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## Anti-windup in mid-ranging control

Staffan Haugwitz, Maria Karlsson, Stéphane Velut and Per Hagander

**Abstract**—The implementation of anti-windup methods in mid-ranging control needs further attention. It is demonstrated how use of standard anti-windup schemes may give unnecessary performance degradation during saturation. The problem is illustrated for two separate systems, control of oxygen concentration in a bio-reactor and temperature control of a cooling system. In the paper, guidelines are derived for how to design the standard anti-windup scheme to recover performance. As an alternative a modified anti-windup scheme for mid-ranging control is presented that minimizes the performance degradation during saturation.

### I. INTRODUCTION

In the process industry most control loops are single-input single-output (SISO). The use of additional inputs of the process for control purposes is often considered to increase control authority, performance or flexibility. When a process has more control inputs than outputs, the question arises how to use the additional degrees of freedom. The general control problem for this type of processes is sometimes referred to as control allocation, see [1], i.e. how the control actions should be distributed among the available control signals to control the process output.

A specific example of control allocation is mid-ranging. As stated in [2] mid-ranging refers to control problems where there are two control inputs and only one output to control. Often the inputs differ significantly in their dynamic effect on the output, and the faster input is in some way more costly to use than the slow one. In general the faster input, in this paper denoted  $u_1$ , is also closer to saturation than the slower input, here denoted  $u_2$ . The mid-ranging idea is to have the fast input  $u_1$  controlling the process output, and to use the effect of the slower input  $u_2$  to gradually reset or mid-range  $u_1$  to its desired value  $u_{1,ref}$ . Thus  $u_2$  indirectly acts to prevent saturation in  $u_1$  and also to control the process mainly using the “cheaper”  $u_2$  in stationarity. With the use of  $u_2$  the operating range of the process is often considerably increased.

The general mid-ranging control structure is seen in Fig. 1. The mid-ranging idea has been around for many years under many different names, for example valve position control [3] or input resetting. It is commonly used in the process industry, but is also a key idea in e.g. the position control of the pickup-head in a CD-player.

The design method for  $C_1$  and  $C_2$  in Fig. 1 depends on the process. Normally PID-controllers are used, but all SISO controllers are possible, including Internal Model Control

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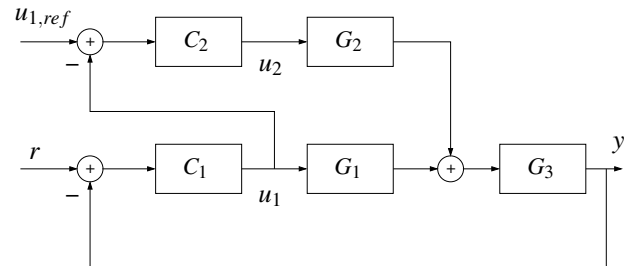


Fig. 1. Mid-ranging control structure, for a process where  $G_3$  represent common dynamics for  $u_1$  and  $u_2$ . In the bio-reactor example  $G_1 = 1$ , which will be used in the sequel.

(IMC) or Linear Quadratic Gaussian (LQG) control. For processes with more complex dynamics and/or time delays, the desired mid-ranging effect can be achieved using multi-variable control design, such as LQG or Model Predictive Control (MPC).

Papers such as [2], [4], [5] present tuning guidelines and applications of mid-ranging control, but there has so far been no discussion on anti-windup for this type of control structure. The main topic of this paper is how to implement anti-windup in mid-ranging control, and it will be demonstrated that standard anti-windup schemes may lead to unnecessary performance degradation during saturation.

### II. PROBLEM STATEMENT

Processes with two input signals and one output signal can be represented with the block diagram in Fig. 1. In many situations like paper drying with steam and infrared light [4] it is reasonable to use the approximation  $G_3 = 1$ , while it is more natural to use  $G_1 = 1$  for other processes like the bio-reactor described in Section III.

In standard applications of mid-ranging control, the dynamics in  $G_1$  is significantly faster than that of  $G_2$ . As a rule of thumb, the controllers  $C_1$  and  $C_2$  should therefore be tuned to keep the dynamics from  $y$  to  $u_1$  an order of magnitude faster than the dynamics from  $y$  to  $u_2$  to avoid exciting cross-couplings, violating control constraints in  $u_2$  or obtaining a high closed-loop bandwidth at frequencies where there are generally large process uncertainties.

