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General Metrics in Relativistic Alpha Field Theory

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Abstract
The notion of an alpha field has been associated to any potential field that can be presented by two dimensionless field parameters α and α'. The problem in this paper is to derive the generalized metrics in the Relativistic Alpha Field Theory (RAFT). In that sense, it is started with the new General Lorentz Transformation model in an alpha field (GLT_α – model) derived by employing the well known group postulates and isotropy of the space. It follows the derivation of the general line element and the related general metric tensor in an alpha field both in the Minkowski and Riemannian metrics. The one section of the paper is devoted to derivation of a general diagonal form of a line element and metric tensor in an alpha field. It has been shown that there exists a simple coordinate transformation procedure in an alpha field that transforms the Riemannian metrics into the Minkowski one and vice versa.

Keywords: General Metrics, Relativistic Alpha Field Theory, General Line Element, General Metric Tensors, Diagonal Forms in Metrics.

1 Introduction

This paper has been written by consideration of the related theories and fundamental laws of physics in the references [1-21]. The notion of an alpha field has been associated to any potential field that can be presented by two dimensionless field parameters α and α'. For an example, to this category belong an electromagnetic field, a gravitational field and their combination (a two-potential field). These parameters should satisfy the field equations of the related potential field in each concrete case. The field parameters α and α' are the functions of the potential energy of the related field. In the case of the multi-potential field, the field parameters α and α' become the functions of the total potential energy in the multi-potential field. This fact opens the possibilities of derivation of the form invariant mathematical descriptions that unify dynamics of one, or two, or more potential fields. Even vacuum (without any potential field) is included, because in that case the field parameters α and α' are equal to one ($\alpha = \alpha' =1$). Following this idea one can derive the form invariant features that unify the well known Einstein's Special and General Theory of Relativity.

The first step in that unification should be the derivation of the related unified metric forms. Thus, the problem in this paper is to derive the generalized metrics in the Relativistic Alpha Field Theory (RAFT). In that sense, it is started with the new General Lorentz Transformation model in an alpha field (GLT_α – model) derived by employing the well known group postulates and isotropy of the space. It follows the derivation of the general line element and the related general metric tensor in an alpha field, both in the Minkowski and Riemannian metrics. The one section of the paper is devoted to derivation of a general diagonal form of a line element and metric tensor in an alpha field. It has been shown that there exists a simple coordinate transformation procedure in an alpha field that transforms the Riemannian metrics into the Minkowski one and vice versa. The presented line elements and metric tensors can be used in Special Relativity for $\alpha = \alpha' = 1$ and in General Relativity by identification of the field parameters α and α' in a gravitational potential field, using the well known Einstein's field equations. Following the considerations in this paper, one can conclude that the unification of the line elements, metric tensors and coordinate transformations in the Einstein's Special and General Theory of Relativity is possible, if one employs the dimensionless field parameters α and α'.

This paper is organized as follows. Derivation of general Lorentz Transformation model in an alpha field (GLT_α – model) is presented in the section 2. The general line element in an alpha field, as function of the dimensionless field parameters α and α', is derived in the section 3. The related general metric tensors in an alpha field are presented in the section 4. Derivations of the general diagonal form of the line element and the metric tensor in an alpha field are pointed out in the section 5. Finally, the conclusion and the reference list are given in the sections 6 and 7, respectively.

2 Derivation of General Lorentz Transformation Model in an Alpha Field (GLT_α - Model) from Group Postulates

The GLT_α – model can be derived, among the others, by employing the group postulates [1] and isotropy of the space. The coordinate transformations between inertial frames form a group. This group is called the proper Lorentz group with the group operation being the composition of transformations. This means performing one transformation after another. In that sense, the following four group axioms should be satisfied:

1. Closure: the composition of two transformations is a transformation. In such a manner a composition of transformations from the inertial frame K to inertial frame K' and then from K' to inertial frame K'' can be replace with a transformation directly from an inertial frame K to inertial frame K'':

$$[\mathrm{K} \to \mathrm{K}'][\mathrm{K}' \to \mathrm{K}''] = [\mathrm{K} \to \mathrm{K}''].\qquad (1)$$

