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ABSTRACT

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Technology choice and unit vs *ad valorem* tax

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Abstract

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1 Introduction

The comparison between *ad valorem* and unit taxes has long been of wide interest in the study of public economics, both for theoretical and policy reasons. In competitive markets, each *ad valorem* and unit taxes yield equal results. In monopolies, Suits and Musgrave (1953) proved the superiority of *ad valorem* taxation. Delipalla and Keen (1992) examined different models of symmetric oligopoly with and without free entry to compare the two types of tax regimes and find that *ad valorem* taxation often provides a better result. Skeath and Trandel (1994) raised this superiority, nowadays widely accepted in the literature, to a higher level by showing that in a monopoly *ad valorem* taxation Pareto-dominates unit taxes, producing larger profits, tax revenues and consumer surplus, and extending this result to symmetric Cournot oligopolies with linear demand, when government's revenue is sufficiently large. Denicolo and Matteuzzi (2000) and later Anderson et al. (2001) found that *ad valorem* taxes are always better than unitary ones in asymmetric Cournot oligopolies with homogeneous goods.

However, this matter may not be fully closed. Anderson et al. (2001) proved that, under Bertrand competition with product differentiation, the superiority of *ad valorem* taxes can be reversed. Blackorby and Murty (2007) considered a special case, in which the government taxes away a monopoly's whole profit in a general equilibrium, to show that *ad valorem* and unitary taxation are equivalent. Lockwood (2004), in a standard model of tax competition, showed that residents in all countries are worse off when competition is in *ad valorem* taxes. Dröge and Schröder (2009) concluded that, where the main objective of the tax is to reduce a negative externality, unit taxes may be superior in terms of welfare effects. Wang and Zhao (2009) showed that with sufficiently differentiated goods and a high enough cost variance and taxes, unit taxes can be superior to *ad valorem* taxes if asymmetric (Cournot or Bertrand) oligopolies are considered.

Most of these examples rely on significant changes to the basic model to achieve reversal of the classical superiority of *ad valorem* taxes. This paper implements a small

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variation to the classical model: the availability of two technologies. With this additional hypothesis, we find that the superiority of *ad valorem* taxes can be reversed in non trivial examples. Even in monopoly settings, where traditional theory indicates that *ad valorem* taxes are a much better choice, it is possible to find cases (although unlikely) where the corresponding unit taxes would be preferable. tax could be a better option for society.

We investigate the welfare superiority of *ad valorem* taxation in monopoly and oligopoly settings where two technologies are available to firms. One can easily think of situations where technology choice is possible, for example in sectors with strong R&D components, transition periods between labor and capital intensive technologies, etc. Technology changes can be understood also as firms investing to improve the efficiency of their productive processes. These investments can take many forms. Some imply minor modifications (such as big or small repairs in machinery, task reorganizations or in-the-job training) while others result in significant changes and costs (for example purchases of new machinery, the building of a new plant, cost reduction task forces, etc). For the sake of this paper, technology choice means a reduction in variable costs at the expense of some fixed costs, or vice versa. We believe our results to be of particular importance for oligopolies although, for completeness, we also study the monopoly case.

The paper proceeds as follows. Section two will discuss unit and *ad valorem* taxes in a monopoly. We will show that in the presence of a technological change that causes an increase in prices, *ad valorem* taxes could do worse than the equivalent unit taxes, while in general *ad valorem* taxes are the better alternative if the new technology decreases marginal costs. To conclude this section we will show that, even with a technology that cuts marginal costs, examples can be provided where unit taxes could be superior to *ad valorem* taxes if conditional taxation is possible. Section three contains some examples that seem to be in contradiction with the results of Denicolo and Matteuzzi (2000) and Anderson et al. (2001), by allowing firms to choose among different technologies. Under these circumstances, we show that unit taxes can be welfare superior to *ad valorem* taxes in asymmetric Cournot oligopolies. Section four concludes.

2 Monopoly

The usual knowledge on taxes has it that for any unit tax t, there is always a Pareto superior *ad valorem* tax. In this section, we will consider the welfare effects of *ad valorem* and unit taxes in the presence of different technological changes and see how the generally accepted result can be reversed. We shall provide three main results:
(a) In proposition 1, we will see how unit taxes may perform better than *ad valorem* taxes if the new technology causes an increase in prices. (b) Propositions 2 and 3 will prove that, under certain conditions, if marginal costs decrease with the new technology, *ad valorem* taxes are always the better alternative. (c) Proposition 4 illustrates a result that is more of a curiosity: even if the new technology decreases marginal costs, unit taxes may be better if the government imposes conditional taxes that change according to the technology implemented.

Proposition 1. *Consider a profit maximizing monopoly. When a technological change causes an increase in prices, it is possible that a Pareto inferior unit tax provides a higher level of welfare than the* ad valorem *tax.*

Proof. Suppose a linear demand[1] $P = a - Q$, a technology with fixed costs F and no

[1]Throughout the paper we use the capital P for the price that consumers pay, and the lower-case p

variable costs, with two possible tax regimes: unit tax, t; and *ad valorem* tax, τ. The basic equilibrium variables (quantity, government income and profit) in both tax regimes are:

$$\text{Unit tax:} \qquad Q_t = \frac{a-t}{2}; \qquad G_t = \frac{t(a-t)}{2}; \qquad \Pi_t = \frac{(a-t)^2}{4} - F. \tag{1}$$

$$\textit{Ad valorem:} \qquad Q_\tau = \frac{a}{2}; \qquad G_\tau = \tau\frac{a^2}{4}; \qquad \Pi_\tau = (1-\tau)\frac{a^2}{4} - F. \tag{2}$$

In order to find an *ad valorem* tax, τ, that is Pareto superior to the unit tax, t, (i) prices must be lower, so that consumers are better off, which is always the case for $t > 0$; (ii) government revenue must be higher; i.e. $\tau \geq 2t(a-t)/a^2$; and (iii) the firm must receive higher profits; i.e. $\tau \leq t(2a-t)/a^2$. In sum, the *ad valorem* tax, τ, will be Pareto superior to the unit tax, t, if $\tau \in [\frac{2t(a-t)}{a^2}, \frac{t(2a-t)}{a^2}]$ for the existing technology.

Let's consider now that a new technology with constant marginal cost c and no fixed costs becomes available. The quantity and profit in both tax regimes are:

$$\text{Unit tax:} \qquad Q_t = \frac{a-t-c}{2}; \qquad \Pi_t = \frac{(a-t-c)^2}{4}. \tag{3}$$

$$\textit{Ad valorem:} \qquad Q_\tau = \frac{a}{2} - \frac{c}{2(1-\tau)}; \qquad \Pi_\tau = (1-\tau)\left(\frac{a}{2} - \frac{c}{2(1-\tau)}\right)^2. \tag{4}$$

Let us call $F_t = c(\frac{2a-2t-c}{4})$, $F_\tau = c(\frac{a}{2} - \frac{c}{4(1-\tau)})$, and $F_0 = \frac{(a-t)^2}{4}$. Then the following statements follow:

(a) Whenever $F > F_t$, the firm with a unit tax regime prefers the technology with $c > 0$; this can be seen by comparing the profits from equations 1 and 3.

(b) Whenever $F > F_\tau$, the firm with the *ad valorem* regime prefers the technology with $c > 0$; this can be seen by comparing the profits from equations 2 and 4.

(c) When $F \leq F_0$, the firm has no negative profits with the zero marginal cost regime. This follows from equation 1.

Comparing equations 3 and 4 enables us to conclude that if $c > t(1-\tau)/\tau$, then $Q_t^c > Q_\tau^c$. And comparing F_t and F_τ yields that whenever $c < 2t(1-\tau)/\tau$, then $F_t < F_\tau$.

