Finite Element Model for Predicting Stiffness of Metal-Plate-Connected Tension-Splice and Heel Joints of Wood Trusses

J. M. Cabrero, K. G. Gebremedhin

Abstract. *A finite element model that predicts axial stiffness of metal-plate-connected (MPC) tension-splice and heel joints of wood trusses is developed. The commercial software ABAQUS was used in developing the model. The model was based on: (1) the assumption that the joints are two-dimensional, (2) plane-strain modeling, and (3) the assumption that the properties of the wood and metal plate are linearly isotropic. The interface between the wood and the teeth of the metal plate is modeled with a finite sliding formulation. Contact surfaces (rather than contact elements) model the slip of the teeth of the metal plate and shear at the wood-tooth interface. The tangential contact properties are set to a specified coefficient of friction while the normal contact properties are set to a "hard" contact formulation, allowing for a possible separation of the nodes after contact is achieved. Model predictions are validated against experimentally measured stiffness values obtained in the literature. The data cover two wood species and three levels of modulus of elasticity (MOE). On the average, the model predicts within 5% of the experimentally measured stiffness values. The unique features of the model include: (1) accounting for friction at the tooth-wood interface, (2) accounting for tooth slip, (3) requiring no empirical factor (such as foundation modulus) in predicting axial stiffness, and (4) using the same methodology in modeling tension-splice and heel joints.*

Keywords. *Axial stiffness, Finite element, Heel joint, Metal-plate-connected joint, Tension splice, Truss joint.*

Conventional methods of design of metal-plate-connected (MPC) wood truss joints assume that connections between the metal-plate connector and wood are either pinned or rigid. In reality, these joints exhibit a semi-rigid behavior, i.e., not purely pinned or rigid but somewhere in between (Amanuel et al., 2000; Gupta and Gebremedhin, 1990; Riley et al., 1993). Therefore, the challenge is to determine the stiffness of these joints so that their semi-rigidity can be accounted for in the design of MPC wood trusses. Modeling the behavior of the connector in truss members is complicated by the composite nature of the metal and wood and the configuration of the system (a row of teeth embedded in wood and a gap existing between the wood elements). The metal-plate connection is the least understood in truss design.

A simplified approach to truss design is to assume the truss joints to be pin-connected, which means that no bending moment is transferred between adjacent members. This assumption violates the continuity of chord members at the joints. To account for the indeterminacy of a truss when analyzed as pin-joined approximations, the Truss Plate Institute (TPI) has provided empirically based Q-factors to modify the bending moment or the buckling length of truss members (TPI, 1995). The Q-factors were developed based upon many years of experience of design and extensive simulated investigation of wood trusses of standard configurations using the Purdue Plane Structures Analyzer (PPSA) (Purdue Research Foundation, 1993). The PPSA is a matrix method of structural analysis that determines the axial forces and bending moments of truss-frame models. The tabulated Q-factors provided by TPI do not cover all ranges and combinations of loading conditions, spans, and geometries. Therefore, theoretical models that provide realistic treatment of joints are needed so that forces and moments can be predicted with greater accuracy. Because of the wide application of MPC wood trusses in commercial, industrial, residential, and agricultural buildings, even a reasonably small improvement in the characterization of truss joints may result in significant cost savings.

The main focus of this research is to develop a simple finite element model for the tooth-wood interface of MPC tension-splice and heel joints based on fundamental principles of contact mechanics that, apart from basic material properties, requires no empirical factors to predict stiffness values. Linear elastic finite elements represent the metal plate, teeth, and wood, while contact surfaces transfer axial and frictional forces between the wood and the teeth of the metal plate as the joint is externally loaded. The commercial software package ABAQUS was used to develop the contact surfaces. Modeling the interface using contact surfaces can have wide engineering applications, such as in modeling the bond between steel and concrete in reinforced concrete structures, modeling the transfer of frictional forces between piles and soil in pile foundations, and modeling the rotational stiffness of MPC wood joints.

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Objectives

The specific objectives of this study were:

- To develop a simple finite element model that predicts axial stiffness values for tension-splice and heel joints of wood trusses, taking into account friction at the tooth-wood interface and tooth slip, and requiring no empirical factor such as foundation modulus to predict stiffness.
- To validate the predicted stiffness against measured values.

Literature Review

Several theoretical studies were conducted to model MPC wood truss joints. Foschi (1977) was the first to propose a theoretical expression that represents the nonlinear load-slip relationship of connectors. Most of the subsequently developed theoretical expressions that represent nonlinear behavior of MPC joints have been based on Foschi's work. Foschi's approach was based on the relative displacement between two points on a joint that initially had the same coordinates: one referring to the metal plate, and the other referring to the wood member. Foschi modeled the wood and the metal plate as rigid bodies connected by specified properties of nonlinear springs.

Triche and Suddarth (1988) developed a finite element based analysis for MPC joints. They employed Foschi's (1977) mathematical load-deformation relationship to predict the relative displacement between the surface of the metal-plate connector and the wood. Conventional frame elements were used to model lumber, special wood-to-plate elements were used to model the nonlinear load-slip behavior between the surface of the metal-plate connector and frame elements, and special plate elements were used to model strain in the metal-plate surface (non-tooth portion of the plate). They reported good agreement of stiffness values between model predictions and experimental results.

