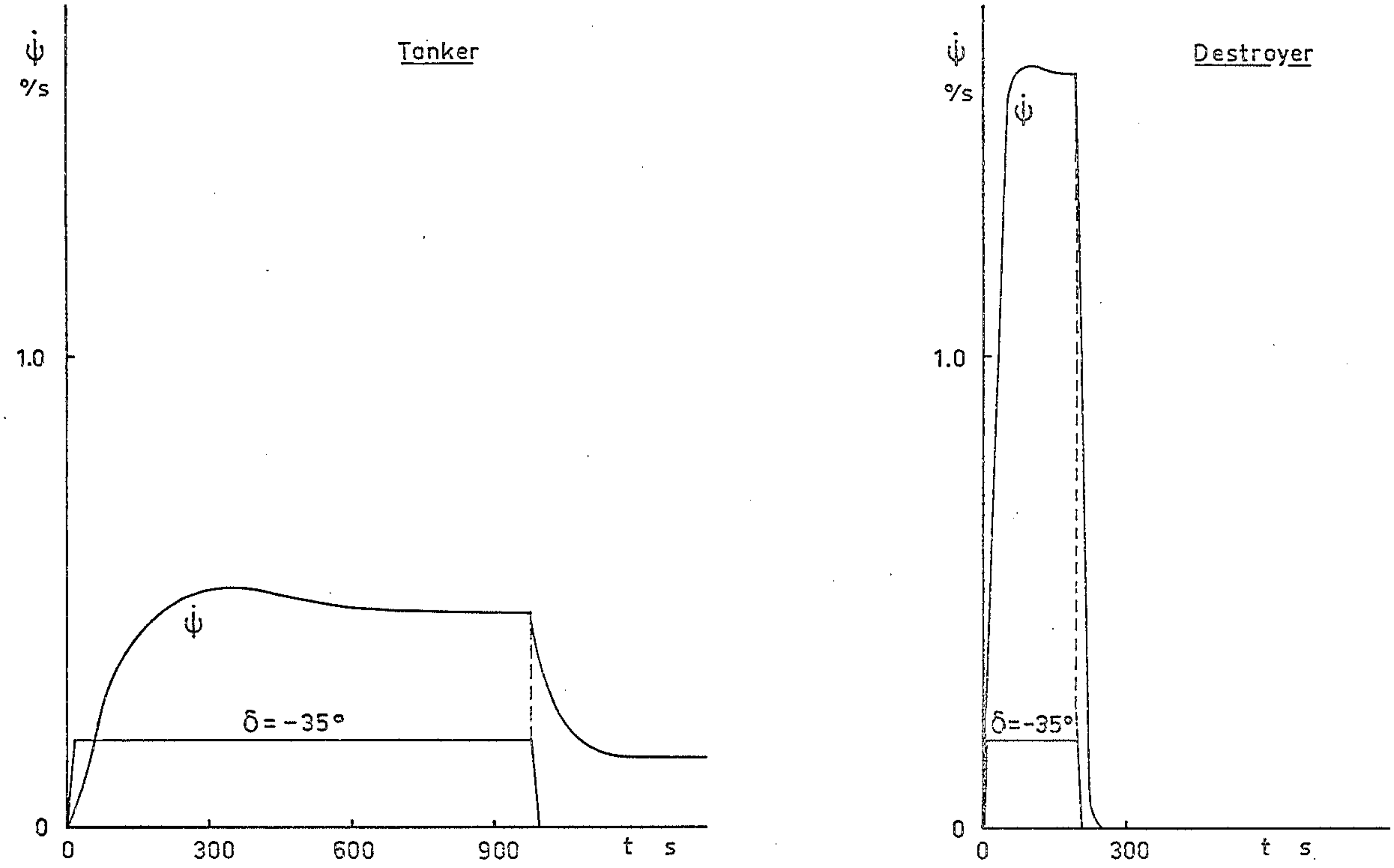



Fig 2.4 Schematic time histories of turning circle manoeuvres with pull-outs

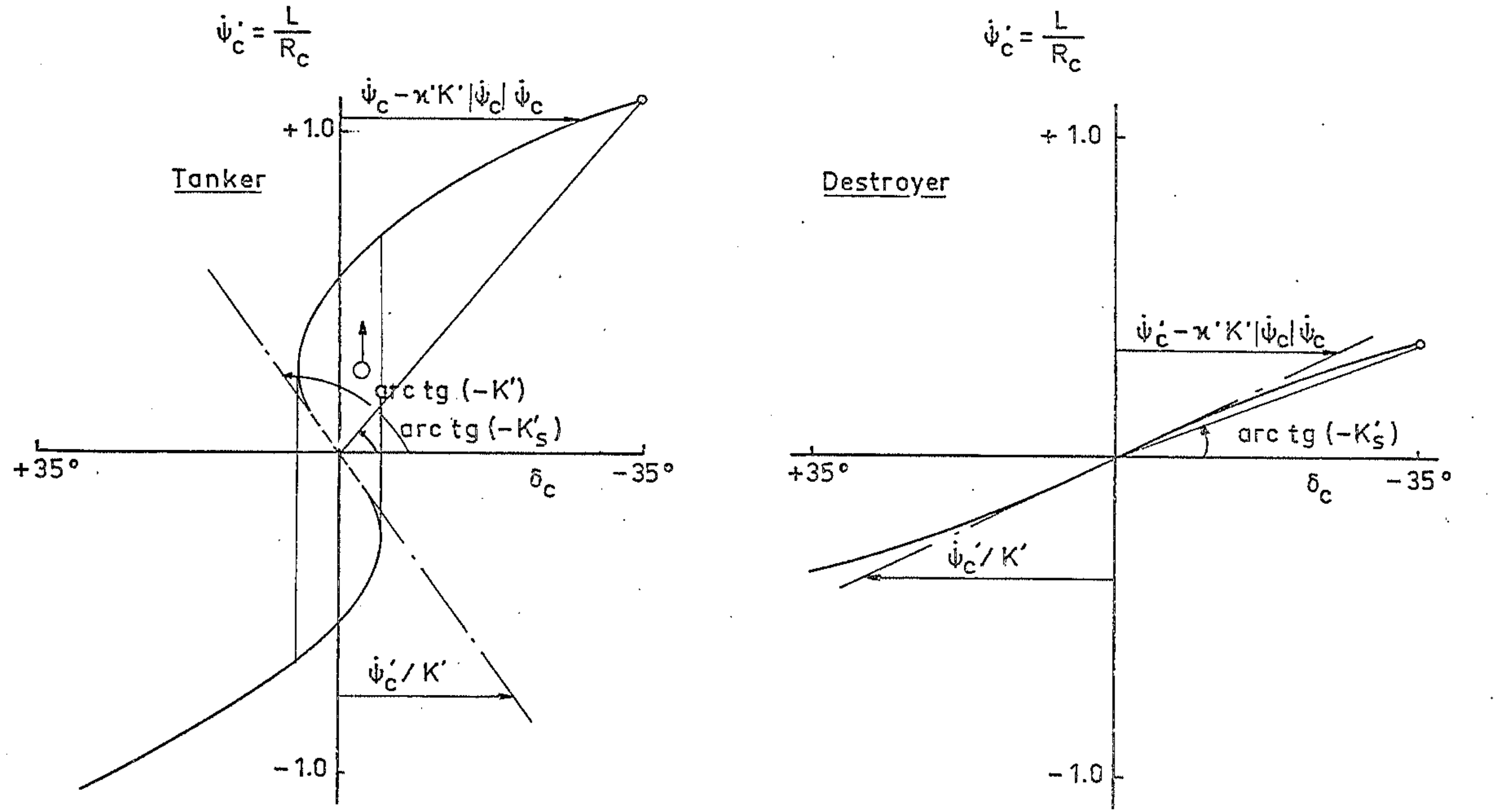



Fig 2.5 Non-dimensional steady-state steering characteristics for the unstable tanker and the stable destroyer. The diagrams also illustrate the definition of K' (and of "effective" $K'_s$) as well as one particular form of approximation to the non-linear characteristics. (Cf Section 2.8.)

branches of the spiral loci represent stable steady-state conditions, whereas the dotted centre branch is the unstable equilibrium condition, which is characteristic for the dynamically unstable ship.

Surely, the handling of a ship is not just turning kinematics, but it involves steering on a straight course as well as repeated changes of heading in fairways or congested areas, etc, etc; at best, it is based on an understanding of the dynamics of the ship.

A good helmsman or auto-pilot will steer a large unstable tanker on a straight course almost as well as if it were stable. The diagram in Fig 2.6 shows a time history of the $\dot{\psi}(\delta)$ locus during a simulator test, plotted on top of the spiral characteristics. In fact it will be possible to control the ship with small helm, well inside the width of the loop, if it is observed that the yaw rate should never be allowed to develop too much before checking rudder be applied; for checking the rudder must be moved over to the opposite side of the dotted curve.

Special trial manoeuvres (such as the zig-zag test by Kraemer (1934)) have been devised to describe the controllability and response of a ship in a relative manner, as compared with other ships or as predicted from ship model basin experiments with a free-sailing scale model. The standard or Kempf zig-zag test procedures - using $10^{\circ}$ opposite helm execute at $10^{\circ}$ change from initial heading, or $20^{\circ}$ helm at $20^{\circ}$ change of heading - both result in oscillations of almost the same "period", which for different ship types may vary within a range from 6 to 14 ship lengths. The "dynamic gain", i e the ratio of double amplitude of yaw rate to that of helm angle, should reflect the inertia in the motion, but the results are obscured by the influence of non-linear damping. The multitude of explicit data collected along these lines has again failed to add very much of useful information on the dynamics of different types of ships.

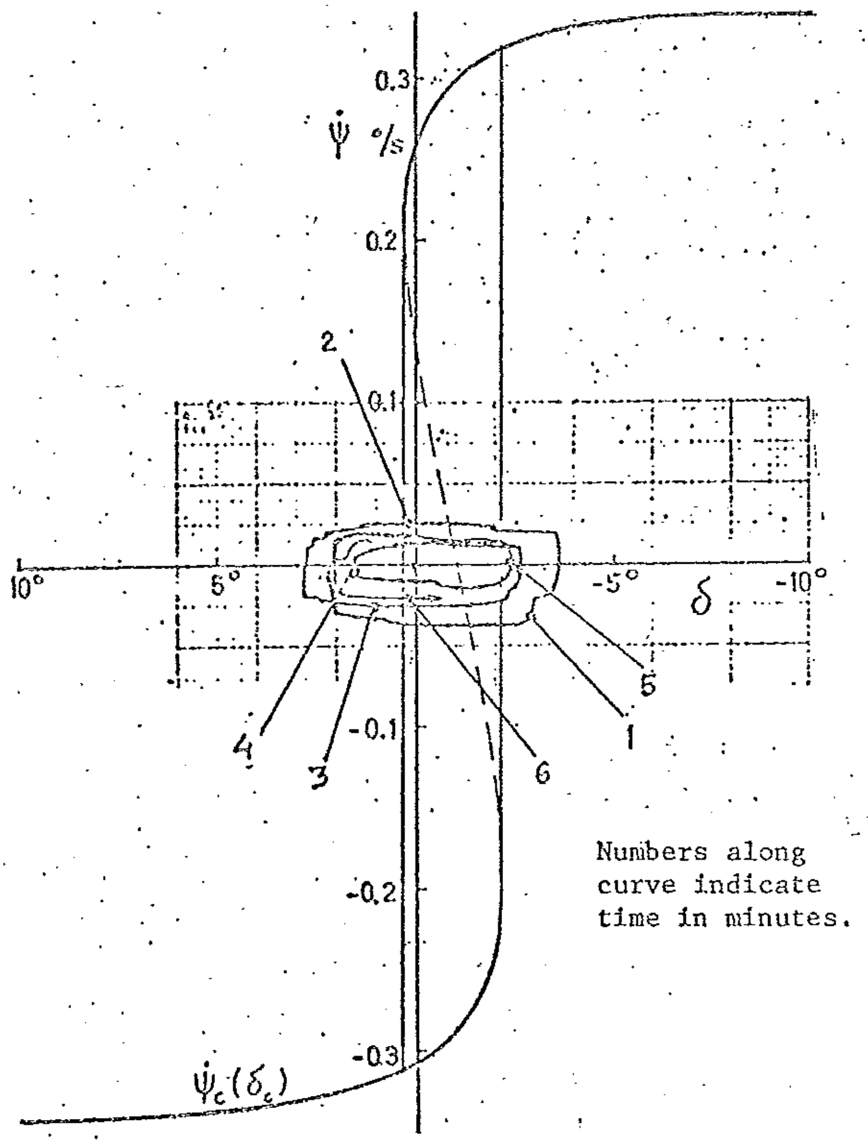



Fig 2.6 Time history of the straight course helm angle - yaw-rate locus of a 200 000 tdw tanker during manual control in the SSPA simulator, plotted on top of the steady-state steering characteristics

## 2.4 The Mathematical Models

In applications to ship motion studies in general the ship may be regarded as a rigid body with six degrees of freedom, i e three translations and three rotations. The following relates to the analysis of the motion proper in response to the control actuations achieved at the rudder and screw.

The ship moves in the interface between two fluids and the generation of waves and vortices are real world facts, since long well-known to be the possible sources of "memory effects" in the dynamics. A functional approach to the handling of such effects within a linear theory has been presented by Bishop, Burcher and Price (1973). In the application of ordinary differential equations the equivalent approach would require coefficients which vary with the reduced frequency, $\omega' = \omega \cdot L/V$. A well-documented experience of calm-water ship manoeuvring analysis has shown that this frequency-dependence may usually be ignored in favour of a quasi-steady low-frequency theory, which more readily accepts the unavoidable complications due to non-linear hydrodynamics.

The formal equations of motion are conveniently written down in Euler form, with reference to body axes in a right-handed system where x is positive forward and y positive to starboard. The same convention infers that a stern rudder for lateral manoeuvring will produce a positive (starboard) yawing moment when switched in negative (starboard) direction. (Cf Fig 2.2.)

For the purpose of this study the problem will be greatly simplified in that the discussion is confined to motions in the horizontal plane only, and in that the influence of heeling in a turn is ignored. Mostly this heeling is small in itself, and in other cases the coupling effect on sway and yaw has been shown to remain small. The turning of a ship is initiated by rudder control but the subsequent motion is largely governed by the hydrodynamic forces due to drift or side-slip. This is in clear contrast to the case of an aircraft, where side-slip is to be kept as small as possible, whereas the horizontal turn is achieved by proper banking. In comparison with the aircraft steering problem, again, the ship problem is complicated by the dominating influence of added inertias and large-value damping non-linearities.

Referring to Fig 2.2 the steering manoeuvre will be described by three second order differential equations for the linear translations u and v and the yaw rate $r = \frac{d\psi}{dt} = \dot{\psi}$. The heading angle $\psi$ may indicate the attitude in relation to a datum line in the $x_0y_0$ plane of the space frame or in relation to the approach direction prior to a certain manoeuvre. From a knowledge of the initial position ship track and heading are obtained by integration over time t, or over the distance $t' = \int\frac{Vdt}{L}$ in ship lengths sailed. Thus

$$x_0(t) = \int_0^t (u\cos\psi - v\sin\psi)dt$$

$$y_0(t) = \int_0^t (u\sin\psi + v\cos\psi)dt \qquad (2.1)$$

$$\psi(t) = \int\dot{\psi}dt$$

The drift velocity v is related to the side-slip angle $\beta = -\arcsin\frac{v}{V}$, and therefore the position of the ship may alternatively be calculated as

$$x_0(t) = \int_0^t V\cos(\psi - \beta)dt$$

$$y_0(t) = \int_0^t V\sin(\psi - \beta)dt \qquad (2.2)$$

The derivation and application of the Euler equations to the ship manoeuvring problem may be found in papers by Davidson & Schiff (1946), Norrbin (1960) and Abkowitz (1964). More or less "complete" non-linear models have been put forth by Abkowitz (1964), by Eda & Crane (1965) and by Norrbin (1970).

As the parameter identification procedure presented in the following chapters will treat the linear problem only, the equations of motion pertinent to the present task are written in the form of eq (2.3) on the next page. The mass inertias proper, displayed in the left side members, have been complemented by those parts of the hydrodynamic forces, which are usually named "added masses", and which will mean - as already pointed out - a substantial increase of virtual inertias.

The first position in each of the right side members is reserved for control forces, which at constant speed and screw loading (or constant speed u and revs n) may be taken as proportional to helm angle $\delta$. (If the rudder is not within the influence of the screw race the rudder force is of course not a function of screw loading.)

The right side members in second and third positions are approximations to damping forces in steady sway or yaw, and they include pure mass forces as well as the contributions due to hydrodynamics. If the forward speed u is fairly constant the four terms may obviously be considered as linear. If speed variation is handled separately their linearization is still valid in terms of side-slip and path curvature. Their coefficients will also appear in the analytical condition for dynamic stability on a straight course.

The mass forces in positions (4) and (5) should be included in the formal linear model although they are mostly small enough to be ignored.

All terms in the positions (5), (6) and (7) are non-linear. The non-linear "residuals" indicated in the last column positions are complicated functions of the hydrodynamic state of motion, and they will be required in the proper description or simulation of "tight" or radical manoeuvres. As an example, the lateral force due to side-slip no longer

| Spec | Linear |  | Linear |  |  |  | Non-Linear |  |  |
|---|---|---|---|---|---|---|---|---|---|
|  | inertia terms |  | control terms | damping terms |  | inertia couplings | inertia complings |  | residual terms |
| Position | 0 |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| X | $(m-X_{\dot{u}})\,\dot{u}$ | = |  |  |  |  | $+(mx_G-\frac{1}{2}X_{rr})\dot{\psi}^2$ | $+(m+X_{vr})v\dot{\psi}$ | $+\tilde{X}$ |
| Y | $(m-Y_{\dot{v}})\,\dot{v}$ | = | $Y^R(u,n)\cdot\delta$ | $+Y_{uv}uv$ | $+(Y_{ur}-m)u\dot{\psi}$ | $+(Y_{\dot{r}}-mx_G)\ddot{\psi}$ |  |  | $+\tilde{Y}$ |
| N | $(mk_{zz}^2-N_{\dot{r}})\,\ddot{\psi}$ | = | $N^R(u,n)\cdot\delta$ | $+N_{uv}uv$ | $+(N_{ur}-mx_G)u\dot{\psi}$ | $+(N_{\dot{v}}-mx_G)\dot{v}$ |  |  | $+\tilde{N}$ |

(2.3)

increases in proportion to the drift velocity v but much faster, due to the additional contribution of cross-flow viscous drag.

It should be especially observed that the force balance along the x-axis is completely non-linear. The added "resistance" in a turn is dominated by the fore-and-aft component of a "centripetal force", in which the mass force is strengthened by a force of hydrodynamic nature. (The centripetal acceleration is usually written as $V^2/R$, where the momentaneous radius is given by $R = V/\dot{\psi}$. The acceleration thus has x and y components equal to $u\dot{\psi}$ and $v\dot{\psi}$ respectively.)

The diagrams in Figs 2.7 and 2.8 illustrate typical records of helm manoeuvre and speed and heading response (top) as well as "linear" and "non-linear" contributions to the sway and yaw accelerations in simulated $10^{\circ}/10^{\circ}$ and $20^{\circ}/20^{\circ}$ zig-zag tests with a container ship. Whereas the linear analysis may possibly have some justification when applied to the $10^{\circ}/10^{\circ}$ test it is quite clear that this is no longer true in the case of the $20^{\circ}/20^{\circ}$ test.

If, thus, even the $10^{\circ}/10^{\circ}$ zig-zag test is far from a linear manoeuvre it would be tempting to revert to an analysis of the routine course-keeping record. Unfortunately, now, such records often display the very-low-frequency limit cycle type steering associated with non-linear elements as shown in Fig 2.6; the non-linear element may be represented by the ship hydrodynamics, the steering engine, or the helmsman's mode of control. A forced bang-bang type small-helm control executed at random intervals will produce a nearly straight course and a response record covering a wide frequency spectrum, however, and it will be seen to furnish an adequate input for linear analysis. (The response frequencies now considered may still be said to be "low" with respect to the hydrodynamic frequency effects mentioned earlier.)

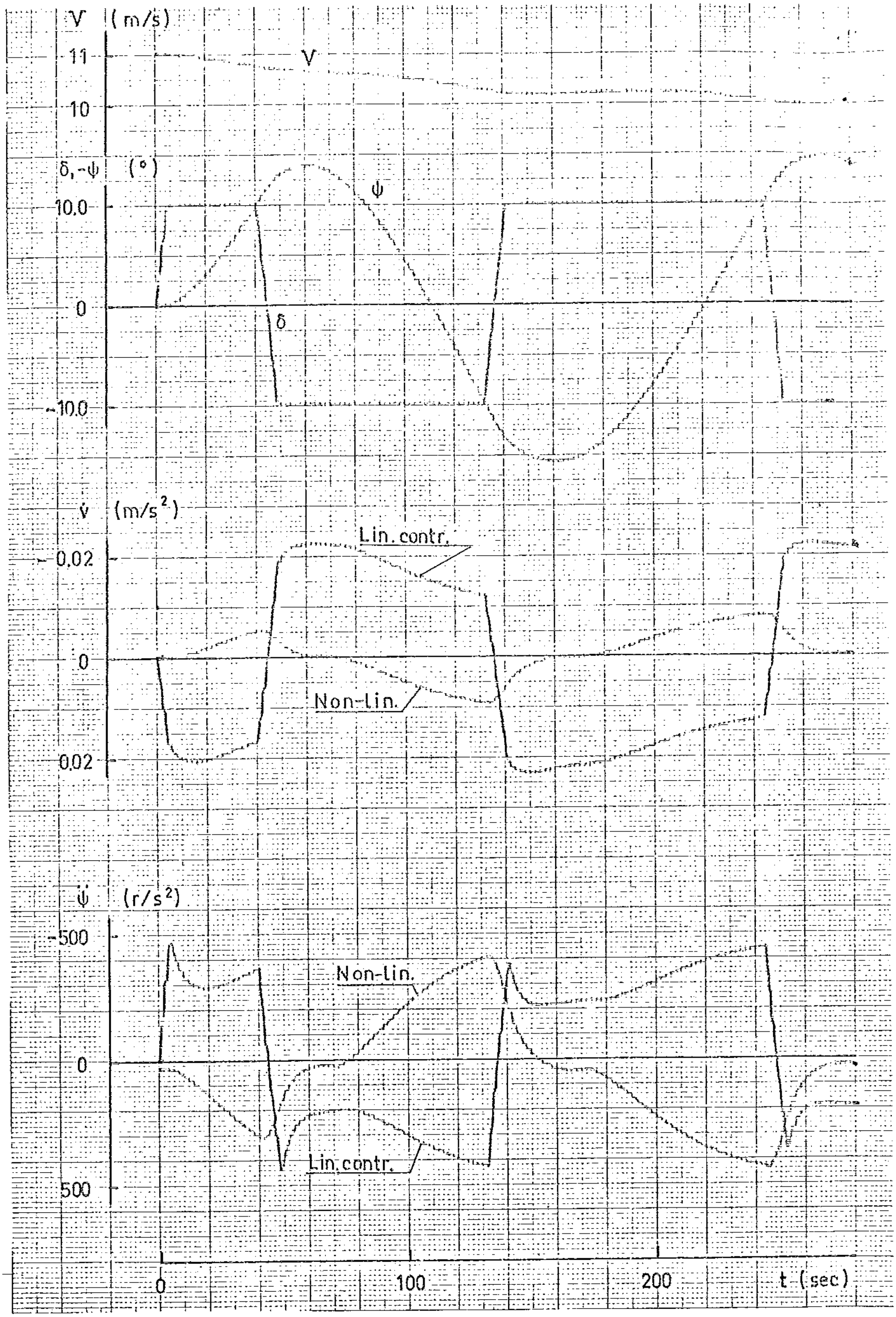



Fig 2.7 Linear and non-linear contributions to sway and yaw accelerations in the mathematical modelling of a $10^o/10^o$ zig-zag manoeuvre with a twin-screw container ship

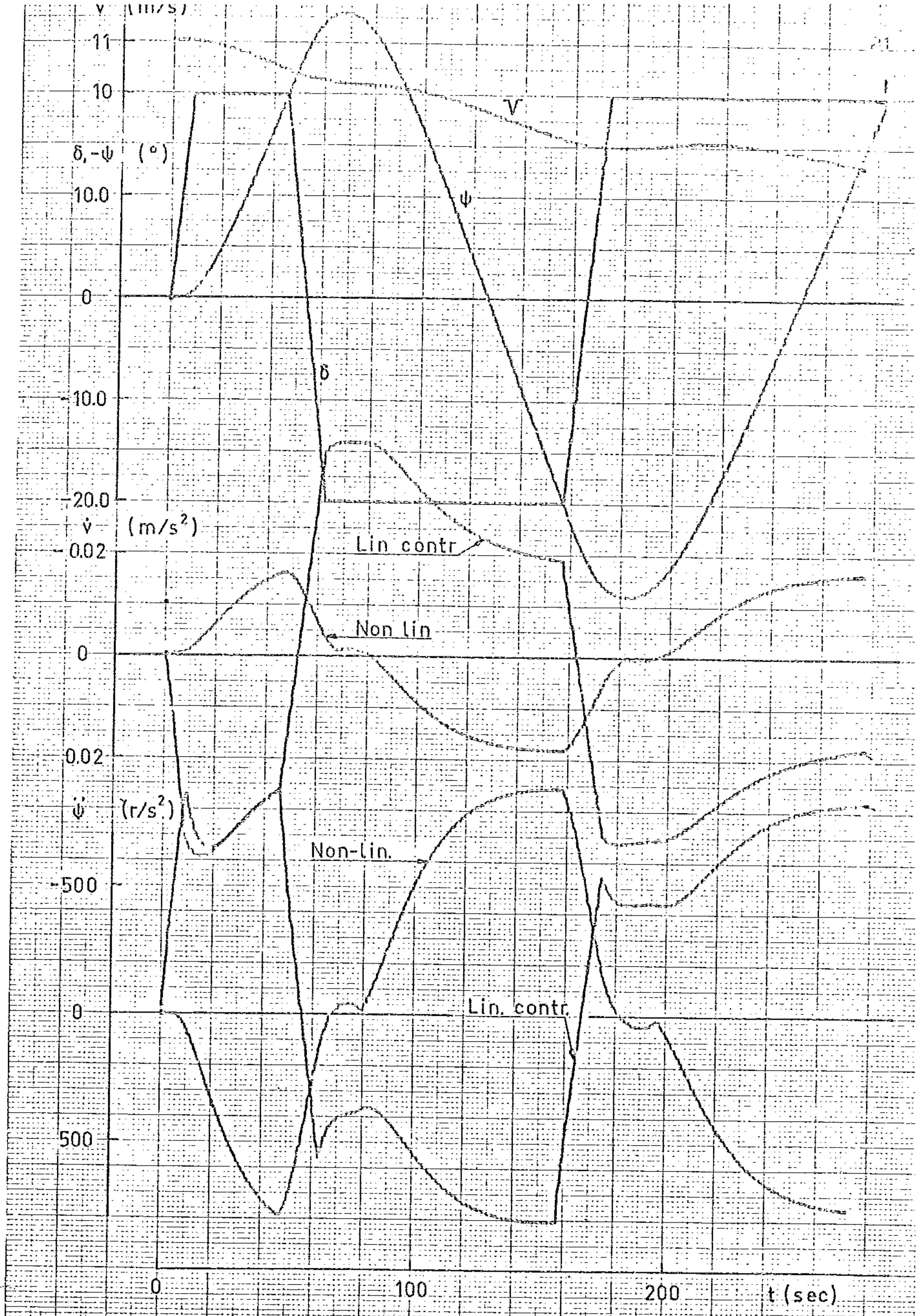



Fig 2.8 Linear and non-linear contributions to sway and yaw accelerations in the mathematical modelling of a 20°/20° zig-zag manoeuvre with a twin-screw container ship

## 2.5 The Linear Model

From the above it follows that the linearized model will include perturbations in sway and yaw only, i e, the Y- and N-equations. That does not mean that speed loss must necessarily be ignored, but it can be handled separately. In general the forces involved are all proportional to speed squared or roughly, in normal steering manoeuvres, to forward speed squared, $u^2$.

It is convenient to introduce non-dimensional coefficients to characterize the ship geometry in the dynamics. The "bis" system - Norrbin (1970) - is particularly handy in more general cases; here all forces are related to the displacement force of the ship, linear accelerations are related to the acceleration of gravity, and the speed is given by the Froude number. In dimension-true equations the non-dimensional coefficients (or "derivatives") appear together with multiplication factors in powers of ship length L.

On the next page the Y- and N-equations (2.4) are presented in this way. The formal forms appear at top and bottom, and the equations may be reduced as indicated in the way of the arrows. The example shown refers to a single screw tanker, where the rudder force is taken to be proportional to rudder angle $\delta$ and to $n^2$, i e to RPM squared. The asymmetric lateral force, which is an effect of the single-screw action, is likewise taken to be proportional to $n^2$. When speed and RPM variations are moderate the rudder force will vary with $\delta$ only, and the asymmetric force will be balanced by a small residual helm $\delta_0$. In the absence of outer disturbances $\delta_0 = -1^\circ$ may be a typical mean helm value on a straight course.

## 2.6 Outer Disturbances in the Linear Model

A record of heading angles and helm positions in a ship experiment will exhibit the influence of water currents and wind loadings. Referring to Fig 2.9 it will be observed that a steady current will not be sensed by

Y

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| — | $ms^{-2}$ | | $m$ | | $s^{-2}$ | | $m^{-1}$ | | $m^2s^{-2}$ | | | | $ms^{-2}$ | | $m^{-1}$ | | $m^2s^{-2}$ | | $m$ | | $s^{-2}$ |
| $1-Y''_{\dot v}$ | $\dot v$ | | $L$ | $Y''_{\dot r}-x''_G$ | $\ddot\psi$ | | $L^{-1}$ | $Y''_{uv}$ | $uv$ | | $1$ | $Y''_{ur}-1$ | $u\dot\psi$ | | $L^{-1}$ | $\frac{1}{2}Y''_{cc\delta}$ | $c^2\delta$ | | $L$ | $\frac{1}{2}Y''_{nn}$ | $n^2$ |
| 1.67 | $\dot v$ | = | 329.2 | −0.050 | $\ddot\psi$ | + | $329.2^{-1}$ | −1.21 | $uv$ | + | 1 | −0.525 | $u\dot\psi$ | + | $329.2^{-1}$ | 0.210 | $c^2\delta$ | + | 329.2 | $8\cdot10^{-7}$ | $n^2$ |
| 1.67 | $\dot v$ | | −16.46 | | $\ddot\psi$ | | −0.00368 | | $uv$ | | −0.525 | | $u\dot\psi$ | | 0.000638 | | $c^2\delta$ | | 0.000263 | | $n^2$ |
| $(m-Y_{\dot v})/m$ | $\dot v$ | | $(Y_{\dot r}-x_G)/m$ | | $\ddot\psi$ | | $Y_{uv}/m$ | | $uv$ | | $(Y_{ur}-m)/m$ | | $u\dot\psi$ | | $\frac{1}{2}Y_{cc\delta}/m$ | | $c^2\delta$ | | $\frac{1}{2}Y_{nn}/m$ | | $n^2$ |

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $\dot v$ | | $(Y_{\dot r}-x_G)/(m-Y_{\dot v})$ | $\ddot\psi$ | | $Y_{uv}/(m-Y_{\dot v})$ | $u$ | $v$ | | $(Y_{ur}-m)/(m-Y_{\dot v})$ | $u$ | $\dot\psi$ | | $\frac{1}{2}Y_{cc\delta}/(m-Y_{\dot v})$ | $c^2/n^2$ | $n^2$ | $\delta$ | | $\frac{1}{2}Y_{nn}/(m-Y_{\dot v})$ | $n^2$ |
| 1 | $\dot v$ | | −9.86 | $\ddot\psi$ | | −0.00220 | $u$ | $v$ | | −0.314 | $u$ | $\dot\psi$ | | 0.000382 | 34 | $n^2$ | $\delta$ | | 0.000158 | $n^2$ |
| 1 | $\dot v$ | = | −9.86 | $\ddot\psi$ | + | −0.00220 | | $v$ | + | −0.314 | | $\dot\psi$ | + | 0.0130 | | 2.0 | $\delta$ | + | 0.000158 | 2.00 |
| 1 | $\dot v$ | | −9.86 | $\ddot\psi$ | | −0.0192 | | $v$ | | −2.74 | | $\dot\psi$ | | 0.0250 | | | $\delta$ | | 0.000316 | |
| 1 | $\dot v$ | | $(Y_{\dot r}-x_G)/(m-Y_{\dot v})$ | $\ddot\psi$ | | $Y_v/(m-Y_{\dot v})$ | | $v$ | | $(Y_r-mu)/(m-Y_{\dot v})$ | | $\dot\psi$ | | $Y_\delta/(m-Y_{\dot v})$ | | | $\delta$ | | $Y_o/(m-Y_{\dot v})$ | |
| — | $ms^{-2}$ | | $m$ | $s^{-2}$ | | $s^{-1}$ | | $ms^{-1}$ | | $ms^{-1}$ | | $s^{-1}$ | | $ms^{-2}$ | | | | | $ms^{-2}$ | |

(2.4)

N

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| — | $s^{-2}$ | | $m^{-1}$ | $ms^{-2}$ | | $m^{-1}s^{-1}$ | | $ms^{-1}$ | | $s^{-1}$ | | $s^{-1}$ | | $s^{-2}$ | | | | | $s^{-2}$ | |
| 1 | $\ddot\psi$ | | $(N_{\dot v}-x_G)/(J-N_{\dot r})$ | $\dot v$ | | $N_v(J-N_{\dot r})$ | | $v$ | | $(N_r-mux_G)/(J-N_{\dot r})$ | | $\dot\psi$ | | $N_\delta/(J-N_{\dot r})$ | | | $\delta$ | | $N_o/(J-N_{\dot r})$ | |
| 1 | $\ddot\psi$ | | −0.00122 | $\dot v$ | | −0.000145 | | $v$ | | −0.0680 | | $\dot\psi$ | | −0.000628 | | | $\delta$ | | −0.0000074 | |
| 1 | $\ddot\psi$ | = | −0.00122 | $\dot v$ | + | −0.0000166 | 8.74 | $v$ | + | −0.00778 | 8.74 | $\dot\psi$ | + | −0.000314 | | 2.0 | $\delta$ | + | −0.0000037 | 2.00 |
| 1 | $\ddot\psi$ | | −0.00122 | $\dot v$ | | −0.0000166 | $u$ | $v$ | | −0.00778 | $u$ | $\dot\psi$ | | −0.00000923 | 34 | $n^2$ | $\delta$ | | −0.0000037 | $n^2$ |
| 1 | $\ddot\psi$ | | $(N_{\dot v}-x_G)/(J-N_{\dot r})$ | $\dot v$ | | $N_{uv}/(J-N_{\dot r})$ | $u$ | $v$ | | $(N_{ur}-mx_G)/(J-N_{\dot r})$ | $u$ | $\dot\psi$ | | $\frac{1}{2}N_{cc\delta}/(J-N_{\dot r})$ | $c^2/n^2$ | $n^2$ | $\delta$ | | $\frac{1}{2}N_{nn}/(J-N_{\dot r})$ | $n^2$ |

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(J-N_{\dot r})/mL^2$ | $\ddot\psi$ | | $(N_{\dot v}-x_G)/mL^2$ | | $\dot v$ | | $N_{uv}/mL^2$ | | $uv$ | | $(N_{ur}-mx_G)\,mL^2$ | | $u\dot\psi$ | | $\frac{1}{2}N_{cc\delta}/mL^2$ | | $c^2\delta$ | | $\frac{1}{2}N_{nn}/mL^2$ | | $n^2$ |
| 0.100 | $\ddot\psi$ | | −0.000122 | | $\dot v$ | | −0.00000166 | | $uv$ | | −0.000778 | | $u\dot\psi$ | | −0.000000923 | | $c^2\delta$ | | −0.00000037 | | $n^2$ |
| 0.0576+0.0424 | $\ddot\psi$ | = | $329.2^{-1}$ | −0.040 | $\dot v$ | + | $329.2^{-2}$ | −0.180 | $uv$ | + | $329.2^{-1}$ | −0.256 | $u\dot\psi$ | + | $329.2^{-2}$ | −0.100 | $c^2\delta$ | + | 1 | $-3.7\cdot10^{-7}$ | $n^2$ |
| $k^2_{zz}-N''_{\dot r}$ | $\ddot\psi$ | | $L^{-1}$ | $N''_{\dot v}-x''_G$ | $\dot v$ | | $L^{-2}$ | $N''_{uv}$ | $uv$ | | $L^{-1}$ | $N''_{ur}-x''_G$ | $u\dot\psi$ | | $L^{-2}$ | $\frac{1}{2}N''_{cc\delta}$ | $c^2\delta$ | | 1 | $\frac{1}{2}N''_{nn}$ | $n^2$ |
| — | $s^{-2}$ | | $m^{-1}$ | | $ms^{-2}$ | | $m^{-2}$ | | $m^2s^{-2}$ | | $m^{-1}$ | | $ms^{-2}$ | | $m^{-2}$ | | $m^2s^{-2}$ | | — | | $s^{-2}$ |

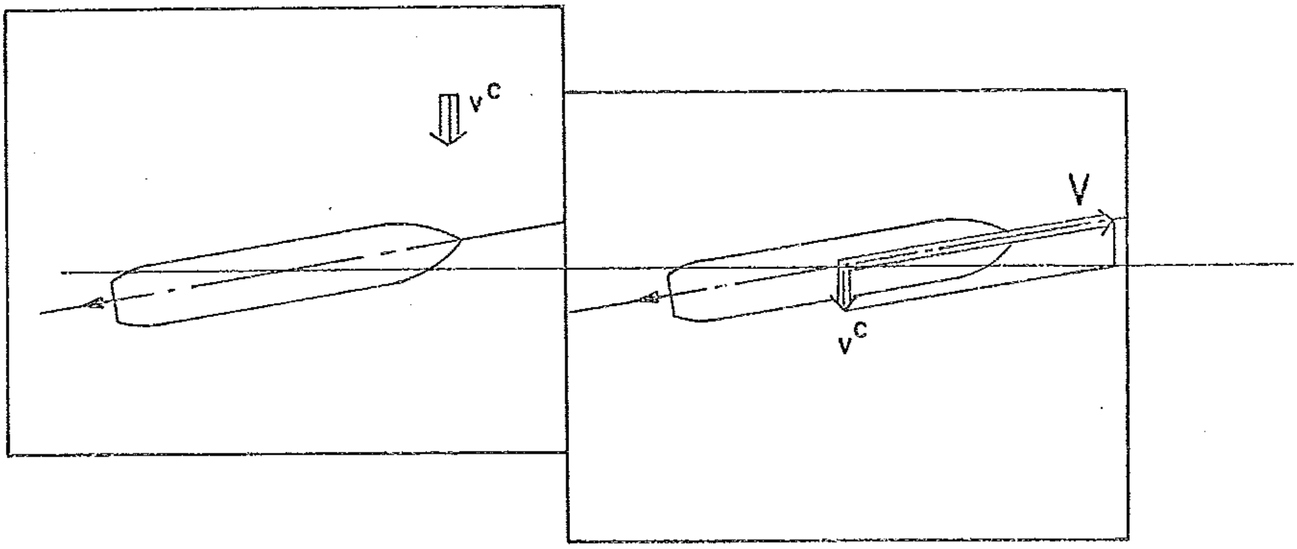
the ship hull; it must of course by corrected for, by heading upstream, when aiming for a certain position. The effects of the steady wind, on the other hand, must be balanced by water forces through a combination of side-slip and correcting helm.