Then, suppose that in an industry with demand $P = a - Q$ there is a monopoly with zero marginal costs and a fixed cost F. The government considers a unit tax t, but prefers a Pareto superior *ad valorem* tax τ. A new technology with constant marginal costs, c appears, such that $t(1-\tau)/\tau < c < 2t(1-\tau)/\tau$. Besides, $F_\tau < F \leq F_0$. Then, since the firm will definitely switch to the technology with the higher marginal cost in both tax regimes, and will produce more with the unit regime, then welfare will be higher with the unit tax[2].

To illustrate the results of proposition 1, we provide a numerical example below.

Numerical example

Let the demand function be $P = 100 - Q$ and the unit tax $t = 10$. Then, the set of *ad valorem* tax Pareto superior to $t = 10$, for a zero marginal cost, is $\tau \in [0.18, 0.19]$. Suppose that $c = 46$, then $c > t(1-\tau)/\tau$ for any $\tau \in [0.18, 0.19]$. Then, it follows that: $F_t = 391$, $F_\tau \in (1646.91, 1654.88)$, and $F_0 = 2025$.

for the price that the producers get. In the absence of taxes, we will use either of them.

[2]It may be worth noting that society would prefer that the firm remained with the old technology. However, the monopoly will decide to switch.

When the fixed cost, F, of the old technology is between 1655 and 2025, and a new technology with zero fixed costs, and constant marginal costs $c = 46$, appears, then welfare will be higher with the unit tax $t = 10$ than with any Pareto superior tax $\tau \in [0.18, 0.19]$.

It may be argued that the government could define a new *ad valorem* tax for the new technology that is Pareto superior to the unit tax. This is true, but in Proposition 4 we show that such a contingent tax can be favorable to the unit tax. ■

The next two propositions will consider what happens if the new technology decreases prices.

Proposition 2. *For both unit and* ad valorem *taxes, if a profit maximizing monopoly prefers a technology that increases its production level ($Q^n \geq Q$), so will society.*

Proof. Let us first consider the case where $Q^n = Q$. If the monopoly prefers the new technology, so will society, since profits increase and both consumer surplus and government revenue remain the same, independently of the tax scheme.

When $Q^n > Q$, consumers are better off because prices are lower. Government revenue will increase under unit taxes (because the quantity sold is increased) and under *ad valorem* taxes (as long as the marginal cost and marginal revenue are positive so that revenues are increasing with quantity). Again, if the monopoly prefers the new technology, so will society since consumers and government are also better off. ■

Note that the reverse is not true. That is, it is possible for society to prefer a price-lowering technology that firms do not want to adopt. One such example can be found for a monopoly with constant marginal costs of 2, facing a demand given by $P = 6 - Q$. Under these conditions, profits are equal to 4, while society's welfare (consumer surplus plus profits) is 6. If a new technology becomes available with zero marginal costs and a fixed cost of 6, the profits of the monopoly would be lower (3), but welfare would be higher (7.5).

Proposition 3. *Let's consider an* ad valorem *tax, τ, which is Pareto superior to the unit tax, t, in a market where a profit maximizing monopoly sells its product. The market shows decreasing marginal income and non-decreasing marginal costs. When a technology change decreases marginal costs by introducing a fixed cost F, the* ad valorem *tax, τ, will be superior to the unit tax, t, for consumers, the firm and welfare, though it may not be so for the government.*

Proof. In order to prove the proposition we will consider the following four possibilities:

(a) The firm does not adopt the technology change under either tax scheme. Then, τ is Pareto superior to t, according to the initial hypothesis.

(b) The firm adopts the technology change in the presence of the unit tax, but not under the *ad valorem* tax. We will see this is an impossible situation.

(c) The firm adopts the technology change in the presence of the *ad valorem* tax, but not under the unit tax. In this case, we will show that τ is still Pareto superior to t.

(d) The firm adopts the technology change under either tax scheme. We will show that production will be larger under *ad valorem* taxes than under unit taxes, meaning profits for the firm would be higher and prices paid by consumers lower, but government revenue may not increase.

Figure 1 contains the gist of the proof. We will prove that area $A + B$ (the increment in profit for the *ad valorem* tax) is higher than area A (the increment in profit for the unitary tax).

Now we consider each case in turn. (a) It needs no proof. (b) Let I' be the marginal revenue associated to the demand. Solving $(1-\tau)I' = I' - t$, we get $I' = t/\tau$. As I' is decreasing, there is only one point, $\hat{x}$, such that $I'(\hat{x}) = t/\tau$. For simplicity's sake, we impose that the marginal cost of the "old" technology (c') be non-decreasing, so it necessarily intersects the marginal revenues at a point such that $\hat{x} < Q_t < Q_\tau$, where Q_t is the production t and Q_τ with tax τ. Let n' be the marginal cost of the "new" technology.

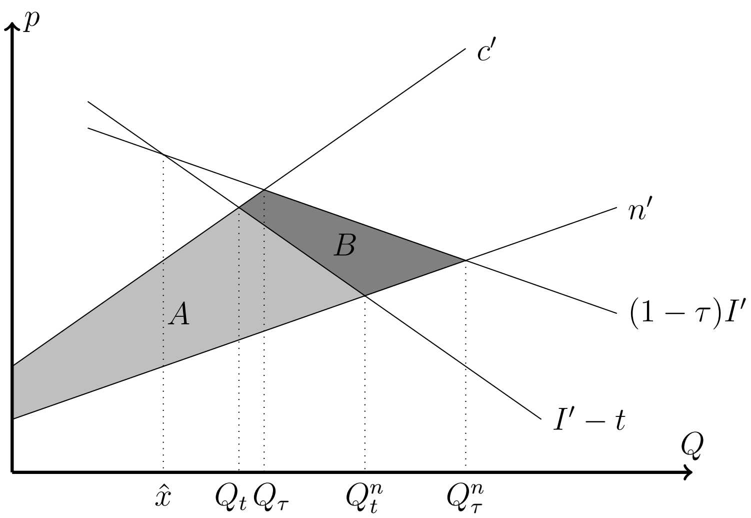

Figure 1: Two different marginal cost curves ($n' < c'$) in two tax regimes (t and τ). Note that Q_t^n can be greater or lower than Q_τ.

The monopolist will adopt the new technology with the unit tax, t, if

$$\int_0^{Q_t} (I' - t - n')dx + \int_{Q_t}^{Q_t^n} (I' - t - n')dx - F > \int_0^{Q_t} (I' - t - c')dx,$$

or

$$\int_0^{Q_t} (c' - n')dx + \int_{Q_t}^{Q_t^n} (I' - t - n')dx > F. \tag{5}$$

Similarly, with the *ad valorem tax*, the firm would adopt the new technology when

$$\int_0^{Q_t} (c' - n')dx + \int_{Q_t}^{Q_\tau} (c' - n')dx + \int_{Q_\tau}^{Q_\tau^n} ((1-\tau)I' - n')dx > F. \tag{6}$$

Remember that $Q_t < Q_\tau$ and $(1-\tau)I' > I' - t$ for any $x > Q_t$; consequently, $Q_t^n < Q_\tau^n$. Also, the monopoly maximizes its profit, c' is non-decreasing and I' is decreasing, which implies that $c' \geq I' - t$ for $x \geq Q_t$ and $(1-\tau)I' \geq n'$ for $x \leq Q_\tau^n$. Therefore, the left hand side in 6 is greater than the left hand side in 5, and that means that if the left hand side in 5 is higher than F, then the left hand side in 6 will also be greater than F. Hence, it is impossible that the firm adopts the new technology in the presence of the unit tax, but not under the *ad valorem* tax.

