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Article in *International Review on Modelling and Simulations* · April 2016

DOI: 10.15866/iremos.v9i2.8161

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## Parameter Estimation of Local Volatility in Currency Option Valuation

S. O. Edeki<sup>1</sup>, M. E. Adeosun<sup>2</sup>, E. A. Owoloko<sup>1</sup>, G. O. Akinlabi<sup>1</sup>, I. Adinya<sup>3</sup>

**Abstract** – In quantitative finance and option pricing, one of the basic determinants of option prices is the volatility of the underlying asset. In this paper, we therefore, present a concise study of volatility in option pricing in the sense of Dupire's approach. Thereafter, we outspread such study via the application of Ito formula to the modelling and valuation of currency option with local volatility.

For the purpose of efficiency, we use the daily historical prices of stock-S&P 500 for a certain period to estimate the corresponding historical volatility. Graphical representation of the analysed daily historical data of stock prices with respect to a local volatility is presented. **Copyright © 2016 Praise Worthy Prize S.r.l. - All rights reserved.**

**Keywords:** Currency Option Valuation, Local Volatility, Implied Volatility, Stochastic Model

### Nomenclature

$\sigma$	Constant volatility
$\sigma(t)$	Time dependent volatility
$\sigma(S(t))$	Stock price dependent volatility
$\sigma(\omega)$	Stochastic volatility
$S_t$	Asset price at time $t$
$\mu(t)$	Drift coefficient
$W_t$	Standard Brownian motion
$r$	Instantaneous risk-free rate
$q$	Dividend yield parameter
$\Lambda(t, S_t)$	Transition probability conditional to the asset price at time $t$
$C$	Call option price
$T$	Time at maturity
$K$	Strike price
$r^d$	Domestic risk-free interest rate
$r^f$	Foreign risk-free interest rate
$\sigma_e$	Exchange rate volatility
$\Xi$	Fair price of a contingent claim
$C^E$	European call option
$P^E$	European put option

### I. Introduction

The price of an option is determined by a number of variables relating to the underlying assets and financial markets. One of the major determinants of the price of an option is the volatility of the underlying asset.

It is important to note that increase in the volatility of an underlying asset increases both the put and the call values of option.

The above sounds contradictory: that is, why an increase in volatility which is a risk-measure should cause an increase in value (price) of the option? The only answer is: options are different from other securities since buyers of option can never lose more than the price they pay for them; they can even earn significant returns from large price movements.

The term 'volatility' refers to a measure of randomness in asset returns, which could just mean an estimate of the historical variance of some market prices estimated by looking at historical data. Empirical measurement of volatility can be categorized into two methods namely: historical and implied volatility [1].

A review of volatility models has been made by many researchers such as Mitra [1] and Taylor [2].

These authors considered stochastic modelling, and classified same into the following categories: constant volatility  $\sigma$ ; time-dependent volatility  $\sigma(t)$ ; local volatility: volatility dependent on the stock price  $\sigma(S(t))$ ; and stochastic volatility: volatility driven by an additional random process  $\sigma(\omega)$ .

In mathematical and quantitative finance, a local volatility model takes volatility as a function of both the present asset price  $S_t$  and time  $t$ . However, in Black-Scholes model, the volatility of the asset price is assumed a constant. But in reality, the realised underlying asset's volatility varies with respect to time. This constancy volatility assumption has drawn the attention of many researchers in finance, actuarial sciences, and other areas of management sciences [3]-[7].

The local volatility was proposed as a result of insufficiency of the time dependent volatility model which by empirical characteristics of volatility does not explain the volatility smile and leverage effect.

In a Local Volatility Model (LVM), the asset price model under a risk-neutral measure satisfies the SDE of the form:

$$dS_t = \mu(t)S_t dt + \sigma(t, S_t)S_t dW_t \quad (1)$$

where  $\mu(t)$  is a drift coefficient,  $\sigma_t = \sigma(t, S_t)$  is a set of diffusion coefficients, and  $W_t$  is a Brownian motion denoting the inflow of randomness into the dynamics.

The LVM is an important simplification of the stochastic volatility model.

Dupire [8], and Derman and Kani [9] developed the concept of local volatility when they noted that there exists a unique diffusion process that is consistent with the risk neutral densities obtained from the market prices of European options. Derman and Kani in [9], further noted the implementation of local volatility function in modelling instantaneous volatility in binomial options pricing model.

The Dupire formula uses the Fokker-Planck equation in the derivation of local volatility as an expected value.

