Facultad de Ciencias Económicas y Empresariales
Universidad de Navarra

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Testing of nonstationarities in the unit circle, long memory processes and day of the week effects in financial data

Luis A. Gil-Alana
Facultad de Ciencias Económicas y Empresariales
Universidad de Navarra

Michael Nazarski

and

Guglielmo M. Caporale
Faculty of Economics and Business Administration
Brunel University

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ABSTRACT

This paper examines a version of the tests of Robinson (1994) that enables one to test models of the form $(1-L^k)^d x_t = u_t$, where k is an integer value, d may be any real number, and u_t is I(0). The most common cases are those with k = 1 (unit or fractional roots) and k = 4 and 12 (seasonal unit or fractional models). However, we extend the analysis to cover situations such as $(1-L^5)^d x_t = u_t$, which might be relevant, for example, in the context of financial time series data. We apply these techniques to the daily Eurodollar rate and Dow Jones index, and find that for the former series the most adequate specifications are either a pure random walk or a model of the form $x_t = x_{t-5} + \varepsilon_t$, implying in both cases that the returns are completely unpredictable. In the case of Dow Jones index, a model of the form $(1-L^5)^d x_t = u_t$ is selected, with d constrained between 0.50 and 1, implying nonstationarity and mean-reverting behaviour.

Luis A. Gil-Alana
University of Navarra
Faculty of Economics
Edificio Biblioteca, E. Este
E-31080 Pamplona, Spain
alana@unav.es

Mike Nazarski
10 Discovery Walk, Reardon Street
E1W 2JG Wapping, London
ENGLAND
mnazar@sgh.waw.pl

Guglielmo M. Caporale
Centre for Empirical Finance
Brunel University
Uxbridge, Middlesex UB8 3PH
ENGLAND
Guglielmo-Maria.Caporale@brunel.ac.uk

1. INTRODUCTION

Nonstationarity is a characteristic of many economic and financial time series. The unit root polynomial is the most natural way of modelling this behaviour: once a time series is first-differenced, it is assumed to be stationary, or, more precisely, integrated of order 0 (denoted by I(0)). For the purpose of the present paper, we define an I(0) process in its most general form, i.e., as a covariance stationary process with a spectral density function that is positive and finite not only at zero but at any frequency on the spectrum.

Following the work of Box and Jenkins (1970), the unit root model became very popular, especially after the seminal paper of Nelson and Plosser (1982). Using the tests of Fuller (1976) and Dickey and Fuller (1979), these authors were unable to reject the presence of a unit root in fourteen US macroeconomic series. Such tests and others that have been proposed later (Phillips, 1987; Phillips and Perron, 1988; Kwiatkowski et al., KPSS, 1992)[1] are based on autoregressive (AR) alternatives, in the sense that they are nested in a model of the form:

$$(1 - \alpha L)\, x_t \;=\; u_t, \quad t \;=\; 1, 2, \ldots, \qquad (1)$$

where L is the lag operator (i.e., $Lx_t = x_{t-1}$), and where the unit root corresponds to the null $\alpha = 1$. However, a problem with these procedures is that their limit distributions are non-standard, and therefore the critical values have to be obtained numerically by performing simulations. This is a consequence of the abrupt change in the asymptotic behaviour of the distributions around the unit root. Note that x_t in (1) is stationary for $|\alpha| < 1$; it is nonstationary but non-explosive for $|\alpha| = 1$; and it becomes explosive for $|\alpha| > 1$. This has motivated the use of other models for testing unit roots. In particular, fractional processes have been considered as an alternative to AR models. In this context, the unit root hypothesis is tested within the model:

$$(1-L)^d \, x_t \;=\; u_t , \quad t \;=\; 1, 2, ..., \qquad (2)$$

where d may be any real number, and the unit root case corresponds to d = 1. Here, the limit behaviour is smooth around the unit root null, and d = 0.5 becomes the crucial parameter to distinguish between stationarity and nonstationarity. Examples of testing procedures using the fractional model (2) are, amongst others, Sowell (1992), Robinson (1994, 1995a,b), Tanaka (1999) and Dolado et al. (2001).

Many series also contain seasonal fluctuations, and stochastic models based on seasonal unit roots have been proposed in recent years. As in the previous case, most of the available procedures (Dickey, Hasza and Fuller, DHF, 1984; Hylleberg, Engle, Granger and Yoo, HEGY, 1990; etc.) are based on AR alternatives, and therefore face the same problem concerning the limit behaviour of the distribution. New approaches based on (seasonal) fractional integration have been proposed by Porter-Hudak (1990) and Robinson (1994). They consider processes of the form:

$$(1-L^k)^d \, x_t \;=\; u_t , \quad t \;=\; 1, 2, ..., \qquad (3)$$

where k is the number of time periods within the year, and L^k is the seasonal lag-operator ($L^k x_t = x_{t-k}$). Note that the polynomial on the left-hand-side of (3) can be expressed in terms of its Binomial expansion, such that, for all real d,

$$(1-L^k)^d \;=\; \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^{kj} \;=\; 1 \,-\, d\,L^k \,+\, \frac{d\,(d-1)}{2} L^{2k} \,-\, ...$$

Clearly, if d = 0 in (3), $x_t = u_t$, and a weakly autocorrelated x_t is allowed for. If d > 0, the process is said to be a long memory one, so named because of the strong association between observations far apart in time, and the higher is d, the stronger will be the association.

The literature on seasonal processes has usually concentrated on the cases of k = 4 (quarterly) or k = 12 (monthly observations). In this paper, we examine different versions of the tests of Robinson (1994) that enable us to test models like (3) for any integer k and any real value d. Thus, we consider unit (and fractional) orders of integration at the zero and the seasonal frequencies, but also at other frequencies in the interval (0, π]. Specifically, we analyse the following processes:

i) I(1) processes: when k = d = 1. (see Dickey and Fuller, 1979; Phillips and Perron, 1988; KPSS, 1992).

ii) I(d) processes: when k = 1 and d is a real value. (see Diebold and Rudebusch, 1989; Baillie, 1996; Gil-Alana and Robinson, 1997).

iii) Seasonal unit roots: when k = 4 and d = 1. (see DHF, 1984; HEGY, 1992; Beaulieu and Miron, 1992, for k = 12).

iv) Seasonal fractional models: when k = 4, 12 and d is a real value. (see Porter-Hudak, 1990; Ray, 1993; Sutcliffe, 1994),

but also processes of the form $(1-L^3)^d x_t = u_t$, or $(1-L^5)^d x_t = u_t$.

Let us focus on the last type of model. If d = 1, this implies that the present value of the series (x_t) depends exclusively on its value five periods before (x_{t-5}), and if d is real, it will depend not only on x_{t-5} but on all past observations which are backwards multiples of 5, i.e., x_{t-10}, x_{t-15}, … This type of model is relevant, for example, in the context of daily financial data, where the value of an asset on a given day of the week may be strongly influenced by its value on the same day of the previous week. There is in fact an extensive literature documenting the presence of calendar anomalies (such as the weekend effect, the day of the week effect, and the January effect) in financial series, both in the US and in other developed markets, dating back to Osborne (1962). Negative Monday returns were found, inter alia, by Cross (1973), French (1980), and Gibbons and Hess (1981), the former two analysing the S&P 500 index, the latter the Dow Jones Industrial Index. Similar findings have been reported for other US financial markets, such as the futures, bond and Treasury bill markets (Cornell, 1985, Dyl and Maberly, 1986), foreign exchange markets (Hsieh, 1988), and for Australian, Canadian, Japanese and UK financial markets (e.g., Jaffe and Westerfield, 1985, Jaffe, Westerfield and Ma, 1989, Agrawal and Tandon, 1994). Effects on stock market volatility have also been documented (Kiymaz and Berument, 2003).