A typical diagram to show the variation of lateral force and yawing moment due to a "relative" wind of given angle $\gamma_R$ off the bow of a modern tanker is reproduced in Fig 2.10, from van Berlekom, Trägårdh and Dellhag (1974).

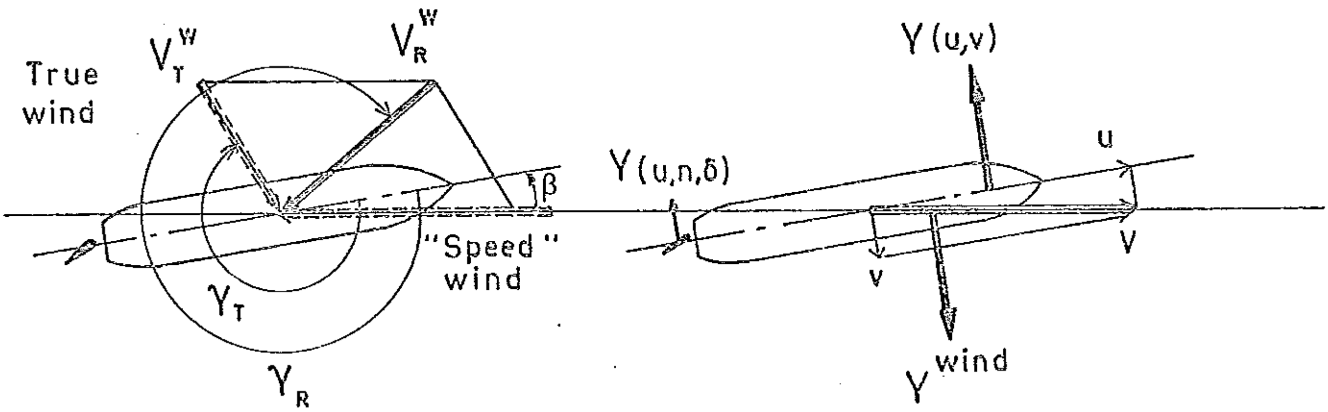
During manoeuvres in a true wind of constant velocity and direction the magnitude as well as the direction of the relative wind will change with the heading of the ship, and the force and moment will be complicated functions of this heading; especially it may be seen that the lateral centre of wind pressure may move rapidly as the ship turns through certain angles. It is important to remember that the ship is no weathercock, however, and that its tendency to luff into or bear away from the wind always depends on the combination of wind and water forces.

Within the application to the linear analysis the wind force and moment may be assumed to vary proportionally to the change of heading, so that $Y^W = Y^W_0 + Y^W_\psi \psi$ and $N^W = N^W_0 + N^W_\psi \psi$. In approach to head-on wind $Y^W_0 = N^W_0 = 0$ whereas $Y^W_\psi$ and $N^W_\psi$ have finite negative values. In a (relative) beam wind $Y^W_\psi \approx 0$, but here the three other coefficients have finite values. The constants corresponding to the initial average wind load will not be distinguished from the residual rudder load; thus, say $F_1 = Y^W_0 - Y_\delta \delta_0$ and $F_2 = N^W_0 - N_\delta \delta_0$.

In the real world the true wind fluctuates in "strength" and direction in a way which is known to be influenced by the special meteorological conditions prevailing. Trial codes usually stipulate that ship performance tests are not to be run in wind conditions above 3 Bft, in which case the fluctuations just mentioned may be described as the effect of a homogeneous isotropic turbulence. The time histories included in




The homogeneous cross-current case: The ship is drifting with the mass of water with no hydrodynamic side-slip ($\beta = 0$) or correcting helm.




The beam-wind case: Helm angle $\delta$ and side-slip $\beta$ are both adjusted to make the ship proceed in force and moment balance. The force $Y^W$ and the corresponding yaw moment vary with magnitude $V_R^W$ and direction $\gamma_R$ of relative wind as examplified in Fig 2.10.

Fig 2.9 "Drift" due to cross-current and due to beam-wind in sailing along a straight path over ground

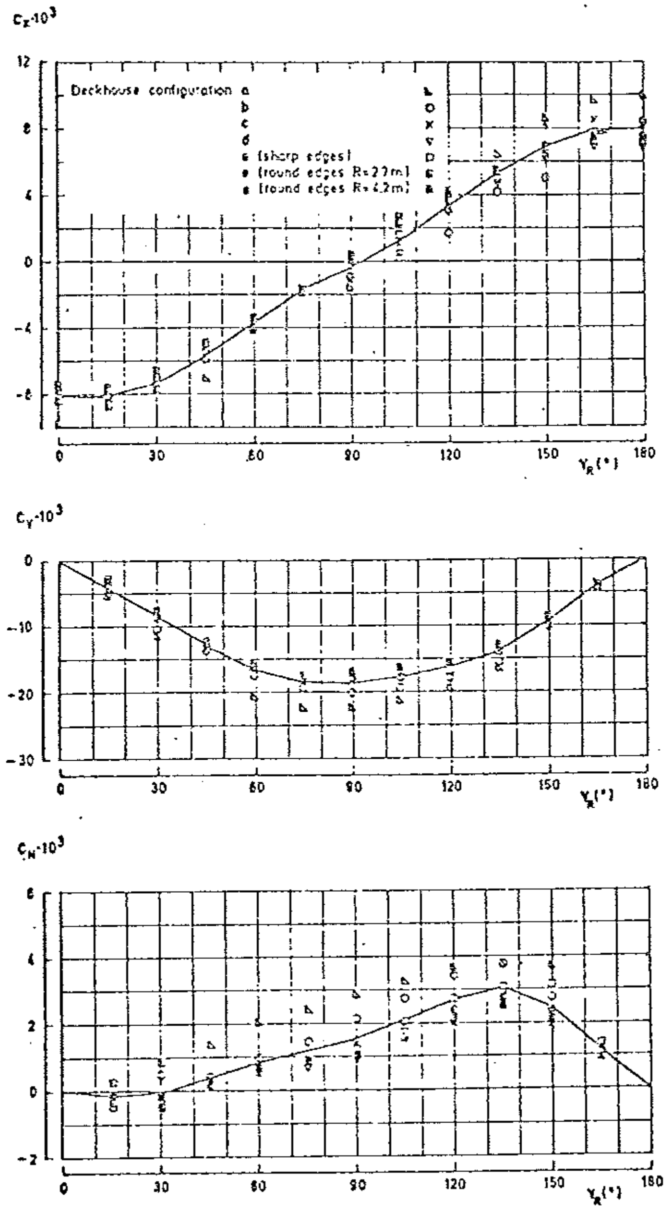
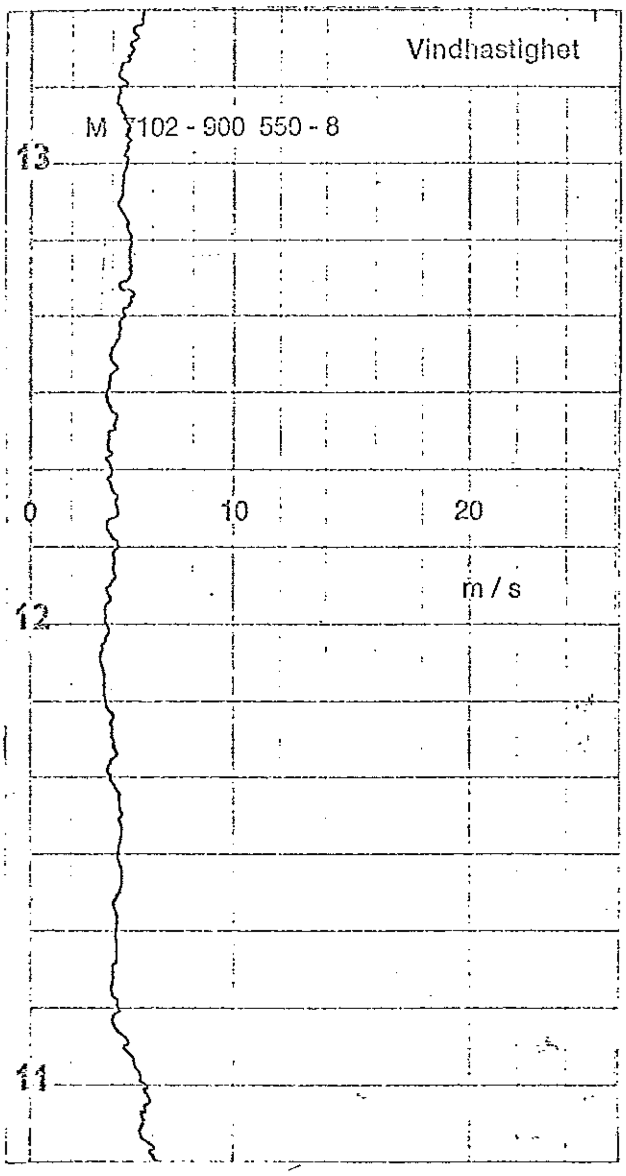
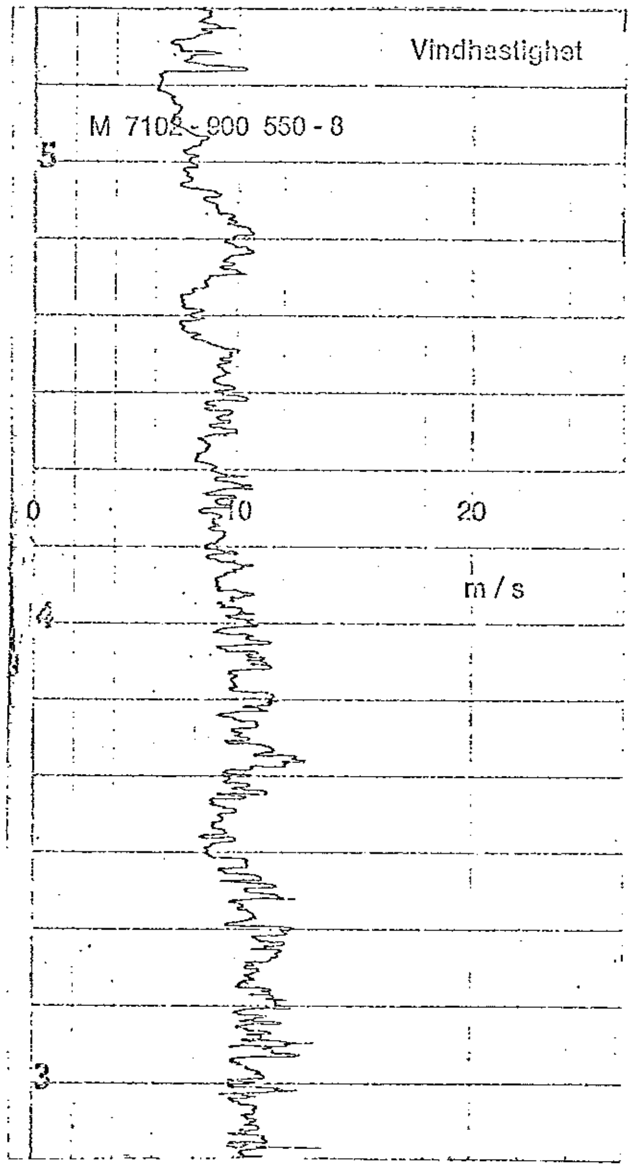



Fig 2.10 Example of wind force components measured on the model of a large tanker

The non-dimensional coefficients are based on product of relative wind stagnation pressure and reference area $L^2$. (From van Berlekom, Trägårdh and Dellhag (1974))




Average wind speed abt 4 m/s




Average wind speed abt 10 m/s

Fig 2.11 Examples of wind speed fluctuations during 2-hour intervals, from anemometer records 10 m above ground at Säve Airport

(By courtesy of the Meteorological Station of the 2nd Helicopter Division in Göteborg and of the Sw Meteorological and Hydrological Institute in Stockholm)

Fig 2.11 will then be fairly typical of the variation of wind velocities close above a flat surface such as a wide field or the open sea. In view of the long time constants seen to characterize the lateral ship response it is reasonable to assume that the variations of wind force and moment are mainly realizations of white noise.

## 2.7 The Linear Model Continued

If v, r ($= \dot{\psi}$) and $\psi$ are considered as three state variables, and if the expressions of the last Section are added to eq (2.4), then the linear equations may be written in matrix form as

$$\begin{bmatrix} m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} & 0 \\ mx_G - N_{\dot{v}} & mk_{zz}^2 - N_{\dot{r}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_r - mu & Y_\psi^W \\ N_v & N_r - mux_G & N_\psi^W \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} Y_\delta \\ N_\delta \\ 0 \end{bmatrix} \delta + \begin{bmatrix} F_1 \\ F_2 \\ 0 \end{bmatrix} \qquad (2.5)$$

In the next Chapter this equation will appear (eq (3.1)) with the coefficients in suitable matrix notation, and formal expressions will be derived for the transfer functions, eq (3.3), (3.5), (3.7) and (3.8), the two latter for the case of no wind load.

Whereas the control function $\delta(t)$ may itself include a heading reference the open ship-and-screw-and-rudder loop has no inherent preference for any one heading. The characteristic equation $s(s^2 + a_1 s + a_2) = 0$ thus has one root equal to zero, and it may be shown that the two other roots normally are real. By convention the root to the right on the real axis is denoted $s_1 = - T_1^{-1}$. The appropriate transfer function from helm to heading may read

$$G_1(s) = G_{\psi\delta} = \frac{K(1 + T_3 s)}{s\{T_1 T_2 s^2 + (T_1 + T_2)s + 1\}} \qquad (2.6)$$

with accepted symbols following Nomoto (1957). The same time constants $T_1$ and $T_2$ will appear in the transfer function from helm to sway velocity:

# TABLE OF TRANSFER FUNCTION CHARACTERISTICS

| | in true dimensions | in non-dim "prime" and "bis" systems | | |
|---|---|---|---|---|
| K | $\frac{\frac{1}{2}Y_{uu\delta}u}{m-Y_{ur}}\cdot\frac{l_\delta-l_v}{l_r-l_v}$ | $K'=\frac{K''}{F_{nL}}$ | $\frac{Y'_\delta}{m'-Y'_r}\cdot\frac{l'_\delta-l'_v}{l'_r-l'_v}=\frac{\frac{1}{2}Y''_{uu\delta}}{1-Y''_{ur}}\cdot\frac{l''_\delta-l''_v}{l''_r-l''_v}$ | $l_r=l'_rL=l''_rL$, cf. eq.2(10) |
| Dim $T^{-1}$ | | $=\frac{L}{V}\cdot K$ | | $l_v=l'_vL=l''_vL$, — " — |
| $K_v$ | $\frac{\frac{1}{2}Y_{uu\delta}\cdot u}{Y_{uv}}\cdot\frac{l_\delta-l_r}{l_r-l_v}$ | $K'_v=\frac{K''_v}{F_{nL}}$ | $\frac{Y'_\delta}{Y'_v}\cdot\frac{l'_\delta-l'_r}{l'_r-l'_v}=\frac{\frac{1}{2}Y_{uu\delta}}{Y''_{uv}}\cdot\frac{l''_\delta-l''_r}{l''_r-l''_v}$ | $l_\delta=N(\delta)\,/\,Y(\delta)$ |
| Dim $LT^{-1}$ | | $=\frac{1}{V}\cdot K_v$ | | |
| $T_3$ | $\frac{m-Y_{\dot v}}{Y_{uv}\cdot u}\cdot\frac{l_\delta-\frac{mx_G-N_{\dot v}}{m-Y_{\dot v}}}{l_v-l_\delta}$ | $T'_3=F_{nL}\cdot T''_3$ | $\frac{m'-Y''_{\dot v}}{Y'_v}\cdot\frac{l'_\delta-\frac{m'x'_G-N'_{\dot v}}{m'-Y'_{\dot v}}}{l'_v-l'_\delta}=\frac{1-Y''_{\dot v}}{Y''_{uv}}\cdot\frac{l''_\delta-\frac{x''_G-N''_{\dot v}}{1-Y''_{\dot v}}}{l''_v-l''_\delta}$ | |
| Dim T | | $=\frac{V}{L}\cdot T_3$ | | |
| $T_{3v}$ | $\frac{mx_G-Y_{\dot r}}{(m-Y_{ur})\cdot u}\cdot\frac{l_\delta-\frac{mk^2_{zz}-N_{\dot r}}{mx_G-Y_{\dot r}}}{l_\delta-l_r}$ | $T'_{3v}=F_{nL}\cdot T''_{3v}$ | $\frac{m'x'_G-Y'_{\dot r}}{m'-Y'_{ur}}\cdot\frac{l'_\delta-\frac{m'k'^2_{zz}-N'_{\dot r}}{m'x'_G-Y'_{\dot r}}}{l'_\delta-l'_r}=\frac{x''_G-Y''_{\dot r}}{1-Y''_{ur}}\cdot\frac{l''_\delta-\frac{k''^2_{zz}-N''_{\dot r}}{x''_G-Y''_{\dot r}}}{l''_\delta-l''_r}$ | |
| Dim T | | $=\frac{V}{L}\cdot T_{3v}$ | | |
| $T_1T_2$ | $\frac{\frac{mx_G-Y_{\dot r}}{Y_{uv}}\cdot\frac{mx_G-N_{\dot r}}{m-Y_{ur}}-\frac{m-Y_{\dot v}}{Y_{uv}}\cdot\frac{mk^2_{zz}-N_{\dot r}}{m-Y_{ur}}}{(l_r-l_v)\cdot u^2}$ | $T'_1T'_2=F^2_{nL}\cdot T''_1T''_2$ | $\frac{\frac{m'x'_G-Y'_{\dot r}}{Y'_v}\cdot\frac{m'x'_G-N'_{\dot r}}{m'-Y'_r}-\frac{m'-Y'_{\dot v}}{Y'_v}\cdot\frac{m'k'^2_{zz}-N'_{\dot r}}{m'-Y'_r}}{l''_r-l''_v}=\frac{\frac{x''_G-Y''_{\dot r}}{Y''_{uv}}\cdot\frac{x''_G-N''_{\dot r}}{1-Y''_{ur}}-\frac{1-Y''_{\dot v}}{Y''_{uv}}\cdot\frac{k''^2_{zz}-N''_{\dot r}}{1-Y''_{ur}}}{l''_r-l''_v}$ | |
| Dim $T^2$ | | $=\frac{V^2}{L^2}T_1\cdot T_2$ | | |
| $T_1+T_2$ | $\frac{\frac{m-Y_{\dot v}}{Y_{uv}}\cdot l_r-\frac{mx_G-Y_{\dot r}}{m-Y_{ur}}\cdot l_v+\frac{mk^2_{zz}-N_{\dot r}}{m-Y_{ur}}-\frac{mx_G-N_{\dot v}}{Y_{uv}}}{(l_r-l_v)\cdot u}$ | $T_1+T_2=F_{nL}(T''_1+T''_2)$ | $\frac{\frac{m'-Y'_{\dot v}}{Y'_v}l'_r-\frac{m'x'_G-Y'_{\dot r}}{m'-Y'_r}l'_v-\frac{m'k'^2_{zz}-N'_{\dot r}}{m'-Y'_r}-\frac{m'x'_G-N'_{\dot v}}{Y'_v}}{l'_r-l'_v}=\frac{\frac{1-Y''_{\dot v}}{Y''_{uv}}l''_r-\frac{x''_G-Y''_{\dot r}}{1-Y''_{ur}}l''_v+\frac{k''^2_{zz}-N''_{\dot r}}{1-Y''_{ur}}-\frac{x''_G-N''_{\dot v}}{Y''_{uv}}}{l''_r-l''_v}$ | |
| Dim T | | $=\frac{V}{L}(T_1+T_2)$ | | |

require additional information. This latter is even more obvious for the unstable ship.

In terms of "stability derivatives" the analytical criterion for dynamic stability on a straight course may be written as an unequality of two lever arms,

$$l_r - l_v = \frac{mx_G - N_{ur}}{m - Y_{ur}} - \frac{N_{uv}}{Y_{uv}} > 0 \qquad (2.10)$$

The physical interpretation of this criterion is that in the motion initiated by a disturbance the yaw damping moment and the sway damping force will tend to stop the yawing. The linearization may in a similar way be applied to the ship in a turn of given radius and with a side-slip $\neq 0$, and it will then be seen that the stability in the turn is increased over that on a straight course. It was seen earlier in this chapter that the unstable ship was in fact inherently stable in a certain turn to port or starboard.

## 2.8 Approximations with Added Non-Linearities

It was seen above that the validity of the linear relations is limited to moderate amplitudes. In particular $\dot{\psi}_c = K\delta_c$ fails to describe the steady state characteristics as derived from spiral tests with marginally stable or unstable ships, and diagrams as in Fig 2.5 suggest that the linear relation be replaced by an expression with a cubic or abs-square term, such as

$$\dot{\psi}_c - \kappa K|\dot{\psi}_c|\dot{\psi}_c = K\delta_c \qquad (2.11)$$

As a first approximation to the non-linear problem these equilibrium conditions may be introduced into the steering equations, which correspond to the transfer functions (2.8) or (2.6); Norrbin (1963, 1965).

In the equation

$$T_1T_2\dddot{\psi} + (T_1 + T_2)\ddot{\psi} + \dot{\psi} - \kappa K|\dot{\psi}|\dot{\psi} = K(\delta + T_3\dot{\delta}) \tag{2.12}$$

the presence of an explicit abs-square term may be argued for with reference to the dominating role of viscous cross-flow resistance and to the fact that the phase difference between yaw and sway remains fairly constant during the course of low-frequency manoeuvres. Thus the equation may be especially useful in standard zig-zag test analysis, and in the analysis of small deviations from a pure turning circle.

Bech & Smitt (1969) generalized the non-linear assumption by use of an unknown function $H(\dot{\psi})$ to be derived by curve fitting to the reversed spiral results, and they also put forth a convenient way to find the time constants from graphical analysis of the zig-zag test phase plot of $\ddot{\psi}(\dot{\psi})$.

In the context of the present work an equation such as (2.12) will be suggested for predictions of non-linear manoeuvres on the basis of linear characteristics, that have been obtained through the parameter identification procedure to be described next. It is anticipated that similar procedures will be developed in a future for a direct evaluation of the most significant non-linear contributions to the original equations of motion, (2.3).

# 3. SYSTEM IDENTIFICATION

Mathematical models for the motion of the ship in the horisontal plane were given in the previous chapter. The problem of determining the parameters of the models from observed motions of the ship will now be discussed. It is assumed that an experiment is performed by changing the rudder in some way, and that the resulting motion is observed. It will first be investigated if the parameters of the models can be determined from such an experiment.

## 3.1 Parameter Identifiability.

A model is called parameter identifiable if its parameters can be determined from input/output data obtained under certain experimental conditions. The identifiability of the models (2.5) and (2.6) will now be explored. Neglecting the disturbances and assuming that the velocity component u is constant, the linearized equations of motion contains 14 parameters m, $x_G$, $k_{zz}$, $u_o$ and 10 hydrodynamic derivatives. The transfer function (2.6) relating heading or yaw rate to the rudder is characterized by four parameters, and the transfer function relating sway velocity to rudder by two additional parameters. Since the input/output data is uniquely characterized by the transfer functions, it is clear from this simple argument that six parameter combinations can be determined if both sway velocity and heading angle are measured, but that only four parameter combinations can be obtained if the heading only is measured.

## 3.2 State Equations.

To investigate the parameter combinations that can be determined further, a state space model is introduced by solving $\dot{v}$, $\dot{r}$ and $\dot{\psi}$ from the equations of motion. This gives

$$\frac{d}{dt}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ 0 \end{bmatrix}\delta + \begin{bmatrix} f_1 \\ f_2 \\ 0 \end{bmatrix} \qquad (3.1)$$

where $f_1$ and $f_2$ are the linearized windforces and

$$\begin{bmatrix} a_{11} & a_{12} & b_{11} \\ a_{21} & a_{22} & b_{21} \end{bmatrix} = \begin{bmatrix} m-Y_{\dot{v}} & -Y_{\dot{r}}+mx_G \\ -N_{\dot{v}}+mx_G & mk_{zz}^2-N_{\dot{r}} \end{bmatrix}^{-1} \begin{bmatrix} Y_v & Y_r-mu_o & Y_\delta \\ N_v & N_r-mx_Gu_o & N_\delta \end{bmatrix} \qquad (3.2)$$

The parameters $a_{13}$ and $a_{23}$ depend on the windforces; they will vanish if there are no disturbances.

The transfer function relating heading angle to rudder is given by

$$G_1(s) = \frac{b_1 s + b_2}{s^3 + a_1 s^2 + a_2 s + a_3} \qquad (3.3)$$

where the parameters are related by

$$\begin{aligned} a_1 &= -a_{11} - a_{22} \\ a_2 &= -a_{12}a_{21} + a_{11}a_{22} - a_{23} \\ a_3 &= -a_{13}a_{21} + a_{11}a_{23} \\ b_1 &= b_{21} \\ b_2 &= a_{21}b_{11} - a_{11}b_{21} \end{aligned} \qquad (3.4)$$

The transfer function relating sway velocity to rudder is given by

$$G_2(s) = \frac{c_1 s^2 + c_2 s + c_3}{s^3 + a_1 s^2 + a_2 s + a_3} \qquad (3.5)$$

where $a_1$, $a_2$ and $a_3$ are given by (3.4) and

$$c_1 = b_{11}$$
$$c_2 = -a_{22}b_{11} + a_{12}b_{21} \tag{3.6}$$
$$c_3 = -a_{23}b_{11} + a_{13}b_{21}$$

Notice that if the effects of wind are neglected, then the transfer functions $G_1$ and $G_2$ are reduced to

$$G_1(s) = \frac{b_1 s + b_2}{s(s^2+a_1 s+a_2)} \tag{3.7}$$

$$G_2(s) = \frac{c_1 s + c_2}{(s^2+a_1 s+a_2)} \tag{3.8}$$

In the presence of wind the transfer function $G_1$ will thus not contain a pure integration.

The state equation (3.1) contains 8 parameters. Since the transfer function $G_1$ relating heading angle to rudder is described by 5 parameters only, namely $a_1$, $a_2$, $a_3$, $b_1$ and $b_2$, it is clear that the parameters of the state model can not be determined from an experiment where only the heading is measured. The parameter combinations that can be determined are given by (3.4). Notice in particular that the parameter $b_{21}$ is identifiable. However, if the sway velocity is also measured, then the transfer function $G_2$ can also be determined. This gives 3 additional parameters, $c_1$, $c_2$ and $c_3$. To analyse the identifiability of the parameters of the state model it must then be investigated if the parameters of the state model can be determined from the equations (3.4) and (3.6). These equations can be solved if there are no pole zero cancellations.

## 3.3 Parameter Estimation.

Having discussed the problem of identifiability, the parameter estimation problem will now be discussed. Both the estimation of parameters in a state model and the estimation of the parameters of a transfer function model can be formulated as a problem of determining the parameters in the stochastic differential equation

$$dx = Axdt + Budt + dw \tag{3.9}$$

It is assumed that the initial state is a gaussian vector with mean value m and covariance $R_0$ and that $\{w(t),\ 0 \leq t \leq \infty\}$ is a Wiener process with incremental covariance $R_1 dt$ which is assumed independent of the initial state. Assume that an input signal has been applied to the system and that the output has been observed at discrete times $t_0, t_1, \ldots, t_N$ with a measuring device which can be characterized by

$$y(t_k) = Cx(t_k) + Du(t_k) + e(t_k) \tag{3.10}$$

$$k = 0,1,\ldots,N$$

The measurement errors $\{e(t_k)\}$ are assumed to be independent and gaussian with zero mean and covariance $R_2$. It is furthermore assumed that the measurement errors are independent of $\{w(t),\ 0 \leq t \leq \infty\}$ and of the initial state.

The model (3.10) implies that the measuring instruments are such that they give an output signal which is the instantaneous value of a linear combination of the state variables. The errors of measurements taken at different times are independent. The equation (3.10) is a good model when the sensor dynamics is considerably faster than the system dynamics and the measurement errors are so small that it is not reasonable to average measurement signals over intervals comparable to the sampling intervals.

In the particular case of ship dynamics the model (3.10) is reasonable because the shortest time constant of interest and the sampling interval is about 5 - 60 s. All sensors have dynamics with time constants shorter than 1 s, and the measurement errors are about $0.1^{\circ}$ in heading, $0.02^{\circ}/s$ in yaw rate and 0.01 m/s in velocity.

## 3.4 Problem Statement.

It is thus assumed that an identifiable model (3.9), (3.10) is given and the problem is to determine the identifiable parameters from observed input-output pairs. The parameters will be determined using the maximum likelihood method.

## 3.5 The Likelihood Function.

To obtain the likelihood function, i.e. the joint probability density of the observed outputs assuming all parameters known, we introduce

$$y_{t_k} = \begin{bmatrix} y(t_0) \\ y(t_1) \\ \vdots \\ y(t_{k-1}) \\ y(t_k) \end{bmatrix} \qquad (3.11)$$

i.e. a vector consisting of all outputs observed up to and including time $t_k$.

Assume that the probability distribution of $y_{t_k}$ has a density $p(y_{t_k})$. It then follows from the definition of conditional probabilities that .

$$p(y_{t_k}) = p\left(y(t_k)\,|\,y_{t_{k-1}}\right)p(y_{t_{k-1}}) \tag{3.12}$$

Repeated use of this formula gives the following formula for the likelihood function

$$L = p(y_{t_k}) = p\left(y(t_k)\,|\,y_{t_{k-1}}\right)p\left(y(t_{k-1})\,|\,y_{t_{k-2}}\right)\cdots$$
$$\cdots\ p\left(y(t_1)\,|\,y(t_0)\right)\,p\left(y(t_0)\right) \tag{3.13}$$

The likelihood function can thus be conveniently written as a product of conditional densities.

In the particular case of a model described by (3.9) and (3.10) all random variables are gaussian and the conditional density is also gaussian. The logarithm of the likelihood function can then be written as

$$-\log L = \frac{1}{2}\sum_{k=0}^{N} \log\det R(t_k) +$$
$$+\frac{1}{2}\sum_{k=0}^{N} \varepsilon^T(t_k)R^{-1}(t_k)\varepsilon(t_k) + \text{const} \tag{3.14}$$

where

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k|t_{k-1}) \tag{3.15}$$

and $\hat{y}(t_k|t_{k-1})$ denotes the conditional mean of $y(t_k)$ given $y_{t_{k-1}}$ and $R(t_k)$ the conditional covariance. We have

$$\hat{y}(t_0|t_{-1}) = Cm + Du(t_0) \tag{3.16}$$

Also notice that $\{\varepsilon(t_k),\ k = 0,\ldots,N\}$ are the innovations of the output process. The conditional mean $\hat{y}(t_k|t_{k-1})$ and the conditional covariance $R(t_k)$ are easily determined recursively through the Kalman-Bucy filtering theory. See e.g.

Åström (1970). We have

$$\hat{y}(t_k|t_{k-1}) = C\hat{x}(t_k|t_{k-1}) + Du(t_k)$$

$$\hat{x}(t_k|t_k) = \hat{x}(t_k|t_{k-1}) + K(t_k)\varepsilon(t_k)$$

$$\frac{d}{dt}\hat{x}(t|t_k) = A\hat{x}(t|t_k) + Bu(t) \qquad t_k \leq t \leq t_{k+1}$$

$$K(t_k) = P(t_k|t_{k-1})C^T[R_2 + CP(t_k|t_{k-1})C^T]^{-1} \tag{3.17}$$

$$P(t_k|t_k) = P(t_k|t_{k-1}) - K(t_k)CP(t_k|t_{k-1})$$

$$\frac{d}{dt}P(t|t_k) = AP(t|t_k) + P(t|t_k)A^T + R_1 \qquad t_k \leq t \leq t_{k+1}$$

$$R(t_k) = R_2 + CP(t_k|t_{k-1})C^T$$

The computation of the likelihood function is thus easily done recursively. A description of the program LISPID, which performs the calculations, is given in the next chapter.

## 3.6 Input-Output Models.

If we are only interested in an input-output model, the parameter estimation can be simplified significantly if the parameters of the pulse transfer function model

$$y(t) + a_1y(t-1)+\ldots+a_ny(t-n) = b_1u(t-1)+\ldots+b_nu(t-n) +$$

$$+ e(t) + c_1e(t-1)+\ldots+c_ne(t-n)$$

are determined directly, as was described in Åström and Bohlin (1965). An interactive program IDPAC, which performs this, is described in Gustavsson et al (1973).

# 4. PARAMETER ESTIMATION PROGRAM LISPID

A program package for LInear System Parameter IDentification (LISPID) has been developed for the computer UNIVAC 1108. All programs, with the exception of two small subroutines, have been coded in Fortran. Several subroutines from the Program Library of the Division of Automatic Control have also been incorporated with LISPID. The program package is an implementation of the computation of the maximum likelihood estimate described in Chapter 3.

## 4.1 Subroutine Structure.

The program package LISPID consists of 52 subroutines, but only the most important are described here. The total number of Fortran statements, including comments, is 9 200. A simplified subroutine structure is shown in Fig. 4.1. The program heads of some of the subroutines are given in Appendix C.

MAIN - The user must supply the main program where data and parameters are organized as described in the subroutine LISPID. It is necessary to make a call to the subroutine DATEXP. A call to the subroutine IOLISP may facilitate the reading of parameters.

LISPID - All administration is performed in this subroutine. After the parameter estimation, data can be analysed, printed and plotted according to the user's desire.

POWBRE, NUFLET - Two different algorithms to minimize the loss function, which is obtained from the parameter estimation. The choice between the two algorithms is controlled by the user.

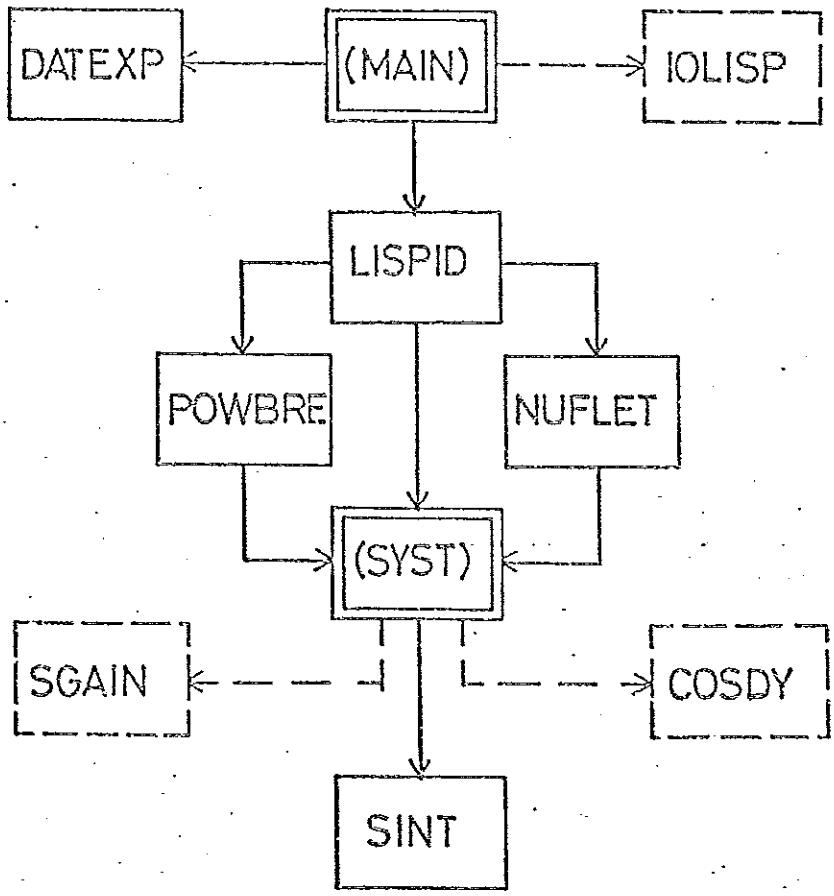



Fig. 4.1. Simplified subroutine structure of LISPID. The main program MAIN and the subroutine SYST must be supplied by the user. IOLISP, SGAIN and COSDY are auxiliary subroutines, which are not necessary to call.