Sasaki and Takemura (1990) developed a model based on replacing MPC joints with a set of three linear elastic springs representing axial, shear, and rotational stiffness characteristics. They performed a matrix analysis for a member having semi-rigid joint connections at its ends by replacing the joints with springs. Their mathematical expression is similar to that of Foschi (1977). Similarly, Cramer et al. (1990) developed a two-dimensional, non-linear, plane-stress, finite element model for a tension-splice joint. Similar to Foschi (1977), they used spring elements to model the wood, metal plate, and the wood-metal interface. The wood was modeled as a linearly elastic and orthotropic material. They concluded that current design assumptions represent realistic approximations only for relatively small plates. In another study, Cramer et al. (1993) developed "a more efficient scheme for computing the stiffness of a metal-plate-connected joint" that accounts for joint eccentricities, nonlinear semi-rigid joint performance, and which includes an automated means to compute the geometric characteristics of each plate-wood contact surface. Their approach was that each plate-wood connection is modeled as a single element with a set of three springs (two translational and one rotational) located at the center of gravity of each plate-wood contact area. The springs were connected to the wood element, which was idealized as a frame member along the wood-member centerline, and to the plate model through rigid links. Their model is semi-analytical because the analysis requires that the stiffness characteristics representing the contact area be computed from the geometric and individual tooth load-slip characteristics of a given plate obtained from testing.

Crovella and Gebremedhin (1990) developed two theoretical models, a two-dimensional linear finite element model and an elastic foundation model, that predict stiffness values of MPC tension-splice joints. The finite element model used linear three-node triangular elements to model the joint. The elements did not account for bending. Experimental tests were conducted to validate the predicted results. It was reported that the finite element model overpredicted the stiffness values due to the properties of the triangular elements used to mesh the domain. The stiffness values predicted by the elastic foundation model were, however, close to actual experimental results. The elastic foundation model requires, as an input, a foundation modulus, which must be obtained from experimental bearing tests.

Groom and Polensek (1992) developed a theoretical model that accurately predicted the ultimate load and failure modes of different joints. Their method was based on a beam on an elastic foundation and included the inelastic behavior of the tooth and wood. A linear step-by-step loading procedure was used to better represent the nonlinear response of the foundation. Their model accurately predicted the load-displacement curves and the ultimate load of MPC joints considering wood-grain orientation and plate geometry.

Riley et al. (1993) developed a semi-analytical model that predicted axial and rotational stiffness values of MPC truss joints based on the concept that a tooth of a metal plate embedded in wood acts as a cantilever beam on an elastic foundation. They assumed the wood to be linearly elastic and neglected friction forces at the tooth-wood interface. Their model requires, as an input, a foundation modulus, which must be obtained from experimental bearing tests.

Vatovec et al. (1995) used the ANSYS finite element program to predict the axial load-deflection relationship of MPC joints. They employed a three-dimensional nonlinear model in which each tooth was represented as a single point consisting of three non-linear spring elements. The metal plate was modeled without the slots that exist between teeth. Their model represented the axial load-deflection relationship "relatively well," but they reported that the rotational response of the model was not validated because of "insufficient boundary conditions" in the experimental work. In addition, the authors employed contact elements that were limited to wood-to-wood interaction, but not tooth-to-wood interaction. In a later study, Vatovec et al. (1996) developed a three-dimension model for a tension-splice joint. In this model, the wood was modeled by a linear elastic isotropic beam and the plate was assumed to be a rigid body. Three uni-axial springs, calibrated against experimental data, were used for the tooth-wood interface. The model required a long computational time because of the high number of degrees of freedom, and it was reported insensitive to variations in the modulus of elasticity of wood or steel or inclusion of the holes of the metal plate.

Riley and Gebremedhin (1999) developed a semi-analytical model that predicted axial and rotational stiffness values of MPC tension-splice and heel joints. In the formulation of this model, the punched teeth were assumed to act like cantilever beams in elastic foundations. The model is semi-analytical because it requires, as an input, a foundation modulus, which needs to be specified. The model is based on the theory that the reaction forces of the foundation (wood) are proportional at every point to the deflection of the beam at that point. In the model, wood was treated as linearly elastic, and friction at the tooth-wood interface was neglected. The model predictions were validated against extensive test data of MPC wood joints. The stiffness predictions from our model are validated against these data (Riley and Gebremedhin, 1999).

Amanuel et al. (2000) developed a finite element model that predicts axial stiffness of MPC tension-splice wood joints. In this study, linear elastic finite elements were used to model the metal-plate surface, teeth, and wood, while nonlinear contact elements were used to model slip at the tooth-wood interface. The commercial computer software ANSYS was used to develop the contact elements. In this study, wood was assumed to be isotropic, and frictional forces between tooth and wood were not considered. The procedure, however, required no empirical factor (such as foundation modulus) to predict joint stiffness. The predictions were within 5% of measured values.

None of the models reported previously (with the exception of the study by Amanuel et al., 2000) have accounted for slip at the tooth-wood interface using contact surfaces. In addition, most of the models reported herein required either an empirical factor (foundation modulus) or do not account for frictional forces. The model proposed herein accounts for tooth slip and friction at the interface, and requires no foundation modulus to predict axial stiffness.

