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Robust Decentralized Global Asymptotic Tracking Control of a Class of Nonlinear Mechanical Systems

Josip Kasac, Tihomir Zilic, Vladimir Milic, Andrej Jokic and Mihael Lobrovic

Abstract— **In this paper, a RISE type of tracking controllers for a class of nonlinear mechanical systems is proposed. The proposed chattering-free controller provides global asymptotic tracking in the presence of external disturbances. The proof of global asymptotic stability is based on a novel approach to the construction of a Lyapunov function which is parameterized by a time-varying function of reference and disturbance vector. The explicit conditions on the controller gains to ensure global asymptotic tracking are obtained. The simulation results on a system of three inverted pendulums interconnected by two springs illustrate the performances of the proposed controller.**

I. INTRODUCTION

The proportional-integral-derivative (PID) controllers are widely used because of their relatively simple implementation and effectiveness in the vast majority of applications. A simple model-free decoupled PID feedback controller with appropriate control gains achieves the constant desired position without any steady-state error. This is the main reason why PID controllers are still used in the regulation of nonlinear mechanical systems [1], [2], [3]. However, PID controllers cannot render asymptotic stability for tracking tasks of mechanical systems. Also, PID controllers perform poor capabilities of dealing with system uncertainties like external disturbances.

In the case of periodic reference signals and periodic external disturbances of known frequency, a model-free control approach for mechanical systems is still possible by using a repetitive learning controllers [4], [5]. Despite the high precision tracking performances, the main disadvantages of repetitive controllers is relatively high sensitivity to the frequency of a periodic signal and aperiodic distortions of signals.

As a robust control approach, sliding mode control has been applied to the trajectory tracking control of robot manipulators [6], [7]. The advantages of using sliding mode control include robustness with regard to parameter variations and arbitrary bounded time-varying disturbances. The main drawback of static sliding mode controllers is chattering phenomenon, caused by the high-frequency control switching, which may excite unmodeled high frequency dynamics and even cause system instability. A standard way for chattering reduction is boundary layer method which provides smooth control action, but with price of losing asymptotic tracking property [8]. In applications of static sliding mode controllers, there is an inherent trade-off between zero tracking error and smooth control law and they cannot be achieved simultaneously.

One approach for chattering reduction without losing asymptotic stability is higher-order sliding-mode control [9]. The main disadvantage of this approach is that the implementation becomes complex, since estimation of additional state information is required using differentiation techniques, state observers or additional sensors [10].

Another approach, which is motivated by simplicity and satisfactory performances of PID type of controllers, is the dynamic sliding PID control [11], [12], or ”the robust integral of the sign of the error” (RISE) type of control [13], [14]. These controllers have a common feature in control configuration: a proportional plus a nonlinear integral term. The part of the controller which includes integration of the signum function provides chattering-free robust cancelation of bounded uncertainties and asymptotic convergence of the tracking error.

In [11] a dynamic sliding PID controller is used to achieve semi-global asymptotic tracking of robot manipulators without external disturbances. However, the proposed controller is time-varying and dependent on the state initial conditions. Also, conditions on controller gains are lower bounded by a state and time dependent function.

A RISE type of controller is used in [15] for asymptotic tracking of mechanical systems with nonlinearly parameterized friction model. In [14] a RISE type of controller in combination with feedforward adaptive controller is used for asymptotic tracking of mechanical systems with structured and unstructured uncertainties. In [16] a combination of RISE feedback controller and neural network is used for asymptotic tracking of mechanical systems. Recently, a saturated version of RISE feedback controllers for a class of second-order nonlinear systems is proposed in [17]. However, all the proposed RISE type of controllers provide only semi-global asymptotic tracking results. Also, some controller gains are not lower bounded and asymptotic stability is guaranteed for sufficiently large values of controller gains.

In this paper, a RISE type of controller is proposed for global asymptotic tracking of a class of nonlinear mechanical systems in the presence of external disturbances. The proof of global asymptotic stability with explicit conditions on controller gains is based on a novel approach to construction of a Lyapunov function which is parameterized by a time-varying function of reference and disturbance vector.

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The authors are with the Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Zagreb HR-10000, Croatia. Emails: jkasac@fsb.hr, tzilic@fsb.hr, vmilic@fsb.hr, ajokic@fsb.hr, mlobrovic@fsb.hr.

Throughout the paper we use the following notation: $\|x\| = \sqrt{x^T x}$ for the Euclidean norm of the vector $x \in \mathbb{R}^n$, $\|x\|_1 = \sum_{i=1}^n |x_i|$ for the 1-norm of the vector, $\|x(t)\|_M = \max_t \|x(t)\|$, $\|x(t)\|_{1M} = \max_t \|x(t)\|_1$, $\lambda_M\{A\}$ and $\lambda_m\{A\}$ for the maximal and minimal eigenvalues, respectively, of the symmetric positive definite matrix A.

This paper is organized as follows. The dynamics of the considered class of mechanical systems and its main properties are presented in Section II. The main results are presented in Section III, where a class of the RISE controllers is introduced and conditions for global asymptotic stability are established. The simulation results are presented in Section IV. Finally, the concluding remarks are emphasized in Section V.

