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Regime switching models of hedge fund returns

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KEYWORDS: Markov switching ARMA-GARCH, forecasting performance

JEL CLASSIFICATION: C13, C15, G32

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The objective of this paper is to analyze the nonlinear behavior of hedge funds returns and assess the forecasting performance of different regime-switching models when applied to hedge funds data. Many authors have argued that nonlinear processes model better the behavior of different financial variables than the linear ones. In particular, Markov regime-switching models have been shown to properly capture the dynamics of several financial assets data series which occasionally exhibit periodical breaks in their behavior associated with events such as financial crises or abrupt changes in government policy. For example, many financial assets present an apparent tendency to behave quite differently during economic downturns due to the sudden changes in fundamentals (e.g. Ang and Bekaert, 2002, Garcia et al., 2003 and Dai et al., 2003). On the other hand, Markov regime-switching models possess other appealing features since they can better capture fat tails, asymmetries, autocorrelation, volatility clustering or mean reversion in financial asset series. Building on the seminal work of Hamilton (1989) regime-switching methods have been widely used in a variety of contexts: to model stock market returns (e.g. Hamilton and Susmel, 1994, Gray, 1996, Kim et al., 1998, Ang and Bekaert, 2002, Ang and Chen, 2002 and Chauvet and Potter, 2000), commodities prices (e.g. Deng, 2000, de Jong, 2005 and Chen and Forsyth, 2008), exchange rates dynamics (e.g. Engel and Hamilton, 1990, Engel, 1994, Bollen et al., 2000, Dewachter, 2001, Marsh, 2000 and Dacco and Satchell, 1999), etc.

On the other hand, motivated by the recent spectacular growth of the hedge funds industry, a large body of literature has focused on modeling hedge funds returns. In the last 20 years this industry has grown from a small number of funds to over 10000 hedge funds and funds of hedge funds managing assets of almost 1.8 trillion (see Figure 1). On many occasions Hedge Funds have made headlines being the protagonist of huge losses (e.g. LTCM, Bearn Stearn, Amaranth), have been accused of posing systemic risk, manipulating prices, being threat for global stability. It is not clear, for example, to what extent hedge funds, through their excessive use of leverage, large and concentrated positions in derivatives or short selling of the stocks of financial firms in troubles, have contributed to deepen the current financial crisis. Moreover, the hedge fund industry itself has experienced unprecedented losses and withdraws of capital during the whole year 2008. Hedge Fund Research (HFR), for example, estimates that a typical hedge fund has fallen by almost a fifth in assets under management in 2008 (see Figure 2), although, some industry executives report that hedge funds portfolios could have fallen by 30-40 percent in the recent financial crisis (The economist, 2008).

[Approximate location of Figures 1 and 2.]

The very first approaches to modeling hedge funds returns consisted in using linear factor models or non-parametric models. More recently, the former one was extended to linear factor models with option-like factors as many authors have shown that various hedge fund strategies exhibit nonlinear risk-return characteristics and non-normal option-like payoffs (e.g. Fung and

Hsieh, 1997 and 2001, Mitchell and Pulvino, 2001, Agarwal and Naik, 2004, Kuenzi and Shi, 2006). A possible explanation of this feature of the hedge funds data may be their frequent use of options or other nonlinear derivatives and dynamic regime-dependent trading strategies. For example, long-short strategy hedge funds are more likely to be long equity during up-markets and short equity during down-markets (Alexander and Dimitriu, 2005). In the light of these new findings, original linear factor models need to be revisited. Amenc et al. (2007a), for example, argue that on general basis, linear factor models fail the test of robustness, giving poor out-of-sample results (see also Amenc et al., 2007b and Hasanhodzic and Lo, 2007). Recently, Kosowski et al. (2007) use a factor model to evaluate hedge fund performance and predict it using Bayesian and Bootstrap analysis. Another advance in this area is the use of state-space modeling approach to modeling and estimating hedge funds returns. Bilio et al. (2006 and 2007) analyze the exposure of hedge fund indexes with a factor model based on regime switching, where non-linearity in the exposure is captured by factor loadings that are state dependent. They use univariate Markov switching (MS) factor models where both the conditional mean and volatility are regime-switching. However, they focus on the factor structure and do not include GARCH component into volatility. Moreover, Giamouridis and Vrontos (2007) employ a multivariate factor-GARCH model to construct optimal hedge fund portfolios. However, they do not include regime-switching into the GARCH equation. (See also Géhin, 2006, Diez de los Rios and Garcia, 2008). Despite the current advances in modeling non-linearities in hedge funds returns, this area of research still represents a methodological challenge.

In this paper, in line with the state-space approach, we use a univariate model of hedge fund returns which does not include factors. In particular, we combine the classical GARCH model of Bollerslev (1986) with the MS model of Hamilton (1989). This makes possible to include MS into the conditional volatility equation. In addition, we consider an ARMA structure of the conditional mean process and we also include MS into the expected return equation. Consideration of the MS parameters in the returns model is motivated for example by Diebold (1986) who notes that the GARCH specification can be improved by including regime dummy variables for the conditional variance intercept. Moreover, Freidman and Laibson (1989) noted that the GARCH model does not differentiate between the persistence of large and small shocks. Therefore, in this paper we combine MS and dynamic heteroscedasticity components and the most general model estimated is the MS-ARMA-GARCH specification.

In the past finance literature, several papers have considered MS-AR-ARCH models during the 90s (Hamilton and Susmel, 1994 and Cai, 1994). The advantage of that specification is that it avoids the 'path dependence' of the conditional distribution of returns since it only depends on the current and some recent lagged values of the regime variable. When the order of the AR-ARCH is low then the exact likelihood can be computed using basic MS techniques. However, when we consider more general MS-GARCH or MS-ARMA-GARCH specifications, the conditional distribution becomes dependent on all past regimes, i.e. it becomes path dependent. The consequence of this feature is that in order to compute the likelihood of a model that includes k regimes and T observations, we have to integrate over k^T regime paths. The exponentially growing number of terms makes this computation infeasible in practice.

In order to deal with this problem, there have been two streams of the MS-ARMA-GARCH literature. Papers in the first stream, propose modifications of the general MS-ARMA-GARCH model, which preserve the MS and GARCH properties but are not path dependent. Gray (1996) and Klaassen (2002) use a 'recombining' structure for the regime path tree, which is obtained by integrating out the lagged regime variable from the GARCH term directly. Thus, the conditional distribution of returns depends only on the current regime and the estimation of their MS-ARMA-GARCH model can be done using the standard MS inference method. Questions related to the covariance stationarity of these models have motivated the paper of Haas et al. (2004), who suggest another modification of the MS-GARCH model for which they prove the stationarity conditions. More recently, Abramson and Cohen (2007) derive the stationarity conditions for general MS-GARCH models that include Gray (1996), Klaassen (2002) and Haas el at. (2004).

The second stream of the research has been to address the estimation problems related to the path dependent MS-ARMA-GARCH model. Dueker (1997) estimate an MS-GARCH model in which the conditional variance depends on all past regimes, and the inference is performed applying a collapsing procedure introduced by Kim (1994). Using this procedure the distribution of returns depends only a limited number of recent regimes 'at the cost of introducing a degree of approximation that does not appear to distort the calculated likelihood that much' (Dueker, 1997). Recently, Henneke et al. (2006) and Bauwens et al. (2007) have suggested a Bayesian MCMC method for the inference of the path dependent MS-ARMA-GARCH and MS-GARCH models, respectively. Moreover, Bauwens et al. (2007) prove the conditions for strict stationarity of the MS-GARCH model, which may be combined with the MS-ARMA stationarity conditions derived by Francq and Zakoian (2001) to verify the stationarity of the MS-ARMA-GARCH model.

In this paper, we focus on the first stream of literature, i.e. we estimate non-'path dependent' MS volatility models because these models can be rapidly estimated which makes the repetition of out-of-sample forecasts feasible in practice. Therefore, first we apply single regime AR and ARMA-GARCH models and then employ the MS-AR specification, the MS-AR-ARCH model of Hamilton and Susmel (1994) and the MS-ARMA-GARCH specification of Klaassen (2002) for several hedge fund securities.

The paper is organized as follows. In Section 1 we present the hedge fund data set. Section 2 introduces the econometric models. Section 3 presents the statistical inference of the models and Section 4 describes the applied forecasting technique. Finally, Section 5 shows the estimation, stationarity and forecasting results and Section 6 concludes.

1 HEDGE FUND DATA

We analyze monthly return data of several hedge fund indices. The monthly data cover a 196 months period between January 1990 and April 2006. We use data obtained from the HFR Inc. The HFRI Monthly Indices are a series of benchmarks designed to reflect hedge fund industry performance by constructing equally weighted composites of constituent funds, as reported by the hedge fund managers listed within HFR Database. The HFRI are fund-weighted (equal-weighted) indices. Unlike asset-weighting, the equal-weighting of indices presents a more general picture of performance of the hedge fund industry. Any bias towards the larger funds potentially created by alternative weightings is greatly reduced, especially for strategies that encompass a small number of funds. For monthly data, estimation of our models is performed for the next ten hedge fund portfolios: 1-Fund Weighted Composite, 2-Equity Market Neutral, 3-Convertible Arbitrage, 4-Event-driven, 5-Merger Arbitrage, 6-Distressed Securities, 7-Equity Hedge, 8-Macro, 9-Relative Value Arbitrage and 10-Fixed Income.

Tables 1 and 2 show some descriptive statistics and serial correlation tests for these data series. Reviewing these tables we see that mean return is positive for all strategies. Standard deviation of returns is the highest for the Equity Hedge and Macro strategies and standard deviation of returns is the lowest for the Equity Market Neutral and Fixed Income strategies. We observe negative skewness, i.e. the distribution is more pronounced to the right tail, for the Merger Arbitrage, Event driven and Convertible Arbitrage strategies. We find that kurtosis is higher than three, i.e. the distribution is more peaked than the normal distribution, for the Merger Arbitrage and Relative Value Arbitrage strategies. Contrarily, we evidence that kurtosis is lower than three, i.e. the distribution is less peaked than the normal distribution, for the Equity Market Neutral and Macro strategies. For most strategies, we find significant autocorrelation of returns and squared returns using the Ljung-Box statistic. These preliminary findings motivate a dynamic specification for the conditional mean and volatility of hedge fund return time series. Finally, we also perform augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests for all data series. Table 3 presents the values of the test statistics and the corresponding 1 percent critical values. We observe that all data series are stationary according to the ADF and PP tests.

2 DYNAMIC MODELS

We consider several univariate specifications of hedge fund returns, which can be applied relatively easily in practice for forecasting purposes. For the simple AR(p) model of the mean, we consider $p = 1, \ldots, 12$ lags of monthly data, while for the more complicated MS or GARCH specifications we restrict our attention to include only one lag into the model formulation. Our restriction of the lag structure can be relaxed to include more lags of the ARMA and GARCH terms to obtain better forecasting performance. In addition, we consider only two regimes of the unobservable MS process, which can be interpreted as two possible latent states driving the mean and volatility dynamics of hedge fund returns, which may capture endogenously extreme observations (outliers or 'jumps') in the observed data set. In the followings, we present five dynamic models of hedge fund returns:

Model 1: AR(p)

In this model, we assume that volatility is constant over time and we only include dynamic structure into the conditional mean process:

$$y_t = c + \textstyle\sum_{k=1}^{p} \phi_k y_{t-k} + \psi u_t, \quad u_t \sim N[0,1], \tag{1}$$

where c, ϕ_k and ψ are real parameters. Rewriting this equation using the lag operator L we get

$$\left[1 - \sum_{k=1}^{p} \phi_k L^k\right] y_t = c + \psi u_t. \tag{2}$$

Then, the AR(p) process is covariance stationary when all roots of the following equation lie outside the unit circle:

$$1 - \sum_{k=1}^{K} \phi_k z^k = 0. \tag{3}$$

For $p = 1$, we obtain the AR(1) model:

$$y_t = c + \phi y_{t-1} + \psi u_t, \quad u_t \sim N[0,1], \tag{4}$$

where c, ϕ and ψ are real parameters. The $|\phi| < 1$ condition ensures stationarity.

Model 2: MS-AR(1)

We extend the previous AR(1) model by considering random parameters whose process is driven by an underlying Markov chain, $\{s_t\}$:

$$y_t = c(s_t) + \phi(s_t) y_{t-1} + \psi(s_t) u_t, \quad u_t \sim N[0,1], \tag{5}$$

where $s_t \in \{0,1\}$ indicates the regime at time t which form a Markov chain with transition probability matrix $P = \{\eta_{ij}\}$. Moreover, c, ϕ and ψ are real parameters. Francq and Zakoian (2001) show that the stationarity condition of the MS-ARMA model depends on the autoregressive part of the model as follows:

$$\sum_{i=0}^{1} \pi_i \log(|\phi(i)|) < 0 \tag{6}$$

where π_i is the *invariant probability measure* (Meyn and Tweedie, 1993) of the regimes.[1]

Model 3: MS-AR(1) and MS-ARCH(1)

In this model, introduced by Hamilton and Susmel (1994), we consider Markov switching both in the conditional mean and conditional volatility and we include heteroscedasticity in each regime by the ARCH equation:

$$\begin{aligned} y_t &= c(s_t) + \phi(s_t) y_{t-1} + \epsilon_t \\ h_t &= \omega(s_t) + \alpha(s_t)\epsilon_{t-1}^2 \\ \epsilon_t &= \sqrt{h_t} u_t, \end{aligned} \tag{7}$$

where $u_t \sim N[0,1]$ is an i.i.d error term, the c, ϕ, $\omega > 0$ and $\alpha > 0$ are real parameters. Finally, $s_t \in \{0,1\}$ indicates the regime at time t, which form a Markov chain with transition probability matrix $P = \{\eta_{ij}\}$. Bauwens, Preminger and Romboust (2007) show that the stationarity condition of the MS-GARCH model is

$$\sum_{i=0}^{1} \pi_i E[\log(\alpha(i) u_t^2 + \beta(i))] < 0 \tag{8}$$

where π_i is the invariant probability measure of the regimes. This condition for the MS-ARCH model reduces to

$$\sum_{i=0}^{1} \pi_i E[\log(\alpha(i) u_t^2)] < 0 \tag{9}$$

Based on this result and the result of Francq and Zakoian (2001) the two conditions (6) and (9) are required for the stationarity of the model.