2. Associativity: the result of the following two transformations is apparently the same:

$$\begin{aligned}&([\mathrm{K} \to \mathrm{K}'][\mathrm{K}' \to \mathrm{K}''])[\mathrm{K}'' \to \mathrm{K}'''] = [\mathrm{K} \to \mathrm{K}'''],\\ &[\mathrm{K} \to \mathrm{K}']([\mathrm{K}' \to \mathrm{K}''][\mathrm{K}'' \to \mathrm{K}''']) = [\mathrm{K} \to \mathrm{K}'''].\end{aligned}\qquad (2)$$

3. Identity element: there is an identity element, a transformation K $\to$ K.

4. Inverse element: for any transformation K $\rightarrow$ K' there apparently exists an inverse transformation K' $\rightarrow$ K.

Let an inertial frame K' is moving in an alpha field with a velocity $\vec{v}_\alpha$ relative to an inertial frame K. Using rotations and shifts operations one can choose the x axis in K and x' axis in K' along the relative velocity vector $\vec{v}_\alpha$ and that the events (t = 0, x = 0) and (t' = 0, x' = 0) coincide. The velocity boost is along the x and x' axes only, therefore nothing happens to the perpendicular coordinates (y, z) and (y', z') and one can just omit them for brevity. The transformation [K$\rightarrow$K'] connects two inertial frames. Therefore it has to transform a linear motion in (t, x) into a linear motion in (t', x') coordinates. The conclusion is that the transformation [K$\rightarrow$K'] must be a linear transformation. This also includes that a relative velocity v_α between K and K' systems should be a constant. Meanwhile, a motion in an alpha field with relative velocity v_α, between reference frames K' and K, generally is not a constant. In order to derive a linear coordinate transformation model, one should assume that in the infinitesimally small space-time regions of an alpha field (dx, dt) and (dx', dt'), a relative velocity v_α is a constant. In that case the General Lorentz Transformation in an alpha field [K$\rightarrow$K'] transforms a linear motion in (dt, dx) into a linear motion in (dt', dx') coordinate system.

Generally, for a relative motion of the systems K and K' in an alpha field, the relative velocity v_α is a composition of the two velocities (v and v_f). Here v is a component of the relative velocity v_α that is equal to the free particle motion. The velocity v_f is a component of the relative velocity v_α that shows an influence of an alpha field to the particle motion. For an example, v_f could be a free fall particle velocity in a gravitational field. Therefore, the velocity v_f should be a function of the field parameters α and α'. Taking into account the previous consideration, the relative velocity v_α , between two systems K and K', can be described as the following composition of its components:

$$\begin{gathered} v_\alpha \;=\; v_{\alpha_x} = v_x + v_{f_x} = v\cos\varphi + v_f\cos\psi = v_x - \frac{\kappa(\alpha-\alpha')_x c}{2}, \\ \varphi=\psi=0 \quad\rightarrow\quad v_\alpha \;=\; v_{\alpha_x} = v - \frac{\kappa(\alpha-\alpha')c}{2}. \end{gathered} \tag{3}$$

Here φ and ψ are angles between vectors ($\vec{v},\vec{v}_\alpha$) and ($\vec{v}_f,\vec{v}_\alpha$), respectively. In the relation (3) we assume that the observation signal is the light with invariant velocity c in both system K, and K'. Finally, we can employ, for the convenience, an observation parameter κ. Thus, one can put $\kappa = 1$ if an observation signal is emitted from the origin of the system K, or $\kappa = -1$ if an observation signal is emitted from the origin of the system K'. The relation (3) can also be obtained by using the relativistic adding law for velocities and putting u' = 0. The structure of the relative velocity v_α in (3) has been confirmed, among the others, by derivation of generalized relativistic Hamiltonian in an alpha field [21] that, after inclusion of the related field parameters α and α', is valid for both an electromagnetic and a gravitational field.