In this paper, standard mid-ranging control for a general process with saturation of  $u_1$  is studied. When  $u_1$  saturates, the unsaturated signal is used as input to  $C_2$ . The controllers  $C_1$  and  $C_2$  can be tuned according to guidelines in for example [2]. When implemented, most PI-controllers have a standard anti-windup scheme [6], see Fig. 2. For more

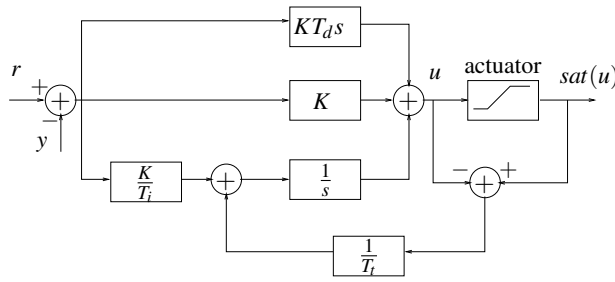


Fig. 2. Basic scheme for anti-windup in a PID-controller.

general anti-windup methods, see for example [7]. The time constant  $T_i$  determines the speed with which the integral term is reset. In general  $T_i$  is chosen as  $T_i \leq T_i$ , where  $T_i$  is the integral time of the controller. In this paper, it will be demonstrated how applying this standard choice of  $T_i$  for  $C_1$  can lead to unnecessary performance degradation when  $u_1$  saturates because of decreased control authority of  $u_2$ . Guidelines will be derived for design of anti-windup schemes for mid-ranging control structures to recover performance.

The paper is organized as follows. In Section III a bio-reactor application of mid-ranging control is presented. The disadvantages of using a standard choice of  $T_i$  will be demonstrated by simulation. In Section IV-A guidelines will be derived for how to choose a suitable time constant  $T_i$  to maintain the same control action in  $u_2$  as in the unsaturated case. A modification that increases control action in  $u_2$  to further reduce performance degradation will be presented in Section IV-B. The guidelines are also applied to a general purpose cooling/heating system in Section V. The aim is to show how the derived guidelines and modified control laws can be used for a process quite different from the bio-reactor. Finally, stability and performance for the proposed tuning are verified using theory for piecewise linear systems in Section VI.

### III. MID-RANGING CONTROL OF A BIO-REACTOR

An example of a process where mid-ranging control can be applied is control of oxygen concentration in a stirred-tank bio-reactor used for cultivations of bacteria. Control of the bio-reactor is described in [8] and [9]. To ensure sufficient oxygen transfer to the cultivation medium, a mechanical stirrer is used with feed-back from the dissolved oxygen concentration to the stirrer speed. Bacterial growth leads to an exponentially increasing demand for oxygen, which causes the stirrer speed to saturate. The resulting decrease in dissolved oxygen concentration causes unnecessary stress on the bacteria, and also disables schemes for substrate feeding and monitoring of the cultivation that depend upon a constant dissolved oxygen concentration. The undesired effects may be avoided by mid-ranging the stirrer speed to a desired value by decreasing the reactor temperature. A lower temperature leads to decreased activity and reproduction of bacteria, thus reducing the oxygen demand. However, caution must be taken in the temperature control since the model is only valid within a limited temperature range, and a too low temperature

may significantly inhibit growth.

A linearized second-order model of the oxygen and temperature dynamics can be described by

$$\dot{x}(t) = Ax(t) + B_1 u_1(t) + B_2 u_2(t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where the first state  $x_1$  denotes the dissolved oxygen concentration which is also the process output  $y$ , and the second state  $x_2$  denotes the reactor temperature. The two control signals  $u_1$  and  $u_2$  represent the stirrer speed and the reference temperature, respectively. The effect of the stirrer speed on  $y$  is much faster than that of the reference temperature. Numerical values for the system matrices used in simulations are given by

$$A = \begin{bmatrix} -2400 & -5200 \\ 0 & -15 \end{bmatrix} \quad (3)$$

$$[ B_1 \ B_2 ] = \begin{bmatrix} 210 & 0 \\ 0 & 15 \end{bmatrix}, \quad C = [ 1 \ 0 ] \quad (4)$$

For efficient use of the bio-reactor, it is essential to choose the reference value  $u_{1,ref}$  for the stirrer speed close to the maximum value  $u_1^{max}$ . This narrow margin limits the control authority of  $u_1$  around its reference value, and requires a sufficiently fast temperature controller to achieve acceptable responses to load disturbances.