Model Formulation

A simple 2-D finite element model that predicts axial stiffness of MPC tension-splice and heel joints of wood trusses was developed using the commercial computer software ABAQUS. Several assumptions were made to simplify the 3-D, composite, orthotropic (wood) problem into a 2-D plain strain problem. As was indicated previously, Vatovec et al. (1996) modeled MPC wood joints as a 3-D problem and reported a long computational time because of the high number of degrees of freedom. The computational time would have increased exponentially had they used contact elements to model the interface. They used uni-axial springs, calibrated against experimental data, to simulate the tooth-wood interface. In this study, a simple, computationally efficient, 2-D model is proposed.

Assumptions

The following assumptions were made in this model:

- The deformation perpendicular to the direction of the axial force was ignored. The deformation in the longitudinal direction (elongation in the direction of the axial force) was assumed to be dominant. Because of this assumption, the model is reduced to a plane strain problem. The same assumption is made for the heel joint, i.e., only force in the axial direction (parallel to the top chord) is considered. This assumption corresponds to the assumption made by Riley and Gebremedhin (1999), whose data were used to validate our model.
- The predicted axial stiffness of the joints is governed by tooth slip and friction at the wood-tooth interface and the resulting deformation of the wood (elongation in the direction of the load).
- For the tension-splice joint, the force in each row of teeth (parallel to the direction of load) was assumed to be the same for simplicity, and is commonly assumed in design practice as well. For the heel joint, however, the force in each row of teeth was assumed to be different. A row of teeth is defined parallel to the top chord, and the row at the interface of the top and bottom chords (the longest row) was taken for calculating stiffness.
- The number of teeth located in the top chord was assumed to be half of the total teeth of the plate (60 teeth), and the other half were assumed to be located in the bottom chord. This is consistent with the assumption made by Riley and Gebremedhin (1999).
- Both wood and steel were assumed to be elastic and isotropic. Wood is an orthotropic material (Goodman and Bodig, 1973), but since the joints are modeled as a 2-D problem, only material properties in the direction of the load were considered.

Geometrical Model

The wood lumber species groups used in this study were spruce-pine fir and southern pine. The actual size of the lumber was 38 mm thick and 89 mm wide (nominal 2 × 4). Plates were 20-gauge steel and were 76.2 × 102 mm for the tension-splice joint and 76.2 × 127 mm for the heel joint. These were the same lumber species groups, lumber size, and steel plate sizes that were used in the experimental study by Riley and Gebremedhin (1999). Figure 1 shows the configuration of the tension-splice joint, and figure 2 shows the heel joint.

Some simplifications were made in modeling the geometry of the plate. The slots (because of punched teeth) were not considered in modeling. The tooth of a metal plate, which is actually twisted and tapered toward the end, was modeled as flat rectangular surface having a thickness equal to the nominal thickness of the plate. A similar assumption was made by Amanuel et al. (2000).

The hole in the wood was assumed to be 0.01 mm smaller than the thickness of the tooth. This technique allows good initial contact between the tooth and the wood. From the start, good contact was established by relocating the nodes on the wood (initially "inside" the steel) to the metal-plate surface. This was accomplished inside the software.

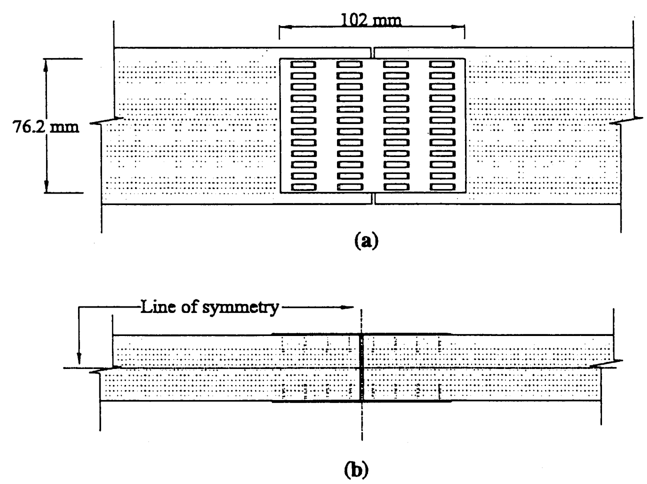
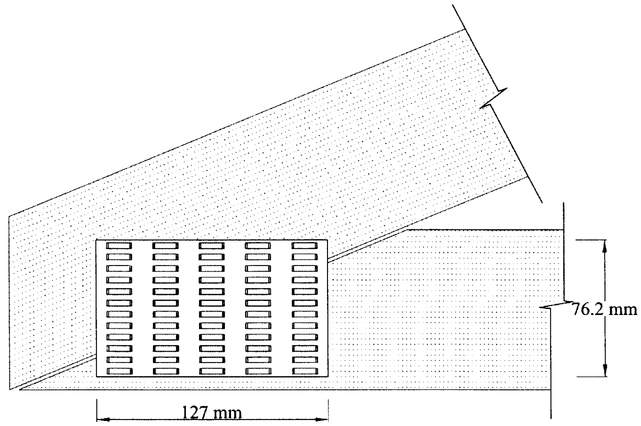
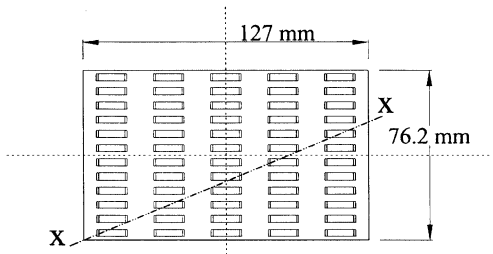


Figure 1. Diagram of a tension-splice joint: (a) top view and (b) side view with line of symmetry shown.