Various explanations have been offered for the observed patterns. Some focus on delays between trading and settlement in stocks (Gibbons and Hess, 1981): buying on Fridays creates a two-day interest-free loan until settlement; hence, there are higher transaction volumes on Fridays, resulting in higher prices, which decline over the weekend as this incentive disappears. Others emphasise a shift in the broker-investor balance in buying-selling decisions which occurs on weekends, when investors have more time to study the market themselves (rather than rely on brokers); this typically results in net sales on Mondays, when liquidity is low in the absence of institutional trading (Miller, 1988). It has also been suggested that the Monday effect largely reflects the fact that, when daily returns are calculated, the clustering of dividend payments around Mondays is normally ignored; alternatively, it could be a consequence of positive news typically being released during the week, and negative ones over the weekend (Fortune, 1998). Additional factors which could be relevant are serial correlation, with Monday prices being affected by Friday ones, and a negative stock performance on

Fridays being given more weight (Abraham and Ikenberry, 1994); measurement errors (Keim and Stambaugh, 1984); size (Fama and French, 1992); volume (Lakonishok and Mayberly, 1990).

Further empirical evidence on weekday effects is provided in the present study using fractional integration techniques. In particular, we use a methodology that allows us to consider fractional processes where the dependence between the observations is a function of a specific day of the week, not only Monday as in earlier studies, but any day of the week. Of particular interest is the order of (fractional) integration of the series. If it is smaller than 1, shocks affecting the weekly structure will be mean reverting, and their effects will disappear in the long run. On the other hand, if it is 1 or higher, shocks will persist forever, and strong measures will be required to bring the variable back to its original level.

The outline of the paper is as follows: Section 2 describes the different versions of the tests of Robinson (1994) we employ. Section 3 presents a Monte Carlo simulation study, examining the size and power properties of these tests in finite samples. Section 4 discusses two empirical applications based on financial data, while Section 5 offers some concluding remarks.

2. TESTING OF NONSTATIONARITIES IN THE UNIT CIRCLE

Following Bhargava (1986), Schmidt and Phillips (1992) and other studies in the parameterisation of unit roots, Robinson (1994) considers the regression model:

$$y_t \;=\; \beta' z_t \;+\; x_t, \qquad t \;=\; 1, 2, \ldots \qquad (4)$$

where y_t is the time series we observe; β is a (kx1) vector of unknown parameters; z_t is a (kx1) vector of regressors, and x_t are the regression errors, taking the form:

$$\rho(L;\theta)\, x_t \;=\; u_t, \quad t \;=\; 1, 2, ..., \qquad (5)$$

where ρ is a scalar function that depends on L and the unknown parameter θ that will adopt different forms as below, and u_t is I(0). The function ρ is specified in such a way that all its roots should be on the unit circle in the complex plane, and therefore it includes polynomials of the form $(1\text{-}L^k)^{d+\theta}$, where k is an integer and d may be a real value. Thus, in what follows, we assume that

$$\rho(L;\theta) \;=\; (1 - L^k)^{d+\theta}. \qquad (6)$$

Robinson (1994) proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o: \;\theta \;=\; 0, . \qquad (7)$$

in a model given by (4) - (6). Based on H_o (7), the estimated β and residuals are:

$$\hat{u}_t \;=\; (1 - L^k)^{d_o}\; y_t \;-\; \hat{\beta}\,' w_t,$$

$$w_t \;=\; (1 - L^k)^{d_o}\; z_t; \quad \hat{\beta} = \left(\sum_{t=1}^{T} w_t \; w_t' \right)^{-1} \sum_{t=1}^{T} w_t \,(1 - L^k)^{d_o}\; y_t.$$

The functional form of the test statistic is given by:

$$\hat{r} \;=\; \frac{T^{1/2}}{\hat{\sigma}^2}\, \hat{A}^{-1/2}\, \hat{a} \qquad (8)$$

where T is the sample size and

$$\hat{A} \;=\; \frac{2}{T} \left(\sum_{j=1}^{*} \psi(\lambda_j)^2 \;-\; \sum_{j=1}^{*} \psi(\lambda_j)\,\hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{*} \hat{\varepsilon}(\lambda_j)\,\hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{*1} \hat{\varepsilon}(\lambda_j)\psi(\lambda_j) \right) \qquad (9)$$

$$\hat{a} \;=\; \frac{-2\pi}{T} \sum_{j=1}^{*} \psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j); \qquad \hat{\sigma}^2 \;=\; \sigma^2(\hat{\tau}) \;=\; \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j);$$

$$\hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j;\hat{\tau}); \qquad \lambda_j = \frac{2\pi\; j}{T}; \qquad \hat{\tau} \;=\; \arg min_{\tau \in T^*}\, \sigma^2(\tau).$$

and the sums over * in the above expressions are over $\lambda \in$ M where M = $\{\lambda$: $-\pi < \lambda < \pi$, $\lambda \notin (\rho_l - \lambda_1, \rho_l + \lambda_1)$, l = 1, 2, …, s$\}$ such that ρ_l, l = 1, 2, …, s < ∞ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$. Also,

$$\psi(\lambda_j) = \mathrm{Re}\left[\log\left(\frac{\partial}{\partial\theta}\log\rho(e^{i\lambda_j};\theta)\right)\right]_{\theta=0}, \qquad (10)$$

and $I(\lambda_j)$ is the periodogram of u_t evaluated under the null. The function g above is a known function coming from the spectral density of u_t,

$$f(\lambda;\sigma^2;\tau) \; = \; \frac{\sigma^2}{2\pi} g(\lambda;\tau), \qquad -\pi < \lambda \leq \pi.$$

Note that these tests are purely parametric, and, therefore, they require specific modelling assumptions about the short memory specification of u_t. Thus, if u_t is a white noise, then g ≡ 1, (and thus, $\hat{\varepsilon}(\lambda_j) = 0$), and if it is an AR process of the form $\phi(L)u_t = \varepsilon_t$, $g = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ.

Based on H_o (7), Robinson (1994) showed that under certain very mild regularity conditions:[2]

$$\hat{r} \;\to_d\; N(0,1) \quad as \quad T \to \infty. \qquad (11)$$

Hence, we are in a classical large sample-testing situation: an approximate one-sided 100α% level test of H_o (7) against the alternative: H_a: $d > d_o$ ($d < d_o$) will be given by the rule: "Reject H_o if $\hat{r} > z_\alpha$ ($\hat{r} < - z_\alpha$)", where the probability that a standard normal variate exceeds z_α is α.

Note that given the functional form of ρ in (6),

$$\psi(\lambda_j) = \mathrm{Re}\left[\left(\frac{\partial}{\partial\theta}\log\rho(e^{i\lambda_j};\theta)\right)\right]_{\theta=0} = \mathrm{Re}\left[\frac{\partial}{\partial\theta}(d+\theta)\log(1-e^{i\lambda_j k})\right] = \mathrm{Re}\left[\log(1-e^{i\lambda_j k})\right] =$$

$$\mathrm{Re}\left[\log(1-\cos\lambda_j k - i\sin\lambda_j k\right] = \log\left|(1-\cos\lambda_j k - i\sin\lambda_j\right| = \log\left(2-2\cos\lambda_j k\right)^{0.5}.$$

In some simple cases, the above formula simplifies. Thus, for example, if k = 1,

$$\psi(\lambda_j) \;=\; \log\left|2\sin\frac{\lambda_j}{2}\right|.$$

If k = 2, and noting that $(1 - e^{i2\lambda}) = (1 - e^{i\lambda})(1 - e^{-i\lambda})$,

$$\psi(\lambda_j) \;=\; \log\left|2\sin\frac{\lambda_j}{2}\right| \;+\; \log\left(\;2\cos\frac{\lambda_j}{2}\right)$$

and similarly, if k = 4,

$$\psi(\lambda_j) \;=\; \log\left|2\sin\frac{\lambda_j}{2}\right| \;+\; \log\left(\;2\cos\frac{\lambda_j}{2}\right) \;+\; \log\left|\,2\cos\lambda_j\right|.$$

A common feature of all these expressions is that they have a finite number (k) of poles across the spectrum, but they are all squared integrable. Figure 1 displays plots of the $\psi(\lambda)$-functions for k = 1, 2, 3, 4, 5 and 6. The poles are clearly identified in the plots.