PI-controllers are used for both  $C_1$  and  $C_2$ . The nominal control parameters in the following discussions are  $K_1 = 10$ ,  $T_{i1} = 1/120$  hours,  $K_2 = 0.02$ ,  $T_{i2} = 10/120$  hours,  $u_1^{max} = 1200$  rpm and  $u_{1,ref} = 0.98u_1^{max}$ .

## IV. ANTI-WINDUP

### A. Standard anti-windup

During control design of  $C_1$  and  $C_2$ , the controllers are tuned mainly for the nominal, i.e. the unsaturated, case. The purpose of  $C_2$  is to influence the process so that  $u_1$  is mid-ranged to its reference value  $u_{1,ref}$ .  $C_2$  also indirectly acts as an anti-windup scheme for  $u_1$ , but the question arises whether  $C_1$  needs additional support in terms of conventional anti-windup to reduce problems due to saturation.

When  $u_1$  saturates at its maximum value  $u_1^{max}$ , the feedback loop of  $u_1$  is broken, and instead all feedback has to go through  $u_2$ . With fast anti-windup  $u_1 \approx u_1^{max}$ , which gives  $u_2 \approx C_2(u_{1,ref} - u_1^{max})$ . When the operating point is chosen such that  $u_{1,ref}$  is close to  $u_1^{max}$ , the control error that  $C_2$  acts on is very small, thus the contribution from  $u_2$  to quickly end the saturation of  $u_1$  is limited. It is then apparent that  $C_2$  in the saturated case has decreased control authority, compared to the case when  $u_1$  is not saturated. The performance is then significantly reduced as seen in Fig. 3 for the bio-reactor example. With fast anti-windup,  $u_2$  adjusts very slowly resulting in a long period of saturation of  $u_1$ . A standard choice of  $T_i \leq T_i$  has consequently negative effects in the mid-ranging control setting.

Without anti-windup on the other hand, the characteristic wind-up phenomenon can be seen, with a large transient peak in the process output. Unless the controllers  $C_1$  and  $C_2$  are

well separated, disturbances may actually cause instabilities if no anti-windup is used.

The problem with fast anti-windup arises from loss of control authority of  $u_2$ . It is possible to fix this problem by increasing the controller bandwidth, but this may not be feasible for all processes due to constraints on actuator dynamics or process uncertainties. Instead it is desirable to find a time constant  $T_i$  of the anti-windup scheme such that the control action in  $u_2$  is the same in the saturated case as in the unsaturated case. That will be achieved by examining the bandwidth from  $r$  to  $u_2$  in the two cases.

Given the block diagram in Fig. 1 with  $G_1 = 1$ , the transfer function from  $r$  to  $u_2$  in the nominal case can be derived as

$$u_2 = -\frac{C_1 C_2}{1 + C_1 G_3 - C_1 G_3 G_2 C_2} r \stackrel{def}{=} G_{nom} r \quad (5)$$

The saturated case is less straight forward. The controllers and the process are governed by

$$y = G_3 sat(u_1) + G_3 G_2 u_2 \quad (6)$$

$$u_1 = C_1(r - y) + \frac{1}{(sT_i)}(sat(u_1) - u_1) \quad (7)$$

$$u_2 = C_2(u_{1,ref} - u_1) \quad (8)$$

In saturation<sup>1</sup>  $sat(u_1) = u_1^{max}$ . The reference control output  $u_{1,ref}$  is assumed to be constant.

$$y = G_3 u_1^{max} + G_3 G_2 u_2 \quad (9)$$

$$u_1 = C_1(r - y) + \frac{1}{(sT_i)}(-u_1) + \frac{1}{(sT_i)}(u_1^{max}) \quad (10)$$

$$u_1 = \frac{sT_i}{(sT_i + 1)} C_1(r - y) + \frac{1}{(sT_i + 1)} u_1^{max} \quad (11)$$

$$u_2 = C_2(u_{1,ref} - u_1) \quad (12)$$

where we define  $C_{aw} = sT_i/(sT_i + 1)$  from the anti-windup scheme. The broken feedback loop reduces the transfer function from  $r$  to  $u_2$  as follows:

$$u_2 = -C_2 u_1 = -C_2(C_{aw} C_1(r - G_3 G_2 u_2)) \quad (13)$$

$$u_2 = -\frac{C_2 C_{aw} C_1}{(1 - C_2 C_{aw} C_1 G_3 G_2)} r \stackrel{def}{=} G_{sat} r \quad (14)$$

Note that the transfer function  $G_{sat}$  is parameterized in the anti-windup time constant  $T_i$ .

