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Hamiltonian of Multipotential Field in Nanorobotics

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***Abstract*—As it is well known, nanorobotics is the field that deals with the controlled manipulation with atomic and molecular-sized objects. In order to control nanorobots in the regions of mechanics, electronics, electromagnetism, photonics and biomaterials we have to have the ability to construct of the related artificial control potential fields. At the nanoscale the control dynamics is very complex because there are very strong interactions between nanorobots, manipulated objects and nanoenvironment. The problem is to design the control dynamics that will compensate or/and control the mentioned interactions. The first step in designing of the control dynamics for nanorobots is the development of the relativistic Hamiltonian (Hamilton functions) that will include external artificial control potential fields. Thus, derivation of the first and second form of the relativistic Hamiltonians for nanorobots control is presented in this paper.**

Index Terms **— Nanorobotics, Relativistic Hamiltonian, Multipotential field, Artificial control field.**

I. Introduction

As it is well known, the nanorobotics belongs to the multidisciplinary field that deals with the controlled manipulation with atomic and molecular-sized objects and therefore sometimes is called molecular robotics [1-10]. Potential applications of the nanorobots are expected in the tree important regions: nanomedicine, nanotechnology and space applications. In nanomedicine the nanorobots can be employed for surgery, early diagnoses, drug delivery at the right place (for destroying a cancer cell), biomedical instrumentation, pharmacokinetics, monitoring of diabetes and genome applications by reading and manipulating DNA [7]. In nanotechnology the nanorobots can be utilized for creation of new materials, nanofabrics for different products, cell probes with small dimensions, computer memory, near field optics, x-ray fabrication, very small batteries and optical antennas. In the space applications it is expected that nanorobots replace of human being in the intergalactic space missions, be hardware and software to fly on satellites and have a high level of an artificial intelligence. The complex tasks of the future nanorobots are sensing, thinking, acting and working cooperatively with the other nanorobots.

In order to control nanorobots in mechanics, electronics, electromagnetic, photonics, chemical and biomaterials regions we have to have the ability to construct the related artificial control potential fields. At the nanoscale the control dynamics is very complex because there are very strong interaction between nano robots and nanoenvironment. Thus, the first step in designing the control dynamics for nanorobots is the development of the relativistic Hamiltonian that will include external artificial potential field. This paper has been written by consideration of the related theories and fundamental laws of physics in [11]-[21] and [30]-[37]. The generally approach to the multidisciplinary nanorobotics field has been presented in [22]-[23]. In the derivation of the Hamiltonians that includes the external potential fields for control in nanorobotics, the new General Lorentz Transformation model (the GLT_α model derived in [24]-[26]) and the new Relativistic Alpha Field Theory (RAFT) in [27]-[29] are employed.

The first form of the Hamilton function has been derived starting with variation principle and using the procedure from [31]-[32]. In that sense, the relativistic invariant term Ldt (where L is the Lagrange function and dt is the differential of the time) is derived by employing product of the two relativistic invariant terms: proper time $d\tau$ and energetic term $m_o c^2$. Here m_o is a rest mass of a sample (particle) and c is the speed of the light in a vacuum. It is shown that the relativistic invariant term Ldt can also be derived starting with the generalized line element ds, since ds^2 is a fundamental invariant of the four dimensional space-time continuum. The obtained first form of the Hamiltonian ***H*** is equal to the general covariant energy equation E_c that usually can be derived by employing the null component of the covariant four-momentum vector P_o. Further, this form of the Hamiltonian can be easily transformed into the expression that includes extended momentum as a function of the field parameters α and α'. This form is also very important, because the obtained Hamiltonian is a linear function of the extended momentum and therefore belongs to the Dirac's like structure of the Hamiltonian [16]-[17]. The obtained result gives the possibility to compare the coefficients of the well known Dirac's Hamiltonian and the first form of the Hamiltonian derived in this paper.

The second form of the Hamiltonian ***H*** has been derived starting with the modification of the some relations in the previous derivation procedure. This form of the Hamiltonian belongs to the usual structure of the classical relativistic Hamiltonians [36]-[37]. The main shortage of the second form of the Hamiltonian is the fact that this form is a nonlinear function of the extended momentum. Thus, the first form of the Hamiltonian has got the important priority, because this form is linear function of the extended momentum. Usually, one can introduce the approximation of the second form of the Hamiltonian. It also has been done in this paper and resulted with the new form of the Hamiltonian as the approximation of its second form.

