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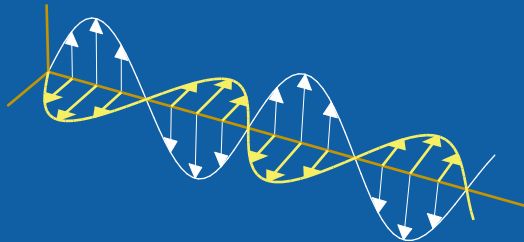
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Time-domain methods for complex media

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Abstract

The objective of this paper is to review the fundamental macroscopic modeling of linear complex materials in a time domain formulation. Based upon a set of basic assumptions, the general form of the constitutive relations for a linear complex media in the time domain is formulated. Furthermore, constraints on the possible form of the susceptibility kernels for a passive medium are presented. For completeness, the corresponding constraints for a reciprocal medium are also given.

1 Introduction

In general, the electromagnetic field is affected by the presence of matter. In a macroscopic formulation of electromagnetics, these effects are modeled by a set of constitutive relations. In this paper a review of these constitutive relations are presented and discussed. Traditionally, the constitutive relations are described as a relation at fixed frequency. The intensified interest in transient phenomena, however, especially wave propagation properties in more complex media, motivates a fresh look at these problems from a different starting point.

Several excellent presentations of the constitutive relations for harmonic waves are found in the literature, see *e.g.*, [5, 7]. However, to make sure that the correct physical behavior is accommodated in the model, it is adequate to start from a formulation in a space-time setting. Unfortunately, the body of information concerning the formulation in the time domain is scant. The consequences of the physical relations, however, have been discussed intensely in recent years, see *e.g.*, [6, 8].

Some of the physical aspects and foundations, *e.g.*, causality and dissipation, in a time domain setting are discussed in this paper. A more detailed version of the material is found in Ref. 4, and the reader is referred to this reference for more details concerning the mathematical analysis.

The dynamics of the macroscopic electromagnetic fields is modeled by the Maxwell equations

$$\begin{cases} \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

These equations, however, are not complete. The reaction of the electromagnetic fields to the presence of matter is modeled by six additional equations — the constitutive relations. These relations relate the electric field \mathbf{E} , the magnetic induction \mathbf{B} , the displacement field \mathbf{D} , and the magnetic field \mathbf{H} to each other, and are completely independent of the Maxwell equations. They have reference to the equations of motion of the constituents, *i.e.*, the charges, of the medium in an electromagnetic field [1].

2 Linear dispersive law

In this section, a set of general physical assumptions is presented that ensures that the correct physics is built into the model. The premises are assumed to hold at each fixed point of the material. This assumption excludes that the material shows spatial dispersion. Such effects are more general than the phenomena modeled in this paper.

The constitutive relations in its most general form are usually given as a relationship between two pairs of fields,¹ *e.g.*, $\{\mathbf{D}, \mathbf{B}\}$ and $\{\mathbf{E}, \mathbf{H}\}$. The constitutive relations used in this paper can formally be written as a general functional dependence

$$\begin{Bmatrix} \mathbf{D} \\ \mathbf{B} \end{Bmatrix} = \mathbf{F} \begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix}$$

All fields are assumed to be quiescent before a fixed time. If space is empty, the vacuum relations between the fields hold, *i.e.*,

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} \\ \mathbf{B} = \mu_0 \mathbf{H} \end{cases}$$

where ϵ_0 and μ_0 are the vacuum permittivity and permeability, respectively. The difference between the non-vacuum relations and the vacuum ones reflects the presence of a medium.

The transformation \mathbf{F} associates with each pair of fields $\{\mathbf{E}, \mathbf{H}\}$ a new pair of fields $\{\mathbf{D}, \mathbf{B}\}$. For physical reasons, the transformation \mathbf{F} is limited to be a *linear dispersive law* defined in the following definition. Here, the spatial dependence, of all vector fields and dyadics is fixed and therefore suppressed.