(INSERT FIGURE 1 ABOUT HERE)

Various forms of the test statistic described above have been applied in empirical studies. For example, Gil-Alana and Robinson (1997) used a version of the above tests with k = 1 and d being a real number, and Gil-Alana (1999) and Gil-Alana and Robinson (2001) extended the analysis to the case of k = 12 and 4 respectively. However, there are no previous empirical studies considering, as in the present paper, the case of k = 5, which is particularly relevant in the context of financial daily data for the reasons outlined in Section 1.

3. A MONTE CARLO SIMULATION STUDY

We start by examining the size of the above versions of the tests of Robinson (1994). We consider six null models of the form:

$$(1 - L^k)\, x_t \;=\; u_t, \quad t \;=\; 1, 2, ...,$$

with k = 1, 2, 3, 4, 5 and 6, with white noise u_t. We generate Gaussian series using the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986), with 10,000 replications in each case. The sample size is T = 50, 100, 300, 500 and 1000 observations, and the nominal size is 10% in Table 1, and 5% in Table 2.

(INSERT TABLES 1 AND 2 ABOUT HERE)

We can see from these two tables that if the sample size is small (e.g., T = 50), there is a significant positive bias, especially for the cases of k = 3, 4 and 6. However, as the sample size increases, the empirical sizes approximate the nominal ones. Thus, for example, if T = 1000, the sizes range between 10.6% and 11.8% in Table 1, and between 5.3% and 6.8% in Table 2.

Tables 3 – 5 display the rejection probabilities of the above versions of the tests when looking at alternatives of the form given in (6) with d = 1 and θ = -1, -0.75, -0.50, -025, 0.25, 0.50, 0.75 and 1, assuming that k is correctly specified, that is, it is the same under both the null and the alternative hypotheses.

(INSERT TABLES 3 -5 ABOUT HERE)

Table 3 reports the results for T = 100, while Tables 4 and 5 refer to T = 300 and 500 respectively. We observe that even for the smallest sample size (T = 100, in Table 3), the rejection probabilities are high even for small departures from the null. The lowest values are obtained for the cases of k = 3 and 5 (with $\theta > 0$) and k = 6 (with $\theta < 0$). However, if $\theta \leq$ -0.50 or $\theta \geq$ 0.50, the values are higher than 0.90 in all cases. If T = 300 (Table 4), the rejection probabilities exceed 0.90 even for $\theta = \pm 0.25$, and they are all 1 with $|\theta| > 0.50$. Finally, if T = 500 (Table 5), the values are equal to 1 in all cases.

In brief, the versions of the tests of Robinson (1994) considered in this paper appear to perform well in finite samples, especially if the sample size is relatively large (i.e., $T \geq 300$).

4. TWO EMPIRICAL APPLICATIONS

Two financial time series are analysed in this section. The first is the Eurodollar rate, daily (from Monday to Friday), for the time period January 9, 1995 - April 23, 2004.[3] The second is the Dow Jones stock price index, daily from January 7th, 2002 to May 7th, 2004. In both cases, if there is no value for a given day, the arithmetic mean using the previous and the following observation was computed. We have chosen to analyse these two series because their statistical properties are those typically found in most daily financial time series. Other financial series with similar features could also have been employed.

The analysis is carried out with the original data, though identical results were obtained when using the log-transformations. Note, however, that these are of interest only in the case of integer-differentiation, which gives a returns series. Deterministic components such as an intercept or an intercept and a linear time trend were also included in the models specified below, but these coefficients were found to be insignificantly different from zero in all cases. Thus, the analysis was carried out on the basis of equation (5) for different ρ-functions and different types of I(0) disturbances.

4.1 The Eurodollar rate

The Eurodollar rate is the bid side of the Eurodollar quote in London. It is collected between 7 and 9 a.m. Eastern time (approximately late morning London time), and it has been obtained from the Federal Reserve Bank of St. Louis database.

(INSERT FIGURE 2 ABOUT HERE)

Figure 2 contains plots of the original series, its first and 5-period differences along with their corresponding correlograms and periodograms. We see that the original series seems to be stationary for the first part of the sample, up to approximately December 2000. Then, it starts decreasing sharply and becomes relatively stable towards the end of 2003 and the beginning of 2004. Both the correlogram and the periodogram indicate nonstationarity, with the autocorrelation values decaying very slowly and with a large peak in the periodogram at the smallest frequency. The plots corresponding to the first differences suggest that the differenced series may be stationary, though there are significant values in the correlogram at some lags far away from zero, and the same happens with the 5-period differences. Finally, the periodogram of the 5-period differences has values close to 0 at some frequencies, indicating that the series may be overdifferenced with respect to them.

Denoting the series by x_t, we start by estimating the model given by (5) and (6) with k = 5, i.e., under H_o (7), we test:

$$(1 - L^5)^d x_t = u_t, \quad t = 1, 2, ...,$$

with d = 0, (0.10), 2, and modelling u_t first as a white noise process, and then allowing for I(0) autocorrelation. In the latter case, we first assumed AR(1) and AR(2) processes, and the null hypothesis was rejected in all cases except when d = 0, thus implying short memory. However, in these cases the coefficients corresponding to the AR parameters were extremely close to the unit root circle, suggesting that these parameters are competing with the fractional differencing one in describing the nonstationarity of the series. Thus, we tried other less conventional forms for the I(0) disturbances, which are very convenient in the context of the present tests. In particular, we used a model due to Bloomfield (1973), where the short-run components are defined exclusively in terms of the spectral density function, which is given by:

$$f(\lambda;\tau) \;=\; \frac{\sigma^2}{2\pi}\;\exp\left(2\sum_{r=1}^{m}\tau_r\cos(\lambda r)\right), \qquad (12)$$

where m indicates the number of parameters required to describe the short-run dynamics. The intuition behind this model is the following. Suppose that u_t is an ARMA process of the form:

$$u_t \;=\; \sum_{r=1}^{p}\phi_r\, u_{t-r} \;+\; \varepsilon_t \;-\; \sum_{r=1}^{q}\theta_r\;\varepsilon_{t-r},$$

where ε_t is a white noise process and all zeros of $\phi(L) = (1 - \phi_1 L - \ldots - \phi_p L^p)$ lie outside the unit circle and all zeros of $\theta(L) = (1 - \theta_1 L - \ldots - \theta_q L^q)$ lie outside or on the unit circle. Clearly, the spectral density function of this process is then:

$$f(\lambda;\,\tau) \;=\; \frac{\sigma^2}{2\pi}\left|\frac{1-\sum_{r=1}^{q}\theta_r\;e^{i r\lambda}}{1-\sum_{r=1}^{p}\phi_r\;e^{i r\lambda}}\right|^2, \qquad (13)$$

where τ now corresponds to all the AR and MA coefficients, and σ^2 is the variance of ε_t. Bloomfield (1973) showed that the logarithm of an estimated spectral density function is often a fairly well-behaved function, and therefore can be approximated by a truncated Fourier series. He showed that (12) approximates (13) well, with p and q being small values, which usually happens in economics. Like the stationary AR(p) model, the

Bloomfield (1973) model has exponentially decaying autocorrelations, and thus can be used for u_t in (5). It is a member of a large family of spectral density functions of which the most famous example is the Fourier transformation providing a spectral density for a given process (see, Wong, 1971), and, while there exist alternative spectral density representations, we have chosen to use the Bloomfield (1973) specification in this paper because it is particularly suited to the functional form of the test statistic we employ. The formulae for Newton-type iterations for estimating the τ_r are very simple (involving no matrix inversion), updating formulae when m is increased are also simple, and we can replace $\hat{A}$ in (9) by the population quantity:

$$\sum_{l=m+1}^{\infty} l^{-2} = \frac{\pi^2}{6} - \sum_{l=1}^{m} l^{-2},$$

which indeed is constant with respect to the τ_r (in contrast to the AR case). The Bloomfield (1973) model, combined with fractional integration, has not been used very much in previous econometric models (though the model itself is well-known in other disciplines – see, e.g., Beran, 1993). Our analysis shows that it is a credible alternative to the fractional ARIMA specifications, which have become conventional in the parametric modelling of long memory.[4]

The test statistic reported in Table 6 is the one-sided statistic given by $\hat{r}$ in (8) for the three types of disturbances. A noteworthy feature emerging from this table is that it decreases monotonically with d. This is to be expected, given the fact that it is a one-sided statistic. Thus, for example, it is desirable that if H_o (7) is rejected with d = 0.75 in favour of alternatives of form d > 0.75, an even more significant rejection should occur when d = 0.50 or 0.25 are tested. It can be seen that the only value of d for which the null hypothesis cannot be rejected corresponds to d = 1, in all three cases of white noise and Bloomfield (with m = 1 and 2) disturbances.

