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Education, lifetime labor supply, and longevity improvements

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Education, lifetime labor supply, and longevity improvements[☆]

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Abstract

This paper presents an analysis of the differential role of mortality for the optimal schooling and retirement age when the accumulation of human capital follows the so-called "Ben-Porath mechanism". We set up a life-cycle model of consumption and labor supply at the extensive margin that allows for endogenous human capital formation. This paper makes two important contributions. First, we provide the conditions under which a decrease in mortality leads to a longer education period and an earlier retirement age. Second, those conditions are decomposed into a Ben-Porath mechanism and a lifetime-human wealth effect vs. the years-to-consume effect. Finally, using US and Swedish data for cohorts born between 1890 and 2000, we show that our model can match the empirical evidence.

Keywords: Mortality decline, Early retirement, Increasing length of schooling,
Ben-Porath mechanism, Years-to-consume effect, Lifetime human wealth effect
JEL classification: I25, J10, J24, J26.

1. Introduction

In many countries, economic development has been accompanied with significant increases in life expectancy and reductions in labor supply. Over the nineteenth and twentieth century record life expectancy (at birth) has increased by 40 years at a rate of 3 months per year (Oeppen and Vaupel, 2002; Lee, 2003), while labor supply has decreased at two extensive margins: later entrance in the labor market and earlier retirement.[1] These historical trends are shown in Figure 1 for males born in the United States from 1850 to 1930. Between the 1850 and 1930 cohorts, average years of schooling increased from 8 to 13.28 (an increase of 5 years), average retirement age decreased from 69 to 63.8 (a declined of 5 years), and the life expectancy at age 5 increased from 52.5 to 66.7 (an increase of 14 years).
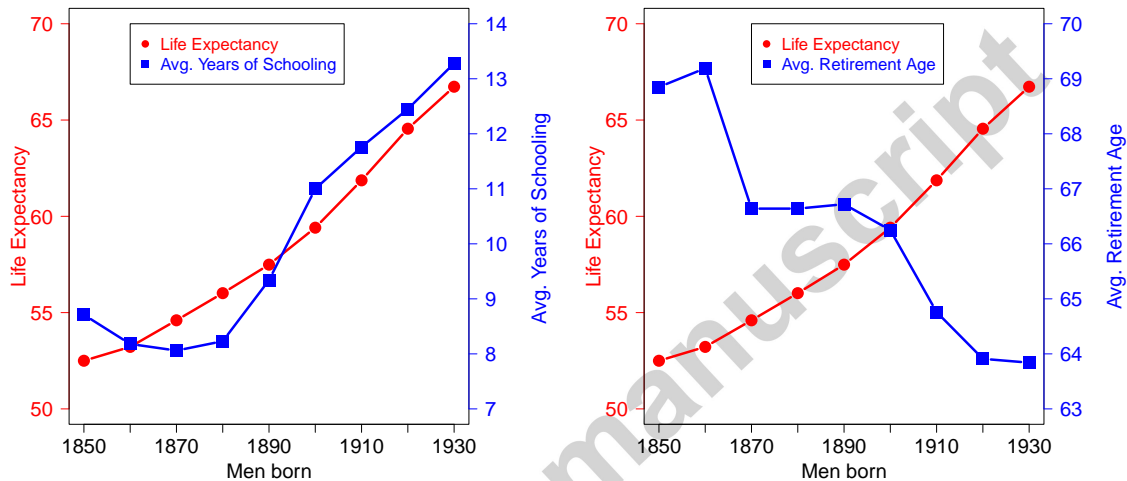
Figure 1: Life expectancy at age 5, average years of schooling, and average retirement age for US men by birth cohort

Source: Hazan (2009). The average retirement age were calculated using the labor force participation rates from age 45 to 80.

These trends are not exclusively of the United States but are, instead, common to most advanced countries. Prior to industrialization, male literacy rates started to increase in the most advanced countries (Cippola, 1969; Cervellati and Sunde, 2005; Boucekkine et al., 2007). This process continued with an expansion of primary education enrollment rates at the end of the nineteenth century and first half of the twentieth century (Benavot and Riddle, 1988). By 1950, the average length of schooling for males was around six years in the most advanced countries and has increased up to twelve years in 2010 (Barro and Lee, 2013). Over the same period, labor force participation rates for old workers started to fall until recently, even before the introduction of pension systems

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[1]Before 1950, most of the gain in life expectancy was due to large reductions in death rates at younger ages (Oeppen and Vaupel, 2002). Still, although at a lower rate, the life expectancy at ages 6 and 16 have also risen for the last 160 years in developed countries like Sweden at a steady pace of $1\frac{3}{4}$ and $1\frac{1}{2}$ months per year, respectively.

(Costa, 1998; Schieber and Shoven, 1999). In 1970, the average retirement age was 68 in OECD countries and has declined to age 63 in 2010 – see OECD (2009).

Existing theoretical models that analyze the effect of mortality on education and retirement implicitly assume a positive causal relationship between the length of schooling, retirement, and life expectancy; for example Boucekkine et al. (2002), Echevarria (2004), Echevarria and Iza (2006), Ferreira and Pessoa (2007), and Zhang and Zhang (2009), among others. This is because an increase in the length of schooling above the optimum needs to be compensated by working longer in order to maintain the same level of consumption. Thus, according to the existing literature, a decline in mortality leads to an increase in education and a postponement of the retirement age, which contradicts the historical empirical evidence. The motivation of our study is therefore to provide possible explanations under which a decrease in mortality leads to an increase in schooling and early retirement.

There are two important dimensions to be considered when modeling the effect of mortality on schooling and retirement. First, the observed positive link between human capital investment and life expectancy that is theoretically replicated through the well-known Ben-Porath (1967)'s mechanism (de la Croix and Licandro, 1999; Kalemli-Ozcan et al., 2000; Zhang et. al., 2001, 2003; Cervellati and Sunde, 2005; Soares, 2005; Zhang and Zhang, 2005; Jayachandran and Lleras-Muney, 2009; Oster et al., 2013), except for Hazan and Zoabi (2006). Second, the recent findings showing that the link between the life expectancy and labor supply depends on the age pattern of mortality improvements. In particular, on the one side, for a given retirement age it has been shown that only improvements in survival during prime-working ages –and not longevity per se– increase human capital investment (Cervellati and Sunde, 2013; de la Croix, 2015). On the other, for a given educational attainment mortality declines during adulthood may cause early retirement, while reductions in mortality at older ages delay retirement (d'Albis et al., 2012; Strulik and Werner, 2012).[2]

In this paper, we set up a life-cycle model of consumption and labor supply at the extensive margin that allows for endogenous human capital formation through the Ben-Porath's mechanism. First, we explain the differential role of mortality on the optimal schooling choice and retirement choice. Second, we use the model to study whether the observed decline in mortality across cohorts born in US and Sweden can produce a monotonic increase in schooling followed by a decline in retirement. Our model has one key feature. Following the literature on human capital formation, individuals have a relative disutility from attending school, or aversion to schooling time, like in Heckman et al. (1998), Bils and Klenow (2000), Card (2001), Oreopoulos (2007), Restuccia and Vandenbroucke (2013). This feature has been shown to be important to account for the substantial difference between the returns to schooling and the marginal cost of schooling (Oreopoulos, 2007).[3] Our model differs from the previous literature

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[2]Empirical investigations of the mortality decline over the last two centuries show that mortality does not improve uniformly across age groups (Lee, 1994; Wilmoth and Horiuchi, 1999; Cutler et al., 2006). Early stages of the mortality transition are mainly characterized by reductions of mortality for infants and children, while recent mortality declines occur at older ages.

[3]The empirical work on the returns to education suggests that the aversion to schooling time captures sizable nonpecuniary effects of schooling. Examples of negative nonpecuniary effects of schooling are high psychic costs of school and higher risk and uncertainty (Carneiro and Heckman, 2003; Carneiro et al., 2003; Cunha et al., 2005; Heckman et al., 2006), while suggested positive nonpecuniary effects of compulsory schooling are fostering trust and the reduction of teen fertility, criminal activity, or smoking

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in two aspects. First, we model the labor supply decision at the extensive margin (retirement) rather than at the intensive margin (hours worked). Thus, we complement the recent work by Restuccia and Vandenbroucke (2013), who have shown that the increase in life expectancy during the last century only counts for 3% of the decline in hours worked in the US, by empirically showing the effect of life expectancy on retirement. Second, since the decline in mortality does not occur uniformly across age-groups, following d'Albis et al. (2012) we model the age-specific mortality rates non-parametrically. Thus, using the derivative of a functional (Ryder and Heal, 1973; d'Albis et al., 2012), we analyze the impact of a mortality decline at any arbitrary age on human capital investment and retirement.

This paper provides two important contributions. First, we find that when there exists an aversion to schooling time, an increase in the length of schooling might lead to a decline in the retirement age. The intuition is as follows. Individuals who are averse to schooling do not maximize their lifetime income, since they prefer to invest less into education and therefore anticipate their entrance in the labor market. Thus, under such a setting, by extending the length of schooling lifetime income rises (income effect), which is used to increase consumption and leisure (i.e., early retirement age). On the other, by extending the length of schooling, the increase in the wage rate raises the marginal benefit of working (substitution effect). The net effect of these two opposite effects depends on the strength of the income effect relative to the substitution effect. Hence, if the income effect dominates over the substitution effect, the optimal length of schooling and the optimal retirement age will be negatively related. Thereby, only when we assume that there exists aversion to schooling time we can replicate that an increase in life expectancy may cause an increase in the length schooling and early retirement, reconciling the empirical facts with the economic theory. Second, using a general utility function we provide the economic intuition of our results by decomposing the differential effect of mortality on schooling and retirement into a *Ben-Porath mechanism* and a *lifetime human wealth* effect vs. the *years-to-consume* effect. The *Ben-Porath mechanism* is the positive effect on the marginal benefit of schooling caused by gains in life expectancy. *Lifetime human wealth* effect stands for the positive impact that a mortality decline has on consumption because it raises the likelihood of receiving a future labor income stream. On the contrary, the *years-to-consume* effect, which is always negative, reflects the overall reduction in consumption due to a longer lifespan.

We also perform a quantitative exercise of our model using US and Swedish mortality data. We restrict the parameters of the model so that it reproduces the years of schooling and retirement age observed in the data for the cohort born in 1890.[4] Then, we compute cohort-specific sequences of years of schooling and retirement ages assuming that each cohort faces a different survival probability. Using a stylized model, we find that improvements in the survival probability may account for a decline in the retirement age around 1.2 years (or 40% of the total decline) and an increase in years of education around 1.2 year (or 30%) between the 1890 and 1930 cohorts. Thereby, our results suggest that the

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(Oreopoulos and Salvanes, 2011), among others.

[4]Starting from this cohort guarantees that in both countries individuals received pension benefits upon retirement from the PAYG pension system. In the United States the Social Security Act was signed in 1935 by Franklin D. Roosevelt. In Sweden the old-age pension system switched from a fully funded system that dates back to 1913 to a pay-as-you-go system in 1935 (Palme, 2005). The universal coverage was achieved in 1946 in Sweden, while more than 60% of workers were covered by the Social Security in 1940 in the US (Schieber and Shoven, 1999).

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effect of increases in life expectancy on the labor supply is stronger than previously suggested. In addition, since in the earlier stage of mortality transition, a decline in mortality mainly occurred to younger people, whereas in the later stage, a decline in mortality has mainly occurred at older ages, we show that the optimal retirement age bottomed out for cohorts born in the 1920s and it is expected to increase from now on.

The paper is organized as follows: Section 2 introduces the model setup and presents the first-order conditions for optimal consumption, length of schooling, and retirement. Furthermore, the relationship between the optimal length of schooling and retirement is explained. In Section 3, we study –using the Volterra derivative of a functional– the differential role of mortality on the optimal length of schooling and retirement. In Section 4, we solve the model numerically and demonstrate, using a simple quantitative exercise, how the mortality transition may increase the length of schooling and reduce the retirement age. Concluding remarks are made in Section 5.

2. The model

We setup a consumer's problem that consists in choosing the optimal number of years of schooling (S), optimal retirement age (R), and the optimal consumption path $(c(x))$ in order to maximize the expected lifetime utility $(V(S, R, c))$. We assume time is continuous. Agents face lifetime uncertainty, which is represented by the survival function

$$p(x) = e^{-\int_0^x \mu(q)dq}, \tag{1}$$

where $p(x)$ is the (unconditional) probability of surviving to age x, $p(0) = 1$, $p(\omega) = 0$, $\omega \in (0, \infty)$ denotes the maximum age, and $\mu(q) \geq 0$ is the mortality hazard rate at age q.

Schooling and labor supply are indivisible and the transitions from schooling to working and from working to retirement are irreversible, as in Boucekkine et al. (2002), Echevarria (2004), Echevarria and Iza (2006), and Cai and Lau (2012). We also assume that agents do not save with a bequest motive in mind and there exists a perfect annuity market, which grants that agents borrow and lend freely at a fixed interest rate. Thereby, consumers optimally choose to purchase annuities (Yaari, 1965). The instantaneous expected utility depends positively on current consumption and negatively on current non-leisure time. The utility of consumption $U(c)$ is an increasing and concave function (i.e. $U_c(\cdot) > 0$, $U_{cc}(\cdot) < 0$).[5] Let $\tilde{\phi}(S, x)$ denote the disutility of non-leisure time at age x of an individual who has completed S years of schooling. Assume $\tilde{\phi}(S, x)$ is a positive and increasing function with respect to age (i.e. $\tilde{\phi}(S, x) > 0$, $\tilde{\phi}_x(S, x) > 0$), which reflects the fact that the disutility of not enjoying leisure is increasing with age (Hazan, 2009; Kalemli-Ozcan and Weil, 2010; d'Albis et al., 2012; Cai and Lau, 2012). After retirement, $\tilde{\phi}(S, x)$ equals zero. Then, assuming that agents discount future utility flows at a subjective discount rate ρ, the expected lifetime utility, conditional on the years of schooling (S), retirement (R), and consumption path (c) is

$$V(S, R, c) = \int_0^\omega e^{-\rho x} p(x) U\big(c(x)\big)dx - \int_0^R e^{-\rho x} p(x) \tilde{\phi}(S, x)dx. \tag{2}$$

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[5]We use subscripts to denote the derivative with respect to the variable in the subscript, and apply the same notation for partial derivatives.