This paper is organized as follows. The second section presents a process of the determination of the dimensionless field parameters α and α'. It is shown that these parameters are functions of the potential energy of the multi-potential field with n-potentials plus an artificial control field of the nanorobot control. The third section shows the derivation of

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the first form of the Hamiltonian equal to the general covariant energy equation. In the fourth section the second form of the Hamiltonian has been derived as nonlinear function of the related extended momentums. A nonrelativistic approximation of the second form of the Hamiltonian is discussed in the fifth section. An application of the nonrelativistic Hamiltonian into the nonrelativistic quantum systems is given in sixth section. The conclusion of the paper with some comments is presented in the seventh section. Finally, the reference list is shown at the end of this paper.

II. Introduction of Field Parameters in Multipotential Field

Field parameters α and α′ have been introduced in [22]-[29] in order to include the influences of a potential field to the well known Lorentz coordinate transformation in Special Relativity that is related to the vacuum without any potential field. Generally, the field parameters α and α′ are dimensionless functions of the space time coordinates that satisfy the related field equation of the potential field in which particle is propagated. Thus, if a particle is present in an electromagnetic field, then the field parameters α and α′ should satisfy the well known Maxwell's field equations. On the other side, if a particle is present in a gravitational field, then the field parameters α and α′ should satisfy the well known Einstein's field equations. Finally, if a particle is in the both mentioned potential fields at the same time, then the field parameters α and α′ should satisfy the both field equations of that multipotential field. If the potential energy U of the particle (sample) in the multipotential field is known, then the field parameters α and α′ can be determined as the dimensionless functions of that potential energy [23]. In this case the obtained solution for the field parameters α and α′ should also satisfy the related field equations of the multipotential field. At the nanoscale control of a particle (sample) motion or/and manipulation we usually have the multi-potential field with n-potentials, plus an artificial control field of the nanorobot that influents to the particle with a potential energy U_c. Thus, the related potential energy of the particle (sample) in that case can be calculated by using the following equation:

$$U = U_1 + U_2 + .. + U_n + U_c = \Sigma U_j + U_c\,, \quad j = 1, 2, .., n. \tag{1}$$

In the relation (1) U_j is a potential energy of the particle in the j-th potential field. Generally, there are four solutions for field parameters α and α′ [27] (like in Dirac's theory) as the dimensionless functions of the total potential energy U of a particle in the related potential field. For the simplicity here will be presented the first solution only [23]:

$$\alpha = 1 + i\sqrt{\frac{2U}{m_0c^2} + \left(\frac{U}{m_0c^2}\right)^2}\,, \qquad \alpha' = 1 - i\sqrt{\frac{2U}{m_0c^2} + \left(\frac{U}{m_0c^2}\right)^2}\,. \tag{2}$$

In the equation (2) m_o is a rest mass of a particle, c is the speed of the light in a vacuum and i = √(-1) is an imaginary unit. In the Hamiltonians, ***H***, derived in the sections III and IV, we have to know the product and the difference of the field parameters α and α′ from (2):

$$\alpha\alpha' = \left(1 + \frac{2U}{m_0c^2} + \left(\frac{U}{m_0c^2}\right)^2\right) = \left(1 + \frac{U}{m_0c^2}\right)^2, \qquad \alpha - \alpha' = 2\,i\sqrt{\frac{2U}{m_0c^2} + \left(\frac{U}{m_0c^2}\right)^2}\,. \tag{3}$$

The relations (2) and (3) are valid for a strong potential field. Meanwhile, in the case of a weak potential field ($U << m_oc^2$) the quadratic term can be neglected and the relations (2) and (3) are reduced to the following forms:

$$\alpha = 1 + i\sqrt{\frac{2U}{m_0c^2}}\,, \quad \alpha' = 1 - i\sqrt{\frac{2U}{m_0c^2}}\,, \quad \alpha\alpha' = \left(1 + \frac{2U}{m_0c^2}\right), \quad \alpha - \alpha' = 2\,i\sqrt{\frac{2U}{m_0c^2}}\,. \tag{4}$$

In the application of the field parameters one can meet both a strong and a weak multipotential field. Therefore, one should employ field parameters α and α′ given by (2) and (3) for a strong potential field. In the case of a weak potential field, the field parameters α and α′ given by the relation (4), should be employed.