Model 4: ARMA(1,1) and GARCH(1,1)

This model suggests a different dynamic structure for the conditional distribution of returns as previous models because it does not include Markovian regimes, however, it considers moving average terms in both equations:

$$\begin{aligned} y_t &= c + \phi y_{t-1} + \epsilon_t + \psi \epsilon_{t-1} \\ h_t &= \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} \\ \epsilon_t &= \sqrt{h_t} u_t, \end{aligned} \tag{10}$$

where $u_t \sim N[0,1]$ is an i.i.d error term, the c, ϕ, ψ, $\omega > 0$, $\alpha > 0$ and $\beta > 0$ are real parameters. The $|\phi| < 1$ and $\alpha + \beta < 1$ conditions ensure stationarity.

Model 5: MS-ARMA(1,1) and MS-GARCH(1,1)

In this section, we extend the previous model by assuming that all parameters are Markov switching. We consider Klaassen's (2002) specification of the MS-ARMA-GARCH model, which has been introduced to handle the path dependence problem mentioned in the introduction.

Klaassen (2002) extended Gray (1996) by including more information into the conditioning set of the expectations of the last two equations of the recombining MS-ARMA-GARCH model:

$$\begin{aligned}
y_t &= c(s_t) + \phi(s_t)y_{t-1} + \epsilon_t(s_t) + \psi(s_t)\epsilon_{t-1}(s_t) \\
h_t(s_t) &= \omega(s_t) + \alpha(s_t)\epsilon^2_{t-1}(s_t) + \beta(s_t)h_{t-1}(s_t) \\
\epsilon_t(s_t) &= \sqrt{h_t(s_t)}u_t \\
\epsilon_{t-1}(s_t) &= E[\epsilon_{t-1}(s_{t-1})|s_t, Y_{t-1}] \\
h_{t-1}(s_t) &= E[h_{t-1}(s_{t-1})|s_t, Y_{t-1}]
\end{aligned} \tag{11}$$

where $u_t \sim N[0,1]$ is an i.i.d error term, the c, ϕ, ψ, $\omega > 0$, $\alpha > 0$ and $\beta > 0$ are real parameters, $s_t \in \{0,1\}$ indicates the regime at time t which form a Markov chain with transition probability matrix $P = \{\eta_{ij}\}$.[2] As we noted in the introduction, for the models of Gray (1996) and Klaassen (2002) conditions for covariance stationarity have been established by Abramson and Cohen (2007). In particular, for our specification, define the 2×2 matrix $V = \{V_{ij}\}$ as $V_{ij} = (\alpha_i + \beta_i)\frac{\pi_j}{\pi_i}\eta_{ij}$. Then, the MS-GARCH equation is stationary when the largest eigenvalue in modulus of V is less than one. Finally, define the 2×2 matrix $M = \{M_{ij}\}$ as $M_{ij} = \phi_i\frac{\pi_j}{\pi_i}\eta_{ij}$. Then, the MS-ARMA equation is stationary when the largest eigenvalue in modulus of M is less than one.

3 INFERENCE

All models presented in the previous section are estimated by maximum likelihood method. The statistical inference of the MS models is done following Kim and Nelson (1999). Denote the vector of returns observed until period t as $Y_t = (y_1, \ldots, y_t)$. Then, all dynamic models presented in Section 2 can be estimated by maximizing numerically the likelihood functions derived in the following subsections.

Model 1: AR(p)
The conditional distribution of $y_t|Y_{t-1}$ is

$$y_t|Y_{t-1} \sim N\left[c + \sum_{k=1}^{p}\phi_k y_{t-k}, \psi^2\right] = N[\mu_t, \psi^2] \tag{12}$$

and the likelihood function is given by:

$$L = \prod_{t=1}^{T} f(y_t|Y_{t-1}) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\psi^2}} \exp\left(-\frac{(y_t - \mu_t)^2}{2\psi^2}\right) \tag{13}$$

Model 2: MS-AR(1)

In the MS-AR(1) model, the transition probability matrix is given by the next four parameters:

$$\begin{aligned} &\Pr[s_t = 1|s_{t-1} = 1] = \eta_{11} \quad \Pr[s_t = 0|s_{t-1} = 1] = \eta_{01} \\ &\Pr[s_t = 0|s_{t-1} = 0] = \eta_{00} \quad \Pr[s_t = 1|s_{t-1} = 0] = \eta_{10} \end{aligned} \tag{14}$$

where $\eta_{11}+\eta_{01} = 1$ and $\eta_{00}+\eta_{10} = 1$. Given $\Pr[s_{t-1} = i|Y_{t-1}]$ the weighting terms $\Pr[s_t = j|Y_{t-1}]$ are calculated as follows:

$$\pi_{jt} = \Pr[s_t = j|Y_{t-1}] = \sum_{i=0}^{1} \Pr[s_t = j|s_{t-1} = i] \Pr[s_{t-1} = i|Y_{t-1}] = \sum_{i=0}^{1} \eta_{ji}\pi^*_{it-1} \tag{15}$$

In order to compute π^*_{it-1}, we need to apply the conditional density of returns as follows:

$$\pi^*_{it-1} = \frac{f(y_{t-1}|s_{t-1} = i, Y_{t-2})\pi_{it-1}}{\sum_{i=0}^{1} f(y_{t-1}|s_{t-1} = i, Y_{t-2})\pi_{it-1}} \tag{16}$$

Therefore, we can compute the weighting terms by iteration.[3] The conditional distribution of $y_t|(Y_{t-1}, s_t = i)$ for $i = 0, 1$ is

$$y_t|(Y_{t-1}, s_t = i) \sim N[c_i + \phi_i y_{t-1}, \psi_i^2] \tag{17}$$

and the likelihood function is given by

$$L = \prod_{t=1}^{T} \pi_{0t} f(y_t|Y_{t-1}, s_t = 0) + \pi_{1t} f(y_t|Y_{t-1}, s_t = 1). \tag{18}$$

Model 3: MS-AR(1) and MS-ARCH(1)

In order to compute h_t in equation (7) given Y_{t-1}, we need to compute $\epsilon_{t-1} = y_{t-1} - c(s_{t-1}) - \phi(s_{t-1})y_{t-2}$. Thus, the conditional distribution of y_t depends on s_{t-1} as well and it can be denoted as $f(y_t|s_t, s_{t-1}, Y_{t-1})$. Therefore, the likelihood of Y_T is given by

$$L = \prod_{t=1}^{T} \sum_{i=0}^{1} \sum_{j=0}^{1} f(y_t|s_t = j, s_{t-1} = i, Y_{t-1}) \Pr[s_t = j, s_{t-1} = i|Y_{t-1}]. \tag{19}$$

Then, in the likelihood we have to compute $\Pr[s_t = j, s_{t-1} = i|Y_{t-1}]$:

$$\Pr[s_t = j, s_{t-1} = i|Y_{t-1}] = \Pr[s_t = j|s_{t-1} = i] \Pr[s_{t-1} = i|Y_{t-1}] = \eta_{ji} \Pr[s_{t-1} = i|Y_{t-1}] \tag{20}$$

and to compute $\Pr[s_t = i|Y_t]$ in equation (20) consider

$$\Pr[s_t = i, s_{t-1} = j|Y_t] = \frac{f(y_t|s_t = i, s_{t-1} = j, Y_{t-1}) \Pr[s_t = i, s_{t-1} = j|Y_{t-1}]}{\sum_{i=0}^{1}\sum_{j=0}^{1} f(y_t|s_t = i, s_{t-1} = j, Y_{t-1}) \Pr[s_t = i, s_{t-1} = j|Y_{t-1}]} \tag{21}$$

and integrate out the past regime s_{t-1} from (21):

$$\Pr[s_t = i|Y_t] = \sum_{j=0}^{1} \Pr[s_t = i, s_{t-1} = j|Y_t] \tag{22}$$

and thus the probability in the likelihood function can be computed by iteration and we can use the invariant probability measure π_i as an initial value for $\Pr[s_0 = i|Y_0]$ in equation (20).

Model 4: ARMA(1,1) and GARCH(1,1)
The conditional distribution of $y_t|Y_{t-1}$ is:

$$y_t|Y_{t-1} \sim N[c + \phi y_{t-1} + \psi \epsilon_{t-1}, h_t] = N[\mu_t, h_t] \tag{23}$$

and the likelihood function is given by:

$$L = \prod_{t=1}^{T} f(y_t|Y_{t-1}) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi h_t}} \exp\left(-\frac{(y_t - \mu_t)^2}{2h_t}\right) \tag{24}$$

Model 5: MS-ARMA(1,1) and MS-GARCH(1,1)
In specification of Model 5, the lagged regimes are integrated out, therefore, the conditional density of y_t in these models depends only on the current regime and the likelihood is given by

$$L = \prod_{t=1}^{T} f(y_t|s_t = i, Y_{t-1}) \Pr[s_t = i|Y_{t-1}] \tag{25}$$

The probability $\Pr[s_t = i|Y_{t-1}]$ can be computed by iteration in the same way as in Model 2 (see equations (15) and (16)).

4 FORECASTING

The one-step-ahead out-of-sample forecast of return y_t is the conditional expectation of y_t given all past observable information, Y_{t-1}. We perform repeated out-of-sample forecasts of returns: First, we estimate the model for selected initial period $[0, t]$, for example for the first half of the full sample period, then, we forecast the next observation $t + 1$.[4] Consecutively, we move the time window to include that observation as well and reestimate the model for the period $[0, t + 1]$ in order to forecast the return of period $t + 2$. We repeat this procedure until time T and this way we include all available information until time t to forecast the time $t + 1$ value.

The forecasting performance of the models is evaluated by comparing the true and the forecasted returns by the normalized root mean squared error (NRMSE) computed as follows:

$$NRMSE = \frac{\sqrt{MSE}}{y_{\max} - y_{\min}} \tag{26}$$

where the mean squared error (MSE) is given by

$$MSE = \frac{\sum_{t=1}^{T}[y_t - E(y_t|Y_{t-1})]^2}{T} \tag{27}$$

and the conditional expectation for different models can be computed as follows:

Model 1: $E(y_t|Y_{t-1}) = c + \sum_{k=1}^{p} \phi_k y_{t-k}$

Model 2: $E(y_t|Y_{t-1}) = \pi_{0t}(c_0 + \phi_0 y_{t-1}) + \pi_{1t}(c_1 + \phi_1 y_{t-1})$

Model 3: $E(y_t|Y_{t-1}) = \pi_{0t}(c_0 + \phi_0 y_{t-1}) + \pi_{1t}(c_1 + \phi_1 y_{t-1})$

Model 4: $E(y_t|Y_{t-1}) = c + \phi y_{t-1} + \psi \epsilon_{t-1}$

Model 5: $E(y_t|Y_{t-1}) = \pi_{0t}(c_0 + \phi_0 y_{t-1} + \psi_0 \epsilon_{t-1}) + \pi_{1t}(c_1 + \phi_1 y_{t-1} + \psi_1 \epsilon_{t-1})$

where $\pi_{it} = \Pr[s_t = i|Y_{t-1}]$ in the previous equations. The main advantage of employing a normalized measure of forecast precision is that we are able to compare forecasting among several hedge fund strategies.

5 RESULTS

5.1 Estimation results

In the first part of this section, we review the general results of the maximum likelihood estimation for each strategies and specification for the 1990-2006 period. In order to compare the different models presented in Section 3, Tables 4, 5 and 6 show the mean log-likelihood (LL), Akaike information criterion (AIC) and Bayes information criterion (BIC) values, respectively, for each specification and strategy. In addition, Tables 7-16 present the specific parameters estimates for each strategy and model.

Reviewing the LL, AIC and BIC values we can see that the most general Model 5 performs better than other specifications when the LL and AIC are considered. However, when the BIC is considered, Models 2 and 1 dominate the rest of the models due to the penalization for the higher number of parameters included. The only security for which Model 5 has the lowest BIC value is Convertible Arbitrage (see Figure 5). For each strategy, we present the evolution of the filtered probability of being in the first regime, $s_t = 0$ for the MS specification with the best BIC on the lower part of Figures 3-12.

[Approximate location of the lower part of Figures 3-12.]