Following Heckman et al. (1998), Bils and Klenow (2000), Card (2001), Oreopoulos (2007), Restuccia and Vandenbroucke (2013), we assume agents may have different preferences between schooling time and working time

$$\tilde{\phi}(S,x) = \begin{cases} \phi(x) + \psi(x) & \text{if } x \leq S, \\ \phi(x) & \text{if } x > S, \end{cases} \tag{3}$$

where $\phi(x) > 0$ (with $\phi_x(x) \geq 0$) is the underlying disutility of non-leisure time and $\psi(x)$ is the relative disutility from attending school or aversion to schooling time. Factor $\psi(x)$ is positive when the agent prefers work to schooling or negative when schooling is preferred to work. We assume that if $\psi(x)$ is positive, the aversion to schooling time increases with age (i.e. $\psi_x(x) \geq 0$), whereas if $\psi(x)$ is negative, our agent has a decreasing preference for schooling, or $\psi_x(x) \leq 0$. As a particular case, notice $\psi(x) = 0$ for all $x \in (0,\omega)$ is implicitly assumed in Boucekkine et al. (2002), Echevarria (2004), Echevarria and Iza (2006), Ferreira and Pessoa (2007), Zhang and Zhang (2009), Kalemli-Ozcan and Weil (2010), and Cai and Lau (2012), among many others.

Labor income, denoted by y, is assumed to be proportional to years of schooling (S). We write labor income at age x conditional on S years of schooling as $y(S,x) = wh(x)$, where $w > 0$ represents the wage rate per unit of human capital and $h(x)$ is the stock of human capital of an individual at age x with S years of schooling.[6] Assume the law of motion of human capital of an individual at age x with S years of schooling accumulates according to a Ben-Porath (1967) technology

$$h_x(x) = \begin{cases} q(h(x)) - \delta h(x) & \text{if } x \leq S \\ -\delta h(x) & \text{otherwise,} \end{cases} \tag{4}$$

where $q(\cdot)$ is the human capital production function (with $q_h(\cdot) > 0$ and $q_{hh}(\cdot) < 0$), and $\delta > 0$ is the human capital depreciation rate, which is assumed constant across age.[7] As a result, the law of motion of financial wealth at age x ($a(x)$) is

$$a_x(x) = \begin{cases} [r + \mu(x)]a(x) + y(S,x) - c(x) & \text{if } S < x < R, \\ [r + \mu(x)]a(x) - c(x) & \text{otherwise,} \end{cases} \tag{5}$$

with boundary conditions $a(0) = 0$ and $a(\omega) = 0$, where r is the real interest rate. Integrating (5) with respect to age, subject to the boundary conditions, we obtain the standard lifecycle budget constraint faced by our individual:

$$\int_0^\omega e^{-rx}p(x)c(x)dx = \int_S^R e^{-rx}p(x)y(S,x)dx \equiv W(S,R), \tag{6}$$

where $W(S,R)$ is the lifecycle earnings (measured at age 0) conditional on S years of schooling and retirement age R. For the sake of comparison with the literature on the impact of mortality on retirement and education, notice that we implicitly assume that

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[6]Note that a constant wage rate per unit of human capital can be used without lost of generality. In AppendixA we show that a sufficient condition for our results to hold is that the wage rate per unit of human capital must be a non-increasing function upon retirement.

[7]The functional form $h(S) = e^{\theta(S)}$, used by Hazan (2009), p. 1834, can be obtained assuming either that $\delta = 0$ or that $q(h(x))$ is equal to $(\theta_x(x) + \delta)h(x)$.

the only pecuniary cost of schooling is foregone labor income (Kalemli-Ozcan et al., 2000; Hazan, 2009; Cai and Lau, 2012; Cervellati and Sunde, 2013). Tuition costs, earnings while in school, and taxes are also modeled in the returns to education literature (Willis, 1986; Card, 2001; Heckman et al., 2006).

2.1. Optimal consumption, length of schooling, and retirement age

Following d'Albis and Augeraud-Véron (2008), Heijdra and Romp (2009), and d'Albis et al. (2012) we obtain our agent's optimal consumption path, length of schooling, and retirement in two steps. First, we derive the optimal consumption path. We define the optimal consumption at age x, conditional on the length of schooling (S) and retirement age (R), as $c(x, S, R)$. Second, based on the conditional consumption path derived in the first step, we obtain the optimal length of schooling and retirement age. Let us define $\hat{V}(S, R)$ as the expected lifetime utility conditional on the optimal consumption path.

In Proposition 1, we characterize the optimal consumption path, the optimal length of schooling, and the optimal retirement age. The proof is given in AppendixA.

Proposition 1. *For the life-cycle model given by (1)-(5), the optimal consumption path, conditional on a length of schooling S and a retirement age R, is characterized by*

$$U_c\big(c(x, S, R)\big) = e^{(\rho-r)x}U_c\big(c(0, S, R)\big). \tag{7}$$

Moreover, an interior optimal length of schooling (S^*) satisfies

$$\int_{S^*}^{R} e^{-r(x-S^*)}\frac{p(x)}{p(S^*)}y_S(S^*, x)dx = y(S^*, S^*) + \frac{e^{(r-\rho)S^*}\psi(S^*)}{U_c\big(c(0, S^*, R)\big)}, \tag{8}$$

and an interior optimal retirement age (R^*) is given by

$$U_c\big(c(0, S, R^*)\big)e^{-rR^*}y(S, R^*) = e^{-\rho R^*}\phi(R^*). \tag{9}$$

Eq. (7) is the standard Euler condition characterizing the consumption path. The left-hand side of Eq. (8) is the marginal benefit of the S^*-th year of schooling, whereas the right-hand side represents the marginal cost of the S^*-th year of schooling. The first term is the foregone earnings, or pecuniary cost of schooling, and the second term represents the nonpecuniary cost (if $\psi(S^*) > 0$) or benefit (if $\psi(S^*) < 0$) from attending schooling. Let us define $f(S, R)$ as the marginal effect of an additional unit of schooling (measured at age S) on lifecycle earnings:

$$f(S, R) \equiv \frac{W_S(S, R)}{e^{-rS}p(S)} = \int_{S}^{R} e^{-r(x-S)}\frac{p(x)}{p(S)}y_S(S, x)dx - y(S, S), \tag{10}$$

or, equivalently, the marginal benefit of the S-th year of schooling minus the foregone labor income at age S (measured at age S). From (4) Eq. (10) can be rewritten, after rearranging, as

$$f(S, R) = \frac{W(S, R)}{e^{-rS}p(S)}\left(\frac{q(h(S))}{h(S)} - \delta - r - \frac{\int_{S}^{R} e^{-(r+\delta)x}\mu(x)p(x)dx}{\int_{S}^{R} e^{-(r+\delta)x}p(x)dx} - \frac{e^{-rR}p(R)y(S, R)}{W(S, R)}\right), \tag{11}$$

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where $q(h(S))/h(S) - \delta$ is the rate of return to education at the S-th unit of schooling (henceforth $r^h(S)$). The third and fourth terms inside the parenthesis represent the average return lost in the capital market from postponing the entrance in the labor market. Specifically, the fourth term is the expected mortality premium –at age 0– lost from the S-th unit of schooling, which hereinafter we denote by $\bar{\mu}_{[S,R]}$. The last term is the income lost at retirement relative to the lifetime wealth. For notational convenience, let us denote the sum of the last three negative terms in (11) as $\bar{r}(S,R)$; that is

$$\bar{r}(S,R) = r + \bar{\mu}_{[S,R]} + \frac{e^{-rR}p(R)y(S,R)}{W(S,R)}. \tag{12}$$

Eq. (12) represents the hurdle rate or annuitized marginal cost of the S-th unit of schooling, expressed in terms of foregone earnings, conditional on the retirement age R.[8] Assuming there is no mortality risk and considering that R tends to infinity, Eq. (12) reduces to the real interest rate. Thus, if $\psi(S^*)$ is zero, we obtain the result that individuals invest in schooling until the marginal return to education equals the return to capital, see Willis (1986).

Substituting (10)-(12) in (8), and rearranging, gives

$$r^h(S^*) = \bar{r}(S^*, R) + \frac{e^{-\rho S^*}p(S^*)\psi(S^*)}{W(S^*,R)U_c(c(0,S^*,R))}. \tag{13}$$

Eq. (13) implies that the return to education at the S^*-th unit of schooling is equal to the sum of the marginal cost of the S^*-th unit of schooling expressed in terms of foregone earnings and the nonpecuniary cost/benefit of schooling. Eq. (13) implies that when working is preferred to schooling ($\psi(S^*) > 0$), individuals underinvest in education since $r^h(S^*) > \bar{r}(S^*, R)$. In contrast, when schooling is preferred to work ($\psi(S^*) < 0$), individuals over-investment in education since $r^h(S^*) < \bar{r}(S^*, R)$. As a consequence, if education were considered a pure investment good ($\psi(S^*) = 0$), $r^h(S^*) = \bar{r}(S^*, R^*)$.

Empirically, the econometric estimations of returns to education report values of $r^h(S^*)$ exceeding those of $\bar{r}(S^*, R^*)$. For example, Card (1999) finds a wide range of rates of returns to education in the US centered around 8% per year, while Heckman et al. (2008) estimate also for the US that the returns to education range between 10 to 15% per year. In contrast, when education is considered a pure investment good, the rate of return to education for an individual with 10 years of education does not exceed 3% per year for a wide range of feasible retirement ages.[9] Several explanations are suggested in the literature for the positive difference between $r^h(S^*)$ and $\bar{r}(S^*, R^*)$. The most common ones are high "psychic cost" of school, uncertainty, and heterogeneity among individuals (Carneiro et al., 2003; Cunha et al., 2005; Heckman et al., 2006), the myopic behavior of adolescents (Oreopoulos, 2007), while credit constraints might be important for going to college decisions (Belley and Lochner, 2007), but not for most students (Carneiro and Heckman, 2002; Heckman et al., 2006). Henceforth, following the literature on returns to education, we assume hereinafter that $\psi(x) > 0$ for all $x \in (0, S)$. As a consequence, $r^h(S^*) > \bar{r}(S^*, R)$.

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[8]The hurdle rate is: the minimum return required to make an individual financially better off from taking one year of school instead of one year of work (Oreopoulos, 2007).

[9]A value of 3% has been calculated based on the wage rate per unit of human capital $\log w(x - S) = \log w(0) + 0.094(x - S) - 0.0013(x - S)^2$ withdrawn from Table 2 (Heckman et al., 2006, p. 326), US death rates of males from the cohort born in year 1900 (Bell et al., 1992), an interest rate of 3%, and no human capital depreciation rate.

Eq. (9) is the optimal retirement age condition. Eq. (9) implies that the marginal benefit of continued working at age R^*, which is equivalent to the additional labor income at age R^* measured in utility terms, equals the marginal cost of working at age R^*, or the disutility of continued working at age R^*. This optimal retirement age condition was first derived by Sheshinski (1978).

The first important results one can obtain from Proposition 1 are the effects of an increase in the optimal length of schooling and retirement age on the optimal consumption path. Differentiating (6) and (7) with respect to S, substituting, and using (10) gives

$$\frac{c_S(0, S^*, R)}{c(0, S^*, R)} = \frac{e^{-rS^*}p(S^*)\sigma(c(0, S^*, R))f(S^*, R)}{\int_0^\omega e^{-rx}p(x)\sigma(c(x, S^*, R))c(x, S^*, R)dx}. \tag{14}$$

where

$$\sigma(c) = -\frac{U_c(c)}{c \cdot U_{cc}(c)} > 0, \tag{15}$$

is the intertemporal elasticity of substitution (IES) for consumption c. Using (6) and (11)-(13), Eq. (14) becomes[10]

$$\frac{c_S(0, S^*, R)}{c(0, S^*, R)} = \frac{\sigma(c(0, S^*, R))}{\sigma(c(\bar{x}, S^*, R))}(r^h(S^*) - \bar{r}(S^*, R)), \tag{17}$$

Assuming a constant IES, Eq. (17) implies that the relative increase in the initial consumption due to an additional unit of schooling is equal to the difference between the return to education and the marginal cost of the S^*-th unit of schooling expressed in terms of foregone earnings. As a consequence, an additional investment in schooling is efficient when $r^h(S^*) > \bar{r}(S^*, R)$, and inefficient when $r^h(S^*) < \bar{r}(S^*, R)$.

To analyze the impact of retirement on the optimal consumption path we differentiate (6) and (7) with respect to R. Substituting and using (9), we have

$$\frac{c_R(0, S, R^*)}{c(0, S, R^*)} = \frac{e^{-rR^*}p(R^*)\sigma(c(0, S, R^*))y(S, R^*)}{\int_0^\omega e^{-rx}p(x)\sigma(c(x, S, R^*))c(x, S, R^*)dx}, \tag{18}$$

which is equivalent to

$$\frac{c_R(0, S, R^*)}{c(0, S, R^*)} = \frac{\sigma(c(0, S, R^*))}{\sigma(c(\bar{x}, S, R^*))}\frac{e^{-rR^*}p(R^*)y(S, R^*)}{W(S, R^*)}. \tag{19}$$

For a constant IES, Eq. (19) states that the relative impact of delaying retirement on the initial consumption is equal to the weight of labor income at age R^* in lifecycle earnings. Thereby, contrary to an increase in the length of schooling, an increase in the retirement age always raises the optimal consumption path because the agent receives an additional labor income at age R^*.

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[10]Applying the mean value theorem for integration, there exists an $\bar{x} \in (0, \omega)$ such that

$$\int_0^\omega e^{-rx}p(x)\sigma(c(x, S^*, R))c(x, S^*, R)dx$$

$$= \sigma(c(\bar{x}, S^*, R))\int_0^\omega e^{-rx}p(x)c(x, S^*, R)dx = \sigma(c(\bar{x}, S^*, R))W(S^*, R). \tag{16}$$

2.2. *Relationship between years of schooling and retirement*

In the previous subsection we have shown the first-order conditions for an optimum of S^* and R^*, separately, and how they impact on the optimal consumption path. In this subsection, we turn to a detailed study about the relationship between the optimal years of schooling and the optimal retirement age.