II. A Class of Nonlinear Mechanical Systems

The model of a class of mechanical systems with n degrees of freedom is represented by

$$M\ddot{q} + D\dot{q} + g(q) = u(t) + d(t), \tag{1}$$

where $q(t) \in \mathbb{R}^n$ is the vector of generalized configuration coordinates, $u(t) \in \mathbb{R}^n$ is the vector of applied generalized forces (e.g., forces in translational and torques in rotational joints), $M \in \mathbb{R}^{n\times n}$ is inertia matrix, $D \in \mathbb{R}^{n\times n}$ is the viscous friction coefficient matrix (note that this matrix is always symmetric) and $g(q) \in \mathbb{R}^n$ is the vector of gravitational and linear elastic forces, obtained as the gradient of the potential energy $U(q)$ of the mechanical system

$$g(q) = \frac{\partial U(q)}{\partial q}, \tag{2}$$

and $d(t) \in \mathbb{R}^n$ is the vector of external disturbances.

The following properties of the dynamic model (1) are important for stability analysis (see e.g. [18], [19], [20]).

Property 1. The inertia matrix M is a positive definite symmetric matrix which satisfies

$$\lambda_m\{M\}\|z\|^2 \le z^T M z \le \lambda_M\{M\}\|z\|^2, \tag{3}$$

for all $z \in \mathbb{R}^n$.

Property 2. There exist positive constants k_{g1} and k_{g2} such that the Jacobian of the vector of gravitational and linear elastic forces

$$G(q) = \frac{\partial g(q)}{\partial q}, \tag{4}$$

satisfies

$$\|G(q)\| \le k_{g1}, \quad \forall\, q \in \mathbb{R}^n, \tag{5}$$

$$\|G(q)\dot{q} - G(q_d)\dot{q}_d\| \le k_{g1}\|\dot{\tilde{q}}\| + k_{g2}\|\tilde{q}\|\,\|\dot{q}_d\|, \tag{6}$$

for all $q, q_d \in \mathbb{R}^n$, where $\tilde{q} = q - q_d$, and the values of the parameters k_{g1} and k_{g2} can be obtained as follows

$$k_{g1} = n\left(\max_{i,j,q}\left|\frac{\partial g_i(q)}{\partial q_j}\right|\right), \tag{7}$$

$$k_{g2} = n^2\left(\max_{i,j,k,q}\left|\frac{\partial^2 g_i(q)}{\partial q_j \partial q_k}\right|\right). \tag{8}$$

The property (6) follows straightforward from Corollary A.1 in [18].

Property 3. If $q_d(t) \in \mathbb{R}^n$ is bounded twice differentiable vector function, then the first and second time derivative of the gravitational vector $g(q_d)$ can be estimated as follows

$$\|\dot{g}(q_d)\|_1 \le \sqrt{n}k_{g1}\|\dot{q}_d\|_{1M}, \tag{9}$$

$$\|\ddot{g}(q_d)\|_1 \le \sqrt{n}k_{g1}\|\ddot{q}_d\|_{1M} + k_{g2}\|\dot{q}_d\|_{1M}^2. \tag{10}$$

The proof of properties (9) and (10) is provided in Appendix.

III. RISE type of Tracking Control

A. Main Result

Proposition 1: Consider the dynamic system (1) in closed loop with decentralized RISE type of controller

$$u = -K_1 s - K_2\tilde{q} - z, \tag{11}$$

$$\dot{z} = K_I s + K_3\tilde{q} + K_\rho \text{sign}(s), \tag{12}$$

where $\tilde{q} = q - q_d(t)$ is the position error, $q_d(t)$ is the time-varying desired position, $s = \dot{\tilde{q}} + \alpha\tilde{q}$ is the sliding variable, α is the constant positive parameter, and $K_1, K_2, K_3, K_I, K_\rho \in \mathbb{R}^{n\times n}$ are constant positive definite diagonal gain matrices. The components of vector function $\text{sign}(s) = [\text{sign}(s_1)\ \text{sign}(s_2)\dots\text{sign}(s_n)]^T$ are defined as follows

$$\text{sign}(s_i) = \begin{cases} 1, & s_i > 0 \\ 0, & s_i = 0 \\ -1, & s_i < 0 \end{cases} \tag{13}$$

for $s_i \in \mathbb{R}$, $i = 1, 2, ..., n$. Assume that external disturbances $d(t)$ are bounded twice continuously differentiable functions, and desired positions $q_d(t)$ are fourth times continuously differentiable function. Then, there exist controller gains such that position error $\tilde{q}$ globally asymptotically converges to zero. The explicit conditions on controller gains are given by (41), (42) and (44). □

Note that since all controller gains are diagonal matrices, the obtained controller is implemented in a fully *decentralized* fashion. The vector signum function $\text{sign}(s)$ satisfies the following property

$$\lambda_m\{K_\rho\}\|s\|_1 \le s^T K_\rho \text{sign}(s) \le \lambda_M\{K_\rho\}\|s\|_1. \tag{14}$$

Note that in the case when $K_\rho = 0$, the controller (11)-(12) becomes the conventional linear PID controller.