Reviewing the specific parameters estimates for the Fund Weighted Composite strategy, we identify all parameters of each model. However, for the ARMA-GARCH specification the MA(1) coefficient is not significant. In addition, the p and q parameters of the transition matrix are higher than 0.9 only for the MS-AR(1) specification (see Figure 3). For the Equity Market Neutral portfolio, most parameters we find significant. Nevertheless, the ARCH(1) parameters are non-significant for Model 5. We find significant and high p and q values for the MS-AR(1) model (see Figure 4). Reviewing the results for the Convertible Arbitrage strategy we see that in the ARMA-GARCH specification there are no dynamics in volatility ($\alpha = 0$ and $\beta = 0$). Moreover, in the MS-ARMA-GARCH (Model 5) specification, some of the switching volatiliy parameters are zero. Thus, regime switching is not confirmed by our results for this strategy (see also Figure 5). Estimation results of the Event Driven strategy show that in Model 4 and 5 there are no GARCH dynamics of volatility and there are problems with the MS specification as well (see Figure 6). For the Merger Arbitrage strategy, we can see that there are no GARCH dynamics in Model 3, 4 and 5. Moreover, for this strategy we do not identify clearly the two regimes (see Figure 7). Observing the Distressed Securities estimation results we can see that there are no volatility dynamics and there are problems with the two-state regime switching specification as well for Models 3-5 (see Figure 8). The Equity Hedge parameter estimates show that some of the volatility parameters of Model 5 are non-significant. However, the elements of the transition matrix of states is higher than 0.9 in Model 2 and 5 (see Figure 9). For the Macro strategy we evidence that some volatility parameters are not significant in Model 5 and the p and q parameters are lower than 0.9 for Models 3 and 5 (see Figure 10). The Relative Value Arbitrage strategy estimation results show that the GRACH dynamics of Model 5 are not significant. However, the regime switching probabilities are higher than 0.9 for Model 5 (see Figure 11). Finally, for the Fixed Income strategy we do not find significant MS volatility dynamics. Nevertheless, we find the ARMA-GARCH parameters significant for Model 4 and the p and q parameters we find higher than 0.9 for Model 2 (see Figure 12).

Finally, Table 17 shows the stationarity diagnostics checks of each model estimated. We find that Models 1, 2 and 4 are stationary for all strategies, while Model 3 and 5 are stationary for all products besides the Distressed securities, Fixed income (both for Model 3) and Relative value arbitrage (for Model 5).

5.2 Forecasting results

Tables 18 and 19 compare the forecasting performance of several dynamic econometric models using the NRMSE measure. In particular, Table 18 shows the NRMSE for the AR(p) model for lags $p = 1, \dots, 12$ and Table 19 presents the forecasting performance of different first-order specifications of Models 1-5. The observed and forecasted hedge fund return series for the best NRMSE model for the May 1998 - April 2006 period can be seen on the upper part of Figures 3-12.

[Approximate location of the upper part of Figures 3-12.]

Reviewing Tables 18 we conclude that for the most hedge fund portfolios the best forecasting performance is achieved when we consider the first-order AR(1) specification and do not consider more lags of the historical return data. Thus, including higher lags into the conditional mean usually does not improve forecast precision. This may motivate our restriction to consider only first-order specifications of the mean and the volatility for Models 2-5.

Table 19 shows that Model 5 dominates the forecasting precision of other specifications because for the Fund Weighted Composite, Equity Market Neutral, Distressed Security, Equity Hedge and Fixed Income strategies the NRMSE is the lowest in case of the MS-ARMA-GARCH formulation. However, for the rest of the hedge fund strategies, the AP(p) and MS-AR(1) formulations yield better forecasting performance. The main reason for this finding is that for the Convertible Arbitrage, Event-driven and Merger Arbitrage strategies we do not find significant regime switching for the model that provides the best forecasts. This could be due to the fact that the sample size of the observed monthly return series is too small to identify two latent regimes. Moreover, for the Macro and Relative Value Arbitrage portfolios we do not find significant GARCH components for the best forecasting specifications.

The advantage of the selection of the normalized NRMSE criterion is that using this measure we can compare the forecast precision across all strategies considered. In particular, reviewing Tables 18 and 19 we evidence that the best forecastable time series are the (1) Relative Value Arbitrage, (2) Distressed Securities and (3) Fund Weighted Composite strategies. Nevertheless, the least forecastable strategies are the (1) Macro, (2) Equity Market Neutral and (3) Fixed Income time series.

6 CONCLUSIONS

In this paper, we apply different and relatively easily applicable dynamic models of the mean and the volatility of the next ten hedge fund indices to forecast returns over the 1990-2006 period: 1-Fund Weighted Composite, 2-Equity Market Neutral, 3-Convertible Arbitrage, 4-Event-driven, 5-Merger Arbitrage, 6-Distressed Securities, 7-Equity Hedge, 8-Macro, 9-Relative Value Arbitrage and 10-Fixed Income.

We consider models for which a repeated out-of-sample forecast procedure is a reliable alternative for practitioners due to its relative simplicity and rapid applicability. We think that forecasting these data series is especially interesting because hedge funds' strategies have been very significantly growing during the last two decades and they are especially impacted by the 2008 global financial crisis.

We employ five combinations of AR, MA, ARCH, GARCH and MS models to capture possible jumps of these volatile investment funds and find that depending on the security considered, different model provides the best forecast. In particular, we find that the best forecastable time series are the (1) Relative Value Arbitrage by MS-AR, (2) Distressed Securities by MS-ARMA-GARCH and (3) Fund Weighted Composite strategies by MS-ARMA-GARCH. while, the least forecastable strategies are the (1) Macro by MS-AR, (2) Equity Market Neutral by MS-ARMA-GARCH and (3) Fixed Income by MS-ARMA-GARCH.

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Notes

[1] In our two state Markov process, the invariant probability measure π_i is defined as follows:

$$\pi_i = \int \Pr[s_t = i | s_{t-1} = j] \pi(dj) = \int \eta_{ij} \pi(dj) = \eta_{i0}\pi_0 + \eta_{i1}\pi_1$$

where $\{\eta_{ij} : i, j = 0, 1\}$ are the transition probabilities of the Markov chain. Then, using the fact that $\pi_0 + \pi_1 = 1$ we obtain that

$$\pi_0 = \frac{1 - \eta_{11}}{2 - \eta_{00} - \eta_{11}} \qquad \pi_1 = \frac{1 - \eta_{00}}{2 - \eta_{00} - \eta_{11}}$$

[2]The expectations of the last two equations are computed using the $\pi_{it-1} = \Pr[s_{t-1} = i | s_t, Y_{t-1}]$ probabilities in order to integrate out s_{t-1}. This modification is motivated by the fact that the $\epsilon_{t-1}(s_t)$ and $h_{t-1}(s_t)$ values are computed in the beginning of period t before observing y_t. Therefore, we can condition on s_t and Y_{t-1} and π_{it-1} is computed as follows:

$$\pi_{it-1} = \Pr[s_{t-1} = i | s_t = j, Y_{t-1}] = \frac{\Pr[s_{t-1} = i | Y_{t-1}] \Pr[s_t = j | s_{t-1} = i]}{\sum_{i=0}^{1} \Pr[s_{t-1} = i | Y_{t-1}] \Pr[s_t = j | s_{t-1} = i]}$$

where we can compute $\Pr[s_{t-1} = i | Y_{t-1}]$ by

$$\Pr[s_{t-1} = i | Y_{t-1}] = \frac{f(y_{t-1} | s_{t-1} = i, Y_{t-2}) \Pr[s_{t-1} = i | Y_{t-2}]}{\sum_{i=0}^{1} f(y_{t-1} | s_{t-1} = i, Y_{t-2}) \Pr[s_{t-1} = i | Y_{t-2}]}$$

[3]In order to compute the weighting terms, we need the initial value of π_{i0}^*, which is approximated by the following invariant probabilities of s_t:

$$\pi_{00}^* = \frac{1 - \eta_{11}}{2 - \eta_{00} - \eta_{11}} \qquad \pi_{10}^* = \frac{1 - \eta_{00}}{2 - \eta_{00} - \eta_{11}}$$

[4]In our application, the initial sample period is January 1990 - May 1998.

Table 1 Descriptive statistics of returns

Strategy	Max	Min	Mean	St.dev	Skewness	Kurtosis
1-Fund Weighted Composite	7.65	-8.70	1.14	1.96	-0.62	2.84
2-Equity Market Neutral	3.59	-1.67	0.74	0.89	0.18	0.40
3-Convertible Arbitrage	3.33	-3.19	0.81	1.02	-1.10	1.92
4-Event-driven	5.13	-8.90	1.17	1.87	-1.31	4.68
5-Merger Arbitrage	3.12	-6.46	0.84	1.23	-2.51	10.83
6-Distressed Securities	7.06	-8.50	1.21	1.72	-0.66	5.65
7-Equity Hedge	10.88	-7.65	1.38	2.52	0.16	1.37
8-Macro	7.88	-6.40	1.25	2.36	0.35	0.61
9-Relative Value Arbitrage	5.72	-5.80	0.96	1.03	-0.84	10.29
10-Fixed Income	5.34	-3.27	0.84	0.97	-0.28	5.11

Table 2 Ljung-Box test of serial correlation

	Returns		Squared returns	
Strategy	LB(10)	p-value	LB(10)	p-value
1-Fund Weighted Composite	19.92	0.030	13.49	0.197
2-Equity Market Neutral	42.60	0.000	34.42	0.000
3-Convertible Arbitrage	85.68	0.000	10.53	0.395
4-Event-driven	17.89	0.057	6.05	0.811
5-Merger Arbitrage	19.07	0.039	17.56	0.063
6-Distressed Securities	56.43	0.000	25.34	0.005
7-Equity Hedge	13.96	0.175	39.00	0.000
8-Macro	18.36	0.049	34.24	0.000
9-Relative Value Arbitrage	28.30	0.002	11.45	0.323
10-Fixed Income	55.43	0.000	34.90	0.000

Notes: The Ljung-Box (LB) test statistic is computed for 10 lags.

Table 3 Unit root tests of data series

Strategy	ADF	PP
1-Fund Weighted Composite	-6.212	-10.907
2-Equity Market Neutral	-4.985	-13.167
3-Convertible Arbitrage	-5.414	-7.558
4-Event-driven	-5.814	-10.862
5-Merger Arbitrage	-5.080	-12.732
6-Distressed Securities	-5.961	-8.077
7-Equity Hedge	-6.397	-12.076
8-Macro	-4.699	-11.742
9-Relative Value Arbitrage	-5.100	-10.655
10-Fixed Income	-5.676	-9.177

Notes: The table presents the augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) test statistics. The null hypothesis of unit root is rejected when the value of this statistic is lower than the critical value. The 1% critical values of ADF and PP are -3.466 and -3.465, respectively.

Table 4 Maximum mean log-likelihood (LL)

Strategy	Model 1	Model 2	Model 3	Model 4	Model 5
1	-2.056	-1.987	-1.982	-2.041	**-1.942**
2	-1.301	-1.223	-1.263	-1.251	**-1.177**
3	-1.243	-1.148	-1.150	-1.242	**-1.104**
4	-1.999	-1.909	-1.908	-1.997	**-1.881**
5	-1.599	-1.353	-1.353	-1.578	**-1.326**
6	-1.821	**-1.731**	-1.821	-1.811	-1.811
7	-2.328	-2.273	-2.275	-2.299	**-2.255**
8	-2.261	-2.202	-2.226	-2.254	**-2.164**
9	-1.406	-1.234	-1.251	-1.296	**-1.198**
10	-1.294	-1.139	-1.288	-1.237	**-1.134**

Notes: Strategies: 1-Fund Weighted Composite, 2-Equity Market Neutral, 3-Convertible Arbitrage, 4-Event-driven, 5-Merger Arbitrage, 6-Distressed Securities, 7-Equity Hedge, 8-Macro, 9-Relative Value Arbitrage, 10-Fixed Income. Number written by **bold** letters denote the model with the best information criterion.

Table 5 Akaike information criterion (AIC)

Strategy	Model 1	Model 2	Model 3	Model 4	Model 5
1	4.142	4.056	4.065	4.144	**4.026**
2	2.633	2.527	2.628	2.564	**2.486**
3	2.516	2.378	2.402	2.525	**2.321**
4	4.028	3.900	3.919	4.035	**3.875**
5	3.229	2.788	**2.767**	3.207	2.775
6	3.672	**3.544**	3.672	3.663	3.663
7	4.687	4.628	4.652	4.658	**4.612**
8	4.553	4.486	4.543	4.568	**4.451**
9	2.843	2.549	2.594	2.653	**2.529**
10	2.618	**2.359**	2.618	2.534	2.390

Notes: $AIC = -2LL + 2K/T$, where K denotes the number of parameters, T is the sample size and LL is the mean log-likelihood. Strategies: 1-Fund Weighted Composite, 2-Equity Market Neutral, 3-Convertible Arbitrage, 4-Event-driven, 5-Merger Arbitrage, 6-Distressed Securities, 7-Equity Hedge, 8-Macro, 9-Relative Value Arbitrage, 10-Fixed Income. Number written by **bold** letters denote the model with the best information criterion.

Table 6 Bayes information criterion (BIC)

Strategy	Model 1	Model 2	Model 3	Model 4	Model 5
1	4.192	**4.190**	4.233	4.244	4.261
2	2.683	**2.661**	2.795	2.664	2.703
3	2.566	2.512	2.569	2.592	**2.505**
4	4.079	**4.034**	4.086	4.102	4.059
5	3.280	2.921	**2.868**	3.291	2.975
6	3.722	**3.678**	3.722	3.730	3.730
7	**4.737**	4.761	4.820	4.759	4.779
8	**4.604**	4.620	4.694	4.669	4.652
9	2.893	**2.683**	2.745	2.754	2.746
10	2.668	**2.493**	2.685	2.635	2.591

Notes: $BIC = -2LL + K \ln T/T$, where K denotes the number of parameters, T is the sample size and LL is the mean log-likelihood. Strategies: 1-Fund Weighted Composite, 2-Equity Market Neutral, 3-Convertible Arbitrage, 4-Event-driven, 5-Merger Arbitrage, 6-Distressed Securities, 7-Equity Hedge, 8-Macro, 9-Relative Value Arbitrage, 10-Fixed Income. Number written by **bold** letters denote the model with the best information criterion.

