Welfare and Output in Third-Degree Price Discrimination: a Note

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Abstract

One main result about the welfare effects of third-degree price discrimination by a monopolist is that an increase in total output is a necessary condition for welfare improvement. This note provides two examples showing that this proposition cannot be generalized to an oligopoly with heterogenous firms. In these examples, price discrimination makes competition more favorable to the low cost firm. This fact induces a cost saving that overcome the welfare loss from consumer misallocations associated to price discrimination.

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I Introduction

One of the well-known conclusions about the welfare effects of third-degree price discrimination by a monopolist is that "an increase in total output is a necessary condition for welfare improvement." To avoid repetition, in our paper this is called "proposition *WO*," or simply, "*WO*." Schmalensee (1981), Varian (1985), and Schwartz (1990) proved *WO* with different levels of generality. This note shows that, although *WO* is valid for a monopolist, it does not extend to every situation with more than one firm.

The logic of *WO* is clear. There is a consumer inefficiency associated with third-degree price discrimination: output is not optimally distributed to consumers because their marginal utilities will be unequal. With a change from uniform price to price discrimination, units of the good are taken away from consumers with a higher valuation of the good, and given to consumers with lower willingness to pay. Proposition *WO* asserts that the *only* way to overcome this consumer inefficiency is a sufficient increase in total output. This is true when there is only one cost function. But, with heterogeneous firms, cost saving by a better redistribution of output among firms can also overcome the consumer surplus inefficiency. When this is the case, it is no longer true that welfare must fall if output decreases when price discrimination is introduced.

Some papers challenge proposition *WO*. Adachi (2002, 2005) shows that *WO* may not hold in the presence of consumption externalities. One example of these externalities is a bar that gives discounts on drinks sold to women to attract more women, hoping to attract more men as well. The willingness to pay of men increases with the number of women. However, an improvement in welfare due to positive externalities is not a big surprise.

Yoshida (2000) refers to proposition *WO* because his result is in stark contrast with it, but not because it is in opposition to it. In Yoshida's model, an upstream monopolist, by price discriminating, can induce production inefficiencies by increasing the price of its product to the more efficient downstream firms, and lowering its price to the less efficient firms. In this way inefficient firms increase their production and efficient firms reduce it. Yoshida shows that with price discrimination total welfare is reduced when the aggregate production of the final good is increased: exactly the opposite of *WO*. But, in Yoshida's model there is no consumer inefficiency to overcome, which is the source of inefficiency in proposition *WO*. Besides, proposition *WO* refers to consumer inefficiencies, that get worse when total quantity is *reduced*, while in Yoshida's model, price discrimination in the input market generates production inefficiencies, that get aggravated as output *rises*. Yoshida proves that, in his model, these inefficiencies overcome the increase in value due to a higher production.

To our knowledge, the welfare effects of price discrimination when competition is present have been studied (see Stole (2003) for a good survey), but *WO* has never been challenged when more than one firm is introduced. Holmes (1989, note 2) is right when he states, without proof, that *WO* "holds for this oligopoly analysis", because in his paper each firm produces with the *same* constant average cost. Corts (1998) considers consumer welfare and firm profit, but does not attempt an analysis of the relation between a reduction in output and welfare. Armstrong and Vickers (2001), when considering *WO* in duopoly, assume symmetric and constant marginal costs between firms. Stole (2003), citing a previous version of this paper –see Galera (2003)–, is aware of the problem of the possible influence of asymmetric costs in *WO*. As most models suppose symmetric costs among firms, sometimes it is affirmed that *WO* is true more generally than the monopoly case, without reference to the costs. See, for example, Layson (1994, p. 323).

The plan of this note is very simple. In the next section we present the examples, and we finish the paper with some conclusions.

II The examples

Both examples present a homogeneus good industry with two firms. One of the firms, Firm L has low cost and the other, Firm H, high cost. They compete in quantities. Consumers can be divided into two separate markets. There are also two possible price regimes: price discrimination, whose variables we denote with a hat (for example $\widehat{Q}$), and uniform price: we use a bar to denote these variables, for instance $\bar{p}$.

II.1 A Cournot duopoly

Example 1. Our first example is a Cournot duopoly. Both firms have constant marginal cost, $c = c^H > c^L = 0$, and sell in two distinguishable markets A and B. In market A, demand is $Q_A = Ap^{-a}$, with $a \leq 1$. In market B, demand is $Q_B = Bp^{-b}$, also with $b \leq 1$. That is, both market demand curves are of the constant elasticity type, and both inelastic. We will show that, within a whole range of values of the parameters a and b, proposition *WO* is no longer true. To prove it, we present three propositions in this example. The first one provides some useful results for the Cournot oligopoly. The second states that total output is always reduced with the introduction of price discrimination for all the relevant parameter values. And the last result says that welfare is reduced for some values of the parameters.