SYST- The dependence of the parameters, which are to be estimated, has to be supplied by the user in this subroutine. The value of the loss function, which is connected with a special set of parameter values, is obtained by calls to the subroutine SINT. For special cases, calls to the auxiliary subroutines SGAIN and COSDY can be made.

SINT- This subroutine transforms continuous system and covariance matrices to discrete form, iterates system equations over data and computes values of the loss function.

DATEXP - A subroutine to compute index values and expand the data vector.

IOLISP - An auxiliary subroutine to read parameters from card reader and print them on line printer.

SGAIN - This auxiliary subroutine can be called by subroutine SYST to compute the discrete, stationary filter gain.

COSDY - An auxiliary subroutine to be called from the subroutine SYST if the time lag between the discontinuity of the inputs and the measurement of the outputs is to be determined.

## 4.2 General Models.

It is possible to handle two different kinds of measurements in LISPID. If the parameter MEAS is put equal to zero by the user, it is assumed that instantaneous measurements have been made, and if MEAS is put equal to one, integrating measurements are assumed.

The type of linear model to be used in LISPID is controlled by the parameter ISYS. A positive value of ISYS means a continuous model, and a negative value means a discrete model. The parameters, which are to be estimated, are collected in a vector $\theta$. It is assumed in the following models that w and e are Wiener processes and that $\tilde{w}$ and $\tilde{e}$ are gaussian processes.

If only measurement noise is assumed, put ISYS equal to 1 or -1.

ISYS = 1, MEAS = 0:

$$\begin{cases} \frac{dx}{dt} = A(\theta)x(t) + B(\theta)u(t) \\ y(t_k) = \tilde{C}(\theta)x(t_k) + \tilde{D}(\theta)u(t_k) + \tilde{e}(t_k) \\ \qquad k = 0,1,\dots,N \end{cases} \tag{4.1}$$

ISYS = 1, MEAS = 1:

$$\begin{cases} \frac{dx}{dt} = A(\theta)x(t) + B(\theta)u(t) \\ dz = C(\theta)x(t)dt + D(\theta)u(t)dt + de \\ y(t_k) = \int_{t_k}^{t_{k+1}} dz \qquad k = 0,1,\dots N \end{cases} \tag{4.2}$$

ISYS = -1, MEAS = 0:

$$\begin{cases} x(t_{k+1}) = \tilde{A}(\theta)x(t_k) + \tilde{B}(\theta)u(t_k) \\ y(t_k) = \tilde{C}(\theta)x(t_k) + \tilde{D}(\theta)u(t_k) + \tilde{e}(t_k) \\ \qquad k = 0,1\dots,N \end{cases} \tag{4.3}$$

If measurement noise and state noise modelled by a gain matrix $\tilde{K}(\theta)$ are assumed, put ISYS equal to 2 or -2.

ISYS = 2, MEAS = 0:

$$
\begin{cases}
dx = A(\theta)x(t)dt + B(\theta)u(t)dt + dw \\
y(t_k) = \tilde{C}(\theta)x(t_k) + \tilde{D}(\theta)u(t_k) + \tilde{e}(t_k) \\
\qquad k = 0,1,\ldots,N
\end{cases}
\tag{4.4}
$$

This model is then transformed by LISPID (see Chapter 3 and Åström (1970)):

$$
\begin{cases}
\hat{x}(t_{k+1}) = \tilde{A}(\theta)\hat{x}(t_k) + \tilde{B}(\theta)u(t_k) + \tilde{K}(\theta)\varepsilon(t_k) \\
y(t_k) = \tilde{C}(\theta)\hat{x}(t_k) + \tilde{D}(\theta)u(t_k) + \varepsilon(t_k) \\
\qquad k = 0,1,\ldots,N
\end{cases}
\tag{4.5}
$$

ISYS = 2, MEAS = 1:

$$
\begin{cases}
dx = A(\theta)x(t)dt + B(\theta)u(t)dt + dw \\
dz = C(\theta)x(t)dt + D(\theta)u(t)dt + de \\
y(t_k) = \int_{t_k}^{t_{k+1}} dz \qquad k = 0,1,\ldots,N
\end{cases}
\tag{4.6}
$$

This model is also transformed into model (4.5) by LISPID.

ISYS = -2, MEAS = 0:

See (4.5)

Put ISYS equal to 3 or -3 if measurement noise and state noise modelled by covariance matrices are assumed.

ISYS = 3, MEAS = 0:

$$\begin{cases} dx = A(\theta)x(t)dt + B(\theta)u(t)dt + dw \\ y(t_k) = \tilde{C}(\theta)x(t_k) + \tilde{D}(\theta)u(t_k) + \tilde{e}(t_k) \\ w \text{ has incremental covariance } R_1(\theta)dt \\ \tilde{e}(t_k) \in N(0, \tilde{R}_2(\theta)) \qquad k = 0,1,\ldots,N \end{cases} \tag{4.7}$$

ISYS = 3, MEAS = 1:

$$\begin{cases} dx = A(\theta)x(t)dt + B(\theta)u(t)dt + dw \\ dz = C(\theta)x(t)dt + D(\theta)u(t)dt + de \\ y(t_k) = \int_{t_k}^{t_{k+1}} dz \qquad k = 0,1,\ldots,N \\ w \text{ and } e \text{ have incremental covariances} \\ R_1(\theta)dt \text{ and } R_2(\theta)dt, \text{ resp. The cross} \\ \text{covariance is } R_{12}(\theta)dt. \end{cases} \tag{4.8}$$

ISYS = -3, MEAS = 0:

$$\begin{cases} x(t_{k+1}) = \tilde{A}(\theta)x(t_k) + \tilde{B}(\theta)u(t_k) + \tilde{w}(t_k) \\ y(t_k) = \tilde{C}(\theta)x(t_k) + \tilde{D}(\theta)u(t_k) + \tilde{e}(t_k) \\ \tilde{w}(t_k) \in N(0, \tilde{R}_1(\theta)) \qquad k = 0,1,\ldots,N \\ \tilde{e}(t_k) \in N(0, \tilde{R}_2(\theta)) \end{cases} \tag{4.9}$$

A statistical test of the residuals to check if the values are suitably small can be performed by putting ISYS equal to 4 or -4. The same model as if ISYS is equal to 3 or -3 is then used.

It is also possible to let the initial state and, if ISYS is equal to 3 or -3, the initial covariance matrix of state estimate errors depend on the parameter vector $\theta$.

If ISYS is equal to 1, -1, 2 or -2, a simplified loss function $V_1(\theta)$ is used (see Åström and Eykhoff (1971)):

$$V_1(\theta) = \frac{1}{N+1} \log \det \left[ \sum_{k=0}^{N} \varepsilon(t_k)\varepsilon^T(t_k) \right] \tag{4.10}$$

The negative logarithm of the likelihood function, divided by N+1, is used as loss function if ISYS is equal to 3, -3, 4 or -4, cf. (3.14).

$$V_2(\theta) = -\frac{\log L}{N+1} = \frac{1}{2(N+1)} \left[ \sum_{k=0}^{N} \log \det R(t_k) + \right.$$
$$\left. + \sum_{k=0}^{N} \varepsilon^T(t_k) R^{-1}(t_k) \varepsilon(t_k) + (N+1) n_y \log 2\pi \right] \tag{4.11}$$

where $n_y$ is the number of measurement signals. In this case the loss function $V_1(\theta)$ is computed too, but is not used for the estimation.

If the sampling interval is not constant, the user must put the parameter ISAMP equal to 3. ISAMP equal to 1 means constant sampling interval, and ISAMP equal to 2 means constant sampling interval but that some measurements are missing.

To obtain the maximum likelihood estimates of the parameters, i.e. a model which performs, in a certain sense, the one-step prediction as well as possible, put the parameters NPRED1 and NPRED2 equal to 1. A model which predicts an arbitrary number of time steps ahead can be obtained by giving NPRED1 and NPRED2 other values.

If the sampling interval is constant, or if possibly some measurements are missing (ISAMP = 1 or 2), and the system and the covariance matrices are time-invariant, the models (4.7), (4.8) and (4.9), when ISYS is equal to 3 or -3, can be reduced to the model (4.5) by calling the subroutine SGAIN from the subroutine SYST. This will save a lot of computing time.

## 4.3 Ship Models.

A special subroutine SHIP to be used as the subroutine SYST of the program package LISPID has been developed. The program head is given in Appendix C.

Parameters of three different ship models can be estimated. The choice of the model is controlled by the parameter IMOD.

Model 1 (IMOD=1 ; cf. (2.5) and (4.7)):

$$\begin{bmatrix} \frac{L}{V^2}\theta_1 & \frac{L^2}{V^2}\theta_2 & 0 \\ \frac{L}{V^2}\theta_3 & \frac{L^2}{V^2}\theta_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d\bar{v} \\ d\bar{r} \\ d\bar{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{V}\theta_5 & \frac{L}{V}\theta_6 & \theta_9 \\ \frac{1}{V}\theta_7 & \frac{L}{V}\theta_8 & \theta_9\theta_{10} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{v}(t) \\ \bar{r}(t) \\ \bar{\psi}(t) \end{bmatrix} dt +$$

$$+ \begin{bmatrix} \alpha_1\theta_{11} & \theta_{13} \\ -\alpha_1\theta_{11}\theta_{12} & \theta_{14} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t-\tau) \\ U1 \end{bmatrix} dt + dw$$

$$\begin{bmatrix} v_1(t_k) \\ v_2(t_k) \\ v(t_k) \\ r(t_k) \\ \psi(t_k) \end{bmatrix} = \begin{bmatrix} \alpha_2 & L_1\alpha_2 & 0 \\ \alpha_2 & -L_2\alpha_2 & 0 \\ \alpha_2 & 0 & 0 \\ 0 & 1/\alpha_1 & 0 \\ 0 & 0 & 1/\alpha_1 \end{bmatrix} \begin{bmatrix} \bar{v}(t_k) \\ \bar{r}(t_k) \\ \bar{\psi}(t_k) \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & \theta_{15} \\ 0 & \theta_{16} \\ 0 & \theta_{15} \\ 0 & \theta_{17} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t_k - \tau) \\ U1 \end{bmatrix} + \tilde{e}(t_k) \qquad k = 0,1,\ldots,N$$

w is a Wiener process with incremental covariance $R_1\,dt$, where

$$R_1 = \begin{bmatrix} |\theta_{18}| & \sqrt{|\theta_{18}||\theta_{19}|}\,\sin\theta_{20} & 0 \\ \sqrt{|\theta_{18}||\theta_{19}|}\,\sin\theta_{20} & |\theta_{19}| & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The measurement errors $\{\tilde{e}(t_k)\}$ are assumed to be independent and gaussian with zero mean and covariance $R_2$, where

$$\tilde{R}_2 = \begin{bmatrix} |\theta_{21}| & 0 & 0 & 0 \\ 0 & |\theta_{22}| & 0 & 0 \\ 0 & 0 & |\theta_{23}| & 0 \\ 0 & 0 & 0 & |\theta_{24}| \end{bmatrix}$$

The initial state is given by

$$\begin{bmatrix} \bar{v}(t_o) \\ \bar{r}(t_o) \\ \bar{\psi}(t_o) \end{bmatrix} = \begin{bmatrix} \theta_{25}/\alpha_2 \\ \alpha_1 \ \theta_{26} \\ \alpha_1 \ \theta_{27} \end{bmatrix}$$

and the initial covariance matrix of state estimate errors is

$$P(t_o) = \begin{bmatrix} |\theta_{28}| & \theta_{31} & \theta_{32} \\ \theta_{31} & |\theta_{29}| & \theta_{33} \\ \theta_{32} & \theta_{33} & |\theta_{30}| \end{bmatrix}$$

The time lag $\tau$ is computed as

$$\tau = T|\sin \theta_{34}|$$

where T is the sampling interval.

U1 is an artificial input signal which consists of only ones to make it possible to estimate the bias on state equations and measurements. $\theta_9$ and $\theta_{10}$ are parameters to describe the wind influence during the experiment (see Chapter 3). $\alpha_1$ and $\alpha_2$ are conversion factors from degrees to radians and from m/s to knots, respectively. The dimensions of inputs, states and measurements are shown in Table 4.1.

| | |
|---|---|
| $\delta$ | deg |
| Ul | - |
| $\bar{v}$ | m/s |
| $\bar{r}$ | rad/s |
| $\bar{\psi}$ | rad |
| $v_1$ | knots |
| $v_2$ | knots |
| v | knots |
| r | deg/s |
| $\psi$ | deg |

Table 4.1 Dimensions of inputs, states and measurements.

Notice that it is possible to estimate only a subset of the 34 parameters of the model. The other parameters can be given arbitrary fixed values. It is not necessary to have measured all the five output signals of the model, but attention has to be paid to the identifiability discussion of Chapter 3. Notice that it is not possible to use $v_1$ and v at the same time.

Hydrodynamic derivatives normalized by use of either the "prime" system or the "bis" system can be estimated in model 1. If the values of the four derivatives $\theta_1$, $\theta_2$, $\theta_3$ and $\theta_4$ are given in the "prime" system, all other derivative estimates are obtained in the "prime" system, and analogous for the "bis" system.

Model 2 (IMOD = 2; cf. (4.7)):

$$\begin{bmatrix} d\bar{v} \\ dx_2 \\ dx_3 \end{bmatrix} = \begin{bmatrix} -\frac{V}{L}\theta_1 & 1 & 0 \\ -\frac{V^2}{L^2}\theta_2 & 0 & 1 \\ -\frac{V^3}{L^3}\theta_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{v}(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} dt +$$

$$+ \begin{bmatrix} \alpha_1 \frac{V^2}{L} \theta_4 & \theta_7 \\ \alpha_1 \frac{V^3}{L^2} \theta_5 & \theta_8 \\ \alpha_1 \frac{V^4}{L^3} \theta_6 & \theta_9 \end{bmatrix} \begin{bmatrix} \delta(t-\tau) \\ U1 \end{bmatrix} dt + dw$$

$$v(t_k) = \begin{bmatrix} \alpha_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{v}(t_k) \\ x_2(t_k) \\ x_3(t_k) \end{bmatrix} + \begin{bmatrix} 0 & \theta_{10} \end{bmatrix} \begin{bmatrix} \delta(t_k-\tau) \\ U1 \end{bmatrix} + \tilde{e}(t_k)$$

$$k = 0,1,\ldots,N$$

$$R_1 = \begin{bmatrix} |\theta_{11}| & \sqrt{|\theta_{11}|\ |\theta_{12}|} \sin\theta_{14} & \theta_{15} \\ \sqrt{|\theta_{11}||\theta_{12}|} \sin\theta_{14} & |\theta_{12}| & \theta_{16} \\ \theta_{15} & \theta_{16} & |\theta_{13}| \end{bmatrix}$$

$$\tilde{R}_2 = |\theta_{17}|$$

$$\begin{bmatrix} \bar{v}(t_o) \\ x_2(t_o) \\ x_3(t_o) \end{bmatrix} = \begin{bmatrix} \theta_{18}/\alpha_2 \\ \theta_{19} \\ \theta_{20} \end{bmatrix}$$

$$P(t_o) = \begin{bmatrix} |\theta_{21}| & \theta_{24} & \theta_{25} \\ \theta_{24} & |\theta_{22}| & \theta_{26} \\ \theta_{25} & \theta_{26} & |\theta_{23}| \end{bmatrix}$$

$$\tau = T|\sin\theta_{27}|$$

The states $x_2$ and $x_3$ are linear combinations of the original states $\bar{v}$, $\bar{r}$ and $\bar{\psi}$, and they have the dimensions of $m/s^2$ resp. $m/s^3$. The transfer function relating the sway velocity $\bar{v}$ to the rudder angle, measured in radians, is given by

$$G_2(s) = \frac{\frac{V^2}{L}\theta_4 s^2 + \frac{V^3}{L^2}\theta_5 s + \frac{V^4}{L^3}\theta_6}{s^3 + \frac{V}{L}\theta_1 s^2 + \frac{V^2}{L^2}\theta_2 s + \frac{V^3}{L^3}\theta_3} \qquad (4.12)$$

or, if the wind parameters $\theta_3$ and $\theta_6$ are equal to zero,

$$G_2(s) = \frac{\frac{V^2}{L}\theta_4 s + \frac{V^3}{L^2}\theta_5}{s^2 + \frac{V}{L}\theta_1 s + \frac{V^2}{L^2}\theta_2} = \frac{K_v(1 + sT_{3v})}{(1+sT_1)(1+sT_2)} \qquad (4.13)$$

The two transfer functions can be compared to (3.5) and (3.8).

<u>Model 3</u> (IMOD = 3; cf. (4.7)):

$$\begin{bmatrix} dx_1 \\ d\bar{r} \\ d\bar{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{V^3}{L^3}\theta_3 \\ 1 & -\frac{V}{L}\theta_1 & -\frac{V^2}{L^2}\theta_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ \bar{r}(t) \\ \bar{\psi}(t) \end{bmatrix} dt +$$

$$\begin{bmatrix} \alpha_1 \frac{V^3}{L^3}\theta_5 & \theta_6 \\ \alpha_1 \frac{V^2}{L^2}\theta_4 & \theta_7 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t-\tau) \\ Ul \end{bmatrix} dt + dw$$

$$\begin{bmatrix} r(t_k) \\ \psi(t_k) \end{bmatrix} = \begin{bmatrix} 0 & 1/\alpha_1 & 0 \\ 0 & 0 & 1/\alpha_1 \end{bmatrix} \begin{bmatrix} x_1(t_k) \\ \bar{r}(t_k) \\ \bar{\psi}(t_k) \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & \theta_8 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t_k - \tau) \\ U1 \end{bmatrix} + \tilde{e}(t_k)$$

$$k = 0, 1, \ldots, N$$

$$R_1 = \begin{bmatrix} |\theta_9| & \sqrt{|\theta_9|\,|\theta_{10}|}\ \sin\theta_{11} & 0 \\ \sqrt{|\theta_9|\,|\theta_{10}|}\ \sin\theta_{11} & |\theta_{10}| & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{R}_2 = \begin{bmatrix} |\theta_{12}| & \sqrt{|\theta_{12}|\,|\theta_{13}|}\ \sin\theta_{14} \\ \sqrt{|\theta_{12}|\,|\theta_{13}|}\ \sin\theta_{14} & |\theta_{13}| \end{bmatrix}$$

$$\begin{bmatrix} x_1(t_o) \\ \bar{r}(t_o) \\ \bar{\psi}(t_o) \end{bmatrix} = \begin{bmatrix} \theta_{15} \\ \alpha_1\theta_{16} \\ \alpha_1\theta_{17} \end{bmatrix}$$

$$P(t_o) = \begin{bmatrix} |\theta_{18}| & \theta_{21} & \theta_{22} \\ \theta_{21} & |\theta_{19}| & \theta_{23} \\ \theta_{22} & \theta_{23} & |\theta_{20}| \end{bmatrix}$$

$$\tau = T\,|\sin\ \theta_{24}|$$

The state $x_1$ is a linear combination of the original states $\bar{v}$, $\bar{r}$ and $\bar{\psi}$, and it has the dimension of $1/s^2$. The transfer function relating $\psi$ to $\delta$ is given by

$$G_1(s) = \frac{\frac{V^2}{L^2}\theta_4 s + \frac{V^3}{L^3}\theta_5}{s^3 + \frac{V}{L}\theta_1 s^2 + \frac{V^2}{L^2}\theta_2 s + \frac{V^3}{L^3}\theta_3} \qquad (4.14)$$

or, if the wind parameter $\theta_3$ is equal to zero,

$$G_1(s) = \frac{\frac{V^2}{L^2}\theta_4 s + \frac{V^3}{L^3}\theta_5}{s[s^2 + \frac{V}{L}\theta_1 s + \frac{V^2}{L^2}\theta_2]} = \frac{K(1+sT_3)}{s(1+sT_1)(1+sT_2)} \qquad (4.15)$$

cf. (2.6), (3.3) and (3.7).

The transfer function relating r to $\delta$ is obtained by multiplying (4.14) or (4.15) by s, eg.

$$G(s) = \frac{K(1+sT_3)}{(1+sT_1)(1+sT_2)} \qquad (4.16)$$

Nomoto's first order model can be identified, using model 3, by giving the wind parameter $\theta_3$ and the parameters $\theta_2$ and $\theta_5$ the fixed value zero. Following transfer function, relating r to $\delta$, will then be obtained:

$$G(s) = \frac{\frac{V^2}{L^2}\theta_4}{s + \frac{V}{L}\theta_1} = \frac{K}{1 + sT_s} \qquad (4.17)$$

A summary of the three ship models, which are described in this section, is also given in Appendix B.

## 4.4 Numerical Aspects.

The maximum of the likelihood function is found by an optimization routine. It is, however, extremely tedious to compute the gradient analytically, so only optimization techniques using the values of the loss function have been tried.

Two different algorithms have proved to be the most suitable for this kind of problems. In the first one, subroutine NUFLET, the gradient is computed numerically using finite differences. Then a quasi-newton method is applied to find the optimum (see Fletcher (1972)). The other algorithm, subroutine POWBRE, does not use numerical gradients, but gets information about the loss function by a special search pattern (see Brent (1973)).

The actual computations are far from straightforward, and although as fast a computer as UNIVAC 1108 is used, the execution times often become rather long, especially when a lot of data points are used.

# 5. PROGRAM DECCON FOR CALCULATION OF THE SWAY VELOCITY FROM DECCA COORDINATES

From a Decca-recording of, say, a manoeuvring test the heading of the ship and the Decca coordinates are available. The Decca coordinates, given in zones and lanes, are usually recorded by a series of discrete photographs, taken at a rate of about 0.05 Hz. As the pointers of the decometer are moving all the time such a recording is, however, less convenient. A more accurate recording can be obtained, e g by photographing the decometers at a steady higher rate or sampling and recording on paper tape by means of a datalogger.

In this part a method will be described, by which the Decca coordinates are transformed into a convenient set of orthogonal coordinates and further used to furnish the sway velocity of the ship during a manoeuvre.

## 5.1 The Decca Navigator System

A brief description of the Decca Navigator System follows below. For further details see Decca (1965).

The wireless direction finding system of Decca Navigator that is used in conjunction with ship navigation is based upon phase comparison. Suppose that there are two radio transmitters, M and S, synchronously transmitting continuous waves of the length $\lambda$. See Fig 5.1

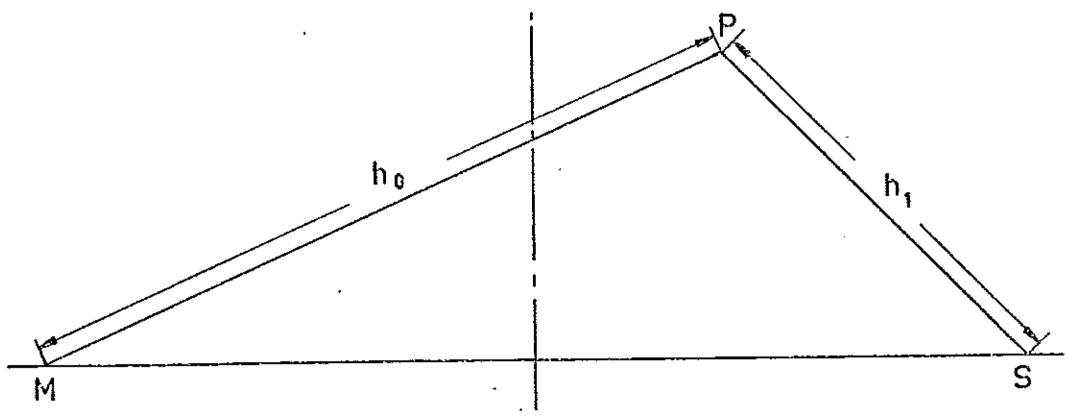



Fig 5.1

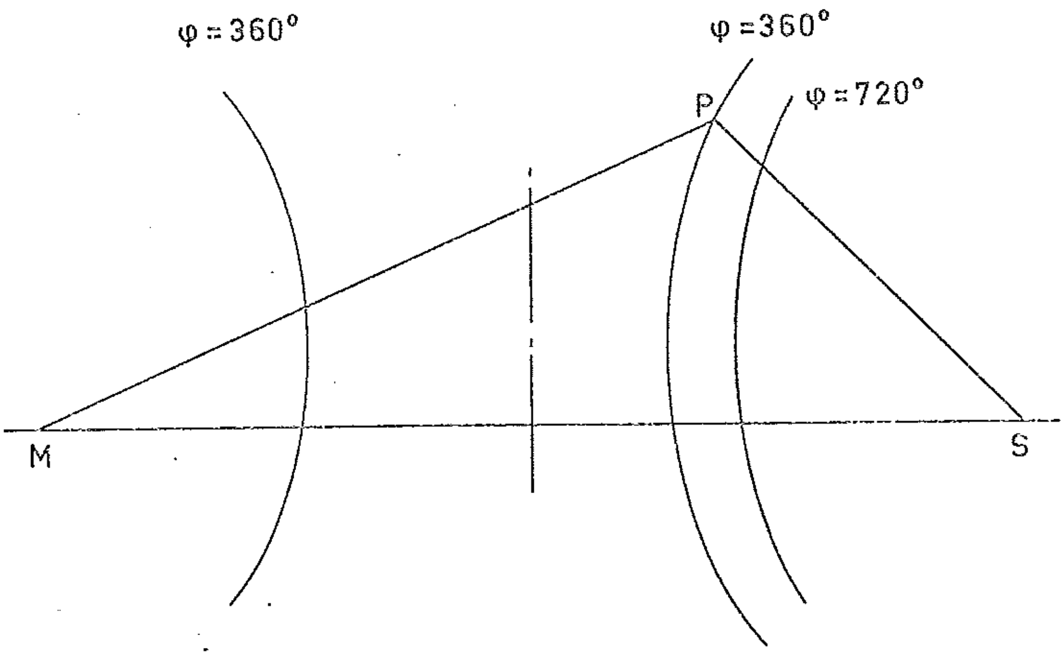



Fig 5.2

The phase shift at a point P is $360(h_0 - h_1)/\lambda$ degrees. If P is on the centre normal of MS then the phase shift is zero. At a point a little aside of the centre normal (to the left or to the right) $h_0 - h_1$ is equal to $\lambda$ and thus the phase shift is $360^{\circ}$. The locus of all the points where the phase shift is $360^{\circ}$, is by definition a hyperbola with its focal points at M and S respectively. A little further aside there is another hyperbola, where the phase shift is $720^{\circ}$. In this way a whole set of hyperbolas is composed. See Fig 5.2.

The area between two consecutive hyperbolas is called a lane and the distance between the hyperbolas the width of the lane. The phase shift of a certain point P can be measured by means of a decometer with the accuracy of about 0.01 lanes.

Thus if the phase shift of a position is determined in two systems of hyperbolas of the type described above, the position may be determined.

A Decca Chain consists of a Head Transmitter, the Master and three Slave Stations, which are denoted by the colours Red, Green and Purple. By using three groups of curves it is possible to avoid unfavourable angles between the hyperbolas in the area considered.

In the description above it has been assumed that all the transmitters are working at the same frequency. This is, however, impossible in practice. Therefore the transmitters are working at different frequencies in the proportions 3:4:5. In the receivers the Master and Slave frequencies are multiplied to the same frequency before the phase comparition.

## 5.2 Computation of Decca Coordinates at a Point

Suppose the Master, see Fig 5.2, is at M and a Slave at S then the Decca coördinate at P is given by, see Decca (1949)

$$L = \frac{MS + MP - SP}{d} \tag{5.1}$$

where $d = \dfrac{\text{velocity of propagation}}{\text{frequency of phase comparition}}$

MS = Distance from Master to Slave
MP = " " " " point in question
SP = " " Slave " " " "

Now L is the so called "Numerical Lane Number" and represents the total number of lanes in the pattern counted to the point P from the extension of the line joining the Slave to the Master. The Decca lines are conventionally divided into groups of 24 Red, 18 Green or 30 Purple lanes called "Zones" and these Zones are denoted by letters advancing progressively from the Master end of the base line from A to J and then repeating.

Moreover the numbering of the lanes within each Zone is from

0 to 23 on the Red Pattern
30 to 47 on the Green Pattern
50 to 79 on the Purple Pattern

starting in each case at the Master end of the Zone.

The conversion of the conventional lane numbering to the "Numerical Lane Number" is explained by the following example:

If the Red Lane Number is e g D12.34 then L = 3 × 24 + 12.34 = 84.34 lanes.

5.3 Conversion from Decca Coordinates to Cartesian Coordinates

The principle underlying the conversion of hyperbolic to Cartesian coordinates is the linear relationship that exists between the x-coordinate, see Fig 5.3, and the distance from the Master to the point in question. (Cf the Decca report on the Omnitrac system.)

Triangle $MPP_1$ gives

$$y^2 = h_0^2 - x^2 \tag{5.2}$$

And triangle $P_1PS$

$$y^2 = h_1^2 - (a_1 - x)^2 \tag{5.3}$$

At P the decometer value D is

$$D = h_0 - h_1 + a_1 \tag{5.4}$$

$P_1$ is defined by

$$P_1 = D - a_1 = h_0 - h_1 \tag{5.5}$$

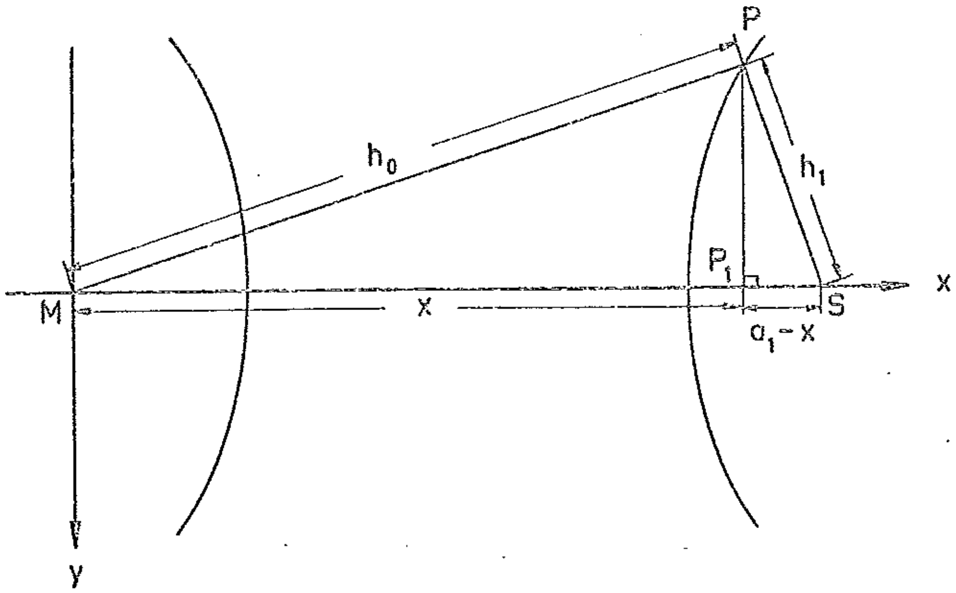



Fig 5.3

Thus

$$x = \frac{p_1}{a_1} \cdot h_0 + \frac{a_1}{2} - \frac{p_1^2}{2a_1} \tag{5.6}$$

Suppose the coordinates of the point P are required in an arbitrary coordinate system (X, Y) with the Master at the origin. Set up two auxiliary coordinate systems ($X_1$, $Y_1$) and ($X_2$, $Y_2$), the positive axes of which go along the baseline to the Slaves $S_1$ and $S_2$, Fig 5.4. The coordinates of $S_1$ and $S_2$ in the (X, Y) system are ($u_1$, $v_1$) and ($u_2$, $v_2$). Applying the above principles to these auxiliary systems and using the "rotation of axes" formula gives two linear equations in x and y, which can be solved in terms of $h_0$ as

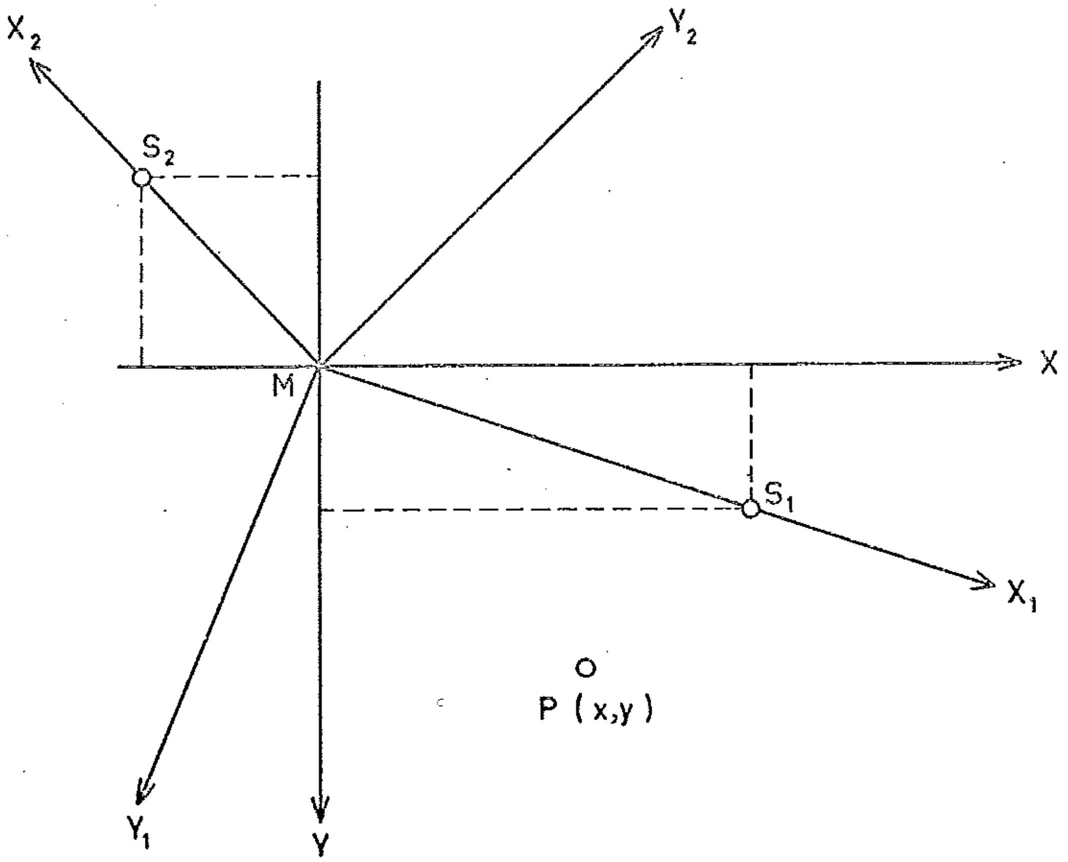



Fig 5.4

$$x = \frac{P_1 v_2 - P_2 v_1}{u_1 v_2 - u_2 v_1} \cdot h_0 + \frac{a_1^2 v_2 - a_2^2 v_1}{2(u_1 v_2 - u_2 v_1)} - \frac{P_1^2 v_2 - P_2^2 v_1}{2(u_1 v_2 - u_2 v_1)} \qquad (5.7)$$

$$y = \frac{P_2 u_1 - P_1 u_2}{u_1 v_2 - u_2 v_1} \cdot h_0 + \frac{a_2^2 u_1 - a_1^2 u_2}{2(u_1 v_2 - u_2 v_1)} - \frac{P_2^2 u_1 - P_1^2 u_2}{2(u_1 v_2 - u_2 v_1)} \qquad (5.8)$$

Moreover there is the relation

$$x^2 + y^2 = h_0^2 \qquad (5.9)$$

This gives two solutions for $h_0$, where one is the correct one and the other is either negative or gives an intersectional point far away from the area considered and can therefore be rejected.

## 5.4 Choice of System of Coordinates

The earth surface may be transformed into two dimensions by a number of transformations. In this case the Mercator Transformation has been used.

It should be noticed that when using the heading of the ship in such a two dimensional system of coordinates the meridian convergence has to be taken into account. That is if the direction of e g the y-axis at the Master is to the geographical north and if the area considered is far aside of the Master, along the x-axis, then the difference between the direction of the y-axis and that to the geographical north is not negligible.