(INSERT TABLES 6 AND 7 ABOUT HERE)

Table 7 displays the 95% confidence intervals of the values of d for which H_o cannot be rejected. These intervals were constructed as follows: We re-computed the tests sequentially for d_o-values = 0, (0.01), 2, choosing the values of d_o for which H_o cannot be rejected at the 5% significance level. Thus, the value corresponding to the lowest statistic in absolute value (which is reported in the table in parenthesis within the square brackets) will be an approximation to the maximum likelihood estimator. We see that the intervals are very narrow, and the lowest statistic in absolute value corresponds to d = 1 or 1.01.

Next, we performed the test assuming that the process contains only one root at the long- run or zero frequency. In other words, we tested for the presence of unit (or fractional) roots in a model given by:

$$(1 - L)^d y_t = u_t, \quad t = 1, 2, ...,$$

for the same values of d and the same type of disturbances as in the previous case. The results are reported in Tables 8 and 9. Starting with the case of a white noise u_t, we see that the unit root cannot be rejected, implying that a simple random walk model may be a plausible alternative for this series, and similar results are obtained when autocorrelated disturbances are employed. Again, the values here are centred around the unit root, and the coefficients for the autocorrelation case were significantly close to 0 in all cases.

(INSERT TABLES 8 AND 9 ABOUT HERE)

The results presented so far suggest that the daily Eurodollar rate can be described either as a pure random walk process, i.e.,

$$y_t = y_{t-1} + \varepsilon_t, \quad t = 1, 2, \ldots \qquad (14)$$

or as 5-period differences, such that:

$$y_t = y_{t-5} + \varepsilon_t, \quad t = 1, 2, \ldots, \qquad (15)$$

both models implying that the data are completely unpredictable.[5] Note that the similarities for the cases of white noise and autocorrelated (Bloomfield) disturbances may be explained by the fact that the short-run dynamics are not important when modelling this series. In fact, the coefficients corresponding to the Bloomfield model (for the unit root case – not reported) were very close to 0, indicating that u_t can be specified as a white noise process. Finally, the plot of the series in Figure 2 also indicates the possible presence of a structural break in the data. An appealing feature of Robinson's (1994) testing procedure described in Section 2 is that it allows one to include deterministic components to take into account the break with no effect on its standard limit distribution. On the other hand, a drawback of this procedure is that the type and the time of the break have to be specified. Visual inspection of Figure 2 suggests that the break is likely to have occurred in December 2000, and we choose as the breakpoint November 29, 2000, which corresponds to the highest value of the series, which then decreases sharply.[6] We experimented with different types of break, including slope and shift breaks. In all cases the results were very similar to those reported, providing evidence of unit roots of the form given in (14) or (15).

Therefore, it appears that the daily structure of the Eurodollar rate is completely unpredictable, supporting the efficiency market hypothesis, according to which, because of arbitrage, it should not be possible, using publicly available information, to make systematic profits over and above transaction costs and risk premia. Thus, evidence of mean reversion would be inconsistent with equilibrium asset pricing models. The fact that both a random walk and a 5-period difference model appear to be equally suitable to capture the stochastic behaviour of the series can be interpreted as indicating that weekday effects do not play a crucial role in this series.[7]

In order to determine which of the two specifications better describe the data we carry out a bootstrap simulation. Instead of standard normal increments, we consider the changes in the Eurodollar rate. For both the random walk (14) and the 5-period random walk (15) we simulate 500 times the future paths for t = 250 … 2425, drawing each time uniformly from the original changes. In Figure 3 we can see the Eurodollar rate and the averages of the simulated paths.

(INSERT FIGURE 3 ABOUT HERE)

One can see that approximately up to observation 1645 the Eurodollar rate can be better modelled as a 5-period random walk, while for the whole sample a random walk specification seems preferable. It also appears that there is a structural break around t = 1645 (a big drop on 19/04/2001 from 4.97 to 4.44).

4.2 The Dow Jones index

The second empirical application is based on the Dow Jones (5) index. This is a market index constructed as a subset of the Dow Jones Industrial average. Of the 30 stocks in the Industrial Average the five with the highest dividend yield during the 12-month period ending in December are selected as the Dow Jones (5). The source of the data is http:// www.djindexes.com.

(INSERT FIGURE 4 ABOUT HERE)

Figure 4 is similar to Figure 2 but refers to the new series. This is also clearly nonstationary. Its first differences may be stationary, while the 5-period differences suggest overdifferencing with respect to some of the frequencies.

(INSERT TABLES 10 AND 11 ABOUT HERE)

We proceed as in the previous case. Thus, we start by performing the tests for the case of (5) and (6) with k = 5, with the same values of d as in the other cases (see Table 10). An interesting result we find is that the unit root null (i.e., d = 1) is rejected for the three types of disturbances in favour of smaller orders of integration. If u_t is white noise, H_o cannot be rejected at d = 0.9, and, assuming autocorrelation in the case of Bloomfield (1973) disturbances, the non-rejection values occur at d = 0.60 with m = 1, and at d = 0.50 with m = 2. Thus, in the three cases, we find evidence of mean reversion. Table 11 displays the 95%-confidence intervals. If u_t is white noise, the values of d where H_o cannot be rejected range between 0.89 and 0.98, and in the case of Bloomfield (1973) disturbances, they range between 0.57 and 0.68 (with m = 1) and between 0.44 and 0.56 (with m = 2).

(INSERT TABLES 12 AND 13 ABOUT HERE)

In Table 12 we assume that the correct model is given by (5) and (6) with k = 1, and the non-rejection values now occur at d = 1 (white noise u_t) along with d = 0.9 with autocorrelated disturbances. Thus, assuming a single pole (or singularity) at the zero frequency, the values of d for which the null cannot be rejected are much higher than in the previous case. However, these values may be biased. Several studies conducted in a hydrological context (Montanari, Rosso and Taqqu, 1995, 1996, 1997) showed that the presence of periodicities might influence the reliability of the estimators of the long memory parameter. Analysing the series of the monthly flows of the Nile River at

Aswan, these authors found that many heuristic estimators gave a positive value for d, indicating long memory where none was present.[8]

(INSERT TABLE 14 ABOUT HERE)

As a final step, we investigated whether the day of the week has any influence on the order of integration of the series. Therefore, we separated the data according to the day of the week, and performed again the tests of Robinson (1994) based on the model given by (5) and (6) with k = 1, testing the degree of dependence at the long run or zero frequency. We find that the unit root null hypothesis cannot be rejected for any series and any type of disturbances, though the lowest statistics are in all cases smaller than 1 (Table 14). Another interesting feature emerging from Table 14 is that the degree of dependence increases with the day of the week. On Mondays, d is around 0.92. It is slightly higher on Tuesdays and Wednesdays, and a bit higher on Thursdays and Fridays. These differences, though small, give further support to the model given in (5) and (6) with k = 5 as an adequate specification for this series, and, given the fact that d is smaller than 1, future values of the series are predictable to some extent.