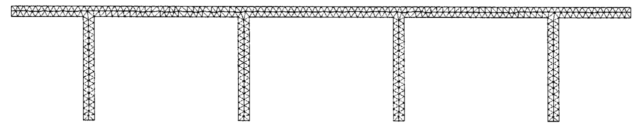


(a)

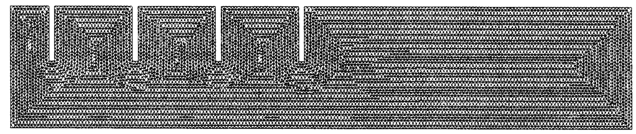


(b)

Figure 2. (a) Diagram of the heel joint studied herein and tested by Riley and Gebremedhin (1999), and (b) teeth layout and axis (X-X) that separates the computational domain.



(a)



(b)

Figure 3. Finite element model: (a) meshed metal plate and (b) meshed wood member. The two figures are not drawn to the same scale.

Boundary Conditions

In modeling the tension-splice joint, only half of the width of the wood member and half of the width of the metal plate in one member (fig. 1b) was considered because of symmetry. Rollers were defined along the line of symmetry of the lumber to allow movement in the direction of the load. The metal plate was hinged at midpoint (at the gap), and displacement perpendicular to the longitudinal direction was restricted to avoid rigid-body motion.

For the heel joint, the computational domain was represented by the metal plate that is located at the top chord (X-X axis in fig. 2b). For simplification, it was assumed that the line passes through a tooth slot at every other row along the plane. Otherwise, more than one section would have to be considered. In this model, the number of teeth modeled was ten because the plate has ten rows.

Load Application

Tensile force was applied at one end of the wood of the tension-splice joint, and compressive force was applied at the top chord of the heel joint (Riley and Gebremedhin, 1999). The forces were applied as uniform pressures. In the model, to achieve uniform load distribution and avoid local stress concentration, the load was applied 50 mm away from the last row of teeth.

Element Used in the Model

The element used in the model, CPE3, is available in the ABAQUS library (ABAQUS, 2004). The element, adapted for plane strain models, is a solid continuum three-node triangular element. Each node has two degrees of freedom, i.e., translation in the x and y directions. Because the elements are triangular in shape, they are less sensitive to geometrical distortions, which could potentially happen to the teeth due to bending.

The mean size of the element is fixed at 0.5 mm. This size ensures two layers of elements for the teeth. The same mesh density was used for both wood and steel components. The resulting mesh is shown in figure 3.

Contact

Load transfer between teeth and wood occurs at the contact interfaces. The contact interface is defined by two surfaces, one for the wood and the other for the metal plate. In this study, the interface was defined by contact surfaces rather than by contact elements. This approach is easier because no matching mesh between the surfaces of contact is required. This method consists of defining the surfaces using a pair of rigid or deformable surfaces. In addition, in this approach, one of the surfaces must be defined as a “master” and the other surface as a “slave” (ABAQUS, 2004). The nodes of the slave surface are constrained from penetrating into the master surface. The nodes of the master surface, however, could penetrate into the slave surface. Loads are transferred to the master nodes according to the contact properties defined and the position of the slave node.

The specification of the surfaces is critical because of the way the interactions (of the surfaces) are discretized. For each node on the slave surface, ABAQUS looks for the closest point on the master surface of the contact pair where the normal of the master surface passes through the node on the slave surface (fig. 4). The interaction is then discretized between the point on the master surface and the node on the slave surface. Actually, only the master surface geometry and orientation are defined. The direction of the slave surface is normal to the master surface. In the model, the surface corresponding to the metal plate is defined as master and that of the wood as slave. It is recommended to define the surface of the stiffer material to be the master surface.

To model the transfer of normal forces, a “hard” contact relationship was chosen. The metal plate is assumed to transfer only compressive forces when in contact, and the contact pressure reduces to zero when the surfaces are disengaged. No tensile forces are transferred through the interface.

When the teeth of the metal plate deform, they press against the wood, and may also slip and transmit tangential forces. At contact, surfaces transmit shear and normal forces

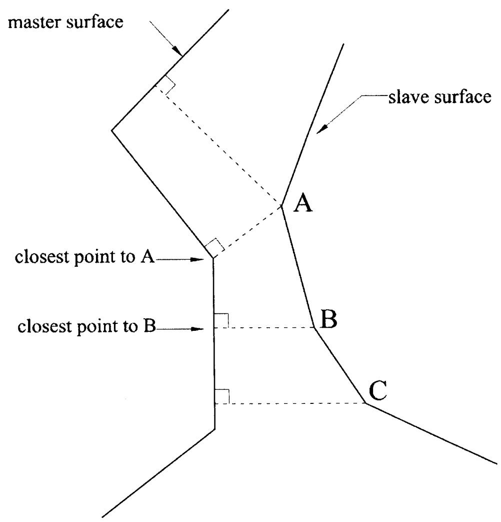


Figure 4. Definition of contact pairs between nodes (ABAQUS, 2004).

across the interface. ABAQUS uses an isotropic Coulomb friction model to account for friction between the contacting surfaces. The critical shear stress, $\tau_{crit.}$, at which sliding of the surfaces begins, is defined in the model. The critical shear stress is defined as:

$$\tau_{crit.} = \mu p \tag{1}$$

where μ is the coefficient of friction, and p is the normal contact pressure.