Fig 5.5 exemplifies the Cartesian coordinates of a ship during a zig-zag test.

## 5.5 Calculation of the Sway Velocity

In Fig 5.6 the system of axes in space is $O_0x_0y_0$, that fixed in the ship is Oxy. The point of reference O lies at the distance $L_{pp}/2$ forward of A P of the ship. The sway velocity v can now be derived as

$$v = \dot{y}_0 \cos(\psi - \psi_0) - \dot{x}_0 \sin(\psi - \psi_0) \qquad (5.10)$$

The derivatives are to be estimated by means of some curve fitting method, e g as

$$f'(x_0) = (f(x_0 - 2h)/12 - 2f(x_0 - h)/3 + 2f(x_0 + h)/3 - f(x_0 + 2h)12)/h \qquad (5.11)$$

This gives, however, rather irregular values of v, see Fig 5.7.

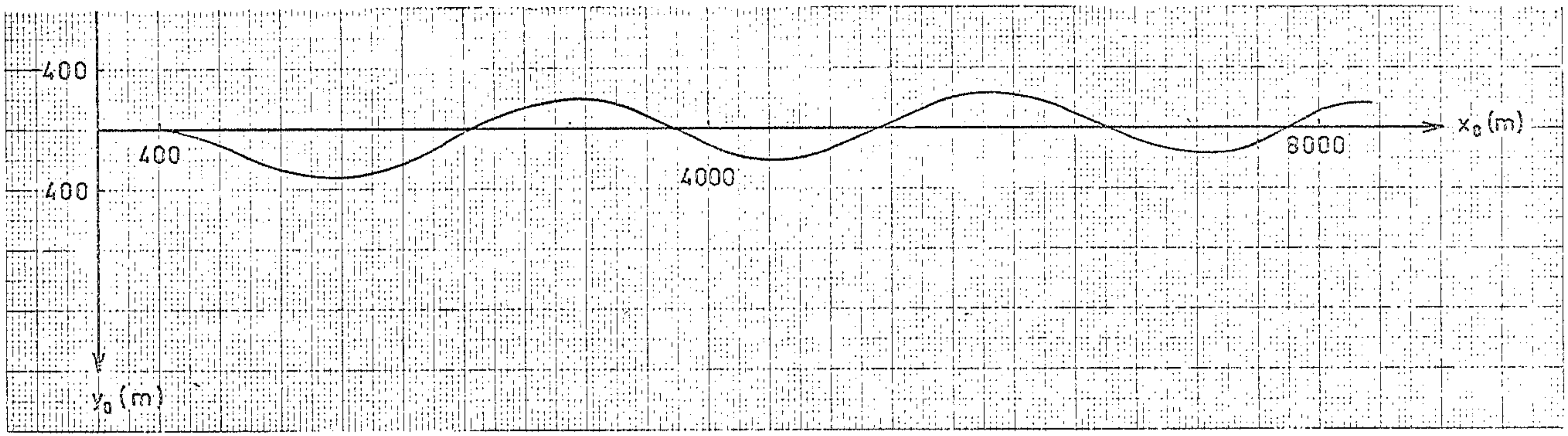



Fig 5.5 $10^\circ/10^\circ$ zig-zag test

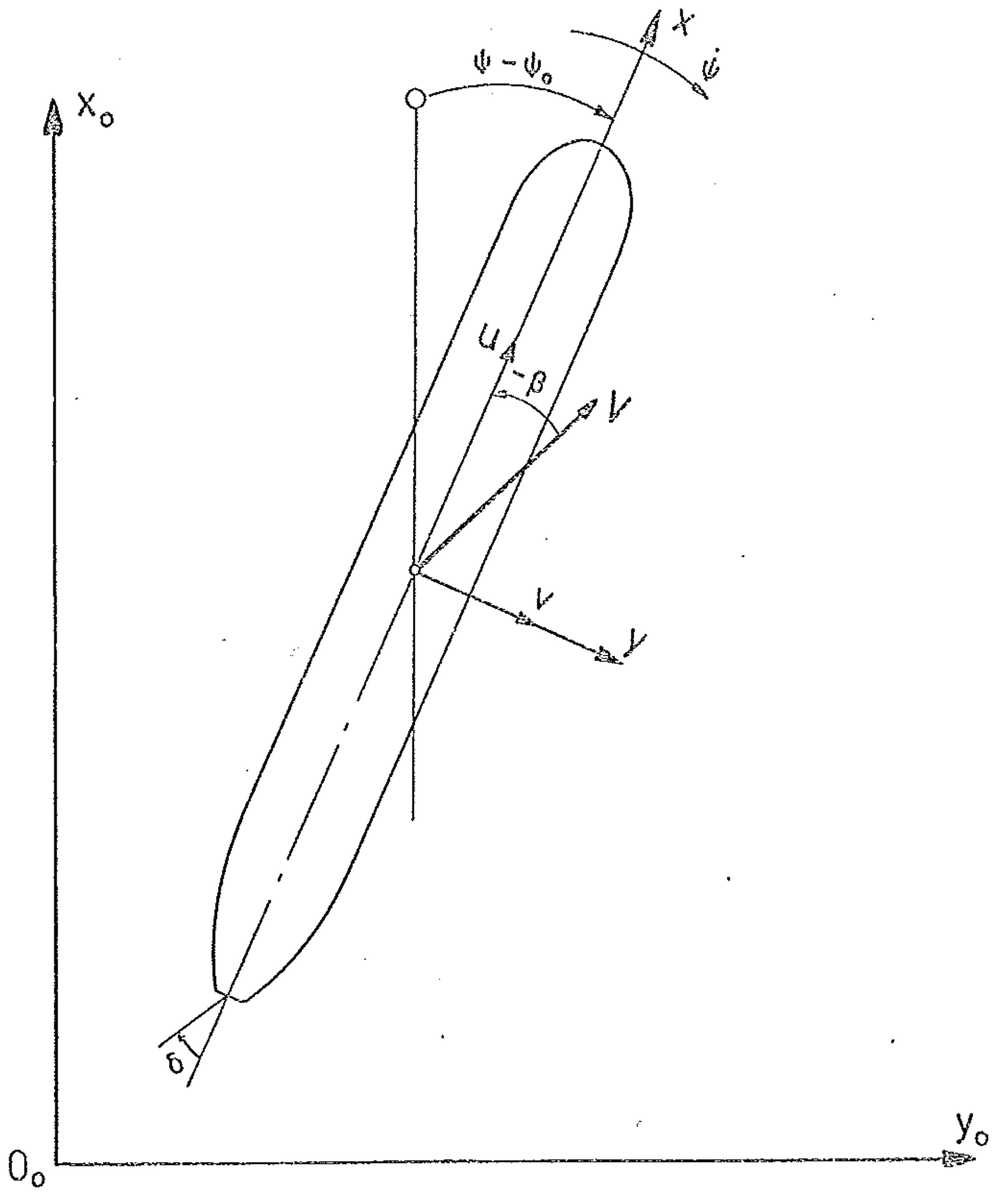



Fig 5.6 Definition of system of axes in space, etc

As the ship movements during for instance a zig-zag test are fairly regular it is suitable to apply some kind of smoothing of the ($x_0$, $y_0$) coordinates, before calculating the sway velocity.

In the process of smoothing, the smoothed value is obtained from observations in the immediate neighbourhood of the point, rather than from the whole set of observations as in the standard least square methods, see e g Guest (1970). Fig 5.8 shows the sway velocity, plotted in Fig 5.7, smoothed by means of Spencer's 15-point formula.

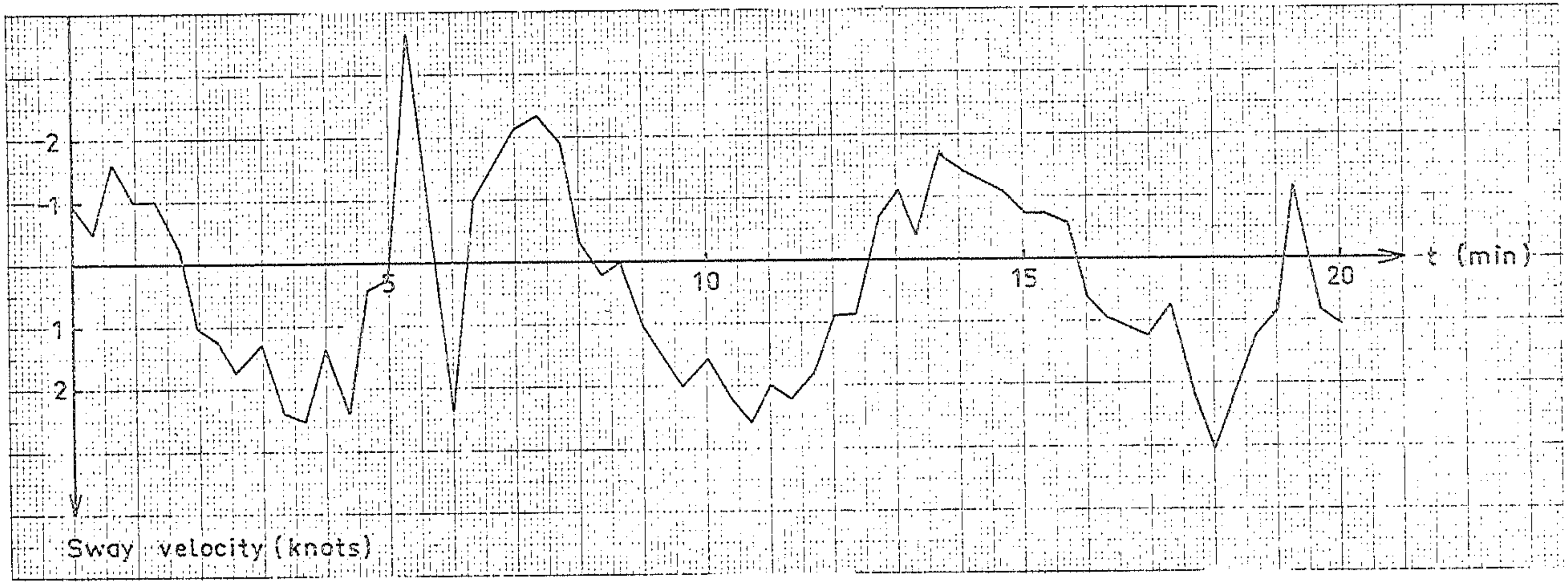



Fig 5.7 Sway velocity calculated from Decca-coordinates

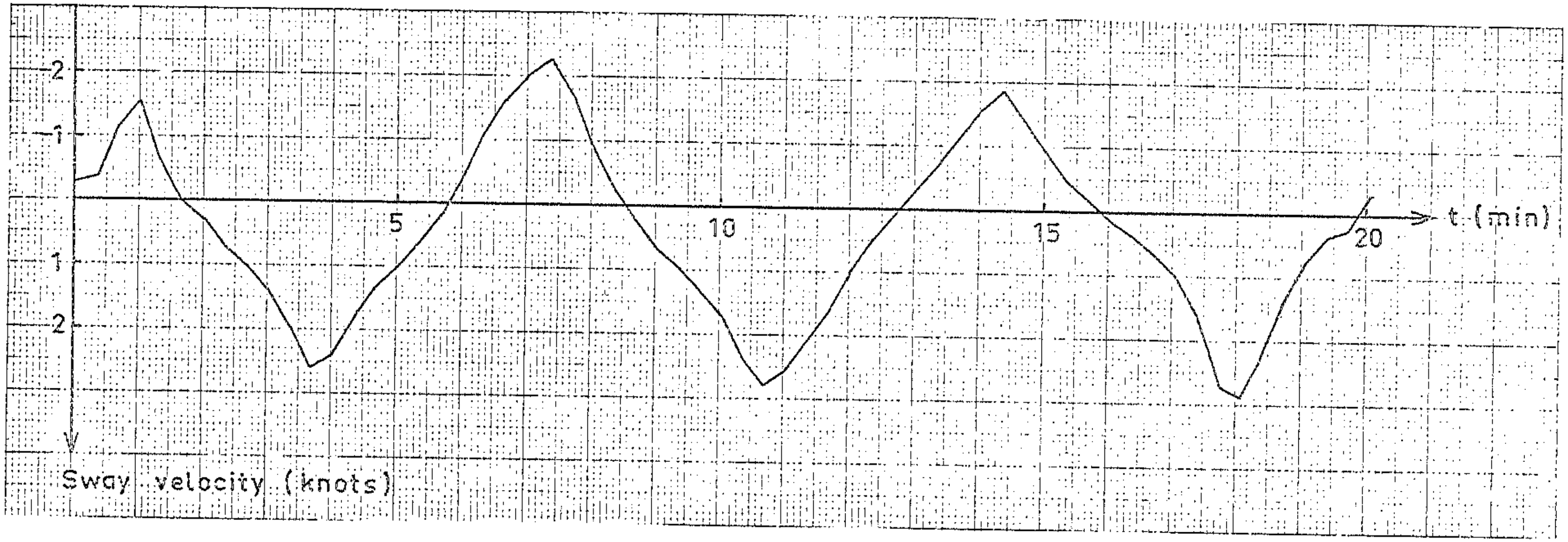



Fig 5.8  Smoothing by means of Spencer's 15-point formula

## 6. THE ATLANTIC SONG EXPERIMENT

The Atlantic Song is a freighter of the Wallenius' lines. It is 197 m long, it weighs 15 000 tons and has a maximum speed of 21 knots. The experiment was made on Sunday, December 21, 1969 off the west coast of Denmark at latitude N 54°17' and longitude E 4°51'. The course was 217°. The wind was about 8 Beaufort (17-20 m/s fresh gale) and the wave height 3.5-4 m. The sight was poor due to a heavy snowfall. The ship had a luffing tendency, and a windgust forced a port yaw. The impact of the waves on the bow induced sudden starboard yaws. The experiment, which lasted for about half an hour, was performed by two students, Mr. Ekwall and Mr. Edvardsson. The speed was 18.5 knots in the beginning of the experiment and was reduced to 18 knots at the end of the experiment due to the rudder motions. In the experiment the rudder angle was perturbed, and the yaw angle was observed. Mr. Ekwall was the coordinator during the experiment. He was standing on the bridge together with the captain, Mr. Tärnsjö. Mr. Edvardsson was at the rudderservo in the machine room. A sampling interval of 15 s was chosen based on a priori knowledge. At each sampling event Mr. Ekwall read the yaw angle from the gyro compass. He also ordered a rudder angle change to be performed by the helmsman Mr. Brand and a reading of the rudder angle to be done by Mr. Edvardsson. The results were recorded in a table. The input was chosen as two periods of a PRBS signal with a length of 64 sampling intervals. The peak to peak variation was about 10°. The signal was changed somewhat to keep a reasonable course.

The input output data recorded are shown in Fig 6.1. The data have partly been analysed before. See Åström and Källström (1972) and (1973). To achieve compatibility with the other experiments new calculations have been performed using the new estimation programs.

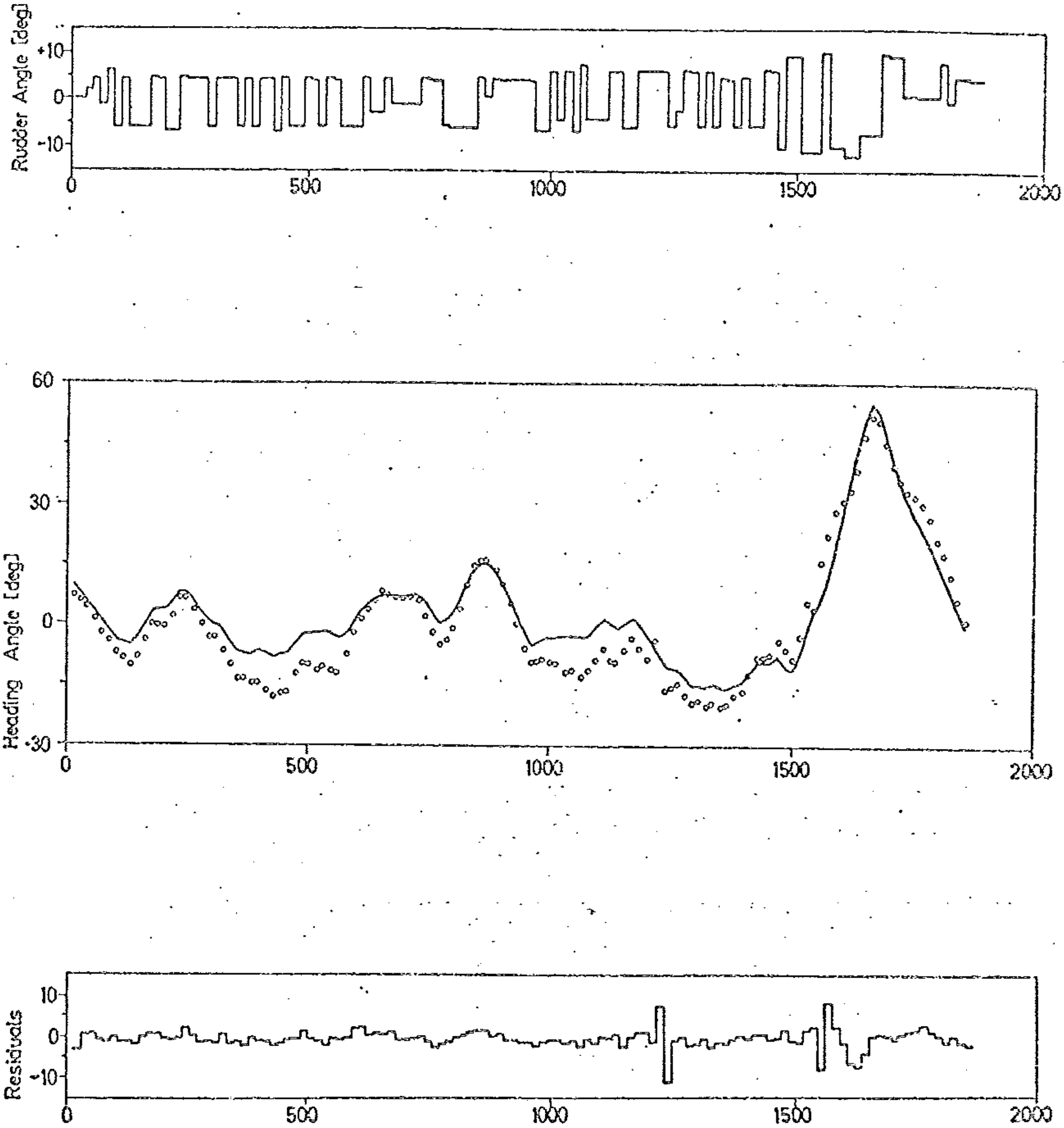



Fig. 6.1. The Atlantic Song experiment, input rudder angle, output heading (measurements dots, model full line) and residuals. The mean values are subtracted and the output is shifted one sampling interval.

As a first attempt to analyse the data the discrete time model

$$y(t) + a_1y(t-1)+\ldots+a_ny(t-n) = b_1u(t-1)+\ldots+b_nu(t-n) +$$

$$+ \lambda[e(t) + c_1e(t-1)+\ldots+c_ne(t-n)] \qquad (6.1)$$

was first fitted to the data by the maximum likelihood method. The program IDPAC was used. Repeating the identification for different values of n the following loss functions were obtained: $V_1 = 668.4$, $V_2 = 294.5$ and $V_3 = 286.9$. An attempt to determine the order by a F-test gives the test quantities $F(1\rightarrow 2) = 50.4$ and $F(2\rightarrow 3) = 1.0$. This indicates that a second order model is appropriate. An application of Akaikes information criterion (see Akaike (1972)) gives the minimum, AIC = 560.5, for a second order model. The model is given below

$$\begin{aligned}
a_1 &= -1.64 \pm 0.05\\
a_2 &= 0.66 \pm 0.05\\
b_1 &= -0.11 \pm 0.03\\
b_2 &= -0.19 \pm 0.04\\
c_1 &= -0.75 \pm 0.10\\
c_2 &= 0.06 \pm 0.10\\
V &= 294.5\\
AIC &= 560.5
\end{aligned} \qquad (6.2)$$

If initial values also are estimated, a second order model is obtained as well. This model has the loss function V = 274.54 and an AIC = 555.7, which indicates that initial values should be estimated. The model parameters are not significantly different from the values given above.

In Fig 6.1 are shown the input, the measured output, the model output and the residuals of the second order model with estimated initial values. The residuals are far from normal. Fig 6.1 shows that the residuals have very large values at times close to 1 250 and 1 600. These large residuals can be traced to bad data.

To avoid any difficulties by trying to replace the bad data points by interpolation or other fudging of the data, only the portion 0-1 200 of the data will be used in the following. Notice, however, that a straightforward ML estimation of the parameters of the discrete time model (6.1), which is very inexpensive to run, is very useful in order to check the measured data.

The following results are thus based on the first 80 input/output pairs. Initial states were estimated because this gave a significant reduction of the loss function and of the AIC. It was attempted to remove both levels and trends from the data. There were no significant changes in the parameters of the second order models obtained. The following second order model was obtained.

$$
\begin{aligned}
a_1 &= -1.60 \pm 0.03 \\
a_2 &= 0.61 \pm 0.03 \\
b_1 &= -0.161 \pm 0.014 \\
b_2 &= -0.285 \pm 0.016 \\
c_1 &= -0.37 \pm 0.14 \\
c_2 &= -0.20 \pm 0.11 \\
V &= 17.96 \\
AIC &= 178.9
\end{aligned}
\tag{6.3}
$$

Notice the significant reduction in loss function as compared with (6.2).

There were some difficulties in estimating a third order model. The algorithm had difficulties to converge, and the parameter $c_3$ had to be fixed to obtain a well-conditioned information matrix. After some attempts the following model was obtained

$$
\begin{aligned}
a_1 &= -1.05 \pm 0.05 & c_1 &= 0.18 \pm 0.13 \\
a_2 &= -0.34 \pm 0.07 & c_2 &= -0.18 \pm 0.13 \\
a_3 &= 0.41 \pm 0.04 & V &= 15.88 \\
b_1 &= -0.15 \pm 0.01 & AIC &= 175.1 \\
b_2 &= -0.38 \pm 0.01 & & \\
b_3 &= -0.13 \pm 0.02 & &
\end{aligned}
\tag{6.4}
$$

A comparison with the model (6.3) gives a test value of F = 3.01. Using the F-test it is thus questionable if the model (6.4) is preferable. Akaike's test indicates that the model (6.4) is better than the model (6.3). The zeros of the polynomials A(z) for the model are 0.95, 0.71 and -0.61. The zeros of the polynomial B are -0.4 and -2.0. The corresponding zeros for the model (6.3) are 0.97, 0.63 and -1.8 for the A and B polynomial respectively. The zero -0.61 of the polynomial A, which is approximatively cancelled by the zero -0.4 of the polynomial B, corresponds to a model of the type

$$z(t+1) = -0.6z(t) + e(t)$$

This is typical for a case when round off noise occurs. Fig. 6.2 shows the covariances of the residuals and the cross covariances between the input and the residuals. The graphs indicate that the second order model is acceptable, but that the residuals are whiter for the third order model.

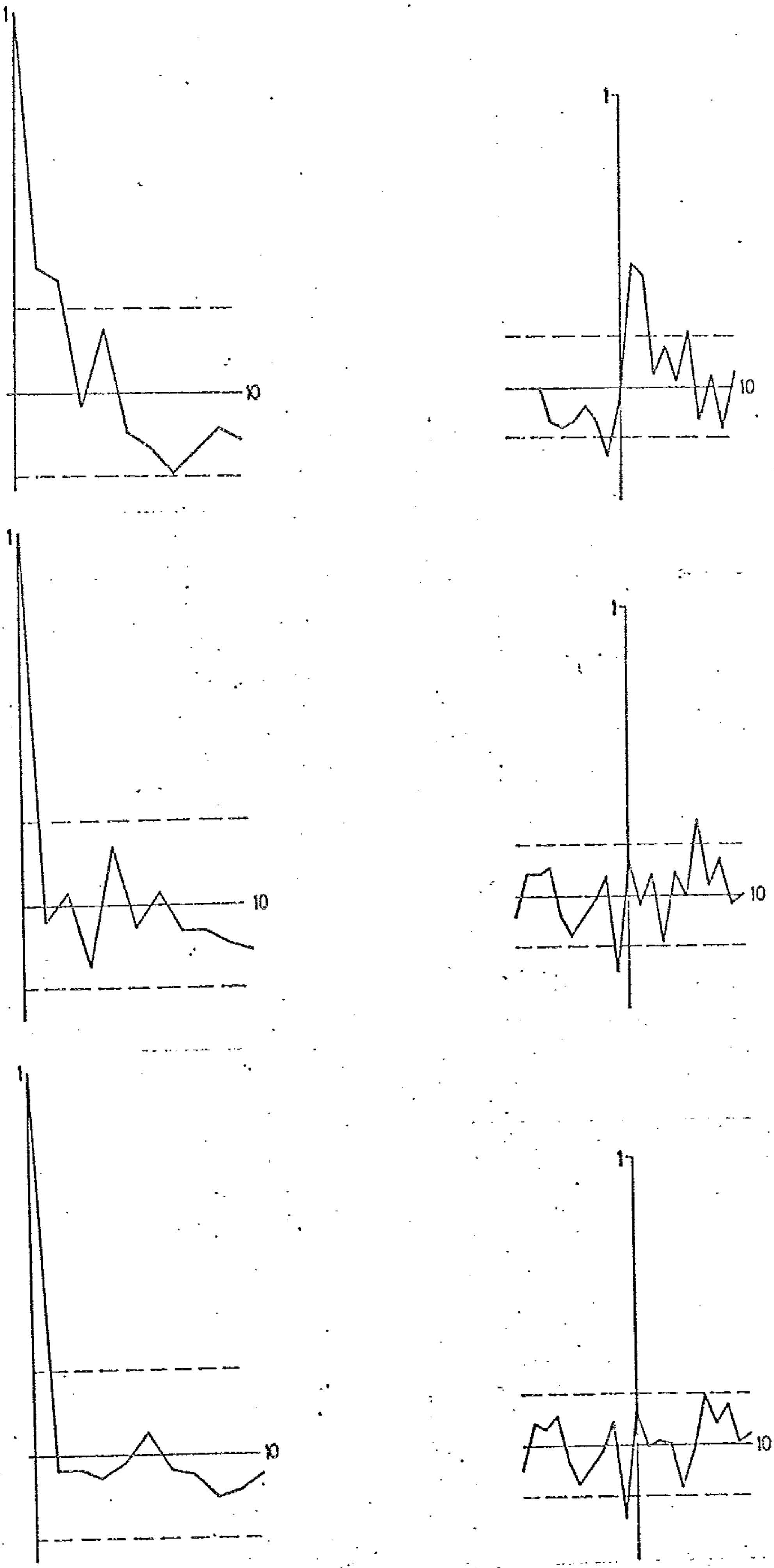



Fig. 6.2. Correlations of residuals and cross correlations of input and residuals for, from top to bottom, 1st, 2nd and 3rd order models.

To summarize, it is thus difficult to distinguish between the second and the third order model. The major improvement by going to a third order model is that the quantization of the heading angle is modelled. Since this is not our major concern, the second order model is accepted. Notice that the polynomial A(z) of the model (6.3) has a zero very close to z = 1. This is expected because the model should ideally contain an integrator. Compare Chapter 3. The deviation is due to the effects of winds and waves that were also discussed in Chapters 2 and 3. The straightforward fitting of a model (6.1) thus gives a model of the Nomoto type with a time constant of 29.4 s and a gain of -0.055 1/s. If a model of higher order is attempted, a pole on the negative real axis is obtained. This is an attempt to model round off errors, which occur due to the quantization of the data. Compare with Fig. 6.2 where the covariances of the second order model indicate an oscillatory behaviour with period 2, which is not present in the third order model.

To get some feeling for the consequences of the model uncertainties Monte Carlo simulations have been performed. A sequence of models have thus been drawn from the distribution of the estimated parameters, and the model responses have been computed. The outputs of such a simulation are shown in Fig. 6.3 and the pulse responses in Fig. 6.4 and Fig. 6.5.

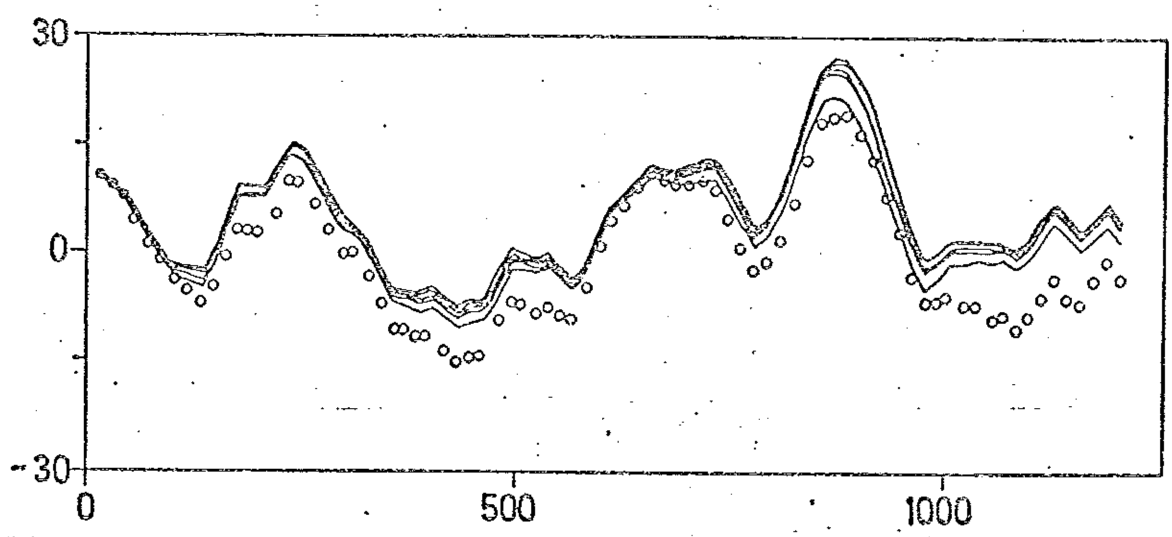



Fig. 6.3. Monte Carlo simulations of the output of the model (6.3). Notice that the levels in the data are removed.

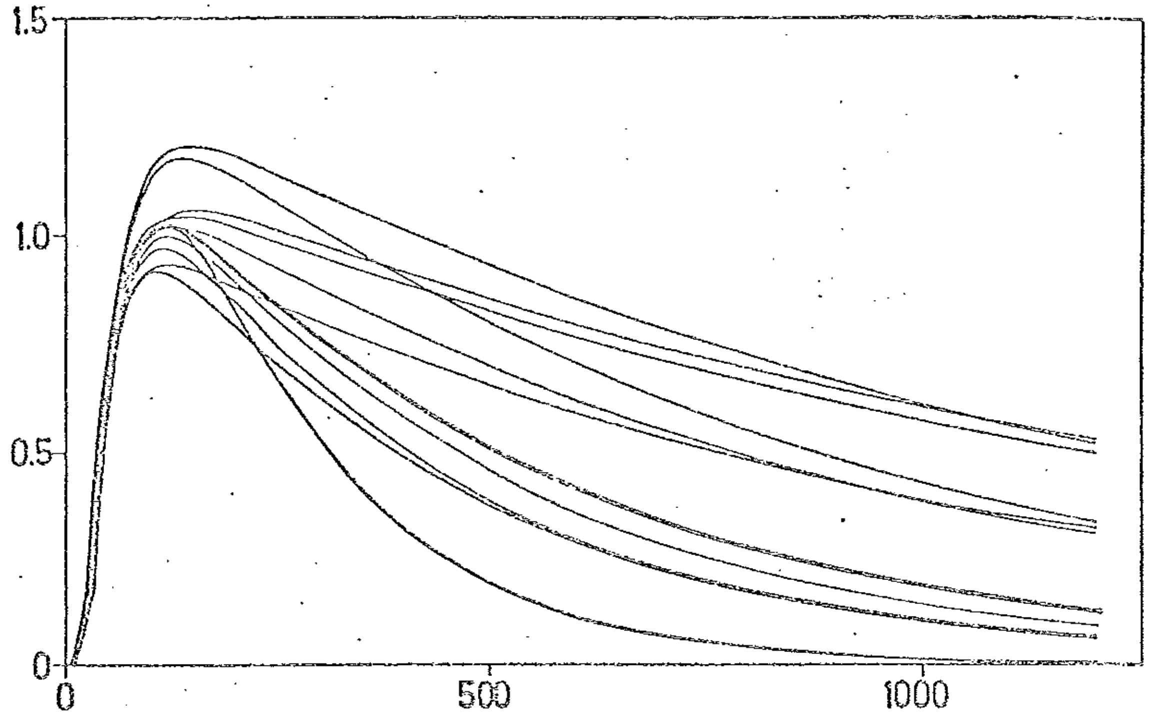



Fig. 6.4. Monte Carlo simulations of impulse responses of the model (6.3).

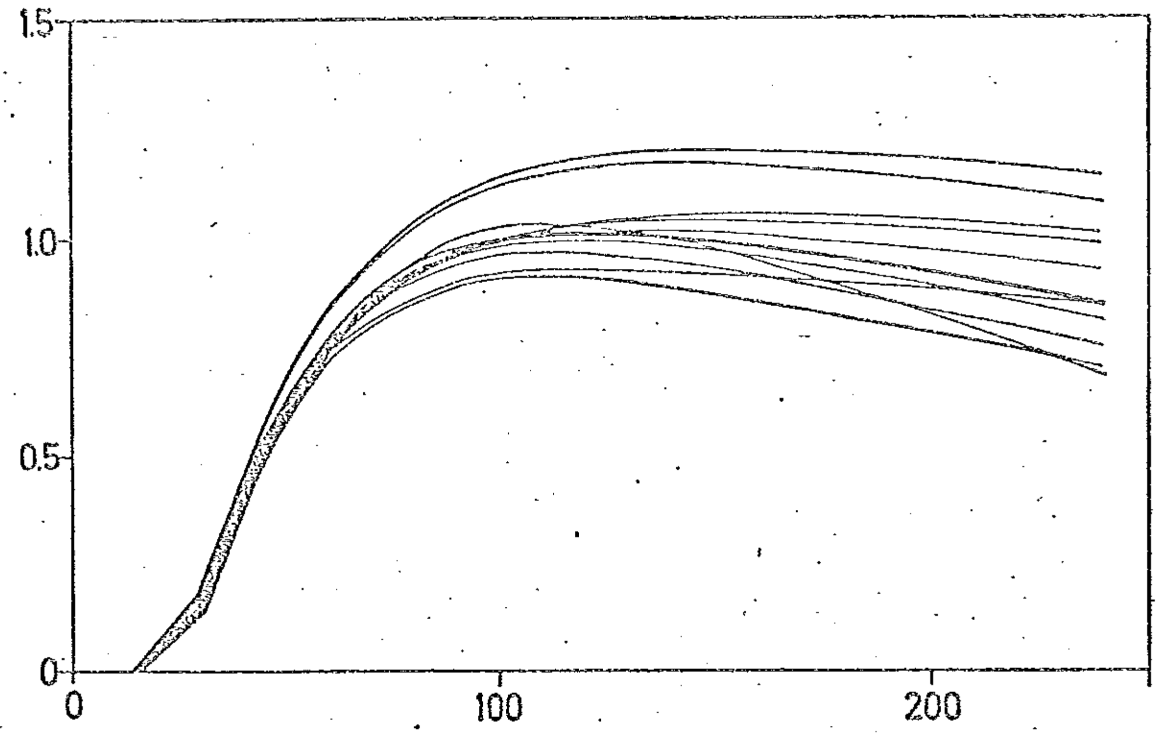



Fig. 6.5. Initial part of the curves in Fig. 6.4.

The fact that the models in Fig. 6.3 are all on the same side of the measurements is natural because of the wind bias and the low frequency nature of the disturbances. For a physical model the impulse responses in Fig. 6.4 should all level off at a constant value. The fact that all the curves go to zero is due to the effects of wind as discussed in Chapter 3. The large spread is thus understandable. Notice, however, in Fig. 6.4 that the initial part of the inpulse responses of the simulated models are all similar.

The first 80 input/output pairs of the Atlantic Song experiment have also been used to estimate the parameters of the model 3 (see Appendix B) by the program LISPID. Both a second order and a third order transfer function (cf.(4.14)) were estimated. The result of the identification of the second order model is shown in Fig. 6.6. The obtained gain and time constant, when the wind influence is eliminated, are given in Table 6.1, where also the result from IDPAC is shown. The third order model obtained is given in Table 6.2, where the wind influence is eliminated too.

| | | From LISPID | From IDPAC |
|---|---|---|---|
| $K'$ | | -1.62 | -1.14 |
| $T_s'$ | | 1.49 | 1.42 |
| $K$ | 1/s | -0.078 | -0.055 |
| $T_s$ | s | 30.9 | 29.4 |
| $\tau$ | s | 12.1 | 15.0 (fixed value) |

Table 6.1. Result of identification of a second order model (see model 3, Appendix B) to the first 80 data points of the Atlantic Song experiment. The values of the transfer function parameters (cf. (4.17)) are given normalized ("prime" system) and non-normalized.

| | | |
|---|---|---|
| $K'$ | | -2.38 |
| $T_1'$ | | 8.32 |
| $T_2'$ | | 1.11 |
| $T_3'$ | | 4.61 |
| $K$ | 1/s | -0.115 |
| $T_1$ | s | 172.3 |
| $T_2$ | s | 22.9 |
| $T_3$ | s | 95.5 |
| $\tau$ | s | 12.0 |

Table 6.2. Result of identification of a third order model (see model 3, Appendix B) to the first 80 data points of the Atlantic Song experiment. The values of the transfer function parameters (cf. (4.15)) are given normalized ("prime" system) and non-normalized.