In order to derive the GLT_α – model in the tensor form one can introduce the differential forms of the displacement four-vectors, dX and dX'. Let these four-vectors are defined in frames K and K' by the relations:

$$
\begin{gathered}
dX \rightarrow K\left(cdt, dx, dy, dz\right) = \left\{dx^{\mu}\right\}, \\
dX' \rightarrow K'\left(cdt', dx', dy', dz'\right) = \left\{dx'^{\mu}\right\}, \ \mu = 0,1,2,3.
\end{gathered} \tag{4}
$$

Here dX has the components in the frame K, and dX' has the components in the frame K'. The variables dx^{μ}, or dx'^{μ} are the related contravariant coordinates in a space-time region.

Because the field parameters α and α' are functions of the space-time coordinates, a particle velocity, v_α, in an alpha field is not a constant. In derivation of the GLT_α – model we supposed that in the infinitesimally small space-time regions of an alpha field dx^{μ}, and dx'^{μ}, $\mu = 0, 1, 2, 3$, the particle velocity v_α is a constant. This means that in the mentioned infinitesimally small intervals of dx^{μ}, and dx'^{μ}, the field parameters α and α', and the free particle velocity v are constants. Now, taking into account the displacement four – vectors (4) and employing the four group postulates [4] one can derive a tensor form of the GLT_α - model for the events on x-axis:

$$
\begin{gathered}
dx'^{\mu} = \Lambda^{\mu}{}_{\nu}\, dx^{\nu}, \qquad \mu, \nu = 0,1,2,3, \\
v_\alpha = \text{const.} \quad \rightarrow \quad x'^{\mu} = \Lambda^{\mu}{}_{\nu}\, x^{\nu}.
\end{gathered} \tag{5}
$$

In this relation the Einstein's summation convention is postulated. The first line in (5) is valid for v_α is a constant in the regions (dx,dt) and (dx',dt'), while the second line in (5) is valid for v_α is a constant in the regions (x,t) and (x',t'). The term $\Lambda^{\mu}{}_{\nu}$ is the element in the μ-th line and ν-th column of the (4x4) transformation matrix $[\Lambda^{\mu}{}_{\nu}]$ of GLT_α - model:

$$
\left[\Lambda^{\mu}{}_{\nu}\right] = \begin{bmatrix} H & -H\beta_\alpha & 0 & 0 \\ -H\beta_\alpha & H & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{6}
$$

In the relation (6), the parameters H and β_α are presented by the equations:

$$
\begin{gathered}
H = \left(1 - \frac{v_\alpha^2}{c^2}\right)^{-1/2} = \left(\alpha\alpha'_x - \frac{v_x^2}{c^2} + \frac{\kappa\left(\alpha - \alpha'\right)_x c\, v_x}{c^2}\right)^{-1/2}, \\
\beta_\alpha = \frac{v_\alpha}{c}, \qquad v_\alpha = v_x - \frac{\kappa\left(\alpha - \alpha'\right)_x c}{2}, \quad \alpha\alpha'_x = 1 - \frac{\kappa^2\left(\alpha - \alpha'\right)_x^2}{4}.
\end{gathered} \tag{7}
$$

If the angles between vectors ($\vec{v}, \vec{v}_\alpha$) and ($\vec{v}_f, \vec{v}_\alpha$) are equal to zero ($\varphi = 0$ and $\psi = 0$, respectively) then $v_x \to v$ and $\kappa(\alpha - \alpha')_x \to \kappa(\alpha - \alpha')$. In that case the relations (7) are transformed into the expressions:

$$H = \left(1 - \frac{v_\alpha^2}{c^2}\right)^{-1/2} = \left(\alpha\alpha' - \frac{v^2}{c^2} + \frac{\kappa(\alpha - \alpha')cv}{c^2}\right)^{-1/2},$$
$$\beta_\alpha = \frac{v_\alpha}{c}, \qquad v_\alpha = v - \frac{\kappa(\alpha - \alpha')c}{2}, \quad \alpha\alpha' = 1 - \frac{\kappa^2(\alpha - \alpha')^2}{4}. \tag{8}$$

It is easy to see that in the case of vacuum (without any potential field) the field parameters α and α' should be equal to one ($\alpha = \alpha' = 1$). For that case the relations (6), (7) or (8) are transformed into the well known Lorenz Transformation model (LT – model) [1] .