In this study, friction was assumed to be equal in all directions. Normally, there is a difference in magnitude between friction when slippage is initiated and when it is underway. Due to lack of data, the same friction coefficient (0.5) was assumed for both cases.

A finite sliding model formulation was applied. This formulation allows the contact surfaces to separate and slide with finite amplitude and arbitrary rotation. As mentioned previously, the hole in the wood is a little bit smaller (by 0.01 mm) than the thickness of the idealized tooth. This ensures an initial contact between wood and tooth. The initial contact is adjusted automatically by "moving" the overpassing nodes to the exact contact position. This technique ensures the necessary good initial contact.

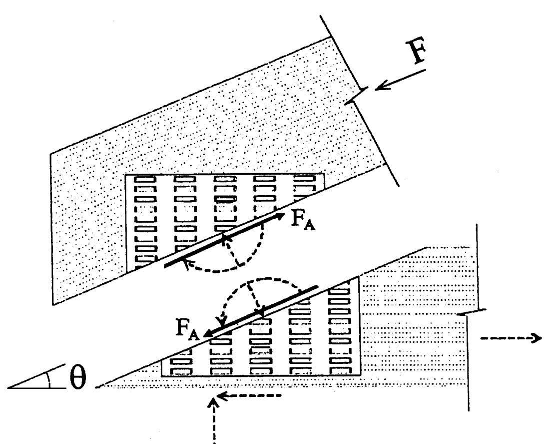


Figure 5. Free-body diagram of the heel joint. Axial displacement of the joint is defined along the wood grain of the top chord. The forces and moments that were ignored in modeling the joint are shown in broken lines.

Transformation of Properties

For the heel joint, the internal force, F_A, is in the plane of the top chord but is at an angle to the grain of the bottom chord (fig. 5). Therefore, the MOE value measured along the grain of the bottom chord needs to be transformed to the plane of F_A using elastic theory. The stiffness matrix of the bottom chord in the direction of force F_A is calculated as:

$$S_\theta = TST^{-1} \tag{2}$$

where S_θ is the stiffness matrix of the bottom chord in the plane of force F_A, S is the stiffness matrix in the direction of grain of the bottom chord, and T is the transformation matrix.

To calculate the MOE in the plane of force F_A, the compliance element ($S_{\theta,1}$) must first be calculated as (Bodig and Jayne, 1982):

$$S_{\theta,1} = \frac{1}{E_L}\cos^4\theta + \left(\frac{1}{G_{LR}} - 2\frac{v_{RL}}{E_R}\right)\sin^2\theta\cos^2\theta + \frac{1}{E_R}\sin^4\theta \tag{3}$$

The MOE in the plane of force F_A is then obtained as the inverse of the compliance element ($S_{\theta,1}$) as (Bodig and Jayne, 1982):

$$E_{\theta,1} = \frac{1}{S_{\theta,1}} \tag{4}$$

where E_L is the MOE in the longitudinal direction, E_R is the MOE in the radial direction, G_{LR} is the shear modulus in the longitudinal-radial plane, and v_{RL} is Poisson's ratio, considering an active strain in the longitudinal direction and a resulting passive strain in the radial plane.

Mechanical properties in the different axes are defined by the following ratios (Bodig and Jayne, 1982):

$$\left.\begin{aligned} &E_L : E_R : E_T >> 20:1.6:1 \\ &G_{LR} : G_{LT} : G_{RT} >> 10:9.4:1 \\ &E_L : G_{LR} >> 14:1 \end{aligned}\right\} \tag{5}$$

where E_T is the MOE in the tangential axis, G_{LT} is the shear modulus in the longitudinal-tangential plane, and G_{RT} is the shear modulus in the radial-tangential plane.

Poisson's ratio in the longitudinal direction, $v_{LR} = 0.4$, and in the radial direction, $v_{RL} = 0.041$ (assuming $\frac{v_{LR}}{E_L} = \frac{v_{RL}}{E_R}$) were used in this study. These values were proposed by Bodig and Jayne (1982) for softwood.

Results and Discussion

Tension-Splice Joint

In analyzing the tension-splice joint, test results (from Riley and Gebremedhin, 1999) of modulus of elasticity of two wood species groups, southern pine and spruce-pine fir, were used. These values were 8.49 MPa for spruce-pine fir, and 10.85 and 15.17 MPa for southern pine. The modulus of elasticity used for steel was 203,000 N/mm^2, and Poisson's ratios used for steel and wood were 0.3 and 0.4, respectively.

The elongation of the joint was calculated at two locations that are vertical to each other. These two locations are identified by the two nodes shown in figure 6. Node N_A is

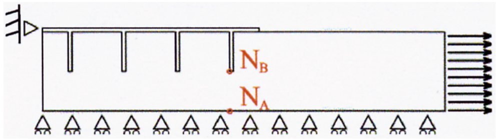


Figure 6. Model definition and node locations where displacements were calculated for the tension-splice joint.

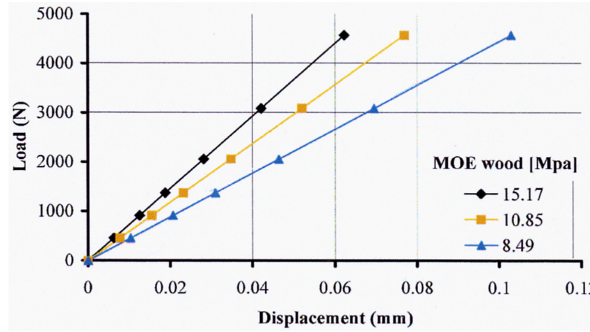


Figure 8. Load-displacement results for the tension-splice joint for three MOE values.

located along the axis of symmetry below the last tooth, and the second node (N_B) is located at the tip end of the last tooth. These two locations correspond to where measurements of elongations were made by Riley and Gebremedhin (1999). In calculating displacements, the contributions of the teeth and wood were taken into account.