The result of a F-test shows distinctly that a second order model is appropriate to the data. Akaike's information criterion gives the same result.

The results thus show that the data from the experiment can be modelled well by a second order model. There are very good agreements between the results obtained when fitting a discrete time model and when fitting a continuous time model. The decrease in loss function obtained when increasing the order of the continuous time model is very insignificant. The corresponding improvement in the discrete time model is more significant. The improvement corresponds to a more accurate modeling of the round off errors. This indicates that if the experiments are performed using heading information only, it is important to have a good resolution of the measurement of heading angle.

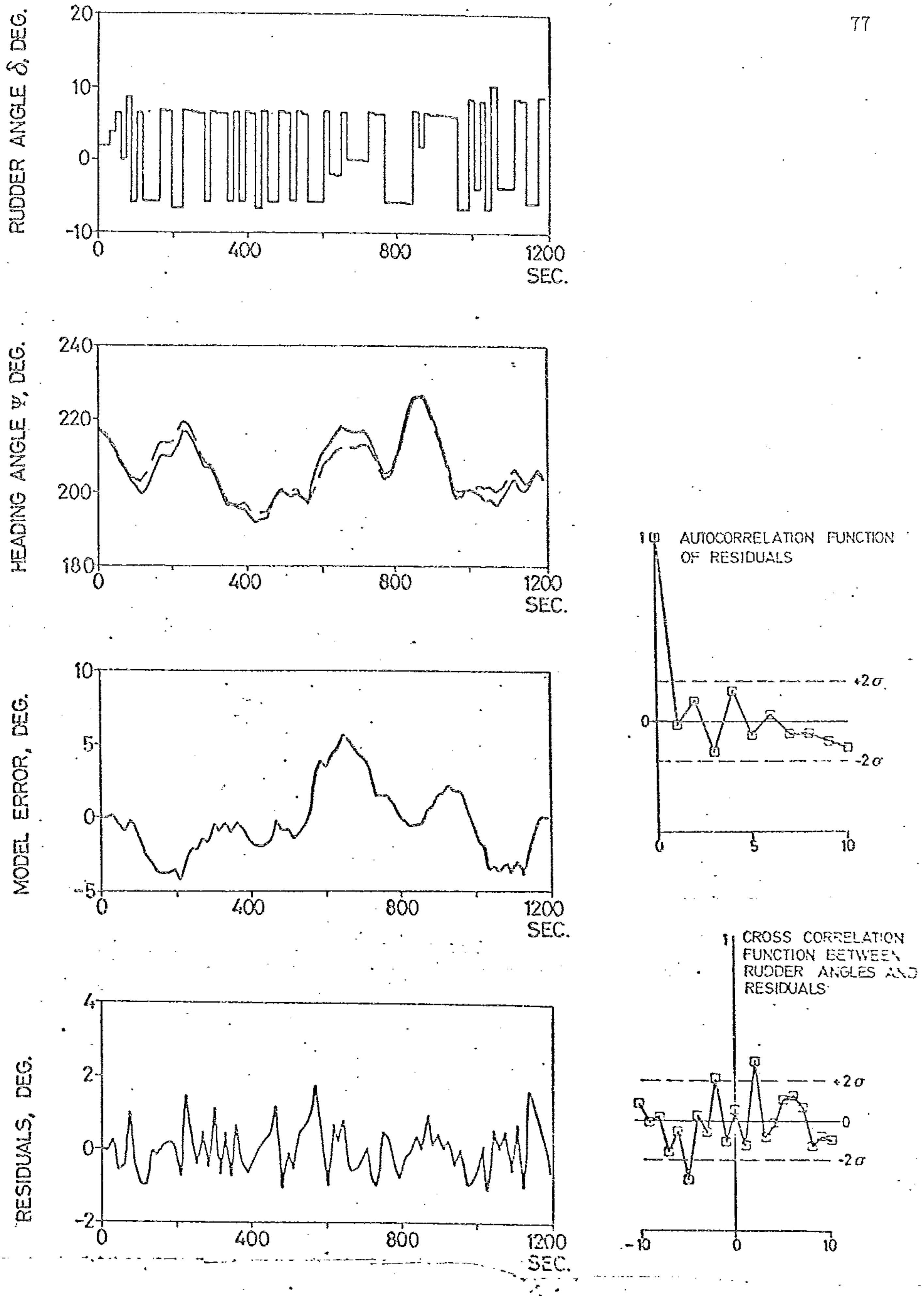



Fig. 6.6. Result of identification of a second order model (see model 3, Appendix B) to the first 80 data points of the Atlantic Song experiment. The dashed heading is the model output, the full line is the measurements.

## 7. THE BORE I EXPERIMENT

The Bore I was delivered in the spring of 1973 by Wärtsilä's Turku Shipyard for the Silja Line. She has an $L_{pp}$ of 115.7 m, a displacement of 8 700 tons and a maximum speed of 22 knots.

The weather conditions during the experiment, carried out in October 1974, were good. During the experiment, which lasted about 15 minutes, the speed was about 18 knots. The rudder angle, the input signal, was chosen as a PRBS signal with a length of 64 sampling intervals. The peak-to-peak amplitude was about 10 degrees. Rudder angle and yaw rate were analogously recorded on a tape recorder. Thus having recorded that input-output pair, the static gain constant K and the time constants $T_1$, $T_2$ and $T_3$ of the transfer function $G_1(s)$, representing the rudder angle-yaw rate relation are identifiable, $G_1(s)$ can be interpreted as

$$G_1(s) = \frac{K(1 + sT_3)}{(1 + sT_1)(1 + sT_2)} \qquad (7.1)$$

(See also model 3, Appendix B, and eq (2.6)).

As the time constant $T_1$ was expected to be something between 10 and 20 seconds the analogous signals were sampled at a rate of 1 Hz.

### 7.1 Results

The constants of the transfer function (7.1) were estimated to

$K = -0.07$ 1/s
$T_1 = 23.3$ s
$T_2 = 2.3$ s
$T_3 = 3.8$ s

Normalizing in the "Prime" system gives

$K' = -0{,}9$

$T'_1 = 1{,}9$

$T'_2 = 0{,}2$

$T'_3 = 0{,}3$

The results of the parameter estimation are illustrated in Fig 7.1, which shows the input-output records (rudder angle and yaw rate), the model output, the model error and the residuals.

It has been suggested by Nomoto that the transfer function (7.1) of a stable ship should be approximated by

$$G_2(s) = \frac{K}{1 + sT} \tag{7.2}$$

(Cf (4.17) and Chapter 2)

By means of a least square method K and T were estimated to

$K = -0{,}07$ 1/s
$T = 15{,}6$ s
$K' = -0{,}8$
$T' = 1{,}4$

The two transfer functions $G_1(s)$ of second order and $G_2(s)$ of first order are plotted in Fig 7.2. As seen from Fig 7.2 the two transfer functions are in accordance with each other within a small range of frequencies around $\omega' = 1{,}1$.

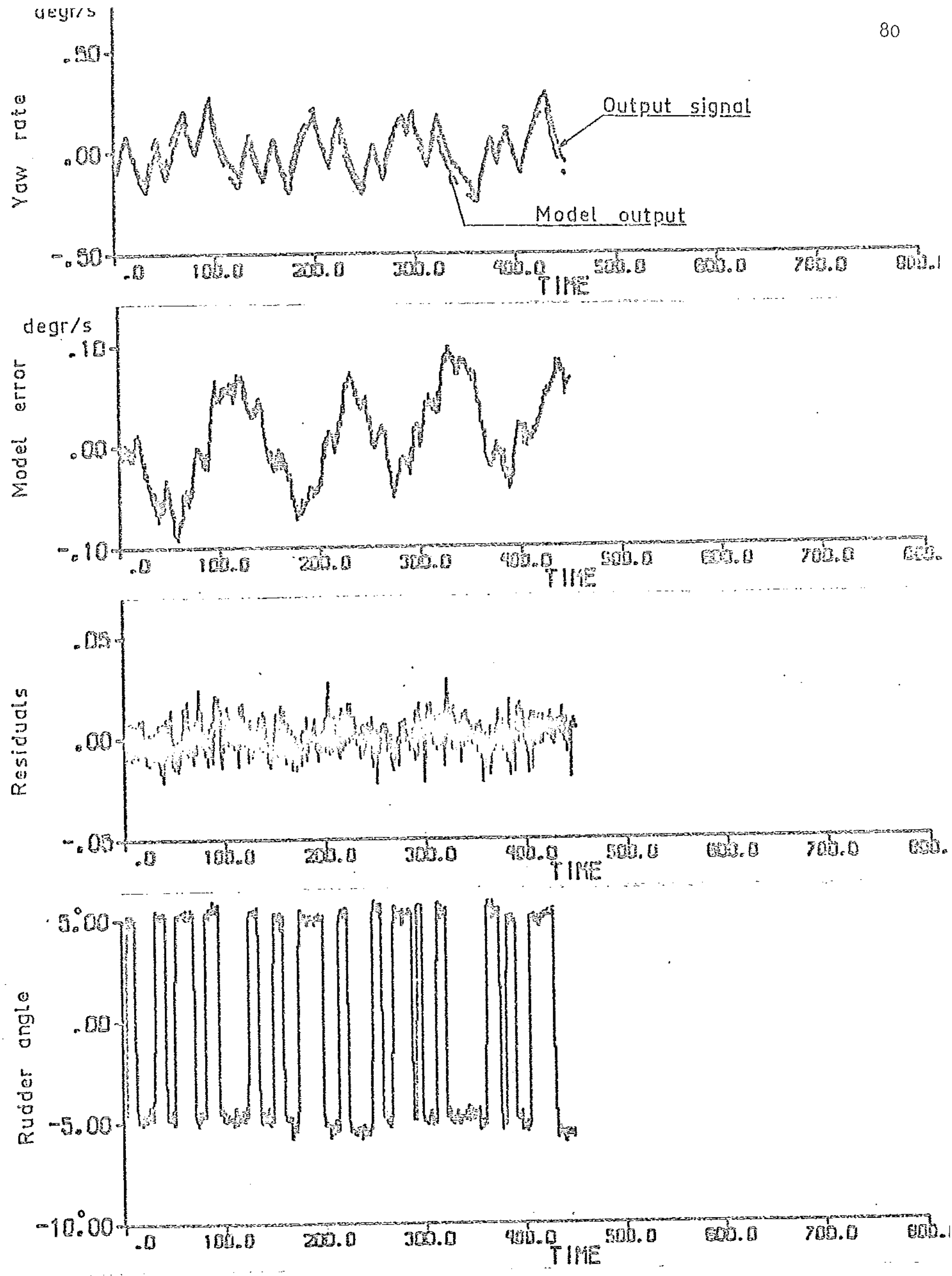



Fig 7.1 Results of the identification of the static gain constant K and the time constants $T_1$, $T_2$ and $T_3$ for the Bore I

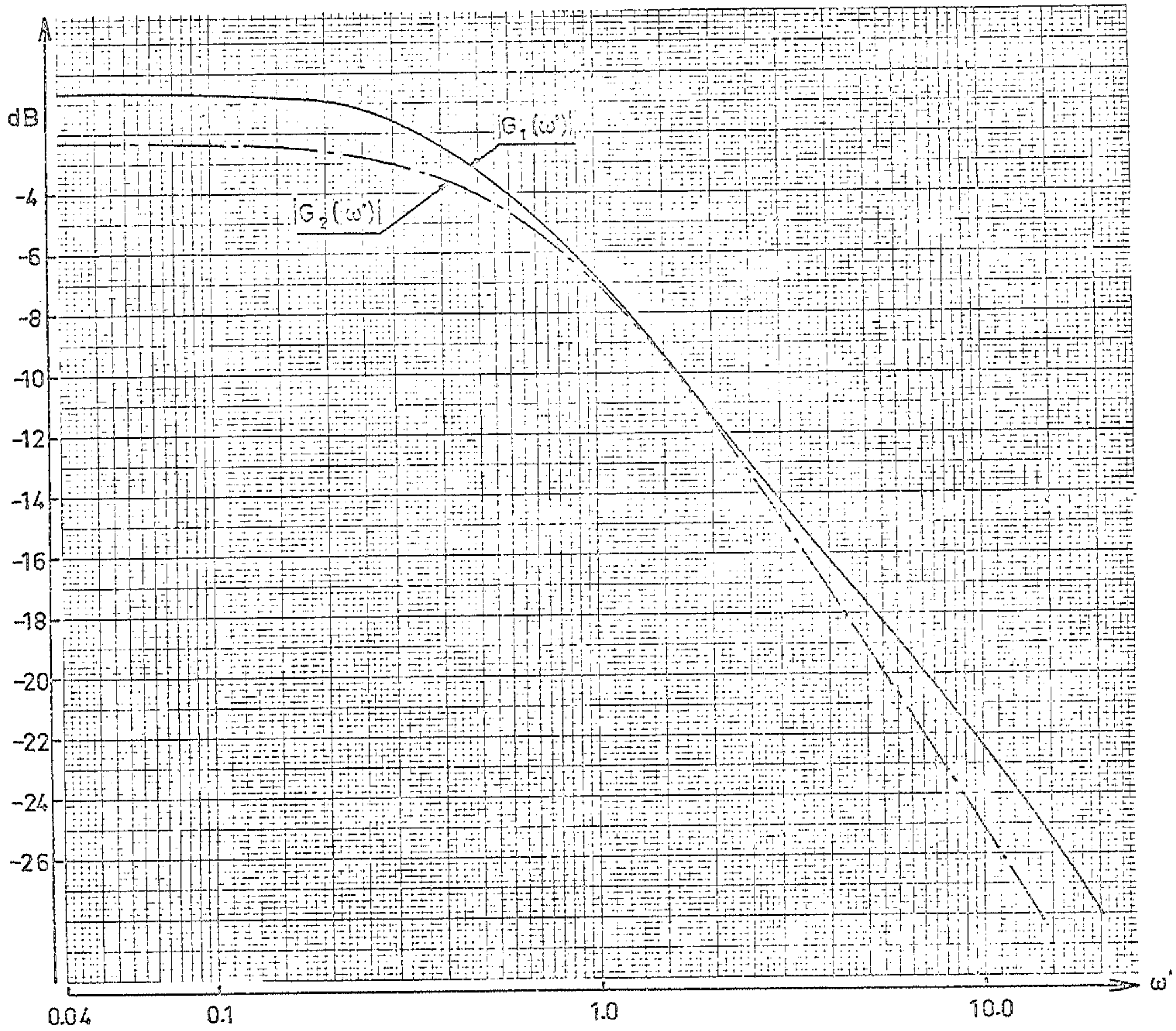



Fig 7.2 Bode diagram showing the transfer functions $G_1(\omega')$ and $G_2(\omega')$ for the Bore I

## 8. THE A K FERNSTRÖM EXPERIMENT

The A K Fernström, a tanker built by Öresunds Shipyard in Malmö, has an $L_{pp}$ of 244 m, a beam of 39 m and her volume displacement is 120 000 $m^3$. The test, a zig-zag test, was performed in connection with the delivery trials by Decca-Navigator AB, Gothenburg.

The weather conditions during the test were fairly good, with a wind of 2 m/s and sea state 1. At the beginning of the test the ship was not quite steady on speed and course, nominally given as 16 knots and 135 degrees, respectively. During the test the initial speed dropped to 13 knots because of rudder movements. The average speed was therefore not more than 14.2 knots. During the test heading, rudder angle and Decca coordinates were recorded. The sway velocity was then calculated from heading and Decca coordinates.

### 8.1 Results

First of all an attempt was made to estimate the static gain constant K and the time constants $T_1$, $T_2$ and $T_3$ of the transfer function $G_1(s)$ representing the rudder-yaw rate relation, without using the sway velocity. $G_1(s)$ can be interpreted as

$$G_1(s) = \frac{K(1+sT_3)}{(1+sT_1)(1+sT_2)} \tag{8.1}$$

(See also model 3, Appendix B)

The constants were estimated to

| | Non-normalized | | Normalized |
|---|---|---|---|
| K | -0.2 | 1/s | -6.2 |
| $T_1$ | 373.8 | s | 11.2 |
| $T_2$ | 4.1 | s | 0.1 |
| $T_3$ | 20.3 | s | 0.6 |

The results of the estimation are shown in Fig 8.1.

The constants K and T in the first-order transfer function $G_2(s)$, where

$$G_2(s) = \frac{K}{1 + sT} \tag{8.2}$$

were estimated to

| | Non-normalized | Normalized |
|---|---|---|
| K | -0.08 1/s | -2.8 |
| T | 148.3 s | 4.4 |

The two transfer functions are plotted in Fig 8.2. On comparison it is obvious that the transfer function (8.2) is a good approximation only within a small range of frequencies.

Using the sway velocity, in this case calculated from Decca coordinates, cf Chapter 5, some of the components of the equations of motion are identifiable. As an initial estimate, results from PMM tests of models of other ships of similar proportions have been used.

A second estimation of the components was made with the sway velocity smoothed by means of Spencer's 15-point formula.

The results of the first estimation are illustrated in Fig 8.3 and Fig 8.4 and the results of the second estimation, where the sway velocity is smoothed, are shown in Fig 8.5 and Fig 8.6.

The estimation of the components of the equations of motion (see model 1, Appendix B) are tabulated in Table 8.1. It should be noticed that the first four components are approximated from PMM tests of other ship models and that no further estimation is made.

The corresponding components of the transfer function (8.1) are:

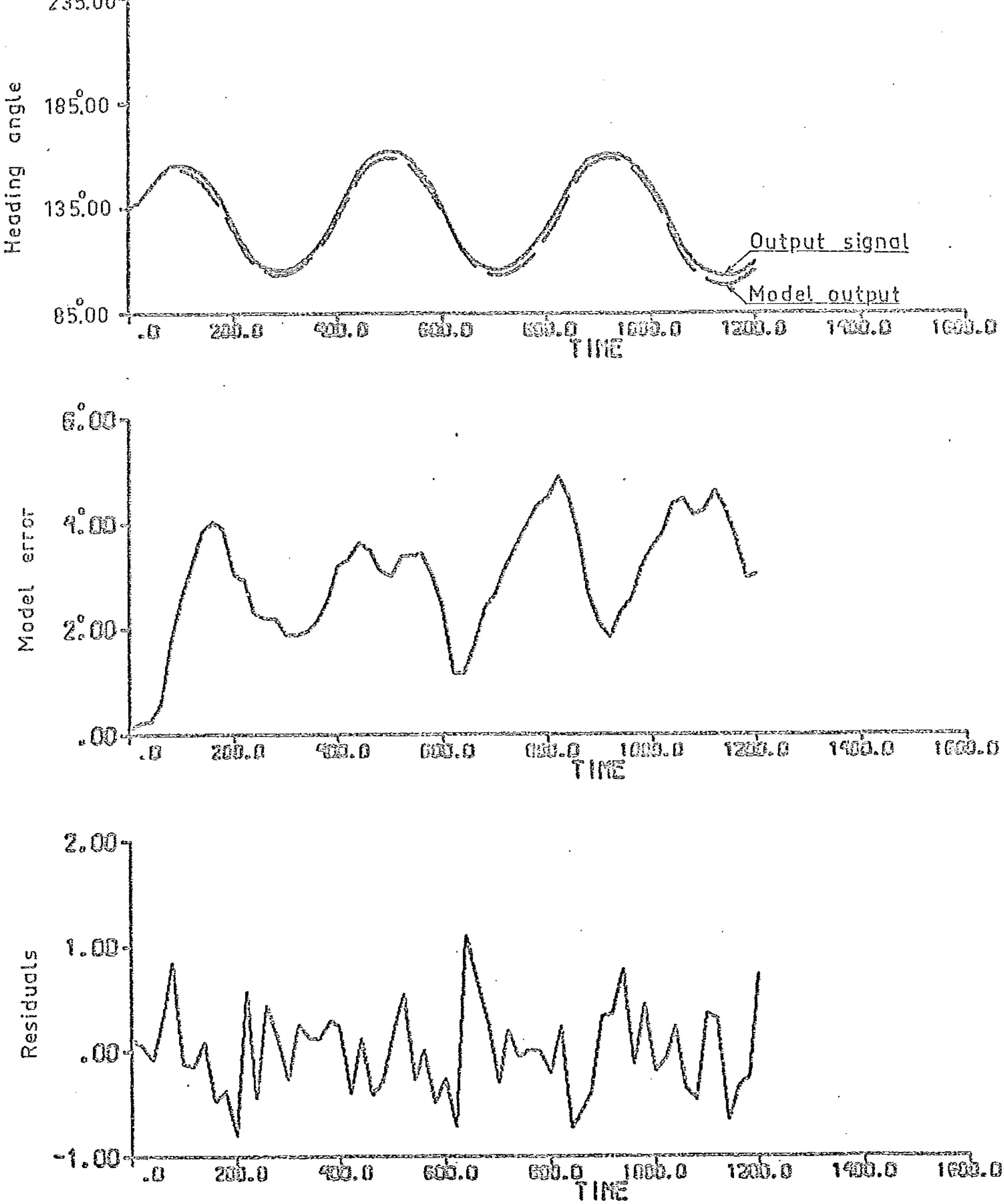



Fig 8.1 Results of the identification of the static gain constant K and the time constants $T_1$, $T_2$ and $T_3$ for the A K Fernström

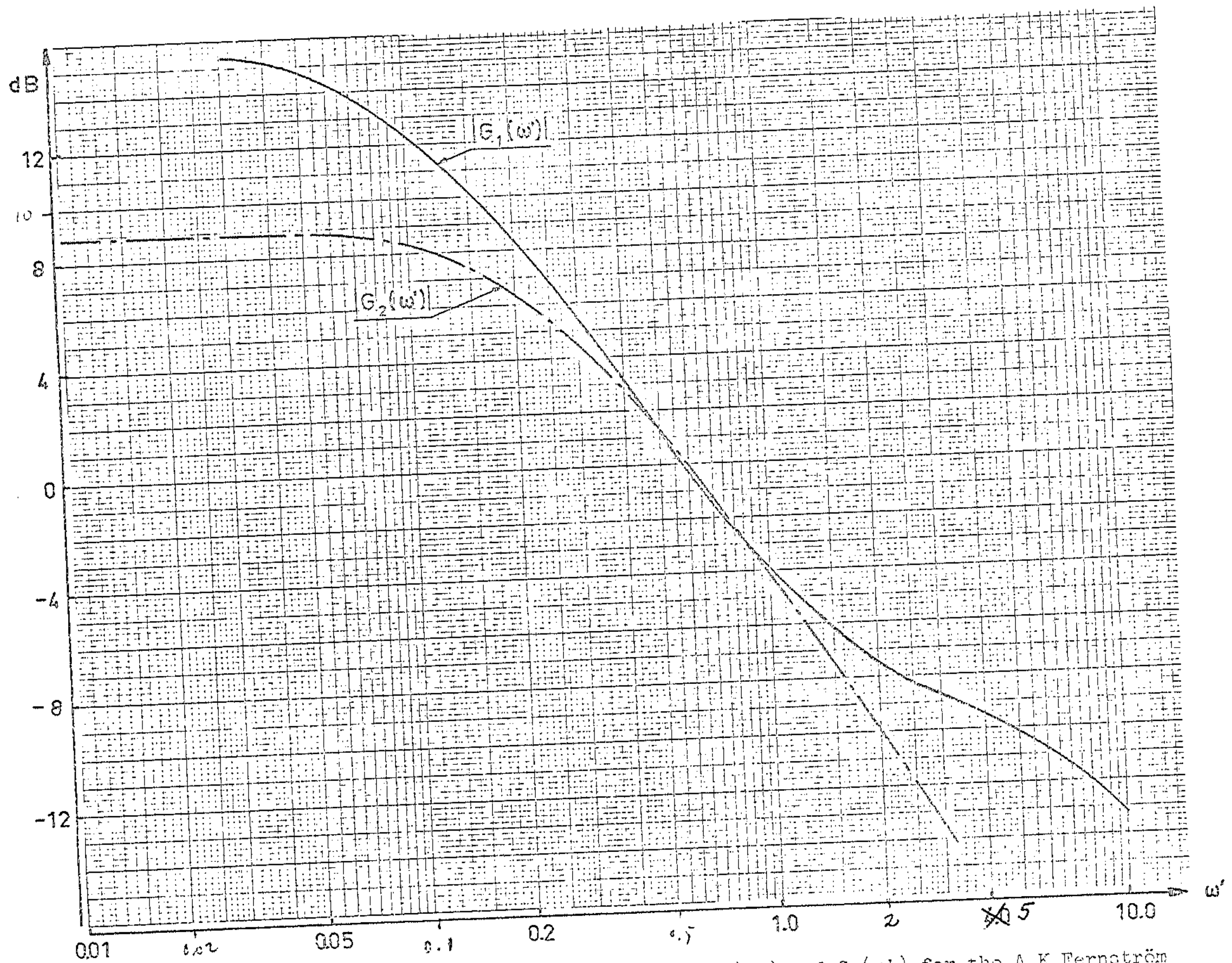



Fig 8.2 Bode diagram showing the transfer functions $G_1(\omega')$ and $G_2(\omega')$ for the A K Fernström

| | Initial estimate | | | Final estimate | | |
|---|---|---|---|---|---|---|
| | Non-normalized | | Normalized | Non-normalized | | Normalized |
| K | 0.1 | 1/s | 3.8 | 1.5 | 1/s | 49.0 |
| $T_1$ | -304.4 | s | -9.1 | -3470.0 | s | -103.8 |
| $T_2$ | 13.4 | s | 0.4 | 25.5 | s | 0.8 |
| $T_3$ | 33.4 | s | 1.0 | 55.2 | s | 1.7 |

and with v smoothed

| | Initial estimate | | | Final estimate | | |
|---|---|---|---|---|---|---|
| | Non-normalized | | Normalized | Non-normalized | | Normalized |
| K | 0.1 | 1/s | 3.8 | 0.45 | 1/s | 15.2 |
| $T_1$ | -304.4 | s | -9.1 | -1142.4 | s | -34.2 |
| $T_2$ | 13.4 | s | 0.4 | 25.8 | s | 0.8 |
| $T_3$ | 33.4 | s | 1.0 | 63.5 | s | 1.9 |

In contrast to the estimation of K, $T_1$, $T_2$ and $T_3$ without using the sway velocity K is positive and $T_1$ is negative, i e the ship is unstable. It thus appears that more information than only heading and rudder angle is needed when the components of the transfer function 8.2 are to be estimated. During a zig-zag test, however, the non-linear forces are stabilizing and a linear analysis will therefore give a more stable ship than really is the case.

Table 8.1 shows that the estimation of $Y\delta$ and $N\delta$ was successful, however, the rest of the components seem too small. The reason for this may be that the non-linear effects are in evidence during a $10^{\circ}/10^{\circ}$ zig-zag test.

There is no crucial difference between the results of the two estimations, i e it is possible to perform the estimation with the sway velocity unsmoothed. Consequently it is not convenient for this purpose to smooth the sway velocity, as in doing so the results of the estimation might be influenced incorrectly.

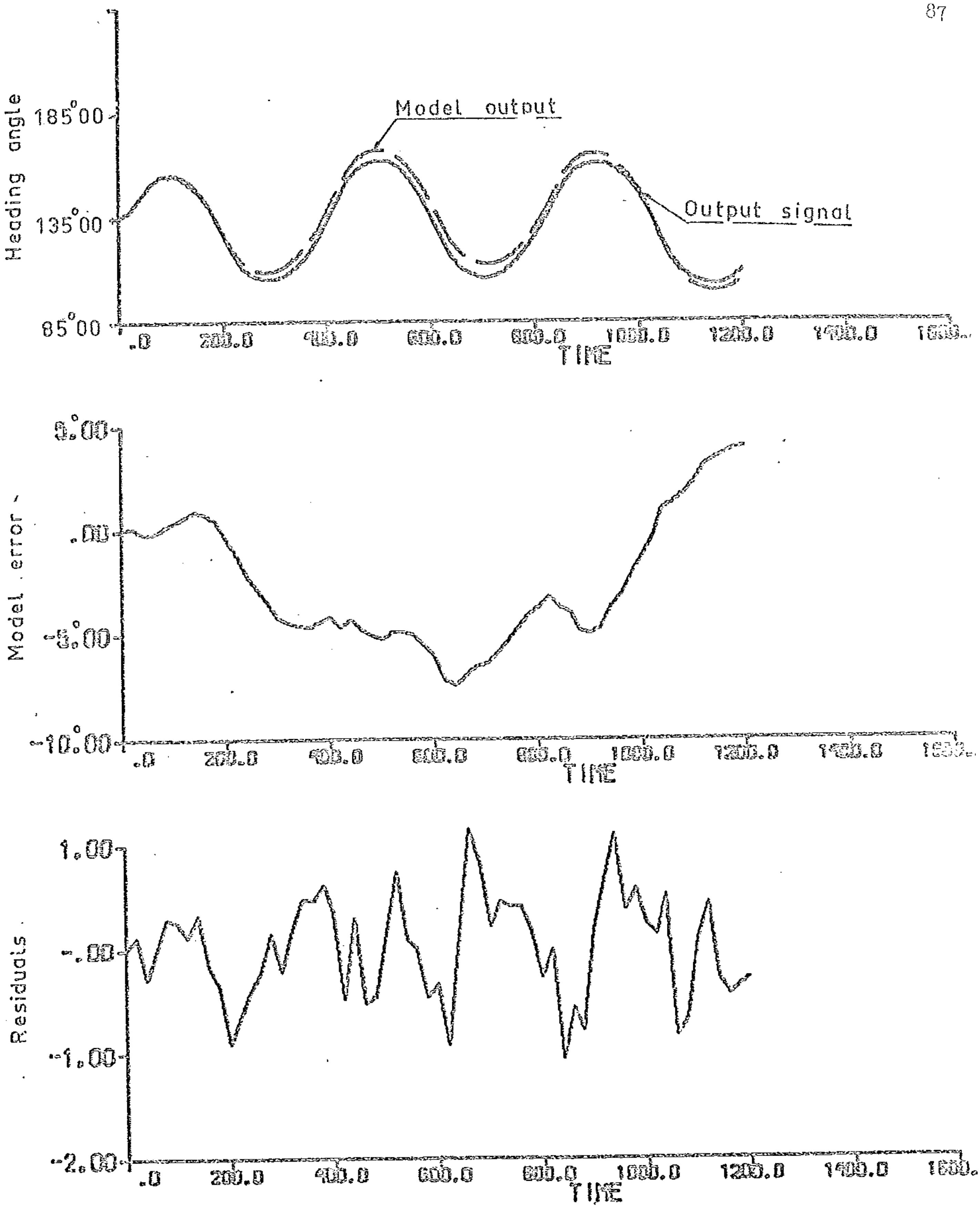



Fig 8.3 Results of the identification of the components of the equations of motion (A K Fernström)

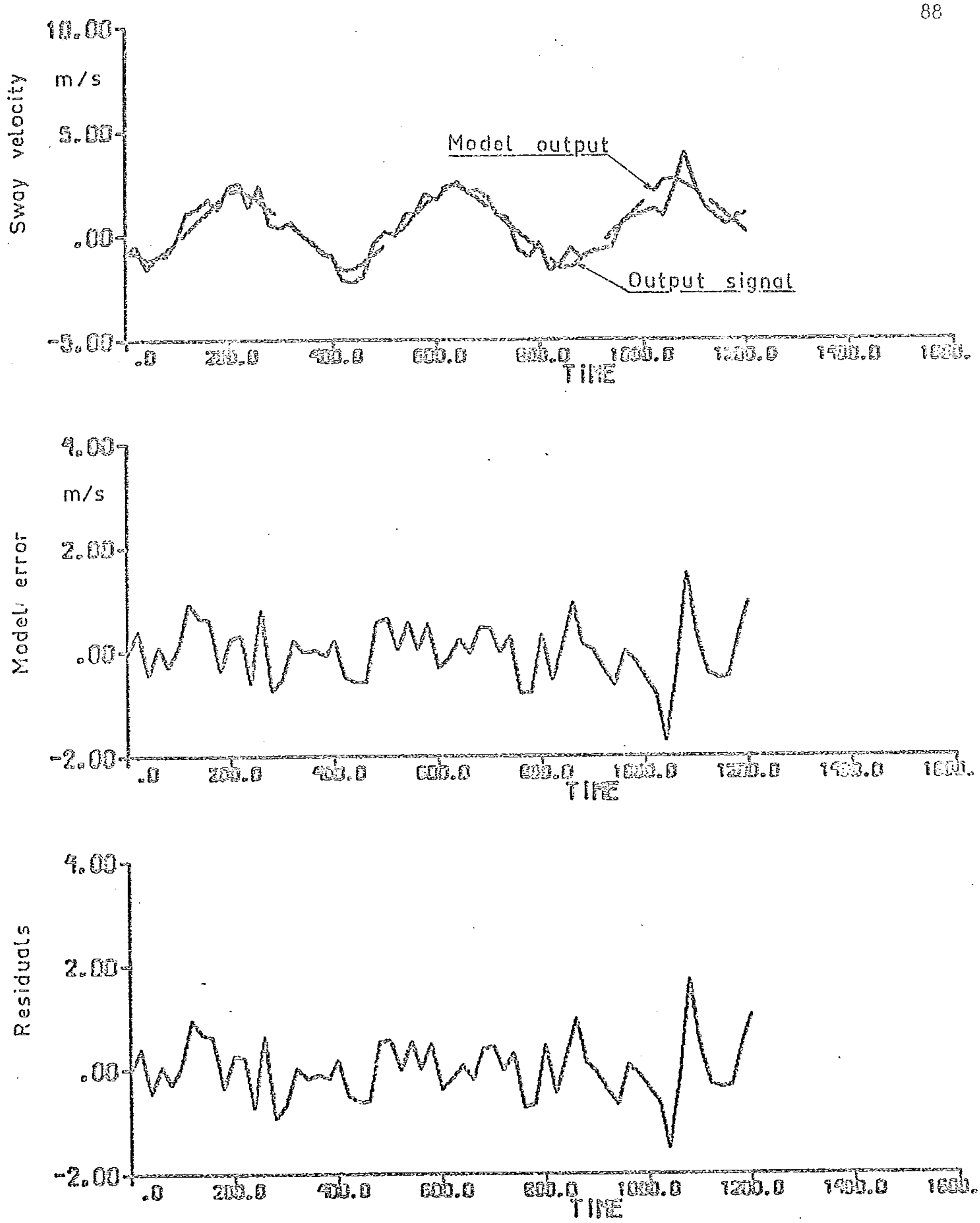



Fig 8.4 Results of the identification of the components of the equations of motion (A K Fernström)

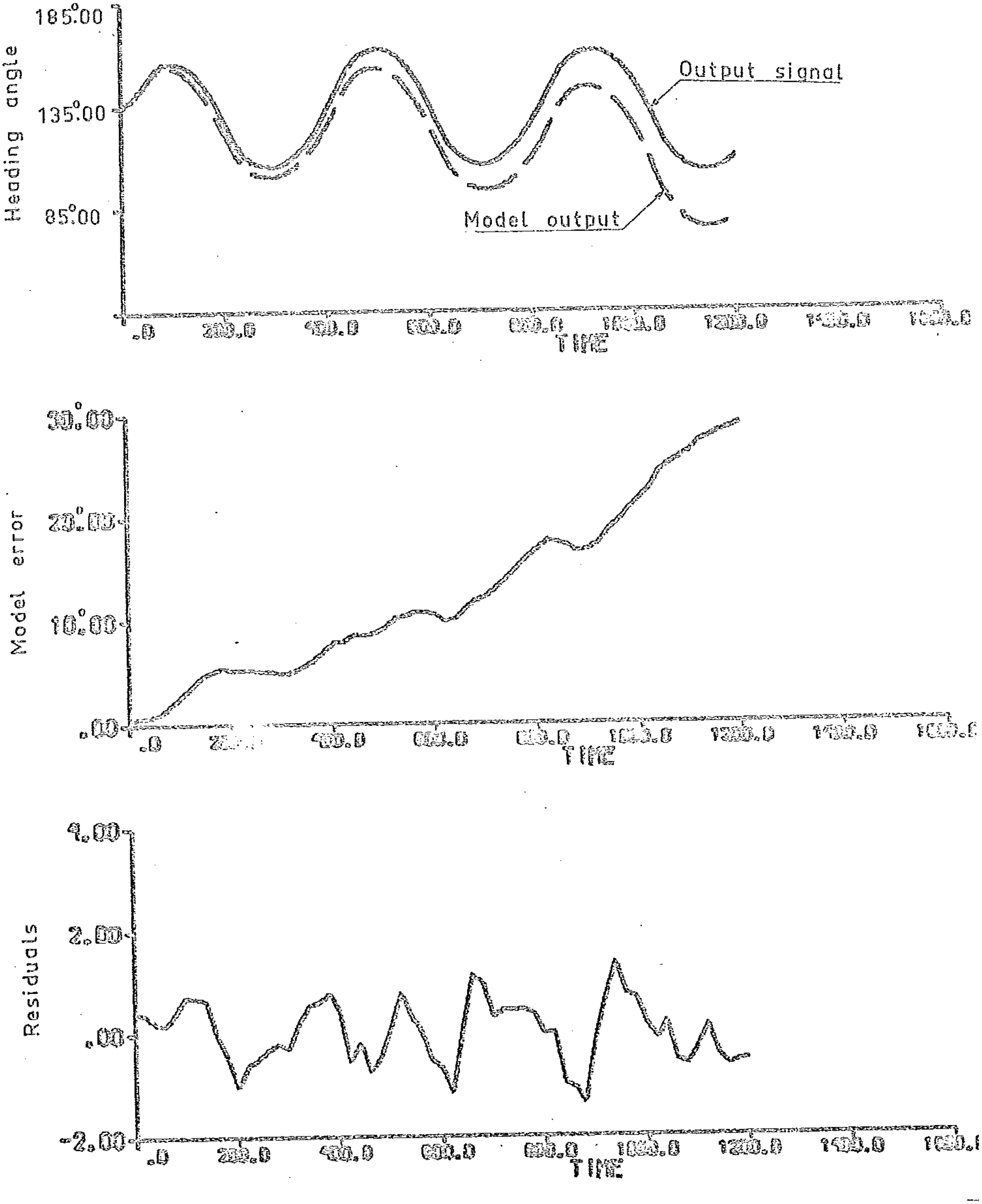



Fig 8.5 Results of the identification of the components of the equations of motion for the A K Fernström. (The sway velocity smoothed)