Definition 2.1. *A transformation \mathbf{F} is said to be a linear dispersive law if it, to every pair $\{\mathbf{E}, \mathbf{H}\}$ that belongs to the class² \mathcal{C}^0 , associates a pair of fields $\{\mathbf{D}, \mathbf{H}\}$ given by*

$$\begin{Bmatrix} \mathbf{D} \\ \mathbf{B} \end{Bmatrix} = \mathbf{F} \begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix}$$

and that satisfies the conditions 1–4 below. Here $\{\mathbf{D}, \mathbf{B}\}$ and $\{\mathbf{D}', \mathbf{B}'\}$ are defined by

$$\begin{Bmatrix} \mathbf{D} \\ \mathbf{B} \end{Bmatrix} = \mathbf{F} \begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix}, \quad \begin{Bmatrix} \mathbf{D}' \\ \mathbf{B}' \end{Bmatrix} = \mathbf{F} \begin{Bmatrix} \mathbf{E}' \\ \mathbf{H}' \end{Bmatrix}$$

where $\{\mathbf{E}, \mathbf{H}\}$ and $\{\mathbf{E}', \mathbf{H}'\}$ both belong to the class \mathcal{C}^0 .

¹Other combinations between different pairs of fields are also frequently used, say between $\{\mathbf{D}, \mathbf{H}\}$ and $\{\mathbf{E}, \mathbf{B}\}$, see [10]. From a *macroscopic* point of view these two choices are equivalent, and does not affect the results presented in this paper.

²A field belonging to class \mathcal{C}^0 is a everywhere continuous field and quiescent before a fixed time τ_0 .

1. The transformation is linear, i.e., for every pair of real numbers α, β

$$\mathbf{F} \left[\alpha \begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix} + \beta \begin{Bmatrix} \mathbf{E}' \\ \mathbf{H}' \end{Bmatrix} \right] = \alpha \mathbf{F} \begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix} + \beta \mathbf{F} \begin{Bmatrix} \mathbf{E}' \\ \mathbf{H}' \end{Bmatrix}$$

2. The transformation is invariant to time translations, i.e., for every fixed time³ $\tau > 0$ the relation

$$\begin{Bmatrix} \mathbf{E}'(t) \\ \mathbf{H}'(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{E}(t - \tau) \\ \mathbf{H}(t - \tau) \end{Bmatrix} \quad \Longrightarrow \quad \begin{Bmatrix} \mathbf{D}'(t) \\ \mathbf{B}'(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{D}(t - \tau) \\ \mathbf{B}(t - \tau) \end{Bmatrix}$$

for all $t \in (-\infty, \infty)$.

3. The transformation satisfies causality, i.e., for every fixed t such that

$$\begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix} = 0 \text{ on } (-\infty, t] \quad \Longrightarrow \quad \begin{Bmatrix} \mathbf{D} \\ \mathbf{B} \end{Bmatrix} = 0 \text{ on } (-\infty, t]$$

4. The transformation is continuous, i.e., for every fixed τ and every $\epsilon > 0$ there exists a $\delta(\epsilon, \tau) > 0$ such that $\max \{|\mathbf{E}(t)|, |\mathbf{H}(t)|\} < \delta(\epsilon, \tau)$ for all $t \in (-\infty, \tau]$ implies $\max \{|\mathbf{D}(\tau)|, |\mathbf{B}(\tau)|\} < \epsilon$.