3 General line Element in an Alpha Field

Generally, in mathematics, a line element can be thought of as the square of the change in a position vector in an affine space. This is equated to the square of the change of the arc length [2-8]. A line element is a function of the metrics and can be related to the curvature tensor. Therefore, the line elements are often used in physics, mainly in the Special and General Theory of Relativity. Thus, in a gravitational field a space-time is modelled as a curved manifold with Riemannian metrics. When one wants to consider space and time derivatives of functions it is agreed that time derivatives are taken with respect to the proper time τ [8-11]. This is because a proper time is an invariant, with consequence that the proper time derivative of any four-vector is itself a four-vector. From the time dilation relation in the GLT_α - model one has very important equation that connects the proper time derivative $d\tau$ with another time derivative dt:

$$\frac{dt}{d\tau} = H = \left(1 - \frac{v_\alpha^2}{c^2}\right)^{-1/2} = \left(\alpha\alpha' - \frac{v^2}{c^2} + \frac{\kappa(\alpha - \alpha')cv}{c^2}\right)^{-1/2}. \tag{9}$$

Here $d\tau$ is a differential of the proper time of the moving particle, H is a transformation factor, as an invariant of the GLT_α – model and v_α is a particle velocity in an alpha field. In the relation (9) we suppose (without losing in a generality) that the angles between velocity vectors ($\vec{v}, \vec{v}_\alpha$) and ($\vec{v}_f, \vec{v}_\alpha$) are equal to zero ($\varphi = 0$ and $\psi = 0$, respectively). The consequences are that $v_x \to v$, $(\alpha\alpha')_x \to \alpha\alpha'$ and $\kappa(\alpha - \alpha')_x \to \kappa(\alpha - \alpha')$.

In order to derive a line element ds^2 of the GLT_α - model one can employ the equation (9) and derivation procedure from the references [5,6,8,11,13 and 20]:

$$d\tau^2 = \frac{-ds^2}{c^2} = \frac{1}{H^2}dt^2 \quad \to \quad ds^2 = -\frac{1}{H^2}c^2dt^2. \tag{10}$$

Including the first form of H from (9) to the equation (10) one obtains the line element in the following general form:

$$H = \left(1 - \frac{v_\alpha^2}{c^2}\right)^{-1/2} \quad \rightarrow \quad ds^2 = -\,c^2 dt^2 + v_\alpha^2 dt^2. \tag{11}$$

Now, one can make the following substitutions into the relation (11):

$$v_\alpha^2 = \left(v_\alpha^1\right)^2 + \left(v_\alpha^2\right)^2 + \left(v_\alpha^3\right)^2, \qquad v_\alpha^\mu = \frac{dx_\alpha^\mu}{dt}, \qquad \mu = 1,2,3. \tag{12}$$

These substitutions transform the second equation in (11) into the first form of the line element of GLT_α – model, valid for a particle motion in an alpha field:

$$ds^2 = -c^2 dt^2 + \left(dx_\alpha^1\right)^2 + \left(dx_\alpha^2\right)^2 + \left(dx_\alpha^3\right)^2. \tag{13}$$