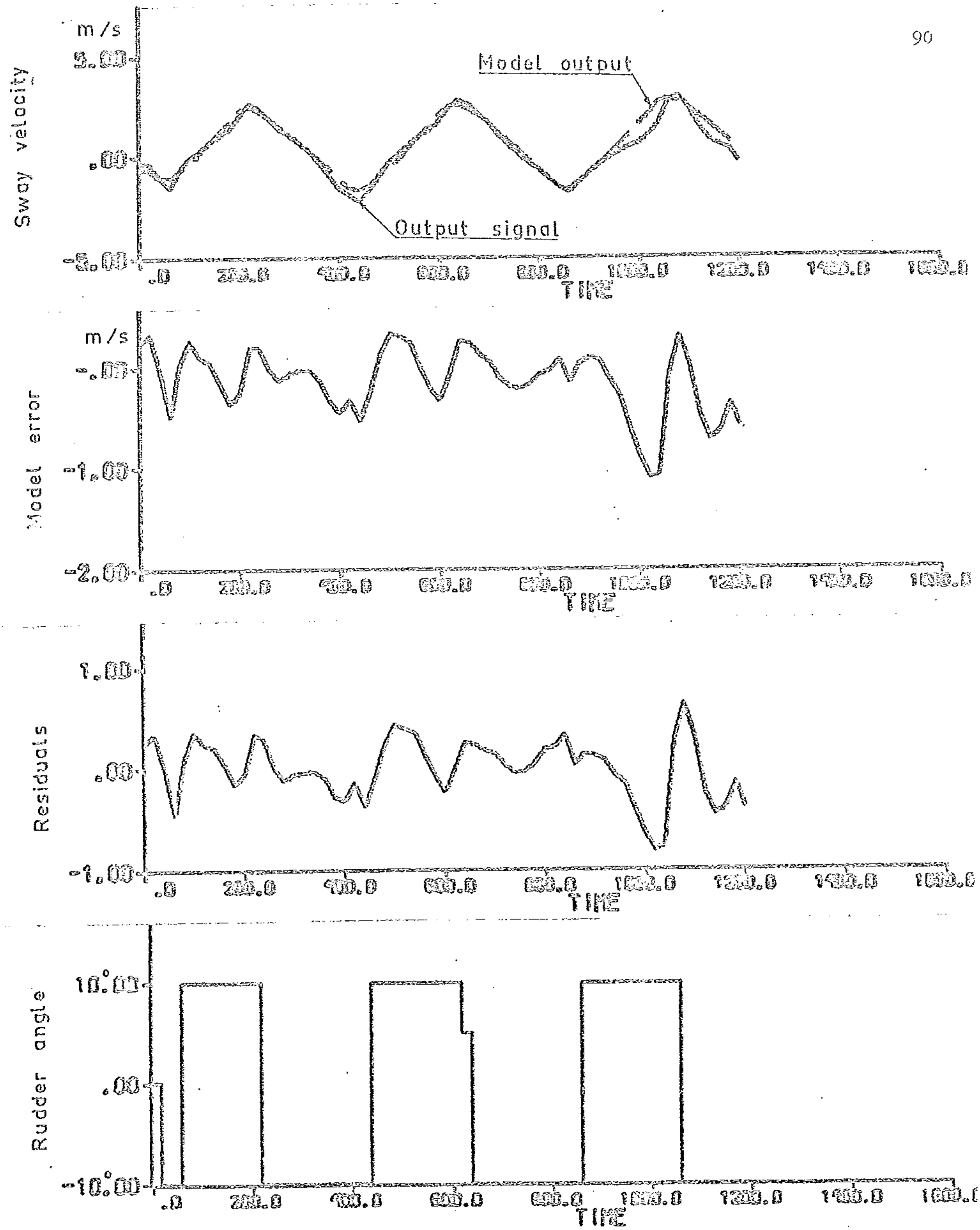



Fig 8.6 Results of the identification of the components of the equations of motion for the A K Fernström. (The sway velocity smoothed)

| Components | "Prime" system $\times 10^2$ | | | "Bis" system | | |
|---|---|---|---|---|---|---|
| $m - Y_{\dot{v}}$ | 3.23 | | | 2.00 | | |
| $mx_G - Y_{\dot{r}}$ | 0.10 | | | 0.06 | | |
| $mx_G - N_{\dot{v}}$ | 0.09 | | | 0.06 | | |
| $I_z - N_{\dot{r}}$ | 0.19 | | | 0.12 | | |
| | Initial est $\times 10^2$ | Final est $\times 10^2$ | Final est v smoothed $\times 10^2$ | Initial est | Final est | Final est v smoothed |
| $Y_{uv}$ | −1.94 | −1.40 | −1.16 | −1.20 | −0.87 | −0.72 |
| $Y_{ur} - m$ | −1.2 | −0.92 | −0.87 | −0.75 | −0.57 | −0.54 |
| $N_{uv}$ | −0.72 | −0.31 | −0.29 | −0.45 | −0.19 | −0.18 |
| $N_{ur} - mx_G$ | −0.37 | −0.19 | −0.21 | −0.23 | −0.12 | −0.13 |
| $Y_{cc\delta}$ | 0.37 | 0.38 | 0.42 | 0.23 | 0.24 | 0.26 |
| $N_{cc\delta}$ | −0.17 | −0.17 | −0.17 | −0.11 | −0.11 | −0.12 |

Table 8.1 Estimated hydrodynamic derivatives from identification of model 1 (see Appendix B) to the A K Fernström data

# 9. THE SEA SPLENDOUR EXPERIMENT

The Sea Splendour is a tanker built for the Salén group by Kockums Mekaniska Verkstads AB in Malmö, Sweden. It is 329 m long, has a beam of 52 m and weighs 255 000 tons. The cargo capacity is 339 000 $m^3$, and the maximum speed is 16 knots.

The experiments were performed on Sunday, June 4, 1972, north of Stavanger, Norway. The course was $140^o$ and the wind was blowing on starboard with a speed of about 10 m/s. The tanker had a ballast corresponding to 50% of the full capacity. The forward draught was about 10 m, and the aft draught was about 13 m during the experiment.

The captain of the ship was Mr. Andersson, and the experiment was carried out by two engineers from Kockums, Mr. J. Eriksson and Mr. N. E. Thorell. During the experiment, which lasted for about 50 minutes, the speed was between 15.5 and 16.0 knots. In the middle of the experiment the course was changed with $20^o$. The sampling interval was 30 s. The input signal was chosen as a PRBS signal, but it was necessary to make a lot of manual changes to avoid large deviations from the desired course. At every sample instant the process computer measured rudder deflection, course, yaw rate, forward velocity, bow and stern sway velocities, and printed them on a typewriter. The course was measured by a Sperry gyro compass, the yaw rate by a rate gyro manufactured by AB ATEW, Sweden, and the velocities by a doppler log, type Ametek Straza. The input-output data obtained during the experiment is shown in Fig. 9.1. Eight consecutive readings were missed during the experiment.

Attempts to estimate the parameters of the model 1, (see Appendix B), failed when all data points were used. This was probably due to nonlinear effects during the course change in the middle of the experiment. Therefore, the experiment was divided into two parts, one before the yaw and one after. The parameters of the model 1 were then

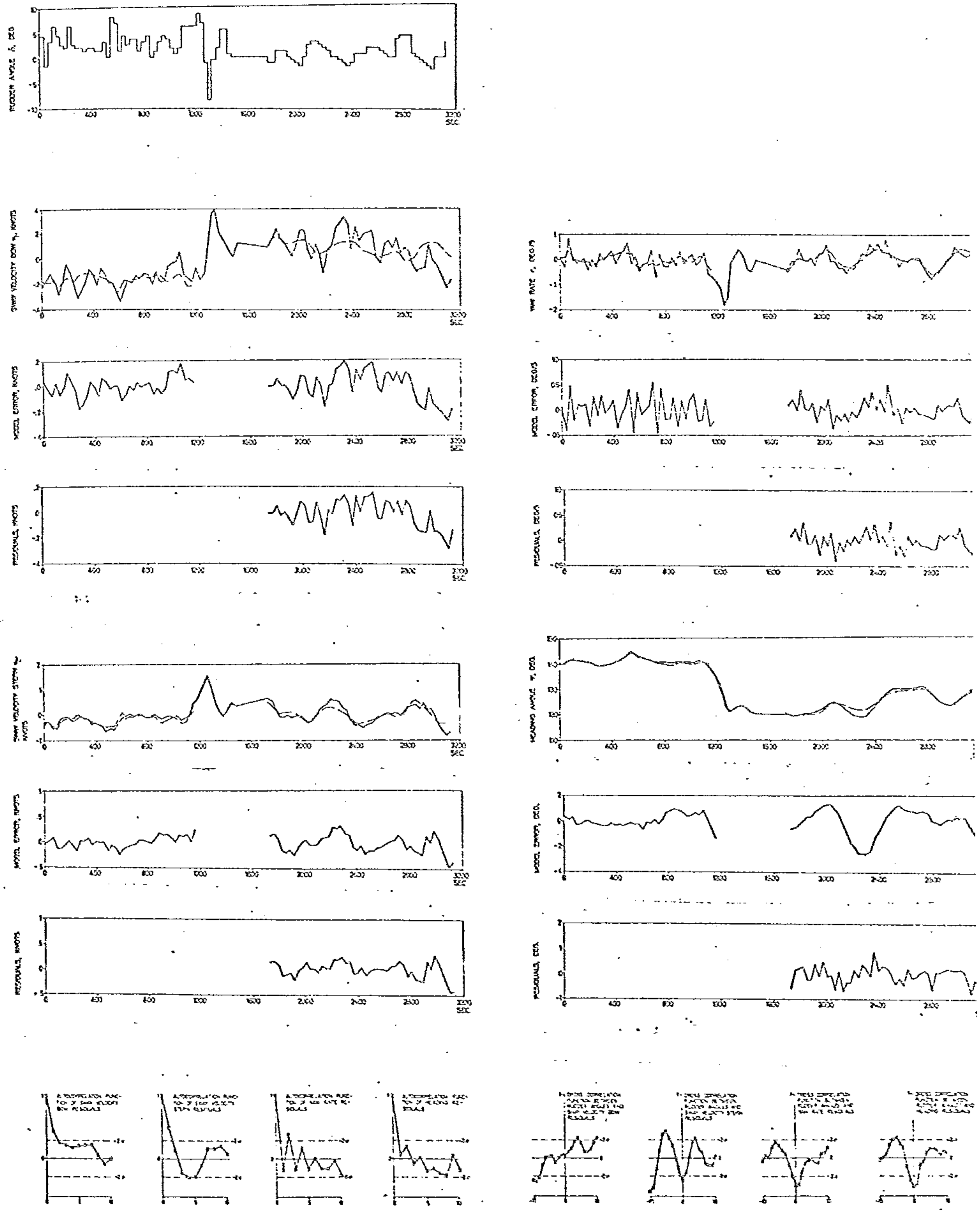


Fig. 9.1. Results of identification of the model 1 (see Appendix B) to the second part of the Sea Splendour data. Simulation of the model to the first part of the rudder angle signal is also shown. The dashed lines are model outputs.

estimated using the second part of the data set. The model obtained was also simulated with the rudder angles from the first part of the data set as input signal. Results of both the identification and the simulation are shown in Fig. 9.1. The parameter values obtained from the identification are given in Tables 9.1, 9.2 and 9.3. The values of the acceleration derivatives and the initial estimates of the other derivatives are adjusted values from model tests with a similar tanker.

| "Prime" system (mass unit $\frac{\rho}{2} L^3$) | | | "Bis System" | | |
|---|---|---|---|---|---|
| $m' - Y'_{\dot{v}}$ | 0.0156 | | $1 - Y''_{\dot{v}}$ | 1.70 | |
| $m' x'_G - Y'_{\dot{r}}$ | 0 | | $x''_G - Y''_{\dot{r}}$ | 0 | |
| $m' x'_G - N'_{\dot{v}}$ | 0 | | $x''_G - N''_{\dot{v}}$ | 0 | |
| $I'_z - N'_{\dot{r}}$ | 0.000963 | | $k''_{zz} - N''_{\dot{r}}$ | 0.105 | |
| | Initial estimates | Final estimates | | Initial estimates | Final estimates |
| $Y'_v$ | -0.0113 | -0.0146 | $Y''_{uv}$ | -1.23 | -1.59 |
| $Y'_r - m'$ | -0.00482 | -0.00565 | $Y''_{ur} - 1$ | -0.525 | -0.616 |
| $N'_v$ | -0.00183 | -0.00168 | $N''_{uv}$ | -0.200 | -0.183 |
| $N'_r - m' x'_G$ | -0.00238 | -0.00115 | $N''_{ur} - x''_G$ | -0.260 | -0.125 |
| $Y'_\delta$ | 0.00181 | 0.00183 | $\frac{1}{2} Y''_{cc\delta}$ | 0.197 | 0.199 |
| $N'_\delta$ | -0.00086 | -0.00060 | $\frac{1}{2} N''_{cc\delta}$ | -0.094 | -0.065 |

Table 9.1. Estimated hydrodynamic derivatives from identification of the model 1 (see Appendix B) to the second part of the Sea Splendour data.

| | | Initial estimates | Final estimates |
|---|---|---|---|
| $K'$ | | -0.72 | -1.63 |
| $K_v'$ | | 0.47 | 0.76 |
| $T_1'$ | | 2.30 | 3.87 |
| $T_2'$ | | 0.36 | 0.54 |
| $T_3'$ | | 1.03 | 0.79 |
| $T_v'$ | | 0.21 | 0.32 |
| $K$ | 1/s | -0.018 | -0.040 |
| $K_v$ | m/s | 3.8 | 6.2 |
| $T_1$ | s | 92.5 | 155.6 |
| $T_2$ | s | 14.6 | 21.5 |
| $T_3$ | s | 41.5 | 31.9 |
| $T_v$ | s | 8.3 | 12.9 |

Table 9.2. Values of normalized ("prime" system) and non-normalized transfer function parameters (cf. (4.13) and (4.15)) computed from initial and final estimates in Table 9.1.

| | | | |
|---|---|---|---|
| Wind parameters | $\theta_9$ | | $-4.4\cdot10^{-2}$ |
| | $\theta_{10}$ | | $7.8\cdot10^{-2}$ |
| Bias parameters | $\theta_{13}$ | | $1.5\cdot10^{-2}$ |
| | $\theta_{14}$ | | $1.4\cdot10^{-4}$ |
| | $\theta_{15}$ | knots | $8.6\cdot10^{-1}$ |
| | $\theta_{16}$ | knots | $8.1\cdot10^{-1}$ |
| | $\theta_{17}$ | deg/s | $-5.2\cdot10^{-4}$ |
| Covariance matrices | $R_1(1,1)$ | | $3.3\cdot10^{-2}$ |
| | $R_1(1,2)$ | | $1.3\cdot10^{-2}$ |
| | $R_1(2,2)$ | | $5.4\cdot10^{-3}$ |
| | $\tilde{R}_2(1,1)$ | $(\text{knots})^2$ | $1.7\cdot10^{-7}$ |
| | $\tilde{R}_2(2,2)$ | $(\text{knots})^2$ | $1.5\cdot10^{-3}$ |
| | $\tilde{R}_2(3,3)$ | $(\text{deg/s})^2$ | $1.0\cdot10^{-5}$ |
| | $\tilde{R}_2(4,4)$ | $(\text{deg})^2$ | $1.0\cdot10^{-2}$ (fixed value) |
| Initial state | $\theta_{25}$ | knots | $-5.3\cdot10^{-1}$ |
| | $\theta_{26}$ | deg/s | $-4.4\cdot10^{-2}$ |
| | $\theta_{27}$ | deg | 121.0 |
| Time lag | $\tau$ | s | 6.2 |

Table 9.3. Parameter values from identification of the model 1 (see Appendix B) to the second part of the Sea Splendour data.

Experiments have also been carried out on two sister ships of the Sea Splendour, viz. the Sea Scout and the Sea Swift. The draught of the first tanker was about 10 m during the experiments, while the other, the Sea Swift, was fully loaded, i.e. the draught was about 20 m. These experiments, however, have not yet been analysed.

## 10. THE USS COMPASS ISLAND EXPERIMENT

The USS Compass Island is a converted merchant ship of the Mariner class. The length between the perpendiculars is 160.93 m, and the beam is 23.16 m. The forward draught was 6.86 m and the aft draught was 8.08 m during the experiment, i.e. a mean draught of 7.47 m and a displacement of 16 650 $m^3$. The approach speed was 10 knots, but the speed was descreased to 8.5-9.0 knots during the experiment, which was a $20^\circ/20^\circ$ zig-zag test performed in calm sea. The experiment is described in Morse and Price (1961), where more information is available. Recorded rudder angles, sway velocities, yaw rates and heading angles are shown in Fig. 10.1. The sampling interval is 6 s, but some measurements are missing. The experiment lasted for about 10 minutes.

The values of the acceleration derivatives and the initial estimates of the other derivatives are obtained from the planar motion mechanism tests performed by the Hydro- and Aerodynamics Laboratory (HyA), Denmark. The tests are described in Chislett and Strøm-Tejsen (1965). The numerical values of the hydrodynamic derivatives are corrected to $x_G = 0$. It should be pointed out that HyA:s model tests were performed at a speed corresponding to 15 knots. Results of other model tests of Mariner class vessels are summarized in Comstock (1967) and Motora (1972).

The first attempt was to estimate the parameters of the model 2 (see Appendix B) and determine the transfer function (4.13) relating the sway velocity to the rudder angle. The measurements of the yaw rates and the heading angles were then not used. The result is given in Table 10.1.

| | | HyA:s model | Identified model |
|---|---|---|---|
| $K_v'$ | | 2.01 | 0.98 |
| $T_1'$ | | 5.70 | 1.75 |
| $T_2'$ | | 0.37 | 0.55 |
| $T_v'$ | | 0.22 | 0.18 |
| $K_v$ | m/s | 9.0 | 4.4 |
| $T_1$ | s | 204.0 | 62.6 |
| $T_2$ | s | 13.3 | 19.5 |
| $T_v$ | s | 7.8 | 6.6 |
| $\tau$ | s | - | 5.4 |

Table 10.1. Result of identification of the model 2 (see Appendix B) to the USS Compass Island data from $20^0/20^0$ zig-zag test. The values of the transfer function parameters (cf. (4.13)) are given normalized ("prime" system) and non-normalized.

The parameters of the model 1 (see Appendix B) have also been estimated. The sway velocity, the yaw rate and the heading angle were then used as output signals from the model. Results of the identification are shown in Fig. 10.1. The parameter values obtained are given in Tables 10.2, 10.3 and 10.4.

The speed dependence of the hydrodynamic derivatives of a 190 000 dwt tanker have been investigated by model tests at HyA (see Smitt and Chislett (1973)). If it is assumed that these results are applicable to the USS Compass Island, the absolute values of $Y_v'$, $N_v'$ and $N_r' - m' x_G'$ of HyA:s model (see Table 10.2) should be decreased with about 5% to be adapted to the speed 8.5-9.0 knots.

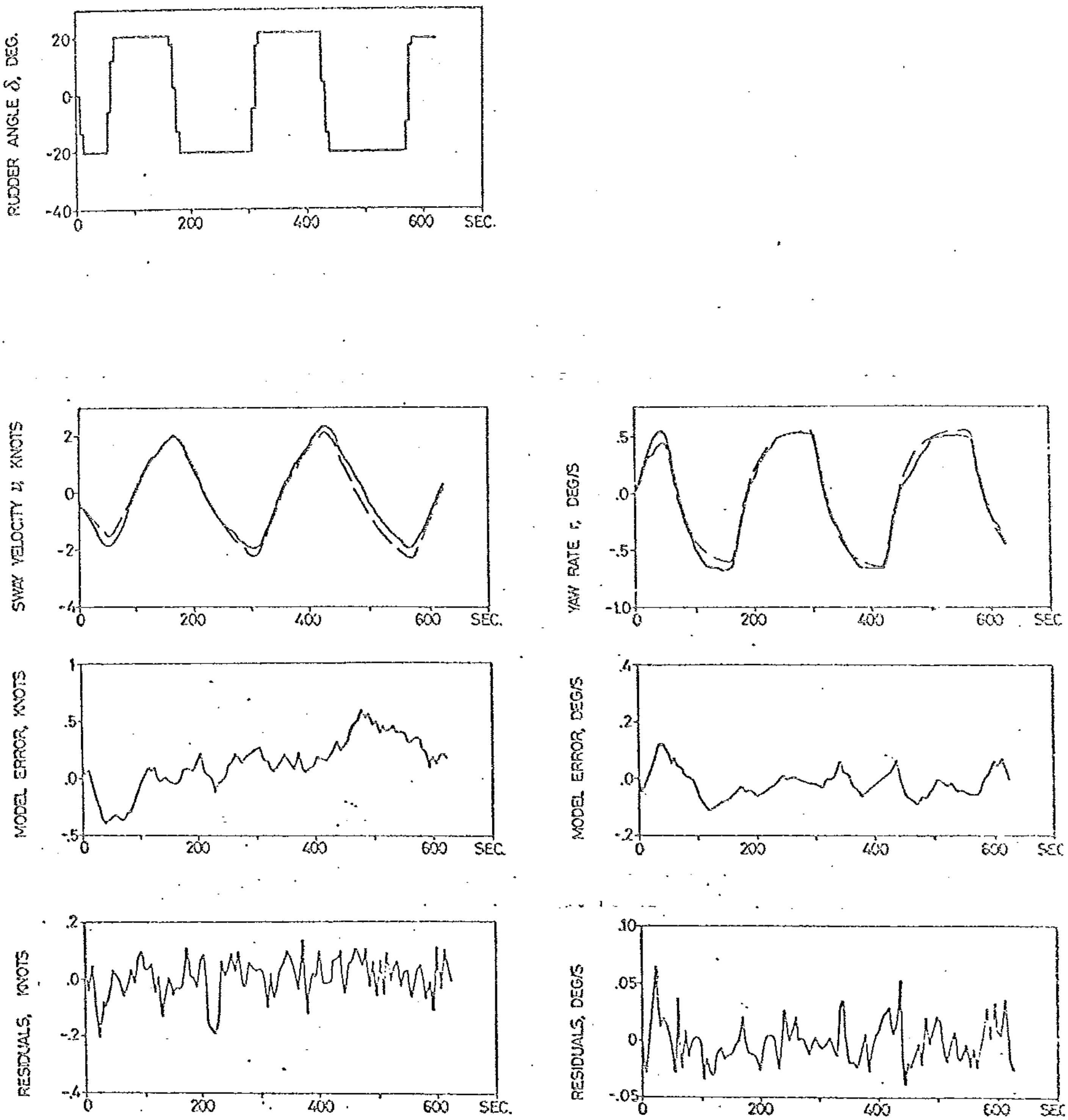



Fig. 10.1. Results of identification of the model 1 (see Appendix B) to the USS Compass Island data from $20^\circ/20^\circ$ zig-zag test. The dashed lines are the model outputs.

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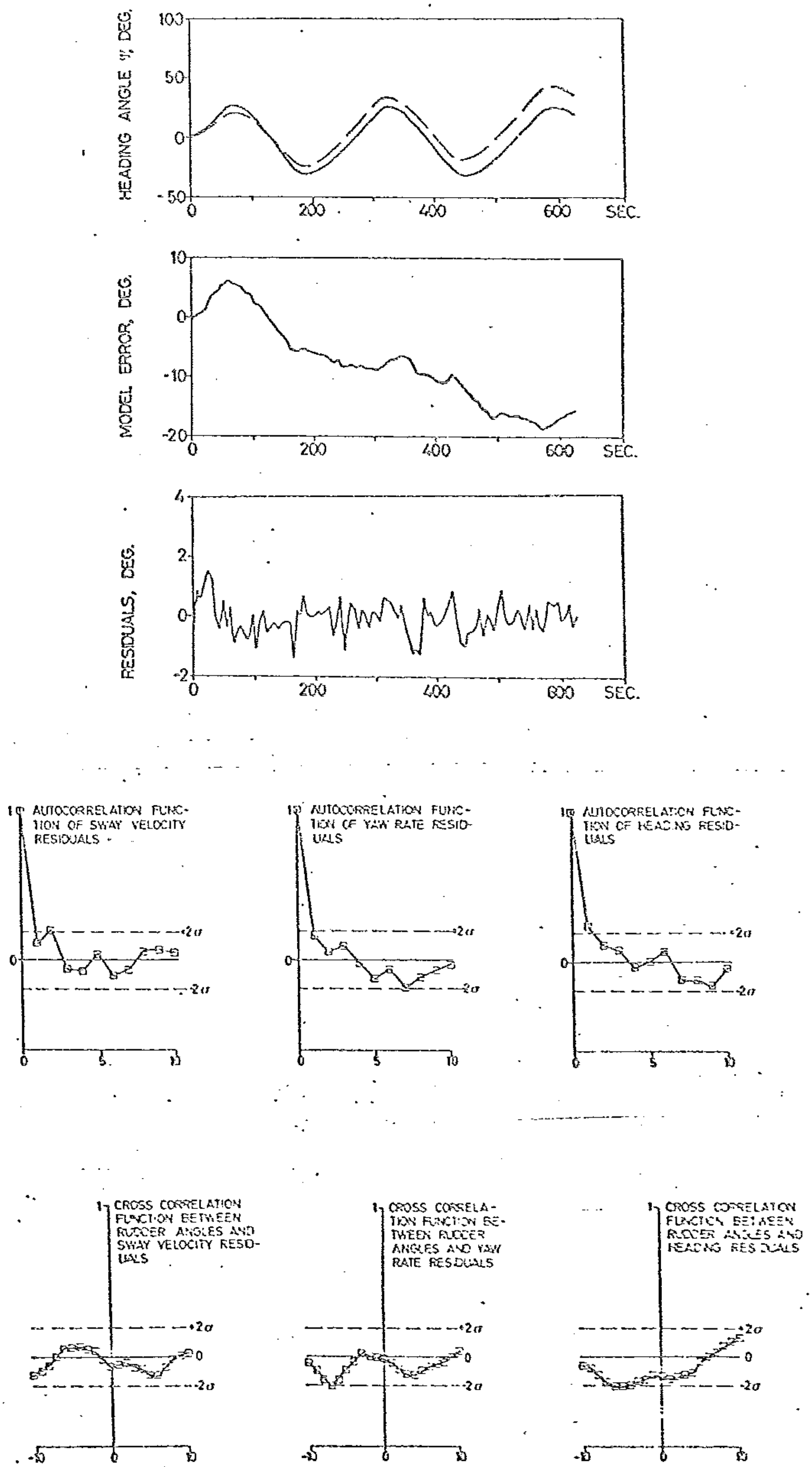


See text on the proceeding page.

| "Prime" system (mass unit $\frac{\rho}{2} L^3$) | | | "Bis" system | | |
|---|---|---|---|---|---|
| $m' - Y'_{\dot{v}}$ | 0.01546 | | $1 - Y''_{\dot{v}}$ | 1.935 | |
| $m' x'_G - Y'_{\dot{r}}$ | 0.00026 | | $x''_G - Y''_{\dot{r}}$ | 0.033 | |
| $m' x'_G - N'_{\dot{v}}$ | 0.00012 | | $x''_G - N''_{\dot{v}}$ | 0.015 | |
| $I'_z - N'_{\dot{r}}$ | 0.00083 | | $k''_{zz} - N''_{\dot{r}}$ | 0.104 | |
| | Initial estimates (HyA:s model) | Final estimates | | Initial estimates (HyA:S model) | Final estimates |
| $Y'_v$ | -0.01160 | -0.00883 | $Y''_{uv}$ | -1.452 | -1.105 |
| $Y'_r - m'$ | -0.00526 | -0.00446 | $Y''_{ur} - 1$ | -0.658 | -0.558 |
| $N'_v$ | -0.00291 | -0.00003 | $N''_{uv}$ | -0.364 | -0.004 |
| $N'_r - m' x'_G$ | -0.00184 | -0.00103 | $N''_{ur} - x''_G$ | -0.230 | -0.129 |
| $Y'_\delta$ | 0.00278 | 0.00435 | $\frac{1}{2} Y''_{cc\delta}$ | 0.348 | 0.544 |
| $N'_\delta$ | -0.00133 | -0.00104 | $\frac{1}{2} N''_{cc\delta}$ | -0.166 | -0.130 |

Table 10.2. Estimated hydrodynamic derivatives from identification of the model 1 (see Appendix B) to the USS Compass Island data from $20^o/20^o$ zig-zag test.

| | | Initial estimates (HyA:s model) | Final estimates |
|---|---|---|---|
| $K'$ | | -3.90 | -1.04 |
| $K_v'$ | | 2.01 | 1.02 |
| $T_1'$ | | 5.70 | 1.70 |
| $T_2'$ | | 0.37 | 0.84 |
| $T_3'$ | | 0.89 | 1.78 |
| $T_v'$ | | 0.22 | 0.43 |
| $K$ | 1/s | -0.109 | -0.029 |
| $K_v$ | m/s | 9.0 | 4.6 |
| $T_1$ | s | 204.0 | 60.6 |
| $T_2$ | s | 13.3 | 30.1 |
| $T_3$ | s | 31.8 | 63.6 |
| $T_v$ | s | 7.8 | 15.2 |

Table 10.3. Values of normalized ("prime" system) and non-normalized transfer function parameters (cf. (4.13) and (4.15)) computed from initial and final estimates in Table 10.2.

| | | | |
|---|---|---|---|
| Bias parameters | $\theta_{13}$ | | $-3.2\cdot10^{-3}$ |
| | $\theta_{14}$ | | $-1.3\cdot10^{-5}$ |
| | $\theta_{15}$ | knots | 3.2 |
| | $\theta_{17}$ | deg/s | $-3.1\cdot10^{-2}$ |
| Covariance matrices | $R_1(1,1)$ | | $5.9\cdot10^{-6}$ |
| | $R_1(1,2)$ | | $-3.5\cdot10^{-6}$ |
| | $R_1(2,2)$ | | $1.6\cdot10^{-5}$ |
| | $\tilde{R}_2(1,1)$ | $(\text{knots})^2$ | $4.2\cdot10^{-4}$ |
| | $\tilde{R}_2(3,3)$ | $(\text{deg/s})^2$ | $7.7\cdot10^{-3}$ |
| | $\tilde{R}_2(4.4)$ | $(\text{deg})^2$ | $1.0\cdot10^{-2}$ (fixed value) |
| Initial state | $\theta_{25}$ | knots | -3.7 |
| | $\theta_{26}$ | deg/s | $5.6\cdot10^{-2}$ |
| | $\theta_{27}$ | deg | $5.4\cdot10^{-1}$ |
| Time lag | $\tau$ | s | 5.3 |

Table 10.4. Parameter values from identification of the model 1 (see Appendix B) to the USS Compass Island data from $20^\circ/20^\circ$ zig-zag test.

Typical plots of N versus v and N versus r, adopted from Comstock (1967), are shown in Fig. 10.2 and 10.3. The nonlinear regions of these plots are probably reached during the $20^\circ/20^\circ$ zig-zag test. This observation, together with the earlier discussed speed dependence, may explain the differences of the values of $N_v'$ and $N_r'-m'x_G'$ in Table 10.2.

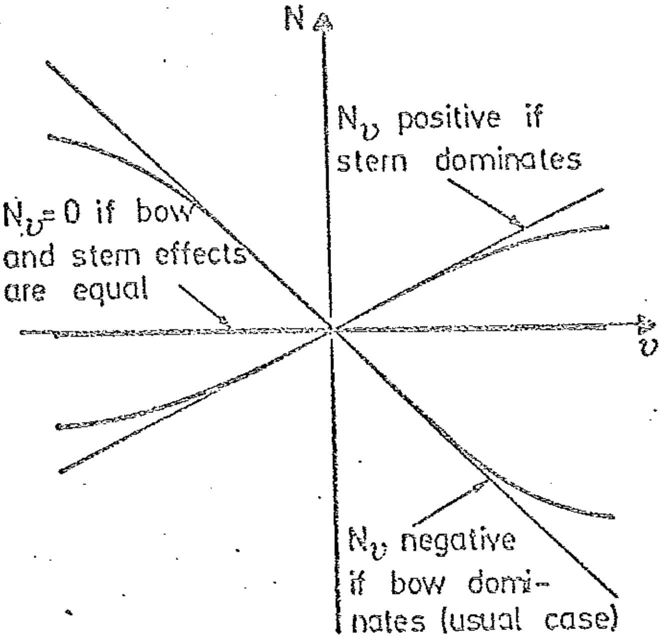


Fig. 10.2. Typical N versus v relationship.

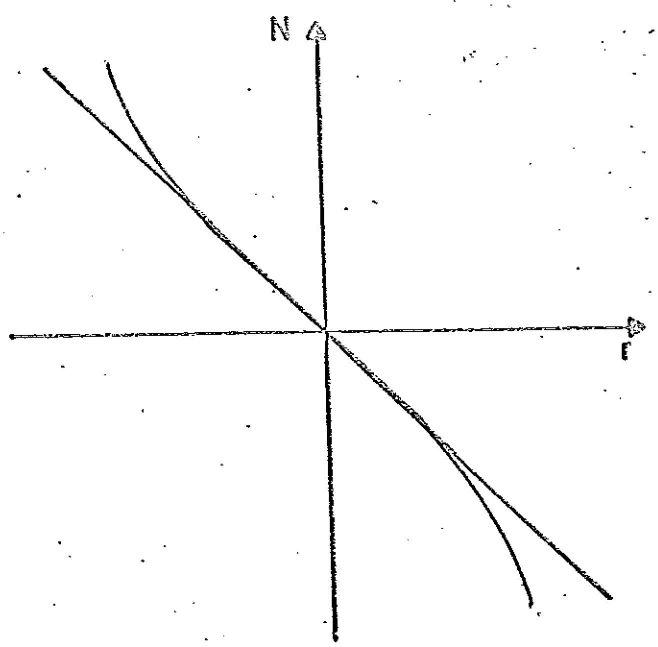


Fig. 10.3. Typical N versus r relationship.

## 11. CONCLUSIONS

To furnish meaningful results it is necessary that steering tests with scale models or ships are analyzed with respect to numerical values of the coefficients of the equations of motion. It is also desirable to have access to a stochastic method of analysis, which can be applied to experiments involving "small motions", i e to true linear manoeuvres, which can be performed in routine service, but which may then also suffer from a fairly large noise-to-signal ratio.

It has been attempted here to apply the maximum likelihood technique of system identification to determine linear ship dynamics. A special computer software system has been written to carry out the identification: A flexible system identification package (LISPID) has been developed and set up for terminal use at the computing centre at the University of Lund. This has proved to be a very versatile way of communication between SSPA and LTH. A method for converting Decca coordinates to sway velocities (DECCON) has also been developed.

The method has been applied to tests on several ships, i e

- A large freighter, the Atlantic Song
- A passenger ferry, the Bore I
- Two tankers, the A K Fernström and the Sea Splendour
- A Mariner class ship, the USS Compass Island

Calculations have been performed on two widely different types of experiments

- Standard type zig-zag tests ($10^{\circ}/10^{\circ}$ and $20^{\circ}/20^{\circ}$)
- Specially designed experiments with PRBS rudder motion

The experimental conditions varied significantly. In some cases the input signal was introduced manually, in other cases the whole experiment was computer controlled. The measurements made were also varied over a large range from "manual" reading of rudder and heading angle only to computer controlled sensing of rudder angle, heading angle, yaw rate and sway velocities. Direct measurement of sway velocity by doppler sonar and estimates of sway velocity by the use of the DECCON program have been attempted.

A theoretical study has confirmed that the transfer function relating heading to rudder angle can be determined by measurements of these two variables only. However, if the hydrodynamic derivatives are desired, then it is necessary to have 1) a priori knowledge or estimate of the acceleration derivatives and 2) information on the sway velocity.