In Ref. 4 it is proved that any linear dispersive law is a mapping from \mathcal{C}^0 into \mathcal{C}^0 and that it has the form of the following Riemann-Stieltjes integrals:

$$\begin{cases} c_0 \eta_0 D_i(t) = \sum_{j=1}^3 \left\{ \int_0^\infty E_j(t-t') d\mathcal{G}_{ij}(t') + \eta_0 \int_0^\infty H_j(t-t') d\mathcal{K}_{ij}(t') \right\} \\ c_0 B_i(t) = \sum_{j=1}^3 \left\{ \int_0^\infty E_j(t-t') d\mathcal{L}_{ij}(t') + \eta_0 \int_0^\infty H_j(t-t') d\mathcal{F}_{ij}(t') \right\} \end{cases} \quad i = 1, 2, 3 \quad (2.1)$$

where $\mathcal{G}_{ij}(t)$ and $\mathcal{F}_{ij}(t)$ are dimensionless tensors of second rank, and $\mathcal{K}_{ij}(t)$ and $\mathcal{L}_{ij}(t)$ are dimensionless pseudotensors of second rank. Furthermore, it is proved in Ref. 4 that $\mathcal{G}_{ij}(t) = \mathcal{F}_{ij}(t) = \mathcal{K}_{ij}(t) = \mathcal{L}_{ij}(t) = 0$ on $(-\infty, 0)$, and $\mathcal{G}_{ij}(t)$, $\mathcal{F}_{ij}(t)$, $\mathcal{K}_{ij}(t)$ and $\mathcal{L}_{ij}(t)$ are of bounded variation on every closed subinterval of $(-\infty, \infty)$. The fact that the tensors vanish for $t < 0$ is due to the causality assumption 3 in Definition 2.1. Moreover, $\mathcal{G}_{ij}(t)$, $\mathcal{F}_{ij}(t)$, $\mathcal{K}_{ij}(t)$ and $\mathcal{L}_{ij}(t)$ are continuous on the right on $(-\infty, \infty)$, i.e. $\mathcal{G}_{ij}(t) = \mathcal{G}_{ij}(t+)$ and similarly for the other tensors. *Conversely*, every set of tensor-valued functions $\mathcal{G}_{ij}(t)$, $\mathcal{F}_{ij}(t)$, $\mathcal{K}_{ij}(t)$ and $\mathcal{L}_{ij}(t)$ defined on $(-\infty, \infty)$, satisfying the properties above, generates, by equation (2.1), a linear dispersive law.

Equation (2.1) expresses the general form of a constitutive relation under the assumptions 1–4 in Definition 2.1. In most material of interest, this form is too general and some smoothness of the functions $\mathcal{G}_{ij}(t)$, $\mathcal{F}_{ij}(t)$, $\mathcal{K}_{ij}(t)$ and $\mathcal{L}_{ij}(t)$ is

³Time shifts $\tau \leq 0$ is not of interest since then $\{\mathbf{E}', \mathbf{H}'\} \notin \mathcal{C}^0$.

generally at hand. If, for example, $\mathcal{G}_{ij}(t)$, $\mathcal{F}_{ij}(t)$, $\mathcal{K}_{ij}(t)$ and $\mathcal{L}_{ij}(t)$ are continuously differentiable on $(0, \infty)$, then all integrals in (2.1) reduce to convolution integrals, which we more conveniently write as

$$\begin{cases} c_0 \eta_0 \mathbf{D}(t) = \boldsymbol{\varepsilon} \cdot \mathbf{E}(t) + (\boldsymbol{\chi}_{ee} * \mathbf{E})(t) + \eta_0 \boldsymbol{\xi} \cdot \mathbf{H}(t) + \eta_0 (\boldsymbol{\chi}_{em} * \mathbf{H})(t) \\ c_0 \mathbf{B}(t) = \boldsymbol{\zeta} \cdot \mathbf{E}(t) + (\boldsymbol{\chi}_{me} * \mathbf{E})(t) + \eta_0 \boldsymbol{\mu} \cdot \mathbf{H}(t) + \eta_0 (\boldsymbol{\chi}_{mm} * \mathbf{H})(t) \end{cases} \quad (2.2)$$