Derivation of local volatility from implied volatility has been used as methods in deriving local volatility in (1) [10], [11]. The historic volatility is calculated from empirical (discrete) price of the associated data while the implied volatility is an estimate made using the Black-Scholes formula. An investigation into NSE NIFTY future options was carried out using the historical and the implied volatility surface [12], [13]. Their results showed how divergent the implied volatility has been and using the GARCH (1,1) model, the historical volatility for the period was estimated.

Bollen and Rasiel [14] considered the comparison of regime-switching, GARCH, and jump-diffusion models to a standard “smile” model; and showed that the former provide vital improvement over the latter on a fixed basis. Higgins [15] on stochastic models, defined an alternative model where the spot/volatility correlation is regarded as a separate mean-reverting stochastic variable (M-RSV) that is auto-correlated with spot price. Zhang [16] developed an approximation scheme for a nonlinear partial differential equation resulting from an uncertain volatility model in option pricing. They also proved the consistency, stability, and monotonicity of the developed method.

In [17], it was shown that information about the stock market, the real economy and monetary policy are usual coincidence with changes in implied volatility.

A computation of the implied volatility in existing volatility models such as the Heston’s model and log-normal model was done by [18]. The dynamics of implied volatility was characterized in [19] and [20].

The assumption that the volatility of an asset price is a constant factor throughout the lifetime of the option in the Black-Scholes model [21]; which is the most frequently used model in pricing financial derivative has been criticized by researchers in the last decades (see [10]).

Due to this shortcoming, it is of necessity to find a model that incorporates the variability of the implied volatility of the model and at the same time having same volatility values as opposed to those implied by the Black-Scholes model.

In view of the above, Ye [22] did a comparative analysis on the relationship between the implied volatility inferred by the Black-Scholes model, the local volatility specified by the LVM and the volatility given by the Dupire’s formula for implied volatility.

In this work, we consider the Dupire’s approach to LVMs and outspread same to the valuation of currency options.

## II. Derivation of the Dupire LVM and its Local Volatility

Considering the Stochastic Differential Equation (SDE) in (1) with  $\mu(t) = r - q$ , ( $r$  as the instantaneous risk-free rate), resulting to risk neutral SDE:

$$\frac{dS_t}{S_t} = (r - q)dt + \sigma(t, S_t)dW_t \quad (2)$$

and suppose the transition probability  $\wedge(t, s_t)$  conditional to the initial asset price  $S_0$ , satisfies the Fokker – Planck equation:

$$\frac{\partial \wedge(t, S)}{\partial t} = \frac{1}{2} \frac{\partial^2 (\partial S)^2 \wedge(t, S)}{\partial S^2} - \frac{\partial (r - q)S \wedge(t, S)}{\partial S} \quad (3)$$

By martingale pricing theorem, considering the price of a call option  $C$  with maturity time  $T$ , and a strike price  $K$ , we have:

$$Ce^{rT} = E^Q \max(S_T - K, 0) \quad (4)$$

$$\Rightarrow C = e^{-rT} \left( \int_K^\infty S \wedge dS - K \int_K^\infty \wedge dS \right) \quad (5)$$

$$\therefore C = e^{-rT} \int_K^\infty S \wedge dS - e^{-rT} \int_K^\infty K \wedge dS$$

Thus, differentiating  $C$  w.r.t. to  $K$  gives:

$$\frac{\partial C}{\partial K} = -e^{-rT} \int_K^\infty \wedge dS \quad (6)$$

$$\Rightarrow K \frac{\partial C}{\partial K} = -e^{-rT} \int_K^\infty K \wedge dS \quad (7)$$

Putting (7) in (5) we have:

$$C = e^{-rT} \int_K^\infty S \wedge dS + K \frac{\partial C}{\partial K} \quad (8)$$

showing that:

$$C - K \frac{\partial C}{\partial K} = e^{-rT} \int_K^\infty S \wedge dS \quad (9)$$

Similarly, if the price of a call option is differentiated twice w.r.t. the strike price  $K$ , and once w.r.t. time  $T$  at maturity date, then the following is obtained respectively:

$$\frac{\partial^2 C}{\partial K^2} = e^{-rT} \wedge(T, S) \quad (10)$$

and:

$$\frac{\partial C}{\partial K} + rC = \int_K^\infty e^{-rT} S(S - K) \frac{\partial \wedge(T, S)}{\partial T} dS \quad (11)$$

Thus, with the application of Fokker-Planck equation, and integration by part, the following is obtained (see [1], [10], [11] for additional information):

$$\frac{\partial C}{\partial T} = \frac{1}{2} \sigma^2 K^2 \frac{\partial^2 C}{\partial S^2} - qC - (r - q)K \frac{\partial C}{\partial K} \quad (12)$$

Note: In a particular case when  $r = q = 0$ , (12) automatically becomes:

$$\frac{\partial C}{\partial T} = \frac{1}{2} \sigma^2 K^2 \frac{\partial^2 C}{\partial S^2} \quad (13)$$

Solving for  $\sigma = \sigma(T, K)$  in (13), we have:

$$\sigma = \frac{1}{K} \sqrt{\frac{2(\partial C / \partial T)}{(\partial^2 C / \partial S^2)}} \quad (14)$$

Eq. (14) is thus, referred to as Dupire's formula for local volatility in terms of  $C(\cdot, \cdot)$  as a volatility surface.