Table 7 Results: 1-Fund Weighted Composite

θ	Model 1	Model 2	Model 3	Model 4	Model 5
c_0	0.86*(0.156)	0.84*(0.079)	1.70*(0.053)	0.82*(0.051)	−0.29*(0.062)
c_1		0.74*(0.067)	−0.95*(0.051)		0.94*(0.050)
ϕ_0	0.25*(0.069)	0.34*(0.061)	0.19*(0.049)	0.33*(0.052)	0.54*(0.041)
ϕ_1		0.17*(0.089)	0.55*(0.055)		0.65*(0.026)
ψ_0	1.89*(0.095)	1.41*(0.064)		−0.04(0.050)	−0.01(0.061)
ψ_1		2.63*(0.119)			−1.32*(0.037)
ω_0			0.86*(0.052)	1.48*(0.050)	1.03*(0.052)
ω_1			0.98*(0.051)		0.11*(0.048)
α_0			0.37*(0.052)	0.20*(0.049)	0.09*(0.043)
α_1			0.99*(0.051)		0.45*(0.049)
β_0				0.40*(0.047)	0.92*(0.057)
β_1					0.31*(0.039)
η_{00}		0.97*(0.021)	0.62*(0.051)		0.67*(0.042)
η_{11}		0.99*(0.011)	0.30*(0.056)		0.71*(0.039)

Notes: Standard errors are reported in parentheses. The * denotes coefficient significant at the 5 percent level. The - denotes that the coefficient was set to zero value in order to estimate correctly the standard errors.

Table 8 Results: 2-Equity Market Neutral

θ	Model 1	Model 2	Model 3	Model 4	Model 5
c_0	0.69*(0.064)	1.58*(0.064)	0.19*(0.057)	0.04(0.034)	0.35*(0.075)
c_1		0.26*(0.071)	0.94*(0.066)		1.20*(0.065)
ϕ_0	0.07(0.063)	−0.37*(0.063)	0.18*(0.055)	0.94*(0.043)	−0.15*(0.067)
ϕ_1		0.08(0.069)	0.52*(0.065)		−0.11(0.061)
ψ_0	0.89*(0.045)	0.81*(0.064)		−0.82*(0.074)	0.20*(0.067)
ψ_1		0.65*(0.060)			0.02(0.067)
ω_0			0.20*(0.064)	0.17*(0.067)	0.09*(0.027)
ω_1			0.34*(0.064)		0.04(0.036)
α_0			0.53*(0.064)	0.19*(0.083)	-
α_1			0.44*(0.064)		0.12(0.068)
β_0				0.60*(0.120)	0.68*(0.067)
β_1					0.86*(0.062)
η_{00}		0.93*(0.065)	0.44*(0.064)		0.98*(0.021)
η_{11}		0.94*(0.064)	0.26*(0.063)		0.98*(0.017)

Notes: Standard errors are reported in parentheses. The * denotes coefficient significant at the 5 percent level. The - denotes that the coefficient was set to zero value in order to estimate correctly the standard errors.

Table 9 Results: 3-Convertible Arbitrage

θ	Model 1	Model 2	Model 3	Model 4	Model 5
c_0	$0.36^*(0.058)$	$0.02(0.067)$	$-0.61^*(0.069)$	$0.38^*(0.081)$	$0.88^*(0.057)$
c_1		$0.91^*(0.062)$	$0.93^*(0.062)$		$-0.75^*(0.054)$
ϕ_0	$0.56^*(0.050)$	$0.53^*(0.074)$	$0.91^*(0.069)$	$0.53^*(0.066)$	$0.34^*(0.043)$
ϕ_1		$0.27^*(0.052)$	$0.29^*(0.050)$		$1.24^*(0.069)$
ψ_0	$0.84^*(0.063)$	$1.08^*(0.078)$		$0.05(0.061)$	$-0.15^*(0.075)$
ψ_1		$0.49^*(0.052)$			$-1.38^*(0.069)$
ω_0			$0.29^*(0.067)$	$0.16^*(0.053)$	-
ω_1			$0.24^*(0.063)$		$0.43^*(0.070)$
α_0			$0.82^*(0.067)$	-	-
α_1			$0.25^*(0.078)$		$0.35^*(0.068)$
β_0				-	$0.89^*(0.034)$
β_1					-
η_{00}		$0.88^*(0.060)$	$0.56^*(0.066)$		$0.87^*(0.050)$
η_{11}		$0.82^*(0.067)$	$0.83^*(0.057)$		$0.68^*(0.065)$

Notes: Standard errors are reported in parentheses. The * denotes coefficient significant at the 5 percent level. The - denotes that the coefficient was set to zero value in order to estimate correctly the standard errors.

Table 10 Results: 4-Event-driven

θ	Model 1	Model 2	Model 3	Model 4	Model 5
c_0	$0.83^*(0.054)$	$-2.20^*(0.052)$	$-2.51^*(0.859)$	$0.93^*(0.052)$	$1.28^*(0.056)$
c_1		$1.18^*(0.054)$	$1.27^*(0.150)$		$-4.45^*(0.070)$
ϕ_0	$0.29^*(0.050)$	$0.64^*(0.052)$	$0.84(0.525)$	$0.20^*(0.051)$	$0.09(0.047)$
ϕ_1		$0.19^*(0.050)$	$0.17^*(0.075)$		$-0.24^*(0.088)$
ψ_0	$1.79^*(0.089)$	$2.96^*(0.052)$		$0.11^*(0.048)$	$0.13^*(0.054)$
ψ_1		$1.41^*(0.055)$			$-0.61^*(0.05)$
ω_0			$5.11(3.376)$	$3.18^*(0.051)$	$2.02^*(0.101)$
ω_1			$1.27^*(0.265)$		$0.73^*(0.054)$
α_0			$0.76(0.550)$	-	-
α_1			$0.39^*(0.100)$		-
β_0				-	-
β_1					$0.00^*(0.052)$
η_{00}		$0.95^*(0.025)$	$0.19(0.194)$		$0.98^*(0.012)$
η_{11}		$0.29^*(0.052)$	$0.94^*(0.040)$		$0.46^*(0.056)$

Notes: Standard errors are reported in parentheses. The * denotes coefficient significant at the 5 percent level. The - denotes that the coefficient was set to zero value in order to estimate correctly the standard errors.

Table 11 Results: 5-Merger Arbitrage

θ	Model 1	Model 2	Model 3	Model 4	Model 5
c_0	$0.66^*(0.058)$	$-1.80^*(0.061)$	$-1.80^*(0.061)$	$0.75^*(0.080)$	$3.96^*(1.067)$
c_1		$1.02^*(0.071)$	$1.02^*(0.071)$		$0.04(0.058)$
ϕ_0	$0.21^*(0.055)$	$1.84^*(0.063)$	$1.84^*(0.063)$	$0.13^*(0.049)$	$-2.51^*(1.140)$
ϕ_1		$0.06(0.044)$	$0.06(0.044)$		$0.93^*(0.057)$
ψ_0	$1.20^*(0.061)$	$1.54^*(0.062)$		$0.20^*(0.056)$	$1.09(0.747)$
ψ_1		$0.70^*(0.061)$			$-0.73^*(0.078)$
ω_0			$1.54^*(0.062)$	$1.26^*(0.107)$	$30.97(18.651)$
ω_1			$0.70^*(0.061)$		$0.51^*(0.059)$
α_0			-	$0.15(0.116)$	$0.24(3.883)$
α_1			-		-
β_0				-	$0.16(3.492)$
β_1					-
η_{00}		$0.82^*(0.059)$	$0.82^*(0.059)$		$0.21(0.137)$
η_{11}		$0.13^*(0.061)$	$0.13^*(0.061)$		$0.95^*(0.021)$

Notes: Standard errors are reported in parentheses. The * denotes coefficient significant at the 5 percent level. The - denotes that the coefficient was set to zero value in order to estimate correctly the standard errors.

Table 12 Results: 6-Distressed Securities

θ	Model 1	Model 2	Model 3	Model 4	Model 5
c_0	$0.62^*(0.053)$	$-0.42^*(0.054)$	$0.62^*(0.053)$	$0.84^*(0.184)$	$0.84^*(0.184)$
c_1		$0.71^*(0.055)$	-		-
ϕ_0	$0.49^*(0.053)$	$1.26^*(0.054)$	$0.49^*(0.053)$	$0.32^*(0.123)$	$0.32^*(0.123)$
ϕ_1		$0.42^*(0.048)$	-		-
ψ_0	$1.49^*(0.076)$	$3.55^*(0.055)$		$0.25^*(0.118)$	$0.25^*(0.118)$
ψ_1		$1.19^*(0.081)$			-
ω_0			$1.49^*(0.076)$	$0.78^*(0.101)$	$0.78^*(0.101)$
ω_1			-		-
α_0			-	-	-
α_1			-		-
β_0					-
β_1					-
η_{00}		$0.95^*(0.026)$	-		-
η_{11}		$0.24^*(0.055)$	-		-

Notes: Standard errors are reported in parentheses. The * denotes coefficient significant at the 5 percent level. The - denotes that the coefficient was set to zero value in order to estimate correctly the standard errors.

Table 13 Results: 7-Equity Hedge

θ	Model 1	Model 2	Model 3	Model 4	Model 5
c_0	$1.16^*(0.066)$	$1.55^*(0.049)$	$-1.62^*(0.627)$	$0.85^*(0.322)$	$1.82^*(0.048)$
c_1		$0.96^*(0.050)$	$1.77^*(0.293)$		$0.61^*(0.048)$
ϕ_0	$0.16^*(0.052)$	$0.10(0.111)$	$0.89^*(0.309)$	$0.36(0.218)$	$0.08(0.048)$
ϕ_1		$0.22^*(0.068)$	$0.07(0.108)$		$0.52^*(0.045)$
ψ_0	$2.48^*(0.117)$	$3.49^*(0.047)$		$-0.14(0.221)$	-
ψ_1		$1.90^*(0.054)$			$-0.32^*(0.048)$
ω_0			$1.94^*(0.948)$	$2.87^*(0.816)$	$14.54^*(0.048)$
ω_1			$2.70^*(0.824)$		$0.45^*(0.046)$
α_0			$0.58^*(0.248)$	$0.26^*(0.107)$	-
α_1			$0.38^*(0.102)$		-
β_0				$0.29(0.154)$	-
β_1					$0.86^*(0.020)$
η_{00}		$0.99^*(0.011)$	$0.09(0.131)$		$0.93^*(0.048)$
η_{11}		$0.97^*(0.026)$	$0.63^*(0.216)$		$0.99^*(0.009)$

Notes: Standard errors are reported in parentheses. The * denotes coefficient significant at the 5 percent level. The - denotes that the coefficient was set to zero value in order to estimate correctly the standard errors.

Table 14 Results: 8-Macro

θ	Model 1	Model 2	Model 3	Model 4	Model 5
c_0	1.04*(0.049)	0.64*(0.048)	3.21*(0.080)	0.78(0.431)	0.53*(0.049)
c_1		1.31*(0.049)	0.43*(0.097)		0.39*(0.049)
ϕ_0	0.17*(0.052)	0.01(0.050)	0.10(0.134)	0.30(0.393)	0.82*(0.040)
ϕ_1		0.16*(0.057)	0.01(0.074)		−1.11*(0.049)
ψ_0	2.32*(0.117)	1.22*(0.048)		−0.16(0.436)	−0.71*(0.036)
ψ_1		2.68*(0.052)			1.79*(0.049)
ω_0			3.26*(0.055)	1.06*(0.532)	1.17*(0.049)
ω_1			2.58*(0.090)		-
α_0			0.63*(0.136)	0.11(0.065)	-
α_1			-		0.21*(0.049)
β_0				0.70*(0.096)	0.84*(0.048)
β_1					0.20*(0.052)
η_{00}		0.97*(0.024)	0.52*(0.181)		0.87*(0.049)
η_{11}		0.94*(0.042)	0.80*(0.075)		0.65*(0.048)

Notes: Standard errors are reported in parentheses. The * denotes coefficient significant at the 5 percent level. The - denotes that the coefficient was set to zero value in order to estimate correctly the standard errors.

Table 15 Results: 9-Relative Value Arbitrage

θ	Model 1	Model 2	Model 3	Model 4	Model 5
c_0	0.70*(0.056)	0.51*(0.060)	0.85*(0.064)	0.17(0.121)	1.07*(0.058)
c_1		0.93*(0.064)	0.54*(0.054)		0.44*(0.063)
ϕ_0	0.27*(0.050)	0.45*(0.055)	0.06(0.064)	0.85*(0.113)	0.30*(0.039)
ϕ_1		0.10(0.064)	0.40*(0.050)		0.27*(0.064)
ψ_0	0.99*(0.060)	0.66*(0.060)		-0.61*(0.187)	-0.27*(0.072)
ψ_1		1.99*(0.064)			-
ω_0			4.75*(0.064)	0.24*(0.079)	0.16*(0.065)
ω_1			0.31*(0.063)		0.32*(0.056)
α_0			-	0.58*(0.161)	1.57*(0.065)
α_1			0.67*(0.062)		0.02(0.063)
β_0				0.31*(0.124)	0.07(0.065)
β_1					0.02(0.064)
η_{00}		0.86*(0.0642)	0.86*(0.064)		0.93*(0.053)
η_{11}		0.98*(0.0146)	0.99*(0.011)		0.94*(0.048)

Notes: Standard errors are reported in parentheses. The * denotes coefficient significant at the 5 percent level. The - denotes that the coefficient was set to zero value in order to estimate correctly the standard errors.