In order to have the same effect of  $u_2$  in saturation as in the nominal case, i.e. the same bandwidth from  $r$  to  $u_2$ , the transfer functions  $G_{nom}$  and  $G_{sat}$  should differ as little as possible. By examining the Bode diagrams of these transfer functions in Fig. 4, a value of  $T_i = 1.2T_{i1}$  is chosen that improves performance during saturation for the bio-reactor compared to the extreme cases of fast and no anti-windup, respectively, see Fig. 3.

The suggested method for choosing  $T_i$  uses information on the nominal process and controllers, with the aim of minimizing the difference in the second control output between the constrained and the nominal case. It is independent of choice of reference value  $u_{1,ref}$  in relation to  $u_1^{min}$  and  $u_1^{max}$ ,

<sup>1</sup>Without loss of generality we consider only the upper limit on  $u_1$ .

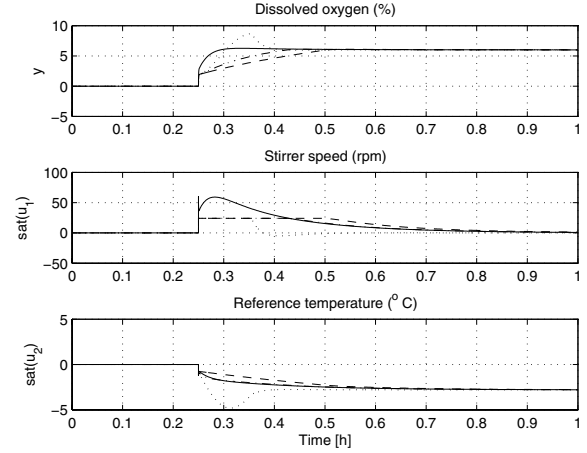


Fig. 3. Step response for bio-reactor example. Fast anti-windup (dashed), no anti-windup (dotted), nominal unsaturated system (solid), and  $T_i$  chosen to minimize the difference in Bode diagrams in Fig. 4 (dashed-dotted)

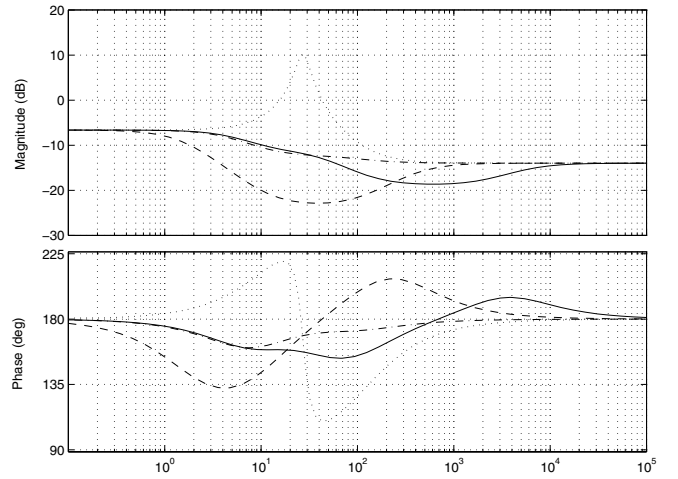


Fig. 4. Bode diagrams of the transfer function from  $r$  to  $u_2$  for the bio-reactor example. Unsaturated case (solid), no anti-windup (dotted), fast anti-windup  $T_i = T_{i1}/3$  (dashed) and  $T_i = 1.2T_{i1}$  that minimizes the distance to the nominal case (dashed-dotted).

and is not limited to a particular disturbance model, or to particular reference trajectories.

### B. Modified anti-windup scheme

So far the main criterion for choosing the anti-windup time constant  $T_i$  has been to obtain the same bandwidth from  $r$  to  $u_2$  in the saturated case as in the nominal case. The control action in  $u_2$  is then similar in the two cases.

Assume now that there is good process knowledge in  $G_3$  and  $G_2$  and constraints on  $u_1$  and  $u_2$  are well known. In this section an easy modification of the anti-windup scheme will be presented that improves the performance during saturation. By manipulating the input signal to  $C_2$  during saturation, the use of  $u_2$  can be increased without having to change the control parameters in  $C_2$ . The modified anti-windup scheme can be seen in Fig. 5.