B. Proof of Main Result

The stability analysis is based on Lyapunov's direct method, and can be divided into three parts. First, error equations for the closed-loop system (1), (11), and (12) are determined. Second, the Lyapunov function candidate is proposed. Then, conditions on the controller gains which ensure the global asymptotic stability are established.

1) Error Equations: By inserting $q = \tilde{q} + q_d(t)$ in dynamic equations (1) the following equation is obtained

$$M\ddot{\tilde{q}} + D\dot{\tilde{q}} + h(q, q_d) = u + f(t), \tag{15}$$

where

$$f(t) = d(t) - M\ddot{q}_d - D\dot{q}_d - g(q_d), \tag{16}$$

$$h(q, q_d) = g(q) - g(q_d). \tag{17}$$

From definition of the sliding variable $s = \dot{\tilde{q}} + \alpha\tilde{q}$, it follows

$$\dot{\tilde{q}} = s - \alpha\tilde{q}, \qquad \ddot{\tilde{q}} = \dot{s} - \alpha s + \alpha^2\tilde{q} \tag{18}$$

By inserting previous equations in (15), we get

$$M\dot{s} + \bar{D}s - \alpha\bar{D}\tilde{q} + h(q, q_d) = u + f(t), \tag{19}$$

where $\bar{D} = D - \alpha M$. By including the control variable (11) into the previous equation, the following error equations are obtained

$$M\dot{s} + \bar{K}_1 s + \bar{K}_2\tilde{q} + z + h(q, q_d) = f(t), \tag{20}$$

$$\dot{z} = K_I s + K_3\tilde{q} + K_\rho \text{sign}(s), \tag{21}$$

where the following notation is introduced

$$\bar{K}_1 = K_1 + \bar{D} = K_1 + D - \alpha M, \tag{22}$$

$$\bar{K}_2 = K_2 - \alpha\bar{D} = K_2 - \alpha D + \alpha^2 M. \tag{23}$$

Taking the time derivative of the first equation and inserting second one, we get

$$M\ddot{s} + \bar{K}_1\dot{s} + K_I s + \bar{K}_2\dot{\tilde{q}} + K_3\tilde{q} + K_\rho \text{sign}(s) + \dot{h}(q, q_d) - \dot{f}(t) = 0 \tag{24}$$

2) Lyapunov Function Candidate: The following step is the construction of a Lyapunov function for the error dynamics (24). First, an output variable $y = \dot{s} + \alpha s = \ddot{\tilde{q}} + 2\alpha\dot{\tilde{q}} + \alpha^2\tilde{q}$ with some $\alpha > 0$ is introduced, and the inner product between (24) and y is made, resulting in the following nonlinear differential form

$$\begin{aligned} &\dot{s}^T M\ddot{s} + \dot{s}^T\bar{K}_1\dot{s} + \dot{s}^T K_I s + \alpha s^T M\ddot{s} + \alpha s^T\bar{K}_1\dot{s} \\ &+\alpha s^T K_I s + \dot{\tilde{q}}^T\bar{K}_2\ddot{\tilde{q}} + 2\alpha\dot{\tilde{q}}^T\bar{K}_2\dot{\tilde{q}} + \alpha^2\dot{\tilde{q}}^T\bar{K}_2\tilde{q} + \tilde{q}^T K_3\ddot{\tilde{q}} \\ &+2\alpha\tilde{q}^T K_3\dot{\tilde{q}} + \alpha^2\tilde{q}^T K_3\tilde{q} + \dot{s}^T K_\rho \text{sign}(s) + \alpha s^T K_\rho \text{sign}(s) \\ &-\dot{s}^T\dot{f}(t) - \alpha s^T\dot{f}(t) + \dot{s}^T\dot{h}(q, q_d) + \alpha s^T\dot{h}(q, q_d) = 0. \end{aligned} \tag{25}$$

Some terms in the differential form (25) can be decomposed in the following way

$$s^T M\ddot{s} = \frac{d}{dt}\left(s^T M\dot{s}\right) - \dot{s}^T M\dot{s}, \tag{26}$$

$$\dot{s}^T M\ddot{s} = \frac{d}{dt}\left(\frac{1}{2}\dot{s}^T M\dot{s}\right), \tag{27}$$

$$\dot{s}^T K s = \frac{d}{dt}\left(\frac{1}{2}s^T K s\right), \tag{28}$$

$$\dot{s}^T\dot{f}(t) = \frac{d}{dt}\left(s^T\dot{f}(t)\right) - s^T\ddot{f}(t), \tag{29}$$

$$\dot{s}_i K_{\rho,ii}\text{sign}(s_i) = \frac{d}{dt}\left(s_i K_{\rho,ii}\text{sign}(s_i)\right), \quad \text{for} \quad i = 1, ..., n, \text{and} \quad s_i \neq 0, \tag{30}$$

and similar decompositions can be obtained for quadratic terms depending on vectors $\tilde{q}$ and $\dot{\tilde{q}}$. Note that the term in (30) which is differentiated in time (right-hand side of (30)) is indeed differentiable for all $s_i \neq 0$. We do not require differentiability at $s_i = 0$, and the equality (30) indeed excludes these points.