Table 16 Results: 10-Fixed Income

θ	Model 1	Model 2	Model 3	Model 4	Model 5
c_0	$0.50^*(0.069)$	$0.56^*(0.068)$	$0.54^*(0.094)$	$0.31^*(0.115)$	$-1.20^*(0.071)$
c_1		$0.43^*(0.069)$	-		$0.86^*(0.078)$
ϕ_0	$0.40^*(0.066)$	$0.38^*(0.068)$	$0.38^*(0.090)$	$0.63^*(0.130)$	$2.38^*(0.067)$
ϕ_1		$0.40^*(0.069)$	-		$0.04(0.061)$
ψ_0	$0.88^*(0.044)$	$0.48^*(0.039)$		-0.31(0.169)	$-2.23^*(0.067)$
ψ_1		$1.33^*(0.096)$			$0.39^*(0.068)$
ω_0			$-0.32^*(0.114)$	$0.09^*(0.032)$	$2.23^*(0.067)$
ω_1			-		$0.21^*(0.056)$
α_0			$-2.63^*(1.015)$	$0.20^*(0.078)$	-
α_1			-		$0.20^*(0.066)$
β_0				$0.72^*(0.066)$	-
β_1					$0.09(0.064)$
η_{00}		$0.92^*(0.052)$	-		$0.17^*(0.069)$
η_{11}		$0.96^*(0.027)$	-		$0.86^*(0.062)$

Notes: Standard errors are reported in parentheses. The * denotes coefficient significant at the 5 percent level. The - denotes that the coefficient was set to zero value in order to estimate correctly the standard errors.

Table 17 Stationarity

	Model 1	Model 2	Model 3		Model 4		Model 5	
Strategy			Mean	Var	Mean	Var	Mean	Var
1	0.25	-1.25	-0.97	-1.66	0.33	0.60	0.60	0.89
2	0.07	-1.70	-1.11	-2.04	0.94	0.79	0.15	0.96
3	0.56	-1.04	-0.41	-1.83	0.53	-	0.86	0.79
4	0.29	-1.58	-0.28	-1.62	0.11	-	0.10	0.00
5	0.21	-2.23	-2.23	-10.51	0.13	0.15	0.80	0.08
6	0.49	-0.80	0.49	1.49	0.32	-	0.32	-
7	0.16	-1.71	-0.85	-1.97	0.36	0.55	0.51	0.85
8	0.17	-2.76	-2.98	-4.34	0.30	0.81	0.69	0.76
9	0.27	-0.99	-2.69	-9.93	0.85	0.89	0.29	1.53
10	0.40	-0.95	0.38	-2.63	0.63	0.92	0.45	0.25

Notes: Strategies: 1-Fund Weighted Composite, 2-Equity Market Neutral, 3-Convertible Arbitrage, 4-Event-driven, 5-Merger Arbitrage, 6-Distressed Securities, 7-Equity Hedge, 8-Macro, 9-Relative Value Arbitrage, 10-Fixed Income. The following values are presented in the table: Model 1: $|\phi| < 1$, Model 2: $\sum_{i=0}^{1} \pi_i \log(|\phi(i)|) < 0$, Model 3: (Mean) $\sum_{i=0}^{1} \pi_i \log(|\phi(i)|) < 0$, (Var) $\sum_{i=0}^{1} \pi_i E[\log(\alpha(i) u_t^2)] < 0$, Model 4: (Mean) $|\phi| < 1$, (Var) $\alpha + \beta < 1$, Model 5: We present the largest eigenvalue in modulus of the M (Mean) and V (Var) matrices that should be less than one for stationarity.

Table 18 NRMSE of one-step-ahead forecasts for Model 1, AR(p) specifications

Strategy	$p = 1$	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$
1	**0.135**	0.136	0.137	0.138	0.138	0.141
2	0.189	0.190	0.192	0.192	0.193	**0.180**
3	**0.146**	0.147	0.148	0.149	0.150	0.150
4	**0.139**	0.140	0.140	0.141	0.141	0.143
5	0.134	0.135	0.135	0.134	**0.133**	**0.133**
6	**0.119**	0.120	0.121	0.121	0.121	0.122
7	**0.153**	0.154	0.156	0.157	0.156	0.157
8	**0.179**	0.180	0.180	0.182	0.182	0.183
9	0.115	**0.113**	0.114	0.114	0.115	0.116
10	**0.150**	0.151	0.151	0.153	0.153	0.156

Strategy	$p = 7$	$p = 8$	$p = 9$	$p = 10$	$p = 11$	$p = 12$
1	0.144	0.147	0.147	0.148	0.151	0.153
2	0.181	0.183	0.184	0.184	0.185	0.186
3	0.151	0.153	0.154	0.155	0.155	0.157
4	0.143	0.145	0.145	0.146	0.147	0.148
5	**0.133**	0.135	0.136	0.136	0.137	0.137
6	0.123	0.123	0.125	0.125	0.127	0.127
7	0.158	0.159	0.161	0.162	0.164	0.165
8	0.184	0.184	0.183	0.185	0.188	0.196
9	0.117	0.118	0.117	0.117	0.117	0.117
10	0.156	0.157	0.156	0.156	0.157	0.157

Notes: Strategies: 1-Fund Weighted Composite, 2-Equity Market Neutral, 3-Convertible Arbitrage, 4-Event-driven, 5-Merger Arbitrage, 6-Distressed Securities, 7-Equity Hedge, 8-Macro, 9-Relative Value Arbitrage, 10-Fixed Income. Numbers written by **bold** letters denote the model with the best forecasting performance.

Table 19 NRMSE of one-step-ahead forecasts of Models 1-5

Strategy	Model 1	Model 2	Model 3	Model 4	Model 5
1	0.135	0.133	0.135	0.137	**0.130**
2	0.189	0.180	0.178	0.191	**0.175**
3	**0.146**	0.152	0.148	0.149	0.148
4	**0.139**	0.142	0.148	**0.139**	0.140
5	**0.134**	0.142	0.147	0.135	0.155
6	0.119	0.121	**0.118**	0.119	**0.118**
7	0.153	0.153	0.153	0.153	**0.146**
8	0.179	**0.176**	0.178	0.180	0.177
9	0.115	**0.110**	**0.110**	0.114	0.112
10	0.150	0.152	0.150	0.154	**0.147**

Notes: Strategies: 1-Fund Weighted Composite, 2-Equity Market Neutral, 3-Convertible Arbitrage, 4-Event-driven, 5-Merger Arbitrage, 6-Distressed Securities, 7-Equity Hedge, 8-Macro, 9-Relative Value Arbitrage, 10-Fixed Income. Numbers written by **bold** letters denote the model with the best forecasting performance. For Model 1, we report the AR(1) specification results presented in Table 18 in order to compare the first order econometric models.

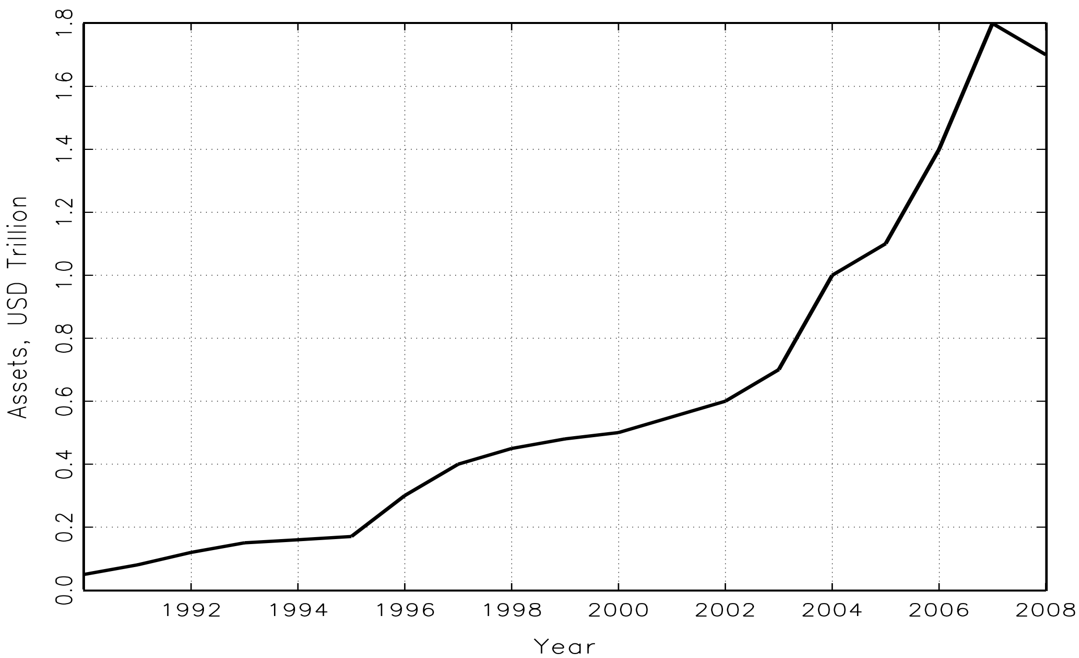


Figure 1 Hedge funds assets' value between 1990 and 2008 (Source: The Economist, 2008)

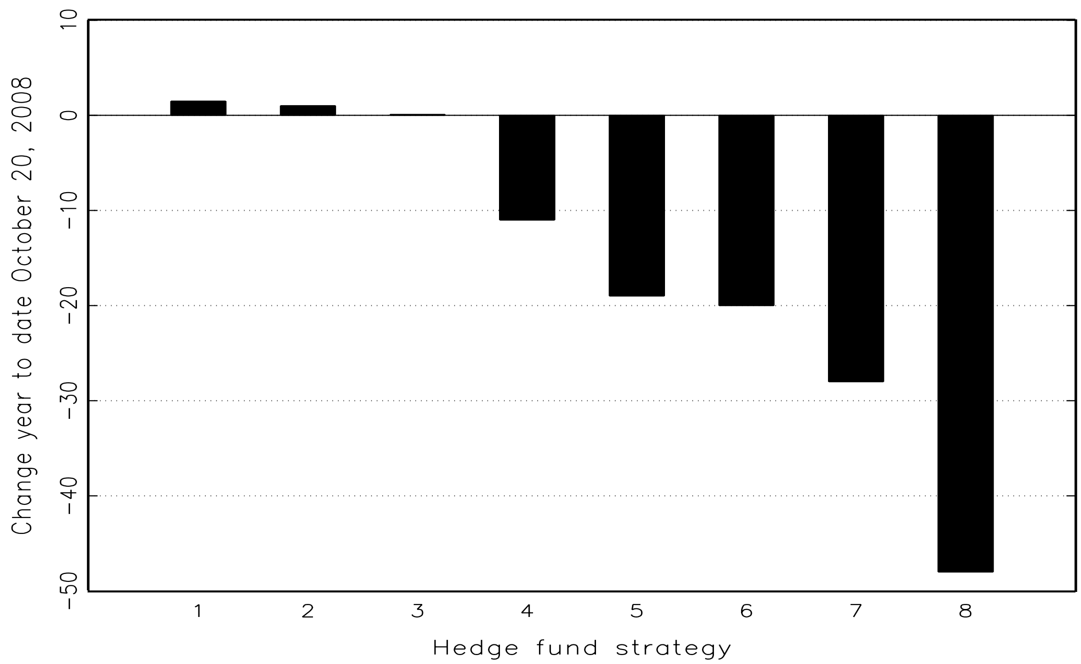


Figure 2 Price changes (%) of several hedge fund strategies over 2008 (Source: The Economist, 2008)

Notes: Strategies: 1-Merger Arbitrage, 2-Macro, 3-Equity Market Neutral, 4-Distressed Securities, 5-Event-driven, 6-Equity Hedge, 7-Relative Value Arbitrage, 8-Convertible Arbitrage.

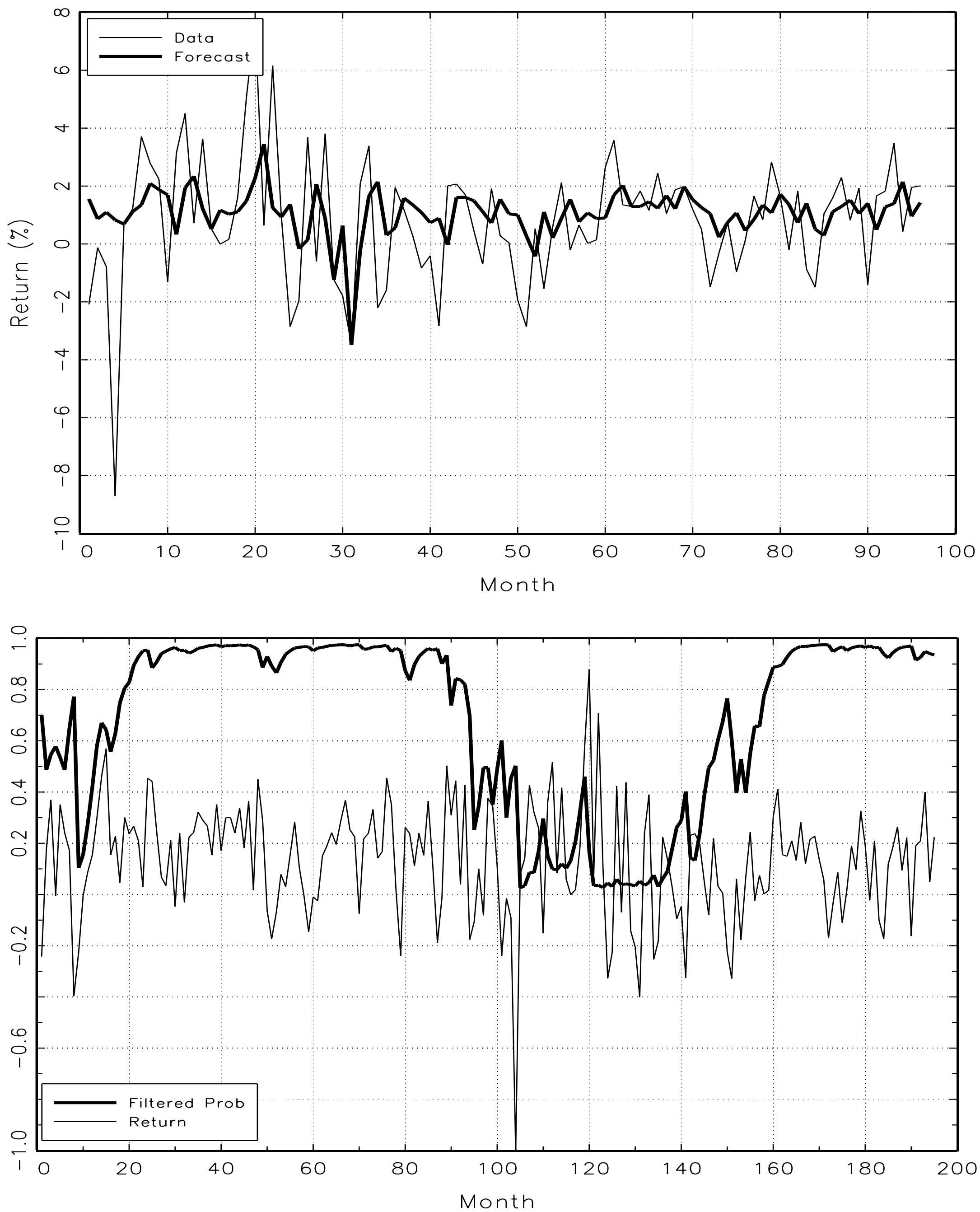


Figure 3 Fund Weighted Composite

Notes: The first figure shows the observed and forecasted return time series for the May 1998 - April 2006 time period using the estimates of the lowest NRMSE model: MS-ARMA(1,1)-GARCH(1,1). The second figure presents the evolution of the probability of being in regime 0, i.e. $\pi_{0t} = \Pr[s_t = 0|Y_{t-1}]$ for the January 1990 - April 2006 period for the model with the lowest BIC: MS-AR(1).