Here, the star symbol $*$ denotes temporal convolution with a scalar product included, *i.e.*,

$$(\boldsymbol{\alpha} * \mathbf{B})(t) = \int_{-\infty}^t \boldsymbol{\alpha}(t - t') \cdot \mathbf{B}(t') dt'$$

and $c_0 = 1/\sqrt{\epsilon_0 \mu_0}$, $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, and μ_0 and ϵ_0 are the free-space constants and the dyadics $\boldsymbol{\varepsilon}$, $\boldsymbol{\xi}$, $\boldsymbol{\zeta}$ and $\boldsymbol{\mu}$, also often called the *optical responses*, are defined by

$$\begin{cases} \epsilon_{ij} = \mathcal{G}_{ij}(0^+) & \zeta_{ij} = \mathcal{L}_{ij}(0^+) \\ \xi_{ij} = \mathcal{K}_{ij}(0^+) & \mu_{ij} = \mathcal{F}_{ij}(0^+) \end{cases} \quad i, j = 1, 2, 3$$

where $\mathcal{G}_{ij}(0^+)$ denotes the limit value at $t = 0$ from the right, which in general is non-zero. Furthermore, the dyadic-valued (susceptibility) functions $\boldsymbol{\chi}_{ee}$, $\boldsymbol{\chi}_{em}$, $\boldsymbol{\chi}_{me}$ and $\boldsymbol{\chi}_{mm}$ are defined by

$$\begin{cases} \chi_{eeij}(t) = \frac{\partial \mathcal{G}_{ij}}{\partial t}(t) & \chi_{meij}(t) = \frac{\partial \mathcal{L}_{ij}}{\partial t}(t) \\ \chi_{emij}(t) = \frac{\partial \mathcal{K}_{ij}}{\partial t}(t) & \chi_{mmij}(t) = \frac{\partial \mathcal{F}_{ij}}{\partial t}(t) \end{cases} \quad i, j = 1, 2, 3$$

The susceptibility dyadics $\boldsymbol{\chi}_{ee}$, $\boldsymbol{\chi}_{em}$, $\boldsymbol{\chi}_{me}$ and $\boldsymbol{\chi}_{mm}$ are, by definition, continuous functions of time in the interval $(0, \infty)$. At $t = 0$, however, these kernels can show a finite jump discontinuity. This fact is often doubted in the literature, see *e.g.*, [3, p. 310] and [10, 11], but it is not in conflict with the causality assumption. It is important to notice that any set of tensors $\mathcal{G}_{ij}(t)$, $\mathcal{F}_{ij}(t)$, $\mathcal{K}_{ij}(t)$ and $\mathcal{L}_{ij}(t)$, which satisfies the requirements given above, gives a causal response. The Debye model, frequently used as a model for water, is an explicit example of a model that is discontinuous at $t = 0$. In fact, the presence of a finite jump discontinuity at $t = 0$ is due to the neglect of the inertia of the charges (and dipoles) in the material.

Several authors have suggested that the constitutive relations should contain explicit time derivatives, see *e.g.*, [9]. It is interesting to note that the assumptions made in Definition 2.1 exclude any such terms in (2.2). More general assumptions, *e.g.*, spatial dispersion, imply that time derivatives can be included. However, if only local effects in the spatial variables are assumed, no such terms are possible.

Additional assumptions on the material imply constraints on the constitutive relations. In the next sections these constraints are discussed.

3 Dissipation

Generally, dispersion implies that energy is absorbed in the medium, *i.e.*, dissipation of electromagnetic energy. The effects of absorption are due to the presence of the dyadic-valued (susceptibility) functions χ_{ee} , χ_{em} , χ_{me} and χ_{mm} . The dyadics $\boldsymbol{\varepsilon}$, $\boldsymbol{\xi}$, $\boldsymbol{\zeta}$ and $\boldsymbol{\mu}$ give the instantaneous contribution to the \mathbf{D} - and \mathbf{B} -fields, and these dyadics are related to the energy density of the electromagnetic field, and not to the absorption of energy in the medium.