By separating terms in the form of the time derivatives on the left side of the equality (25) and the rest of the terms on the right side, we obtain Lyapunov function candidate in the following form

$$\begin{aligned} V =\ & \frac{1}{2}\dot{s}^T M\dot{s} + \alpha s^T M\dot{s} + \frac{1}{2}s^T(\alpha\bar{K}_1 + K_I)s \\ &+ \frac{1}{2}\dot{\tilde{q}}^T\bar{K}_2\dot{\tilde{q}} + \tilde{q}^T K_3\dot{\tilde{q}} + \frac{1}{2}\tilde{q}^T\alpha(\alpha\bar{K}_2 + 2K_3)\tilde{q} \\ &+ \sum_{\{i|s_i\neq 0\}}[K_{\rho,ii}\text{sign}(s_i) - \dot{f}(t)]s_i, \end{aligned} \tag{31}$$

Note that with (26)-(30), for all $\left[\tilde{q}\ \dot{\tilde{q}}\ \ddot{\tilde{q}}\right]^T \neq 0$, we have

$$\frac{d}{dt}V\left(\tilde{q}, \dot{\tilde{q}}, \ddot{\tilde{q}}, t\right) = -W\left(\tilde{q}, \dot{\tilde{q}}, \ddot{\tilde{q}}, t\right), \tag{32}$$

where

$$\begin{aligned} -W =\ & -\dot{s}^T(\bar{K}_1 - \alpha M)\dot{s} - \alpha s^T K_I s \\ &- \dot{\tilde{q}}^T(2\alpha\bar{K}_2 - K_3)\dot{\tilde{q}} - \alpha^2\tilde{q}^T K_3\tilde{q} \\ &- \alpha s^T K_\rho \text{sign}(s) + \alpha s^T\dot{f}(t) - s^T\ddot{f}(t) \\ &- \dot{s}^T\dot{h}(q, q_d) - \alpha s^T\dot{h}(q, q_d). \end{aligned} \tag{33}$$

Careful inspection shows that the candidate Lyapunov function, as defined by (32), is a continuously differentiable function. More precisely, the complicating non-differentiable terms which include sign functions are carefully omitted in the definition of V, what has preserved continuity, implies differentiability, and therefore (32) is well-defined.

We define $x = \left[\tilde{q}\ \dot{\tilde{q}}\ \ddot{\tilde{q}}\right]^T$. Note that $V(x, t) = 0$ if and only if $x = x^* = 0$ (see (34)). To prove global asymptotic stability to x^*, it is now sufficient to show that $V(x, t) > 0$ and $W(x, t) > 0$ for all $x \neq x^*$.

3) Stability criterion determination: The following step is determination of sufficient conditions for positive definiteness of the function V and W.

We consider function V, which can be rearranged in the following form

$$\begin{aligned} V =\ & \frac{1}{2}\left(\dot{s} + \alpha s\right)^T M\left(\dot{s} + \alpha s\right) + \frac{1}{2}s^T(\alpha\bar{K}_1 + K_I - \alpha^2 M)s \\ &+ \frac{1}{2}\begin{bmatrix}\tilde{q} \\ \dot{\tilde{q}}\end{bmatrix}^T\begin{bmatrix}\alpha(\alpha\bar{K}_2 + 2K_3) & K_3 \\ K_3 & \bar{K}_2\end{bmatrix}\begin{bmatrix}\tilde{q} \\ \dot{\tilde{q}}\end{bmatrix} \\ &+ \sum_{\{i|s_i\neq 0\}}[K_{\rho,ii}\text{sign}(s_i) - \dot{f}(t)]s_i, \end{aligned} \tag{34}$$

and by using properties (3) and (14) we get

$$\begin{aligned} V \geq\ & \frac{1}{2}\left(\alpha\lambda_m\{\bar{K}_1\} + \lambda_m\{K_I\} - \alpha^2\lambda_M\{M\}\right)\|s\|^2 \\ &+ \frac{1}{2}\begin{bmatrix}\|\tilde{q}\| \\ \|\dot{\tilde{q}}\|\end{bmatrix}^T R\begin{bmatrix}\|\tilde{q}\| \\ \|\dot{\tilde{q}}\|\end{bmatrix} \\ &+ (\lambda_m\{K_\rho\} - \max_t\|\dot{f}(t)\|_1)\|s\|_1 \geq 0, \end{aligned} \tag{35}$$

where

$$R = \begin{bmatrix}\alpha(\alpha\lambda_m\{\bar{K}_2\} + 2\lambda_m\{K_3\}) & \lambda_M\{K_3\} \\ \lambda_M\{K_3\} & \lambda_m\{\bar{K}_2\}\end{bmatrix} > 0 \tag{36}$$