An example: let a particle (sample) is an electron that is present in the weak two-potential electromagnetic and gravitational field and is manipulated (positioned) with a nanorobot by artificial potential control energy U_c. We also assumed that a gravitational potential field belongs to a spherically symmetric non-rotating and non-charged body with a mass M. For that case the potential energy of the electron with rest mass m_o in the mentioned multipotential field is given by the relation:

$$U = qV - \frac{m_0GM}{r} + U_c\,. \tag{5}$$

Here q is the electric charge of the electron, V is a scalar potential of the electromagnetic field, G is the gravitational constant and r is a radial position of the electron related to the center of gravity of the mass M. Applying (5) to the first line of the relations (4) one obtains the related dimensionless field parameters in the form:

$$\alpha = 1 + i\sqrt{2\left(\frac{q\,V}{m_0c^2} - \frac{GM}{rc^2} + \frac{U_c}{m_0c^2}\right)}\,, \qquad \alpha' = 1 - i\sqrt{2\left(\frac{q\,V}{m_0c^2} - \frac{GM}{rc^2} + \frac{U_c}{m_0c^2}\right)}\,. \tag{6}$$

For that example the second line of the relations (4) is transformed into the expressions:

$$\left(\alpha\alpha'\right)=\left(1+\frac{2qV}{m_0c^2}-\frac{2GM}{rc^2}+\frac{2U_c}{m_0c^2}\right),$$
$$\alpha-\alpha'=2\,i\sqrt{2\left(\frac{qV}{m_0c^2}-\frac{GM}{rc^2}+\frac{U_c}{m_0c^2}\right)}. \tag{7}$$

In order to control of a particle (a sample) motion or/and manipulation in a multipotential field with n-potentials, one can divide an artificial control potential energy U_c into two parts:

$$U_c = U_{c_1}+U_{c_2}\,,\quad U_{c_1}=-\sum U_j\,,\quad j=1,2,...,n,$$
$$U_{c_2}=f(e),\qquad e=w-x. \tag{8}$$

Here e is a vector of the control errors of a particle position or/and orientation, w is a desired vector of a particle position or/and orientation and x is a vector of a measured particle position or/and orientation. As we can see from the relation (8) the first part of the artificial control potential energy U_{c1} has been used for compensation of the influences of the natural multipotential field to the particle position or/and orientation. The second part of the artificial control potential energy U_{c2} can be used for control of the particle position or/and orientation. Therefore, this part of the artificial control potential energy should be a function of the vector of the control errors of position or/and orientation. This function should be determined by applying some of the control methods.

III. Derivation of the First Form of Hamiltonian

An alpha field is a potential field where the influence to the dynamics of a particle motion in that field can be described by two dimensionless field parameters α and α'. In order to develop a Hamiltonian that includes dimensionless field parameters α and α' of an alpha field, one can start with the variation principle [26]-[27]:

$$\int_1^2 L dt = \text{extreme}. \tag{9}$$

Here L is the Lagrange function and dt is the differential of the time. The term Ldt should be a relativistic invariant. We know that the differential of the proper time dτ of the moving particle is the relativistic invariant. It is given in the GLT_α – model by the relation [24]:

$$d\tau=\frac{1}{H}dt=\left(\alpha\alpha'-\frac{v^2}{c^2}+\frac{\kappa\left(\alpha-\alpha'\right)c\,v}{c^2}\right)^{1/2}dt. \tag{10}$$

Here v is a particle velocity and c is the speed of the light both in vacuum without any potential field. Parameters α and α' are dimensionless field parameters of a multipotential field in which a particle is propagating. The parameter κ can have two values [27], $\kappa = \pm 1$and $\kappa^2 = 1$. The system K' is moving relative to the system K with velocity v_α:

$$v_\alpha=v-\frac{\kappa\left(\alpha-\alpha'\right)c}{2}, \tag{11}$$

The parameter H is determined in the reference [24] by the following form:

$$H=\left(1-\frac{v_\alpha^2}{c^2}\right)^{-1/2}=\left(\alpha\alpha'-\frac{v^2}{c^2}+\frac{\kappa\left(\alpha-\alpha'\right)c\,v}{c^2}\right)^{-1/2}. \tag{12}$$

In this relation v_α is a particle velocity in a multipotential field. Now, the relativistic invariant term Ldt can be obtained by employing product of the two relativistic invariant terms: the proper time dτ from (10) and the rest-mass energetic term, m_0c^2, of the particle standing in a vacuum without any potential field (a rest-mass energy). Thus, the Lagrange function valid in the GLT_α – model may be described by the following expression:

$$L=-m_0c^2\left(\alpha\alpha'-\frac{v^2}{c^2}+\frac{\kappa\left(\alpha-\alpha'\right)c\,v}{c^2}\right)^{1/2}. \tag{13}$$