Let us denote $c(0, S^*, R^*)$ as c^* and $c(\bar{x}, S^*, R^*)$ as $\bar{c}$. Applying the implicit-function theorem to the first order condition for S^* around the point (S^*, R^*), we can examine the impact on the optimal length of schooling of a change in the retirement age. Totally differentiating (8) with respect to R and S, taking $\psi(S^*)$ as common factor, and rearranging, we obtain

$$\left.\frac{dS^*}{dR}\right|_{R=R^*} = \frac{\frac{f_R(S^*,R^*)}{f(S^*,R^*)} - \frac{1}{\sigma(c^*)}\frac{c_R^*}{c^*}}{\frac{\psi_S(S^*)}{\psi(S^*)} + \frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*} - \frac{f_S(S^*,R^*)}{f(S^*,R^*)}}. \tag{20}$$

Similarly, applying the implicit-function theorem, we totally differentiate (9) with respect to S and R to examine the impact on the optimal retirement age of a change in the length of schooling

$$\left.\frac{dR^*}{dS}\right|_{S=S^*} = \frac{-\frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*} + r^h(S^*) + \delta}{\frac{1}{\sigma(c^*)}\frac{c_R^*}{c^*} + \delta + \frac{\phi_R(R^*)}{\phi(R^*)}}. \tag{21}$$

Provided (S^*, R^*) is an interior solution of our problem, substituting (10) and (19) in (20)-(21), and using (12), we have[11]

$$\text{sign}\left[\frac{dS^*}{dR^*}\right] = \text{sign}\left[\frac{\bar{r}(S^*, R^*) + \sigma(\bar{c})\delta}{1 - \sigma(\bar{c})} - r^h(S^*)\right], \tag{22}$$

Eq. (22) implies that the length of schooling S^* and the retirement age R^* may be either positively or negatively related. On the one hand, looking at Eq. (21), we have that an additional year of schooling after age S^* increases the labor income at age R^* by $r^h(S^*) + \delta$, which increases the marginal benefit of working. As a consequence, our individual optimally postpones the retirement age in order to reap the benefits of schooling. On the other hand, the increase in education may also change the marginal utility of consumption, and hence the marginal benefit of working, by $-\frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*}$.[12] Thus, the net effect of a change in education on retirement depends upon the strength of the income effect, reflected by the IES (İmrohoroğlu and Kitao, 2009; Keane, 2011), and the difference between $r^h(S^*)$ and $\bar{r}(S^*, R^*)$. At the extreme cases, when $\sigma(\bar{c})$ tends to one or $\bar{r}(S^*, R^*) = r^h(S^*)$, we have that the sign of $\frac{dS^*}{dR}\big|_{R=R^*}$ and $\frac{dR^*}{dS}\big|_{S=S^*}$ depend on the sign of $\bar{r}(S^*, R^*) + \delta$, which is always positive.

Figure 2 summarizes the result obtained in Eq. (22). For any given wage rate per unit of human capital, Figure 2 is divided into two shaded areas. A dark gray area that contains the combination of $(r^h(S^*), \sigma(\bar{c}))$ values for which S^* and R^* are positively related,

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[11]From now on, we use the term dS^*/dR^* as a shorthand notation for $\frac{dS^*}{dR}\big|_{R=R^*}\Big/\frac{dR^*}{dS}\big|_{S=S^*}$.

[12]Differentiating (7) with respect to S, we have

$$\frac{1}{\sigma(c^*)}\frac{c_S^*}{c^*} = \frac{1}{\sigma(c(x, S^*, R^*))}\frac{c_S(x, S^*, R^*)}{c(x, S^*, R^*)} \text{ for all } x \in [0, \omega).$$
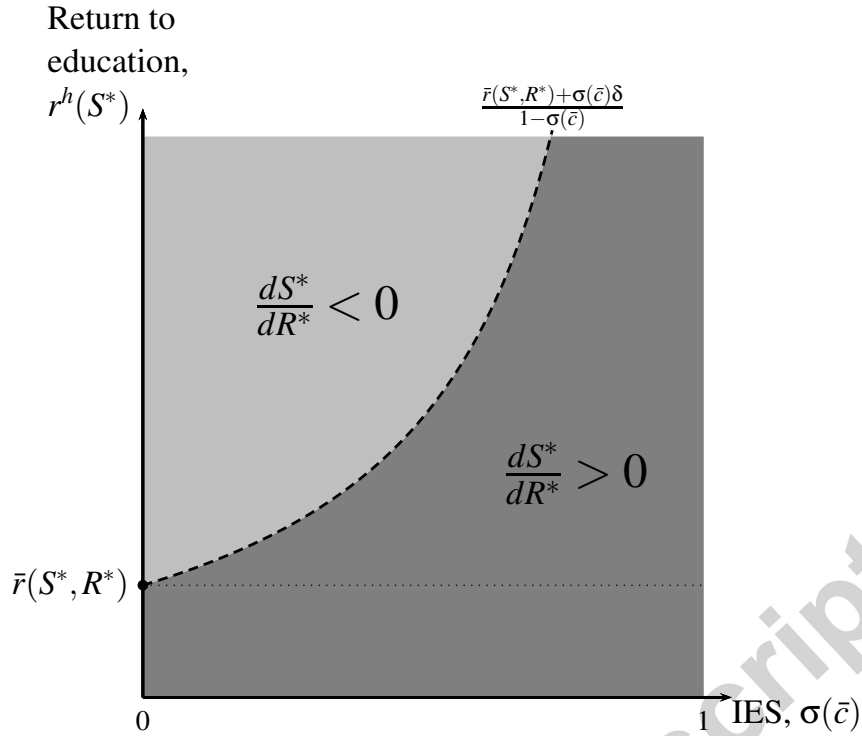
Figure 2: Relationship between S^* and R^* by return to education and intertemporal elasticity of substitution

and a light gray area with the combination of $(r^h(S^*), \sigma(\bar{c}))$ values for which S^* and R^* are negatively related. It is clear looking at Figure 2 that S^* and R^* are positively related whenever the return to education is equal to, or lower than, $\bar{r}(S^*, R^*)$ (dark gray area below the horizontal dotted line in Figure 2). In this region, an increase in the retirement age leads to an increase in the optimal length of schooling (Ben-Porath, 1967), as well as an increase in schooling yields an increase in the retirement age (Boucekkine et al., 2002; Echevarria and Iza, 2006). However, when the return to education is higher than $\bar{r}(S^*, R^*)$, S^* and R^* can either be positively or negatively related. The black dashed line in Figure 2 delimits the combination of $(r^h(S^*), \sigma(\bar{c}))$ values at which S^* and R^* are not related to each other; i.e. $\frac{dS^*}{dR^*} = 0$. The light gray area, located at the upper-left corner, is characterized by low IES and high return to education. In this area, the income effect dominates. Thus, for a sufficiently high return to education and low IES, when a positive income shock increases the optimal years of schooling, the optimal retirement age decreases, since individuals purchase more leisure time, and the positive effect on years of schooling gets reinforced. The same effect would take place if the positive income shock initially reduces the retirement age. Notice, however, the negative relation between S^* and R^* vanishes as the return to education approaches the dashed line, which eventually occurs when the length of schooling is sufficiently large. On the contrary, in the dark gray area, where the strength of the income effect diminishes –as a consequence of a positive income shock that raises the retirement age– the optimal years of schooling increases and the rise in the retirement age gets also reinforced.

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3. Differential impact of mortality decline on optimal schooling years and retirement age

In this Section, we study the impact of a mortality decline at an arbitrary age (x_0) on the optimal length of schooling (S^*) and the optimal retirement age (R^*). For clarity of exposition, we make explicit the dependence of the optimal schooling and retirement age on each other and on the underlying mortality schedule; i.e. $S^* \equiv S^*(R^*; \mu)$ and $R^* \equiv R^*(S^*; \mu)$.[13]

Eqs. (23a)-(23b) below show how the effect of a mortality decline at an arbitrary age x_0 is characterized by the sum of two partial effects:

$$\text{sign}\left[\frac{-dS^*}{d\mu(x_0)}\right] = -\text{sign}\left[S^*_{\mu(x_0)}(R^*; \mu) + \frac{dS^*}{dR^*}R^*_{\mu(x_0)}(S^*; \mu)\right], \tag{23a}$$

$$\text{sign}\left[\frac{-dR^*}{d\mu(x_0)}\right] = -\text{sign}\left[R^*_{\mu(x_0)}(S^*; \mu) + \frac{dR^*}{dS^*}S^*_{\mu(x_0)}(R^*; \mu)\right]. \tag{23b}$$

See the proof in AppendixC. The first partial effect is the impact of a mortality decline at x_0 on S^* and R^* –respectively– holding all other variables unchanged, while the second partial effect is the impact of retirement (resp. schooling) on schooling (resp. retirement) that is mediated by a change in mortality. Thus, as shown in (23a) and (23b), the effect of mortality on S^* and R^* are intertwined.

Combining the partial effects, presented in AppendixB, according to Eqs. (23a) and (23b), Proposition 2 gives under the strict concavity of the expected lifetime utility, the sign of a decline in mortality at an arbitrary age x_0 on the optimal length of schooling and retirement age. See AppendixC for the proof.

Proposition 2. *Assuming the strict concavity of $\hat{V}(S, R)$, for the life-cycle model given by (1)-(5),*

(a) the sign of $\frac{-dS^*}{d\mu(x_0)}$ is the same as that of

$$\begin{cases} a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dS^*} + \int_{x_0}^{R^*} e^{-r(x-x_0)}\frac{p(x)}{p(x_0)}y_S(S^*, x)dx & \text{if } S^* < x_0 < R^*, \\ a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dS^*} & \text{otherwise,} \end{cases} \tag{24}$$

(b) and the sign of $\frac{-dR^*}{d\mu(x_0)}$ is the same as that of

$$\begin{cases} a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dR^*} + \frac{dS^*}{dR^*}\int_{x_0}^{R^*} e^{-r(x-x_0)}\frac{p(x)}{p(x_0)}y_S(S^*, x)dx & \text{if } S^* < x_0 < R^*, \\ a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dR^*} & \text{otherwise.} \end{cases} \tag{25}$$

First, Eq. (24) shows that the total impact of a decline in mortality at an arbitrary age x_0 on the optimal length of schooling is given by three factors: (i) the "lifetime human wealth" effect versus the "years-to-consume" effect, which is reflected by the financial wealth at age x_0, i.e. $a(x_0)$; (ii) the total impact of years of schooling on the initial consumption; and (iii) the effect of mortality on the marginal benefit of schooling or Ben-Porath mechanism. Factors (i) and (ii) only have an impact when $r^h(S^*) \neq \bar{r}(S^*, R^*)$.

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[13]Let the continuous function $\mu : [0, \omega) \to \mathbf{R}^+, x_0 \mapsto \mu(x_0)$ represents the mortality hazard rate at any age $x_0 \in [0, \omega)$.

Second, Eq. (25) shows that the total impact of a decline in mortality at an arbitrary age x_0 on the optimal retirement age is also given by another three factors: (i) the "lifetime human wealth" effect versus the "years-to-consume" effect; (ii) the total impact of retirement on the initial consumption; and (iii) a *Modified Ben-Porath mechanism.* Notice in the last term of Eq. (25) that the Ben-Porath mechanism is modified by the relationship between S^* and R^*; i.e., $\frac{dS^*}{dR^*}$. Thus, if S^* and R^* are negatively related, agents anticipate their retirement age and enjoy more leisure time when a decline in mortality causes an increase in the marginal benefit of schooling. In contrast, if S^* and R^* are positively related, agents postpone their retirement age in order to reap the benefits of schooling. This is because in the former alternative the income effect dominates over the substitution effect, whereas in the latter the substitution effect dominates over the income effect.

During the schooling period and the retirement period, the effect of mortality on the marginal benefit of schooling is null. Thereby, in these two periods, the sign of the total impact of a decline in mortality on the optimal length of schooling and retirement age solely depend on the lifetime human wealth effect versus the years-to-consume effect and the total impact of years of schooling and retirement on the initial consumption:

$$\text{sign}\left[\frac{-dS^*}{d\mu(x_0)}\right] = \text{sign}\left[a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dS^*}\right],$$

$$\text{sign}\left[\frac{-dR^*}{d\mu(x_0)}\right] = \text{sign}\left[a(x_0)\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dR^*}\right],$$

for all $x_0 \in [0, S^*] \cap [R^*, \omega]$. On the one side, from (5) we know that $a(x_0) < 0$ during the schooling period, while $a(x_0) > 0$ during the retirement period. On the other side, combining (17), (19)-(21) we have

$$\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dS^*} = (1 - \lambda^R)\frac{r^h(S^*) - \bar{r}(S^*, R^*)}{\sigma(\bar{c})} + \lambda^R\left(r^h(S^*) + \delta\right), \tag{26}$$

where

$$\lambda^R = \frac{1}{\sigma(c^*)}\frac{c_R^*}{c^*} \Big/ \left(\frac{1}{\sigma(c^*)}\frac{c_R^*}{c^*} + \delta + \frac{\phi_R(R^*)}{\phi(R^*)}\right). \tag{27}$$

Since λ^R takes values between zero and one, assuming $r^h(S^*) - \bar{r}(S^*, R^*) > 0$, it is straightforward to show that (26) is always positive. Therefore, like Cai and Lau (2012) a decline in mortality during the schooling period has a negative impact on education, while a decline in mortality during the retirement period has a positive impact on education. On the contrary, the total impact on the initial consumption of an increase in the optimal retirement age is *a priori* ambiguous. In particular, it can be shown

$$\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{dc^*}{dR^*} \begin{cases} \geq 0 \Leftrightarrow \frac{dS^*}{dR^*} \geq -\frac{c_R^*}{c_S^*}, \\ \leq 0 \Leftrightarrow \frac{dS^*}{dR^*} \leq -\frac{c_R^*}{c_S^*}. \end{cases} \tag{28}$$

Consequently, according to (22), the total impact of a decline in mortality –during the schooling and retirement periods– on the optimal retirement age coincides with that on the length of schooling if, and only if, the relationship between the optimal years of education and retirement age is positive; otherwise, when $\frac{dS^*}{dR^*} < 0$, both alternatives are possible.

The direct consequence of this ambiguity is that the impact of a reduction of $\mu(x_0)$ on $R^* - S^*$ is in general ambiguous. In the next section we perform a simple numerical analysis using observed mortality data in order to show the impact of $\mu(x_0)$ on $R^* - S^*$ under different scenarios.