that will be satisfied when

$$\alpha\lambda_m\{\bar{K}_1\} + \lambda_m\{K_I\} > \alpha^2\lambda_M\{M\}, \tag{37}$$

$$\alpha(\alpha\lambda_m\{\bar{K}_2\} + 2\lambda_m\{K_3\})\lambda_m\{\bar{K}_2\} > \lambda_M\{K_3\}^2, \tag{38}$$

$$\lambda_m\{K_\rho\} > \|\dot{f}(t)\|_{1M}. \tag{39}$$

The following step is the condition which ensures that the time derivative of the Lyapunov function is negative definite function, i.e. $W \geq 0$. Applying properties (3) and (14) we get

$$\begin{aligned} W \geq\ & (\lambda_m\{\bar{K}_1\} - \alpha\lambda_M\{M\}) \|\dot{s}\|^2 + \alpha\lambda_m\{K_I\}\|s\|^2 \\ & + (2\alpha\lambda_m\{\bar{K}_2\} - \lambda_M\{K_3\}) \left\|\dot{\tilde{q}}\right\|^2 + \alpha^2\lambda_m\{K_3\}\|\tilde{q}\|^2 \\ & + \left(\alpha\lambda_m\{K_\rho\} - \max_t(\alpha\|\dot{f}(t)\|_1 + \|\ddot{f}(t)\|_1)\right)\|s\|_1 \\ & - \|\dot{s} + \alpha s\| \left\|\dot{h}(q, q_d)\right\| \geq 0 \end{aligned} \tag{40}$$

The last term in previous expression can be estimated using property (6) and triangle inequality $\left\|\dot{\tilde{q}}\right\| \leq \|s\| + \alpha\|\tilde{q}\|$,

$$\begin{aligned} & \left\|\dot{s}^T + \alpha s^T\right\| \left\|\dot{h}(q, q_d)\right\| \leq \|\dot{s} + \alpha s\| \; \|G(q)\dot{q} - G(q_d)\dot{q}_d\| \\ & \leq (\|\dot{s}\| + \alpha\|s\|) \left(k_{g1}\|\dot{\tilde{q}}\| + k_{g2}\|\dot{q}_d\| \, \|\tilde{q}\|\right) \\ & \leq (\|\dot{s}\| + \alpha\|s\|) \left(k_{g1}\|s\| + \bar{k}_{g2}\|\tilde{q}\|\right) \\ & \leq \alpha k_{g1}\|s\|^2 + k_{g1}\|s\| \, \|\dot{s}\| + \bar{k}_{g2}\|\tilde{q}\| \, \|\dot{s}\| + \alpha\bar{k}_{g2}\|\tilde{q}\| \, \|s\| \end{aligned}$$

where $\bar{k}_{g2} = \alpha k_{g1} + k_{g2}\|\dot{q}_d\|_M$ is introduced.

If the following conditions are satisfied

$$2\alpha\lambda_m\{\bar{K}_2\} > \lambda_M\{K_3\}, \tag{41}$$

$$\alpha\lambda_m\{K_\rho\} > \alpha\|\dot{f}(t)\|_{1M} + \|\ddot{f}(t)\|_{1M}, \tag{42}$$

then (40) is equivalent to

$$W \geq \frac{1}{2} \begin{bmatrix} \|s\| \\ \|\dot{s}\| \\ \|\tilde{q}\| \end{bmatrix}^T Q \begin{bmatrix} \|s\| \\ \|\dot{s}\| \\ \|\tilde{q}\| \end{bmatrix} \geq 0 \tag{43}$$

when

$$Q = \begin{bmatrix} Q_{11} & k_{g1} & \alpha\bar{k}_{g2} \\ k_{g1} & Q_{22} & \bar{k}_{g2} \\ \alpha\bar{k}_{g2} & \bar{k}_{g2} & Q_{33} \end{bmatrix} > 0 \tag{44}$$

where

$$Q_{11} = 2\alpha(\lambda_m\{K_I\} - k_{g1}) > 0, \tag{45}$$

$$Q_{22} = 2(\lambda_m\{\bar{K}_1\} - \alpha\lambda_M\{M\}) > 0, \tag{46}$$

$$Q_{33} = 2\alpha^2\lambda_m\{K_3\} > 0. \tag{47}$$

The vector norms on the right-hand side of inequality (42) can be estimated using (16), (9) and (10),

$$\begin{aligned} \|\dot{f}(t)\|_{1M} \leq\ & \lambda_M\{M\}\|\dddot{q}_d\|_{1M} + \lambda_M\{D\}\|\ddot{q}_d\|_{1M} \\ & + \sqrt{n}k_{g1}\|\dot{q}_d\|_{1M} + \|\dot{d}(t)\|_{1M}, \end{aligned} \tag{48}$$

$$\begin{aligned} \|\ddot{f}(t)\|_{1M} \leq\ & \lambda_M\{M\}\|q_d^{(4)}\|_{1M} + \lambda_M\{D\}\|\dddot{q}_d\|_{1M} \\ & + \sqrt{n}k_{g1}\|\ddot{q}_d\|_{1M} + k_{g2}\|\dot{q}_d\|_{1M}^2 + \|\ddot{d}(t)\|_{1M} \end{aligned} \tag{49}$$

It can be seen that the conditions (37)-(39) are trivially implied by the conditions (41)-(42) and (46). So, the conditions (41)-(42), including (44), are the final stability conditions which guarantee global asymptotic stability.