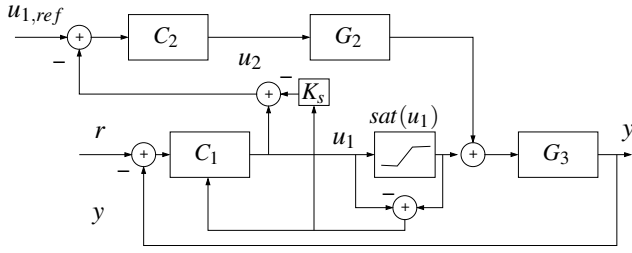


Fig. 5. Mid-ranging control structure with modified anti-windup scheme. The gain  $K_s$  increases the input signal to  $C_2$ , thus generating higher control action in  $u_2$  to improve performance when  $u_1$  saturates.

$$\tilde{u}_1 = u_1 + K_s(u_1 - \text{sat}(u_1)) \quad (15)$$

$$u_2 = C_2(u_{1,ref} - \tilde{u}_1) = C_2(u_{1,ref} - u_1 - K_s(u_1 - \text{sat}(u_1))) \quad (16)$$

In the unsaturated case, the last term is zero as in the standard anti-windup scheme. When  $u_1$  saturates,  $\text{sat}(u_1) = u_1^{max}$  and  $u_1 - u_1^{max} > 0$ . The signal to  $C_2$  is increased by the positive term  $K_s(u_1 - u_1^{max})$ . The modification allows for a standard anti-windup scheme with a small  $T_t \leq T_i$  for fast integral reset, eliminating the windup phenomenon. The difference  $u_1 - u_1^{max}$  is small using fast anti-windup, but with a suitable choice of  $K_s$  performance can be improved. Note that the additional input to  $C_2$ ,  $u_1 - \text{sat}(u_1)$ , is the same signal that is used for standard anti-windup of  $C_1$ .

It is important to note that with this modification, the bandwidth of the transfer function from  $r$  to  $u_2$  is increased. Caution should be used to avoid increasing the bandwidth into high frequency regions where there might be process uncertainties and also explicit constraints in  $u_2$ . With the modification, the response from  $r$  to  $u_2$  will be

$$u_2 = -\frac{C_2(K_s + 1)C_{aw}C_1}{(1 - C_2(K_s + 1)C_{aw}C_1G_2)}r \stackrel{def}{=} G_{K_s}r \quad (17)$$

As can be seen in Fig. 6, the low frequency part of  $G_{K_s}$  is not affected by  $K_s$ .

To find a suitable value of  $K_s$  a standard integral squared error cost function is used as performance criterion,

$$J = \int_0^{t_f} ((r - y)^2 + \rho u_2^2) dt \quad (18)$$

where  $\rho$  is a weight to compensate for scaling, or to suppress large use of  $u_2$ . The objective is to find  $K_s$  that minimizes the cost function  $J$ , for example by numerical search or simulations. Step responses in dissolved oxygen for the bio-reactor with the modified anti-windup scheme are seen in Fig. 7. The performance with  $K_s = 3$  is almost as good as the unsaturated case, due to the extra use of  $u_2$ . In this case  $\rho = 0.34$ , if less control action in  $u_2$  is desired,  $\rho$  should be increased. The optimal value of  $K_s$  depends on the reference  $r$ ,  $C_2$  and the choice of  $\rho$ . A larger reference step  $r$  will give larger control error ( $r - y$ ), thus favoring a higher  $K_s$ . For a faster  $C_2$ , a lower  $K_s$  is needed for the same performance. The value of  $K_s$  will also vary depending on the choice of  $T_t$ , which should be small ( $T_t \leq T_i$ ) to allow fast anti-windup.

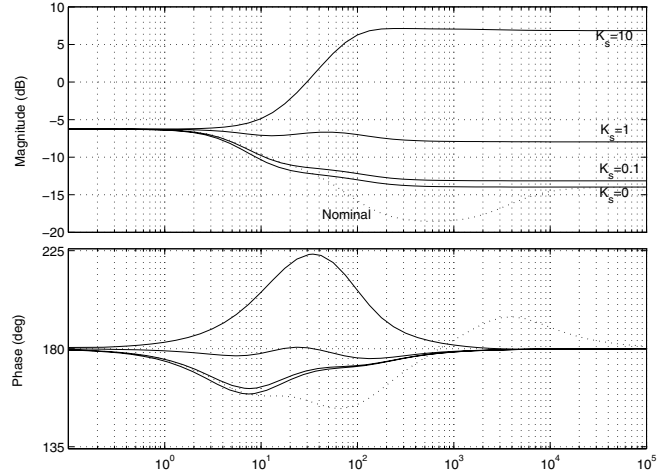


Fig. 6. Bode diagram for transfer function from  $r$  to  $u_2$  for bio-reactor with modified anti-windup scheme and different values of  $K_s$ .  $T_t = T_{i1}$  has been used in all cases.