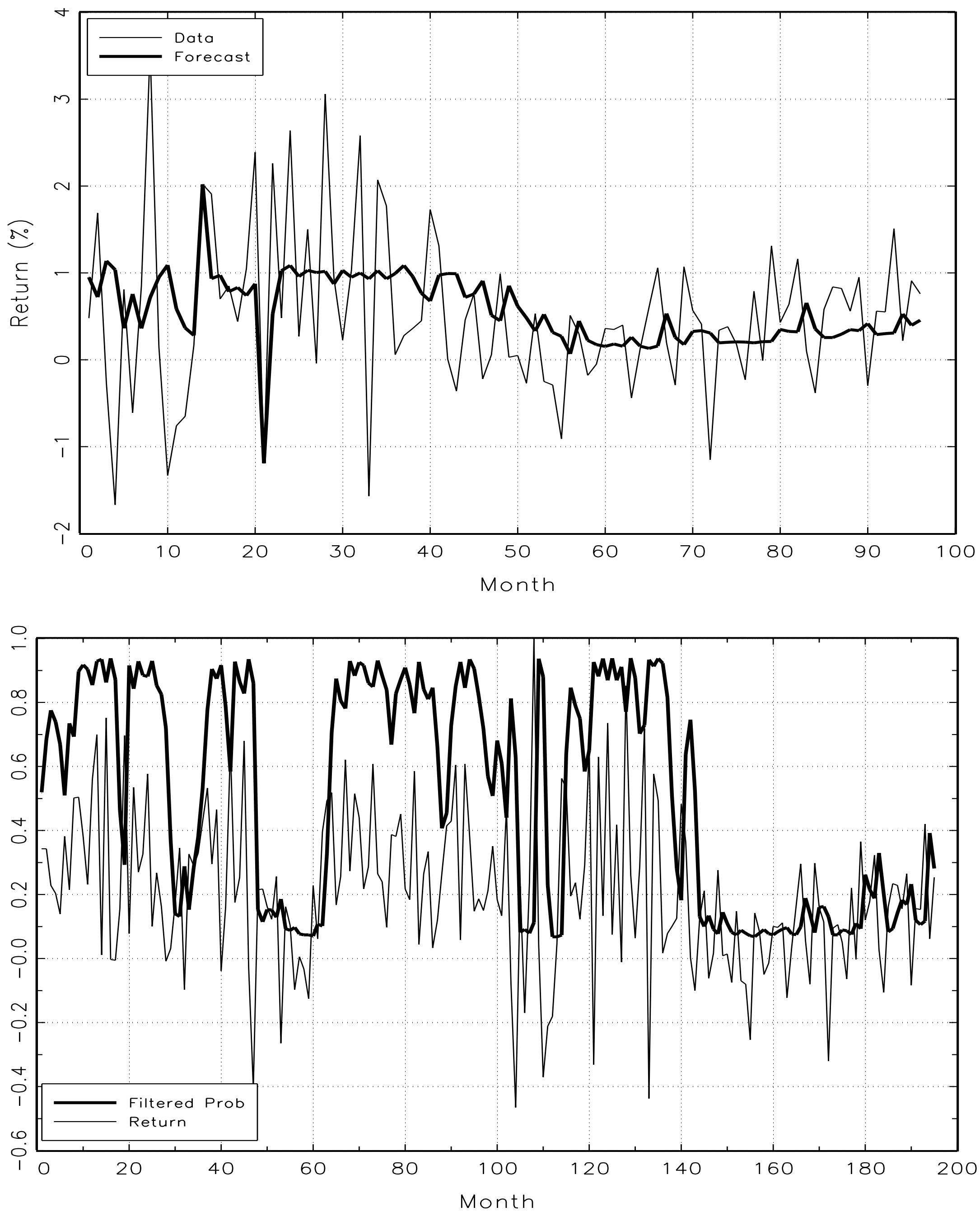
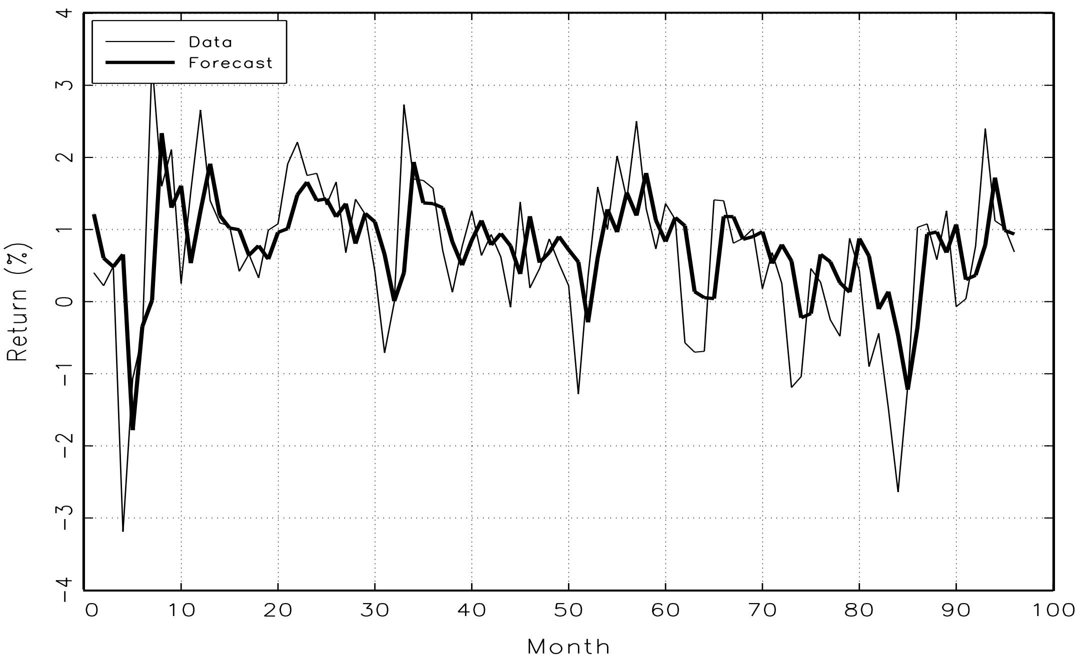


Figure 4 Equity Market Neutral

Notes: The first figure shows the observed and forecasted return time series for the May 1998 - April 2006 time period using the estimates of the lowest NRMSE model: MS-ARMA(1,1)-GARCH(1,1). The second figure presents the evolution of the probability of being in regime 0, i.e. $\pi_{0t} = \Pr[s_t = 0|Y_{t-1}]$ for the January 1990 - April 2006 period for the model with the lowest BIC: MS-AR(1).



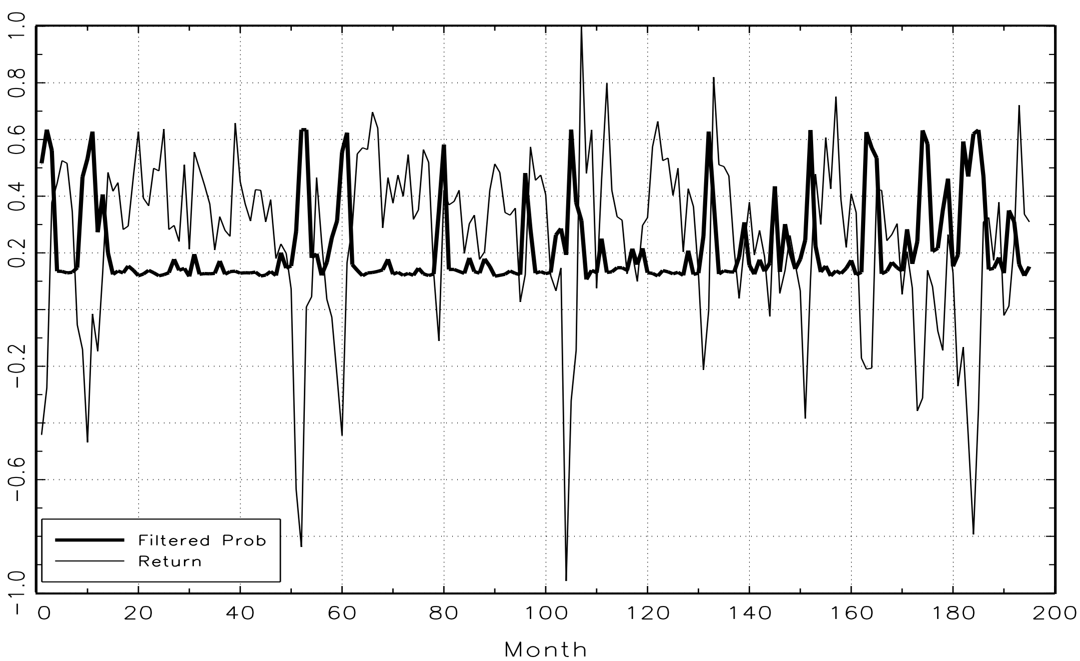


Figure 5 Convertible Arbitrage

Notes: The first figure shows the observed and forecasted return time series for the May 1998 - April 2006 time period using the estimates of the lowest NRMSE model: AR(1). The second figure presents the evolution of the probability of being in regime 0, i.e. $\pi_{0t} = \Pr[s_t = 0|Y_{t-1}]$ for the January 1990 - April 2006 period for the model with the lowest BIC: MS-ARMA(1,1)-GARCH(1,1).

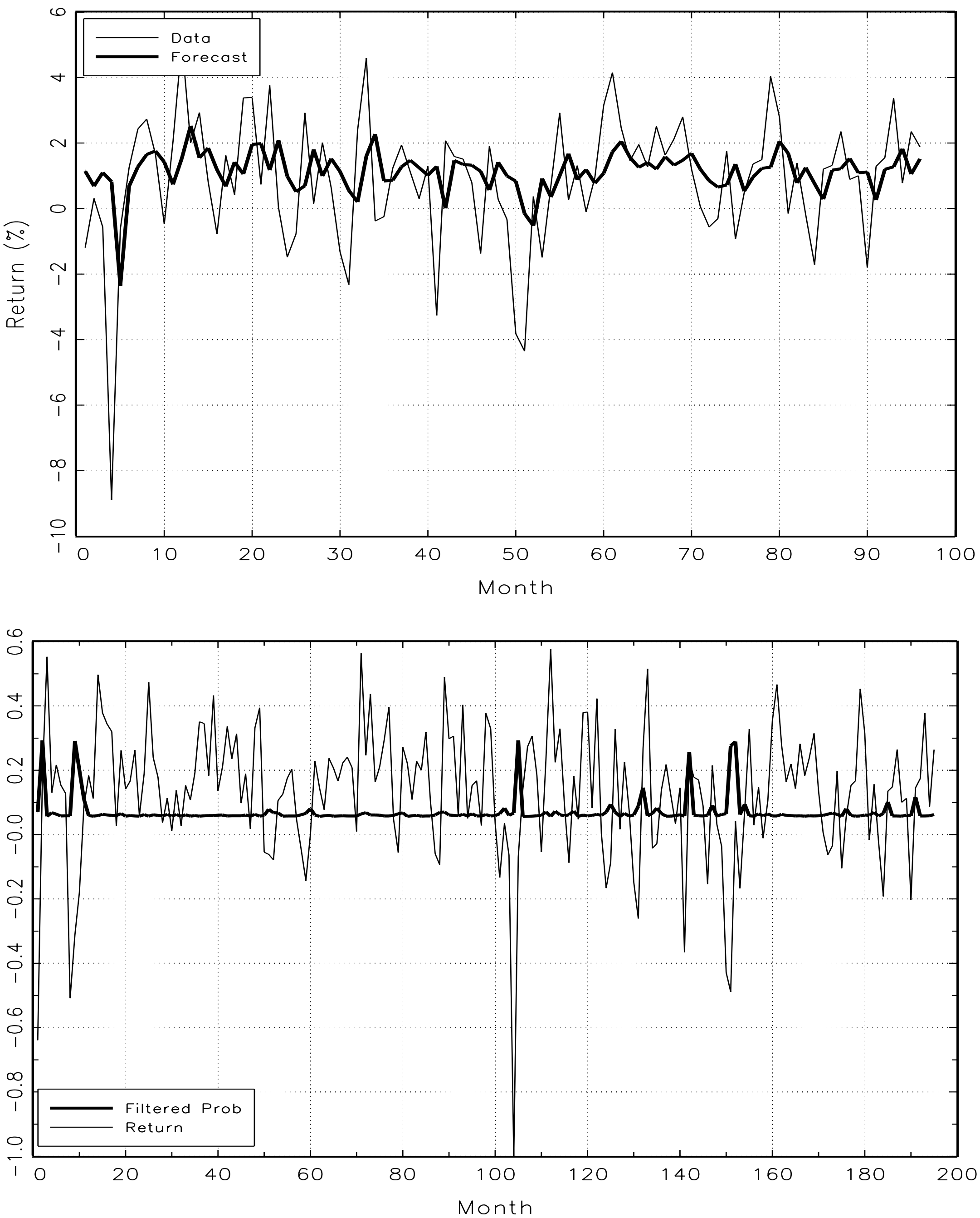


Figure 6 Event-driven

Notes: The first figure shows the observed and forecasted return time series for the May 1998 - April 2006 time period using the estimates of the lowest NRMSE model: AR(1). The second figure presents the evolution of the probability of being in regime 0, i.e. $\pi_{0t} = \Pr[s_t = 0|Y_{t-1}]$ for the January 1990 - April 2006 period for the model with the lowest BIC: MS-AR(1).