The gains $\bar{K}_1$ and $\bar{K}_2$ in conditions (46) and (41) can be replaced by original gains K_1 and K_2 from definition (22), so that

$$2\alpha\lambda_m\{K_2\} > 2\alpha^2(\lambda_M\{D\} - \alpha\lambda_m\{M\}) + \lambda_M\{K_3\}, \tag{50}$$

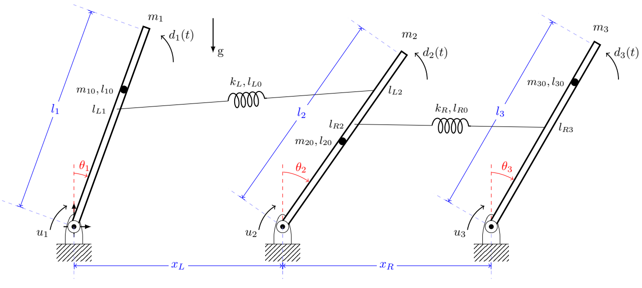

Fig. 1. Three pendulums interconnected by two springs. The origin of coordinate system is in the joint of base 1.

$$Q_{22} = 2(\lambda_m\{K_1\} + \lambda_m\{D\} - 2\alpha\lambda_M\{M\}) > 0. \tag{51}$$

The gain tuning procedure can be summarized as follows: (a) Chose some $\alpha > 0$; (b) Find K_ρ which satisfies (42); (c) Find K_1, K_3 and K_I which satisfy (44); (d) Find K_2 which satisfies (50).

IV. Simulation Example

We consider a system of three pendulums interconnected by two springs, as shown in Fig. 1, with dynamic equations

$$J_i\ddot{\theta}_i + D_i\dot{\theta}_i + g_i(q) = u_i(t) + d_i(t), \tag{52}$$

where $g_i(q) = \dfrac{\partial U(q)}{\partial q_i}$, $q = [\theta_1\ \theta_2\ \theta_3]^T$, and $\theta_i(t)$ is the deflection angle of the i-th pendulum measured from its y-axis, J_i is the inertia moment of the i-th pendulum, D_i is the coefficient of viscous friction of the i-th joint, $u_i(t)$ is the input torque of the i-th motor, $d_i(t)$ is the i-th torque of external disturbance, for $i = 1, 2, 3$.

The potential energy $U(q) = U(\theta_1, \theta_2, \theta_3)$ of the system is

$$\begin{aligned} U(q) = & h_1\cos(\theta_1) + h_2\cos(\theta_2) + h_3\cos(\theta_3) \\ & + \frac{k_L}{2}\left(\sqrt{L_{11} + L_{12}} - l_{L0}\right)^2 \\ & + \frac{k_R}{2}\left(\sqrt{L_{21} + L_{22}} - l_{R0}\right)^2, \end{aligned}$$

where

$$\begin{aligned} & L_{11} = (x_L - l_{L1}\sin(\theta_1) + l_{L2}\sin(\theta_2))^2, \\ & L_{12} = (l_{L1}\cos(\theta_1) - l_{L2}\cos(\theta_2))^2, \\ & L_{21} = (x_R - l_{R2}\sin(\theta_2) + l_{R3}\sin(\theta_3))^2, \\ & L_{22} = (l_{R3}\cos(\theta_3) - l_{R2}\cos(\theta_2))^2, \\ & h_i = m_i g\frac{l_i}{2} + m_{i0}gl_{i0}, \\ & J_i = \frac{m_i l_i^2}{3} + m_{i0}l_{i0}^2. \end{aligned}$$

Parameter m_i is the mass of the i-th pendulum, m_{i0} is the point mass on the i-th pendulum, l_{i0} is the distance from the i-th pendulum base to the point mass, l_i is the length of the i-th pendulum, for $i = 1, 2, 3$. Parameters l_{Li}, $i = 1, 2$ (left spring), is the distance from the spring's i-th end to the i-th pendulum base, l_{Ri}, $i = 2, 3$ (right spring) is the distance from the spring's i-th end to the i-th pendulum base, l_{L0}, l_{R0} are left and right length of the unstretched spring, respectively, x_L is the distance between the pendulums' bases 1 and 2, x_R is the distance between the pendulums' bases 2 and 3, k_L, k_R are left and right spring constants and g is the gravity acceleration.

The parameter values used in simulations are shown in Table I. The desired reference trajectories are $q_{d1}(t) = \sin(t)$, $q_{d2}(t) = 0.5\sin(2t)$, $q_{d3}(t) = 0.3\sin(3t)$. The external disturbances are $d_1(t) = 6\sin(2t)$, $d_2(t) = 4\sin(3t+0.1)$, $d_3(t) = 8\sin(t+0.3)$. The initial conditions are: $q_1(0) = 0.6$, $q_2(0) = 0.4$, $q_3(0) = -0.2$.