In the case of the Dow Jones index, therefore, we find evidence of nonstationarity, but also of shocks dying away in the long run, which would imply a degree of predictability apparently inconsistent with market efficiency. We also show that this is a function of the day of the week being considered: it appears that there are significant weekday effects, resulting in predictable values throughout the past history of the series.

5. CONCLUSIONS

This paper has considered a version of the tests of Robinson (1994) that enables one to test models of the form $(1-L^k)^d x_t = u_t$, where k is an integer value, d can be any real number, and u_t is I(0). The most common cases are those with k = 1 (unit or fractional roots) and k = 4 and 12 (seasonal unit or fractional models). However, we extend the analysis to cover situations such as $(1-L^5)^d x_t = u_t$, which might be relevant, for example, in the context of daily financial data. Our Monte Carlo experiments show that these tests perform well against fractional alternatives if the sample size is relatively large (e.g., $T \geq 300$).

Two empirical applications were carried out to shed light on day of the week effects in financial series. This is an important issue, as the existence of such predictable patterns might enable investors to devise trading strategies which result in excess returns, thereby violating market efficiency.[9] First, we examined the Eurodollar rate, and found no evidence of fractional integration either at the long run or zero frequency, or in the more elaborated version based on $(1-L^5)^d$. In fact, the most adequate specifications for this series were a pure random walk model ($x_t = x_{t-1} + \varepsilon_t$) or its 5-period difference ($x_t = x_{t-5} + \varepsilon_t$), implying that the series is unpredictable.

The second application focused on the Dow Jones (5) daily index. Here, using a model with a single pole at the zero frequency, the unit root cannot be rejected. However, using the version based on $(1-L^5)^d$, the hypothesis of a unit root (i.e., d = 1) was decisively rejected in favour of smaller degrees of integration, implying mean-reverting behaviour. The value of d is found to range between 0.50 and 1, indicating nonstationarity but with shocks disappearing in the long run. Finally, it was also found that the degree of dependence between the observations is higher at the end of the week.

In the presence of such mean-reverting behaviour, i.e. if asset prices over time move back to some “fundamental” value, their changes are highly predictable, implying that there are unexploited profit opportunities, which might indicate that investors are not fully rational, and the market is not efficient.

For further research, it may be of interest to apply the same type of model as the one employed here to other financial daily time series data. Also, it would be interesting to develop procedures to estimate the fractional differencing parameter in the context of the models presented here. In the seasonal case (k = 4 or 12) some attempts have been made by some authors. Ooms (1995) proposes Wald tests based on the same model as in Robinson (1994), but requiring efficient estimates of the fractional differencing parameters (he uses a modified periodogram regression estimation procedure due to Hassler, 1994). Also, Hosoya (1997) establishes the limit theory for long memory processes with the singularities not restricted at the zero frequency and proposes a set of quasi log-likelihood statistics to be applied in raw time series. Arteche and Robinson (2000) and Arteche (2002) propose a model for the cyclical component in raw time series. Specifically, they estimate d at any frequency in the spectrum, thereby including seasonal or cyclical structures. Unlike previous methods, Robinson’s (1994) tests do not require estimation of the long memory parameter, since the differenced series have short memory under the null. More recently, Giraitis, Hidalgo and Robinson (2001) extend the estimation to the frequency parameter, i.e. assuming that the pole occurs at an unknown frequency. The robustness of our results to using such methods will be the object of future research.

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ENDNOTES

1. Note that, strictly speaking, KPSS is not a unit root test, since the null hypothesis is stationarity (around either a level or a linear trend) while the alternative is a unit root.

2. These conditions are technical and refer to the functional form of $\psi(\lambda_j)$ in the specification of the test statistic. They are satisfied by the model given by (4) - (6).

3. The week corresponding to the September 11 attacks in 2001 was removed from the analysis.

4. Empirical applications using the model of Bloomfield (1973) with I(d) processes can be found in Velasco and Robinson (2000) and Gil-Alana (2001b).

5. Similar conclusions were obtained with the log-transformed data implying that the returns are also unpredictable.

6. Other breakpoints were also considered, obtaining very similar results.

7. We also investigated the possible presence of day of the week effects by applying both versions of the tests to the data according to the day of the week. In all cases we found evidence of unit roots and hence of no weekday effects.

8. In another recent paper, Montanari, Taqqu and Teverowsky (1999) performed an extensive Monte Carlo investigation in order to find out how reliable the estimators of long memory are in the presence of periodicities, and they concluded that the best results are those obtained using likelihood-type methods.

9. Note, however, that, because of transaction costs, profits might still not be gained. Furthermore, in addition to returns, stock price volatility and the risk profile of investors will also determine their buying-selling decisions.

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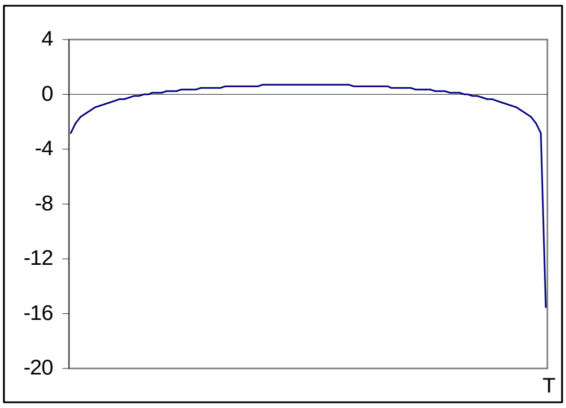
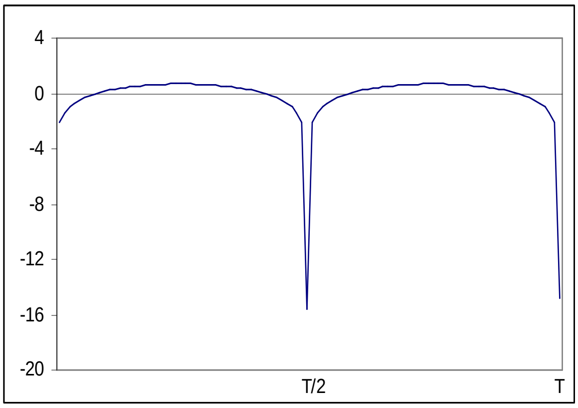
FIGURE 1