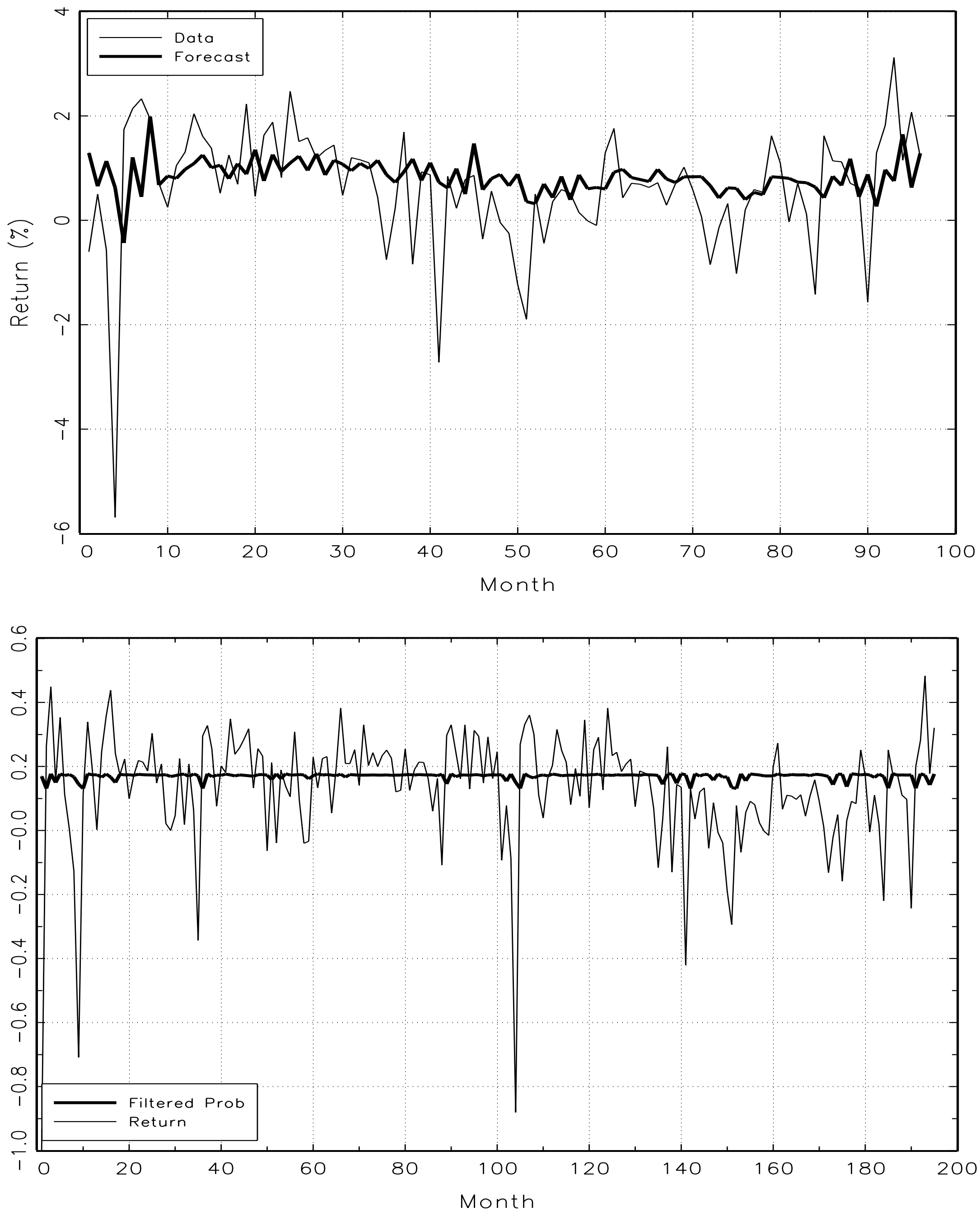


Figure 7 Merger Arbitrage

Notes: The first figure shows the observed and forecasted return time series for the May 1998 - April 2006 time period using the estimates of the lowest NRMSE model: AR(5). The second figure presents the evolution of the probability of being in regime 0, i.e. $\pi_{0t} = \Pr[s_t = 0|Y_{t-1}]$ for the January 1990 - April 2006 period for the model with the lowest BIC: MS-AR(1).

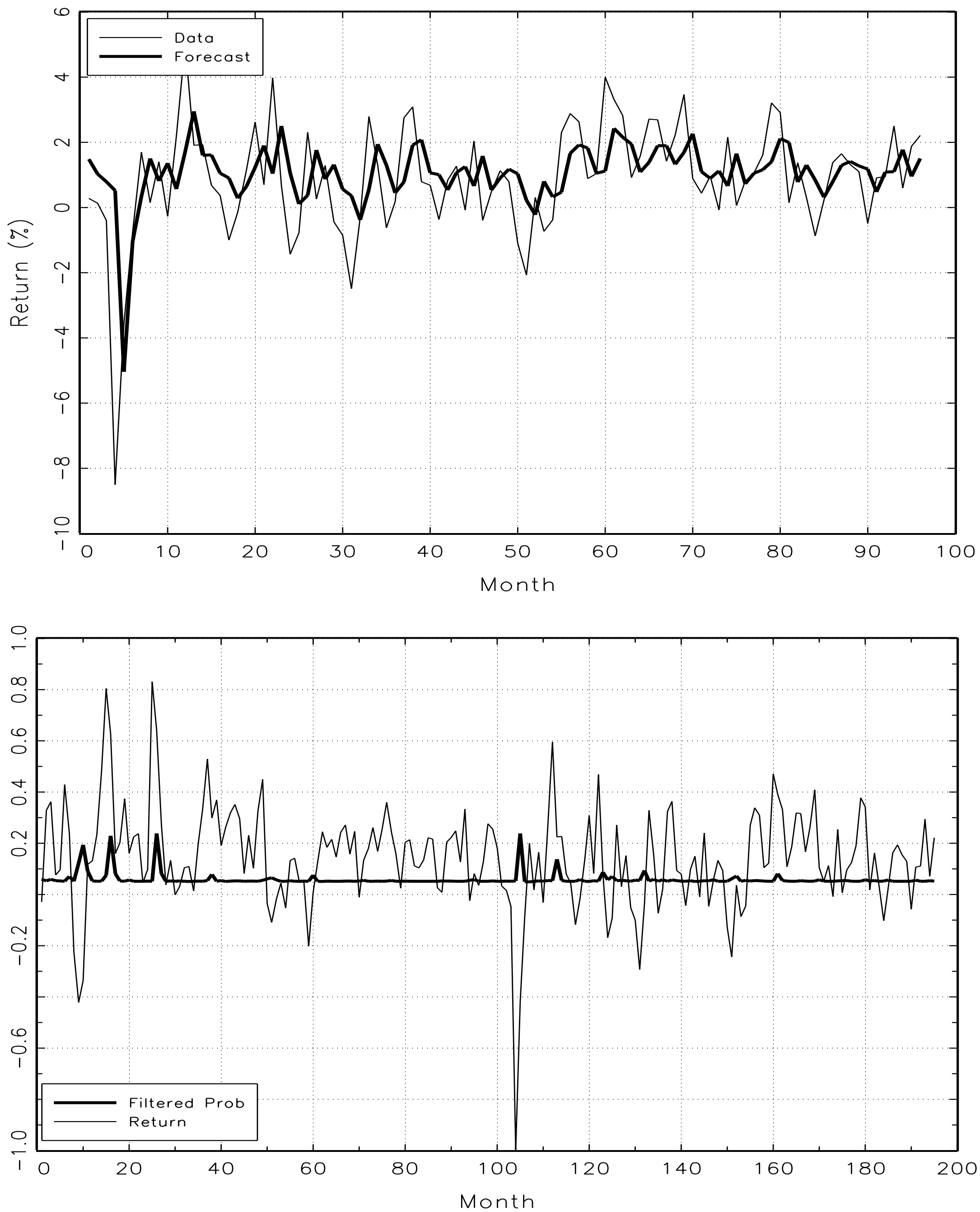
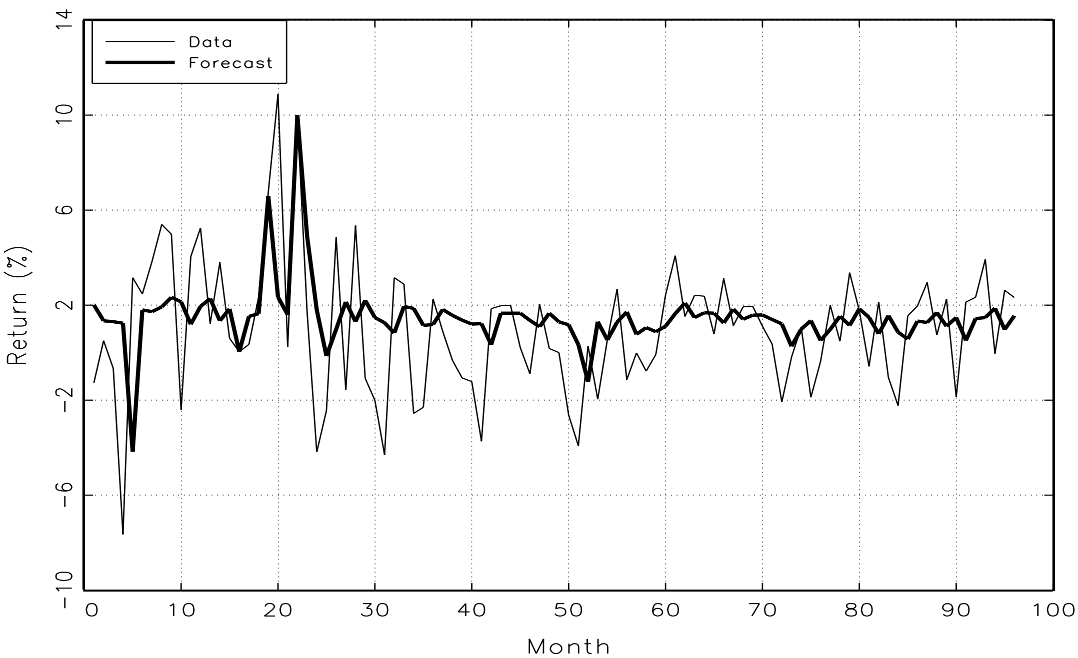


Figure 8 Distressed Securities

Notes: The first figure shows the observed and forecasted return time series for the May 1998 - April 2006 time period using the estimates of the lowest NRMSE model: MS-ARMA(1,1)-GARCH(1,1). The second figure presents the evolution of the probability of being in regime 0, i.e. $\pi_{0t} = \Pr[s_t = 0|Y_{t-1}]$ for the January 1990 - April 2006 period for the model with the lowest BIC: MS-AR(1).



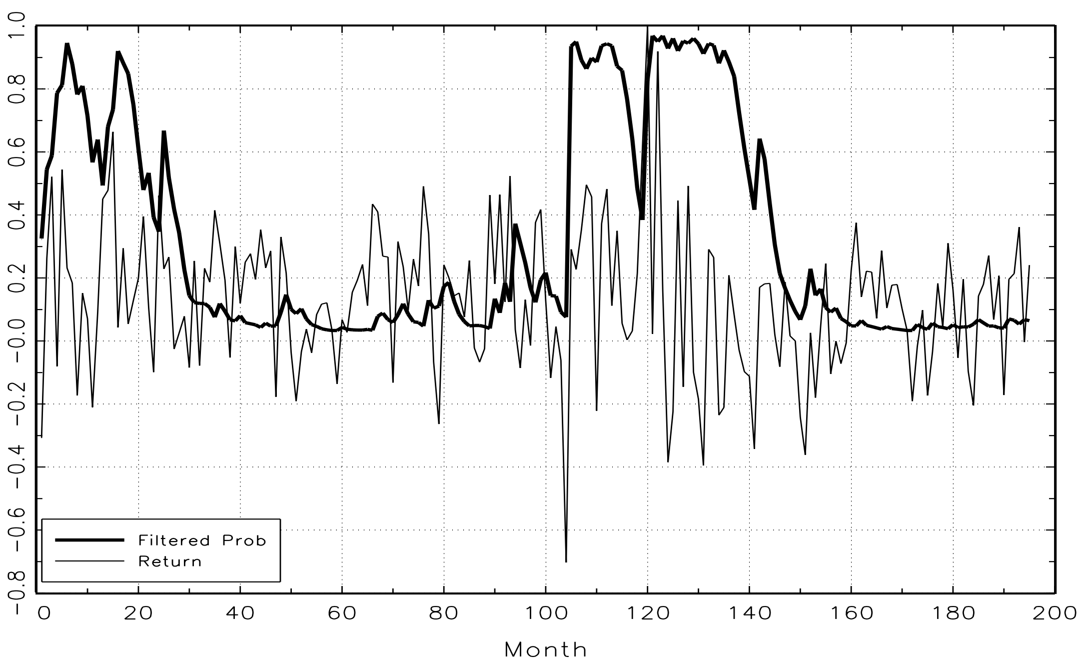


Figure 9 Equity Hedge

Notes: The first figure shows the observed and forecasted return time series for the May 1998 - April 2006 time period using the estimates of the lowest NRMSE model: MS-ARMA(1,1)-GARCH(1,1). The second figure presents the evolution of the probability of being in regime 0, i.e. $\pi_{0t} = \Pr[s_t = 0|Y_{t-1}]$ for the January 1990 - April 2006 period for the model with the lowest BIC: MS-AR(1).