In the case of a vacuum without any potential field, the field parameters α and α' from (2) or (4) satisfy the relation α = α' = 1. For that case the relation (13) is transformed into the well-known equation, valid in the Special Relativity [11]:

$$L=-\,m_0c^2\left(1-\frac{v^2}{c^2}\right)^{1/2}. \tag{14}$$

For the small particle velocity v, the Lagrange function (13) is transformed into the new relation:

$$L=-m_0\sqrt{\alpha\alpha'}c^2+\frac{m_0v^2}{2\sqrt{\alpha\alpha'}}-\frac{m_0\kappa\left(\alpha-\alpha'\right)c\,v}{2\sqrt{\alpha\alpha'}}+.. \tag{15}$$

The relativistic invariant term Ldt can also be derived starting with the line element ds of the GLT_α – model, since ds^2 is a fundamental invariant of the four dimensional space-time continuum [26]:

$$ds^2=-\,\alpha\alpha'c^2dt^2-\kappa\left(\alpha-\alpha'\right)c\,v\,dt^2+v^2dt^2. \tag{16}$$

This line element can be rewritten into the following form:

$$-ds^2=c^2\left[\alpha\alpha'-\frac{v^2}{c^2}+\frac{\kappa\left(\alpha-\alpha'\right)c\,v}{c^2}\right]dt^2,$$
$$ds=i\,c\left(\alpha\alpha'-\frac{v^2}{c^2}+\frac{\kappa\left(\alpha-\alpha'\right)c\,v}{c^2}\right)^{1/2}dt. \tag{17}$$

Now, starting with the second equation in (17), the relativistic invariant term Ldt can be described in the new form:

$$Ldt=i\,m_0c\,ds\,. \tag{18}$$

Thus, the integral property of the motion S is given by the relation:

$$S=\int_1^2 Ldt=\int_1^2 i\,m_0c\,ds=\text{extreme}. \tag{19}$$

After substitution the line element ds from the second equation in (17) to (19) we obtain the relation:

$$S=\int_1^2\left[-m_0c^2\left(\alpha\alpha'-\frac{v^2}{c^2}+\frac{\kappa\left(\alpha-\alpha'\right)c\,v}{c^2}\right)^{1/2}\right]dt,$$
$$S=\int_1^2\left[L\right]dt. \tag{20}$$

From this relation one can recognize the Lagrange function L in the form equal to (13).

Now, we are ready for the derivation of the Hamiltonian ***H*** following the well known procedure [31]-[32]:

$$\boldsymbol{H} = \Sigma \frac{\partial L}{\partial \dot{q}_i} \dot{q}_i - L = \frac{\partial L}{\partial v} v - L\,, \tag{21}$$

where $\dot{q}_i$ are generalized velocities.

Applying (21) to the relation (13) one obtains the following expression:

$$\boldsymbol{H} = \frac{m_0 \alpha\alpha' c^2 + \frac{1}{2}\left(m_0 \kappa(\alpha-\alpha') c v\right)}{\left(\alpha\alpha' - \frac{v^2}{c^2} + \frac{\kappa(\alpha-\alpha') c v}{c^2}\right)^{1/2}}. \tag{22}$$

It follows the substitution of the parameter H from (12):

$$H = \left(\alpha\alpha' - \frac{v^2}{c^2} + \frac{\kappa(\alpha-\alpha') c v}{c^2}\right)^{-1/2}, \tag{23}$$

to the relation (22). As the result of this substitution, the Hamiltonian ***H*** from (22) is transformed into the first form equal to the general covariant energy equation E_c:

$$\boldsymbol{H} = E_c = H m_0 \alpha\alpha' c^2 + \frac{H m_0 \kappa(\alpha-\alpha') c v}{2}. \tag{24}$$

In the case of a vacuum without any potential field, the field parameters α and α' satisfy the relation α = α' = 1. For that case the relation (24) is transformed into the well-known equation, valid in the Special Relativity [11]:

$$\boldsymbol{H} = H_E m_0 c^2, \quad H_E = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \tag{25}$$

Here H_E is the well known Einstein's parameter. The relation (24) is very important, because this Hamiltonian ***H*** is a linear function related to the relativistic momentum P = $Hm_o v$ and therefore belongs to the Dirac's like structure of the Hamiltonian.

