Examples of measurement uncertainty evaluations in accordance with the revised GUM

2016 J. Phys.: Conf. Ser. 772 012008

(http://iopscience.iop.org/1742-6596/772/1/012008)

Examples of measurement uncertainty evaluations in accordance with the revised GUM

B Runje[1], **A Horvatic**[1,4], **V Alar**[1], **S Medic**[2] and **A Bosnjakovic**[3]

[1]University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Ivana Lucica 1, 10000 Zagreb, Croatia
[2]Karlovac University of Applied Science, Trg J.J.Strossmayera 9, 47000 Karlovac, Croatia
[3]Institute of Metrology of Bosnia and Herzegovina, Augusta Brauna 2, 71000 Sarajevo,Bosnia and Herzegovina

E- mail: amalija.horvatic@fsb.hr

Abstract. The paper presents examples of the evaluation of uncertainty components in accordance with the current and revised Guide to the expression of uncertainty in measurement (GUM). In accordance with the proposed revision of the GUM a Bayesian approach was conducted for both type A and type B evaluations.The law of propagation of uncertainty (LPU) and the law of propagation of distribution applied through the Monte Carlo method, (MCM) were used to evaluate associated standard uncertainties, expanded uncertainties and coverage intervals. Furthermore, the influence of the non-Gaussian dominant input quantity and asymmetric distribution of the output quantity y on the evaluation of measurement uncertainty was analyzed. In the case when the probabilistically coverage interval is not symmetric, the coverage interval for the probability P is estimated from the experimental probability density function using the Monte Carlo method. Key highlights of the proposed revision of the GUM were analyzed through a set of examples.

1. Introduction

The Guide to the expression of uncertainty in measurement (GUM) was last amended in 2008. The revision included minor changes and corrections. The new revision JCGM 100 201X introduces an entirely new approach to estimating measurement uncertainty in order to address the problem of measuring uncertainty evaluation in situations not covered in the current JCGM 100 : 2008 version of the Guide. Key conceptual changes in the evaluation of measurement uncertainty with respect to the current version of the GUM include the application of the Bayesian approach and conditional probabilities for the estimation of type A standard uncertainty as well as the problem solving assessment of measurement for the cases when the number of measurements is less than four [3,4]. Changes in the GUM in accordance with the norm JCGM 100 201X are listed below.

[4]To whom any correspondence should be addressed.

2. Changes of the Guide to the expression of uncertainty in measurement in accordance with the document JCGM 100 201x cd

In accordance with the proposed revision, the simplest way to eliminate the disadvantages of the GUM method is through adopting the Bayesian approach and applying the Monte Carlo simulation in the process of calculating measurement uncertainty. Key changes can be stated as follows:

- The current approach to estimating type A uncertainty components relies on the frequentist view while the new version of the GUM uses the Bayesian approach [5].
  Calculation of type A standard uncertainty in accordance with the Guide from the year 2008 and in accordance with revised GUM from year 2015 is shown with expressions (1) and (2) where s_i represents the standard deviation.

$$u(x_i) = \frac{s_i}{\sqrt{n}} \tag{1}$$

$$u(x_i) = \left(\frac{n-1}{n-3}\right)^{1/2} \cdot \frac{s_i}{\sqrt{n}} \tag{2}$$

  From expression (2) it is clear that the minimum required number of measurements for the evaluation of the standard deviation is four. This requirement is considered to be an example of good measurement practice [5].
- In the cases where the number of measurements is between one and four, the evaluation of standard uncertainty is based on the results and experiences of some earlier measurements. Knowledge based on previous results is expressed through a pooled standard deviation estimated from a sufficiently large number of measurements $n_p$were sufficient means greater than or equal to 20 measurements. In the cases were $1 \leq n < 4$, standard uncertainty is estimated by the expression (3).
-

$$u(x_i) = \left(\frac{n_p-1}{n_p-3}\right)^{1/2} \cdot \frac{s_p}{\sqrt{n_p}} \tag{3}$$

- In the process of evaluating type B uncertainty components, where the low availability of input data is assumed, in the current GUM, the quantity is described using the symmetrical rectangular probability density function (PDF). Standard uncertainty is calculated as the standard deviation of the rectangular PDF with assigned degrees of freedom. The rectangular distribution is displayed through the use of fixed endpoints a and b. In the revised GUM, the degrees of freedom were not assigned to the standard uncertainty due to the fact that the distribution was based on available knowledge.