The results indicate clearly that stochastic system identification techniques provide a powerful tool for determining linear ship dynamics. Thus, e g, the results from the Atlantic Song experiment show that for a stable ship with a dominating time constant of about 30 s the second order transfer function - corresponding to the "first-order theory" - can be determined from an experiment with a duration of 20 minutes (40 time constants) even under adverse weather conditions. It is known, of course, that this transfer function can alternatively be obtained from a zig-zag test by the use of a deterministic method, but that approach will be more sensitive to weather conditions.

From the Sea Splendour experiment, where a proper PRBS rudder motion was used, there is good agreement between the a priori estimates based on model tests for a similar tanker and the results of the system identification. The agreement is not so good when a zig-zag test is used. The result of the identification of the USS Compass Island data from a $20^{\circ}/20^{\circ}$ zig-zag test shows that particularly the derivatives $N'_v$ and $N'_r$ are badly underestimated, probably due to non-linear effects. The same tendency, but not so distinct, can be seen from the result of an identification applied to the A K Fernström $10^{\circ}/10^{\circ}$ data.

Again the results indicate that more accurate estimates can be obtained if an experiment with PRBS rudder motion is used in the place of a standard zig-zag test. The zig-zag test has its own value as a comparative test, of course. If a zig-zag test has to be used for the identification purpose the $10^{\circ}/10^{\circ}$ test is to be preferred to the $20^{\circ}/20^{\circ}$ test; a $5^{\circ}/5^{\circ}$ test would be even better. Unfortunately the standard zig-zag tests do not usually cover as long a time as could be desired for this kind of analysis.

One problem met with when using a PRBS as input signal is that of large deviations from the desired course, which are often obtained if adjustments of the rudder angle are not made. This problem can be solved, however, by a feedback from the heading error to the rudder angle added to the PRBS signal. (Identification of closed loop systems is treated in Gustavsson et al (1974).)

Within the limited scope of this study there was no possibility to make special tests using proper experimental design and appropriate measurements, but existing equipment had to be used. A careful analysis of the data also shows effects due to imperfect measurement, such as quantization etc. In spite of this, the results obtained are quite promising. In more recent experiments improvements have been made in the data recording. It has not been possible to analyse these new experiments within the limited scope of the project.

During the course of this project very good and efficient working relations have been established between SSPA and LTH, which will now be further exploited. There are several areas which appear to be interesting and worthwhile continuations:

- Investigation of methods for identification of non-linear systems.
- Investigation of efficient experimental procedures.
- Design of proper equipment for measurement and data analysis.
- Comparison of results of model tests and tests of full scale ships.

The calculations performed so far give evidence to the fact that the non-linear effects must be considered in most cases. Non-linear effects are clearly noticeable in the zig-zag experiments and in the $20^{o}$ turn covered by the Sea Splendour experiment. Although the non-linear terms are important in many problems of design and prediction the non-linear analysis is in general very difficult. Hopefully, however, special methods can be used for the particular equations describing the ship dynamics.

The measurements are the key factor for the results. It is therefore extremely important to give specifications of proper measuring equipment. If the methods presented here are to be used in a routine manner it is highly desirable to have portable box of equipment with proper measuring instruments and a computer with appropriate software, designed so that it can easily be used onboard any ship.

## 12. ACKNOWLEDGEMENT

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The diagrams have been drawn by Miss B-M Carlsson (LTH) and Miss A-L Strömberg (SSPA), and the paper has been expertly typed by Mrs M Moore (LTH) and Mrs B Karsberg (SSPA).

## 13. REFERENCES

Abkowitz, M A (1964), "Lectures on Ship Hydrodynamics - Steering and Manoeuvrability", HyA Report No Hy-5

Akaike, H (1972), "Use of an Information Theoretic Quantity for Statistical Model Identification", Proc 5th Hawaii International Conference on System Sciences, Western Periodicals Co

Åström, K J and Bohlin, T (1965), "Numerical Identification of Linear Dynamic Systems from Normal Operating Records", Proc IFAC Symp Theory on Self-Adaptive Control Systems, Teddington, England

Åström, K J (1970), "Introduction to Stochastic Control Theory", Academic Press, New York

Åström, K J and Eykhoff, P (1971), "System Identification - A Survey", Automatica 7, p 123

Åström, K J and Källström, C (1972), "Identification and Modelling of Ship Dynamics", Report 7202. Div of Automatic Control, Lund Inst of Techn

Åström, K J and Källström, C (1973), "Application of System Identification Techniques to the Determination of Ship Dynamics", Proc 3rd IFAC Symp on Identification and System Parameter Estimation, the Hague/Delft, the Netherlands

Bech, M and Smitt, L Wagner (1969), "Analogue Simulation of Ship Manoeuvres Based on Full Scale Trials on Free-Sailing Model Tests", HyA Report No Hy-14 van Berlekom, W B, Trägårdh, P and Dellhag, A (1974), "Large Tankers - Wind Coefficients and Speed Loss Due to Wind and Sea", Paper presented at RINA Spring Meeting, London

Bishop, R E D, Burcher, R K and Price, W G (1973), "The Use of Functional Analysis in Ship Dynamics", Proc Roy Soc Series A, Vol 332

Brent, R P (1973), "Algorithms for Minimization without Derivatives", Prentice Hall, Englewood Cliffs, New York

Burcher, R K (1972), "Further Analysis of Model-Ship Correlation Data", Appendix III to Manoeuvrability Committee Report, Proc 13th ITTC, Berlin-Hamburg

Chislett, M S and Strøm-Tejsen, J (1965), "Planar Motion Mechanism Tests and Full-Scale Steering and Manoeuvring Predictions for a MARINER Class Vessel", Report Hy-6, HyA, Lyngby, Denmark

Comstock, J P (1967), "Principles of NAVAL ARCHITECTURE", SNAME, New York

Davidson, K S M and Schiff, L (1946), "Turning and Course-Keeping Qualities", Trans SNAME

The Decca Navigator Company Ltd (1949), "Field Computation of Decca Patterns", London

The Decca Navigator Company Ltd (1959), Report on the "Omnitrac" System, London

The Decca Navigator Company Ltd (1965), "The Decca Navigator System as an Aid to Survey", Surrey, England

Eda, H and Crane, C L Jr (1965), "Steering Characteristics of Ships in Calm Water and Waves", Trans SNAME, Vol 73

Fletcher, R (1972), "FORTRAN Subroutines for Minimization by Quasi-Newton Methods", Report AERE-R7125, Harwell

Goodman, A (1960), "Experimental Techniques and Methods of Analysis used in Submerged Body Research", Proc Third Symposium on Naval Hydrodynamics, Scheveningen

Guest, P G (1961), "Numerical Methods of Curve Fitting", Cambridge University Press, London

Gustavsson, I, Selander, S and Wieslander, J (1973), "IDPAC User's Guide", Report 7331. Division of Automatic Control, Lund Inst of Techn

Gustavsson, I, Ljung, L and Söderström, T (1974), "Identification of Linear, Multivariable Process Dynamics Using Closed Loop Experiments", Report 7401. Div of Automatic Control, Lund Inst of Techn

Kraemer, O (1934), "Richtlinien für Steuerversuche", Werft-Reederei-Hafen, Heft 1, 1934

Morse, R V and Price, D (1961), "Maneuvering Characteristics of the Mariner Class Ship in Calm Seas", Sperry Gyroscope Company Report GJ-2233-1019

Motora, S (1972), "Maneuverability, State of the Art", International Jubilee Meeting on the Occasion of the 40th Anniversary of the Netherlands Ship Model Basin, Wageningen, the Netherlands

Nomoto, K et al (1957), "On the Steering Qualities of Ships", Intern Shipb Progr, Vol 4, No 35

Nomoto, K (1960), "Analysis of Kempf's Standard Manoeuvre Test and Proposed Steering Quality Indices", Proc First Symposium on Ship Maneuverability, Washington D C (DTMB Report 1461)

Norrbin, N H (1960), "A Study of Course Keeping and Manoeuvring Performance", SSPA Publ No 45, 1960, and Proc First Symposium on Ship Maneuverability, Washington D C 1960 (DTMB Report 1461, Oct 1960)

Norrbin, N H (1963), "On the Design and Analysis of the Zig Zag Test on Base of Quasi-Linear Frequency Response", Proc 10th ITTC, Vol II, London, 1963

Norrbin, N H (1965), "Zig Zag Test Technique and Analysis with Preliminary Statistical Results", BSRA Transl 2188 from Swedish original SSPA Allmän Rapport No 12, 1965

Norrbin, N H (1970), "Theory and Observations on the Use of a Mathematical Model for Ship Manoeuvring in Deep and Confined Waters", Proc Eighth Symposium on Naval Hydrodynamics, Pasadena, California, 1970, and SSPA Publ No 68, 1971

Smitt, L Wagner (1969), "The Reversed Spiral Test - a Note on Bech's Spiral Test and Some Unexpected Results of its Application to Coasters", HyA Report Hy-10

Smitt, L Wagner and Chislett, M S (1973), "Course Stability while Stopping", Report Hy-16, HyA, Lyngby, Denmark

Strøm-Tejsen, J and Chislett, M S (1966), "A Model Testing Technique and Method of Analysis for the Prediction of Steering and Manoeuvring Qualities of Surface Vessels", HyA Report Hy-7

# APPENDIX A

## LIST OF SYMBOLS

| | |
|---|---|
| AIC | Akaike's information criterion |
| A, B, C, D | System matrices |
| $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$ | Transformed system matrices |
| $C_x$, $C_y$, $C_N$ | Wind load coefficients (cf Fig 2.10) |
| D | Decometer value |
| $F_1$, $F_2$ | Wind and asymmetry force and moment |
| G, $G_1$, $G_2$ | Transfer functions |
| $H(\dot{\psi})$ | Non-linear function in steady-state $\dot{\psi}(\delta)$ characteristics |
| I | Mass moment of inertia about z-axis (also denoted $I_z$) |
| K | Static gain (spec in yaw equation; $K_s$ = "effective" K) |
| $K_v$ | Static gain in sway equation |
| K | Filter gain matrix |
| $\tilde{K}$ | Transformed filter gain matrix |
| L | Numerical lane number |
| L | Length of ship ($L = L_{pp}$) |
| $L_1$ | Distance from centre of mass to the forward doppler log transmitter |
| $L_2$ | Distance from centre of mass to the aft doppler log transmitter |
| L | Likelihood function |
| N | Number of sampling events minus one |
| N | Yawing moment around z-axis |
| P | Covariance matrix of state estimate errors |
| R | Radius of turning path ($R_c$, do in constant turn) |

| | |
|---|---|
| $R_0, R_1, R_{12}, R_2$ | Covariance matrices |
| $\tilde{R}_1, \tilde{R}_2$ | Transformed covariance matrices |
| $T$ | Sampling interval |
| $T$ | Time const (spec $T = T_1 + T_2 + T_3$; $T_s$ = "effective" T) |
| $T_1, T_2$ | Time constants in ship response |
| $T_3$ | Time constants of rudder motion in yaw equation |
| $T_{3v}$ | Do in sway equation |
| U1 | Artificial input signal consisting of only ones |
| $V, V_1, V_2, V_3$ | Loss functions |
| $V$ | Ship speed |
| $V^W$ | Wind speed ($V_R^W$ = relative wind speed) |
| $X, Y$ | Hydrodynamic forces along x- and y-axes |
| $a_1, a_2$ | Distance between Master and Slave |
| $a_i, b_i, c_i$ | $i = 1, \ldots, n$ Coefficients of transfer functions or pulse transfer functions |
| $a_{i,j}, b_{i,j}$ | $i = 1, 2$ $j = 1, 2, 3$ Coefficients of state space model |
| $e, w$ | Noise |
| $\tilde{e}, \tilde{w}$ | Transformed noise |
| $f_1, f_2$ | Normalized forms of $F_1$ and $F_2$ |
| $g$ | Acceleration of gravity |
| $h_0$ | Distance from a point to the Master |
| $h_1$ | Distance from a point to the Slave |
| $k_{zz}$ | Radius of gyration in yaw ($k'_{zz} = k_{zz}/L$) |
| $m$ | Mass of ship |
| $n$ | Rate of revs of propeller; r.p.s. = RPM/60 |

| | |
|---|---|
| $p$ | Probability density |
| $r$ | Angular velocity of yaw; spec $r = \dot{\psi}$ |
| $s$ | Variable of Laplace transforms |
| $t$ | Coordinate of time |
| $t_k$ | Discrete times ($k = 0, 1, \ldots, N$) |
| $u$, $v$ | Forward and lateral speed of ship through water |
| $u$ | Input |
| $v_1$ | Sway velocity of bow |
| $v_2$ | Sway velocity of stern |
| $x$, $y$, $z$ | Coordinates of system fixed in body |
| $x_0$, $y_0$, $z_0$ | Coordinates of space frame |
| $x$ | State vector |
| $\hat{x}$ | State estimate |
| $x_G$ | x-coordinate of centre of gravity |
| $y$ | Output |
| $\hat{y}$ | Estimated output |
| $y_{t_k}$ | Vector consisting of all outputs observed up to and including $t_k$ |
| $z$ | Output |
| $\theta$ | Parameter vector |
| $\theta_i$ | Elements of parameter vector |
| $\chi$ | Reference angle |
| $\alpha_1$ | Conversion factor from degrees to radians |
| $\alpha_2$ | Conversion factor from m/s to knots |
| $\beta$ | Angle of drift of side-slip; $\beta = -\arctan \frac{v}{u}$ |
| $\gamma$ | Wind inflow angle ($\gamma_R$ = relative wind angle) |

| | |
|---|---|
| $\delta$ | Rudder angle |
| $\varepsilon$ | Innovations or residuals |
| $\kappa$ | Factor in non-linear steady-state $\dot{\psi}(\delta)$ characteristics |
| $\lambda$ | Wave length |
| $\tau$ | Time lag |
| $\psi$ | Heading angle, or heading angle deviation |
| $\psi_0$ | Reference angle |
| $\omega$ | Angular frequency ($\omega' = \omega L/V$, reduced frequency) |

A dot above a variable stands for derivation with respect to time. Partial derivatives of forces and moments are designated by the proper subscript attached to the force or moment symbol.

APPENDIX B

SHIP MODELS IN LISPID

A summary of the three ship models, which are described in Section 4.3, is given in this Appendix.

Model 1.

$$\begin{bmatrix} \frac{L}{V^2}\theta_1 & \frac{L^2}{V^2}\theta_2 & 0 \\ \frac{L}{V^2}\theta_3 & \frac{L^2}{V^2}\theta_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d\bar{v} \\ d\bar{r} \\ d\bar{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{V}\theta_5 & \frac{L}{V}\theta_6 & \theta_9 \\ \frac{1}{V}\theta_7 & \frac{L}{V}\theta_8 & \theta_9\,\theta_{10} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{v}(t) \\ \bar{r}(t) \\ \bar{\psi}(t) \end{bmatrix} dt +$$

$$\begin{bmatrix} \alpha_1\,\theta_{11} & \theta_{13} \\ -\alpha_1\,\theta_{11}\,\theta_{12} & 14 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t-\tau) \\ U1 \end{bmatrix} dt + d\bar{w}$$

$$\begin{bmatrix} v_1(t_k) \\ v_2(t_k) \\ v(t_k) \\ r(t_k) \\ \psi(t_k) \end{bmatrix} = \begin{bmatrix} \alpha_2 & L_1\alpha_2 & 0 \\ \alpha_2 & -L_2\alpha_2 & 0 \\ \alpha_2 & 0 & 0 \\ 0 & 1/\alpha_1 & 0 \\ 0 & 0 & 1/\alpha_1 \end{bmatrix} \begin{bmatrix} \bar{v}(t_k) \\ \bar{r}(t_k) \\ \bar{\psi}(t_k) \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & \theta_{15} \\ 0 & \theta_{16} \\ 0 & \theta_{15} \\ 0 & \theta_{17} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t_k - \tau) \\ U1 \end{bmatrix} + \tilde{e}(t_k)$$

$$k = 0,1,\ldots,N$$

$$R_1 = \begin{bmatrix} |\theta_{18}| & \sqrt{|\theta_{18}||\theta_{19}|}\sin\theta_{20} & 0 \\ \sqrt{|\theta_{18}||\theta_{19}|}\sin\theta_{20} & |\theta_{19}| & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{R}_2 = \begin{bmatrix} |\theta_{21}| & 0 & 0 & 0 \\ 0 & |\theta_{22}| & 0 & 0 \\ 0 & 0 & |\theta_{23}| & 0 \\ 0 & 0 & 0 & |\theta_{24}| \end{bmatrix}$$

$$\begin{bmatrix} \bar{v}(t_o) \\ \bar{r}(t_o) \\ \bar{\psi}(t_o) \end{bmatrix} = \begin{bmatrix} \theta_{25} & /\alpha_2 \\ \alpha_1 & \theta_{26} \\ \alpha_1 & \theta_{27} \end{bmatrix}$$

$$P(t_o) = \begin{bmatrix} |\theta_{28}| & \theta_{31} & \theta_{32} \\ \theta_{31} & |\theta_{29}| & \theta_{33} \\ \theta_{32} & \theta_{33} & |\theta_{30}| \end{bmatrix}$$

$$\tau = T|\sin\theta_{34}|$$

<u>Model 2.</u>

$$\begin{bmatrix} d\bar{v} \\ dx_2 \\ dx_3 \end{bmatrix} = \begin{bmatrix} -\frac{V}{L}\theta_1 & 1 & 0 \\ -\frac{V^2}{L^2}\theta_2 & 0 & 1 \\ -\frac{V^3}{L^3}\theta_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{v}(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} dt + \begin{bmatrix} \alpha_1 \frac{V^2}{L}\theta_4 & \theta_7 \\ \alpha_1 \frac{V^3}{L^2}\theta_5 & \theta_8 \\ \alpha_1 \frac{V^4}{L^3}\theta_6 & \theta_9 \end{bmatrix} \begin{bmatrix} \delta(t-\tau) \\ U1 \end{bmatrix} dt + dw$$

$$v(t_k) = [\alpha_2 \quad 0 \quad 0] \begin{bmatrix} \bar{v}(t_k) \\ x_2(t_k) \\ x_3(t_k) \end{bmatrix} + [0 \quad \theta_{10}] \begin{bmatrix} \delta(t_k-\tau) \\ U1 \end{bmatrix} + \tilde{e}(t_k)$$

$$k = 0,1,\ldots,N$$

$$R_1 = \begin{bmatrix} |\theta_{11}| & \sqrt{|\theta_{11}||\theta_{12}|}\,\sin\theta_{14} & \theta_{15} \\ \sqrt{|\theta_{11}||\theta_{12}|}\,\sin\theta_{14} & |\theta_{12}| & \theta_{16} \\ \theta_{15} & \theta_{16} & |\theta_{13}| \end{bmatrix}$$

$$\tilde{R}_2 = |\theta_{17}|$$

$$\begin{bmatrix} \bar{v}(t_o) \\ x_2(t_o) \\ x_3(t_o) \end{bmatrix} = \begin{bmatrix} \theta_{18}/\alpha_2 \\ \theta_{19} \\ \theta_{20} \end{bmatrix}$$

$$P(t_o) = \begin{bmatrix} |\theta_{21}| & \theta_{24} & \theta_{25} \\ \theta_{24} & |\theta_{22}| & \theta_{26} \\ \theta_{25} & \theta_{26} & |\theta_{23}| \end{bmatrix}$$

$$\tau = T|\sin\theta_{27}|$$

Model 3.

$$\begin{bmatrix} dx_1 \\ d\bar{r} \\ d\bar{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{V^3}{L^3}\theta^3 \\ 0 & -\frac{V}{L}\theta_1 & -\frac{V^2}{L^2}\theta_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ \bar{r}(t) \\ \bar{\psi}(t) \end{bmatrix} dt + \begin{bmatrix} \alpha_1 \frac{V^3}{L^3}\theta_5 & \theta_6 \\ \alpha_1 \frac{V^2}{L^2}\theta_4 & \theta_7 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t-\tau) \\ U1 \end{bmatrix} dt + dw$$

$$\begin{bmatrix} r(t_k) \\ \psi(t_k) \end{bmatrix} = \begin{bmatrix} 0 & 1/\alpha_1 & 0 \\ 0 & 0 & 1/\alpha_1 \end{bmatrix} \begin{bmatrix} x_1(t_k) \\ \bar{r}(t_k) \\ \bar{\psi}(t_k) \end{bmatrix} + \begin{bmatrix} 0 & \theta_8 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t_k-\tau) \\ U1 \end{bmatrix} + \tilde{e}(t_k)$$

$$k = 0,1,\ldots,N$$

$$R_1 = \begin{bmatrix} |\theta_9| & \sqrt{|\theta_9||\theta_{10}|}\,\sin\theta_{11} & 0 \\ \sqrt{|\theta_9||\theta_{10}|}\,\sin\theta_{11} & |\theta_{10}| & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{R}_2 = \begin{bmatrix} |\theta_{12}| & \sqrt{|\theta_{12}||\theta_{13}|}\,\sin\theta_{14} \\ \sqrt{|\theta_{12}||\theta_{13}|}\,\sin\theta_{14} & |\theta_{13}| \end{bmatrix}$$

$$\begin{bmatrix} x_1(t_o) \\ \bar{r}(t_o) \\ \bar{\psi}(t_o) \end{bmatrix} = \begin{bmatrix} \theta_{15} \\ \alpha_1\theta_{16} \\ \alpha_1\theta_{17} \end{bmatrix}$$

$$P(t_o) = \begin{bmatrix} |\theta_{18}| & \theta_{21} & \theta_{22} \\ \theta_{21} & |\theta_{19}| & \theta_{23} \\ \theta_{22} & \theta_{23} & |\theta_{20}| \end{bmatrix}$$