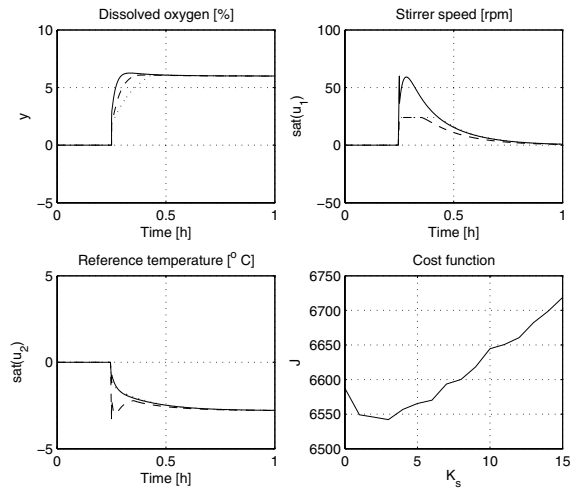


Fig. 7. Step response with modified anti-windup scheme for bio-reactor. Nominal case (solid), standard anti-windup with  $T_t = 1.2T_{i1}$  (dotted), modified anti-windup with  $T_t = T_{i1}$  and  $K_s = 3$  (dashed). Lower right: cost function (18) evaluated for different values of  $K_s$  using  $\rho = 0.34$ .

Dead-beat anti-windup should however be avoided since it gives  $u_1 = u_1^{max}$ , which renders the modification ineffective.

As an alternative to mid-ranging control, Model Predictive Control could be used for the same types of processes. In MPC a similar cost function (18) is evaluated for a specified prediction horizon, and control action is then redistributed to  $u_2$  as  $u_1$  saturates. Performance of the closed-loop system can be further improved using MPC, but at the cost of the computational effort involved in on-line optimization.

The main advantages with the proposed simple modification of standard PID-control are that it is only active as long as  $u_1$  saturates, and that the calculation is very simple and it can be implemented in most commercial PID packages. There is no need to change to a different set of control parameters in  $C_2$  during saturation. A downside is the potential risk of over-using  $u_2$  with the risk of exciting

unmodelled dynamics or violating constraints in  $u_2$ . It is therefore essential to use a value of  $\rho$  that reflects the given process and its constraints. The modified closed-loop system should also be evaluated in simulations to verify performance.

## V. COOLING SYSTEM

In this section the previously derived guidelines are applied on a different process to show that the results derived are in fact general. The process is a multi-purpose cooling system, which should transfer heat released from an exothermic reaction inside a heat exchange reactor [10].

The control variables  $u_1$  and  $u_2$  are the desired positions of two control valves and the output  $y$  is the inlet temperature of the cooling water for the reactor. The process can be approximated with two transfer functions with a common denominator, due to the cross-couplings introduced by the recycle loops. The dynamics correspond to the water temperatures of the cold and warm side of the heat exchanger and the cold side of the reactor, respectively.

$$y = \frac{-7.8(s+0.14)(s+0.036)u_1 - 1.1(s+1.9)(s+0.14)u_2}{(s+0.016)(s+0.017)(s+1.1)} \quad (19)$$

The process is well suited for mid-ranging control, due to the different dynamics from  $u_1$  to  $y$  and  $u_2$  to  $y$ .

The reference temperature for the cooling water is calculated by an MPC controller for the chemical reactor, see [10]. The objective of the cooling system controller is to track the given reference temperature as well as possible. The cooling temperature is solely determined by  $u_1$ . Large reference changes or disturbances may cause saturation of  $u_1$ . To improve performance and increase the operating region of the hydraulic equipment,  $u_2$  is manipulated to mid-range  $u_1$  to the desired value  $u_{1,ref}$ . To have suitable control margin  $u_{1,ref} = 0$ , which corresponds to an actual valve position of 50%. The constraints in  $u_1$  are thus  $\pm 0.5$ .