4. Quantitative exercise

In this Section we study numerically the impact of the epidemiological transition on the optimal years of schooling and retirement age when both variables are endogenous. We thus abstract from the effect that a pension system or any policy reform may have on our decision variables. This is a strong simplification but it allows us to focus on the main point of the article: the effect of mortality declines on education and retirement.

For comparability with the existing literature, our quantitative exercise exploits the data used by Hazan (2009). Moreover, we extend the analysis by using death rates of Swedish males born between 1890 and 2000 in order to see the effect that an alternative observed mortality pattern may have on labor supply. Our analysis delivers two important results. First, when the life expectancy rises, our model is capable of producing a decline in the optimal retirement age and an increase in years of schooling. Second, since in the earlier stage of mortality transition, a decline in mortality belongs mainly to younger people, whereas in the later stage, a decline in mortality decline has mainly occurred at older ages, we show that the optimal retirement age stops declining after the cohort born in year 1920 and increases thereafter.

4.1. Data

To estimate the marginal effect of the decline in mortality on the length of schooling and retirement age, we just need data on mortality rates. Nevertheless, to realistically match the effective retirement age and years of schooling for the cohort born in 1890 (our baseline cohort), we also collect data on labor force participation rates and years of schooling for Swedish males and US males.

4.1.1. Retirement and years of schooling data

The effective retirement age for Swedish males is based on employment rates, taken from census data, for the period 1910-1985 and labor force surveys for the period 1975-2004 published by Statistics Sweden.[14] Years of schooling for birth cohorts born in Sweden is calculated based on the number of students by educational attainment reported by de la Croix et al. (2008).[15] US data on labor force participation rates and on years of schooling for cohorts born between 1890 and 1930 are taken from Hazan (2009).

4.1.2. Mortality data

We combine the data reported by Haines (1998) and Bell et al. (1992) to produce the probability of dying at each age for US males born between 1890 and 2000. Notice,

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[14]Employed and working age population in Sweden during the period 1910-1985 is available at http://www.scb.se/tidsseriehafteforvarvsarbetandefob1910-1985. Before the 1980s, unemployment rates above age 45 were roughly constant and lower than 2 percent (Ljungqvist and Sargent, 1995; Holmlund, 2009). Therefore, we do not expect a significant error by combining both datasets.

[15]Information on number of students and enrollment rates by educational attainment in Sweden from year 1768 to 2002 are withdrawn from http://perso.uclouvain.be/david.delacroix/data/swedish-educ-data.pdf.

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though, that US mortality data for cohorts born before 1933 are not based on complete death registrations and census data.[16] The probability of dying at each age for Swedish males born between 1890 and 1911 is taken from Human Mortality Database (2013). Deaths rates for cohorts born after year 1911 are constructed applying the Lee-Carter model (Lee and Carter, 1992).[17]

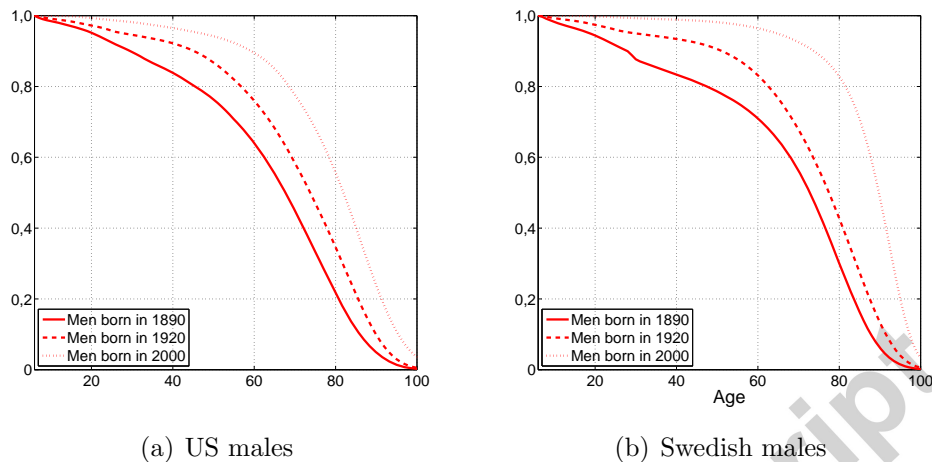(a) US males   (b) Swedish males

Figure 3: Conditional survival probability at age 6 across cohorts: US and Sweden, Male cohorts 1890–2000.

Note: Survival probabilities for cohorts born after 1920 is based on forecasted data.

Figure 3 presents the cohort survival probabilities conditional on reaching age 6 for males born in 1890, 1920, and 2000 in (a) US and (b) Sweden. Both panels show the progressive increase in the conditional survival probability for younger cohorts in US and Sweden. This pattern implies that the life expectancy from age 6 of US males (i.e. the area below the survival probability curve) is expected to increase from 56.9 years for the cohort born in 1890 to 72.8 years for the cohort born in 2000.[18] In Sweden, the expected increase in life expectancy at age 6 is greater than that of US, rising from 59.7 years for Swedish males born in year 1890 to 80.9 for those born in 2000. Therefore, the life expectancy gap between both countries is expected to increase from 2.8 for cohorts born in 1890 to 8.1 years for cohorts born in 2000.

Figure 4 shows the contribution of the mortality decline by stage of life on the increase in life expectancy across males born in US and Sweden from 1890 to 2000. We distinguish

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[16]Death rates based on complete death registrations and census data for cohorts born before year 1900 are only available for a small set of countries: Belgium, Denmark, Finland, France, Iceland, Italy, Netherlands, Sweden and Switzerland. The same standards of quality for the US demographic data start in 1933. This information can be drawn from the Human Mortality Database.

[17]Future age-specific death rates from year 2011 are projected applying the Lee-Carter method to actual Swedish period-death rates from year 1945 up to 2011. Thus, it is assumed that the log of the death rate is explained by the following multiplicative process:

$$\log m_{\tau,x} = a_x + k_\tau b_x + \epsilon_{t,x}, \text{where } \epsilon_{\tau,x} \text{ is } i.i.d.(0, \sigma_\epsilon),$$

where $m_{\tau,x}$ is the death rate at age x in year τ, a_x, b_x are age-specific constants, and k_τ is a time-varying index obtained through the singular-value decomposition of a matrix of death rates.

[18]Life expectancy at age 6 is calculated as: $e_t(6) = \int_6^\omega \frac{p_t(s)}{p_t(6)} ds$, where t denotes the birth cohort.

three periods: schooling from ages 6 to 15 (green area), working period from ages 16 to 65 (blue area), and retirement for ages above 65 (red area). The sum of the three areas gives the total increase in life expectancy for each cohort from 1890 to 2000 compared to cohort born in 1890.

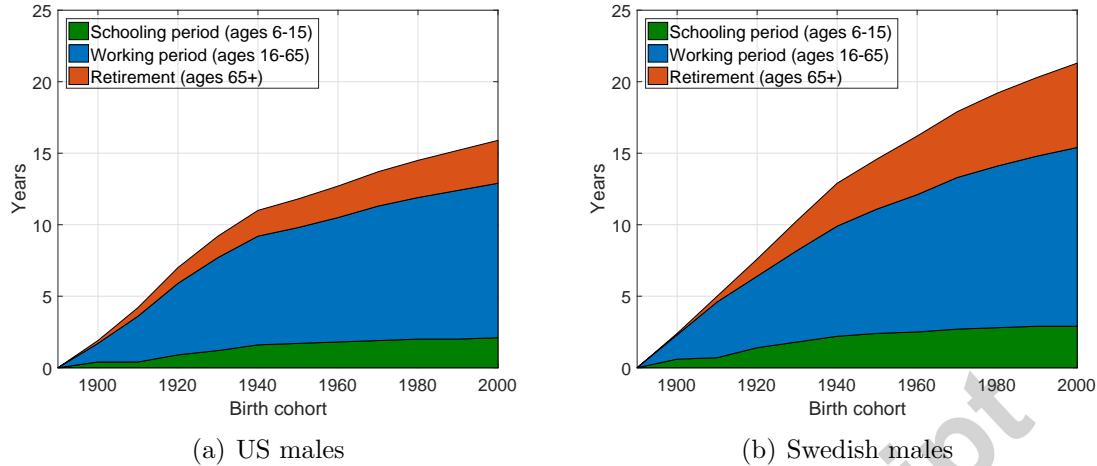(a) US males

(b) Swedish males

Figure 4: Decomposition of the increase in life expectancy –at age 6– since cohort 1890 by stage of life: US and Sweden, Male cohorts 1890–2000.

Note: Authors' calculations based on the decomposition of life expectancy proposed by Arriaga (1984).

According to Figure 4, most years-gained occur during (prime) working ages for the oldest cohorts. Specifically, five additional years were gained, on average, in both countries at (prime) working ages between the cohort born in 1890 and that in 1920, while the total contribution of the mortality reduction during the schooling period and the retirement period was slightly above two years of age. Between the cohort born in 1920 and that in year 2000 a further increase of 5.8 and 7.5 years at (prime) working ages is expected in the US and Sweden, respectively. Hence, as it is shown in Figure 4, the increasing life expectancy gap between both countries for younger cohorts is mainly due to the faster increase in Sweden compared to that in the US in the life expectancy at age 65. In particular, Swedish males born in 2000 are expected to gain at least three more years of age than their counterparts in US.

In the next section we perform a numerical simulation to show the consequences of the different mortality transition on the optimal length of schooling and retirement age.

4.2. Numerical simulation

To solve the model, we follow Cervellati and Sunde (2013) and consider a constant intertemporal elasticity of substitution (CIES) utility function,

$$U(c) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}, \text{ with } \sigma \in (0, 1]. \tag{29}$$

The underlying disutility of non-leisure time is assumed constant $\phi(x) = \phi$. For simplicity, we assume a constant aversion to schooling time, $\psi(x) = \psi$, for all $x \in (0, S)$. Since our individual devotes her full time to education while she is in school, we use the following simplified version of the Ben-Porath human capital production function

$$q\big(h(x)\big) = \xi h(x)^\gamma, \text{ with } \xi > 0 \text{ and } \gamma \in (0, 1), \tag{30}$$

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where ξ is a scaling factor and γ is the returns to scale in human capital investment. Similar to Hazan (2009) and Cervellati and Sunde (2013) the wage rate is assumed to be constant.

To shed light on the effects of mortality on S^* and R^*, we introduce further simplifying assumptions. We assume zero discounting, $r = \delta = \rho = 0$, so that the inverse of $\bar{r}(S^*, R^*)$ coincides with the expected lifetime labor supply (ELW)[19]

$$\bar{r}(S^*, R^*) = \left[\int_{S^*}^{R^*} \frac{p(x)}{p(S^*)} dx \right]^{-1}. \tag{32}$$

Integrating (4) and using (30) the return to education at age S becomes

$$r^h(S^*) = \xi / \left(h(0) + (1 - \gamma)\xi S^* \right). \tag{33}$$

As in Cervellati and Sunde (2013), we set $\gamma = 0.65$, $\sigma = 0.5$, the wage rate per unit of human capital to one, and fix $h(0) = 1$. In order to show the importance on the results of the relationship between S^* and R^*, we run three alternative simulations combining three different returns to education function. In particular, we set ξ to 1, .25, and 0.075 so as to have a return to education after 13 years in school around 18%, 11.5% (Heckman et al., 2008), and 5.5%.[20] Finally, to isolate the effect of the decline in mortality, we restrict the parameters of the model so that it reproduces the years of schooling and retirement age observed in the data for the cohort born in 1890 in the United States. Table 1 reports the values of ϕ and ψ for US and Sweden, which are calculated so as to have an initial optimal length of schooling of 9.3 years and an optimal retirement age of 66.7 years for males born in 1890 (Hazan, 2009).

Table 1: VALUES OF ϕ AND ψ

CIES	Human capital scaling factor	Underlying disutility of leisure		Aversion to schooling time		Ratio	
σ	ξ	ϕ		ψ		ψ/ϕ	
		US	SWE	US	SWE	US	SWE
0.50	1.000	0.03	0.03	0.27	0.29	9.0	9.3
0.50	0.250	0.34	0.36	1.67	1.82	4.9	5.0
0.50	0.075	1.01	1.07	1.59	1.75	1.6	1.6

[†] Note: US and SWE stand for United States of America and Sweden, respectively.

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[19]Under the assumptions zero discounting and constant wage rate, the annuitized marginal cost of the S-th unit of schooling becomes

$$\bar{r}(S^*, R^*) = \frac{\int_{S^*}^{R^*} \mu(x)p(x)dx}{\int_{S^*}^{R^*} p(x)dx} + \frac{p(R^*)}{\int_{S^*}^{R^*} p(x)dx}. \tag{31}$$

Since it follows from (1) that $-dp(x) = \mu(x)p(x)dx$, substituting it in (31), and rearranging, gives Eq. (32).

[20]Notice that when $h(0) = 1$, ξ is equal to the initial return to education $r^h(0)$.

4.3. Results

We divide this Section in three parts. First, we study the effect of the aversion to schooling time assumption for modeling the impact of mortality on total years worked. Second, we analyze the differential effect of mortality declines at different stages of life on the optimal years of schooling and optimal retirement age. Finally, we estimate quantitatively the importance of the aversion to schooling time for explaining the evolution of the years worked.

4.3.1. The importance of the aversion to schooling time assumption

To show the impact of mortality on the labor supply for different relationship between S^* and R^*, we compute for each country three simulations that results from three different human capital scaling factors. The parameters of the model are held constant across cohorts and fixed at the values that reproduce the years of schooling and retirement age observed in the data for the cohort born in 1890 in the United States. Hence, in each controlled experiment, any variation in S^* and R^* across cohorts is solely due to changes in mortality.

Table 2: Absolute change in years of schooling and retirement age between 1890 and 1930

		Observed	Simulated		
Human capital scaling factor	ξ		1.000	0.250	0.075
Aversion to schooling time ratio	ψ/ϕ		9.0	4.9	1.6
			I	**II**	**III**
US Survival					
Retirement	$R^*_{1930} - R^*_{1890}$	-2.88	-1.80	-1.17	0.92
Schooling	$S^*_{1930} - S^*_{1890}$	3.93	0.68	1.17	2.72
Years worked[†]		-6.81	-2.48	-2.34	-1.80
Swedish Survival					
Retirement	$R^*_{1930} - R^*_{1890}$	-2.73	-1.68	-1.09	0.83
Schooling	$S^*_{1930} - S^*_{1890}$	3.93	0.69	1.19	2.76
Years worked[†]		-6.66	-2.38	-2.28	-1.93

[†] Note: Years worked is calculated as the difference between the optimal retirement age and the optimal length of schooling.