(c) Let's assume the firm adopts the new technology if taxes are *ad valorem*, but not unitary. We know that (i) $Q_\tau > Q_t$ and $Q_\tau^n > Q_\tau$, so that prices are lower and consumers are better off with *ad valorem* taxes. (ii) If the firm switches to the new technology under *ad valorem* taxation it must be that $\Pi_\tau^n > \Pi_\tau$ and, by hypothesis $\Pi_\tau > \Pi_t$, so the firm is better off since it has higher profits. (iii) By hypothesis, $G_\tau > G_t$. Since the marginal cost cannot be negative, the marginal revenue (which is decreasing) will be positive. Hence, revenue, pQ, increases and so does government collection. Also, the fixed cost taken on by the firm does not diminish welfare, as we saw in proposition 2. In sum the *ad valorem* tax is Pareto superior to the unit tax since consumers, the firm, and the government are all better off.

(d) The firm adopts the new technology under both tax schemes. We can see that (i) consumers are better off, since $c' > n'$ and $(1-\tau)I' > I' - t$ imply $Q_\tau^n > Q_t^n$, so that prices are lower with *ad valorem* taxes. (ii) The firm is better off because in (b) we proved that the increment in profits will be larger with *ad valorem* taxes than with unit taxes. But (iii) it is easy to find examples where government revenue decreases with *ad valorem* taxes. For instance, consider a case where $P = 10 - Q$, $c = 2$ with an *ad valorem* tax $\tau = 0.2$ that is is Pareto superior to $t = 1.4$. If the marginal cost decreases to $c = 0$, government revenue will be higher with t than with τ.

This concludes the proof. ■

As we saw in proposition 3, when a cost-reducing technology is adopted, government revenue may increase more under unitary than *ad valorem* taxes. If taxes were to be designed so that government revenue was to remain constant under both technologies, would the *ad valorem* tax be socially preferable? The answer is not clear, as we will see in proposition 4. That is, unit taxes may be better under some circumstances. To keep government revenue constant, we must consider a conditional tax that changes according to the technology implemented, which to our knowledge is not used in real life. Yet, it provides an interesting theoretical result.

Proposition 4. *Let's consider a monopoly with a linear demand. The monopoly can choose between two alternative technologies, both with constant marginal costs. A conditional tax that depends on the technology implemented by the firm is enacted so that the government revenue is the same independently of the tax scheme. Under these conditions welfare will be higher under unit taxes for some values of the fixed cost, F.*

Proof. Suppose a linear demand $P = a - Q$ and a monopolist with two available technologies. The first has fixed cost F and no variable costs, while the second technology has constant marginal cost c and no fixed costs. The government considers two possible conditional tax regimes, depending on the technology: unit tax $\{t_0, t_c\}$, and *ad valorem* tax $\{\tau_0, \tau_c\}$. The government first chooses the type of tax, and then the firm chooses the technology.

The government designs the taxes so that the revenue is the same in the four possibilities (c and τ, 0 and τ, c and t; and c and τ). From equations 1 through 4 we get government revenue for all four cases:

$$\text{Unit tax:} \qquad G_{t_0} = \frac{t_0(a-t_0)}{2}; \qquad G_{t_c} = \frac{t_c(a-c-t_c)}{2}. \tag{7}$$

$$\textit{Ad valorem:} \qquad G_{\tau_0} = \tau_0\frac{a^2}{4}; \qquad G_{\tau_c} = \frac{\tau_c}{4}\left(a^2 - \left(\frac{c}{(1-\tau_c)}\right)^2\right). \tag{8}$$

We want to find the following scenario. If the government enacts the *ad valorem* tax, then the firm prefers the technology with no fixed costs because $\Pi_{\tau_c} > \Pi_{\tau_0}$. When the government chooses the unit tax, then the firm chooses the technology with fixed costs, because $\Pi_{t_0} > \Pi_{t_c}$. Besides, in order to prove the proposition we need that welfare is

	τ_c	t_c	τ_0	t_0
P	280	292	230	244
Q	180	168	230	216
G	6048	6048	6048	6048
Π	28512	28224	28452	28256
W	50760	48384	60950	57632

Table 1: A conditional tax for a monopolist.

higher with the unitary tax. We provide an example below to show that this possibility is real. And this example proves the proposition.

The demand is $P = 460 - Q$; $c = 88$. The result of this example holds for any $F \in (18341, 18432)$, but let us suppose that $F = 18400$. The objective of the government is to raise $G = 6048$. Then, from equations 7 and 8, we get the following: $\tau_c = 0.12$, $\tau_0 = \frac{1512}{13225}$, $t_c = 36$ and $t_0 = 28$. In Table 1, we use equations 1, 2, 3 and 4 to find the quantities, Q, and profits, Π. The prices that consumers pay, $P = 460 - Q$, are easily obtained from the quantities. The values for the government, G, we already knew. Welfare comes from $W = (a - c)Q - Q^2/2$, or $W = aQ - Q^2/2 - F$.

It is clear from Table 1 that when the government chooses the *ad valorem* tax, the firm prefers a profit 28512 to 28452. And when the government prefers the unitary tax, the firm will choose the technology with fixed costs because it gets a profit 28256 rather than the profit with the other technology, 28224. Taking into account the incentives for the firm, welfare is higher with the unitary tax. Welfare would be even higher if the government could force the adoption of the fixed cost technology, but we suppose that the firm is the one that chooses its technology. ■

3 Oligopoly

According to Denicolo and Matteuzzi (2000) and Anderson et al. (2001), *ad valorem* taxes are welfare superior to unit taxes under Cournot competition in the presence of cost asymmetries. In this section, we will provide evidence that seems to be in contradiction with this result. The reason for this difference is that in our case firms can *choose* among different technologies. We will see how, under these conditions, we may find a different number of firms adopting the new technology depending on whether taxes are *ad valorem* or unitary. Let's start with a numerical example to illustrate this possibility and we will formalize later.

3.1 Numerical example

In this example we show that, in an asymmetric oligopoly, a unitary tax may be welfare superior to an *ad valorem* tax, if firms can choose the technology freely.

Let's consider a demand function such that $P = 70 - Q$ in a market with three firms that engage in Cournot competition. In this market, there is an existing technology, with costs $c(x) = 89 + 9x$; and a second (new) technology becomes available; this "new" technology has a fixed cost $F = 305$ and no variable costs. Let r denote the number of firms that adopt the new technology.

If the government imposes an *ad valorem* tax of 10%, the demand (as perceived by the firms) will be $p = (1 - \tau)(70 - Q) = 63 - 0.9Q$. Solving Cournot[3], we obtain the figures shown in Table 2. Each column represents the Cournot solution for a given number of adopters. For example, when $r = 1$, only one firm adopts the new technology. In this case, and with $\tau = 10\%$,the price will be 22.5; total quantity, 47.5; profits for firms with the old technology will amount to 51.6, while profits for firms with the new technology will amount to 150.6; total welfare will be 1488.9, and government revenue will be 106.87.

	$r = 0$	$r = 1$	$r = 2$	$r = 3$
P	25	22.5	**20**	17.5
p	22.5	20.25	**18**	15.75
Q	45	47.5	**50**	52.5
Π_{old}	113.5	51.6	**1**	−38,37
Π_{new}	–	150.6	**55**	−29,37
W	1465.5	1488.9	**1461**	1381,87
G	112.5	106.87	**100**	91,87

Table 2: Cournot oligopoly with demand $p = 70 - Q$, *ad valorem* tax $\tau = 0.1$, and two cost functions $c_1(x) = 89 + 9x$ and $c_2(x) = 305$. The parameter r indicates the number of adopters of the new technology.

As we can see from Table 2, two firms will switch to the new technology in this example. To see this, let's compare the profit levels for firms with the various numbers of adopters. If one firm was to adopt the new technology ($r = 1$), its profits would increase from 113.5 to 150.6. If another firm switches ($r = 2$), its profits would go from 51.6 to 55. If the last firm switches ($r = 3$), its profits would go from 1 to negative, which is not a feasible solution. Then, in the equilibrium (noted in bold), two out of the three firms will adopt the new technology.