The predicted node displacement (Δ_h) is equal to one-half the displacement of the joint. The corresponding force (F_h) is equal to the reaction force. The effective stiffness was, therefore, calculated by dividing F_h by Δ_h. The location of node N_A accounts for the displacement due to bending of all teeth, axial extension of the plate, and deformation of the wood. The location of node N_B, however, does not take into account the deformation of the wood. The predicted stiffness values are compared (table 1) against values measured by Riley and Gebremedhin (1999). The maximum difference between the predicted and measured stiffness values is 4% at node N_A and 10% at node N_B. On the average, the stiffness at node N_A is less than 2% and that at N_B is less than 9% different from the measured values (table 1). It can be concluded that the model predicts axial stiffness of MPC tension-splice joints reasonably well.

Contour plots of displacement of the joint are shown in figure 7. The load-displacement relationships for the three MOE values used in the model are shown in figure 8. The relationships are almost linear, and it appears that no initial slip occurred due to lack of initial contacts between teeth and wood.

The modeling technique used herein provides some insight into what is happening at the tooth-wood interface. Figure 9 shows the contact pressure in the teeth and wood surfaces. Note that high pressure is produced at the tooth-wood interface closest to the metal-plate surface (base of tooth), and some at the tooth bottom end. No negative pressure (tension) is shown in the figure when a tension force was applied from left to right, which validates the assumption that no tension force is transmitted between the contact surfaces.

The pressure profile, shown in figure 9, is different from those reported by Riley and Gebremedhin (1999) and by Groom and Polensek (1992). These studies assumed that the punched teeth act like cantilever beams in an elastic foundation. The pressure profile reported by Riley and Gebremedhin (1999) is shown in figure 10. Similarly, the maximum pressure is at the base of the tooth but decreases toward the tip of the tooth. In our model, the top segment of the tooth presses against the wood in one direction, whereas the bottom segment presses in the opposite direction, as shown in figure 11. Pressure varies with tooth location, being lower on the tooth closest to the joint gap.

The shear due to contact friction is shown in figure 12. The friction profile is different for each tooth. The friction force is lower for the tooth closer to the gap and higher for the tooth farthest from the gap. The stress distribution in the metal-plate surface and wood are shown in figures 13a and 13b, respectively. The metal plate closest to the gap is more stressed than the surface farther away, and the wood close to the tooth base and tip show higher stresses than any other point in the wood.

Table 1. Measured and predicted stiffness values for MPC tension-splice wood truss joint (values in the parentheses are percent differences from measured values).

Lumber Species Group	Wood MOE (MPa)	Measured Stiffness[a] (kN/mm)	Predicted Stiffness at Specified Nodes (kN/mm)	
			N_A	N_B
Spruce-pine fir	8.49	21.3	22.22 (+4%)	19.48 (-9%)
Southern pine	10.85	29.8	29.69 (-0.4%)	26.7 (-10%)
Southern pine	15.17	35.9	36.67 (+2%)	33.41 (-7%)

[a] From Riley and Gebremedhin (1999).

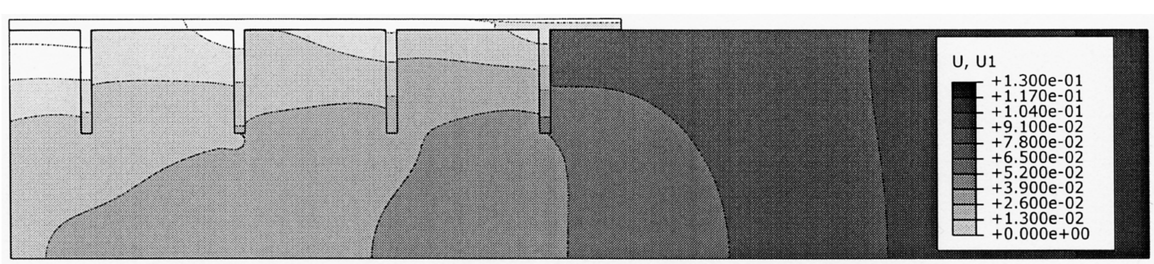


Figure 7. Contour plot of axial displacement (longitudinal direction) in the wood.