Table 2 reports the absolute change in S^* and R^* for cohorts born between year 1890 and 1930 using US and Swedish survival probabilities. The first important result from Table 2 is that the model is capable of predicting that a decline in mortality leads to an increase in schooling followed by a decline in the retirement age. The extent to which the optimal retirement age decreases and the optimal length of schooling increases depends on the ratio ψ/ϕ, or equivalently ξ. Higher (lower) values of ψ/ϕ yield a stronger (weaker) decline in the optimal retirement age and a smaller (higher) increase in the optimal years of schooling. For instance, if $\xi = 1$ or $\psi/\phi = 9.0$ (column I), the decline in mortality would account for 62.6% (-1.80/-2.88) and 61.6% (-1.68/-2.73) of the decline in the retirement age, while it would account for 17.5% (0.68/3.93) and 17.5% (0.69/3.93) of the increase in years of schooling in the US and Sweden, respectively. Instead, if we assume $\xi = 0.25$ or $\psi/\phi = 4.9$ (column II), the model predicts that the decline in mortality would account

for 40.7% (-1.17/-2.88) and 40.0% (-1.09/-2.73) of the decline in the retirement age, whereas it would account for 29.7% (1.17/3.93) and 30.2% (1.19/3.93) of the increase in the years of schooling in the US and Sweden, respectively. The second result is that the optimal years worked, or the difference between the retirement age and the years of schooling, are negatively related to the value of ψ/ϕ. According to Proposition 2, this is because the decline in mortality only affects positively on the optimal years of schooling, while it can have a negative or a positive effect on the optimal retirement age. As explained in Section 3, this result is due to the negative relationship between S^* and R^* or, equivalently, because the income effect dominates over the substitution effect. Under the assumptions of zero discounting and a constant wage rate, the sign of the relation between S^* and R^*, or Eq. (22), is given by

$$\text{sign}\left[\frac{dS^*}{dR^*}\right] = \text{sign}\left[\frac{1}{(1-\sigma)(1+\psi/\phi)} - 1\right] \begin{cases} < 0 & \text{if } \frac{\psi}{\phi} > \frac{\sigma}{1-\sigma}, \\ > 0 & \text{otherwise} . \end{cases} \tag{34}$$

Hence, for $\sigma = 0.50$, a value of ψ/ϕ higher (lower) than one implies a negative (positive) relationship between S^* and R^*. Note therefore that the condition $dS^*/dR^* < 0$, although necessary, is not sufficient to guarantee that a decline in mortality yields earlier retirement. For instance, according to (34), in all simulations the sign of dS^*/dR^* is negative, while the model only predicts an early retirement age in columns I and II. The increase in the retirement age shown in column III is therefore explained by the weak negative relationship between S^* and R^* that results from an aversion to schooling time value of 1.6. In the next subsection, Figures 5 and 6 give further intuitions about the effect of the weak negative relationship between S^* and R^* for the impact of mortality on the years worked.

4.3.2. Contribution of mortality at different stages of life

We complement the empirical analysis by showing the contribution of mortality improvements at different stages of life on the optimal years of schooling and optimal retirement age for our two alternative mortality schedules: males born in US and Sweden. In these simulations, we divide the lifespan in three periods: childhood (ages 6-15), adulthood (ages 16-65), and retirement (ages 65+), which are closely related to the stages of life delimited by the observed length of schooling and retirement age. In order to show the contribution of mortality improvements at each stage of life, we calculate the optimal schooling and retirement considering exclusively the gains in survival during either childhood, or adulthood, or retirement across cohorts ceteris paribus the probability of dying in the other stages of life. We recalculate the survival probability to age x based on the new probability of dying in each age interval.

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Figure 5: Contribution of mortality decline by stage of life on (a) the optimal length of schooling and (b) the optimal retirement age across cohorts by average return to education, US mortality of males born in 1890–2000

Note: Results for cohorts born after 1930 are based on forecasted mortality rates.
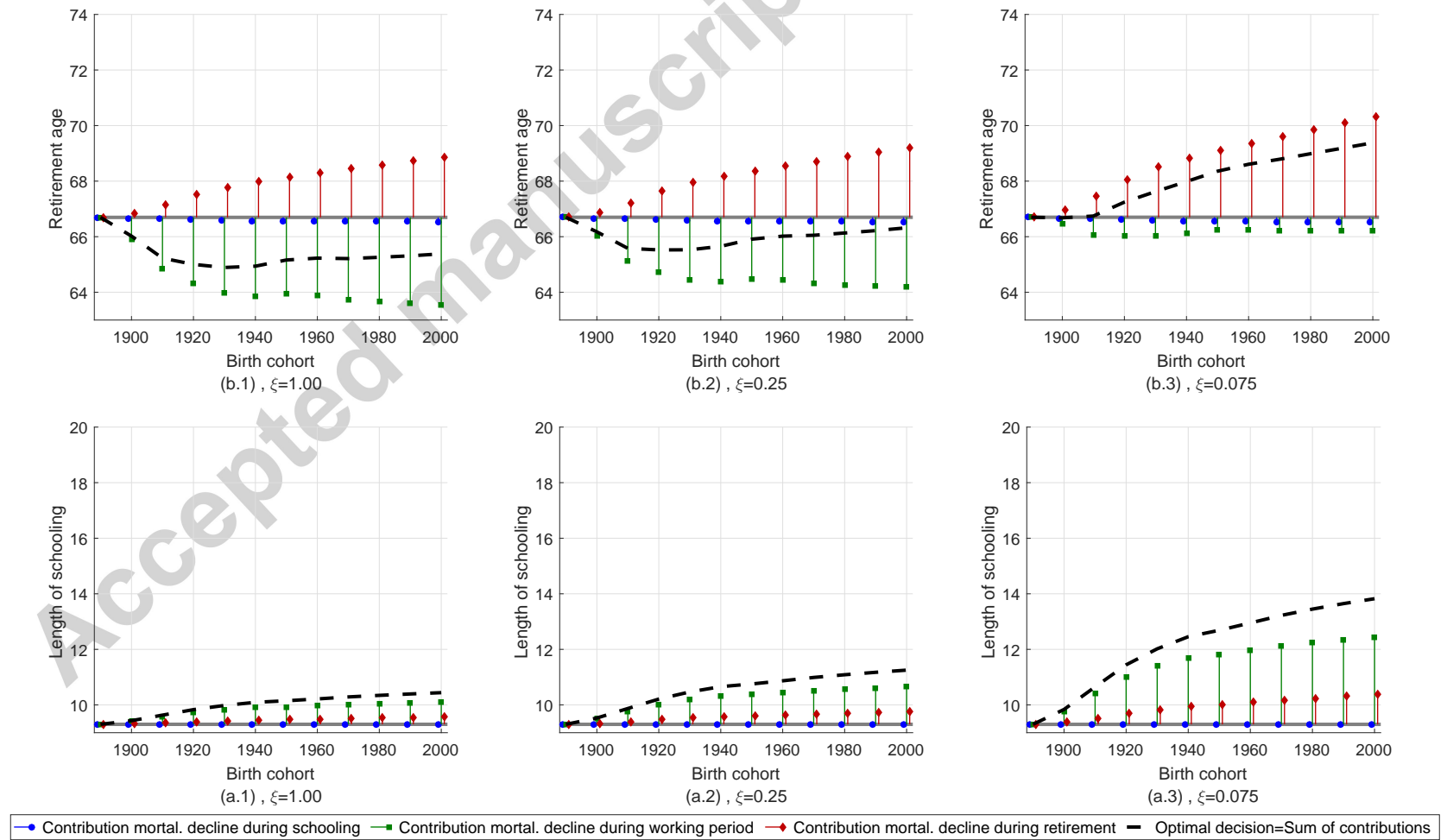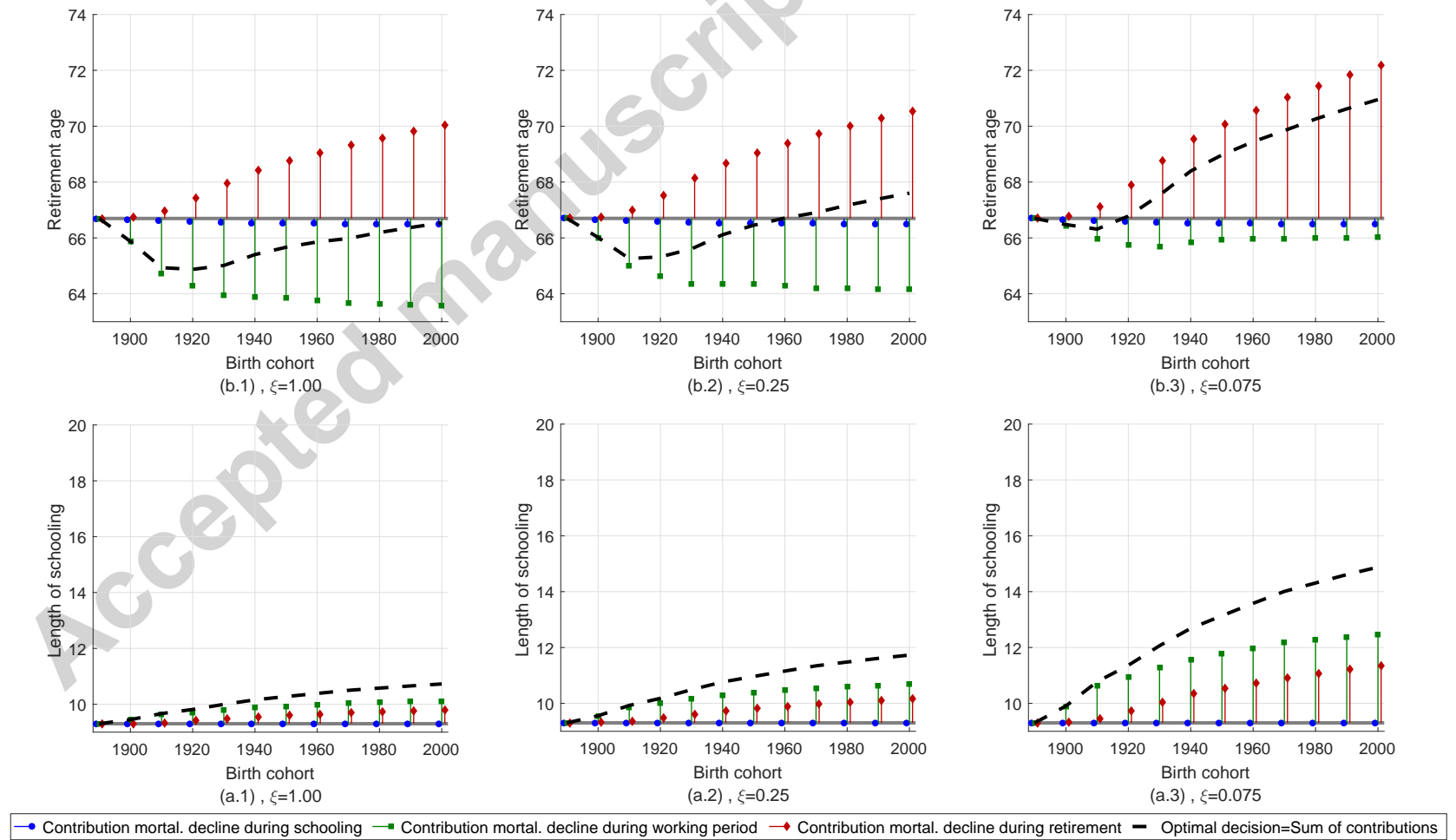
Figure 6: Contribution of mortality decline by stage of life on (a) the optimal length of schooling and (b) the optimal retirement age across cohorts by average return to education, Swedish mortality of males born in 1890–2000

Note: Results for cohorts born after 1930 are based on forecasted mortality rates.

Figures 5 and 6 show how mortality improvements during childhood (circled blue arrows), adulthood (squared green arrows), and retirement (red-diamond arrows) affect on the optimal years of schooling and the retirement age by different functional form of the returns to education. The height of each arrow represents the change from the baseline (solid gray line) in the endogenous variable, while the position of the marker represents the direction of the effect. The evolution of the optimal length of schooling and optimal retirement age –represented by the dashed black line– equals the sum of the heights of the three arrows. Note that the difference between the top and bottom panels for each human capital scaling factor ξ gives the total number of years worked, conditional on reaching the retirement age.

The bottom panels in Figures 5 and 6 show how mortality improvements raise the optimal years of schooling. According to Proposition 2, a decline in mortality during adulthood raises the marginal benefit of schooling and hence the optimal length of schooling, because the likelihood of receiving a future labor earning increases, also known as the Ben-Porath's mechanism. A decline in mortality during retirement also leads to an increase in the length of schooling, because agents continue studying in order to finance the consumption after retirement through an increase in lifetime earnings. That is, the years-to-consume effect dominates over the lifetime human wealth effect. The contribution of mortality improvements during childhood on the optimal length of schooling is negative but negligible. The sum of these three effects of mortality on the optimal length of schooling is represented by the black dashed line.

The panels at the top in Figures 5 and 6 show how mortality improvements change the optimal retirement age. The parameter values assumed in both Figures imply that the income effect dominates over the substitution effect. As a consequence, the relationship between S^* and R^* is negative, or $\frac{dS^*}{dR^*} < 0$. Thus, a decline in mortality during adulthood leads to early retirement (see the second term in Eq. 25). Individuals use the additional income to increase consumption and enjoy more leisure time. Note, however, that the strength of the negative effect decreases (see the length of the green arrows) with lower returns to education. This is because the negative relation between education and retirement vanishes when $r^h(S^*)$ tends to $\bar{r}(S^*, R^*)$ (see Figure 2). Consequently, if the relation between S^* and R^* were positive, a decline in mortality during adulthood would lead to a delay in the retirement age. On the other hand, mortality declines late in life leads to a delay in the optimal retirement age because individuals need more earnings to finance the additional consumption due to a longer retirement period. This positive effect on retirement is indicated by the upward green squared arrow. Similar to the effect on the length of schooling, the contribution of mortality improvements during childhood on the optimal retirement age is also negative but negligible. The net effect of mortality improvements at different stages of life on the optimal retirement age is represented by the black dashed line. Notice that in all cases we observe a turning point in the evolution of the optimal retirement age after the cohorts born in the 1920s. This is because mortality improvements for cohorts born before the 1920s mainly occurred during childhood and adulthood, whereas the improvements in mortality for more recent cohorts mainly occur at older ages.