Following Denicolo and Matteuzzi (2000) and Anderson et al. (2001), this *ad valorem* tax should be superior to unitary taxation in an asymmetric oligopoly. However, in the example below we will show precisely the opposite result and see how imposing a unit tax may actually increase welfare in this situation, rather than decrease it. The reason for this discrepancy is that in those models firms have not the possibility of choosing among different technologies.

Let us now assume that the government imposes a unit tax of $t = 2.2$, so the perceived demand function would be $p = 70 - 2.2 - Q = 67.8 - Q$. To compare the possible solutions ($r = 0, r = 1, r = 2, r = 3$) under the unit tax, we construct Table 3, similar to Table 2, that we used for the case with an *ad valorem* tax. In Table 3 we can see that in the optimal solution ($r = 1$), one firm adopts the new technology since $\Pi_{old}(r = 0) = 127.09 < 155.1 = \Pi_{new}(r = 1)$ and $\Pi_{new}(r = 2) = 63.64 < 66 = \Pi_{old}(r = 1)$.

A comparison of Tables 2 and 3 provides several insights. First, more firms adopt the new technology under *ad valorem* taxation ($r = 2$) than under unit taxes ($r = 1$). Second, both welfare and government revenues are higher under unitary taxation ($G = 101.97$ and $W = 1463.2$) than under *ad valorem* taxation ($G = 100$ and $W = 1461$). Hence, when firms are free to choose their technology, the superiority of *ad valorem* taxation over unitary taxation in an asymmetric Cournot oligopoly may no longer be maintained.

[3]As it is well known, for a Cournot oligopoly with linear demand $p = a - bQ$ and n firms with constant marginal costs, c_i, the price and quantities in equilibrium are $p = \frac{a + \sum c_i}{1+n}$ and $q_i = \frac{p - c_i}{b}$

	$r=0$	$r=1$	$r=2$	$r=3$
P	25.9	**23.65**	21.4	19.15
p	23.7	**21.45**	19.2	16.95
Q	44.1	**46.35**	48.6	50.85
Π_{old}	127.09	**66**	15.04	−25.8
Π_{new}	–	**155.1**	63.64	−17.7
W	1450.7	**1463.2**	1430.22	1351.64
G	97.02	**101.97**	106.92	111.87

Table 3: Cournot oligopoly with demand $p = 70 - Q$, unitary tax $t = 2.2$, and two cost functions $c_1(x) = 89 + 9x$ and $c_2(x) = 305$. The parameter r indicates the number of adopters of the new technology.

In the next sections, we will formalize the intuition provided in this example. First, we will consider the case of two technologies without taxes, and then we will introduce taxes.

3.2 Two technologies without taxes

In this subsection we see that excess adoption of a lower marginal cost technology may be pervasive in oligopolies.

Let's consider a market with a demand function given by $p = a - bQ$ and n firms engaging in Cournot competition. Firms have two technologies at their disposal. The "old" technology has a cost function such that $c(x) = K + hx$, while the "new" technology has costs given by $c(x) = F + K + cx$, with $c < h$ and $K, F > 0$. Firms will adopt the new technology, with lower marginal costs, if it's profitable for them. As before, $r \in \{0, \dots, n\}$ will denote the number of adopters. The equilibrium values are given by[4]:

$$p(r) = \frac{a + rc + (n-r)h}{1+n}; \; q_h(r) = \frac{p(r) - h}{b}; \; q_c(r) = \frac{p(r) - c}{b}. \tag{9}$$

From here we get

$$q_h(r) > 0 \Leftrightarrow a + rc - (r+1)h > 0. \tag{10}$$

Profits for each type of firm will be given by:

$$\Pi_h(r) = \frac{1}{b}\left(\frac{a - r(h-c) - h}{1+n}\right)^2 - K = B_h(r); \tag{11}$$

$$\Pi_c(r) = \frac{1}{b}\left(\frac{a + (n-r)(h-c) - c}{1+n}\right)^2 - K - F = B_c(r) - F. \tag{12}$$

Let's calculate now the threshold values for adoption of the new technology from the firm's point of view. Suppose there are already $r-1$ adopters. An additional firm will adopt the new technology if its profits are higher; that is, $\Pi_h(r-1) < \Pi_c(r)$. Suppose that now there are r adopters. No firm will not adopt the new technology if its profits are higher were it to stay with the old technology; that is, $\Pi_h(r) > \Pi_c(r+1)$.

[4]Where h denotes the old technology and c the new one.

Therefore, joining these two conditions and solving for F, we will find the equilibrium number of adopters (r) when F is such that:

$$B_c(r+1) - B_h(r) < F < B_c(r) - B_h(r-1).$$

Let $f(r) = B_c(r) - B_h(r-1)$, so that the expression above becomes

$$f(r+1) < F < f(r), \tag{13}$$

where

$$f(r) = \frac{(h-c)\left[2n(a-c) + (h-c)n(n-2r)\right]}{b(n+1)^2}. \tag{14}$$

Let's calculate now the threshold values for adoption of the new technology from society's point of view. Let $W(r)$ be total welfare when there are r adopters. Then,

$$W(r) = \frac{1}{2b}\left(a^2 - p(r)^2\right) - rcq_c(r) - (n-r)hq_h(r) - nK - rF.$$

Society would prefer an additional firm to adopt the new technology if the social welfare associated with r adopters is higher than the social welfare of having $r-1$ adopters. That is, society would prefer r adopters to $r-1$ if $W(r) > W(r-1)$; and would prefer r adopters to $r+1$ if $W(r) > W(r+1)$. Let us call $V(r) = W(r) + rF$.

Therefore, joining these two conditions and solving for F, we will find the equilibrium number of adopters (r) when F is such that:

$$V(r+1) - V(r) < F < V(r) - V(r-1)$$

Let $s(r) = V(r) - V(r-1) = W(r) - W(r-1) - F$. Then, the optimal number of adopters from society's point of view is r whenever

$$s(r+1) < F < s(r). \tag{15}$$

In order to find an explicit expression for $s(r)$, notice, from equation 9, that $p(r) - p(r-1) = \frac{h-c}{1+n}$ and $q_c(r) - q_c(r-1) = q_h(r) - q_h(r-1) = \frac{h-c}{b(1+n)}$. Then rearranging $W(r) - W(r-1) - F = s(r)$, yields:

$$s(r) = \frac{(h-c)\left[(n+2)(a-c) + (h-c)\left((n+1)^2 - r(2n+3) - \frac{1}{2}\right)\right]}{b(n+1)^2} \tag{16}$$

Comparing the threshold values for society and firms we can conclude that when $F \in [s(r), f(r)]$, there are too many adopters from society's point of view, and when $F \in [f(r), s(r)]$, there are too few adopters from society's point of view.