Proposition 1 *In a Cournot duopoly, where firms H and L have constant marginal cost $c = c^H > c^L = 0$, the price and equilibrium quantities, if they exist, satisfy the following equations:*

(1) $$q^L = \varepsilon Q; \qquad q^H = (1-\varepsilon)Q.$$

(2) $$p = \frac{c}{2 - \frac{1}{\varepsilon}}$$

Proof. For $k = L, H$, the profit of firm k is $\pi^k(q^k) = \left(P(Q) - c^k\right) q^k - F^k$, where F^k is the fixed cost. The first order condition is: $P'(Q)q^k + P(Q) - c^k = 0$. Arranging from here, as $c^L = 0$, $P'(Q) \equiv 1/Q'(p)$, and $\varepsilon \equiv -pQ'(p)/Q$, we have equation 1. Adding the first order conditions across firms, we get $P'(Q)Q + 2P(Q) - c = 0$, and thus, $-p/\varepsilon + 2p - c = 0$. Rearranging, we obtain expression 2. □

Without loss of generality, we normalize the equilibrium uniform price to be $\bar{p} = 1$. We get the following proposition:

Proposition 2 *If* $0.5 < a < 1$, $0.5 < b < 1$ *and* $a \neq b$, *then total output is reduced; if* $a > 1$, $b > 1$ *and* $a \neq b$, *total output is increased.*

Proof. First we get total output in both regimes. As $\bar{p} = 1$, in the uniform price market total output is $\overline{Q} = A + B$. With price discrimination, total output is $\widehat{Q} = A\hat{p}_A^{-a} + B\hat{p}_B^{-b}$. Now we find the normalized cost. In the whole market, at uniform price $\bar{p} = 1$, elasticity is easily seen to be $\bar{\varepsilon} = (aA + bB)/(A + B)$. Applying equation (2) to this market, we get that

(3) $$c = 2 - \frac{A+B}{aA+bB} = \frac{A(2a-1) + B(2b-1)}{aA+bB}.$$

And finally we get the equilibrium prices. Applying again equation (2) to markets A and B, with elasticities a and b respectively, we get

(4) $$\hat{p}_A = \frac{ac}{2a-1}, \qquad \hat{p}_B = \frac{bc}{2b-1}.$$

Now, we will use a generalization for real exponents of the Bernouilli inequality –see, for example, Weisstein, E. W., (2005). This inequality states the following: "If $0 \neq x > -1$ and $0 < a < 1$ are real values, then $(1+x)^a < 1+ax$; if $0 \neq x > -1$ and $a > 1$, then $(1+x)^a > 1+ax$." It is clear that if $x = 0$, or $a = 1$, the inequality becomes an equality. In order to apply this inequality to our purposes, we need to know that $\frac{1}{\hat{p}_k} - 1 > -1$ for $k = A, B$. Under our hypothesis, this is clear because as $0.5 < a$ and $0.5 < b$, then $c > 0$ and $\hat{p}_k > 0$ for $k = A, B$. Now, with some algebra, we obtain:

$$\frac{1}{\hat{p}_A} = \frac{2a-1}{ac} = 1 + \frac{1}{\hat{p}_A} - 1 = 1 + B\frac{a-b}{a(A(2a-1)+B(2b-1))}. \tag{5}$$

A similar expression can be obtained for $1/\hat{p}_B$. Applying the Bernoulli inequality, for $a < 1$ and $b < 1$, we have:

$$\hat{p}_A^{-a} = \left(\frac{1}{\hat{p}_A}\right)^a < 1 + B\frac{a-b}{A(2a-1)+B(2b-1)}; \tag{6}$$

and

$$\hat{p}_B^{-b} = \left(\frac{1}{\hat{p}_B}\right)^b < 1 + A\frac{b-a}{A(2a-1)+B(2b-1)}. \tag{7}$$

These inequalities are valid for all the parameter values, except when $a = b$. Total output with price discrimination is:

$$\widehat{Q} = A\hat{p}_A^{-a} + B\hat{p}_B^{-b} < A(1 + B\frac{a-b}{A(2a-1)+B(2b-1)}) + B(1 + A\frac{b-a}{A(2a-1)+B(2b-1)}) = A + B. \tag{8}$$

This result proves the proposition. That is, total output is reduced under price discrimination. It is clear that if $a < 1$ and $b = 1$, total output is also reduced. With $a > 1$ and $b > 1$, it is easy to get the inverse result. $\square$

Now, we proceed to our last result for this example.