Since $dx_\alpha^1 = dx_\alpha, dx_\alpha^2 = dy_\alpha, dx_\alpha^3 = dz_\alpha,$ one can describe the first form of the general line element of the GLT_α - model by the following equation:

$$ds^2 = -c^2 dt^2 + dx_\alpha^2 + dy_\alpha^2 + dz_\alpha^2. \tag{14}$$

This line element has a diagonal form with the Minkowski metrics. Meanwhile, the coordinates dx_α, dy_α and dz_α are the functions of the field parameters α and α' as it follows from (3) and (12):

$$\begin{gathered} dx_\alpha = dx - \frac{\kappa\left(\alpha - \alpha'\right)_x c}{2} dt, \qquad dy_\alpha = dy - \frac{\kappa\left(\alpha - \alpha'\right)_y c}{2} dt, \\ dz_\alpha = dz - \frac{\kappa\left(\alpha - \alpha'\right)_z c}{2} dt. \end{gathered} \tag{15}$$

Now, applying the second form of H from (9) to the equation (10) one obtains the second general form of the line element as the explicit function of the dimensionless field parameters α and α':

$$\begin{gathered} H = \left(\alpha\alpha' - \frac{v^2}{c^2} + \frac{\kappa\left(\alpha - \alpha'\right) c\, v}{c^2}\right)^{-1/2} \rightarrow \\ ds^2 = -\,\alpha\alpha' c^2 dt^2 - \kappa\left(\alpha - \alpha'\right) v c\, dt^2 + \left(v\right)^2 dt^2. \end{gathered} \tag{16}$$

Further, one can make the following substitutions into the relation (16):

$$\kappa(\alpha-\alpha')v = \kappa(\alpha-\alpha')_x v^1 + \kappa(\alpha-\alpha')_y v^2 + \kappa(\alpha-\alpha')_z v^3,$$
$$(v)^2 = \left(v^1\right)^2 + \left(v^2\right)^2 + \left(v^3\right)^2, \qquad v^\mu = \frac{dx^\mu}{dt}, \qquad \mu = 1,2,3. \tag{17}$$

These substitutions transform the equation (16) into the second form of a line element of GLT_α – model, valid for a particle motion in an alpha field:

$$ds^2 = -\alpha\alpha' c^2 dt^2 - \kappa(\alpha-\alpha')_x\, cdt\, dx^1 - \kappa(\alpha-\alpha')_y\, cdt\, dx^2 -$$
$$-\kappa(\alpha-\alpha')_z\, cdt\, dx^3 + \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2. \tag{18}$$

Since $dx^1 = dx$, $dx^2 = dy$ and $dx^3 = dz$, we can describe the second form of the general line element of the GLT_α – model (18) by the following equation:

$$ds^2 = -\alpha\alpha' c^2 dt^2 - \kappa(\alpha-\alpha')_x\, cdtdx - \kappa(\alpha-\alpha')_y\, cdtdy -$$
$$-\kappa(\alpha-\alpha')_z\, cdtdz + dx^2 + dy^2 + dz^2. \tag{19}$$

This line element has a nondiagonal form with the Riemannian metrics. It can be shown that the substitutions of the coordinates (15) into the line element (14) and including relations for $\alpha\alpha'$ from (7) or (8), the diagonal form of the line element (14) with Minkowski metrics can be transformed into the nondiagonal form of the line element (19) with Riemannian metrics.

If a particle is moving in a vacuum (without any potential field) then the field parameters α and α' satisfy the relation $\alpha = \alpha' = 1$. For that case the second form of the general line element (19) is transformed into the well-known line element valid in the Special Relativity:

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2. \tag{20}$$

As it is the well known, the line elements (14), (19) and (20) are a space like if $ds^2 > 0$, a time like if $ds^2 < 0$, and a null (or light) like if $ds^2 = 0$.

