Controlling and synchronizing space time chaos

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Control and synchronization of continuous space-extended systems is realized by means of a finite number of local tiny perturbations. The perturbations are selected by an adaptive technique, and they are able to restore each of the independent unstable patterns present within a space time chaotic regime, as well as to synchronize two space time chaotic states. The effectiveness of the method and the robustness against external noise is demonstrated for the amplitude and phase turbulent regimes of the one-dimensional complex Ginzburg-Landau equation. The problem of the minimum number of local perturbations necessary to achieve control is discussed as compared with the number of independent spatial correlation lengths. [S1063-651X(99)00806-5]

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In the last decade, control and synchronization of chaos have attracted the attention of the scientific community. In both cases, a chaotic dynamics is conveniently disturbed by means of an external perturbation (usually small as compared with the unperturbed dynamics), in order to force the appearence of a goal behavior $g(t)$ compatible with the natural evolution of the system. In the former case, the goal dynamics corresponds to one unstable periodic orbit embedded within the chaotic attractor [1], in the latter case it corresponds to compensating for the difference of the same system due to different initial conditions.

Since the first proposals for control [2] and synchronization [3] of chaos, many other approaches have been suggested for chaos control [4,5], while the concept of chaotic synchronization has been recently extended to that of phase synchronization [6] and lag synchronization [7]. The transition between different types of synchronization processes has been extensively studied in a pair of symmetrically coupled chaotic oscillators [7,8]. On the other hand, the control of chaos has been shown to be effective even in the case of delayed dynamical systems [9], by the use of the adaptative technique [5].

The huge body of literature devoted to these issues is justified by the large interest that they have in practical applications, such as communicating with chaos [10], secure communication processes [11,12], and experimental control of chaos in many areas such as, e.g., chemistry [13], laser physics [14], electronic circuits [15], and mechanical systems [16].

Only recently, control mechanisms have been investigated in space-extended systems. After some preliminary attempts [17] to control spatiotemporal chaos, attention has been directed to the control of two-dimensional patterns [18], coupled map lattices [19,20], or particular model equations, such as the complex Ginzburg-Landau equation [21] and the Swift-Hohenberg equation for lasers [22]. Furthermore, synchronization has been proved in extended systems with unidirectional (drive-response) configuration [23].

However, while for concentrated systems the different proposed techniques have easily found experimental verifications, in the extended case there are not yet experimental counterparts to the quite large body of theoretical proposals [17–23]. The main reason for this lack of experiments is that almost all proposed methods used space-extended perturbations, that is, perturbations which had to be applied at any point of the system. The few examples of global control [19], or control with a finite number of local perturbations [20], have so far been limited to discrete systems, i.e., to coupled map lattices. The most relevant problem in passing from concentrated to space-extended continuous systems arises, indeed, when considering that an extended continuous system is an intrinsically infinite dimensional system. Therefore, while control or synchronization of a concentrated system implies a perturbation on a single control parameter, or a single state variable, in the case of a continuous extended system it is still unclear whether the perturbation itself should be extended in space, i.e., should affect all points of the considered system. This last requirement would, indeed, be very difficult to realize experimentally, thus frustrating the possibility of implementing control and/or synchronization of space time chaotic states.

In this paper, we show that both control and synchronization can be achieved in a continuous extended system by means of a *finite* number of local controllers, i.e., by a finite number of nonextended perturbations, each affecting a different point in the system. The minimum number of controllers will be derived, and the robustness of both processes against the presence of noise will be verified.

For the sake of exemplification, and without lack of generality, we refer to the one dimensional complex Ginzburg-Landau equation

$$\dot{A}=A+(1+i\mu_1)A_{xx}-(1+i\mu_2)|A|^2A, \qquad (1)$$

where $A(x,t)\equiv\rho(x,t)e^{i\psi(x,t)}$ is a complex field of amplitude ρ and phase ψ, and the dot denotes a temporal derivative. A_{xx} stands for the second derivative of A with respect to the space variable $0\leqslant x\leqslant L$, where L represents the system length, and μ_1, and μ_2 are suitable real control parameters. The boundary conditions are chosen to be periodic.

Equation (1) describes the universal dynamical features of an extended system close to a Hopf bifurcation [24], and it has been used to describe many different situations in laser physics [25], fluid dynamics [26], chemical turbulence [27], bluff body wakes [28], etc. Different chaotic regimes were identified in Eq. (1) in different regions of the parameter space (μ_1,μ_2) [29]. In fact, Eq. (1) has plane wave solutions of the type

$$A_q=\sqrt{1-q^2}e^{i(qx+\omega t)}, \tag{2}$$

where $-1\leqslant q\leqslant 1$, q being the wave number in Fourier space, and the dispersion relation is

$$\omega=-\mu_2-(\mu_1-\mu_2)q^2. \tag{3}$$

In the parameter region $\mu_1\mu_2>-1$, there exists a critical value of the wave number $q_c=\sqrt{(1+\mu_1\mu_2)/[2(1+\mu_2^2)+1+\mu_1\mu_2]}$, such that all the plane waves in the range $-q_c\leqslant q\leqslant q_c$ are linearly stable. Outside this range, they become unstable through the so called Eckhaus instability [30]. Since q_c vanishes as the product $\mu_1\mu_2$ approaches -1, all plane waves become unstable when crossing from below the line $\mu_1\mu_2=-1$ in parameter space. Such a line is called the Benjamin-Feir or Newell line. Above this line, Ref. [29] identifies three different turbulent regimes—namely, phase turbulence (PT), amplitude turbulence (AT), or defect turbulence—and bichaos. In the following we will concentrate on PT and AT, since they have received special attention in the scientific community [31].