Result 1 *When $A = B$, for all values with a and b over the curve in Figure 1, and not in the diagonal, welfare is increased when price discrimination is introduced. A similar result is valid when $A \neq B$.*

Proof. We have already got the equilibrium prices, $\bar{p}$, $\hat{p}_A$ $\hat{p}_B$. For these prices, we will now compute changes in the set of variables that have an influence on welfare. Total income with uniform price is $A + B$. As elasticity is $\bar{\varepsilon} = (aA + bB)/(A + B)$, total cost is $c(1-a)A + c(1-b)B$. So, total profit is $\bar{\pi} = A(1 - c(1 - a)) + B(1 - c(1 - b))$. With a price discrimination regime, total income in market A is $\hat{p}_A A \hat{p}_A^{-a} = A\hat{p}_A^{1-a}$, and total cost is $c(1-a)A\hat{p}_A^{-a}$. Total profit is the difference between these values, $\hat{\pi}_A = A\hat{p}_A^{1-a} - c(1-a)A\hat{p}_A^{-a}$.

For market B the expressions are similar. The increment in consumer surplus (CS) is:

$$\Delta CS = \int_{\hat{p}_A}^{1} Ap^{-a}dp + \int_{\hat{p}_B}^{1} Bp^{-b}dp = A\frac{1-\hat{p}_A^{1-a}}{1-a} + B\frac{1-\hat{p}_B^{1-b}}{1-b}. \tag{9}$$

We have then that the welfare gain is

(10)
$$\Delta W = \Delta CS + \hat{\pi}_A + \hat{\pi}_B - \bar{\pi} = \frac{Aa}{1-a}(1-\hat{p}_A^{1-a}) + Ac(1-a)(1-\hat{p}_A^{-a}) + \frac{Bb}{1-b}(1-\hat{p}_B^{1-b}) + Bc(1-b)(1-\hat{p}_B^{-b}).$$

Fixing the parameters A and B, total increment in welfare is $\Delta W = W(a,b)$. We have tried without success to simplify analytically ΔW. But, although this function $W(a,b)$ is not simple, it can be studied with numerical methods. We represent $W(a,b) = 0$ in Figure 1 for $A = B$. It is easy to show, by numerical example, that the values of a and b over these curves violate proposition WO. □

[INSERT FIGURE 1 ABOUT HERE]

II.2 Partial competition

This example has the following economic rationale. Suppose that a high cost firm, Firm H, can sell only in a segment of the whole market. If a lower cost firm, Firm L, that operates in the whole market, is bound to a uniform price, then Firm L will suffer, in the whole market, the consequences of competition with firm H in H's particular market. Hence Firm L may prefer a soft competition with Firm H. This strategy of Firm L allows Firm H to expand its production. The result may be an inefficiency in production. If we allow price discrimination, then Firm L can limit the contest with Firm H to H's own segment, without hampering its profit in the other segments of the market. Thus, with price discrimination this inefficiency is avoided. This situation is parallel to that studied by Gelman and Salop (1983), but changing capacity limitations to market demand limitations.

Example 2. There are two markets, A and B, with demands $Q = a - p_A$ and $Q = b - p_B$. There are also two firms. Firm L has zero costs and sells in both markets. Firm H sells only in market A and has a constant marginal cost that we normalize to $c = 1$. Let q_A, q_B be the quantities of the first firm in both markets and x_A the quantity of the second firm. There is Cournot competition in market A. We find the following proposition:

Proposition 3 *For all points* (a,b) *inside the triangle with vertices* $(2,2)$, $(2,\frac{54}{17})$ *and* $(5,4)$, *total output is reduced and welfare is increased, when price discrimination is introduced.*

Proof. Whatever price restrictions Firm L has, Firm H maximizes $\pi^H = (a-1-x_A-q_A)x_A$ at $x_A = \frac{1}{2}(a-1-q_A)$. That is, $x_A = a - x_A - q_A - 1$, or

$$x_A = p_A - 1. \tag{11}$$

With *uniform pricing*, both markets must have the same price. That is, $\bar{p} = a - \bar{q}_A - \bar{x}_A$ and $\bar{p} = b - \bar{q}_B$. Then Firm L maximizes: $\bar{\pi}^L = \bar{p}(a - \bar{x}_A - \bar{p} + b - \bar{p})$ at $\bar{p} = \frac{1}{4}(a - \bar{x}_A + b)$. Applying equation 11, $\bar{x}_A = \bar{p} - 1$, we obtain $\bar{p} = (a+b+1)/5$. With *price discrimination* is easy to see that the equilibrium prices are $\hat{p}_A = (a+1)/3$ and $\hat{p}_B = b/2$. We can use equation 2, or any other method, to find them.