IV. Derivation of the Second Form of Hamiltonian

In order to develop a relativistic Hamiltonian in the second form one can start with the solution (24). Applying the square operation to the relation (24) and including the equality [11]:

$$\frac{\kappa^2 (\alpha-\alpha')^2}{4} = 1 - \alpha\alpha', \quad \kappa^2 = 1, \tag{26}$$

the equation (24) is transformed into the new form:

$$\frac{\boldsymbol{H}^2}{c^2} - P^2 = m_0 \alpha\alpha' c^2, \qquad P = H m_o v. \tag{27}$$

This is the energy – momentum equation of a particle moving in an alpha field with a relativistic momentum P. Dividing this relation with (αα') we obtain the following expression:

$$\begin{gathered} \left(\frac{\boldsymbol{H}}{\sqrt{\alpha\alpha'}}\right)^2 \frac{1}{c^2} - \left(\frac{P}{\sqrt{\alpha\alpha'}}\right)^2 = m_0 c^2, \quad \boldsymbol{H}_e = \frac{\boldsymbol{H}}{\sqrt{\alpha\alpha'}}, \\ P_e = \frac{P}{\sqrt{\alpha\alpha'}} \quad \rightarrow \quad \frac{\boldsymbol{H}_e^2}{c^2} - P_e^2 = m_0 c^2. \end{gathered} \tag{28}$$

In the relation (28) $\boldsymbol{H}_e$ is the extended Hamiltonian and P_e is the extended momentum. Now, one can call the relation (3) from where we can derive the following equations:

$$\begin{gathered} \frac{1}{\sqrt{\alpha\alpha'}} \cong \left(1 - \frac{U}{m_0 c^2}\right) \rightarrow \frac{P}{\sqrt{\alpha\alpha'}} \cong \left(P - \frac{PU}{m_0 c^2}\right), \\ \frac{\boldsymbol{H}}{\sqrt{\alpha\alpha'}} \cong \left(\boldsymbol{H} - \frac{\boldsymbol{H}}{m_0 c^2} U\right), \\ \boldsymbol{H} \cong m_0 c^2 \quad \rightarrow \quad \frac{\boldsymbol{H}}{\sqrt{\alpha\alpha'}} \cong (\boldsymbol{H} - U). \end{gathered} \tag{29}$$

Applying (29) to the equation (28) we obtain the second form of the Hamiltonian ***H***:

$$\boldsymbol{H} = c\sqrt{m_0^2 c^2 + \left(P - \frac{PU}{m_0 c^2}\right)^2} + U. \tag{30}$$

This Hamiltonian is a function of the extended momentum P_e and potential energy U of the particle in the related multipotential field. The extended momentum P_e is given by the relation:

$$P_e = \left(P - \frac{PU}{m_0 c^2}\right). \tag{31}$$

As one can see from (31) the extended momentum P_e includes the interaction of the relativistic momentum P with the potential energy U. In the case of a vacuum without any potential field (U=0), the field parameters α and α' satisfy the relation α = α' = 1. For that case the relation (30) is transformed into the well-known equation, valid in the Special Relativity [11]:

$$\boldsymbol{H} = c\sqrt{m_0^2 c^2 + P^2}, \qquad P = H_E m_0 v, \tag{32}$$

where H_E is the well known Einstein's parameter, given by (25).

V. Nonrelativistic Aproximation of the Hamiltonian in an Alpha Field

If the extended momentum P_e is a small enough in the sense that the following expression is vanishing:

$$\frac{P_e^4}{4 m_0^4 c^4} = \frac{1}{4 m_0^4 c^4}\left(P - \frac{PU}{m_0 c^2}\right)^4 \cong 0, \tag{33}$$

then the relation (30) can be transformed into the approximate equation:

$$\boldsymbol{H} \cong c\sqrt{m_0^2 c^2 \left(1 + \frac{1}{2 m_0^2 c^2}\left(P - \frac{PU}{m_0 c^2}\right)^2\right)^2} + U. \tag{34}$$

It is very easy to see that from the relation (34) one obtains the approximate Hamiltonian in the form:

$$\boldsymbol{H} \cong m_0c^2 + \frac{1}{2m_0}\left(P - \frac{PU}{m_0c^2}\right)^2 + U. \qquad (35)$$

In the nonrelativistic case (v << c) and a weak potential field ($\alpha\alpha' \ll 1$) the parameter H from (12) is close to one ($H \approx 1$) and momentum $P = m_o v$. For that case the relation (35) is reduced to the form:

$$\boldsymbol{H} \cong m_0c^2 + \frac{1}{2m_0}\left(P - \frac{vU}{c^2}\right)^2 + U. \qquad (36)$$