Ψ(λ)-functions for different values of k and T = 100

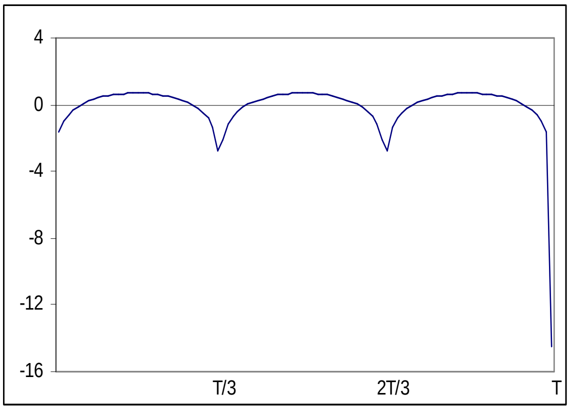
k = 1



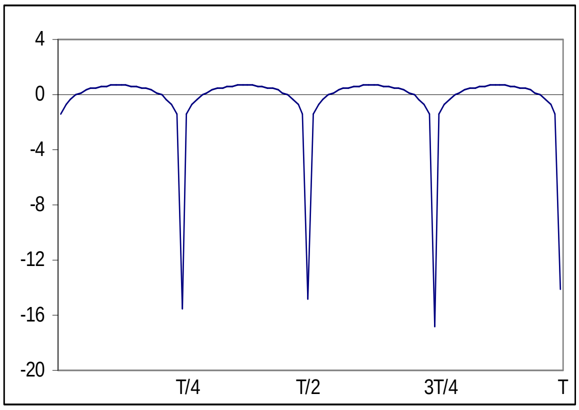
k = 2



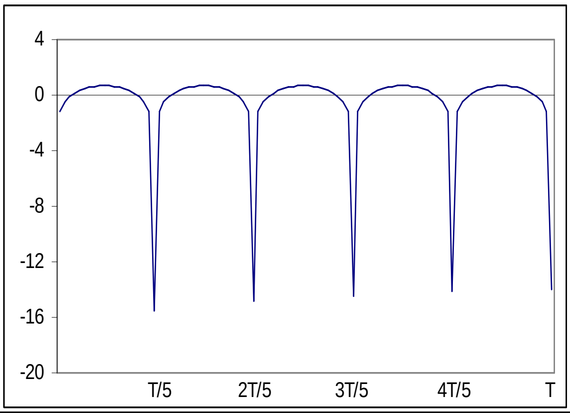
k = 3



k = 4



k = 5



k = 6

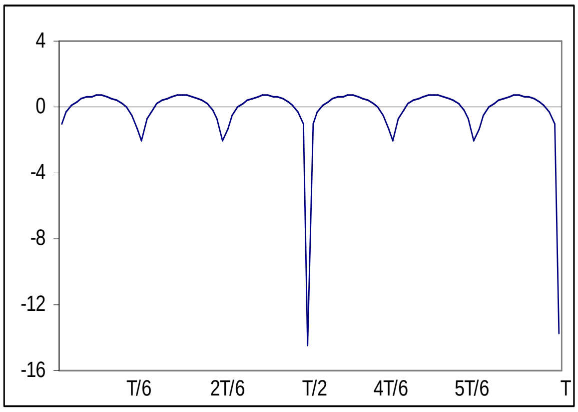


TABLE 1

Sizes of the different versions of the tests with a nominal size of 10%

k / T	50	100	300	500	1000
1	0.153	0.132	0.110	0.104	0.106
2	0.299	0.214	0.143	0.128	0.113
3	0.377	0.233	0.191	0.168	0.116
4	0.367	0.261	0.152	0.156	0.114
5	0.251	0.204	0.139	0.132	0.110
6	0.328	0.230	0.143	0.124	0.118

The nominal size is 10% and 10,000 replications were used in each case.

TABLE 2

Sizes of the different versions of the tests with a nominal size of 5%

k / T	50	100	300	500	1000
1	0.073	0.063	0.056	0.053	0.053
2	0.102	0.097	0.074	0.065	0.057
3	0.145	0.111	0.069	0.094	0.066
4	0.135	0.105	0.098	0.094	0.068
5	0.131	0.111	0.091	0.077	0.056
6	0.161	0.112	0.089	0.078	0.055

The nominal size is 5% and 10,000 replications were used in each case.

TABLE 3

Rejection frequencies of the different versions of the tests of Robinson (1994), with T = 100

k / θ	-1.00	-0.75	-0.50	-0.25	0.25	0.50	0.75	1.00
1	1.000	1.000	0.997	0.795	0.898	0.999	1.000	1.000
2	0.995	0.994	0.979	0.619	0.943	0.999	1.000	1.000
3	1.000	1.000	0.991	0.726	0.515	0.816	0.932	0.998
4	0.917	0.893	0.741	0.649	0.950	0.999	1.000	1.000
5	0.982	0.973	0.903	0.526	0.541	0.878	0.959	0.998
6	0.828	0.731	0.511	0.443	0.992	1.000	1.000	1.000

10,000 replications were used in each case.

TABLE 4

Rejection frequencies of the different versions of the tests of Robinson (1994), with T = 300

k / θ	-1.00	-0.75	-0.50	-0.25	0.25	0.50	0.75	1.00
1	1.000	1.000	1.000	0.998	0.999	1.000	1.000	1.000
2	1.000	1.000	1.000	0.997	0.999	1.000	1.000	1.000
3	1.000	1.000	1.000	0.996	0.994	1.000	1.000	1.000
4	1.000	1.000	1.000	0.962	0.998	1.000	1.000	1.000
5	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000
6	1.000	1.000	1.000	0.998	1.000	1.000	1.000	1.000

10,000 replications were used in each case.

TABLE 5

Rejection frequencies of the different versions of the tests of Robinson (1994), with T = 500

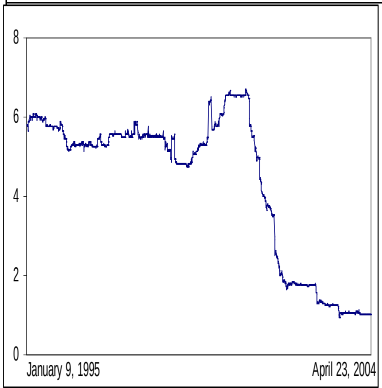
k / θ	-1.00	-0.75	-0.50	-0.25	0.25	0.50	0.75	1.00
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

10,000 replications were used in each case.

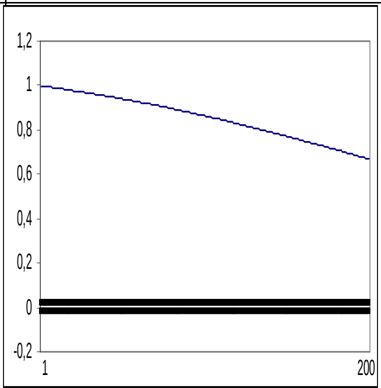
FIGURE 2

Eurodollar rate and its differences with their corresponding correlograms and periodograms

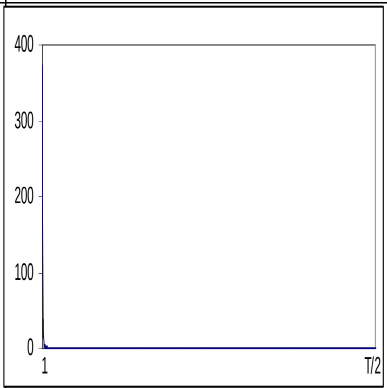
Original time series (y_t)