### III. Modelling Currency Option with Local Volatility

Suppose we take  $S = S_t$  to be the spot exchange rate in terms of domestic currency, with the assumption of lognormal distribution. Let  $r^d$  and  $r^f$  be domestic and foreign risk-free interest rates respectively, and  $\sigma_e$  as the exchange rate volatility.

Then,  $S$  satisfies the SDE:

$$\frac{dS}{S} = (r^d - r^f) dt + \sigma_e dW_t \quad (15)$$

Suppose  $\Xi$  is the fair price of the contingent claim having  $S$  as the underlying asset, then by Ito formula, one gets:

$$\frac{\partial \Xi}{\partial t} + (r^d - r^f) S \frac{\partial \Xi}{\partial S} + \frac{1}{2} \sigma_e^2 S^2 \frac{\partial^2 \Xi}{\partial S^2} - r^d \Xi = 0 \quad (16)$$

Now, associated to (15) and (16) are the European call and European put ( $C^E$  and  $P^E$ ) defined and denoted for an initial spot price,  $S_0$  by:

$$C^E(S_0, T, K, r^f) = S_0 e^{-Tr^f} N(\Theta_1) - K e^{-Tr^d} N(\Theta_2) \quad (17)$$

$$P^E(S_0, T, K, r^f) = K e^{-Tr^d} N(-\Theta_2) + S_0 e^{-Tr^f} N(-\Theta_1) \quad (18)$$

where:

$$\Theta_1 = \frac{1}{\sqrt{\sigma_e^2 T}} \ln \left( \frac{S_0}{K} e^{\left( r^d - r^f + \frac{\sigma_e^2}{2} \right) T} \right) \quad (19)$$

$$\Theta_2 = \frac{1}{\sqrt{\sigma_e^2 T}} \ln \left( \frac{S_0}{K} e^{\left( r^d - r^f - \frac{\sigma_e^2}{2} \right) T} \right) \quad (20)$$

### IV. Discussion and Concluding Remark

Whenever:

$$\sigma = \sigma_e = \frac{1}{K} \sqrt{\frac{2(\partial C / \partial T)}{(\partial^2 C / \partial S^2)}}$$

(see (14)) is used in (15) through (20), then the theoretical fair prices of the currency options in the sense of local volatility associated to Dupire's formula will be obtained, provided the radicand in (14) is well defined.

For the purpose of estimation, we use daily historical prices of stock from SPDR S&P 500 ETY (SPY), spanning September 17, 2015 to December 17, 2015.

The analysed data gives a historical volatility of  $0.114884 = \sigma_H$ . Below is the graphical representation (Fig. 1) of the various prices of the underlying asset (stock); classified as high, low, and close.

It is noteworthy saying that LVMs are very vital in all option markets whose underlying volatility is basically a function of both the underlying asset and the interest rate.

They are very easy to calibrate, and have the stock price as the only source of randomness.

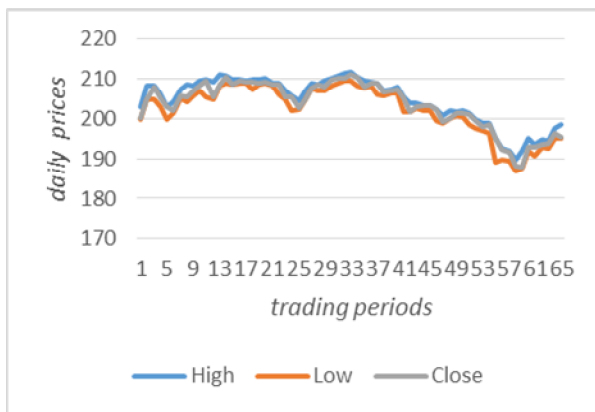


Fig. 1. Price Chart

However, volatility in LVMs is a deterministic function of random stock price, thereby making them less useful in pricing forward-start options since their values are based on the random nature of the volatility itself.

### Acknowledgements

The authors are very much grateful to Covenant University for financial and resourceful support.