The upper bounds on inertia matrix and derivatives of gravity vector are estimated as follows: $\lambda_m\{M\} = 0.168$, $k_{g1} = 10.57$ and $k_{g2} = 10.57$. The controller gains are chosen in agreement with stability conditions: $\alpha = 10$, $K_1 = \text{diag}\{10, 10, 10\}$, $K_2 = \text{diag}\{10, 10, 10\}$, $K_3 = \text{diag}\{25, 25, 25\}$, $K_I = \text{diag}\{30, 30, 30\}$.

In Fig. 2. we can see a comparison of the proposed RISE controller ($K_\rho = 50$) with the linear PID controller ($K_\rho = 0$) for the case of stabilization ($q_{d1} = 0$, $q_{d2} = 0$, $q_{d3} = 0$) in the presence of external disturbances. It can be seen that the RISE controller provides asymptotic convergence toward constant reference state despite the influence of large disturbances. The asymptotic convergence is possible only if the control variables completely compensate the external disturbances, what can be seen in the figure. More precisely, the right-hand side of the dynamic equations (1), $u(t)+d(t)$, converge to zero, so that $u(t) \to -d(t)$. On the other hand, linear PID control cannot asymptotically stabilize the system in the presence of external disturbances.

Fig. 3. shows the system response in the case of stabilization by using the conventional sliding-mode controller

$$u = -K_1 s - K_\rho \text{sign}(s). \qquad (53)$$

The system response is very similar as in the case of RISE controller, but fundamental difference appears in comparison of control variables. Contrary to conventional sliding-mode controller, the proposed RISE controller provides continuous chattering-free control variables without losing asymptotic convergence.

Fig. 4. shows the performances of the proposed RISE controller for the case of the trajectory tracking ($K_\rho = 120$) in the presence of external disturbances. It can be seen that the RISE controller provides asymptotic convergence toward reference trajectory by continuous chattering-free control variables. In this case, control vector asymptotically compensates the function $f(t)$, defined by (16), which includes dependence on reference trajectory and external disturbances.

V. Conclusions

In this paper, a class of globally stable chattering-free RISE type of controllers for a class of nonlinear mechanical systems has been presented. The stability conditions provides an explicit procedure for tuning of controller gains in terms of a few parameters extracted from the system dynamics. The future work will be oriented toward the extension of the proposed approach to the more general class of Euler-Lagrange systems. Also, a more advanced optimization-based procedure for tuning of controller gains will be applied.

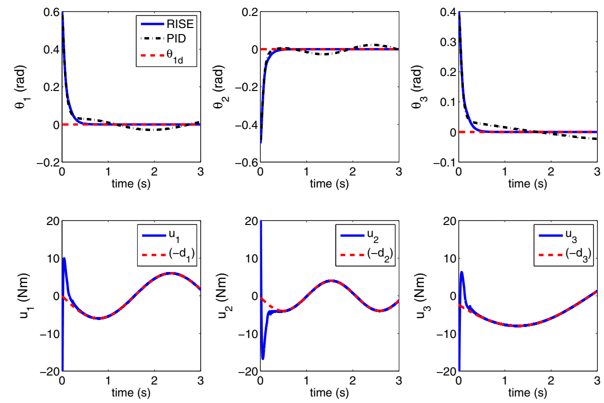

Fig. 2. Stabilization in the presence of external disturbances. Comparison of the RISE controller ($K_\rho = 50$) and the linear PID controller ($K_\rho = 0$).

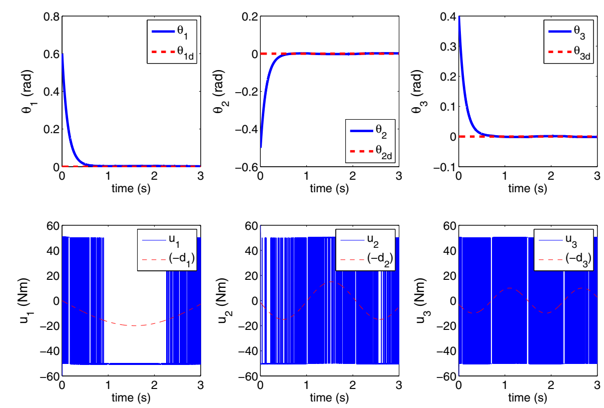

Fig. 3. Stabilization in the presence of external disturbances with the conventional sliding-mode controller.

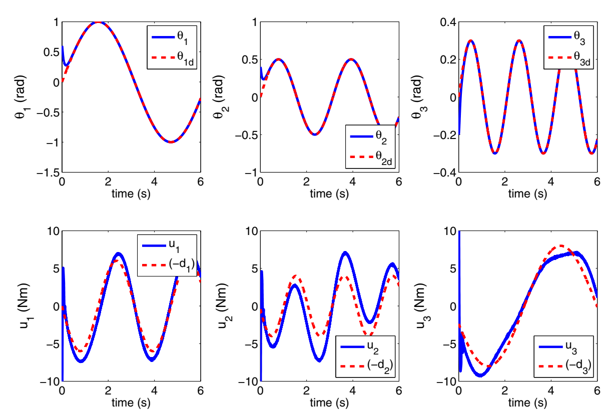

Fig. 4. Trajectory tracking in the presence of external disturbances with the RISE controller.