This is a nonrelativistic approximation of the Hamiltonian in an alpha field.
Now, for an example, if an electron is moving with constant velocity v << c in an electromagnetic field, then one should use the following relations:

$$\begin{aligned} &U = qV \quad \rightarrow \quad \frac{Uv_x}{c^2} = \frac{q}{c}\frac{Vv_x}{c} = \frac{q}{c}A_x, \\ &\frac{Uv_y}{c^2} = \frac{q}{c}\frac{Vv_y}{c} = \frac{q}{c}A_y, \quad \frac{Uv_z}{c^2} = \frac{q}{c}\frac{Vv_z}{c} = \frac{q}{c}A_z. \end{aligned} \qquad (37)$$

Here (A_x, A_y, A_z) is a vector potential of the related electromagnetic field. Including (37) into the equation (36), one obtains the well known Hamiltonian of a electron moving in an electromagnetic field:

$$\begin{aligned} \boldsymbol{H} \cong\ & m_0c^2 + \frac{1}{2m_0}\left(P_x - \frac{q}{c}A_x\right)^2 + \frac{1}{2m_0}\left(P_y - \frac{q}{c}A_y\right)^2 + \\ & + \frac{1}{2m_0}\left(P_z - \frac{q}{c}A_z\right)^2 + qV. \end{aligned} \qquad (38)$$

In the case where quantum mechanical effects are not present one can employ classic Hamiltonian canonic forms for designing equations of the particle (sample) motion:

$$\dot{P}_i = -\frac{\partial \boldsymbol{H}}{\partial q_i}, \qquad \dot{q}_i = \frac{\partial \boldsymbol{H}}{\partial P_i}. \qquad (39)$$

In the relation (39) q_i and P_i are generalized coordinates and momentums, respectively.

VI. Application of Nonrelativistic Hamiltonian to Quantum Systems

In order to apply nonrelativistic Hamiltonian into the nonrelativistic quantum systems one should use the following two steps. The first one is to reduce the Hamiltonian from (36) to the kinetic and potential energy only:

$$\boldsymbol{H} \cong \frac{1}{2m_0}\left(P - \frac{vU}{c^2}\right)^2 + U. \qquad (40)$$

This is Hamiltonian without rest-mass energy, m_oc^2.

The second step is to introduce the related Hamiltonian operator:

$$\begin{aligned} \hat{\boldsymbol{H}} = -\frac{\hbar^2}{2m_0}\nabla_e^2 + U(\mathbf{r}) = -\frac{\hbar^2}{2m_0}\Big[&\left(\frac{\partial}{\partial x} - i\frac{v_xU}{\hbar c^2}\right)^2 + \\ + \left(\frac{\partial}{\partial y} - i\frac{v_yU}{\hbar c^2}\right)^2 + &\left(\frac{\partial}{\partial z} - i\frac{v_zU}{\hbar c^2}\right)^2\Big] + U(\mathbf{r}). \end{aligned} \qquad (41)$$

Here ∇_e^2 is the extended Laplacian operator, ħ is the reduced Planck's constant (Planck's constant divided by 2π) and $\mathbf{r} = (x, y, z)$ is the particle position in three-dimensional space. For a general quantum system one can employ time dependent Schrödinger equation [30] and [35]:

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r}, t) = \hat{\boldsymbol{H}}\Psi(\mathbf{r}, t). \qquad (42)$$

Here $\Psi(\mathbf{r},t)$ is the wave function, which is the probability amplitude for different configurations of the system. Applying the Hamiltonian operator from (41) to (42) we obtain the time dependent Schrödinger equation for a single particle in three dimensional space:

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m_0}\nabla_e^2\Psi(\mathbf{r}, t) + U(\mathbf{r})\Psi(\mathbf{r}, t). \qquad (43)$$

Here $\Psi(\mathbf{r},t)$ is the wave function, which is the amplitude for the particle to have a given position $\mathbf{r}$ at any given time t, and $U(\mathbf{r})$ is the potential energy of the particle at each position $\mathbf{r}$.