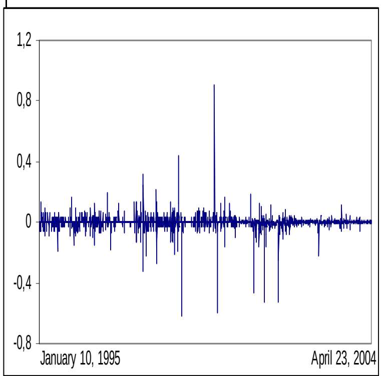
Correlogram y_t



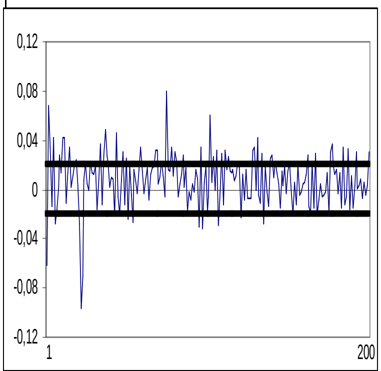
Periodogram y_t



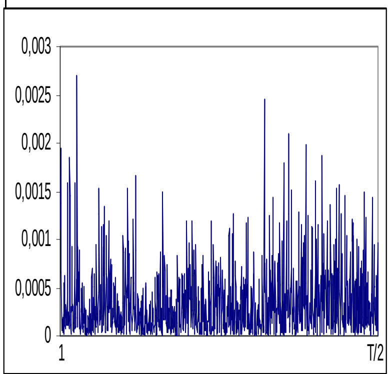
(1 - L) y_t



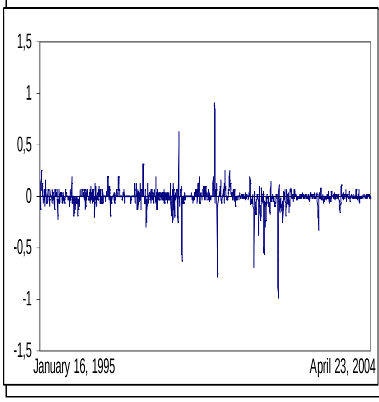
Correlogram (1 - L) y_t



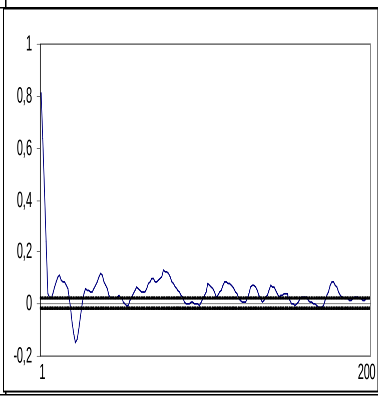
Periodogram (1 - L) y_t



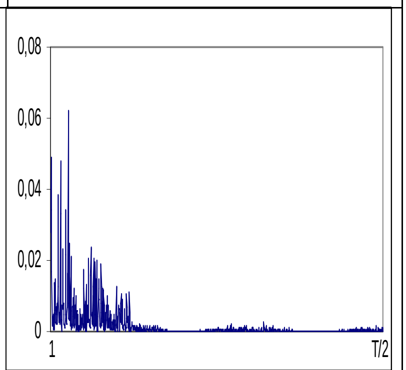
(1 - L^5) y_t



Correlogram (1 - L^5) y_t



Periodogram (1 - L^5) y_t



The large sample standard errors under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.02.

TABLE 6

Testing the order of integration in $(1-L^5)^d\ x_t = u_t$, in the Eurodollar rate

d	White noise	Bloomfield (m=1)	Bloomfield (m=2)
0.00	154.26	65.04	41.09
0.10	146.90	64.23	38.37
0.20	138.16	63.91	34.56
0.30	126.42	61.05	34.63
0.40	110.07	53.70	33.05
0.50	88.65	43.72	29.39
0.60	64.27	32.66	23.63
0.70	41.14	23.48	22.48
0.80	22.68	15.70	14.23
0.90	9.59	8.35	9.09
1.00	**0.79***	**0.91***	**1.02***
1.10	-5.10	-4.67	-3.35
1.20	-9.17	-8.63	-6.78
1.30	-12.07	-11.49	-9.04
1.40	-14.23	-13.66	-11.22
1.50	-15.88	-15.32	-13.42
1.60	-17.18	-16.67	-14.98
1.70	-18.22	-17.26	-16.67
1.80	-19.09	-18.65	-16.98
1.90	-19.81	-19.41	-17.09
2.00	-20.42	-20.07	-17.99

*: Non-rejection values at the 5% significance level.

TABLE 7

95% confidence intervals in the Eurodollar rate

Disturbances	Confidence intervals
White noise	[0.99 (1.01) 1.03]
Bloomfield (m = 1)	[0.99 (1.01) 1.04]
Bloomfield (m = 2)	[0.97 (1.00) 1.03]

TABLE 8

Testing the order of integration in $(1-L)^d x_t = u_t$, in the Eurodollar rate

d	White noise	Bloomfield (m=1)	Bloomfield (m=2)
0.00	211.50	135.90	101.62
0.10	203.87	128.47	94.80
0.20	193.97	116.85	79.30
0.30	178.86	103.78	68.84
0.40	154.69	87.18	58.28
0.50	119.71	65.13	50.96
0.60	79.70	45.06	31.30
0.70	45.58	27.32	19.78
0.80	22.52	14.72	10.73
0.90	8.53	5.97	5.07
1.00	**0.05***	**0.74***	**0.12***
1.10	-5.35	-3.22	-2.50
1.20	-9.04	-5.63	-3.81
1.30	-11.69	-7.15	-5.59
1.40	-13.69	-8.54	-8.19
1.50	-15.24	-9.91	-8.34
1.60	-16.48	-10.58	-9.05
1.70	-17.44	-11.41	-12.12
1.80	-18.33	-12.05	-13.37
1.90	-19.04	-12.56	-13.87
2.00	-19.64	-13.27	-15.03

*: Non-rejection values at the 5% significance level.

TABLE 9

95% confidence intervals in the Eurodollar rate

Disturbances	Confidence intervals
White noise	[0.98 (1.00) 1.02]
Bloomfield (m=1)	[0.99 (1.02) 1.06]
Bloomfield (m=2)	[0.96 (1.00) 1.04]

FIGURE 3

Eurodollar rate and the averages of its simulated paths for t = 251 … 2425

The upper line represents the average of 500 5-period random walks
The bottom line is the average of 500 random walks

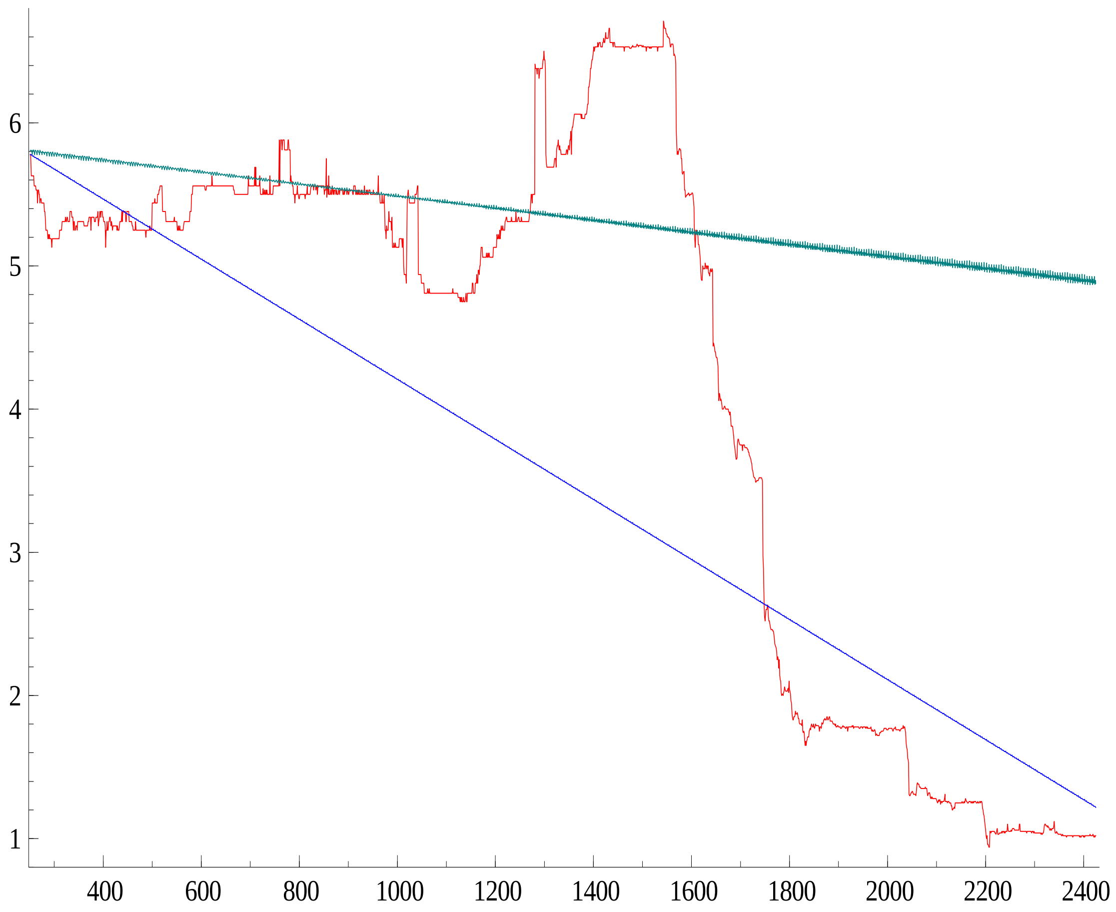
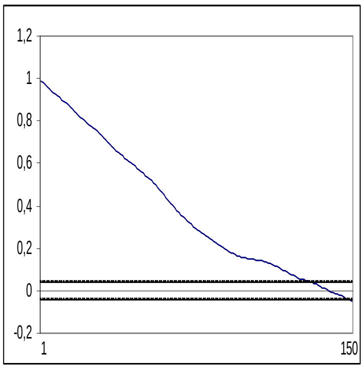