They also wish to thank the anonymous referees for their valuable time, constructive and helpful comments. AMS: 91B25, 91B70, 97M30.

### References

- [1] S. Mitra. A review of volatility and option pricing (2009), submitted to Elsevier, <http://arxiv.org/pdf/0904.1292.pdf>.
- [2] S.J. Taylor. Stochastic Volatility Modeling, (2008), <https://phps.portals.mbs.ac.uk>
- [3] O.O. Ugbebor and S.O. Edeki. On Duality Principle in Exponentially Levy Market, *Journal of Applied Mathematics and Bioinformatics (JAMB)*, (2013), 3 (2), 159-170.
- [4] S.O. Edeki, I. Adinya and O.O. Ugbebor. The Effect of Stochastic Capital Reserve on Actuarial Risk Analysis via an Integro-differential Equation, *IAENG International Journal of Applied Mathematics*, (2014), 44 (2), 83-90.
- [5] M.E. Adeosun, S.O. Edeki, and O.O. Ugbebor. Stochastic Analysis of Stock Market Price Models: A Case Study of the Nigerian Stock Exchange (NSE), *WSEAS Transactions on Mathematics*, (2015), 14, 353-363.
- [6] S.O. Edeki, O.O. Ugbebor, E.A. Owoloko, "On a Dividend-Paying Stock Options Pricing Model (SOPM) Using Constant Elasticity of Variance Stochastic Dynamics", *International Journal of Pure and Applied Mathematics*, 106 (4), (2016): 1029-1036.
- [7] S.O. Edeki, O.O. Ugbebor, and E.A. Owoloko. Analytical Solutions of the Black-Scholes Pricing Model for European Option Valuation via a Projected Differential Transformation Method, *Entropy*, (2015), 17 (11), 7510-7521.
- [8] B. Dupire. Pricing with a Smile, *Risk*, (1994), 7, 18-20.
- [9] E. Derman, and I. Kani. Riding on a Smile. *Risk*, (1994), 7 (2), 139-145.
- [10] M.H.A. Davis. The Dupire Formula. Finite Difference Methods Course material, *Imperial College London*, (2011).
- [11] F. D. Rouah. Derivatoin of Local Volatility. [www.javaquant.net](http://www.javaquant.net), (1998)
- [12] E. Derman, I. Kani and M. Kamal. Trading and Hedging Local Volatility, *Goldman Sachs Quantitative Strategies Research Notes*, (1996).
- [13] N.R. Parasurama, and J.P. Ramudu. Historical and implied

- volatility: An investigation into NSE NIFTY futures. *Australian Journal of Business and Management Research*, (2011), 1 (7), 112-120.
- [14] N.P.B Bollen, and E. Rasiel. The performance of alternative valuation models in the OTC currency options market, *Journal of International Money and Finance*, (2003), 22 (1), 33-64. ISSN 0261-5606.
- [15] M. Higgins. Stochastic Spot/Volatility Correlation in Stochastic Volatility Models and Barrier Option Pricing (2014), <http://arxiv.org/pdf/1404.4028.pdf>.
- [16] K. Zhang, and S. Wang. A computational scheme for uncertain volatility model in option pricing, *Applied Numerical Mathematics*, (2009), 59 (8), 1754-1767.
- [17] C. J. Neely. Using implied volatility to measure uncertainty about interest rates, *Federal reserve bank of St. Louis Review*, May/June (2005), 87(3), 407-425.
- [18] H. Berestycki, J. Busca and I. Florent. Computing the Implied Volatility in Stochastic Volatility Models, *Communications on Pure and Applied Mathematics*, (2004), LVII, 1352-1373, Wiley periodicals, Inc.
- [19] R. Cont and J. da Fonseca. Dynamics of implied Volatility. *Quantitative Finance* (2002), 2, 45-60, Institute of physics publishing.
- [20] R. W. Lee. Implied Volatility: Statics, Dynamics and Probability Interpretation In Recent Advances in Applied Probability, Springer (2004).
- [21] F. Black and M. Scholes. The pricing of options and corporate liability, *J. Political Economy*, (1973), 81, 637-659.
- [22] H. Ye. A Comparison of Local Volatility and Implied Volatility. Department of Mathematics Uppsala University (2011).

### Authors' information

<sup>1</sup>Department of Mathematics, Covenant University, Canaanland, Otta, Nigeria.

E-mail: [soedeki@yahoo.com](mailto:soedeki@yahoo.com)

<sup>2</sup>Department of Mathematics & Statistics, Osun State College of Technology, Esa Oke, Nigeria.

<sup>3</sup>Department of Mathematics, University of Ibadan, Ibadan, Nigeria.