The source-free Maxwell equations imply the Poynting theorem

$$\nabla \cdot \mathbf{S} + \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} = 0$$

where $\mathbf{S} = \mathbf{E} \times \mathbf{H}$.

For fields⁴ in \mathcal{C}^1 , integrate the Poynting theorem over an arbitrary volume $V_{\mathbf{r}}$ centered around \mathbf{r} and bounded by the surface $S_{\mathbf{r}}$ (outward directed normal $\hat{\mathbf{n}}$), use the constitutive relations (2.2), and integrate over time. The result, after use of the divergence theorem, is

$$\mathcal{E}(\tau) = \iiint_{V_{\mathbf{r}}} \{w_{opt}(\tau) + w_d(\tau)\} dv = - \iint_{S_{\mathbf{r}}} \int_0^\tau \mathbf{S}(t) \cdot \hat{\mathbf{n}} dt dS$$

where dv is the volume measure and

$$w_{opt}(\tau) = \frac{1}{c_0 \eta_0} \int_{-\infty}^\tau \left\{ \mathbf{E}(t) \cdot \left[\boldsymbol{\varepsilon} \cdot \frac{\partial \mathbf{E}}{\partial t}(t) + \eta_0 \boldsymbol{\xi} \cdot \frac{\partial \mathbf{H}}{\partial t}(t) \right] \right. \\ \left. + \eta_0 \mathbf{H}(t) \cdot \left[\boldsymbol{\zeta} \cdot \frac{\partial \mathbf{E}}{\partial t}(t) + \eta_0 \boldsymbol{\mu} \cdot \frac{\partial \mathbf{H}}{\partial t}(t) \right] \right\} dt$$

$$w_d(\tau) = \frac{1}{c_0 \eta_0} \int_{-\infty}^\tau \left\{ \mathbf{E}(t) \left[\frac{\partial}{\partial t} (\chi_{ee} * \mathbf{E})(t) + \eta_0 \frac{\partial}{\partial t} (\chi_{em} * \mathbf{H})(t) \right] \right. \\ \left. + \eta_0 \mathbf{H}(t) \left[\frac{\partial}{\partial t} (\chi_{me} * \mathbf{E})(t) + \eta_0 \frac{\partial}{\partial t} (\chi_{mm} * \mathbf{H})(t) \right] \right\} dt$$

The notion of dissipation of the total electro-magnetic energy in a medium can now be defined. A medium is said to be *dissipative* if the energy *entering* through a sphere $S_{\mathbf{r}}$ is always non-negative at all times for all fields, or stated differently, no net production of electro-magnetic energy inside $S_{\mathbf{r}}$ is possible—the medium is passive. The formal definition is

Definition 3.1. *A bianisotropic medium is dissipative at a point \mathbf{r} in the region V if and only if for all τ the total energy $\mathcal{E}(\tau) \geq 0$ for every electromagnetic field $\{\mathbf{E}, \mathbf{H}\}$ in \mathcal{C}^1 in every (sufficiently small) volume $V_{\mathbf{r}}$, such that $V_{\mathbf{r}} \subset V$, around the point \mathbf{r} . The bianisotropic medium is dissipative in V if and only if it is dissipative at all points in V .*

⁴A field belonging to class \mathcal{C}^1 is a everywhere continuously differentiable field and quiescent before a fixed time τ_0 .

Note that dissipation is a local property in space. Thus, in principle, parts of the medium can be dissipative, others not. Since the integrand is continuous in the spatial variables, this definition is equivalent to

$$w_{opt}(\tau) + w_d(\tau) \geq 0$$

for all fields $\in \mathcal{C}^1$.

