Contributions to the Metamathematics of Arithmetic

Fixed Points, Independence, and Flexibility

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Abstract

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This thesis concerns the incompleteness phenomenon of first-order arithmetic: no consistent, r.e. theory T can prove every true arithmetical sentence. The first incompleteness result is due to Gödel; classic generalisations are due to Rosser, Feferman, Mostowski, and Kripke. All these results can be proved using self-referential statements in the form of provable fixed points. Chapter 3 studies sets of fixed points; the main result is that disjoint such sets are creative. Hierarchical generalisations are considered, as well as the algebraic properties of a certain collection of bounded sets of fixed points. Chapter 4 is a systematic study of independent and flexible formulae, and variations thereof, with a focus on gauging the amount of induction needed to prove their existence. Hierarchical generalisations of classic results are given by adapting a method of Kripke's. Chapter 5 deals with end-extensions of models of fragments of arithmetic, and their relation to flexible formulae. Chapter 6 gives Orey-Hájek-like characterisations of partial conservativity over different kinds of theories. Of particular note is a characterisation of partial conservativity over $\mathrm{I}\Sigma_1$. Chapter 7 investigates the possibility to generalise the notion of flexibility in the spirit of Feferman's theorem on the 'interpretability of inconsistency'. Partial results are given by using Solovay functions to extend a recent theorem of Woodin.