We can calculate

$$f(r) - s(r) = \frac{(h-c)\left[(a-c)(n-2) + (h-c)\left(3r - 2n - \frac{1}{2}\right)\right]}{b(n+1)^2}. \tag{17}$$

and enunciate the following proposition

Proposition 5. *We consider a Cournot market with linear demand, $p = a - Q$, and $n \geq 1$ firms that may choose between these two cost functions: $c_1(q) = K + hq$ and $c_2(q) = F + K + cq$, with $0 \leq c < h$ and K, $F > 0$. When $n = 1$, there is insufficient adoption of the low-cost technology for some values of F, but never excess adoption. If $n = 2$, there is both insufficient and excess adoption. When $n \geq \frac{4a+c-5h}{2a+2c-4h}$ there is only excess adoption.*

Proof. Remember that there is excess adoption when $F \in [s(r), f(r)]$, and insufficient adoption when $F \in [f(r), s(r)]$. From equation 17, we can see first that for $n = 1$, $f(1) - s(1) < 0$. For $n = 2$, $f(1) - s(1) < 0$, but $f(2) - s(2) > 0$; so that if only one firm adopts the low cost technology, there can be insufficient adoption, but if the two firms adopt the new technology, there may be excess adoption. Let us consider $n > 2$. It is clear that $f(r) - s(r)$ is increasing in r. Then, any value of n that satisfies $f(1) - s(1) > 0$ will result in excess adoption for any other value of r. From equation 17, the value of n that satisfies this condition is given by $n \geq \frac{4a+c-5h}{2a+2c-4h}$. For instance, when $n = 3$, as long as $3 \geq \frac{4a+c-5h}{2a+2c-4h}$, that is $2a \geq 7h - 5c$, only excess adoption is possible. ■

At this point, a numerical example may be useful to clarify. Let's consider a demand function such that $p = 12 - Q$ in a market with two firms that engage in Cournot competition. In this market, there is an existing technology (which implies variable costs $h = 3$ but no fixed costs), and a second (new) technology becomes available (this "new" technology has a fixed cost F and no variable costs, $c = 0$). Let r denote the number of firms that adopt the new technology. Solving, we get the results provided in Table 4.

	$r = 0$	$r = 1$	$r = 2$
p	6	5	4
q_{old}	3	2	–
q_{new}	–	5	4
Π_{old}	9	4	–
Π_{new}	–	$25 - F$	$16 - F$
W	36	$53.5 - F$	$64 - 2F$

Table 4: Cournot oligopoly with demand $p = 12 - Q$, and two cost functions $c_1(x) = 3x$ and $c_2(x) = F$. The parameter r indicates the number of adopters of the new technology.

From Table 4 we can find the values of F that would cause an additional firm to adopt the new technology. We represent this situation in Figure 3. This graph consists of three lines. Along the middle one, we find the threshold values of F that determine the optimal number of adopters of the new technology from the point of view of society (10.5 and 17.5) and firms themselves (12, 16). As we can see in the upper line, from the point of view of firms, it would be optimal to have 0 adopters for values of $F > 16$, 1 adopter for $F \in [12, 16]$ and 2 adopters for $F < 12$. Society, however, would prefer to have 0 adopters if $F > 17.5$; 1 if $F \in [10.5, 17.5]$; and 2 if $F < 10.5$ (see bottom line). This means there are two sets of values of F for which the optimal decision for r is different from the point of view of firms and society.

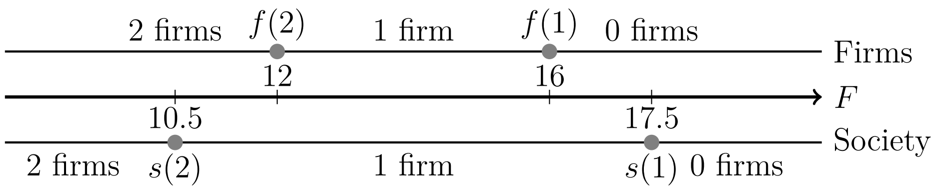

Figure 2: Optimal thresholds for technology adoption.

(i) For values of $F \in [16, 17.5]$, society would prefer $r = 1$, but we will see no firms adopting the new technology. The reason for this shortage is similar to that of Proposition 2 in the case we saw before for monopoly. That is, although society benefits (W increases if r changes from 0 to 1), the increase in revenues for the firm is not enough to compensate the increase in fixed costs (profits decrease from 9 to $25 - F$).

(ii) For values of $F \in [10.5, 12]$, society would prefer $r = 1$, but we will see two firms adopting the new technology. The reason for this excess is *business stealing*. That is, although the new adopter benefits (profits increases from 4 to $16 - F$ if r changes from 1 to 2), that increase in profits is not enough to compensate the increase in fixed costs from society's point of view (W decrease from $53.5 - F$ to $64 - 2F$). In the end, part of the additional revenue for the new adopter comes from clients "stolen" from the other firm.

3.3 Taxes

We will see that the *ad valorem* tax is more prone to excess adoption than the unitary tax. This may render the unitary tax welfare superior to the *ad valorem* tax.

As we have seen, one of the problems of technology choice is that it may lead to excess of adoption of a particular technology. In the monopoly case (see proposition 3), we concluded that *ad valorem* taxes give more incentives to adopt a low marginal cost technology than unitary taxation. Intuitively, this result may be extended to oligopolies, since any firm in an oligopoly acts as a monopoly for the residual demand. Since *ad valorem* taxes favor excess of entry, when we face this problem in the oligopoly, it seems straightforward that the solution offered by unit taxes (which have the opposite effect) may bring us closer to the option preferred by the society. In this section, we will study this possibility in more detail.

In particular, we will be concentrating on cases where the number of adopters is different depending on the tax regime implemented, because (as Denicolo and Matteuzzi (2000) and Anderson et al. (2001) showed) if the number of adopters is the same under both regimes, *ad valorem* taxes will be superior. Different patterns of technology choice is a necessary (but not sufficient) condition for unit tax superiority.

Let's consider a demand function such that $P = a - Q$ in a market with n firms that engage in Cournot competition. In this market, there is an existing technology (which implies a variable cost $c(x) = K + hx$) and a new technology becomes available (this "new" technology –it may be an improvement in the current technology– has higher fixed costs but lower variable costs, such that $c(x) = F + K + cx$, with $c < h$ and $K \geq 0$, $F > 0$)[5]. Let r denote the number of firms that adopt the new technology.

After an *ad valorem* tax, τ, is imposed the demand perceived by the firms becomes $p = (1 - \tau)(a - Q)$. Using equations 9, 11, 12, 13 and 14, substituting a for $a(1 - \tau)$, and b for $1 - \tau$, we find that equation 13 becomes

$$f(\tau, r) = \frac{(h - c)\left[2n(a(1 - \tau) - c) + (h - c)n(n - 2r)\right]}{(1 - \tau)(n + 1)^2}. \tag{18}$$

Doing the same with a unitary tax, t, the perceived demand is $p = a - t - Q$, and substituting now a for $a - t$ and b for 1 we obtain:

$$f(t, r) = \frac{(h - c)\left[2n(a - t - c) + (h - c)n(n - 2r)\right]}{(n + 1)^2}. \tag{19}$$

[5]The parameter K allows us to set the number of firms in the long run equilibrium, by adjusting the profits very near to zero.

Now, to be able to compare, we will find the level of unit taxes that yields the same government revenue with r adopters ($G(t, r)$) as the *ad valorem* tax would with $r+1$ adopters ($G(\tau, r+1)$). That is, we find t such that $G(t, r) = G(\tau, r+1)$ for a given τ.

Let's first calculate $G(\tau, r+1) = \tau P(\tau, r+1)Q(\tau, r+1)$, where $P(\tau, r+1) = \frac{p(\tau, r+1)}{1-\tau}$. From equation 9 we obtain

$$p(\tau, r+1) = \frac{a(1-\tau) + (r+1)c + (n-r-1)h}{1+n}.$$

We turn now to calculating $G(t, r) = t(a - t - p(t, r))$, and from equation 9 we obtain

$$p(t, r) = \frac{a - t + rc + (n-r)h}{1+n}.$$

To find the value of t such that $G(t, r) = G(\tau, r+1)$, we solve for t to get:

$$t(\tau, r) = \frac{D - \sqrt{D^2 - 4n(n+1)G(\tau, r+1)}}{2n}, \tag{20}$$

where $D = n(a-h) + r(h-c)$. Note that $t(\tau, r)$ is continuous where it exists.