TABLE I

PARMAMETERS

param.	values	units	param.	values	units
m_1	0.1	kg	l_1	0.75	m
m_2	0.2	kg	l_2	0.75	m
m_3	0.3	kg	l_3	0.75	m
m_{10}	0.6	kg	l_{10}	0.375	m
m_{20}	0.7	kg	l_{20}	0.375	m
m_{30}	0.8	kg	l_{30}	0.375	m
l_{L0}	1	m	l_{R0}	1	m
l_{L1}	0.40	m	l_{L2}	0.40	m
l_{R2}	0.40	m	l_{R3}	0.40	m
k_L	20	$\mathrm{N\,m^{-1}}$	k_R	20	$\mathrm{N\,m^{-1}}$
D_1	10^{-3}	$\mathrm{N\,s\,m^{-1}}$	D_2	10^{-3}	$\mathrm{N\,s\,m^{-1}}$
D_3	10^{-3}	$\mathrm{N\,s\,m^{-1}}$	g	9.81	$\mathrm{m\,s^{-2}}$
x_L	1.0	m	x_R	1.0	m

APPENDIX

This Appendix presents a proof of Property 3. The following definitions will be used [21]: $\|x\|_\infty = \max_i |x_i|$ for the ∞-norm of the vector $x \in \mathbb{R}^n$, $\|A\|_1 = \max_j \sum_{i=1}^{n} |a_{ij}|$ for the induced 1-norm of the matrix $A \in \mathbb{R}^{n\times n}$. Also, the following relations between different vector norms will be used:

$\|x\| \le \|x\|_1 \le \sqrt{n}\|x\|$ and $\|x\|_1 \le n\|x\|_\infty$ for $x \in \mathbb{R}^n$.

The first derivative of the gravity vector can be estimated as follows

$$\begin{aligned}\|\dot{g}(q_d)\|_1 &\le \sqrt{n}\,\|G(q_d)\dot{q}_d\| \le \sqrt{n}\,\|G(q_d)\|\,\|\dot{q}_d\| \\ &\le \sqrt{n}k_{g1}\,\|\dot{q}_d\| \le \sqrt{n}k_{g1}\,\|\dot{q}_d\|_{1M}\,. \end{aligned} \tag{54}$$

The second derivative of the gravity vector can be estimated as follows

$$\begin{aligned}\|\ddot{g}(q_d)\|_1 &= \left\|G(q_d)\ddot{q}_d + \dot{G}(q_d)\dot{q}_d\right\|_1 \\ &\le \|G(q_d)\|_1\,\|\ddot{q}_d\|_1 + \left\|\dot{G}(q_d)\right\|_1 \|\dot{q}_d\|_1 \\ &\le \sqrt{n}k_{g1}\,\|\ddot{q}_d\|_{1M} + \left\|\dot{G}(q_d)\right\|_1 \|\dot{q}_d\|_{1M}\,. \end{aligned} \tag{55}$$

The induced L_1 matrix norm in the previous expression can be estimated as follows

$$\begin{aligned}\left\|\dot{G}(q_d)\right\|_1 &= \max_j \sum_{i=1}^{n} \left|\dot{q}_d^T \frac{\partial G_{ij}(q_d)}{\partial q_d}\right| \\ &\le \|\dot{q}_d\|_1 \max_j \sum_{i=1}^{n} \left\|\frac{\partial G_{ij}(q_d)}{\partial q_d}\right\|_1 . \end{aligned} \tag{56}$$

Further, by definition of L_1 vector norms, it follows

$$\begin{aligned}\left\|\frac{\partial G_{ij}(q_d)}{\partial q_d}\right\|_1 &= \sum_{k=1}^{n} \left|\frac{\partial G_{ij}(q_d)}{\partial q_{k,d}}\right| \\ &\le n \max_k \left|\frac{\partial G_{ij}(q_d)}{\partial q_{k,d}}\right|, \end{aligned} \tag{57}$$

and

$$\sum_{i=1}^{n} \left|\frac{\partial G_{ij}(q_d)}{\partial q_{k,d}}\right| \le n \max_i \left|\frac{\partial G_{ij}(q_d)}{\partial q_{k,d}}\right|. \tag{58}$$

By inserting (57) in (56) and applying (58), it follows

$$\begin{aligned}\left\|\dot{G}(q_d)\right\|_1 &\le n^2 \left(\max_{i,j,k,q_d} \left|\frac{\partial G_{ij}(q_d)}{\partial q_{k,d}}\right|\right) \|\dot{q}_d\|_1 \\ &\le n^2 k_{g2} \|\dot{q}_d\|_{1M} \end{aligned} \tag{59}$$

where k_{g2} is defined by (8). Finally, by inserting (59) in (55), the property (10) is obtained.

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