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Bratt, Gunilla; Johannesson, Rolf; Zigangirov, Kamil

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*Total number of authors:*  
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LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00

# Decoding Procedure Capacities for the Gilbert-Elliott Channel <sup>1</sup>

Gunilla Bratt and Rolf Johannesson  
 Dept. of Information Theory  
 Lund University  
 P.O. Box 118  
 S-221 00 LUND, Sweden

Kamil Sh. Zigangirov  
 Dept. of Telecommunication Theory  
 Lund University  
 P.O. Box 118  
 S-221 00 LUND, Sweden

*Abstract* — Sequential decoding for the Gilbert-Elliott channel is considered. The decoding procedure capacity  $C_D$  is defined to be the supremum of the rates for which there exists a code that gives arbitrarily small decoding error probability. For different assumptions of the decoder's knowledge of the channel states expressions for  $C_D$  are derived.

## I. INTRODUCTION

Assume that a tree code is used together with sequential decoding to communicate over the Gilbert-Elliott channel. Let  $P(\mathcal{E})$  denote the average probability of decoding error over the ensemble of random, infinite depth tree codes. In this paper we address the question: "When will  $P(\mathcal{E}) \rightarrow 0$ ?"

Consider the Gilbert-Elliott channel model and denote the error probabilities in the Good and Bad states by  $e_G$  and  $e_B$ , respectively. Furthermore, let  $P_G$  and  $P_B$  denote the fraction of time spent in the Good and Bad states, respectively.

## II. DECODING PROCEDURE CAPACITY

Let us define the *decoding procedure assumptions*,  $D$ . The optimistic assumption,  $D = o$ , assumes that the decoder has a complete knowledge of the channel state, which could be given by a genie. The pessimistic assumption,  $D = p$ , assumes that the decoder neither is given any channel state information nor tries to make any estimate of it. Given the decoding procedure assumption  $D$  and the use of the Gilbert-Elliott channel, let  $C_D$  denote the supremum of the rates for which we can guarantee that there exists a code that gives an arbitrarily small decoding error probability  $P(\mathcal{E})$ . We will call  $C_D$  the *decoding procedure capacity*.

We have proved that the decoding procedure capacities are given by

$$C_o = P_G \cdot C_{BSC}(e_G) + P_B \cdot C_{BSC}(e_B)$$

and

$$\begin{aligned} C_p &= P_G \cdot (C_{BSC}(e_G) - h(b)) + P_B \cdot (C_{BSC}(e_B) - h(g)) \\ &= C_o - (P_G \cdot h(b) + P_B \cdot h(g)), \end{aligned}$$

where  $b$  and  $g$  denote the transition probabilities from Good to Bad and from Bad to Good, respectively, in the channel model.

**Theorem 1** *Given the Gilbert-Elliott channel and the decoding procedure assumptions, the use of a rate  $R$  random, infinite depth tree code with the stack decoder, then for any rate  $R < C_D$  and  $\eta \in \mathbb{Z}^+$ ,*

$$P(N \geq \eta) \rightarrow 0 \text{ if } \eta \rightarrow \infty,$$

where  $N$  is the number of computations in an incorrect subtree.

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When we wish to transmit over an ordinary Discrete Memoryless Channel at rates (above  $R_{comp}$  and) close to its capacity, it is sufficient to allow the number of computations of sequential decoding to go to infinity to be able to guarantee that  $P(\mathcal{E})$  can be chosen arbitrarily small. We will show that this is also sufficient for transmission close to rates  $C_D$ , which is the motivation why we call these rates "decoding procedure capacities".

**Theorem 2** *Given the assumptions of Theorem 1, then for any rate  $R < C_D$  the average probability of decoding error*

$$P(\mathcal{E}) \rightarrow 0,$$

if the number of computations,  $N$ , is allowed to go to  $\infty$ .

Since the important condition in Theorem 2 is that  $R < C_D$ , it is clear that the theorem's statement, given the decoding procedure assumptions, is equivalent to stating that the maximal transmission rate over the Gilbert-Elliott channel is at least the rate  $C_D$ .

In the pessimistic case we can interpret this as follows. For arbitrarily small  $P(\mathcal{E})$ , there exists a code such that the transmission rate will be (at least)  $C_p$ , even without any knowledge of the channel state or any attempt to estimate it.

## III. CHANNEL CAPACITY

A common method to lowerbound  $C_{GE}$  is to calculate  $C_{BSC}(\bar{e})$ , where  $\bar{e} = P_G \cdot e_G + P_B \cdot e_B$ , but it turns out that  $C_p$  is a better lower bound for channels with a stable behaviour. The optimistic case helps us to find a stronger result:

**Theorem 3** *Given that the receiver has a complete channel state knowledge, then the channel capacity for the Gilbert-Elliott channel  $C_{GE}^R$  is equal to*

$$C_{GE}^R = C_o.$$

From the proof of Theorem 3 follows immediately

**Corollary 4** *Given that both transmitter and receiver have complete knowledge of the channel state sequence then for the channel capacity of the Gilbert-Elliott channel  $C_{GE}^{TR}$  we have*

$$C_{GE}^{TR} = C_{GE}^R.$$

It should be noted that the capacities  $C_{GE}^{TR}$  and  $C_{GE}^R$ , in contradiction to what is the case for  $C_D$ , are parameters purely dependent of the channel's properties and that nothing is assumed about the decoding method. In the derivations of  $C_D$  we assume sequential decoding, but by deriving them we show that they are achievable rates as such, given the decoding procedure assumptions.

## REFERENCES

- [1] Gunilla Bratt: "Sequential Decoding for the Gilbert-Elliott Channel — Strategy and Analysis". Ph.D. Thesis. Dept. of Information Theory, Lund University, Lund, Sweden, 1994.