We can enunciate the following

Proposition 6. *Suppose a Cournot oligopoly, with demand $P = a - Q$, and n firms that choose freely between technologies with cost functions $C(x) = K + hx$ and $C(x) = F + K + cx$ (with $0 \leq c < h$ and $K, F > 0$). If $n > \frac{4a+c-5h}{2a+2c-4h}$, then for a small enough* ad valorem *tax, τ, and for each $r \leq n/2 - c/(h-c)$, there is a unitary tax, $t(\tau, r)$, revenue equivalent to τ, such that $t(\tau, r)$ is welfare superior to τ for some intervals of F.*

Proof. Using equation 13 for τ and t we can represent graphically adoption of the new technology under both tax schemes.

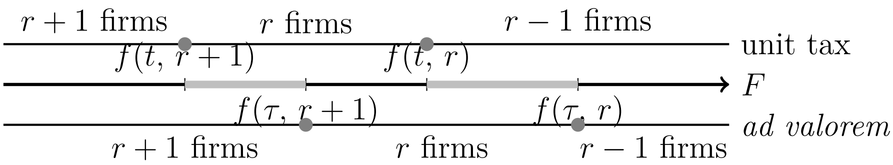

Figure 3: Thresholds for technology adoption under unit and *ad valorem* tax regimes.

As we can see in the graph, $r+1$ firms will adopt the new technology under *ad valorem* taxes if $f(\tau, r+2) < F < f(\tau, r+1)$. On the other hand, r firms will adopt the new technology under unit taxes when $f(t, r+1) < F < f(t, r)$. Hence, whenever $F \in [f(t, r+1), f(\tau, r+1)]$ there will be $r+1$ adopters with the *ad valorem* tax and r adopters with the unit tax.

We will see next that as τ increases, so does the interval. When $\tau = t = 0$, it is clear that $f(t, r+1) = f(\tau, r+1)$. On the other hand, as τ grows the lower bound ($f(t, r+1)$) of the interval decreases while its upper bound ($f(\tau, r+1)$) increases. To see that, note that $\frac{\partial f(t,r)}{\partial \tau} < 0$ because $\frac{\partial f(t,r)}{\partial t} < 0 \; \forall r$ and $\frac{\partial t}{\partial \tau} > 0$.[6] On the other hand, $\frac{\partial f(\tau,r)}{\partial \tau} = \frac{n(h-c)}{(n+1)^2(1-\tau)^2}\left((h-c)(n-2r) - 2c\right) > 0$ if $r \leq n/2 - c/(h-c)$.

[6]If *ad valorem* taxes increase, unit taxes must increase as well to keep government revenue constant; that is, $\frac{\partial t(\tau,r)}{\partial \tau} > 0$, because $\frac{\partial G(\tau,r+1)}{\partial \tau} > 0$.

We must now prove that there is some $\tau^* > 0$ such that welfare is higher with r adopters and unit tax than with $r+1$ adopters and an *ad valorem* tax for any $\tau < \tau^*$. For that, we consider $W(t, r) - W(\tau, r+1)$ as a function of τ. We know, on the one hand, that for $\tau = t = 0$, $W(\tau, r+1) = W(t, r+1)$. We also know (from equation 17) that $W(0, r+1)$ will be lower than $W(0, r)$ for $n > \frac{4a+c-5h}{2a+2c-4h}$, because there is excess adoption of the lower marginal cost technology for any $r \geq 1$. We can then deduce that since $W(t, r) - W(\tau, r+1)$ is a continuous function and takes positive values for $\tau = 0$, it is positive for some interval $[0, \tau^*]$, with $\tau^* > 0$. This completes the proof. ■

Proposition 6 states the possibility of finding welfare superiority of unit taxes over *ad valorem* taxes in a Cournot asymmetric oligopoly, without considering its likelihood. We turn now to the study of this question. As we can see from equation 20, $t(\tau, r)$ is a function of all the parameters of the model; i.e., a, h, c, n and r. The resulting expression of $f(t(\tau, r), r)$ (even in simplified form) is unmanageably long, making its analytical study difficult and notationally cumbersome. To circumvent this problem, we will use graphical analysis to determine the intervals for which unit taxes are superior to *ad valorem* taxes in the space τ - F. In B, we present figures 4, 5, 6 and 7. Each graph corresponds to a demand $p = 100 - Q$, and several values of the number of firms n, and the marginal costs, c and h. In each graph the shaded area for any value of τ represents the intervals of F where unit taxes are welfare superior to *ad valorem* taxes.

Let's describe the axis and different functions of the graphs. Along the horizontal axis we represent the *ad valorem* tax (τ) and along the vertical axis we have the fixed cost (F). In each graph there will be three functions, for any value of r:

(i) The upper line in every area is $f(\tau, r)$, which represents the value of F that changes the equilibrium number of adopters from $r-1$ to r under *ad valorem* taxes.

(ii) The lower line in every area is $f(t, r)$, which represents the value of F that changes the equilibrium number of adopters from $r-1$ to r under under unit taxes.

(iii) The right-most line in every area is a curve that represents the value of F for which society is indifferent between r adopters with *ad valorem* taxes and $r-1$ adopters with revenue equivalent unitary taxes. The equation of this curve is given by: $W(t, r) - W(\tau, r+1) = 0$. Calling $Q(\tau, r+1) = Q_\tau$ and $q(\tau, r+1) = q_\tau$, we get

$$W(\tau, r+1) = aQ_\tau - \frac{Q_\tau^2}{2} - (r+1)cq_\tau^c - (n-(r+1))hq_\tau^h - nK - (r+1)F. \tag{21}$$

And, with $Q(t, r) = Q_t$, $q(t, r) = q_t$ and $W(t, r) = W(t(\tau, r), r)$, we get

$$W(t, r) = aQ_t - \frac{Q_t^2}{2} - rcq_t^c - (n-r)hq_t^h - nK - rF. \tag{22}$$

In the graphs, the shaded areas represent values of τ and F such that unitary taxes are superior to *ad valorem* taxes for a given set of parameters, linear demand and constant marginal costs. These areas share a common feature: in them, there would be one more adopter of the new technology if taxes are *ad valorem* (as opposed to unitary).

As we can see in the graphs, for a given set of parameter values, areas where *ad valorem* taxes would be preferable co-exist with areas where unit taxes are the better alternative, so it is difficult to establish a general rule of superiority. The areas seem to grow narrower as F (the fixed cost associated with adopting the new technology) decreases, probably because as (Anderson et al., 2001, p. 238) argued, *ad valorem* taxation "leads to relatively less production from high-cost firms." The number of areas and the distance between them largely depends on the assumptions of the particular model.

4 Final remarks

This paper has presented non trivial examples of cases where the social optimality of *ad valorem* taxes is reversed, with unitary taxes being a better alternative. All the models rely on technology choice. By this, we understand any change in technology, either small or large. For example, small technology changes could include a case where all firms share the same technology, but small differences exist in marginal and fixed cost due to previous choices.

In general, our findings suggest that, in monopolies where technological change causes a decrease in marginal costs, *ad valorem* taxes remain superior. However, equivalent unit taxes could be more beneficial in monopolies where technological change results in a price increase. We see this as unlikely, but for example mini-mills in the steel market have higher marginal costs and lower fixed costs than blast furnaces.

Stronger results are given for oligopolies where firms face a technology choice. We find situations where *ad valorem* taxes (but not unit taxes) lead to excessive technology adoption, resulting in the superiority of unit taxes.

This paper does not pretend to establish the likelihood of the unit tax superiority in the presence of technology choice. Instead, we illustrate that this superiority is possible. We think that this is important because it contradicts the established wisdom of the superiority of *ad valorem* taxes in asymmetric Cournot oligopolies.