- Coverage interval is calculated from the probability density function of the output quantity y.
  - ✓ In the cases where the functional relationship between output and input quantities is linear and the central limit theorem is met, probability distribution described by y and $u(y)$ is approximately normal, one can assume that when k = 2 this results in an interval with a probability of about P = 95%.
  - ✓ In other cases, Gaussian inequality (k = 3) is applied for the symmetrical distribution while the Chebisheva inequality (k = 4.5) is applied for the asymmetric distribution [5].
  - ✓ In the case of asymmetric distribution, the probabilistically symmetric coverage interval is not sufficient. It is necessary to apply the method of the Monte Carlo simulation and estimate coverage interval from experimental probability density function for a given probability P.

3. Examples of measurement uncertainty evaluations

The values of the uncertainty component obtained through type A evaluation in accordance with the GUM from the year 2008 and a revised Guide from the year 2015, depending on the number of measurements, are given in Table 1.

Journal of Physics: Conference Series **772** (2016) 012008 doi:10.1088/1742-6596/772/1/012008

Table 1. The values of the uncertainty component obtained through type A evaluation, depending on the number of measurements.

Number of measurements, n	Standard uncertainty JCGM 100 : 2008	Standard uncertainty JCGM 100 : 201X	
5	$u^*(x_i) = \frac{s_i}{\sqrt{n}}$	$u(x_i) = \left(\frac{n-1}{n-3}\right)^{1/2} \cdot \frac{s_i}{\sqrt{n}}$	$u(x_i) = 1.41 \cdot u^*(x_i)$
10			$u(x_i) = 1.13 \cdot u^*(x_i)$
20			$u(x_i) = 1.06 \cdot u^*(x_i)$
30			$u(x_i) = 1.04 \cdot u^*(x_i)$

In accordance with the new revision of the GUM the standard uncertainty, calculated on the basis of $n = 20$, increased by 5.7% compared to the version of the Guide from 2008.

The following example illustrates the estimation of the standard uncertainty of the thermal expansion coefficient of the steel gauge blocks [6]. According to information available in the literature it is known that the thermal expansion coefficient of steel gauge blocks α equals $(11 \times 10^{-6} \pm 1 \times 10^{-6})\ \mathrm{K}^{-1}$. According to the current GUM the input quantity will be described with a priori rectangular probability distribution, with the lower bound being $a = 10 \times 10^{-6}\ \mathrm{K}^{-1}$ and the upper bound being $b = 2 \times 10^{-6}\ \mathrm{K}^{-1}$. The question of the number of the degrees of freedom which should be attributed to standard measurement uncertainty begins to be raised. The probability density functions (pdf) of input quantity α, in accordance with current and revised GUM, obtained by using the Monte Carlo simulation (MCS) with a number of simulation $M = 100000$, are given in Figures 1 and 2.

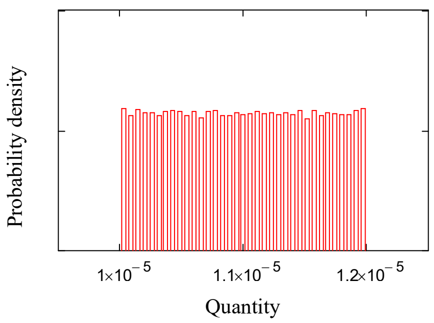


Figure 1. Probability density function of input quantity α, in accordance with the current GUM.