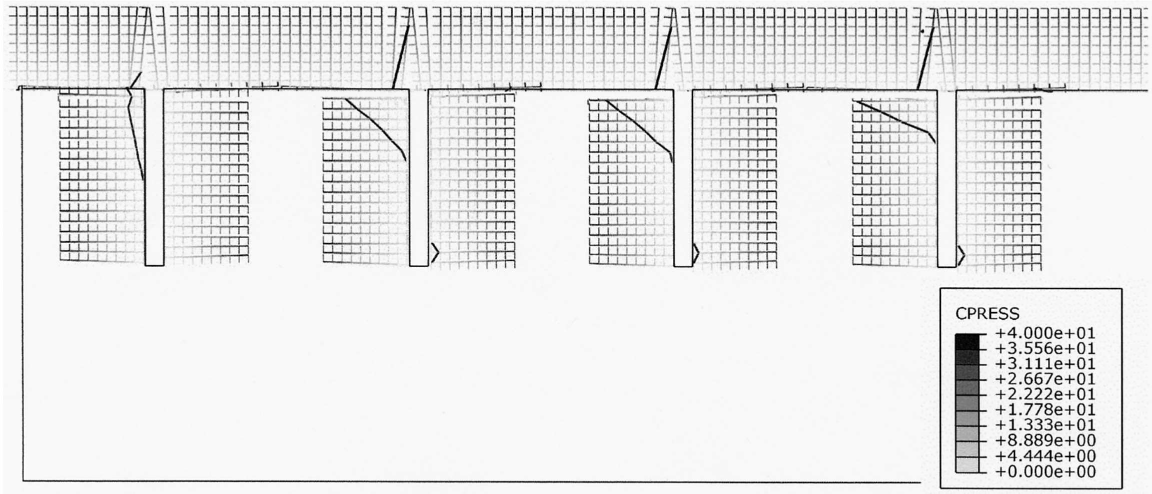


Figure 9. Contact pressure distribution.

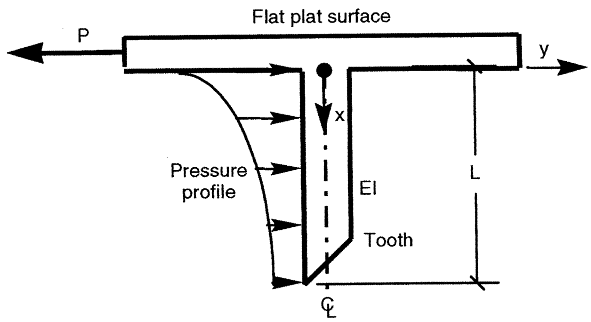


Figure 10. Pressure profile on a tooth of a metal plate assumed as a cantilever beam in an elastic foundation (Riley and Gebremedhin, 1999).

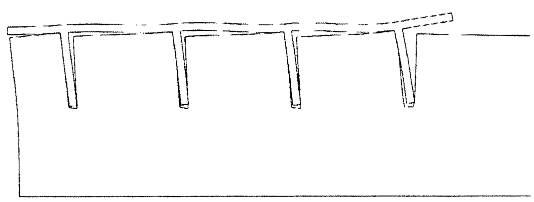


Figure 11. Scaled deformed shape of the metal-plate connector (scale of the deformed shape is 25).

Heel Joint

The heel joint is much more complex than the tension-splice joint because of eccentric loading to the metal plate, which creates rotation. As pointed out previously, the axial stiffness of the joint is calculated based on the assumption that the deformation of the joint is a result of the compressive force in the top chord. The deformation that exists perpendicular to the compressive force is ignored. This assumption reduced the model to a plane strain problem, and thus allowed the heel joint to be modeled in the same way as the tension-splice joint. Riley and Gebremedhin (1999) also ignored the perpendicular deformation in computing axial stiffness of the heel joint.

Heel joints made of different lumber properties (MOE) and slopes were analyzed. These were the same joints tested by Riley and Gebremedhin (1999).

The displacement of the heel joint was calculated by superposition, separately calculating the displacements of the top and bottom chords. The geometrical model for both chords is the same, but the MOE values for the bottom chord need to be transformed, as explained previously. Since the MOE of the top chord was in the direction of load and the grain of the wood, no transformation was necessary.

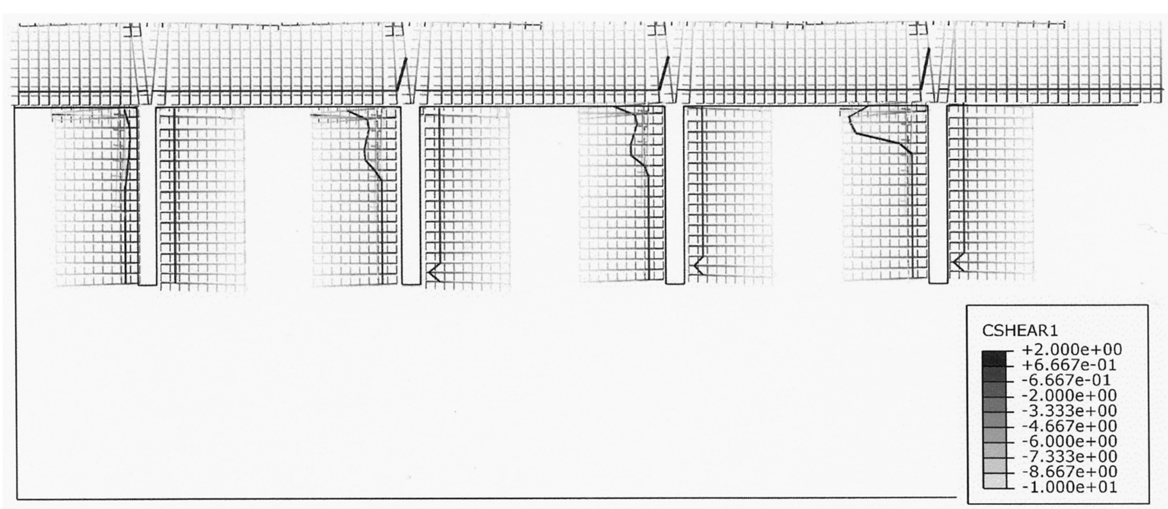


Figure 12. Contact shear distribution due to friction.