PI-controllers are used for both  $C_1$  and  $C_2$ . The nominal control parameters in the following discussions are  $K_1 = -0.03, T_{i1} = 1.0$  seconds,  $K_2 = -0.05, T_{i2} = 30$  seconds.

### A. Anti-windup for the cooling system

Also for the cooling system, a fast time constant  $T_i$  of the anti-windup scheme is not always advantageous. In Fig. 8 the closed-loop system is simulated when there is a step in the temperature reference with  $r = 10^\circ\text{C}$  above the linearized operating point. The response is slow for fast anti-windup, whereas slow anti-windup gives faster response, but a larger overshoot due to windup.

Bode diagrams for the transfer functions (5) and (14) for different values of  $T_i$  are evaluated. By visual inspection  $T_i = 4T_{i1}$  gives the saturated system a similar frequency response as the unsaturated system, see Fig. 9. As predicted, fast anti-windup gives lower gain, whereas slow anti-windup gives higher gain than the unsaturated case.

The step response for  $T_i = 4T_{i1}$  is seen in Fig. 8. Note that the control action in  $u_2$  is almost identical for the unsaturated

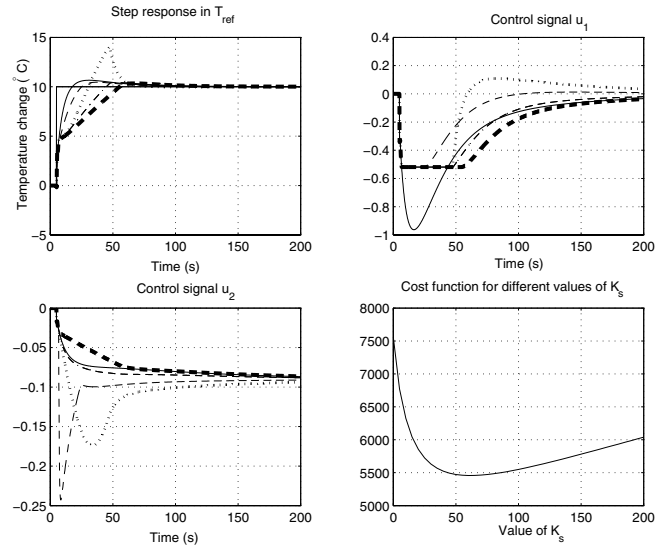


Fig. 8. Step responses for the cooling system. Nominal case without any constraints (solid), with fast anti-windup  $T_i = 0.5$  (thick dashed), with no anti-windup (dotted), the case with  $T_i = 4T_{i1}$  and  $K_s = 0$  (dash-dot) and finally the case with the modified structure with fast anti-windup  $T_i = 0.5$  and  $K_s = 60$  (thin dashed). Lower right: cost function (18) evaluated for different values of  $K_s$  using  $\rho = 150$  when  $T_i = 0.5T_{i1}$ .

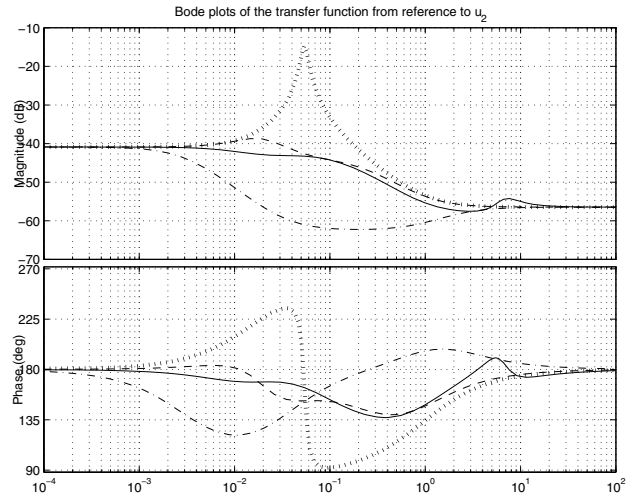


Fig. 9. Bode plots of the transfer function from  $r$  to  $u_2$  for cooling system. Unsaturated case (solid), the saturated cases when  $T_i = 4T_{i1}$  (dashed),  $T_i = 0.5T_{i1}$  (dash-dot) and  $T_i = 50T_{i1}$  (dotted).

case and the case when  $T_i = 4T_{i1}$ , which was the purpose of our choice of  $T_i$ .

### B. Modified anti-windup scheme for the cooling system

To improve performance of the cooling system when  $u_1$  saturates, the modified anti-windup scheme presented in Section IV-B can be used. It is important to increase the control action in  $u_2$  with caution, since higher controller bandwidth might excite unmodelled dynamics or also cause  $u_2$  to saturate.