4 General Metric Tensors in an Alpha Field

The differential form of the contravariant displacement four-vector, dX_α, of the GLT_α – model, presented in the first form of the line element (14), can be defined in the frame K by the relation:

$$dX_\alpha \rightarrow K\left(cdt, dx_\alpha, dy_\alpha, dz_\alpha\right) = \left\{dx_\alpha^\mu\right\}, \qquad \mu = 0,1,2,3. \tag{21}$$

Following (13), (14) and (21) one can derive a matrix expression of the components of the general covariant metric tensor $g_{\mu\nu}$, valid for the first form of the line element (14) and the coordinate system (21):

$$\left[g_{\mu\nu}\right] = \mathrm{diag}\left[-1 \quad 1 \quad 1 \quad 1\right] = \left[\eta_{\mu\nu}\right]. \tag{22}$$

This matrix expression of the metric tensor is diagonal and belongs to the well known Minkowski metrics $\eta_{\mu\nu}$, as we expected that should be. Therefore, the related line element (14) is also called a diagonal line element. The related components of the contravariant general metric tensor $g^{\mu\nu}$ in an alpha field can be derived by inversion of the covariant one using (22). As the result of that inversion one obtains a matrix expression of the components of the general contravariant metric tensor $g^{\mu\nu}$ equal to (22):

$$\left[g^{\mu\nu}\right] = \left[g_{\mu\nu}\right] = \left[\eta^{\mu\nu}\right] = \left[\eta_{\mu\nu}\right]. \tag{23}$$

The determinants and traces of the matrices of the components of the metric tensors (22) and (23) are presented by the relations:

$$\begin{aligned} &\det\left[g_{\mu\nu}\right] = \det\left[\eta_{\mu\nu}\right] = -1, \qquad \det\left[g^{\mu\nu}\right] = \det\left[\eta^{\mu\nu}\right] = -1, \\ &T_R\left[g_{\mu\nu}\right] = T_R\left[\eta_{\mu\nu}\right] = 2, \qquad T_R\left[g^{\mu\nu}\right] = T_R\left[\eta^{\mu\nu}\right] = 2. \end{aligned} \tag{24}$$

Now, one can recall the well known condition that should be satisfied by any metric tensor [9-12]:

$$\sqrt{-\det\left[g_{\mu\nu}\right]} = 1. \tag{25}$$

From the relations (24) one can conclude that the condition (25) is satisfied.

The differential form of the contravariant displacement four-vector, dX, of the GLT_α –model, presented in the second form of the general line element (19), can be defined in the frame K by the relation:

$$dX \to K\left(cdt, dx, dy, dz\right) = \left\{dx^\mu\right\}, \qquad \mu = 0,1,2,3. \tag{26}$$

Applying displacement four-vector dX from (26), the second form of the general line - element (19) is transformed into the equation:

$$\begin{aligned} ds^2 = &-\alpha\alpha'\left(dx^0\right)^2 - \kappa\left(\alpha-\alpha'\right)_x dx^0 dx^1 - \kappa\left(\alpha-\alpha'\right)_y dx^0 dx^2 - \\ &-\kappa\left(\alpha-\alpha'\right)_z dx^0 dx^3 + \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2. \end{aligned} \tag{27}$$

As it is the well known, the Riemannian line element can be introduced by the following expression:

$$\begin{aligned} ds^2 = &g_{00}\left(dx^0\right)^2 + 2g_{01}dx^0dx^1 + 2g_{02}dx^0dx^2 + 2g_{03}dx^0dx^3 + \\ &+ g_{11}\left(dx^1\right)^2 + g_{22}\left(dx^2\right)^2 + g_{33}\left(dx^3\right)^2. \end{aligned} \tag{28}$$

Here **g** is the related metric tensor of the Riemannian manifold. Comparing the equations (27) and (28), one can conclude that non-null components of the metric tensor **g** in the line element (27) are determined by the following relations:

$$g_{00} = -\alpha\alpha', \quad g_{01} = g_{10} = b_x = \frac{-\kappa(\alpha - \alpha')_x}{2}, \quad g_{02} = g_{20} = b_y = \frac{-\kappa(\alpha - \alpha')_y}{2},$$

$$g_{03} = g_{30} = b_z = \frac{-\kappa(\alpha - \alpha')_z}{2}, \qquad g_{11} = 1, \qquad g_{22} = 1, \qquad g_{33} = 1. \tag{29}$$