The dissipation concept constrains the form of the dyadic-valued (susceptibility) functions χ_{ee} , χ_{em} , χ_{me} and χ_{mm} , and the dyadics $\boldsymbol{\varepsilon}$, $\boldsymbol{\xi}$, $\boldsymbol{\zeta}$ and $\boldsymbol{\mu}$. In Ref. 2,4 it is shown that in a passive medium the following constraints must hold:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^t \quad \boldsymbol{\xi} = \boldsymbol{\zeta}^t \quad \boldsymbol{\mu} = \boldsymbol{\mu}^t$$

and

$$\begin{pmatrix} \boldsymbol{\varepsilon} & \boldsymbol{\xi} \\ \boldsymbol{\zeta} & \boldsymbol{\mu} \end{pmatrix} \quad \begin{pmatrix} \chi_{ee}(0^+) & \chi_{em}(0^+) \\ \chi_{me}(0^+) & \chi_{mm}(0^+) \end{pmatrix} \quad \text{positive semi-definite}$$

Here t denotes the transpose of the dyadic. Additional constraints can be found in Ref. 4.

4 Reciprocity

The reciprocity principle dates back to Lorentz. In this paper, reciprocity is defined as a local property in space. Thus, in principle, parts of the medium could be reciprocal, others not.

Definition 4.1. *A medium is defined to be reciprocal at a point \mathbf{r} in the region V if and only if*

$$\iint_{S_{\mathbf{r}}} \{ \epsilon_{ijk}(E_j^a * H_k^b)(\tau) + \epsilon_{ijk}(H_j^a * E_k^b)(\tau) \} \hat{n}_i dS = 0$$

holds for all τ and all electromagnetic fields $\{\mathbf{E}^a, \mathbf{H}^a\}$ and $\{\mathbf{E}^b, \mathbf{H}^b\}$ in \mathcal{C}^1 and for every (sufficiently small) closed surface $S_{\mathbf{r}}$, such that $S_{\mathbf{r}} \subset V$, around the point \mathbf{r} . The medium is reciprocal in V if and only if it is reciprocal at all points in V .

In this definition ϵ_{ijk} is the Levi-Civita symbol. Note that the surface integral in Definition 4.1 has the same value on both sides of any bounding surface of the material, irrespective of whether the material parameters are continuous at the surface or not.

The following result gives a necessary and sufficient condition for reciprocity [4]. The medium is reciprocal at a point \mathbf{r} if and only if the constitutive relations at \mathbf{r} satisfy

$$\begin{cases} \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^t \\ \boldsymbol{\xi} = -\boldsymbol{\zeta}^t \\ \boldsymbol{\mu} = \boldsymbol{\mu}^t \end{cases} \quad \begin{cases} \chi_{ee}(t) = \chi_{ee}^t(t) \\ \chi_{em}(t) = -\chi_{me}^t(t) \\ \chi_{mm}(t) = \chi_{mm}^t(t) \end{cases}$$

Reciprocity and dissipation in a material therefore imply that $\boldsymbol{\xi} = \boldsymbol{\zeta} = \mathbf{0}$. The general constitutive relations for such a material are therefore

$$\begin{cases} c_0\eta_0\mathbf{D}(t) = \boldsymbol{\varepsilon} \cdot \mathbf{E}(t) + (\boldsymbol{\chi}_{ee} * \mathbf{E})(t) + \eta_0(\boldsymbol{\chi}_{em} * \mathbf{H})(t) \\ c_0\mathbf{B}(t) = \eta_0\boldsymbol{\mu} \cdot \mathbf{H}(t) + (\boldsymbol{\chi}_{me} * \mathbf{E})(t) + \eta_0(\boldsymbol{\chi}_{mm} * \mathbf{H})(t) \end{cases}$$

Notice that no coupling between the electric and the magnetic parts occurs in the optical response (momentary response). This coupling can only occur in the dispersive parts $\boldsymbol{\chi}_{em}$ and $\boldsymbol{\chi}_{me}$. The bianisotropic effect is therefore a memory effect.

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