For every time independent Hamiltonian operator $\hat{\boldsymbol{H}}$ there exists a set of quantum states $|\Psi_n\rangle$ known as energy eigenstates and corresponding real numbers E_n satisfying the eigenvalue equation:

$$\boldsymbol{H}\,|\Psi_n\rangle = E_n\,|\Psi_n\rangle. \qquad (44)$$

This is the time independent Schrödinger equation. For the case of a single particle, the Hamiltonian $\boldsymbol{H}$ is the following linear operator:

$$\boldsymbol{H} = -\frac{\hbar^2}{2m_0}\nabla_e^2 + U = \frac{1}{2m_0}\left(P - \frac{vU}{c^2}\right)^2 + U. \qquad (45)$$

This is a self-adjoint operator when U is not too singular and does not grow too fast. Self-adjoint operators have the property that eigenvalues are real in any basis, and their eigenvectors form a complete set, either discrete or continuous. The presented Schrödinger equations describe a particle dynamics without spin effects. For inclusion of the spin effects one should employ the related Dirac's equations [16]- [17].

Dynamics of the quantum feedback systems and control concepts and applications are presented in [31]-[32], respectively.

VII. Conclusion

At the nanoscale the control dynamics is very complex. This is because there are very strong interactions between nanorobots (particles), manipulated objects (samples or particles) and nanoenvironment. The first step in designing of the control dynamics for nanorobots is the development of the relativistic Hamiltonian that includes external artificial control potential field. In that sense, the presented first and second form of the relativistic Hamiltonian can be applied to nanorobots control of a particle (a sample) motion in a multipotential field. The derived Hamiltonians are the functions of the dimensionless field parameters α and α' that describes the influences of the multipotential field to the particle (sample) motion. At the small enough distances between particles (nanorobot tip - sample distances) a quantum mechanical effects can be appeared. Thus, at Angstrom-scale distances, a quantum mechanical effect called tunneling causes electrons to flow across the tip/sample gap and current can be detected. This current is a function of the tip/sample distance and can be employed for a feedback position control of a tip or/and a sample. In the case of quantum systems control the derived Hamiltonian has been transformed into the related Hamiltonian operator. This has been done in the sixth section. The applications of the derived Hamiltonians in the dynamics of the nanorobot control will be presented in the next papers.

References

[1] A. A. G. Requicha, Nanorobotics. *Laboratory for Molecular Robotics and Computer Science Department*, University of Southern California, LosAngeles,CA90089-0781requicha@usc.edu, web-site http://www-lmr.usc.edu/~lmr, 2008.

[2] M. Gómez - López, J.A. Preece, J.F. Stoddart, The art and science of self-assembling molecular machines. *Nanotechnology*, Vol. 7, No. 3, Sept. 1996, pp. 183-192.

[3] K.E.Drexler, Nanosystems. *New York, NY: John Wiley & Sons*, 1992.

[4] G. Binnig, C.F. Quate, Ch. Gerber, Atomic Force Microscopy, *Phys. Rev. Lett. 56,* 3 March, 1986, pp. 930-933.

[5] J.A. Stroscio, D.M. Eigler, Atomic and molecular manipulation with the scanning tunneling microscope, *Science*, Vol. 254, No. 5036, 29 Nov. 1991, pp. 1319-1326.

[6] R. Wiesendanger, Scanning Probe Microscopy Methods and Aplications. *Cambridge University Press*, Cambridge, U.K., 1994.

[7] R.A. Jr. Freitas, Nanomedicine. Volume I: Basic Capabilities, *Landes Bioscience*, Georgetown, TX, 1999.

[8] M. Namdi, A. Ferreira, G. Sharma, C. Mavroidis, Prototyping Bio-Nanorobots using Molecular Dynamics Simulation and Virtual Reality. *Microelectronics Journal,* 2007.

[9] A. Dubey, C. Mavroidis, S.M. Tomassone, Molecular Dynamics Studies of Viral-Protein Based Nano-Actuators. *Journal of Comput. and Theoretical Nanoscience,* Vol. 3, No. 6, 2006, pp. 885-897.

[10] C. Mavroidis, Bionano Machines for Space Applications. *Final Phase II Report to the NASA Institute of Advanced Concepts,* July, 2006.

[11] A. Einstein. *Scientific American,* Vol. 182, No. 4, 1950.

[12] S. Weinberg, Dreams of A Final Theory : The Search for The Fundamental Laws of Nature. *Pantheon Books,* 1992, p. 334.

[13] D.J. Gross, The Status and Future Prospects of String Theory, *Nuclear Physics B (Proceedings Supplement)* 15 :43. , 1990.