FIGURE 4

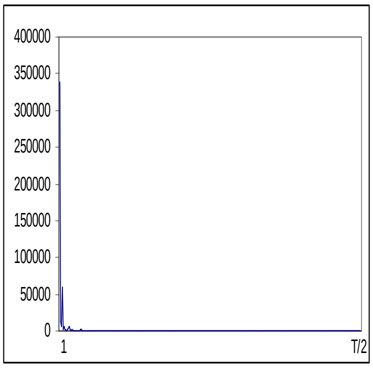
Dow Jones index and its differences with their corresponding correlograms and periodograms

Original time series (y_t)

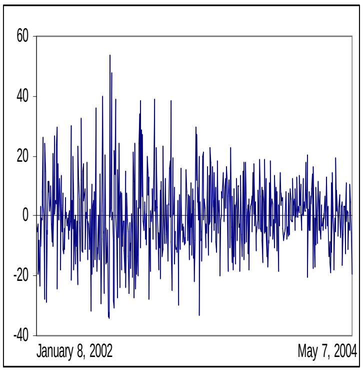
Correlogram y_t



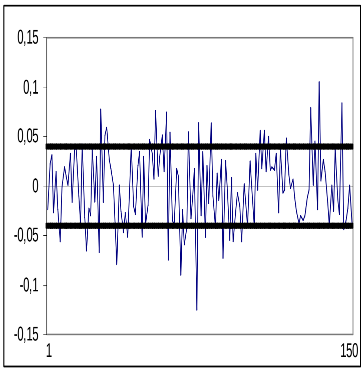
Periodogram y_t



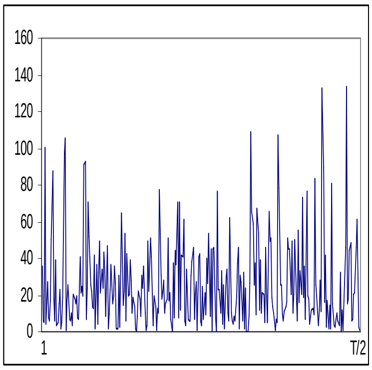
$(1 - L)\ y_t$



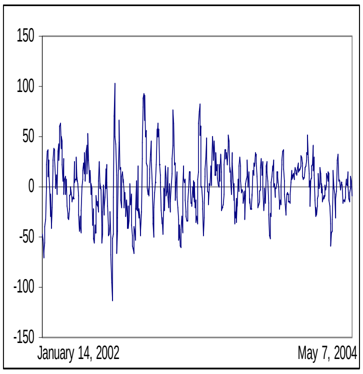
Correlogram $(1 - L)\ y_t$



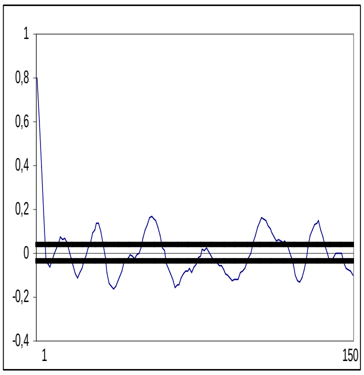
Periodogram $(1 - L)\ y_t$



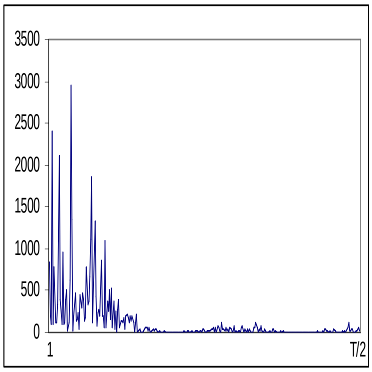
$(1 - L^5)\ y_t$



Correlogram $(1 - L^5)\ y_t$



Periodogram $(1 - L^5)\ y_t$



The large sample standard errors under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.04

TABLE 10

Testing the order of integration in $(1-L^5)^d\ x_t = u_t$, in the Dow Jones index

d	White noise	Bloomfield (m=1)	Bloomfield (m=2)
0.00	55.54	24.51	8.30
0.10	49.74	19.00	6.94
0.20	41.38	13.89	5.01
0.30	34.55	10.13	3.24
0.40	28.13	6.89	1.90
0.50	21.73	3.74	**0.20***
0.60	15.51	**0.73***	-1.57
0.70	9.80	-1.98	-2.83
0.80	4.90	-4.27	-4.09
0.90	**0.96***	-6.13	-4.60
1.00	-2.07	-7.51	-4.78
1.10	-4.36	-8.60	-4.83
1.20	-6.06	-9.41	-5.06
1.30	-7.35	-10.07	-5.76
1.40	-8.34	-10.58	-6.04
1.50	-9.12	-10.99	-7.11
1.60	-9.74	-11.33	-8.06
1.70	-10.25	-11.62	-8.87
1.80	-10.76	-11.86	-9.21
1.90	-11.03	-12.08	-9.56
2.00	-11.33	-12.26	-10.04

*: Non-rejection values at the 5% significance level.

TABLE 11

95% confidence intervals in the Dow Jones index

Disturbances	Confidence intervals
White noise	[0.89 (0.93) 0.98]
Bloomfield (m=1)	[0.57 (0.62) 0.68]
Bloomfield (m=2)	[0.44 (0.50) 0.56]

TABLE 12

Testing the order of integration in $(1-L)^d x_t = u_t$, in the Dow Jones index

d	White noise	Bloomfield (m=1)	Bloomfield (m=2)
0.00	82.73	49.67	32.69
0.10	75.28	42.02	28.12
0.20	64.63	33.77	22.14
0.30	54.84	28.46	16.80
0.40	44.50	21.99	16.30
0.50	33.59	17.16	9.63
0.60	23.08	11.85	9.51
0.70	14.12	7.32	4.83
0.80	7.28	3.75	2.51
0.90	2.42	**1.06***	**1.55***
1.00	**-0.95***	**-0.63***	**0.002***
1.10	-3.30	-2.22	-2.89
1.20	-4.99	-3.25	-3.27
1.30	-6.25	-4.16	-3.36
1.40	-7.21	-4.79	-3.80
1.50	-7.96	-5.44	-5.20
1.60	-8.57	-5.75	-5.87
1.70	-9.07	-5.13	-6.34
1.80	-9.48	-5.43	-7.09
1.90	-9.83	-6.82	-7.88
2.00	-10.13	-6.99	-9.11

*: Non-rejection values at the 5% significance level.

TABLE 13

95% confidence intervals in the Dow Jones index

Disturbances	Confidence intervals
White noise	[0.92 (0.97) 1.02]
Bloomfield (m=1)	[0.88 (0.96) 1.04]
Bloomfield (m=2)	[0.90 (1.00) 1.04]

TABLE 14

95% confidence intervals for the values of d, at the zero frequency, for each day of the week

	White noise	Bloomfield (m = 1)	Bloomfield (m = 2)
Monday	[0.82 (0.92) 1.05]	[0.74 (0.90) 1.15]	[0.71 (0.92) 1.16]
Tuesday	[0.83 (0.92) 1.06]	[0.74 (0.92) 1.17]	[0.72 (0.92) 1.16]
Wednesday	[0.84 (0.93) 1.06]	[0.77 (0.96) 1.20]	[0.76 (0.95) 1.22]
Thursday	[0.84 (0.93) 1.07]	[0.78 (0.95) 1.22]	[0.77 (0.97) 1.24]
Friday	[0.84 (0.94) 1.07]	[0.77 (0.96) 1.21]	[0.77 (0.97) 1.24]