Now we proceed with welfare. Total willingness to pay in market A is $\frac{1}{2}a^2 - \frac{1}{2}p_A^2$, for whatever price regime. The increment in total willingess to pay in this market is thus $(\bar{p}^2 - \hat{p}_A^2)/2$. The same is true for market B. As the cost of firm H is 1, total cost, by equation 11 is $\bar{x}_A = \bar{p} - 1$ with uniform price, and $\hat{x}_A = \hat{p}_A - 1$ with price discrimination. Applying all this, we find the increment in welfare to be

$$\widehat{W} - \overline{W} = \bar{p}^2 - \frac{1}{2}\left(\hat{p}_A^2 + \hat{p}_B^2\right) + \bar{p} - \hat{p}_A. \tag{12}$$

The increment of output in the industry is $\widehat{Q} - \overline{Q} = a - \hat{p}_A + b - \hat{p}_A - (a+b-2\bar{p}) = 2\bar{p} - \hat{p}_A - \hat{p}_B$. With some algebra we find that:

$$\widehat{Q} - \overline{Q} = \frac{2a-3b+2}{30}; \text{ and } \widehat{W} - \overline{W} = \frac{51b-14a-134}{60}\frac{2a-3b+2}{30}. \tag{13}$$

If the values of a and b satisfy $\frac{2a+2}{3} < b < \frac{14a+134}{51}$, then $\widehat{Q} < \overline{Q}$ and $\widehat{W} > \overline{W}$. Firm H will sell a positive quantity only if $a > 2$. These inequalities mean that for all (a,b) inside the triangle of vertices $(2,2)$, $(2,54/17)$ and $(5,4)$, welfare is increased with reduction in output. An example of a point with integer values in this triangle is $a = 3$ and $b = 3$. □

III Conclusion

We conclude with only one remark. It is known for a monopolist that an increase in total output is a necessary condition for welfare improvement. This proposition presents a test that only requires knowledge of observable magnitudes. But to have policy relevance, the policy-maker should be sure that this proposition is still valid under imperfect competition, because pure monopolies are rare. The extension to oligopoly has not previously been revoked because most papers on price discrimination with imperfect competition deal with symmetric cost firms. Unfortunately, this note shows that a generalization is not possible with heterogeneous firms. Then, the policy-maker needs knowledge of variables such as costs; and costs are often hard to measure.

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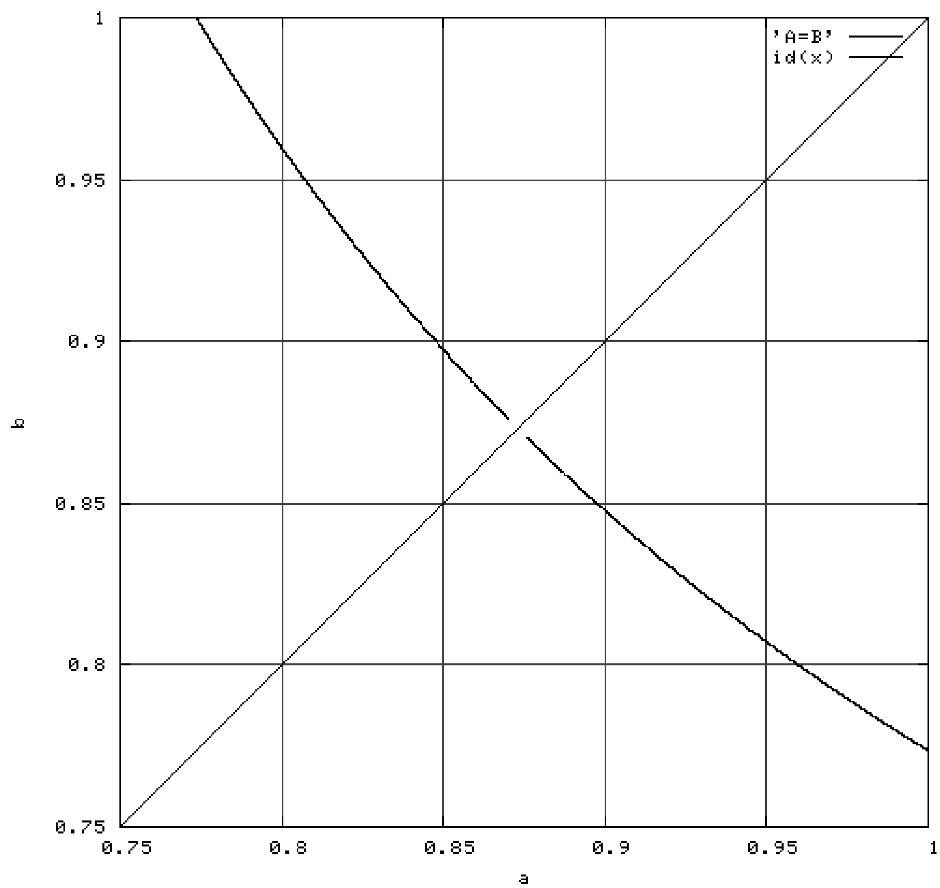

Figure 1: A Cournot model with $A = B$