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Appendix A Code

This code, written in *maxima*, generates the graphs in B. The code is written for a case with eight firms. The number of lines should be modified if using a different number of firms (lines should be deleted to use less firms or added to incorporate a greater number of firms). In the code, the second line sets the values for the basic parameters of the models. Lines 3 to 20 come from equations 9 to 22. Two notes are in order. First, X in the first line is used to find the intersection, in lines 21 and 22, between $W(t, r) - W(\tau, r+1)$ and $f(t, r)$, or $f(\tau, r)$. If for some value of X no intersection point is found, then other values should be provided. Second, when $f(\tau, r) < f(t, r)$, the area does not exists and the graph is not generated. In this case, the last lines should be deleted until the graph is generated.

```
1   kill(all)$load(draw)$ X:0.4;
2   [a, h, c, n]:[100, 2, 0, 8]$
3   pz(z,r):=(a*(1-z)+r*c+(n-r)*h)/(1+n)$
4   Pz(z,r):=pz(z,r)/(1-z)$
5   Qz(z,r):=a-Pz(z,r)$
6   qzc(z,r):=(pz(z,r)-c)/(1-z)$
7   qzh(z,r):=(pz(z,r)-h)/(1-z)$
8   Gz(z,r):=z*Pz(z,r)*Qz(z,r)$
9   D(r):=n*(a-h)+r*(h-c)$
10  t(z,r):=(D(r)-sqrt(D(r)^2-4*n*(n+1)*Gz(z,r+1)))/(2*n)$
11  pt(z,r):=(a-t(z,r)+r*c+(n-r)*h)/(1+n)$
12  Pt(z,r):=pt(z,r)+t(z,r)$
13  Qt(z,r):=a-Pt(z,r)$
14  qtc(z,r):=pt(z,r)-c$
15  qth(z,r):=pt(z,r)-h$
16  ft(z,r):=(h-c)*n*(2*(a-t(z,r)-c)+(h-c)*(n-2*r))/(n+1)^2$
17  fz(z,r):=(h-c)*n*(2*(a*(1-z)-c)+(h-c)*(n-2*r))/((1-z)*(n+1)^2)$
18  Wt(z,r):=a*Qt(z,r)-Qt(z,r)^2/2-r*c*qtc(z,r)-(n-r)*h*qth(z,r)$
19  Wz(z,r):=a*Qz(z,r)-Qz(z,r)^2/2-r*c*qzc(z,r)-(n-r)*h*qzh(z,r)$
20  W(z,r):=Wz(z,r+1)-Wt(z,r)$
21  for r:0 step 1 thru n-1 do a[r+1]:find_root(ft(z,r+1)-W(z,r),z,0,X);
22  for r:0 step 1 thru n-1 do b[r+1]:find_root(fz(z,r+1)-W(z,r),z,0,X);
23  draw2d(grid=true,terminal='eps,
24  filled_func=ft(z,1),fill_color=gray, explicit(fz(z,1),z,0,a[1]),
25  filled_func=fz(z,1),fill_color=gray, explicit(W(z,0),z,a[1],b[1]),
26  filled_func=ft(z,2),fill_color=gray, explicit(fz(z,2),z,0,a[2]),
27  filled_func=fz(z,2),fill_color=gray, explicit(W(z,1),z,a[2],b[2]),
28  filled_func=ft(z,3),fill_color=gray, explicit(fz(z,3),z,0,a[3]),
29  filled_func=fz(z,3),fill_color=gray, explicit(W(z,2),z,a[3],b[3]),
30  filled_func=ft(z,4),fill_color=gray, explicit(fz(z,4),z,0,a[4]),
31  filled_func=fz(z,4),fill_color=gray, explicit(W(z,3),z,a[4],b[4]),
32  filled_func=ft(z,5),fill_color=gray, explicit(fz(z,5),z,0,a[5]),
33  filled_func=fz(z,5),fill_color=gray, explicit(W(z,4),z,a[5],b[5]),
34  filled_func=ft(z,6),fill_color=gray, explicit(fz(z,6),z,0,a[6]),
35  filled_func=fz(z,6),fill_color=gray, explicit(W(z,5),z,a[6],b[6]),
36  filled_func=ft(z,7),fill_color=gray, explicit(fz(z,7),z,0,a[7]),
37  filled_func=fz(z,7),fill_color=gray, explicit(W(z,6),z,a[7],b[7]),
38  filled_func=ft(z,8),fill_color=gray, explicit(fz(z,8),z,0,a[8]),
```

```
filled_func=fz(z,8),fill_color=gray, explicit(W(z,7),z,a[8],b[8]))$
```

Appendix B Graphs

Figures 4, 5 and 6 represent a Cournot oligopoly with 3, 6, and 8 firms respectively. In all cases, the demand is given by $p = 100 - Q$, and there are two cost functions available: $c_1(x) = K + hx$ and $c_2(x) = F + K + cx$. The figures have four panels, representing different sets of parameter values. The panel in the NW corner corresponds to parameter values $c = 0$ and $h = 2$; $c = 0$ and $h = 12$ in the NE; $c = 10$ and $h = 12$ in the SW; and $c = 50$ and $h = 52$ in the SE. The value of K is arbitrary and does not play any role in the graphs, but it may be used to guarantee that n is the number of firms in the long run.

In every graph, the *ad valorem* tax (τ) is represented along the horizontal axis, and the fixed cost F is represented along the vertical axis. At any $\tau = \tau_0$, the shaded areas indicate the values of F such that welfare will be greater with the unit tax than with the *ad valorem* tax. The reason is that in those areas r firms enter with *ad valorem* tax, and $r - 1$ firms with unit tax. For instance, the first shadowed area (starting from high values of F) corresponds to the case where only one firm adopts with *ad valorem* tax, but no firm adopts the new technology with unit tax ($r = 1$). As F decreases for $\tau = \tau_0$ (that is, as we move downwards along the line given by $\tau = \tau_0$), the second shadowed area corresponds to the case $r = 2$, etc.

As we can see comparing figures 4, 5 and 6, the graphs seem to indicate that as the number of firms in the market increases, so does the likelihood of finding a reversal of the superiority of the *ad valorem* tax. The other parameters, h and c, also influence the shape of the areas.

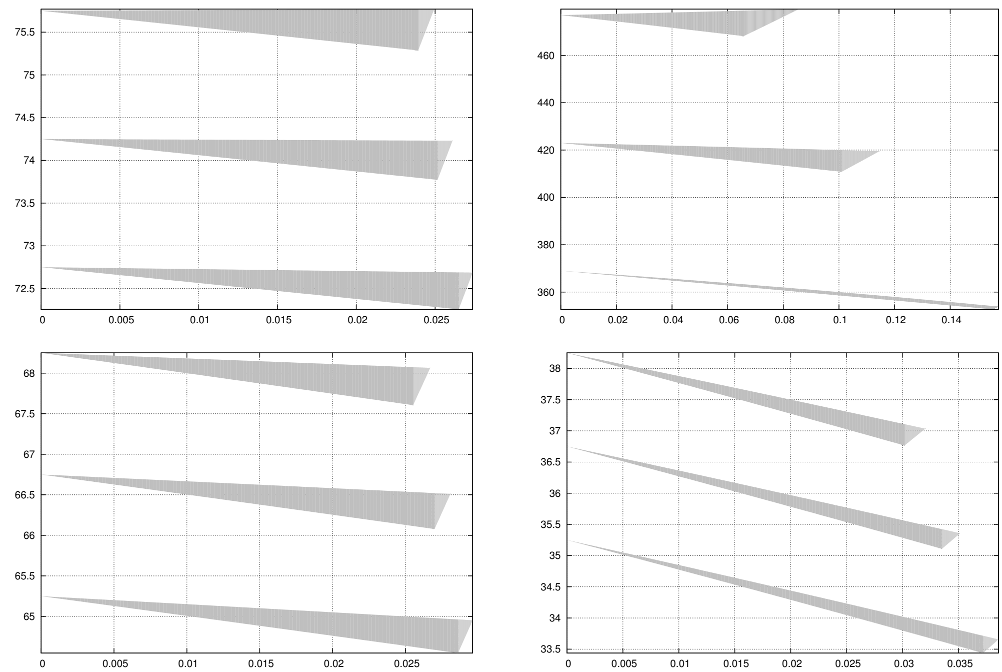

Figure 4: Cournot oligopoly with three firms. The *ad valorem* tax (τ) is represented along the horizontal axis, and the fixed cost F is represented along the vertical axis. For every value of τ, the shaded area represents the values of F with unit tax superiority. In the graph each of the three functions is represented by a contour of the shaded area. The top outline of the shaded area represents $f(\tau, r)$. The bottom outline represents $f(t, r)$, and the one on the right is $W(t, r) - W(\tau, r+1) = 0$.