4.3.3. Aversion to schooling time values

To assess how reasonable our aversion to schooling time assumption is, we compute the nonpecuniary cost of schooling that results from our theoretical model and compare it to the values derived in the literature. To do so, we pin down the values of ϕ and ψ

that corresponds to the observed length of schooling $(\hat{S}_t)$ and retirement age $(\hat{R}_t)$ across cohorts and human capital scaling factors.

Assuming a flat wage rate and no discounting, from Eq. (13) the returns to schooling at the optimal schooling decision must satisfy

$$r^h(S^*) = \bar{r}(S^*, R^*)\left[1 + (\psi/\phi)\right], \tag{35}$$

where the ratio (ψ/ϕ) gives information about the importance of the nonpecuniary cost of schooling for the optimal schooling decision. Thus, provided the values of $(\hat{S}_t, \hat{R}_t)$ and $p_t(x)$, from (35) we can calculate the evolution of this ratio as

$$\widehat{(\psi/\phi)}_t = \frac{r^h(\hat{S}_t)}{\bar{r}(\hat{S}_t, \hat{R}_t)} - 1. \tag{36}$$

From Eq. (32) we know that the marginal cost of the $\hat{S}_t$-th unit of schooling (i.e. $\bar{r}(\hat{S}, \hat{R})$) is, by definition, the inverse of the expected lifetime labor supply (ELW). Column III in Table 3 shows that the expected lifetime labor supply has slightly decreased from 42.7 years to 41.3 between cohort 1890 and 1930. Thus, our assumed marginal cost of the $\hat{S}_t$-th unit of schooling, or hurdle rate, is close to 2.4% –column IV in Table 3–. A value of 2.4% is between 1.2% and 2.6% estimated for the US; see Table 5 in Oreopoulos (2007).[21]

Table 3: Observed average length of schooling $(\hat{S})$, average retirement age $(\hat{R})$, expected lifetime labor supply (ELW), and three hypothetical returns to schooling in the US

Cohort	$\hat{S}+6$	$\hat{R}$	$ELW^\dagger$	$\bar{r}(\hat{S}, \hat{R})$	$r^h(\hat{S})$		
					$\xi = 1.00$	$\xi = .250$	$\xi = .075$
	I	**II**	**III**	**IV=1/III**	**V**	**VI**	**VII**
1890	15.3	66.7	42.7	2.3%	23.4%	13.8%	6.0%
1900	17.0	66.3	42.0	2.4%	20.6%	12.7%	5.8%
1910	17.8	64.8	41.7	2.4%	19.5%	12.3%	5.7%
1920	18.5	63.9	41.5	2.4%	18.7%	12.0%	5.7%
1930	19.3	63.8	41.3	2.4%	17.7%	11.6%	5.6%

† Note: The expected lifetime labor supply (ELW) is defined as $\int_{\hat{S}_t}^{\hat{R}_t} \frac{p_t(x)}{p_t(\hat{S}_t)}dx$, where $p_t(x)$ is the probability of surviving to age x conditional on reaching age 6 for the cohort born in year t.

Using our estimated hurdle rate, Figure 7 shows the evolution of the ratio $\widehat{(\psi/\phi)}_t$ that corresponds to the observed length of schooling $(\hat{S}_t)$ and retirement age $(\hat{R}_t)$ in the US across cohorts for different human capital scaling factors. We obtain that the ratio $\widehat{(\psi/\phi)}_t$ might range between 1 and 9 according to our different scaling factors. However, since estimated returns to compulsory schooling range between 9.5% and 17.4% (Oreopoulos, 2007), a scaling factor of 0.25 seems to be the most likely case according to column

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[21]The hurdle rate values reported by Oreopoulos (2007) are estimated for individuals between age 14 and 16. Thereby, our initial length of schooling of 15.3 years is between the range of ages used by Oreopoulos (2007).

VI in Table 3. In Figure 7, the red dashed curve depicts the evolution of $\widehat{(\psi/\phi)}_t$ that corresponds to the human capital scaling factor of 0.25, which takes values between 4.9 (cohort 1890) and 3.8 (cohort 1930). A human capital scaling factor of 0.25 implies that by staying an additional year in school lifetime wealth would have increased by 11.4% for the cohort born in 1890, while it would have increased by 9.1% for the cohort born in 1930. These values are also within the empirically estimated increase in lifetime wealth from an additional year of compulsory schooling that range between 8.5% and 17.6% (Oreopoulos, 2007).

In sum, we conclude, based on the comparison of our results to the existing empirical literature on the returns to schooling, that the most likely value of ψ/ϕ is 4.9. Therefore, our "aversion to schooling time" is significant and implies that the observed decline in mortality would account for 40.7% of the decline in the retirement age and 29.7% of the increase in the years of schooling in the US.
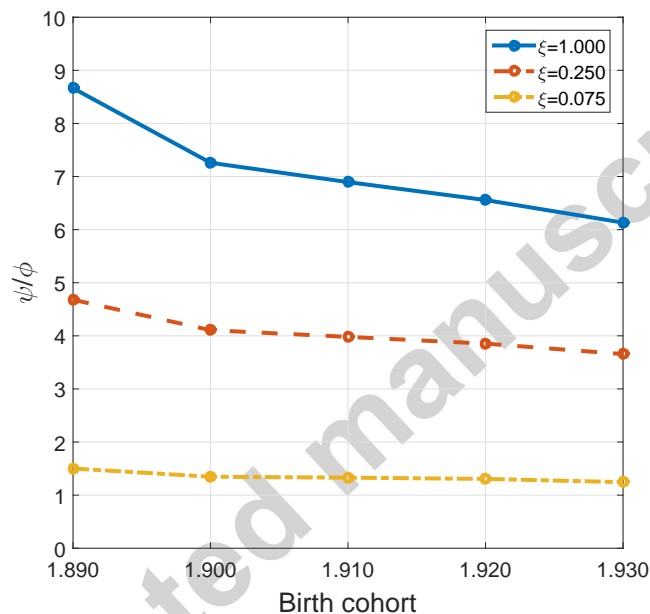

Figure 7: Values of $\widehat{(\psi/\phi)}_t$ that match the observed length of schooling and retirement age by cohort for different ξ values

5. Conclusion

Existing theoretical models predict a causal positive relation between increasing life expectancy and human capital investments and retirement age. However, one salient feature of the economic development during the last two centuries is the negative relation between life expectancy and the extensive labor supply. To reconcile the empirical evidence with economic theory, we develop a lifecycle model with endogenous human capital investment and labor supply in which mortality declines may cause higher schooling and early retirement.

This article makes two important contributions. First, we show that the 'aversion to schooling time' assumption is a necessary, although not sufficient, condition for a decline in mortality to cause higher education and early retirement age. The intuition is as follows. Individuals who are averse to schooling time do not maximize their lifetime income,

since they prefer to anticipate their entrance into the labor market. As a consequence, we show that a higher human capital investment, triggered by a decrease in mortality, raises lifetime income (i.e., positive income effect). This positive income effect reduces the marginal benefit of working. Thus, in this setting, when the income effect dominates over the substitution effect, a decline in mortality might lead to higher education and early retirement age. Second, we derive the differential impact of the mortality decline at any arbitrary age on education and retirement. We show that a mortality decline after retirement always results in higher human capital investment and late retirement, whereas a mortality decline at younger ages leads to higher human capital investment and may, or may not, cause early retirement. Using mortality data for cohorts born between year 1890 and 1930 in US and Sweden, we show for reasonable values of the 'aversion to schooling time' and the intertemporal elasticity of substitution that improvements in the survival probability may account for a 40% of the decline in the retirement age and 30% of the increase in the years of education.

For simplicity, our model abstracts from realistic features like the existence of a pension system, the intervention of governments in the access to all levels of education, the introduction of mandatory years of schooling and retirement ages. Still, our results offer an explanation to the empirical evidence, collected during the last centuries in US and Sweden, on the evolution of education and retirement. Moreover, our model is robust to the introduction of the above mentioned features. For instance, if education and retirement are negatively related, positive spillovers from education and publicly provided education will increase the marginal benefit of schooling and will reduce even further the retirement age. Similarly, the existence of pension incentives for early retirement and the overall increase in the labor-augmenting technological progress during the last century would have induced earlier retirement ages and higher increases in the marginal benefit of schooling. Therefore, our results suggest some interesting directions for future research. In particular, first, a logical extension of our framework is the introduction of a pension systems. Second, the implementation of the model in a computable general equilibrium setting in order to analyze the effect of changes in wages and interest rates. The implementation of these issues can provide researchers and policy-makers a better understanding of the effect of changes in the population on modern economic growth.

AppendixA. Proof of Proposition 1

We first derive the optimal length of schooling condition (S^*) and optimal retirement age (R^*) introduced in Eqs. (8)-(9). Second, we study the conditions for a maximum in S^* and R^*. Substituting the conditional optimal consumption, $c(x, S, R)$, into (2) and differentiating it with respect to S, we obtain

$$V_S(S, R) = \int_0^\omega e^{-\rho x} p(x) U_c\big(c(x, S, R)\big) c_S(x, S, R) dx - e^{-\rho S} p(S) \psi(S). \qquad \text{(A.1)}$$

Substituting (7) in (A.1), and rearranging, gives

$$V_S(S, R) = U_c\big(c(0, S, R)\big) \int_0^\omega e^{-rx} p(x) c_S(x, S, R) dx - e^{-\rho S} p(S) \psi(S). \qquad \text{(A.2)}$$

Differentiating (6) with respect to S, and simplifying, we have

$$\int_0^\omega e^{-rx} p(x) c_S(x, S, R) dx$$

$$= e^{-rS} p(S) \left(\int_S^R e^{-r(x-S)} \frac{p(x)}{p(S)} y_S(S, x) dx - y(S, S) \right). \quad \text{(A.3)}$$

Substituting (A.3) in (A.2), and taking $U_c\big(c(0, S, R)\big) e^{-rS} p(S)$ as common factor, we obtain

$$V_S(S, R) = U_c\big(c(0, S, R)\big) e^{-rS} p(S) \times$$

$$\left(\int_S^R e^{-r(x-S)} \frac{p(x)}{p(S)} y_S(S, x) dx - y(S, S) - \frac{e^{(r-\rho)S} \psi(S)}{U_c\big(c(0, S, R)\big)} \right). \quad \text{(A.4)}$$

Setting $V_S(S, R)$ to zero and simplifying, the first-order condition for an optimal length of schooling is given by (8).

Applying a similar approach, the derivative of (2) with respect to R becomes

$$V_R(S, R) = p(R) \left(U_c\big(c(0, S, R)\big) e^{-rR} y(S, R) - e^{-\rho R} \phi(R) \right). \quad \text{(A.5)}$$

Then, setting $V_R(S, R)$ to zero and simplifying, the first-order condition for an optimal retirement age is given by (9).

Let $\hat{V}(S, R)$ be the expected lifetime utility conditional on the optimal consumption path. Also, let c^* be optimal initial consumption condition on $S = S^*$ and $R = R^*$. In order for $\hat{V}$ to be strictly concave at $S = S^*$ and $R = R^*$ it needs to satisfy

$$\hat{V}_{SS} < 0, \hat{V}_{RR} < 0, \quad \text{(A.6a)}$$

$$\begin{vmatrix} \hat{V}_{SS} & \hat{V}_{SR} \\ \hat{V}_{RS} & \hat{V}_{RR} \end{vmatrix} > 0. \quad \text{(A.6b)}$$

Substituting (10) in (A.4), differentiating with respect to S around (S^*, R^*), using (15), and simplifying gives

$$\hat{V}_{SS}(S^*, R^*) = U_c\big(c^*\big) e^{-rS^*} p(S^*)$$

$$\times \left(f_S(S^*, R^*) - f(S^*, R^*) \left(\frac{\psi_S(S^*)}{\psi(S^*)} + \frac{1}{\sigma(c^*)} \frac{c_S^*}{c^*} \right) \right). \quad \text{(A.7)}$$

Since the sign $\left[\frac{\psi_S(S^*)}{\psi(S^*)} \right]$ and sign $[c_S^*]$ is the same as the sign of $f(S^*, R^*)$, a necessary and sufficient condition for $S^* < R^*$ to be a maximum of $\hat{V}(S, R^*)$ is

$$f_S(S^*, R^*) < f(S^*, R^*) \left(\frac{\psi_S(S^*)}{\psi(S^*)} + \frac{1}{\sigma(c^*)} \frac{c_S^*}{c^*} \right). \quad \text{(A.8)}$$

For $f(S^*, R^*) > 0$ notice that $\frac{f_S(S^*, R^*)}{f(S^*, R^*)} - \frac{\psi_{S^*}(S^*)}{\psi(S^*)} < \frac{1}{\sigma(c^*)} \frac{c_S^*}{c^*}$. It is also worth noticing that for $f(S^*, R^*) = 0$, $f_S(S^*, R^*) < 0$.

Differentiating (A.5) with respect to R, at $S = S^*$ and $R = R^*$, gives

$$\hat{V}_{RR}(S^*, R^*) = U_c\big(c^*\big) e^{-rR^*} p(R^*) y(S^*, R^*)$$

$$\times \left(-\frac{1}{\sigma(c^*)} \frac{c_R^*}{c^*} + \frac{y_R(S^*, R^*)}{y(S^*, R^*)} - \frac{\phi_R(R^*)}{\phi(R^*)} \right). \quad \text{(A.9)}$$

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Provided $y_R(S^*, R^*) = \frac{w_{R-S}(R^*-S^*)}{w(R^*-S^*)} - \delta \leq 0$ and since $\phi_x(x) \geq 0$, we conclude that $\hat{V}_{RR}(S^*, R^*) < 0$ is strictly negative. Note that these results are derived assuming a general functional form for the wage rate per unit of human capital. Finally, the second-order conditions of the maximum in $S = S^*$ and $R = R^*$ is satisfied, if (A.8) holds, $y_R(S^*, R^*) \leq 0$, and

$$\frac{\hat{V}_{SR}(S^*, R^*)}{-\hat{V}_{SS}(S^*, R^*)} \times \frac{\hat{V}_{RS}(S^*, R^*)}{-\hat{V}_{RR}(S^*, R^*)} < 1. \tag{A.10}$$

Since $\frac{dS^*}{dR}\big|_{R=R^*} = \frac{\hat{V}_{SR}(S^*, R^*)}{-\hat{V}_{SS}(S^*, R^*)}$ and $\frac{dR^*}{dS}\big|_{S=S^*} = \frac{\hat{V}_{RS}(S^*, R^*)}{-\hat{V}_{RR}(S^*, R^*)}$, it follows from (A.10) that the impact of a change in S^* on R^* differs from the impact of a change in R^* on S^*; i.e. $\frac{dS^*}{dR}\big|_{R=R^*} \times \frac{dR^*}{dS}\big|_{S=S^*} < 1$.