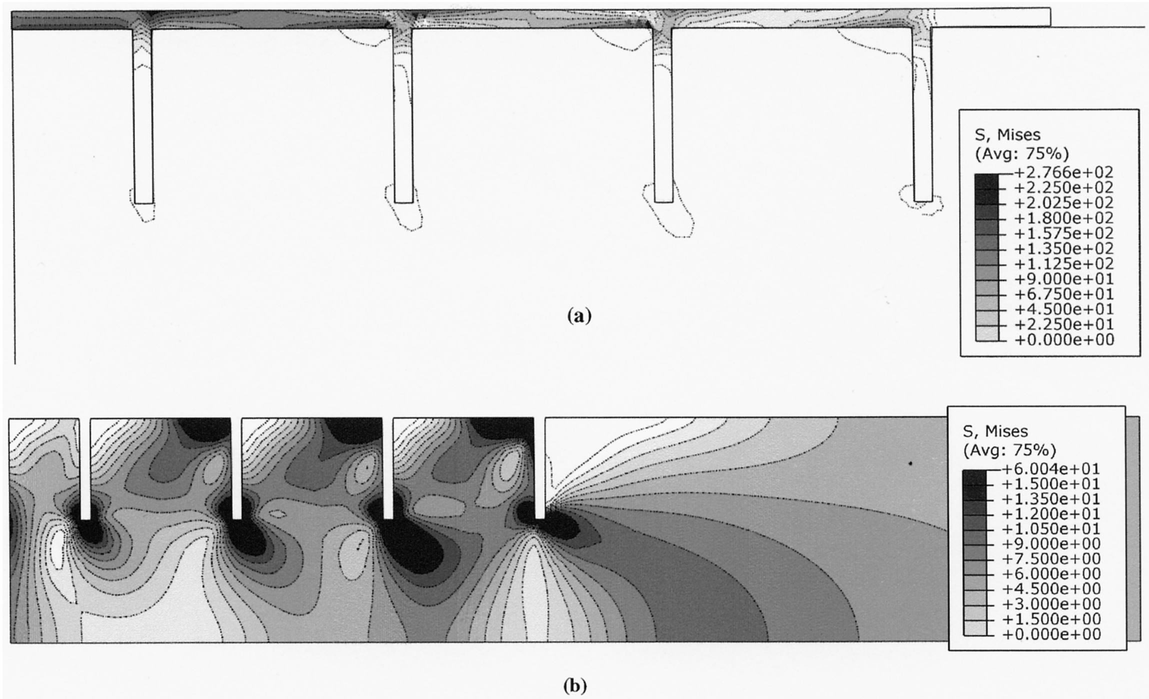


Figure 13. Stress contour (Von Mises reference stress): (a) in the metal plate and (b) in the wood.

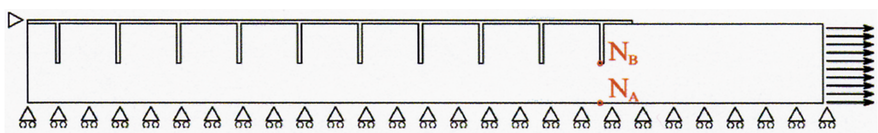


Figure 14. Model definition and node locations where displacements were calculated for the heel joint.

Table 2. Measured and predicted stiffness values for MPC heel wood truss joint (values in parentheses are percent differences from measured values).

Lumber Species Group	Slope of Top Chord	Wood MOE (MPa)	Measured Stiffness[a] (kN/mm)	Predicted Stiffness at Specified Nodes (kN/mm)	
				N_A	N_B
Spruce-pine fir	5:12	10.25	3.24	3.59 (+11%)	3.51 (+8%)
Southern pine	5:12	10.35	3.60	3.61 (+0.3%)	3.53 (-2%)
Southern pine	3.5:12	9.30	4.36	4.13 (-6%)	3.78 (-14%)

[a] From Riley and Gebremedhin (1999).

The displacement for the top chord is represented by Δ_{TC} and that of the bottom chord is represented by Δ_{BC}. The stiffness of the heel joint was calculated as:

$$k = \frac{F}{\Delta_{TC} + \Delta_{BC}} \tag{6}$$

where F is the top chord compressive force.

Displacements of the joint were calculated at two locations (fig. 14), and the stiffness values were calculated at these two locations. The maximum difference between the predicted and measured stiffness values was 11% higher at node N_A and 14% lower at node N_B. On the average, the difference was less than 2% at node N_A and less than 3% at N_B (table 2). Even though the model predicted accurately for an MOE of 10.35 MPa, there was no clear trend in joint stiffness with varying lumber MOE. This may be because of the assumptions made to ignore rotation and deformation perpendicular to the direction of the axial force.

Conclusions

The following specific conclusions can be drawn from the study:

- A simple and relatively accurate finite element model that predicts axial stiffness of MPC tension-splice and heel joints of wood trusses was developed using the ABAQUS (2004) commercial computer software. Both joints were modeled as two-dimensional plane strain problems by accounting for tooth slip and for friction at the tooth-wood interface. The approach required no empirical factor such as foundation modulus.
- The model predictions were compared against measured stiffness values available in the literature. On the average, the model predicted within 5% (between -10% and +4%) of measured values for the tension-splice joint, and within 0.5% (between -14% and +11%) for the heel joint.

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