$$\tau = T|\sin\theta_{24}|$$

## PROGRAM HEADS

```
      SUBROUTINE LISPID(SYST,TH,IERR)
C
C     ADMINISTRATION SUBROUTINE FOR LINEAR SYSTEM PARAMETER
C     IDENTIFICATION.
C     AUTHOR, C. KALLSTROM 1973-09-12.
C     REVISED, C. KALLSTROM 1974-03-15.
C     REVISED, T. ESSEBO 1974-04-17.
C
C SYST- A SUBROUTINE XXX(TH,NTH,VLOSS,DUM1,DUM2,IDUM,ICONT,IER),
C       WHICH COMPUTES THE LOSS VLOSS FOR THE PARAMETER VECTOR TH OF
C       DIMENSION NTH.THIS SUBROUTINE THE USER MUST SUPPLY. FROM
C       LISPID XXX IS ALWAYS CALLED WITH ICONT=0,WITH ONE EXCEPTION:
C       IMMEDIATELY AFTER THE MINIMIZING XXX IS CALLED ONCE WITH
C       ICONT=-1. THIS COULD BE USED TO SAVE THE FINAL PARAMETER
C       VECTOR.  PUT IER=0 IN XXX IF
C       THE COMPUTATIONS ARE OK, PUT IER=-1 TO TERMINATE THE
C       MINIMIZING.
C TH-   PARAMETER VECTOR OF DIMENSION NTH, AT INPUT CONTAINING
C       ESTIMATED MINIMUM POINT. AT RETURN CONTAINING COMPUTED
C       MINIMUM POINT ,IF LOOP .GT. 0 AND IF IERR IS RETURNED EQUAL
C       TO 0.
C IERR- ERROR PARAMETER,ALWAYS PRINTED IF .NE. 0 :
C       IERR=0 IF OK.
C       IERR=1 IF NTH AND/OR LOOP HAS AN ILLEGAL VALUE.
C       IERR=2 IF ONE OR MORE VARIABLES OF THE COMMON BLOCKS /DATA/
C       AND /SYSPAR/ HAVE ILLEGAL VALUES.
C       IERR=3 IF TROUBLES TO PLOT CURVES (NPLOT MUST BE EQUAL TO 1).
C       NO PLOTTING IS INITIATED.
C       IERR=4 IF THE SAME TROUBLES AS FOR IERR=3,BUT THE PLOTTING
C       IS STARTED.
C       IERR=5 IF IER IN THE CALL OF XXX IS RETURNED -1.
C       IERR=6 IF THE COMPUTATIONS OF GRADIENT G AND SECOND DERIVATIVE
C       MATRIX V2 HAVE FAILED, OR IF V2 IS SINGULAR.
C       IERR=7 IF BOTH IERR=5 AND IERR=6.
C
C     DESCRIPTION OF THE COMMON BLOCKS:
C
C     /SYSPAR/
C     MUST BE ASSIGNED VALUES BY THE USER.
C NP-   NUMBER OF SAMPLE EVENTS (MAX 2000,MIN 2).
C IT-   PUT IT=1 IF THE TIMES FOR THE SAMPLE EVENTS ARE SUPPLIED IN
C       /DATA/  ELSE PUT IT=0.
C ISYS- DESCRIBES THE SYSTEM MODEL TO BE USED.
C       ISYS .GT. 0 MEANS A CONTINUOUS MODEL,ISYS .LT. 0 MEANS A
C       DISCRETE MODEL.
C       ISYS=1,-1 : ONLY MEASUREMENT NOISE.
C       ISYS=2,-2 : MEASUREMENT NOISE,AND STATE NOISE MODELLED BY
C       FIX GAIN AK.
C       ISYS=3,-3 : MEASUREMENT AND STATE NOISE MODELLED BY COVARIANCE
C       MATRICES R1,(R12) AND R2.
C       ISYS=4,-4 : AS ISYS=3,-3 BUT THE MODEL IS ONLY SIMULATED AND
C       THE RESIDUALS ARE TESTED AGAINST CHISQ.
C MEAS- PUT MEAS=0 IF INSTANTANEOUS MEASUREMENTS,PUT MEAS=1 IF
C       INTEGRATING MEASUREMENTS.
C       IF MEAS=1 ,THE VALUE YMS(1,*) IN /DATA/ MUST BE THE
C       INTEGRATED VALUE FROM TIME1 TO TIME2,AND SO ON.
C ISAMP-DESCRIBES THE SAMPLE INTERVAL.
C       ISAMP=1 : CONSTANT SAMPLE INTERVAL.
```

```
C         ISAMP=2 : CONSTANT SAMPLE INTERVAL, BUT SOME MEASUREMENTS
C         ARE MISSING. IT MUST BE EQUAL TO 1.
C         ISAMP=3 : VARIABLE SAMPLE INTERVAL. THE PARAMETER IT MUST BE
C         EQUAL TO 1 IF ISYS .GT. 0.
C TSAMP-SAMPLE INTERVAL TO BE USED IF IT=0 .PUT TSAMP=-1. TO INDICATE
C         THAT THE SAMPLE INTERVAL MUST BE COMPUTED FROM TIM IN /DATA/
C NPRED1-
C NPRED2-THE LOSS FUNCTION VALUE IS OBTAINED AS A MEAN VALUE OF LOSSF
C         WHEN PREDICTING NPRED1,NPRED1+1, ... ,NPRED2 STEPS AHEAD.
C         NPRED1:(MAX NPRED2,MIN 1) .NPRED2: (NO MAX,MIN NPRED1).
C         PUT NPRED1=NPRED2=1 TO OBTAIN MAXIMUM LIKELIHOOD ESTIMATES.
C NX-     NUMBER OF STATES (MAX 20,MIN 1).
C NU-     NUMBER OF INPUT SIGNALS (MAX 20,MIN 0).
C         PUT NU=0 IF NO INPUT SIGNAL IS APPLIED.
C NY-     NUMBER OF MEASUREMENT SIGNALS (MAX 20,MIN 1)
C NTH-    NUMBER OF PARAMETERS IN THE OPTIMIZATION.
C NOMAT-VECTOR OF DIMENSION 6 DESCRIBING THE SYSTEM AND COVARIANCE
C         MATRICES. PUT NOMAT(1)=0 IF NO B,ELSE PUT NOMAT(1)=1.
C         PUT NOMAT(2)=0 IF NO C, ELSE PUT NOMAT(2)=1.
C         PUT NOMAT(3)=0 IF NO D, ELSE PUT NOMAT(3)=1.
C         PUT NOMAT(4)=0 IF NO R1, ELSE PUT NOMAT(4)=1
C         PUT NOMAT(5)=0 IF NO R12, ELSE PUT NOMAT(5)=1.
C         PUT NOMAT(6)=0 IF NO R2, ELSE PUT NOMAT(6)=1.
C
C       /DATA/ AND /INDEX/
C       /DATA/ CONSISTS OF VECTOR V IN WHICH THE MEASUREMENTS
C       ,SYSTEM MATRICES AND RESULTING OUTPUTS ARE STORED IN
C       CONSECUTIVE ORDER. THE RELATIVE ADDRESSES OF THE DIFFERENT
C       VECTORS AND MATRICES OF /DATA/ ARE STORED IN /INDEX/.
C       SUBROUTINE DATEXP WILL COMPUTE THE PARAMETERS OF /INDEX/
C       FROM /SYSPAR/ AND CREATE THE NECESSARY AREA NEEDED IN
C       /DATA/. A CALL OF DATEXP MUST BE MADE (PREFERABLY IN THE
C       MAIN PROGRAM) BEFORE ANYTHING CAN BE STORED IN /DATA/.
C       THE ORGANIZATION OF /DATA/ :
C       UIN(NP,NU)      -INPUT SIGNALS
C       YMS(NP,NY)      -MEASUREMENTS
C       TIM(NP)         -TIMES FOR SAMPLE EVENTS (IF IT=1)
C       YMOD(NP,NY)     -OUTPUT FROM DETERMINISTIC MODEL
C       ERRMOD(NP,NY)   -MODEL ERRORS
C       EPS(NP,NY)      -RESIDUALS
C       IEPS(NP)        -INTEGER VECTOR DESCRIBING THE RESIDUALS
C         IEPS(1)=0 IF OK,I.E. THE RESIDUAL AT TIME1 HAS CONTRIBUTED TO
C         THE LOSS FUNCTION.
C         IEPS(1)=1 IF THE RESIDUAL AT TIME1 HAS NOT CONTRIBUTED TO THE
C         LOSS FUNCTION.
C         IEPS(1)=2 IF RESIDUAL TOO LARGE WHEN TESTING AGAINST CHISQ.
C         (ONLY USED WHEN ISYS=4,-4).
C         IEPS(1)=3 IF THE COVARIANCE MATRIX OF RESIDUALS IS SINGULAR
C         WHEN COMPUTING THE CONTRIBUTION TO THE LOSS FUNCTION IN
C         SUBROUTINE KALMAN.
C       A(NX,NX),B(NX,NU),C(NY,NX),D(NY,NU),R1(NX,NX),R12(NX,NY),
C       R2(NY,NY),AK(NX,NY),P0(NX,NX),X0(NX) - SYSTEM AND
C       COVARIANCE MATRICES ,INITIAL ERROR COVARIANCE MATRIX
C       AND INITIAL STATE
C       AD(NX,NX),BD(NX,NU),...,AKD(NX,NY),P(NX,NX),X(NX) -USED
C       AS INTERNAL STORAGE (MAINLY IN SINT)
C       RR(NY,NY),PPD(NX,NX),DELTAR(NY,NY),RPD(NY,NY),RNY(NY,NY),
C       V1(NTH,NTH),V2(NTH,NTH) -USED AS STORAGE AND WORK
C       ARRAYS IN DIFFERENT PARTS OF THE PROGRAM.
C
C       THE USER MUST SUPPLY THE VALUES OF UIN,YMS ,(TIM).
C       THE SYSTEM AND COVARIANCE MATRICES A,B,..AK AND P0,X0
C       MUST ALSO BE ASSIGNED VALUES THAT MAY DEPEND ON THE
C       PARAMETER VECTOR TH. THE DEPENDENCE MUST BE SUPPLIED
```

```
C      IN SUBROUTINE XXX (OR IN A ROUTINE CALLED BY XXX).
C
C      THE RELATIVE ADDRESS OF A MATRIX IN /DATA/ IS GIVEN BY
C      A POINTER IN /INDEX/ WITH THE SAME NAME AS THE MATRIX
C      BUT PREFIXED WITH IX. THIS POINTER POINTS TO THE CELL IN
C      V BEFORE THE FIRST CELL OF THE MATRIX (OR VECTOR).
C      EX.  COVARIANCE MATRIX R12 HAS ADDRESS IXR12+1 IN V.
C      MATRICES ARE STORED COLUMN-WISE IN V.
C
C      BY USING THE UNIVAC FORTRAN V FACILITY  DEFINE PROCEDURES
C      IT IS POSSIBLE TO REFER TO MATRICES AND VECTORS OF
C      /DATA/ AS IF THEY WERE DECLARED IN A DIMENSION STATEMENT.
C      A DEFINE STATEMENT MUST APPEAR BEFORE THE FIRST EXECUTABLE
C      STATEMENT OF A PROGRAM.
C      EX.  MATRIX R12
C      DEFINE R12(I,J)=V(IXR12+NX*J-NX+I)
C       .
C       .
C      R12(1,3)=FX(I)
C       .
C      XP=R12(3,3)
C
C      EXCEPTION: DEFINE PROCEDURE NAMES MUST NOT APPEAR AS
C      FORMAL ARGUMENTS IN SUBROUTINE CALLS ,THUS:
C      CALL SUB(V(IXR12+1),NX,NY)
C
C      FOR FURTHER INFORMATION SEE: UNIVAC 1100 FORTRAN V
C      MANUAL ,P. 4-2
C
C      /LISP/
C      CONTAINS RESULTS FROM THE COMPUTATIONS.
C VLOS1-VLOS1=DET(SUM(EPS*EPST))/NVLOS1, ALWAYS COMPUTED. USED AS
C        STANDARD LOSS IF ISYS=1,-1,2,-2.
C NVLOS1-NUMBER OF CONTRIBUTIONS TO VLOS1.
C VLOS2-VLOS2=-LOG(L(TH,R))/NVLOS2 ,ONLY COMPUTED AND USED AS STANDARD
C        LOSS IF ISYS=3,-3,4,-4.
C NVLOS2-NUMBER OF CONTRIBUTIONS TO VLOS2.
C VLOSS-OBTAINED MINIMUM LOSS.
C VLOSDT-OBTAINED LOSS WHEN SIMULATING THE DETERMINISTIC MODEL.
C G-     GRADIENT VECTOR OF DIMENSION NTH.
C STDEV-VECTOR OF DIMENSION NTH CONTAINING ESTIMATED
C        STANDARD DEVIATIONS.
C      IABSIS,....,NVLDT1- INTERNAL VARIABLES.
C
C      /LISCON/
C      THE VARIABLES HAVE STANDARD VALUES ASSIGNED BY A CALL OF
C      SUBROUTINE LISDAT ,BUT THESE VALUES CAN BE OVERROLLED BY THE USER.
C LOOP- NUMBER OF CALLS TO THE MINIMIZING ALGORITHM (NO MAX,MIN -1).
C        PUT LOOP=0 FOR SIMULATON ONLY. PUT LOOP=-1 TO PRINT AND/OR
C        PLOT DATA IN /DATA/. IN THIS CASE ONLY THE
C        FOLLOWING INPUT ARGUMENTS MUST HAVE VALUES:LOOP,NPRI(3),NPLOT,
C        NP,IT,ISAMP,NU,NY AND TIM,UIN,YMS IN /DATA/. STANDARD:1
C NPRI-  VECTOR OF DIMENSION 3. PUT NPRI(1)=1 TO PRINT SYSTEM AND
C        COVARIANCE MATRICES FOR THE INITIAL PARAMETER VECTOR,ELSE PUT
C        NPRI(1)=0. PUT NPRI(2)=1 TO PRINT SYSTEM AND COVARIANCE
C        MATRICES FOR THE FINAL PARAMETER VECTOR, ELSE PUT NPRI(2)=0.
C        PUT NPRI(3)=1 TO PRINT INPUT SIGNALS,MEASUREMENTS,MODEL
C        OUTPUTS ,MODEL ERRORS AND RESIDUALS ,ELSE PUT NPRI(3)=0.
C      STANDARD: 1,1,1
C NPLOT-PUT NPLOT=1 TO PLOT CURVES ON PLOTTER,ELSE PUT NPLOT=0.
C      STANDARD:0
C ITRAN-ONLY USED IF ISYS=1,2 AND MEAS=0. PUT ITRAN=1 TO SAMPLE THE
C        MATRICES A AND B BY SUBROUTINE TRANS INSTEAD OF SUBROUTINE
C        COSA. STANDARD: 0.
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C NMAX- MAXIMUM NUMBER OF TERMS USED IN SUBROUTINE TRANS.STANDARD: 500.
C EPST- TEST QUANTITY USED IN SUBROUTINE TRANS. STANDARD: 1.E-6.
C EPSK- TEST QUANTITY USED IN SUBROUTINE KALMAN.STANDARD: 1.E-9.
C CHISG-ONLY USED IF ISYS=4,-4 .TEST QUANTITY USED IN SUBROUTINE KALMAN
C        TO TEST IF RESIDUAL TOO LARGE.STANDARD: 6.0.
C IMIN- DECIDES WHICH MINIMIZING ALGORITHM TO BE USED.
C        IMIN=1 NUFLET (STANDARD)
C        IMIN=2 POWBRE
C DFN,HZ, ... ,XM(50) - PARAMETER VALUES FOR NUFLET
C DIST, .. ,IPRIN -PARAMETERS FOR POWBRE
C IND-   NUMBER OF COLUMNS PER PAPER WIDTH WHEN PRINTING VECTORS AND
C        MATRICES BY SUBROUTINE MPRI. STANDARD: 8.
C IF-    IF=0 MEANS FORMAT G16.8 WHEN PRINTING VECTORS AND MATRICES BY
C        SUBROUTINE MPRI, IF=1  MEANS FORMAT E16.8.STANDARD: 0.
C IPLOT-IPLOT=0 MEANS ORDINARY PLOT WHEN PLOTTING THE INPUT SIGNALS BY
C        SUBROUTINE RITA, IPLOT=1 MEANS HISTOGRAM PLOT.STANDARD: 1.
C ITEXT-ITEXT=0 MEANS NO TEXT WHEN PLOTTING CURVES ,ITEXT=1 MEANS
C        STANDARD TEXT. STANDARD: 1.
C IIY-   IIY=1 MEANS THAT BOTH MEASUREMENTS AND MODEL OUTPUTS ARE
C        PLOTTED IN THE SAME DIAGRAM, IIY=0 MEANS THAT ONLY MODEL
C        OUTPUTS ARE PLOTTED. STANDARD: 1.
C SX-    LENGTH OF X-AXIS IN CM WHEN PLOTTING CURVES (NO MAX,MIN 2.)
C        STANDARD: 16.
C SY-    LENGTH OF Y-AXIS IN CM WHEN PLOTTING CURVES (MIN 2.). TOTAL
C        WIDTH OF THE PLOT IN Y-DIRECTION IS ALWAYS (3*SY + 4) CM.
C        STANDARD: 6.
C IDH-   IDH=0 MEANS THAT THE STEP LENGTHS WHEN COMPUTING G AND V2 BY
C        SUBROUTINE GRASD ARE CHOSEN AS HH*TH ,IDH=1 MEANS THAT THE
C        STEP LENGTHS MUST BE SUPPLIED IN VECTOR DH. STANDARD: 0.
C HH-    SEE ABOVE. STANDARD: 0.01.
C DH-    SEE ABOVE. IF IDH=0 ,DH CONTAINS THE COMPUTED STEP LENGTHS.
C IACC-  IACC=1 MEANS THAT G AND V2 ARE COMPUTED WITH ACCURACY
C        ORDO(DH**2), IACC=2 MEANS ACCURACY ORDO(DH**4).
C        IACC=0 MEANS NO COMPUTATION OF G AND V2 .STANDARD:0.
C NPLOTC:0 NOTHING PRINTED OR PLOTTED IN RESLIS
C       :1 TEST QUANTITIES PRINTED
C       :2 1+CORRELATIONS PRINTED
C       :3 1+CORRELATIONS PLOTTED
C       :4 1+2+3
C      STANDARD: 2
C NOL-   NUMBER OF TIME LAGS (MAX 50,MIN 0).STANDARD:10
C INU-   INTEGER VECTOR OF DIMENSION 20. PUT INU(I)=1 IF INPUT
C        I IS TO BE USED WHEN COMPUTING CROSS CORRELATIONS,ELSE PUT
C        INU(I)=0.
C INY-   INTEGER VECTOR OF DIMENSION 20 . PUT INY(K)=1 IF OUTPUT
C        I IS TO BE USED IN RESLIS ,ELSE PUT INY(I)=0.
C      STANDARD VALUES OF INU AND INY:1,1,...1,1
C
C      NOTE 1: THE PLOTTING CAN ALSO BE CONTROLLED BY THE COMMON BLOCK
C      /RITFIG/ DESCRIBED IN SUBROUTINE RITA.
C
C      NOTE 2: THE EXTERNAL SYMBOLS PLCOM,PLTSEG AND RESSEG ARE
C      NAMES OF PROGRAM SEGMENTS THAT ARE LOADED INTO MAIN STORAGE
C      BY REQUEST OF LISPID ,USING THE ROUTINE LODSEG .
C
C      SUBROUTINES REQUIRED
C      (SYST)
C         SINT
C            SAMP
C               COSA
C                  NORM
C                  EXPAN
C            TRANS
C            NORM
```

```
C             LISY
C             KALMAN
C                NORM
C                DESYM
C                SOLVS
C             LOS
C                NORM
C                DESYM
C       MPRI
C       SYSPRI
C          MPRI
C       NUFLET
C          (SYST)
C       POWBRE (WITH SUBROUTINES)
C          (SYST)
C       LIPLOT
C          SCL
C          RITA
C             AXEL
C             DOTLIN
C       GRASD
C          (SYST)
C       NORM
C       SYMIN
C       EIGS
C       RESLIS
C          RESPLT
C          REST
C             CSGFT
C                FINO
C             DESYM
C             SOLVS
C       LODSEG
C
      DIMENSION TH(1),NOLD(9),EVAL(40),INTV(1)
C
      LOGICAL ILLCO
C
      COMMON/SYSPAR/NP,IT,ISYS,MEAS,ISAMP,TSAMP,NPRED1,NPRED2,
     *NX,NU,NY,NTH,NOMAT(6)
C
      COMMON/DATA/V(1)
C
      COMMON/INDEX/IXYMS,IXTIM,IXYMOD,IXERMD,IXEPS,IXIEPS,IXA,IXB,
     *IXC,IXD,IXR1,IXR12,IXR2,IXAK,IXP0,IXX0,IXAD,IXBD,IXCD,
     *IXDD,IXR1D,IXR12D,IXR2D,IXAKD,IXP,IXX,IXRR,IXPPD,
     *IXDLTR,IXRPD,IXRNY,IXV1,IXV2,NX2,NY2,NXNY,NXNU,NYNU
C
      COMMON/LISP/VLOS1,NVLOS1,VLOS2,NVLOS2,VLOSS,VLOSDT,G(40),
     *STDEV(40),IABSIS,IMZ,ICOS,ML,NOM(6),NOTRAN(6),NRR,NVLDT1
C
      COMMON/LISCON/LOOP,NPRI(3),NPLOT,ITRAN,NMAX,EPST,EPSK,CHISQ,IMIN,
     *DFN,NZ,EPZ,MODE,MAXFN,IPRINT,XM(50),DIST,SCALX,TEPS,NSTP,ILLC,
     *IPRIN,IND,IF,IPLOT,ITEXT,IIY,SX,SY,IDH,HH,DH(40),IACC,NPLOTC,NOL,
     *INU(20),INY(20),ICHK,LDUM1,LDUM2,RDUM1,RDUM2
C
      DEFINE IEPS(I)=INTV(IXIEPS+I)
      DEFINE TIM(I)=V(IXTIM+I)
      DEFINE YMS(I,J)=V(IXYMS+NP*J-NP+I)
      DEFINE YMOD(I,J)=V(IXYMOD+NP*J-NP+I)
      DEFINE EPS(I,J)=V(IXEPS+NP*J-NP+I)
      DEFINE V1(I,J)=V(IXV1+NTH*J-NTH+I)
      DEFINE COV(I,J)=V(IXDLTR+NY*J-NY+I)
      DEFINE RNY(I,J)=V(IXRNY+NY*J-NY+I)
```

```
      DEFINE RPU(I,J)=V(IXRPD+NY*J-NY+I)
      DEFINE RR(I,J)=V(IXRR+NY*J-NY+I)
C
      EQUIVALENCE (INTV,V)
C
      EXTERNAL PLCOM,PLTSEG,RESSEG
C
```

```
      SUBROUTINE SINT(NTIME,NSTOP,INIT,IV,VLOSS,IERR)
C
C     SUBROUTINE TO BE USED FOR LINEAR SYSTEM PARAMETER
C     IDENTIFICATION TOGETHER WITH SUBROUTINE LISPID.
C     THE SUBROUTINE CAN PERFORM ONE OR MORE OF FOLLOWING TASKS:
C     1.       TRANSFORMATION OF CONTINUOUS SYSTEM AND COVARIANCE
C          MATRICES TO DISCRETE MATRICES
C     2.       ITERATION OF THE DISCRETE
C          SYSTEM EQUATIONS
C     3.       ITERATION OF THE DISCRETE KALMAN BUCY FILTERING
C          EQUATIONS
C     4.       PREDICTION
C     5.       COMPUTATION OF LOSS FUNCTION
C
C     AUTHOR, C. KALLSTROM 1973-06-26.
C     REVISED, T. ESSEBO 1974-04-17.
C
C     NTIME-   COUNTER FOR MEASUREMENT EVENTS (MAX NP, MIN 1)
C              RETURNED CONTAINING THE VALUE OF NSTOP (IF IERR=0).
C     NSTOP-   (NO MAX, MIN NTIME)
C              THE EQUATIONS ARE ITERATED FROM NTIME TO MIN(NSTOP,NP).
C              NSTOP IS NOT CHANGED IN SINT
C     INIT-    PUT INIT=1 AT THE FIRST CALL OF SINT TO INITIALIZE,
C              THEN PUT INIT=0.
C     IV-      CONTROLS THE REJECTING OF MEASUREMENTS AND THE UPDATING OF
C              LOSS FUNCTIONS (MAX 4, MIN 0).
C              PUT IV=0 IF NO REJECTING OF MEASUREMENTS.
C              PUT IV=1 TO REJECT THE MEASUREMENTS NTIME TO STOP.
C              PUT IV=2 TO REJECT MEASUREMENTS REACHED BY PREDICTION
C              FROM NTIME TO NSTOP.
C              PUT IV=3 IF BOTH IV=1 AND IV=2.
C              PUT IV=4 TO COMPUTE THE FINAL LOSS,
C              RETURNED IN VLOSS.ONLY THE ARGUMENTS VLOSS AND IERR ARE
C              USED.
C     VLOSS-   RETURNED LOSS WHEN IV=4.
C              IF ISYS=1, -1, 2, -2,  VLOSS=(DET(SUM(EPS*EPST)))/N,
C              WHERE N IS THE NUMBER OF CONTRIBUTIONS.
C              IF ISYS=3, -3, 4, -4,  VLOSS=-LOG(L(TH,R))/N.
C     IERR- ERROR PARAMETER.
C              IERR=-1 IF THE COVARIANCE MATRIX IS SINGULAR WHEN
C                      COMPUTING VLOSS. ONLY USED WHEN IV=4 AND ISYS=
C                      1,-1,2 OR -2. VLOSS CONTAINS A ROUGH ESTIMATE
C                      OF THE REAL LOSS.
C              IERR=0 IF OK.
C              IERR=1 IF IV HAS AN ILLEGAL VALUE.
C              IERR=2 IF NTIME AND/OR NSTOP HAVE ILLEGAL VALUES.
C              IERR=3 IF A SAMPLING INTERVAL IS NONPOSITIVE.
C              IERR=4 IF NO CONVERGENCE IN TRANS
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C                  (GIVE NMAX A LARGER VALUE OR INCREASE EPST).
C           IERR=5 ITERATION IS TERMINATED DUE TO REFERENCE OF
C                  MEASUREMENTS BEYOND INDEX NP
C
C     NOTE 1: THE SYSTEM MATRICES A,B,..,R2,AK IN /DATA/ ARE
C     NEVER CHANGED BUT THEY ARE TRANSFERRED TO MATRICES AD,BD,..
C     ,AKD ACCORDING TO PARAMETER ISAMP ,AND IT IS THESE
C     MATRICES THAT ARE USED FOR THE ITERATIONS AND PREDICTION.
C     THE INITIAL STATES X0 AND P0 ARE ONLY TRANSFERED TO X AND P
C     WHEN INIT=1.
C
C     NOTE 4: THE  COMMON BLOCKS ARE DESCRIBED IN SUBROUTINE
C     LISPID (THE COMMON BLOCK /KAL1/ IS DESCRIBED IN KALMAN).
C
C
C     SUBROUTINES REQUIRED
C           SAMP
C              COSA
C                 NORM
C                 EXPAN
C              TRANS
C                 NORM
C           LISY
C           KALMAN
C              NORM
C              DESYM
C              SOLVS
C           LOS
C              NORM
C              DESYM
C
C
      DIMENSION U(20),Y(20),XPD(20),UPD(20),YPD(20),INTV(1)
C
      COMMON/SYSPAR/NP,IT,ISYS,MEAS,ISAMP,TSAMP,NPRED1,NPRED2,
     *NX,NU,NY,NTH,NOMAT(6)
C
      COMMON/DATA/V(1)
C
      COMMON/INDEX/IXYMS,IXTIM,IXYMOD,IXERMD,IXEPS,IXIEPS,IXA,IXB,
     *IXC,IXD,IXR1,IXR12,IXR2,IXAK,IXP0,IXX0,IXAD,IXBD,IXCD,
     *IXDD,IXR1D,IXR12D,IXR2D,IXAKD,IXP,IXX,IXRR,IXPPD,
     *IXDLTR,IXRPD,IXRNY,IXV1,IXV2,NX2,NY2,NXNY,NXNU,NYNU
C
      COMMON/LISP/VLOS1,NVLOS1,VLOS2,NVLOS2,VLOSX,VLOSDT,G(40),
     *STDEV(40),IABSIS,IM2,ICOS,ML,NOM(6),NOTRAN(6),NRR,NVLDT1
C
      COMMON/LISCON/LOOP,NPRI(3),NPLOT,ITRAN,NMAX,EPST,EPSK,CHISQ,IMIN
     *DFN,HZ,EPZ,MODE,MAXFN,IPRINT,XM(50),DIST,SCALX,TEPS,NSTP,ILLC,
     *IPRIN,IND,IF,IPLOT,ITEXT,IIY,SX,SY,IDH,HH,DH(40),IACC,NPLOTC,NCL
     *INU(20),INY(20),ICHK,LDUM1,LDUM2,RDUM1,RDUM2
C
      COMMON/KAL1/ YMODD(20),E(20),DUM1(20),R(20,20),AKT(20,20),
     *TEST,DELTAV
C
      EQUIVALENCE (INTV,V)
C
      DEFINE IEPS(I)=INTV(IXIEPS+I)
      DEFINE TIM(I)=V(IXTIM+I)
      DEFINE UIN(I,J)=V(NP*J-NP+I)
      DEFINE YMS(I,J)=V(IXYMS+NP*J-NP+I)
      DEFINE EPS(I,J)=V(IXEPS+NP*J-NP+I)
      DEFINE X(I)=V(IXX+I)
      DEFINE X0(I)=V(IXX0+I)
      DEFINE RR(I,J)=V(IXRR+NY*J-NY+I)
      DEFINE RPD(I,J)=V(IXRPD+NY*J-NY+I)
```

```
      SUBROUTINE DATEXP
C
C     SUBROUTINE FOR LISPID TO COMPUTE PARAMETERS FOR /INDEX/ AND
C     EXPAND PROGRAM SIZE FOR /DATA/ .
C
C     AUTHOR, T. ESSEBO 1974-06-17.
C
C     NOTE: DATEXP SHOULD BE CALLED IN THE MAIN PROGRAM AS SOON AS
C     NP,IT,NX,NU,NY,NTH IN /SYSPAR/ HAVE BEEN ASSIGNED PROPER
C     VALUES (AND BEFORE ANYTHING IS STORED IN /DATA/ ).
C
C     SUBROUTINE REQUIRED
C         MCORE
C
      COMMON/SYSPAR/NP,IT,ISYS,MEAS,ISAMP,TSAMP,NPRED1,NPRED2,
     *NX,NU,NY,NTH,NOMAT(6)
C
      COMMON/DATA/V(1)
C
      COMMON/INDEX/IXYMS,IXTIM,IXYMOD,IXERMD,IXEPS,IXIEPS,IXA,IXB,
     *IXC,IXD,IXR1,IXR12,IXR2,IXAK,IXP0,IXX0,IXAD,IXBD,IXCD,
     *IXDD,IXR1D,IXR12D,IXR2D,IXAKD,IXP,IXX,IXRP,IXPPD,
     *IXDLTR,IXRPD,IXPNY,IXV1,IXV2,NX2,NY2,NXNY,NXNU,NYNU
C
      DATA MAXCOR/38000/
C
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```
      SUBROUTINE IOLISP(TH,IR,IP)
C
C     SUBROUTINE TO READ DATA FROM CARD READER FOR LINEAR SYSTEM
C     PARAMETER IDENTIFICATION AND/OR PRINT DATA ON LINE PRINTER.
C
C     AUTHOR, C. KALLSTROM 1974-02-15.
C     REVISED, C. KALLSTROM 1974-03-18.
C     REVISED, T. ESSEBO 1974-04-17.
C
C     TH=   PARAMETER VECTOR OF DIMENSION NTH FOR LISPID.
C     IR=   PUT IR=0 IF NO READING.
C           PUT IR=1 TO READ DATA FROM CARD READER. IF IR=1 ZEROS ARE
C           MOVED TO A,B,C,D,R1,R12,R2,AK,X0 AND P0 OF COMMON BLOCK
C           /SYS/ AND TO PAR OF COMMON BLOCK /COMSY/ BEFORE THE READIN
C           PARAMETER VALUES ARE ALSO MOVED FROM PAR TO TH AND VOLD IS
C           PUT EQUAL TO 1.E10.
C     IP=   PUT IP=0 IF NO PRINTING.
C           PUT IP=1 TO PRINT DATA.
C           PUT IP=2 TO PRINT THE SAME DATA AS FOR IP=1 ,BUT IF ISYS=2
C           OR -2 AND SUBROUTINE SGAIN IS CALLED IN SYST TO COMPUTE TH
C           DISCRETE ,STATIONARY FILTER GAIN AK OR IF ISYS=3,-3,4 OR -
C           SOME MORE DATA IS PRINTED.
C
C     DATA TO BE READ IF IR=1:
C     NPAR          (I2)
C     NTH           (I2)
C     PAR(1)        (E20.10)
C       .             .
C     PAR(NPAR)     (E20.10)
C     ITH(1)        (I2)
C       .             .
C     ITH(NTH)      (I2)
C     SCAL(1)       (E20.10)
C       .             .
C     SCAL(NTH)     (E20.10)
C     IMIN          (I2)
C     IF IMIN<0 2 PARAMETERS WILL BE READ:
C     IMIN=-1 (NUFLET)
C     HZ            (E20.10)
C     EPZ           (E20.10)
C     IMIN=-2 (POWBRE)
C     DIST          (E20.10)
C     TEPS          (E20.10)
C     LOOP          (I2)
C     NPRI(1)       (I2)
C     NPRI(2)       (I2)
C     NPRI(3)       (I2)
C     NPLOT         (I2)
C     NP            (I5)
C     IT            (I2)
C     ISYS          (I2)
C     MEAS          (I2)
C     ISAMP         (I2)
C     TSAMP         (E20.10)
C     NPRED1        (I2)
C     NPRED2        (I2)
C     NX            (I2)
C     NU            (I2)
C     NY            (I2)
C     NOMAT(1)      (I2)
C       .             .
C     NOMAT(6)      (I2)
C     IACC          (I2)
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```
C      HH              (E20.10)
C      NPLOTC          (I2)
C      NOL             (I2)
C      TS              (E20.10)
C      EPSI            (E20.10)
C      NKAL            (I5)
C      ICHK            (I2)
C      ICR             (I2)
C
C      IF ICR=0 NO MORE DATA WILL BE READ.
C      IF ICR=1 ,FOLLOWING DATA WILL ALSO BE READ:
C      NMAX            (I5)
C      EPST            (E20.10)
C      EPSK            (E20.10)
C      CHISQ           (E20.10)
C      IPLOT           (I2)
C      ITEXT           (I2)
C      IIY             (I2)
C      SX              (E20.10)
C      SY              (E20.10)
C
C      ATTENTION: IT IS ONLY SENSIBLE TO CALL IOLISP WITH IP=2 ,IF
C      LISPID EARLIER IS CALLED AT LEAST ONCE.
C
C      SUBROUTINE REQUIRED
C           MPRI
C
       DIMENSION TH(1)
C
       COMMON/SYSPAR/NP,IT,ISYS,MEAS,ISAMP,TSAMP,NPRED1,NPRED2,
      *NX,NU,NY,NTH,NOMAT(6)
C
       COMMON/DATA/V(1)
C
       COMMON/INDEX/IXYMS,IXTIM,IXYMOD,IXERMO,IXEPS,IXIEPS,IXA,IXB,
      *IXC,IXD,IXR1,IXR12,IXR2,IX K,IXP0,IXX0,IXAD,IXBD,IXCD,
      *IXDD,IXR1D,IXR12D,IXR2D,IXAKD,IXP,IXX,IXRR,IXPPD,
      *IXDLTR,IXRPD,IXRNI,IXV1,IXV2,NX2,NY2,NXNY,NXNU,NYNU
C
       COMMON/LISCON/LOOP,NPRI(3),NPLOT,ITRAN,NMAX,EPST,EPSK,CHISQ,IMIN,
      *DFN,HZ,EPZ,MODE,MAXFN,IPRINT,XM(50),DIST,SCALX,TEPS,NSTP,ILLC,
      *IPRIN,IND,IF,IPLOT,ITEXT,IIY,SX,SY,IDH,HH,DH(40),IACC,NPLOTC,NOL,
      *INU(20),INY(20),ICHK,LDUM1,LDUM2,RDUM1,RDUM2
C
       COMMON/COMSY/PAR(40),NPAR,ITH(40),SCAL(40),TS,EPSI,NKAL,VOLD,
      *IERRSY
C
       COMMON/KAL1/DUMM1(60),RCOV(20,20),AKT(20,20),DUMM2(2)
C
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      SUBROUTINE SGAIN(TS,EPSI,NKAL,IERROR)
C
C     SUBROUTINE TO BE USED FOR LINEAR SYSTEM PARAMETER IDENTIFICATION
C     TO SAMPLE SYSTEM AND COVARIANCE MATRICES (IF ISYS .GT. 0) AND
C     TO COMPUTE THE DISCRETE, STATIONARY FILTER GAIN AK.
C     AUTHOR, C. KALLSTROM 1974-02-15.
C     REVISED, T. ESSEBO 1974-04-16.
C
C     TS - SAMPLING INTERVAL (NOT USED IF ISYS .LT. 0).
C     EPSI - TEST QUANTITY. THE ITERATIONS ARE TERMINATED WHEN
C            NORM(P(T)-P(T-1))/NORM(P(T)) .LE. EPSI.
C     NKAL - MAXIMUM NUMBER OF ITERATIONS TO BE DONE (NO MAX,MIN 2).
C     IERROR - ERROR PARAMETER.
C              IERROR=0 IF OK.
C              IERROR=1 IF TS IS NONPOSITIVE.
C              IERROR=2 IF NO CONVERGENCE IN TRANS.
C              IERROR=3 IF NO CONVERGENCE WHEN THE KALMAN BUCY
C              FILTERING EQUATIONS ARE ITERATED.
C
C     NOTE 1: FOLLOWING VARIABLES ARE USED (BUT NOT CHANGED) IN THE
C     SUBROUTINE
C     COMMON BLOCK /SYSPAR/:ISYS, MEAS,NX,NY,NOMAT
C     COMMON BLOCK /DATA/: A,C,R1,R12,R2,P0(ACCORDING TO VECTOR NOMAT)
C     COMMON BLOCK /LISCON/: NMAX,EPST,EPSK
C
C     NOTE 2: FOLLOWING MATRICES OF THE COMMON BLOCK /DATA/ ARE USED
C     AS STORAGE IN THE SUBROUTINE: AD,CD,R1D,R12D,R2D,AKD,P
C
C     NOTE 3: IF IERROR=0 ,THE COMPUTED STATIONARY FILTER GAIN IS STORED
C    IN AK  OF THE COMMON BLOCK /DATA/. IF IERROR=3 ,THE LAST COMPUTED
C     (NOT STATIONARY) FILTER GAIN IS STORED IN AK.
C
C     NOTE 4: ADDITIONAL INFORMATION CAN BE OBTAINED FROM THE COMMON
C     BLOCK /SGA/
C
C     NOTE 5: COMMON BLOCK /TRANS2/ ,WHICH IS SHARED WITH
C     SUBROUTINE TRANS CONTAINS WORK ARRAYS.
C
C     SUBROUTINES REQUIRED
C        TRANS
C           NORM
C        KALMAN
C           NORM
C           DESYM
C           SOLVS
C        NORM
C
      COMMON/SYSPAR/NP,IT,ISYS,MEAS,ISAMP,TSAMP,NPRED1,NPRED2,
     *NX,NU,NY,NTH,NOMAT(6)
C
      COMMON/DATA/V(1)
C
      COMMON/INDEX/IXYMS,IXTIM,IXYMOD,IXERMD,IXEPS,IXTEPS,IXA,IXB,
     *IXC,IXD,IXR1,IXR12,IXR2,IXAK,IXP0,IXX0,IXAD,IXBD,IXCD,
     *IXDD,IXR1D,IXR12D,IXR2D,IXAKD,IXP,IXX,IXPP,IXPPD,
     *IXDLTR,IXRPD,IXPNY,IXV1,IXV2,NX2,NY2,NXNY,NXNU,NYNU
C
      COMMON/LISCON/LOOP,NPRI(3),NPLOT,ITRAN,NMAX,EPST,EPSK,CHISQ,THIN,
     *DEN,HZ,EPZ,MODE,MAXFN,IPRINT,YN(50),DIST,SCALX,TEPS,NSTP,ILLC,
     *IPRIN,IPD,IF,IPLOT,ITEXT,ITY,SX,SY,IPN,HH,DH(40),IACC,NPLOTC,NOL,
     *INU(20),INY(20),ICHK,LDUM1,LDUM2,RDUM1,RDUM2
C
      COMMON/TRANS2/DUM1(1600),S1(400)
C
      COMMON/KAL1/DUMM1(460),AKT(20,20),DUMM2(2)
C
      COMMON/SGA/NNTR(6),NOKAL(6),XKAL(20),YKAL(20),EPSIL,IKC,IK
C
      DEFINE AK(I,J)=V(IXAK+NX*J-NX+I)
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      SUBROUTINE COSDY(A,B,FI,GAM1,GAM2,NX,NU,TSAMP,TAU,IA)
C
C     SUBROUTINE TO TRANSFORM A CONTINUOUS LINEAR SYSTEM DX/DT=A*X(T) +
C     B*U(T)  TO A DISCRETE SYSTEM X(T+TSAMP)=FI*X(T) + GAM1*U(T) +
C     GAM2*U(T+TAU) , WHERE U IS A PIECEWISE CONSTANT CONTROL VARIABLE
C     DISCONTINUOUS AT THE TIMES K*TSAMP+TAU ,K=0,1,2,...
C
C     AUTHOR, C. KALLSTROM 1974-03-01.
C
C     A-     CONTINUOUS SYSTEM MATRIX OF ORDER NX*NX ,NOT DESTROYED.
C     B-     CONTINUOUS SYSTEM MATRIX OF ORDER NX*NU ,NOT DESTROYED.
C            NOT REFERENCED IF NU=0.
C     FI-    COMPUTED DISCRETE SYSTEM MATRIX OF ORDER NX*NX.
C     GAM1-  COMPUTED DISCRETE SYSTEM MATRIX OF ORDER NX*NU
C            NOT USED IF NU=0.
C     GAM2-  COMPUTED DISCRETE SYSTEM MATRIX OF ORDER NX*NU
C            NOT USED IF NU=0.
C     NX-    NUMBER OF STATES (MAX 20 ,MIN 1).
C     NU-    NUMBER OF INPUTS (MAX 20 ,MIN 0).
C            PUT NU=0 IF NO B.
C     TSAMP-SAMPLING INTERVAL (NO MAX ,MIN 0.).
C     TAU-   TIME LAG BETWEEN THE SAMPLING EVENT AND NEXT DISCONTINUITY
C            OF U (NO MAX ,NO MIN). NOT USED IF NU=0.
C     IA-    DIMENSION PARAMETER OF A,B,FI,GAM1 AND GAM2.
C
C     NOTE: IF TAU .GE. TSAMP ,GAM1 CONTAINS THE ORDINARY
C            DISCRETE SYSTEM MATRIX GAMMA AND GAM2 CONTAINS ZEROS.
C            IF TAU .LE. 0. ,GAM1 CONTAINS ZEROS AND GAM2 CONTAINS
C            THE ORDINARY DISCRETE SYSTEM MATRIX GAMMA.
C
C     SUBROUTINE REQUIRED
C            COSA
C               NORM
C               EXPAN
C
      DIMENSION A(IA,1),B(IA,1),FI(IA,1),GAM1(IA,1),GAM2(IA,1)
C
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      SUBROUTINE SHIP(TH,NTH1,VLOSS,DUM1,DUM2,IDUM,ICONT,IER)
C
C     SUBROUTINE TO SUPPLY A PARAMETER DEPENDENT SHIP MODEL AND TO
C     COMPUTE A LOSS FUNCTION VALUE.
C
C     AUTHOR, C. KALLSTROM 1974-02-13.
C     REVISED, C. KALLSTROM 1974-03-05.
C     REVISED, I. ESSEBO 1974-04-16.
C
C     TH-    PARAMETER VECTOR OF DIMENSION NTH1.
C     NTH1-  NUMBER OF PARAMETERS (MAX 40,MIN 2).
C     NTH1 IS EQUAL TO NTH IN /SYSPAR/ .
C     VLOSS-COMPUTED LOSS FUNCTION VALUE.
C     DUM1,DUM2,IDUM- DUMMY ARGUMENTS.
C     ICONT-CONTROLS THE COMPUTATION OF LOSS FUNCTION AND DERIVATIVES.
C            IN SUBROUTINE SHIP IT IS ONLY POSSIBLE TO COMPUTE LOSS
C            FUNCTION VALUE, SO ICONT MUST BE EQUAL TO 0 WHEN SUBROUTINE
C            SHIP IS CALLED TO COMPUTE LOSS FUNCTION.
C            ICONT=-1 WILL CAUSE THE PARAMETER VECTOR Q TO BE
C            PRINTED ON LOGICAL UNIT 1.
C     IER-   ERROR PARAMETER:
C            IER=0 IF OK.
C            IER=-1 IF ERROR IN SHIP
C
C     COMMON BLOCK /COMSY/:
C     Q-     PARAMETER VECTOR OF DIMENSION NPAR. VECTOR TH(I)/SCAL(I)
C            IS A PART OF VECTOR Q.
C     NPAR-  NUMBER OF PARAMETERS (MAX 40,MIN 2). NTH MUST BE .LE. NPAR.
C     ITH-   INTEGER VECTOR OF DIMENSION NTH CONTAINING THE INDICES OF
C            THE PARAMETERS OF VECTOR Q, FOR WHICH THE MINIMIZATION
C            IS PERFORMED.
C     SCAL-  SCALING VECTOR OF DIMENSION NTH. TH(I)=Q(I)*SCAL(I).
C            SCAL(I) MUST NOT BE EQUAL TO 0.
C     TS-    SAMPLE INTERVAL, ONLY USED IN SUBROUTINE SGAIN IF ISYS=2.
C     EPSI-  TEST QUANTITY, ONLY USED IN SUBROUTINE SGAIN IF ISYS=2.
C     NVAL-  MAXIMUM NUMBER OF ITERATIONS, ONLY USED IN
C            SUBROUTINE SGAIN IF ISYS=2 (NO MAX,MIN 2).
C     VOLD-  PUT VOLD INITIALLY EQUAL TO A LARGE POSITIVE VALUE (E.G.
C            1.E+10). WHEN SUBROUTINE SHIP IS CALLED ,THE LOSS FUNCTION
C            VALUE VLOSS IS STORED IN VOLD, IF THE COMPUTATION IS SUCCES
C            FULL , ELSE VLOSS IS PUT EQUAL TO 10.*VOLD AND THE VALUE IS
C            THEN RESTORED IN VOLD.
C     IERRSY-ERROR PARAMETER:
C            IERRSY=0 IF OK.
C            IERRSY=1 IF THE COMPUTATION OF VLOSS HAS FAILED.
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C
C     COMMON BLOCK /SHP/:
C     VS-   TOTAL VELOCITY OF THE SHIP IN M/S .
C     SL-   SHIP LENGTH IN M.
C     SL1-  DISTANCE IN M FROM THE CENTRE OF MASS TO THE FORWARD
C           DOPPLER LOG TRANSMITTER. ONLY USED IF IMOD=1 AND IY(1)=1.
C     SL2-  DISTANCE IN M FROM THE CENTRE OF MASS TO THE AFT DOPPLER
C           LOG TRANSMITTER. ONLY USED IF IMOD=1 AND IY(2)=1.
C     IMOD- CONTROLS WHAT SHIP MODEL TO BE USED:
C           IMOD=1 MODEL CONTAINING HYDRODYNAMIC DERIVATIVES (34 PARAM.)
C                  POSSIBLE OUTPUTS: SWAY VELOCITY OF BOW,STERN AND
C                  CENTRE OF MASS, YAWRATE AND HEADING.
C           IMOD=2 MODEL TO ESTIMATE THE TRANSFER FUNCTION FROM RUDDER
C                  TO SWAY VELOCITY OF CENTRE OF MASS (27 PARAMETERS).
C                  ONLY POSSIBLE OUTPUT: SWAY VELOCITY OF CENTRE OF MASS
C           IMOD=3 MODEL TO ESTIMATE THE TRANSFER FUNCTION FROM RUDDER
C                  TO YAW RATE (24 PARAMETERS). POSSIBLE OUTPUTS:
C                  YAW RATE AND HEADING.
C     IY-   INTEGER VECTOR OF DIMENSION 5.
C           PUT IY(1)=1 IF THE SWAY VELOCITY OF BOW IS ONE OUTPUT,
C           ELSE PUT IY(1)=0 .
C           PUT IY(2)=1 IF THE SWAY VELOCITY OF STERN IS ONE OUTPUT,
C           ELSE PUT IY(2)=0 .
C           PUT IY(3)=1 IF THE SWAY VELOCITY OF CENTRE OF MASS IS ONE
C           OUTPUT, ELSE PUT IY(3)=0 .
C           PUT IY(4)=1 IF THE YAW RATE IS ONE OUTPUT, ELSE PUT IY(4)=0.
C           PUT IY(5)=1 IF THE HEADING IS ONE OUTPUT, ELSE PUT IY(5)=0 .
C     LPAR- PUT LPAR=1 IF THE PARAMETER VECTOR SHOULD BE SAVED
C           IF THE COMPUTATIONS ARE RUNNING OUT OF TIME ,ELSE
C           PUT LPAR=0.
C     MSL-  MAX TIME IN MS LEFT IN THE JOB THAT WILL NOT TERMINATE
C           THE OPTIMIZING (IF LPAR=1).
C
C     NOTE 1: IF ISYS=2 OR =2, THE SUBROUTINE SGAIN IS CALLED TO COMPUTE
C     THE DISCRETE, STATIONARY FILTER GAIN AK.
C
C     NOTE 2: COMMON BLOCK /TRANS2/ WHICH IS SHARED WITH
C     SUBROUTINE TRANS ,CONTAINS WORK ARRAYS .
C
C     SUBROUTINE REQUIRED
C        COSDY
C           COSA
C              NORM
C              EXPAN
C        SGAIN
C           TRANS
C              NORM
C           KALMAN
C              NORM
C              DESYM
C              SOLVS
C           NORM
C        SINT (WITH SUBROUTINES)
C
      DIMENSION TH(1),NNTR(6)
C
      COMMON/SYSPAR/NP,IT,ISYS,MEAS,ISAMP,TSAMP,NPRED1,NPRED2,
     *NX,NU,NY,NTH,NOMAT(6)
C
      COMMON/DATA/V(1)
C
      COMMON/INDEX/IXYMS,IXTIM,IXYMOD,IXERMD,IXEPS,IXIEPS,IXA,IXB,
     *IXC,IXD,IXR1,IXR12,IXR2,IXAK,IXPU,IXX0,IXAD,IXBD,IXCD,
     *IXDD,IXR1D,IXR12D,IXR2D,IXAKD,IXP,IXX,IXRR,IXPPD,
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     *IXDLTR,IXRPD,IXRNY,IXV1,IXV2,NX2,NY2,NXNY,NXNU,NYNU
C
      COMMON/LISCON/LOOP,NPRI(3),NPLOT,ITRAN,NMAX,EPST,EPSK,CHISQ,IMIN,
     *DFN,HZ,EPZ,MODE,MAXFN,IPRINT,XM(50),DIST,SCALX,TEPS,NSTP,ILLC,
     *IPRIN,INU,IF,IPLOT,ITEXT,IIY,SX,SY,IDH,HH,DH(40),IACC,NPLOTC,NOL,
     *INU(20),INY(20),ICHK,LDUM1,LDUM2,RDUM1,RDUM2
C
      COMMON/TRANS2/S1(400),DUM3(1600)
C
      COMMON/COMSY/W(40),NPAR,ITH(40),SCAL(40),TS,EPSI,NKAL,VOLD,IERRSY
C
      COMMON /SHP/VS,SL,SL1,SL2,IMOD,IY(5),LPAR,MSL
C
      DEFINE A(I,J)=V(IXA+NX*J-NX+I)
      DEFINE B(I,J)=V(IXB+NX*J-NX+I)
      DEFINE C(I,J)=V(IXC+NY*J-NY+I)
      DEFINE D(I,J)=V(IXD+NY*J-NY+I)
      DEFINE R2(I,J)=V(IXR2+NY*J-NY+I)
      DEFINE R1(I,J)=V(IXR1+NX*J-NX+I)
      DEFINE X0(I)=V(IXX0+I)
      DEFINE P0(I,J)=V(IXP0+NX*J-NX+I)
      DEFINE BD(I,J)=V(IXBD+NX*J-NX+I)
C
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