AppendixB. Partial effect

Following the same order as the derivation of the first-order conditions, we first study the partial impact that a mortality decline has on the optimal consumption path and, second, we continue with the analysis of the partial effect of a mortality decline on the optimal length of schooling and retirement age.

To study the effect of mortality on our variables of interest, we make use of the derivative of a functional (Ryder and Heal, 1973; d'Albis et al., 2012) to obtain, through (1), that

$$-\frac{\partial p(x)}{\partial \mu(x_0)} = \begin{cases} p(x) & \text{if } x_0 \leq x, \\ 0 & \text{if } x_0 > x. \end{cases} \tag{B.1}$$

Eq. (B.1) means that a mortality decline at age x_0 has no effect on the survival probability before age x_0, but it increases the survival probability at ages above or equal to x_0. From (B.1) we derive the impact that a mortality decline at an arbitrary age x_0 has on the optimal consumption path and, in particular, on the initial optimal consumption (c^*). Differentiating (6) and (7) with respect to $-\mu(x_0)$, substituting, and rearranging gives

$$\frac{1}{c^*} \frac{-\partial c^*}{\partial \mu(x_0)} = -\frac{\sigma(c^*)}{\sigma(\bar{c})} \frac{e^{-rx_0} p(x_0) a(x_0)}{W(S^*, R^*)}. \tag{B.2}$$

Notice in Eq. (B.2) that if the IES is constant across the lifecycle, the relative impact of a mortality decline at age x_0 on the initial consumption is minus the ratio between the financial wealth position at age x_0 and lifecycle earnings. Thereby, the sign of the impact of a decline in mortality at age x_0 on the optimal consumption path is equal to minus the sign of the financial wealth at age x_0. Moreover, according to Eq. (B.2) a decline in mortality at two different ages does not necessarily have the same impact on consumption.[22]

Eq. (B.2) is the extension of Eq. (B.5) in d'Albis et al. (2012) to a model with endogenous human capital investment. Like d'Albis et al. (2012) we show that the optimal consumption path increases with a decline in mortality at age x_0 when $a(x_0) < 0$, while

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[22]The partial impact of a mortality decline at all ages on the initial consumption is

$$\frac{1}{c^*} \frac{-\partial c^*}{\partial \mu} = \int_0^\omega \frac{1}{c^*} \frac{-\partial c^*}{\partial \mu(x)} d\mu(x) = -\frac{\sigma(c^*)}{\sigma(\bar{c})} \frac{\int_0^\omega e^{-rx} p(x) a(x) d\mu(x)}{W(S^*, R^*)}. \tag{B.3}$$

the optimal consumption declines when $a(x_0) > 0$, for all $x_0 \in [0, \omega)$. The intuition is simple. On the one hand, a decline in mortality increases the number of years the agent is expected to live. As a consequence, agents compensate a longer lifespan with an overall reduction in consumption. This effect, which is always negative, is named the "years-to-consume" effect. On the other hand, a mortality decline during the working period raises the likelihood of receiving a future labor income stream, which leads to an overall increase in the consumption path. This other effect, which is always positive, is named the "lifetime human wealth" effect. For a better understanding, Proposition 3 gives the net result of these two opposite effects using a CIES utility function.

Proposition 3. *For the life-cycle model given by (1)-(5), if $U_c(c)$ is a power function, the overall result of $\frac{1}{c^*}\frac{-\partial c^*}{\partial \mu(x_0)}$ is the same as that of*

$$g(x_0) = \frac{\int_{S^*}^{R^*} e^{-rx}\left[\frac{-\partial p(x)}{\partial \mu(x_0)}\right]y(S^*, x)dx}{\int_{S^*}^{R^*} e^{-rx}p(x)y(S^*, x)dx} - \frac{\int_{x_0}^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}{\int_0^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}, \tag{B.4}$$

where $\sigma \in [0, 1]$ is the intertemporal elasticity of substitution. Moreover, there exists a critical point x_c within the open interval (S^*, R^*) such that

$$\begin{cases} g(x_0) > 0 & \text{for all } x_0 < x_c, \\ g(x_0) = 0 & \text{for all } x_0 = x_c, \\ g(x_0) < 0 & \text{for all } x_0 > x_c. \end{cases} \tag{B.5}$$

Proof. If $U_x(x) = x^{-\frac{1}{\sigma}}$, where σ is the intertemporal elasticity of substitution, from (7) we have $c(x, S, R) = c(0, S, R)e^{\sigma(r-\rho)x}$, for all $x \in (0, \omega)$, substituting it in (6), and simplifying, we obtain

$$c(0, S, R) = \frac{\int_S^R e^{-rx}p(x)y(S, x)dx}{\int_0^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}. \tag{B.6}$$

Taking logarithms at both sides of (B.6) and differentiating with respect to $-\mu(x_0)$ gives

$$\frac{1}{c(0, S, R)}\frac{-\partial c(0, S, R)}{\partial \mu(x_0)}$$
$$= \frac{\int_S^R e^{-rx}\left[\frac{-\partial p(x)}{\partial \mu(x_0)}\right]y(S, x)dx}{\int_S^R e^{-rx}p(x)y(S, x)dx} - \frac{\int_{x_0}^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}{\int_0^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}, \tag{B.7}$$

Note that the right-hand side of (B.7) at $S = S^*$ and $R = R^*$ is (B.4). Now, from (B.4), we obtain $g(S^*) > 0$, $g(R^*) < 0$,

$$g'(x_0) = -\frac{e^{-rx_0}p(x_0)y(S^*, x_0)}{\int_{S^*}^{R^*} e^{-rx}p(x)y(S^*, x)dx} + \frac{e^{-[(1-\sigma)r+\sigma\rho]x_0}p(x_0)}{\int_0^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}, \tag{B.8}$$

and

$$g''(x_0) = \frac{\left[r + \mu(x_0) - \frac{y_{x_0}(S^*, x_0)}{y(S^*, x_0)}\right]e^{-rx_0}p(x_0)y(S^*, x_0)}{\int_{S^*}^{R^*} e^{-rx}p(x)y(S^*, x)dx}$$
$$- \frac{[(1-\sigma)r + \sigma\rho + \mu(x_0)]\,e^{-[(1-\sigma)r+\sigma\rho]x_0}p(x_0)}{\int_0^{\omega} e^{-[(1-\sigma)r+\sigma\rho]x}p(x)dx}, \tag{B.9}$$

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for any x_0 within the interval (S^*, R^*). Since $g(\cdot)$ is a continuous function in (S^*, R^*), $g(S^*) > 0$, and $g(R^*) < 0$ imply that there exists at least a critical age x_c within the interval (S^*, R^*) such that $g(x_c) = 0$.

In order to prove that x_c is unique, we show that there exists only one local optimum in the interval (S^*, R^*). At a local optimum (denoted by $\tilde{x}_0$, with $g'(\tilde{x}_0) = 0$), from (B.8) and (B.9) we obtain

$$g''(\tilde{x}_0) = \frac{\left[\sigma\left(r - \rho\right) - \frac{y_{x_0}(S^*, x_0)}{y(S^*, x_0)}\right] e^{-r\tilde{x}_0} p(\tilde{x}_0) y(S^*, \tilde{x}_0)}{\int_{S^*}^{R^*} e^{-rx} p(x) y(S^*, x) dx}. \tag{B.10}$$

Let $\tilde{x}_0^i$ and $\tilde{x}_0^{ii}$ be two possible candidates, which satisfy that $g'(\tilde{x}_0^i) = 0$ and $g'(\tilde{x}_0^{ii}) = 0$ with $\tilde{x}_0^i < \tilde{x}_0^{ii}$. Provided that $y(S^*, x)$ is strictly concave within the interval (S^*, R^*), $\tilde{x}_0$ is unique ($\tilde{x}_0^i = \tilde{x}_0^{ii}$) either because $\sigma\left(r - \rho\right) - \frac{y_{x_0}(S^*, x_0)}{y(S^*, x_0)} < 0$ or $\sigma\left(r - \rho\right) - \frac{y_{x_0}(S^*, x_0)}{y(S^*, x_0)} > 0$ for all $\tilde{x}_0 \in (S^*, R^*)$, or $\tilde{x}_0^i$ is a local maximum and $\tilde{x}_0^{ii}$ a local minimum, which proves that x_c is unique. ∎

The first component of (B.4) is the "lifetime human wealth" effect, while the second component represents the "years-to-consume" effect. An illustration of the shape of both effects across the life-cycle is given in Figure B.8. Notice the lifetime human wealth effect dominates the years-to-consume effect up to age $x_c \in (S^*, R^*)$, the year at which the financial wealth is zero, $a(x_c) = 0$. Therefore, a mortality decline early in life leads to an overall increase in consumption. In contrast, a decline in mortality at ages above x_c leads to an overall decline in consumption because the years-to-consume effect dominates the lifetime human wealth effect. Though for simplicity we have not modeled any retirement pension system, our results are robust to the introduction of a more general and realistic framework. Indeed, the introduction of an income during the retirement period will extend the lifetime human wealth effect up to age ω, shifting the age x_c toward older ages.

The partial impact of a decline in mortality on the length of schooling and retirement age is given in Proposition 4.

Proposition 4. *For the life-cycle model given by (1)-(5),*

(a) *the sign of* $-S^*_{\mu(x_0)}$ *is the same as that of*

$$\frac{r^h(S^*) - \bar{r}(S^*, R^*)}{\sigma(\bar{c})} a(x_0), \tag{B.11}$$

when $x_0 \leq S^*$ *and* $x_0 \geq R^*$, *and*

$$\frac{r^h(S^*) - \bar{r}(S^*, R^*)}{\sigma(\bar{c})} a(x_0) + \int_{x_0}^{R^*} e^{-r(x-x_0)} \frac{p(x)}{p(x_0)} y_S(S^*, x) dx, \tag{B.12}$$

when $x_0 \in (S^*, R^*)$, *and*

(b) *the sign of* $-R^*_{\mu(x_0)}$ *is the same as that of* $a(x_0)$.

Proof. Given the implicit-function theorem holds, there is one unique function $\Gamma(R; \mu)$ that equals S^* for any $(R; \mu)$ around $(R^*; \mu(x_0))$. Provided the optimal length of schooling condition (8)

$$\hat{V}_S(\Gamma(R; \mu), R; \mu) = \hat{V}_S(S^*, R^*; \mu(x_0)) = 0, \tag{B.13}$$
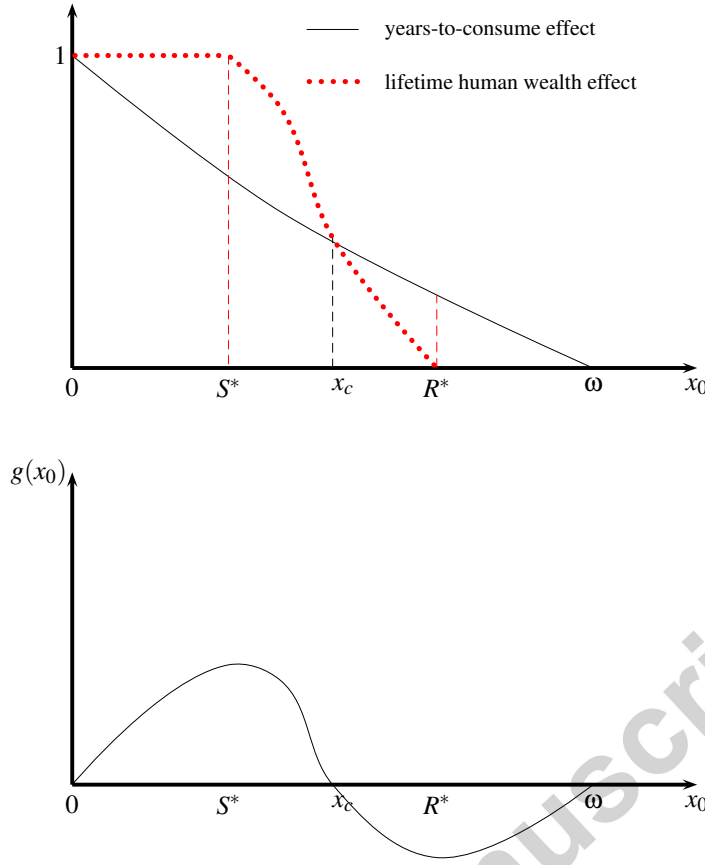
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Figure B.8: The lifetime human wealth and years-to-consume effect

and assuming $\hat{V}(S, R; \mu)$ is strictly concave around the point $(S^*, R^*; \mu(x_0))$. Then, we differentiate (B.13) with respect to a mortality decline at an arbitrary age x_0, $-\mu(x_0)$, to obtain the marginal impact of a mortality decline on the optimal length of schooling. Applying the Chain rule in (B.13) we obtain

$$-S^*_{\mu(x_0)} = \frac{\frac{-\partial f(S^*, R^*)}{\partial \mu(x_0)} - \frac{f(S^*, R^*)}{\sigma(c^*)} \frac{1}{c^*} \frac{-\partial c^*}{\partial \mu(x_0)}}{f(S^*, R^*) \left(\frac{\psi_S(S^*)}{\psi(S^*)} + \frac{1}{\sigma(c^*)} \frac{c^*_S}{c^*} \right) - f_S(S^*, R^*)}. \tag{B.14}$$

Thus, from (A.8) we have

$$\text{sign}\left[-S^*_{\mu(x_0)} \right] = \text{sign}\left[\frac{-\partial f(S^*, R^*)}{\partial \mu(x_0)} - \frac{f(S^*, R^*)}{\sigma(c^*)} \frac{1}{c^*} \frac{-\partial c^*}{\partial \mu(x_0)} \right]. \tag{B.15}$$

Note from (B.1) that the first term on the right-hand side of (B.15) is zero whenever $x_0 \leq S^*$ or $x_0 \geq R^*$. Substituting (11)-(12) and (B.2) in (B.15), and taking $e^{-r(x_0 - S^*)} \frac{p(x_0)}{p(S^*)}$ as common factor gives

$$\text{sign}\left[-S^*_{\mu(x_0)} \right] = \text{sign}\left[\frac{r^h(S^*) - \bar{r}(S^*, R^*)}{\sigma(\bar{c})} a(x_0) \right]. \tag{B.16}$$

when $x_0 \leq S^*$ and $x_0 \geq R^*$, and

$$\text{sign}\left[-S^*_{\mu(x_0)} \right] = \text{sign}\left[\frac{r^h(S^*) - \bar{r}(S^*, R^*)}{\sigma(\bar{c})} a(x_0) + \int_{x_0}^{R^*} e^{-r(x - x_0)} \frac{p(x)}{p(x_0)} y_S(S^*, x) dx \right]. \tag{B.17}$$

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when $x_0 \in (S^*, R^*)$. This proves Proposition 4(a).