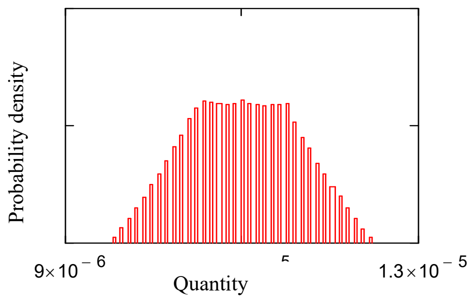


Figure 2. Probability density function of input quantity α, in accordance with the revised GUM.

In this case the standard deviation of input quantity α is characterized by a trapezoidal distribution and equals $6.241 \times 10^{-7}\ \mathrm{K}^{-1}$. This value is larger than the standard deviation of α which is characterized by a rectangular distribution ($5.788 \times 10^{-7}\ \mathrm{K}^{-1}$).

In the following section, the use of the current and revised GUM methods for the output quantity y, which is described using the equation $y = x^2$, are shown [5].
The input quantity is defined by the Gaussian probability density function (as shown in Figure 3) with the mean being 3 and the standard deviation being 5. Figure 4 shows the probability density function of the output quantity y. The input and output quantities obtained using the Monte Carlo simulation (MCS) with a number of simulation $M = 100000$, are given in Figures 3 and 4. Figure 4 also displays the shortest 95 % confidence interval evaluated according to the revised GUM. It is clearly visible that the 95 % confidence interval evaluated according to the current GUM is clearly not reliable because it includes infeasible (negative) values of the output y.

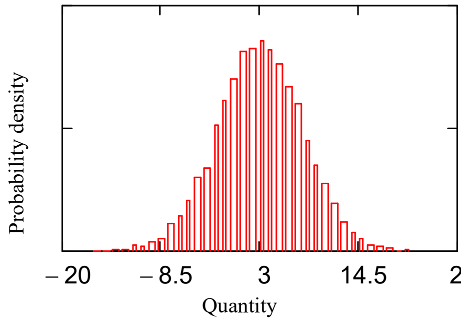


Figure 3. Probability density function for input quantity *x*.

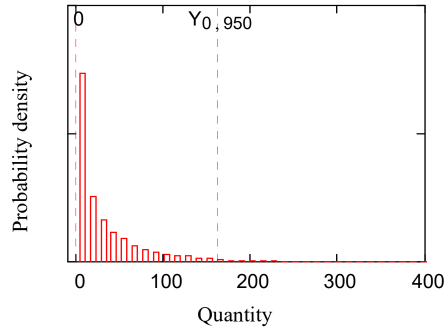


Figure 4. Probability density function and coverage interval for output quantity $y=x^2$.

The following example is an example of the evaluation of the measurement uncertainty that was performed for the calibration procedure of the micrometer setting rod. The mathematical model is provided in expression 4 while the input values and probability density functions in simulation of value L_X is provided in Table 2.

Mathematical measurement model:

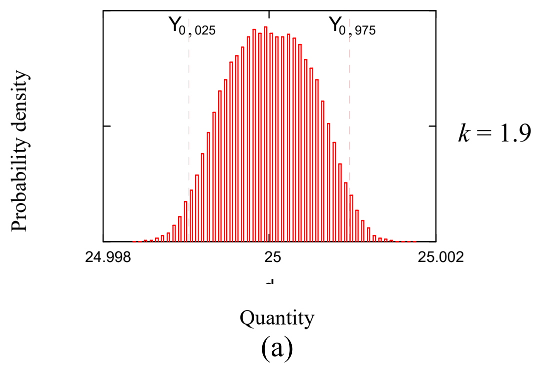
$$L_X = L_{ix} + \delta L_{ix} + \delta L_T + \delta L_E + \delta L_A + \delta L_P \qquad (4)$$