The cost function (18) is used. Based on available control authority in  $u_2$ , a reasonable choice of  $\rho$  is 150. If less control action in  $u_2$  is desired,  $\rho$  should be increased. From

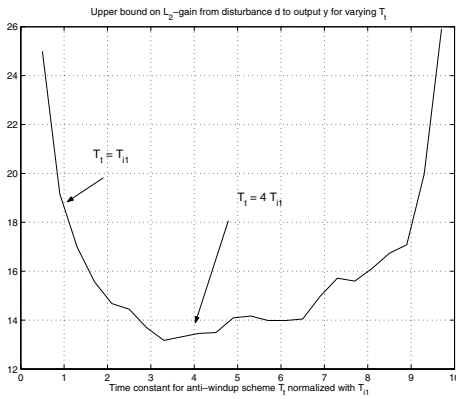


Fig. 10. An upper bound on the  $L_2$ -gain from  $d$  to  $y$  for varying  $T_i$  for the cooling system.

simulations, the cost function is evaluated for varying  $K_s$ , see Fig. 8. Standard fast anti-windup is used,  $T_i = 0.5T_{i1}$ . There is a clear minimum for  $K_s = 60$ . With the modified anti-windup scheme the performance is significantly improved, due to the increased use of  $u_2$  during the short time  $u_1$  is saturated.

## VI. GLOBAL STABILITY AND PERFORMANCE ANALYSIS

Input-output stability for the bio-reactor using the proposed method for choice of  $T_i$  has been ensured using the circle criterion in [9]. We can also use the fact that the closed-loop system is piecewise linear for both the conventional anti-windup scheme discussed in Section IV-A and the modified scheme in section IV-B. Various tools are available for stability and performance analysis of such systems, see for example [11] and [12].

The proposed choice of  $T_i$  is based on local analysis and showed improvement compared to the case with a standard choice of  $T_i \leq T_i$ . To provide a global analysis, the cooling system from Section V is studied using algorithms from [11]. Stability can be derived using a piecewise quadratic Lyapunov function.

The performance is verified by calculating an upper bound of the  $L_2$ -gain from a disturbance  $d$  on the control signal  $u_1$  to  $y$  for various values of  $T_i$ , see Fig. 10. We see that the extreme cases with fast or slow anti-windup have a significantly larger  $L_2$ -gain from  $d$  to  $y$ . Note also that the standard choice  $T_i \leq T_i$  also gives unnecessarily low performance. The results from the local analysis indicating that  $T_i = 4T_{i1}$  gives better results are confirmed by the global analysis.

## VII. SUMMARY

We have presented two rather different applications of mid-ranging control, a bio-reactor and a cooling system. We have examined the effects of anti-windup for the fast controller  $C_1$  when the corresponding control output  $u_1$  is subject to saturation. It has been shown that default choices of anti-windup constant  $T_i$  are not applicable to the mid-ranging control structure since they decrease the control authority of the slow control variable  $u_2$ . However, removing the

anti-windup mechanism leads to the characteristic windup phenomenon and potentially even to instability.

We have suggested a method to choose the anti-windup parameter  $T_i$  that aims to minimize the difference in control action of  $u_2$  between the nominal and the saturated case. This is achieved by examining the Bode diagrams of the transfer function from  $r$  to  $u_2$  for the nominal case as well as the saturated case for different values of  $T_i$ . The advantage of this method is that it deviates as little as possible from the original control design to avoid excitation of unmodelled process dynamics and cross-couplings between the control variables. The method improves the performance for both of the studied processes when  $u_1$  saturates, compared to using either no anti-windup or a default choice of  $T_i$ .

However, the saturation naturally leads to inferior performance compared to the unsaturated case. If the process configuration allows for increased bandwidth of the slow controller  $C_2$  when  $u_1$  saturates, we suggest a modified anti-windup structure that does not change the nominal controller design, but gives an increased control input to  $C_2$  when  $u_1$  saturates. This structure is parameterized in a constant  $K_s$ . Using numerical optimization of an integral square cost function (18) to determine  $K_s$ , it is here possible to obtain a performance comparable to the nominal case without altering the nominal design.

The stability and performance of the global system with saturation can be analysed using theory of piecewise linear systems. The result from the global analysis supports the local analysis on how to choose a suitable anti-windup scheme.

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