Following (28) and (29) one can derive a matrix expression of the components of the general covariant metric tensor $g_{\mu\nu}$ in an alpha field, related to the second form of the general line element (27):

$$\left[g_{\mu\nu}\right] = \begin{bmatrix} -\alpha\alpha' & b_x & b_y & b_z \\ b_x & 1 & 0 & 0 \\ b_y & 0 & 1 & 0 \\ b_z & 0 & 0 & 1 \end{bmatrix}. \tag{30}$$

This metric tensor is symmetric and has ten non-zero elements, as we expected that should be. The matrix expression of the metric tensor (30) is nondiagonal and belongs to the well known Riemannian metrics $g_{\mu\nu}$. Therefore, the related line elements (19) and (27) are also called nondiagonal line elements.

The related components of the contravariant general metric tensor $g^{\mu\nu}$ in an alpha field can be derived by inversion of the covariant one using (30). As the result of that inversion one obtains the components of the general contravariant metric tensor $g^{\mu\nu}$ in an alpha field:

$$g^{00} = \frac{-1}{\alpha\alpha' + b^2}, \qquad g^{01} = g^{10} = \frac{b_x}{\alpha\alpha' + b^2}, \qquad g^{02} = g^{20} = \frac{b_y}{\alpha\alpha' + b^2},$$

$$g^{03} = g^{30} = \frac{b_z}{\alpha\alpha' + b^2}, \qquad g^{11} = \frac{\alpha\alpha' + b_y^2 + b_z^2}{\alpha\alpha' + b^2}, \qquad g^{12} = g^{21} = \frac{-b_x b_y}{\alpha\alpha' + b^2},$$

$$g^{13} = g^{31} = \frac{-b_x b_z}{\alpha\alpha' + b^2}, \qquad g^{22} = \frac{\alpha\alpha' + b_x^2 + b_z^2}{\alpha\alpha' + b^2}, \qquad g^{23} = g^{32} = \frac{-b_y b_z}{\alpha\alpha' + b^2},$$

$$g^{33} = \frac{\alpha\alpha' + b_x^2 + b_y^2}{\alpha\alpha' + b^2}, \qquad b^2 = b_x^2 + b_y^2 + b_z^2 . \tag{31}$$

The related quantities of the parameters b_x, b_y, and b_z are given by (29). The determinants and traces of the metric tensors (30) and (31) are presented by the relations:

$$\det\left[g_{\mu\nu}\right] = -\left(\alpha\alpha' + b^2\right), \qquad \det\left[g^{\mu\nu}\right] = -1/\left(\alpha\alpha' + b^2\right),$$
$$T_R\left[g_{\mu\nu}\right] = 3 - \alpha\alpha', \qquad T_R\left[g^{\mu\nu}\right] = \quad 2 + \frac{\alpha\alpha' - 1}{\alpha\alpha' + b^2}. \tag{32}$$

Now, we recall the well known condition (25) that should be satisfied by any metric tensor [9-12]. Including the determinant of the matrix of the metric tensor components $g_{\mu\nu}$ from (32) into the relation (25) one obtains the important relation between field parameters α and α':

$$\sqrt{-\det\left[g_{\mu\nu}\right]} = \sqrt{\alpha\alpha' + b^2} = \sqrt{\alpha\alpha' + \frac{\kappa^2\left(\alpha - \alpha'\right)^2}{4}} = 1,$$
$$\kappa^2 = 1, \qquad \rightarrow \qquad \left(\frac{\alpha + \alpha'}{2}\right)^2 = 1. \tag{33}$$

This relation can be employed in the procedure of determination of the field parameters α and α' in each particular alpha field. The condition (33) is also satisfied for $\alpha = \alpha' = 1$ that is related to the particle motion in a vacuum (without any potential field). This case belongs to the Special Theory of Relativity.