Similarly, given that the implicit-function theorem holds, there is one unique function $\Upsilon(S; \mu)$ that equals R^* for any $(S; \mu)$ around $(S^*; \mu(x_0))$. Provided the optimal retirement age condition (9)

$$\hat{V}_R(S, \Upsilon(S; \mu); \mu) = \hat{V}_R(S^*, R^*; \mu(x_0)) = 0. \tag{B.18}$$

Assuming $\hat{V}(S, R; \mu)$ is strictly concave around the point $(S^*, R^*; \mu(x_0))$. We obtain after differentiating (B.18) with respect to $-\mu(x_0)$, and applying the Chain rule,

$$-R^*_{\mu(x_0)} = \frac{-\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial \mu(x_0)}}{\frac{1}{\sigma(c^*)}\frac{c^*_R}{c^*} + \delta + \frac{\phi_R(R^*)}{\phi(R^*)}}. \tag{B.19}$$

From (A.9), we have

$$\operatorname{sign}\left[-R^*_{\mu(x_0)}\right] = -\operatorname{sign}\left[\frac{-\partial c^*}{\partial \mu(x_0)}\right]. \tag{B.20}$$

Now, in order to prove Proposition (4)(b) we show that the sign of $-R^*_{\mu(x_0)}$ is that of $a(x_0)$.

Differentiating (6) at $S = S^*$ and $R = R^*$ with respect to $-\mu(x_0)$, and rearranging, gives

$$\int_{x_0}^{\omega} e^{-rx}p(x)c(x, S^*, R^*)dx + \int_0^{\omega} e^{-rx}p(x)\frac{-\partial c(x, S^*, R^*)}{\partial \mu(x_0)}dx = \int_{S^*}^{R^*} e^{-rx}\frac{-\partial p(x)}{\partial \mu(x_0)}y(S^*, x)dx. \tag{B.21}$$

The intertemporal budget constraint at age x_0 can be expressed as

$$e^{-rx_0}p(x_0)a(x_0)$$
$$= \begin{cases} \int_{x_0}^{\omega} e^{-rx}p(x)c(x, S^*, R^*)dx - \int_{x_0}^{R^*} e^{-rx}p(x)y(S^*, x)dx & \text{if } S^* < x_0 < R^*, \\ \int_{x_0}^{\omega} e^{-rx}p(x)c(x, S^*, R^*)dx & \text{otherwise.} \end{cases} \tag{B.22}$$

Substituting (B.22) into (B.21), we obtain

$$\int_0^{\omega} e^{-rx}p(x)\frac{-\partial c(x, S^*, R^*)}{\partial \mu(x_0)}dx = -e^{-rx_0}p(x_0)a(x_0). \tag{B.23}$$

Differentiating (7) at $S = S^*$ and $R = R^*$ with respect to $-\mu(x_0)$, and simplifying, gives

$$\frac{1}{c(x, S^*, R^*)}\frac{-\partial c(x, S^*, R^*)}{\partial \mu(x_0)} = \frac{\sigma(c(x, S^*, R^*))}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial \mu(x_0)}. \tag{B.24}$$

Substituting (B.24) into (B.23), and rearranging, we get (B.2). Finally, substituting (B.2) in (B.20), we have

$$\operatorname{sign}\left[-R^*_{\mu(x_0)}\right] = -\operatorname{sign}\left[\frac{-\partial c^*}{\partial \mu(x_0)}\right] = \operatorname{sign}\left[a(x_0)\right], \tag{B.25}$$

which proves Proposition 4(b). ∎

In Proposition 4(b) we obtain the same "consumption-leisure" relationship as in d'Albis et al. (2012). That is, given that consumption and leisure are normal goods, Proposition 4(b) implies that if a mortality decline yields an increase in consumption

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because the lifetime human wealth effect dominates the years-to-consume effect, agents anticipate their optimal retirement age in order to enjoy more leisure time. Similarly, when the decline in mortality implies that the years-to-consume effect dominates the lifetime human wealth effect, agents diminish their consumption and postpone their optimal retirement age.

Proposition 4(a) extends the years-to-consume effect and lifetime human wealth effect reasoning to the accumulation of human capital. In this regard, we obtain unambiguous results concerning the sign on the optimal length of schooling of a mortality decline at ages before the entrance into the labor market, S^*, and after the optimal retirement age, R^*. Specifically, Proposition 4(a) implies that when $r^h(S^*) > \bar{r}(S^*, R^*)$, if a mortality decline yields an increase in consumption because the lifetime human wealth effect dominates the years-to-consume effect, agents reduce their investment in education. Recall that this happens during the schooling period as Figure B.8 shows. In contrast, a decline in mortality after the optimal retirement age leads to more years of schooling. Instead, if $r^h(S^*) = \bar{r}(S^*, R^*)$ –as frequently assumed in the literature– a decline in mortality during the schooling period or during retirement period does not have an impact on the optimal length of schooling.

During the working period, Proposition 4(a) shows that a decline in mortality positively affects education through the second term in Eq. (B.12), which reflects the effect of a mortality decline at age x_0 on the marginal benefit of schooling (measured at age x_0), also known as the Ben-Porath mechanism. Actually, this is the only component driving the effect of mortality on the length of schooling when $r^h(S^*) = \bar{r}(S^*, R^*)$, but it is not the case whenever $r^h(S^*) \neq \bar{r}(S^*, R^*)$. In other words, under a model of human capital investment with a fixed retirement age, only improvements in survival during prime-working ages triggers the Ben-Porath mechanism (Cervellati and Sunde, 2013).

AppendixC. Proof of Proposition 4

Given the implicit-function theorem holds, there are two unique functions $\Gamma(R; \mu)$ and $\Upsilon(S; \mu)$ that are equal to S^* and R^*, respectively, for any $(S, R; \mu)$ around $(S^*, R^*; \mu(x_0))$. Provided the optimal length of schooling and retirement age conditions, we have:

$$\hat{V}_S(\Gamma(R; \mu), \Upsilon(S; \mu); \mu) = \hat{V}_S(S^*, R^*; \mu(x_0)) = 0, \tag{C.1a}$$

$$\hat{V}_R(\Gamma(R; \mu), \Upsilon(S; \mu); \mu) = \hat{V}_R(S^*, R^*; \mu(x_0)) = 0. \tag{C.1b}$$

For notational simplicity, hereinafter we skip the arguments. Writing the system of equations (C.1) in differential form we have

$$\hat{V}_{S\Gamma}d\Gamma + \hat{V}_{S\Upsilon}d\Upsilon + \hat{V}_{S\mu}d\mu = 0, \tag{C.2a}$$

$$\hat{V}_{R\Gamma}d\Gamma + \hat{V}_{R\Upsilon}d\Upsilon + \hat{V}_{R\mu}d\mu = 0. \tag{C.2b}$$

If $\hat{V}_S(\cdot)$ and $\hat{V}_R(\cdot)$ are continuously differentiable with respect to $(S, R; \mu)$, (S^*, R^*) is a solution of the system at the mortality value $\mu(x_0)$, and the Jacobian matrix of the system (C.1) evaluated at $(S^*, R^*; \mu(x_0))$ is not singular, or $|J| \neq 0$, then the system can be locally solved at $(S^*, R^*; \mu(x_0))$.

The solution of (C.2), by Cramer's rule, is

$$\begin{pmatrix} \frac{dS^*}{d\mu(x_0)} \\ \frac{dR^*}{d\mu(x_0)} \end{pmatrix} = \frac{1}{|J|} \begin{pmatrix} -\hat{V}_{RR}\hat{V}_{S\mu(x_0)} + \hat{V}_{RS}\hat{V}_{R\mu(x_0)} \\ \hat{V}_{SR}\hat{V}_{S\mu(x_0)} - \hat{V}_{SS}\hat{V}_{R\mu(x_0)} \end{pmatrix}. \tag{C.3}$$

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Taking $\hat{V}_{RR}\hat{V}_{SS}$ as common factor in the right-hand side of (C.3) gives

$$
\begin{pmatrix} \frac{dS^*}{d\mu(x_0)} \\ \frac{dR^*}{d\mu(x_0)} \end{pmatrix} = \frac{\hat{V}_{RR}\hat{V}_{SS}}{|J|} \begin{pmatrix} \frac{\hat{V}_{S\mu(x_0)}}{-\hat{V}_{SS}} + \frac{\hat{V}_{RS}}{-\hat{V}_{SS}}\frac{\hat{V}_{R\mu(x_0)}}{-\hat{V}_{RR}} \\ \frac{\hat{V}_{R\mu(x_0)}}{-\hat{V}_{RR}} + \frac{\hat{V}_{SR}}{-\hat{V}_{RR}}\frac{\hat{V}_{S\mu(x_0)}}{-\hat{V}_{SS}} \end{pmatrix}.
\tag{C.4}
$$

Provided the strict concavity of $\hat{V}(S,R;\mu)$ at $(S^*,R^*;\mu(x_0))$ and multiplying both sides of (C.4) by -1, since we are interested in the effect of a decline in mortality rather than an increase in mortality, we obtain

$$
\text{sign}\left[\frac{-dS^*}{d\mu(x_0)}\right] = -\,\text{sign}\left[S^*_{\mu(x_0)}(R^*;\mu) + \frac{dS^*}{dR^*}R^*_{\mu(x_0)}(S^*;\mu)\right],
\tag{C.5}
$$

$$
\text{sign}\left[\frac{-dR^*}{d\mu(x_0)}\right] = -\,\text{sign}\left[R^*_{\mu(x_0)}(S^*;\mu) + \frac{dR^*}{dS^*}S^*_{\mu(x_0)}(R^*;\mu)\right].
\tag{C.6}
$$

This completes the proof of Eqs. (23a) and (23b).

We now differentiate (A.4) and (A.5) with respect to $-\mu(x_0)$ at (S^*,R^*), respectively,

$$
-\hat{V}_{S\mu(x_0)} = U_c\big(c^*\big)e^{-rS^*}p(S^*)\left(\frac{-\partial f(S^*,R^*)}{\partial\mu(x_0)} - \frac{f(S^*,R^*)}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial\mu(x_0)}\right),
\tag{C.7}
$$

$$
-\hat{V}_{R\mu(x_0)} = U_c\big(c^*\big)e^{-rR^*}p(R^*)y(S^*,R^*)\left(-\frac{1}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial\mu(x_0)}\right).
\tag{C.8}
$$

Substituting (C.7)-(C.8) in (C.5), taking $\frac{U_c(c^*)e^{-rS^*}p(S^*)}{-\hat{V}_{SS}(S^*,R^*)}$ as common factor, and using (11), (17) and (19), we get

$$
\text{sign}\left[\frac{-dS^*}{d\mu(x_0)}\right] = \text{sign}\left[\frac{-\partial f(S^*,R^*)}{\partial\mu(x_0)} - \frac{f(S^*,R^*)}{\sigma(c^*)}\frac{1}{c^*}\frac{-\partial c^*}{\partial\mu(x_0)}\left(1 + \frac{dR^*}{dS^*}\frac{c^*_R}{c^*_S}\right)\right].
\tag{C.9}
$$

Using (10), (17), and (B.2) we obtain, after rearranging,

$$
\text{sign}\left[\frac{-dS^*}{d\mu(x_0)}\right] = \text{sign}\left[\int_{S^*}^{R^*}e^{-r(x-S^*)}\frac{-\partial}{\partial\mu(x_0)}\left[\frac{p(x)}{p(S^*)}\right]y_S(S^*,x)dx\right.
$$
$$
\left. -e^{-r(x_0-S^*)}\frac{p(x)}{p(S^*)}a(x_0)\frac{1}{\sigma(c^*)c^*}\left(c^*_S + c^*_R\frac{dR^*}{dS^*}\right)\right].
\tag{C.10}
$$

Finally, taking $e^{-r(x_0-S^*)}\frac{p(x)}{p(S^*)}$ as common factor and using the fact that $\frac{dc^*}{dS^*} = c^*_S + c^*_R\frac{dR^*}{dS^*}$ we obtain Proposition 2(a), or Eq. (24).

Using the same steps for the sign of $\frac{-dR^*}{d\mu(x_0)}$ but now taking $-\hat{V}_{RR}$ as common factor, it can be shown that

$$
\text{sign}\left[\frac{-dR^*}{d\mu(x_0)}\right] = \text{sign}\left[\frac{dS^*}{dR^*}\int_{S^*}^{R^*}e^{-r(x-S^*)}\frac{-\partial}{\partial\mu(x_0)}\left[\frac{p(x)}{p(S^*)}\right]y_S(S^*,x)dx\right.
$$
$$
\left. -e^{-r(x_0-S^*)}\frac{p(x)}{p(S^*)}a(x_0)\frac{1}{\sigma(c^*)c^*}\left(c^*_R + c^*_S\frac{dS^*}{dR^*}\right)\right].
\tag{C.11}
$$

Similarly, taking $e^{-r(x_0-S^*)}\frac{p(x)}{p(S^*)}$ as common factor and using the fact that $\frac{dc^*}{dR^*} = c^*_R + c^*_S\frac{dS^*}{dR^*}$ we obtain Proposition 2(b), or Eq. (25).

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