[14] R.Penrose, W. Rindler, Spinors and space-time, Vol. 1: Two-Spinor Calculus and Relativistic Fields, Vol. 2: Spinor and Twistor Methods in Space-Time Geometry, *Cambridge University Press*, Cambridge, 1984.

[15] L. De Broglie, Mécanique ondulatoire, Paris, 1928, E. C. Kemble, The Fundamental Principles of Quantum Mechanics, p. 13, *McGraw-Hill, New York,* 193.

[16] P.A.M. Dirac, The Principles of Quantum Mechanics, *Oxford,* 1947.

[17] P.A.M. Dirac, Directions in Physics, *John Wiley & Sons, New York,* 1978.

[18] W. Heisenberg, Der Begriff Abgeschlossene Theorie in der Modernen Naturwissenschaft, *Dialectica 2*, 1948, pp. 331-336.

[19] W. Pauli, Relativitätstheorie, *Teubner, Leipzog*, 1921.

[20] S.W. Hawking, R. Penrose, The Nature of Space and Time, *Princeton University Press*, 1996.

[21] S. Weinberg, The First Three Minutes, *Harper Collins Publishers, inc.*, 1995.

[22] B. Novakovic, D. Majetic, J. Kasac, D. Brezak, Derivation of Hamilton functions including artificial control fields in nanorobotics, 12th International Scientific Conference on Production Engineering CIM2009, Zagreb, 2009, pp. 139-146.

[23] B. Novakovic, J. Kasac, M. Kirola, Dynamic Model of Nanorobot Motion in Multipotential Field, *Strojarstvo* 52, 2010.

[24] B. Novakovic, D. Novakovic, A. Novakovic, A New General Lorentz Transformation model, *CASYS'99, Ed. by D. M. Dubois, Publ. by The American Institute of Physics, AIP-CP517,* 2000, pp. 437-450.

[25] B. Novakovic, D. Novakovic, A. Novakovic, A New Approach to Unification of Potential Fields Using GLT Model, *CASYS'01, Ed. by D. M. Dubois, Publ. by CHAOS, Liège, Belgium, International Journal of Computing Anticipatory Systems*, vol.11, 2002, pp.196-211.

[26] B. Novakovic, D. Novakovic, A. Novakovic, A Metric Tensor of the New General Lorentz Transformation model, CASYS'2000, Ed. by D.M. Dubois, *Publ. by CHAOS, Liège, Belgium, International Journal of Computing Anticipatory Systems,* vol.10, 2001, pp.199-217.

[27] B. M. Novakovic, Relativistic alpha field theory - Part I. *International Journal of New Technology and Research*, Vol.**1**, No.**5**, IJNTR01050015, 2015, pp.23-30.

[28] B. M. Novakovic, Relativistic alpha field theory - Part II. *International Journal of New Technology and Research*, Vol.**1**, No.**5**, IJNTR01050016, 2015, pp.23-30.

[29] B. M. Novakovic, Relativistic alpha field theory-Part III. *International Journal of New Technology and Research*, Vol.**1**, No.**5**, IJNTR01050017, 2015. pp.39-47.

[30] D.J. Griffiths, Introduction to the Quantum Mechanics, 2nd edition, *Benjamin Cummings*, 2004.

[31] I. Supek, Theoretical Physics and Structure of Matter, Part 1, Zagreb, *Skolska knjiga*,1992.

[32] I. Supek, Theoretical Physics and Structure of Matter, Part 2, Zagreb, *Skolska knjiga*,1990.

[33] O. Klein, W. Gordon, and V. Fock, *Physic* 37, 895 ; 40,117; 38,242; 39, 226,1926.

[34] D. Dubois, Computational Derivation of Quantum Relativist Electromagnetic Systems with Forward-Backward Space-Time Shifts, *CASYS'99, Ed. by D. M. Dubois. Published by The American Institute of Physics, AIP-CP517*, 2000, pp. 417-429.

[35] Schrödinger equation, from the Wikipedia, the free encyclopedia, http://en.wikipedia.org/wiki/.

[36] M. Yanagisawa, H. Kimura, Transfer Function Approach to Quantum Control – Part I : Dynamics of Quantum Feedback Systems, *IEEE Trans. On Automatic Control,* vol. 48, no. 12, 2003, pp. 2107-2120.

[37] M.Yanagisawa, H. Kimura, Transfer Function Approach to Quantum Control – Part II: Control Concepts and Applications, *IEEE Trans. on Automatic Control,* vol. 48, no. 12, 2003, pp. 2121-2132.

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