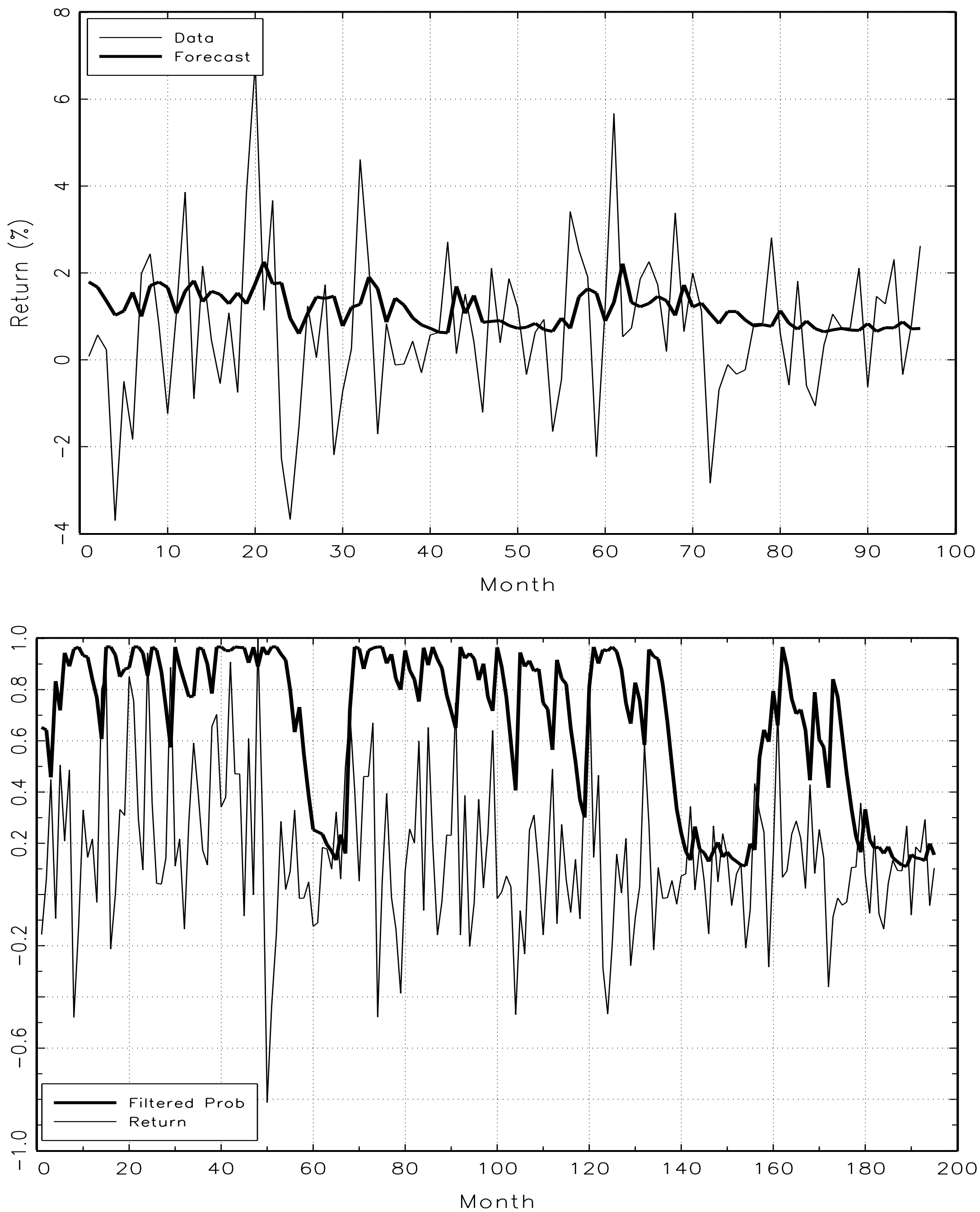


Figure 10 Macro

Notes: The first figure shows the observed and forecasted return time series for the May 1998 - April 2006 time period using the estimates of the lowest NRMSE model: MS-AR(1). The second figure presents the evolution of the probability of being in regime 0, i.e. $\pi_{0t} = \Pr[s_t = 0|Y_{t-1}]$ for the January 1990 - April 2006 period for the model with the lowest BIC: MS-AR(1).

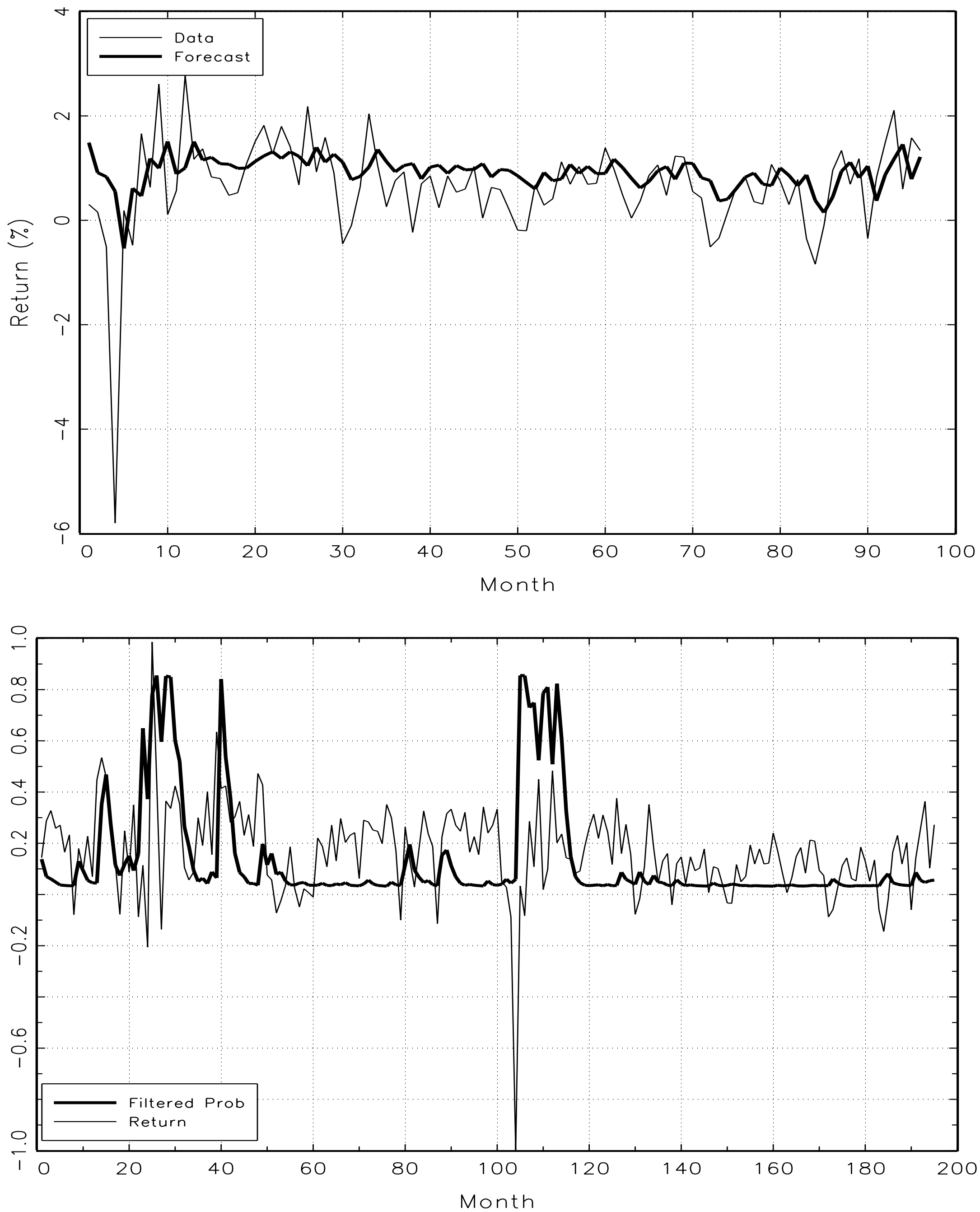


Figure 11 Relative Value Arbitrage

Notes: The first figure shows the observed and forecasted return time series for the May 1998 - April 2006 time period using the estimates of the lowest NRMSE model: MS-AR(1). The second figure presents the evolution of the probability of being in regime 0, i.e. $\pi_{0t} = \Pr[s_t = 0|Y_{t-1}]$ for the January 1990 - April 2006 period for the model with the lowest BIC: MS-AR(1).

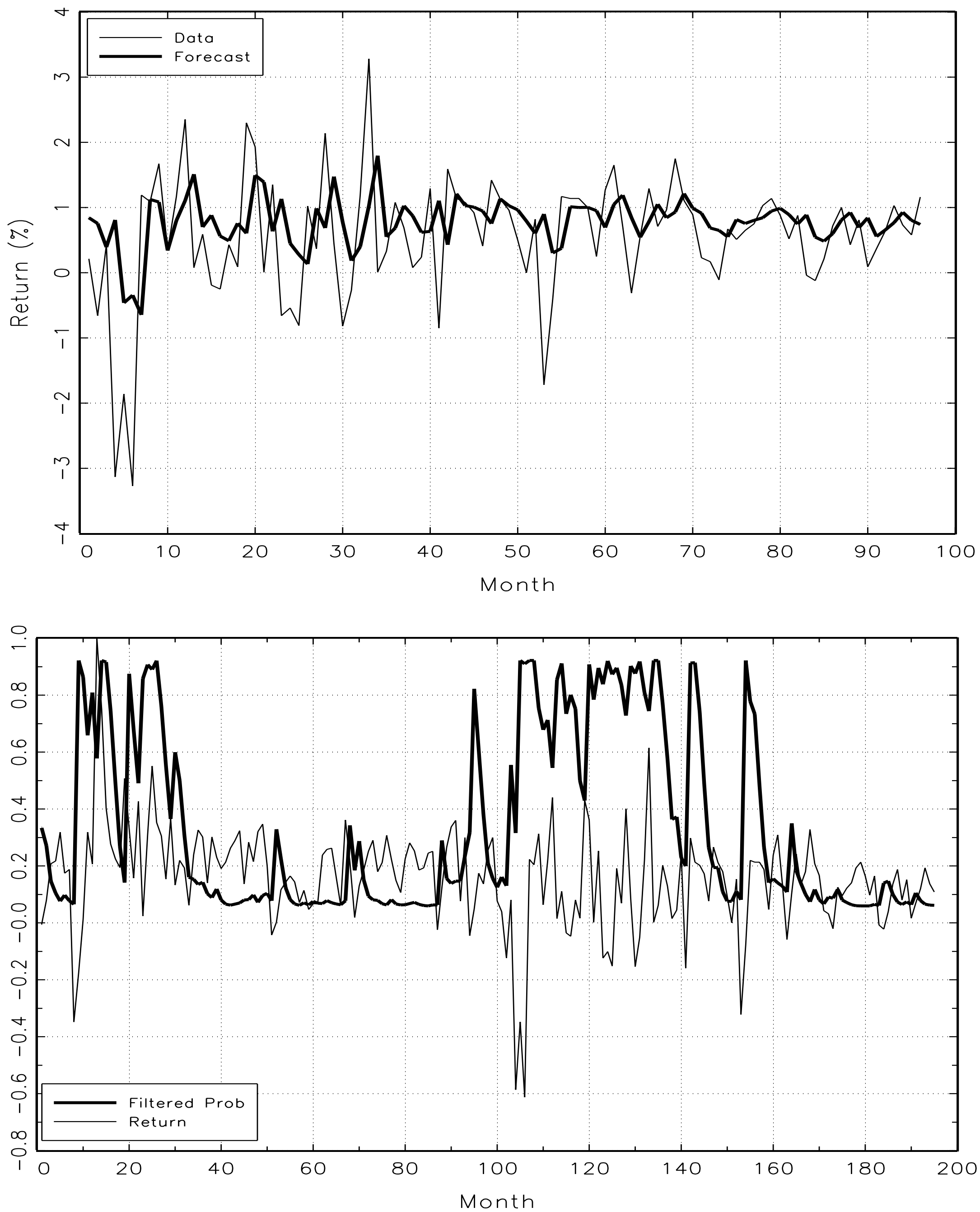


Figure 12 Fixed Income

Notes: The first figure shows the observed and forecasted return time series for the May 1998 - April 2006 time period using the estimates of the lowest NRMSE model: MS-ARMA(1,1)-GARCH(1,1). The second figure presents the evolution of the probability of being in regime 0, i.e. $\pi_{0t} = \Pr[s_t = 0|Y_{t-1}]$ for the January 1990 - April 2006 period for the model with the lowest BIC: MS-AR(1).