If a particle is moving in a vacuum (without any potential field) then the field parameters α and α' satisfy the relation $\alpha = \alpha' = 1$. For that case the parameters $b_x = b_y = b_z = 0$, and the second form of the general line element (27) is transformed into the form (20). In that case the covariant Riemannian metric tensor **g** (30) is transformed into the well known Minkowski metric tensor $\boldsymbol{\eta}$ (22). As it is the well known, this metric tensor is valid in the Special Relativity.

5 Derivation of a General Diagonal form of line Element and Metric Tensor in an Alpha Field

The problem is to derive a general diagonal form of the line element in an alpha field in the following expression:

$$ds^2 = -\left(dz^0\right)^2 + \left(dz^1\right)^2 + \left(dz^2\right)^2 + \left(dz^3\right)^2. \tag{34}$$

In this relation dz^μ, $\mu = 0,1,2,3$, are the new contravariant coordinates of the system. From (34) one can see that diagonal line element generates a diagonal matrix of the components of the metric tensor equal to the well known form of the Minkowski metric tensor structure (22). As we know, this metric structure is used in the Special Relativity. In order to solve the mentioned problem, one can start with the nondiagonal second form of the general line element (27) that belongs to the Riemannian metrics:

$$ds^2 = -\alpha\alpha'\left(dx^0\right)^2 + 2b_x dx^0 dx^1 + 2b_y dx^0 dx^2 +$$
$$+ 2b_z dx^0 dx^3 + \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2. \tag{35}$$

The contravariant coordinates dx^{μ} , $\mu = 0, 1, 2, 3$, in (35) are given by (26). The related parameters b_x, b_y, and b_z have the forms given by (29). Now, one can employ the following coordinate transformation procedure:

$$dz^0 = a\,dx^0 = \sqrt{\alpha\alpha' + b^2}\;dx^0, \quad dz^1 = b_x dx^0 + dx^1, \quad dz^2 = b_y dx^0 + dx^2, \\ dz^3 = b_z dx^0 + dx^3, \qquad b^2 = b_x{}^2 + b_y{}^2 + b_z{}^2. \tag{36}$$

Applying (36) to the general diagonal form of the line element (34) one obtains the non-diagonal form of the line element (35). This confirms that the coordinate transformations (36) transform the non-diagonal line element (35) into the diagonal one (34) and vice versa.

It can be shown that the coordinate transformations (36) are equal to the coordinate transformations (15) if the condition $(\alpha\alpha' + b^2) = 1$ is satisfied. Since this condition is always satisfied by (33), one can conclude that the following relations are valid:

$$dx_{\alpha}^0 = dz^0, \quad dx_{\alpha}^1 = dz^1, \quad dx_{\alpha}^2 = dz^2, \quad dx_{\alpha}^3 = dz^3\,. \tag{37}$$

Thus, the first form of the general line element of the GLT_{α} – model, given in (13), belongs to the general diagonal form of the line element in an alpha field (34). Both diagonal forms have the Minkowski metrics (22).

6 Conclusion

The presented line elements and metric tensors can be used in Special Relativity for $\alpha = \alpha' = 1$ and in General Relativity by identification of the field parameters α and α' in a gravitational potential field, using the well known Einstein's field equations. Generally, these line elements and tensors can be employed in any alpha field if the identification of the field parameters α and α' is possible in that potential field. The coordinate transformations (36) transform the Riemannian metrics (35) into the Minkowski metrics (34), and vice versa. Following the considerations in this paper, one can conclude that the unification of the line elements, metric tensors and coordinate transformations in the Einstein's Special and General Theory of Relativity is possible, if one employs the dimensionless field parameters α and α'. For solution of some problems (like identification of the field parameters α and α' in a gravitational field) the presented models should be transformed into the spherical polar coordinates, which are appropriate to these problems. This will be done in the next paper, together with the identification of the dimensionless field parameters α and α' in a gravitational field. In the future works, the presented approach will be applied to the other items in the Special and General Theory of Relativity.

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