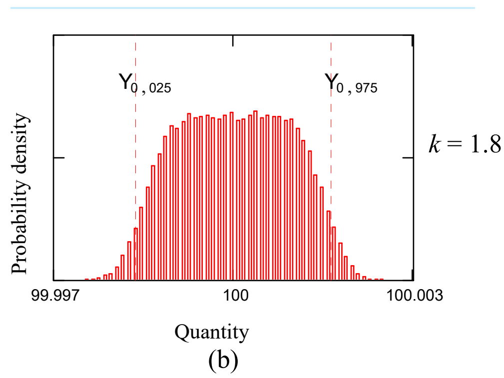
L_X	-	actual (corrected) length of the setting rod
L_{ix}	-	measured length of the setting rod
L	-	nominal length of setting rod
δL_{ix}	-	influence of the maximum permissible error
δL_T	-	influence of temperature
δL_E	-	influence of elastic deformation
δL_A	-	influence of Abbe error
δL_P	-	influence of misalignment of measuring probes

Table 2: Input values and probability density functions in simulation of value L_X

Input value x_i		Probability density function $g(x_i)$
measured length of the setting rod	L_{ix}	Normal distribution (M, 0; 0.17 µm)
influence of the maximum permissible error	δL_{ix}	Rectangular distribution (M; - 0.29 + 5.8 *L*; 0.29 + 5.8 *L*) µm
influence of temperature	δL_T	Normal distribution (M; 0; 0.20 + 0.7 *L*) µm
influence of elastic deformation	δL_E	Normal distribution (M; 0 µm; 0.21 µm)
influence of Abbe error	δL_A	Rectangular distribution (M; - 0.03 µm; 0.03 µm)
influence of misalignment of measuring probes	δL_P	Rectangular distribution (M;-0.11 µm; 0.11 µm)

It may be noted from Table 2 that the uncertainty element $u(\delta L_{ix})$, especially in case of longer setting rods, will have substantially greater contribution to uncertainty compared to others. In other words, for a certain length of the setting rod the conditions of the Central Limit Theorem cease to be valid. In applying the GUM method, it is difficult to predict at which moment the required conditions of Central Limit Theorem are not met any more. For the mentioned example, the measurement uncertainty was calculated by applying the MCS method.





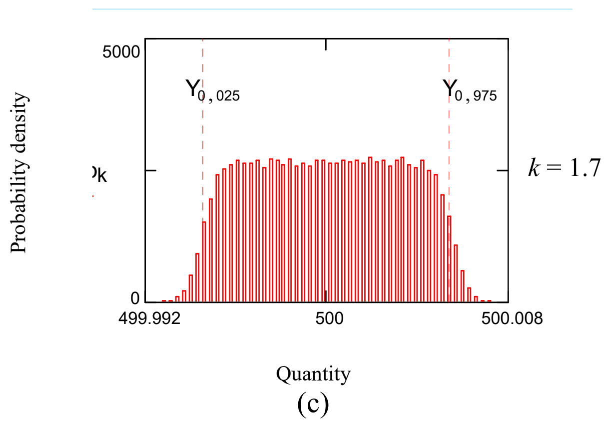
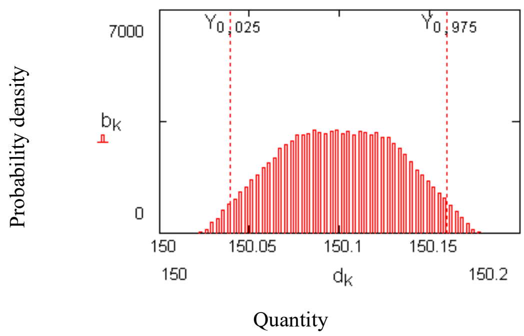


Figure 5. Probability density functions and coverage interval: a) for 25 mm setting rod, b) 100 mm setting rod and c) 500 mm setting rod

By applying the MCS method it has been determined that, depending on the length of the setting rod, the output distributions change their appearance from the normal-like to trapezoidal-like distributions as seen in figures 5a, 5b and 5c. For setting rods of nominal lengths 25 mm, 100 mm and 500 mm the coverage interval and coverage factor k are directly determined from the experimental probability density function. By MCS method it has been determined that the value of the coverage factor k changes regarding the length of setting rod. The coverage interval and coverage factor are directly determined from the experimental probability density function obtained by combining different probability density functions of input values. In measuring the length, the expanded measurement uncertainty is very often expressed in dependence on the length. This example indicates the problems that may result in expressing the expanded measurement uncertainty.