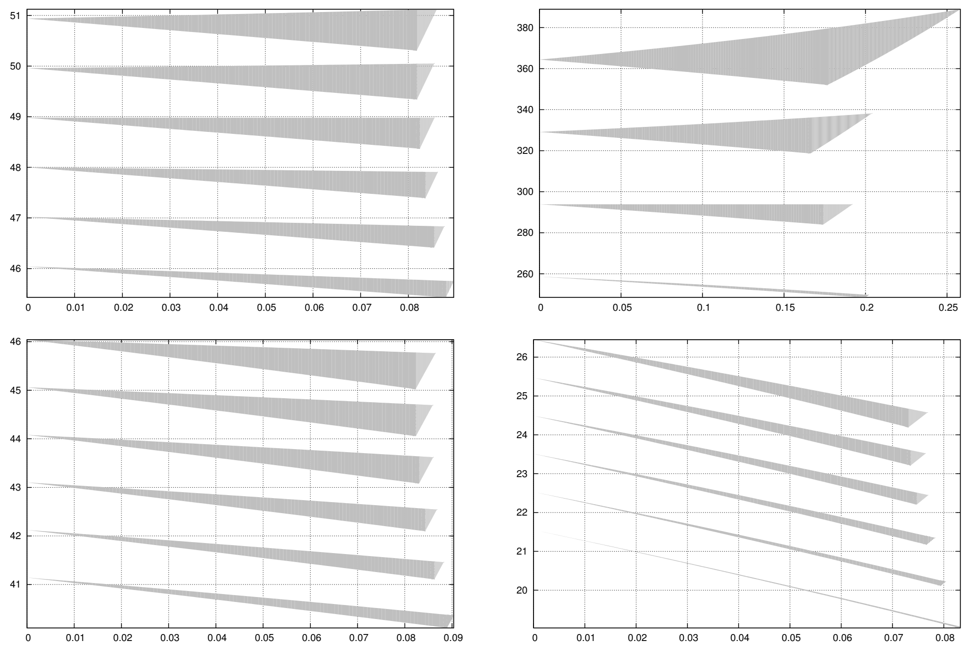

Figure 5: Cournot oligopoly with six firms. Observe that in the NE panel there are only four shaded areas; the reason is that when $r = 5, 6$, $f(\tau, r) < f(t, r)$ for any $\tau > 0$.

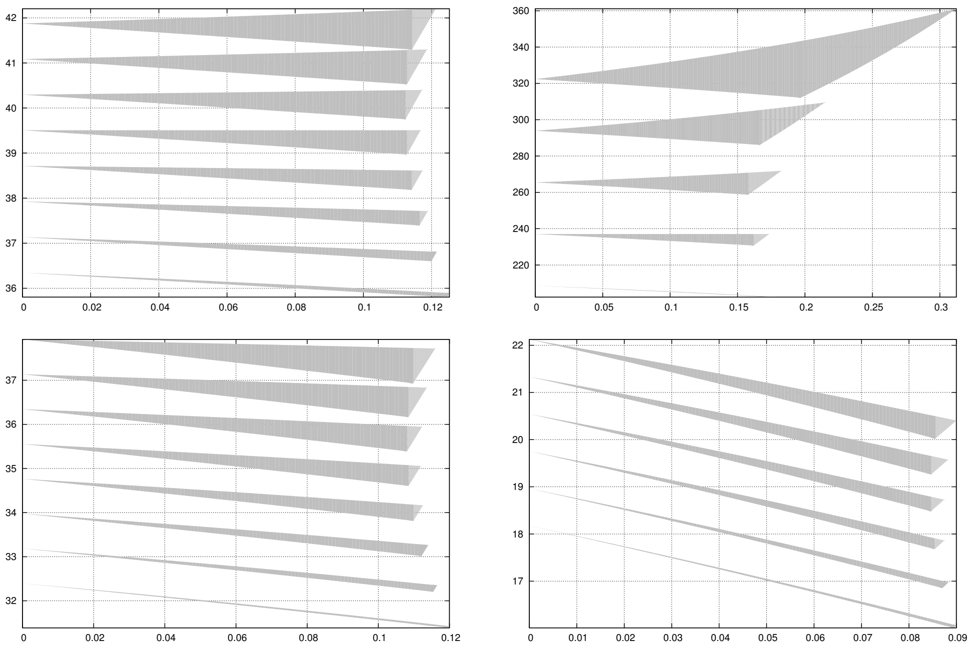

Figure 6: Cournot oligopoly with eight firms. Observe that in the NE panel there are only five shaded areas, and in the SE panel there are only six areas. The reason again is that $f(\tau, r) < f(t, r)$.

Figure 7 represents an oligopoly with a structure very similar to that of the NW panel in the previous figures (demand $p = 100 - Q$, $c = 0$ and $h = 2$), but with a larger number of firms. This figure is made with the code in A, but completing the last lines appropriately. In the graph, the panel on the left represents an oligopoly with 16 firms, and the panel on the right an oligopoly with 24 firms. The numbers in the shaded areas indicate the number of firms that adopt the low cost technology under *ad valorem* taxes. The overlapping of shaded areas means that if r firms enter in the presence of unit taxes, $r + n$ firms (with $n > 1$) enter with *ad valorem* taxes. Note that in these overlapping areas the function in equation 20 does not hold, since the function is valid only if when r firms adopt the new technology with unitary taxes, $r + 1$ firms do so with *ad valorem* taxes. However, from a welfare point of view, this situation seems to favor unit taxes even more than the case we consider in our model.

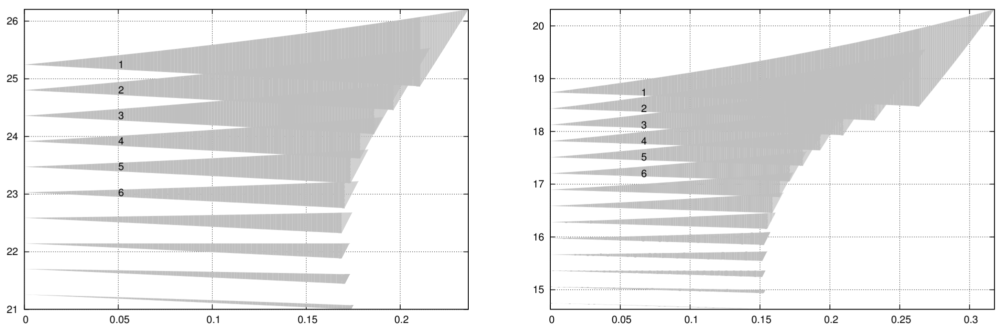

Figure 7: Cournot oligopoly with 16 firms (left panel), and 24 firm (right panel). As before, with $n = 16$ firms, if $r > 10$, then $f(\tau, r) < f(t, r)$. The same happens with $n = 24$ firms when $r > 14$.