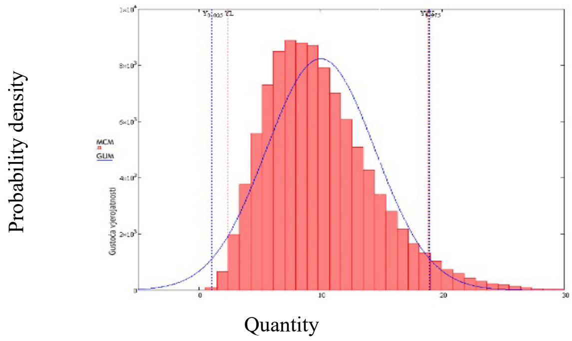
The following example, the evaluation of measurement uncertainty for the calibration of the vernier caliper given in document EA 4/02, also shows that the conditions necessary for the application of GUM method are not always fulfilled. The method used for calculating the coverage factor is clearly related to the fact that uncertainty of measurement associated with the result is dominated by two influences: the mechanical effects and the finite resolution of the vernier scale. Thus the assumption of a normal distribution for the output quantity is not justified [7]. The probability density function (pdf) for the 150 mm length vernier caliper is presented in Figure 6.

In the final example (Figure 7) a comparison is made between the 95 % coverage intervals estimated using the GUM and MCS methods with an asymmetric PDF as the output quantity.



	$\bar{x}$	$\tilde{x}$	s	95% coverage interval
GUM	150.10	150.10	0.03	[150.04; 150.16]
MCS	150.10	150.10	0.03	[150.04; 150.16]

Figure 6. Probability density function and coverage interval for symmetric non-normal distribution.



	$\bar{x}$	$\tilde{x}$	s	95% coverage interval
GUM	9.995	9.342	4.467	[1.061; 18.929]
MCS	9.995	9.342	4.467	[2.425; 18.830]

Figure 7. Probability density function and coverage interval for asymmetric distribution.

4. Conclusions

Key highlights of the proposed revision of the GUM can be stated as follows:

1. Adoption of the Bayesian approach in the calculation of the A and B types of uncertainty components as well as the application of the Monte Carlo simulation.
2. In accordance with the revision of the GUM, the value of the type A standard uncertainty components is higher than in the existing version of the Guide from the year 2008.
3. The coverage interval is calculated from the probability density function of the output quantity *y*.
4. The coverage interval no longer depends on the degrees of freedom calculated using the Welch-Satterthwaite formula.

5. References

[1] JCGM 100:2008, Evaluation of measurement data — Guide to the expression of uncertainty in measurement

[2] JCGM 101:2008, Evaluation of measurement data — Supplement 1 to the “Guide to the expression of uncertainty in measurement” — Propagation of distributions using a Monte Carlo method

[3] JCGM 100 201X CD, Evaluation of measurement data | Guide to uncertainty in measurement, Committee draft

[4] JCGM 110 201X CD, Evaluation of measurement data | Examples of uncertainty evaluation, Committee draft

[5] Cox M and Harris P, Revision of the Guide to the expression of uncertainty in measurement - impact on national metrology institutes and industry *UK CCRI BIPM*, (*Teddington, UK, 16-17 May 2013)*

[6] Lira I 2016, *Eur. J. Phys.* **37** 025803

[7] EA - 4/02 (2013) Evaluation of the Uncertainty of Measurement in Calibration