PT is the dynamical regime encountered just above the Benjamin-Feir line, and it is characterized by the fact that the chaotic behavior of $A(x,t)$ is essentially dominated by the dynamics of the phase $\psi(x,t)$, whereas the amplitude $\rho(x,t)$ changes smoothly, and is always bounded away from zero. By further moving away from the Benjamin-Feir line, a transition is encountered toward AT, wherein the amplitude dynamics becomes dominant over the phase dynamics, leading to large amplitude oscillations which can occasionally drive $\rho(x,t)$ to zero. The vanishing of ρ causes the occurrence of a space-time defect.

Both PT and AT are characterized by the fact that the spatial autocorrelation function decays exponentially with a spatial correlation length ξ which is smaller than the system size L, that is,

$$C(x,x')=\langle A(x,t)A^*(x',t)\rangle_t\simeq e^{-|x-x'|/\xi}, \tag{4}$$

where $\langle\ \rangle_t$ denotes the temporal average. In two spatial dimensions it has been theoretically predicted [25] and experimentally verified [32] that defects have a dynamical role in mediating the shrinking process of ξ, thus in the passage from regular to turbulent behavior.

Within a domain of size ξ, the dynamics remains space correlated. Therefore, once ξ has been measured, the main features of the space time chaotic dynamics can be captured by considering a collection of $N=\mathrm{int}(L/\xi)+1$ uncorrelated domains. A single local perturbation within each domain is sufficient to assure the collapse of $A(x,t)$ onto any goal pattern $g(x,t)$ compatible with the natural evolution of the system.

In fact, we expect that the number of local perturbations necessary to slave $A(x,t)$ to a general goal pattern $g(x,t)$ be smaller than N, because of nonlinear constraints within the system, which make each correlation domain interacting with all the others. Therefore, in the following, we first demonstrate that the above sufficient condition holds for a judicious choice of the local perturbations, and then we will move to show that the necessary condition for the control can, in fact, be obtained with a number of local controllers smaller than N.

Let us begin with the problem of control of space time chaos. For this purpose, we set $\mu_1=2.1$ and $\mu_2=-1.3$ in Eq. (1) in order to enter the AT regime. In the following, we will numerically solve Eq. (1) with $L=64$, periodic boundary conditions, and random initial conditions. The numerical code is based on a semi-implicit scheme in time with finite differences in space. The precision of the code is first order in time and second order in space. In all the simulations we use a space discretization $dx=0.125$ (512 mesh points) and a time step for the integration $dt=0.001$. For the selected μ_1 and μ_2, the spatial correlation length is $\xi=4.39$, corresponding roughly to 35 points of the mesh $(N=17)$. Control of space time chaos here implies the emergence of some unstable periodic pattern out from the AT regime. In this case, the goal pattern $g(x,t)$ is represented by any of the plane wave solutions (2), which are unstable in AT.

In order to control the system to the desired goal pattern, to the right hand side of Eq. (1) we add a perturbative term $U(x,t)$ of the type

$$U(x,t)=0 \quad \text{for} \quad x\neq x_i$$
$$U(x,t)=U_i(t) \quad \text{for} \quad x=x_i \tag{5}$$

where $i=1,\ldots,M$ and $x_i=1+(i-1)\nu$ are the positions of M local controllers, mutually separated by a distance ν $(x_{i+1}-x_i=\nu)$.

For the time being, we will use $\nu=\xi$, so that $M=N$, in order to show that a sufficient condition for a robust control is that the number of controllers equals the number of correlation domains. Later on, we will show that control can also be achieved for $\nu>\xi$ $(M<N)$, and we will therefore prove that the minimum requested number of local controllers is, in fact, smaller than the number of correlation domains, thus making our approach of some help for overcoming the encountered difficulties in practical experimental implementations.

The strength of the M perturbations $U_i(t)$ is selected by the following algorithm. At each controller position x_i and at each integration time t_n, the ith controller measures the distance $\delta_i(t_n)$ between the actual dynamics $A(x_i,t_n)$ and the goal pattern $g(x_i,t_n)$:

$$\delta_i(t_n)=A(x_i,t_n)-g(x_i,t_n). \tag{6}$$

Then the controller evaluates the local variation rates

$$\lambda_i(t_n)=\ln\left|\frac{\delta_i(t_n)}{\delta_i(t_{n-1})}\right|, \tag{7}$$

and selects the perturbation as

$$U_i(t_n)=K_i(t_n)(g(x_i,t_n)-A(x_i,t_n)), \tag{8}$$

where

$$\frac{1}{K_i(t_n)}=\frac{1}{K_0}[1-\tanh(\sigma\lambda_i(t_n))],\quad \sigma>0,\qquad K_0>0. \tag{9}$$

The algorithm of Eqs. (6)–(9) is an extension of the adaptive algorithm introduced in Ref. [5], and successfully applied also to chaos synchronization [12], targeting of chaos [33], filtering of noise from chaotic data sets [34], and control of delayed dynamical systems [9].

The adaptive nature of the algorithm is clear when one considers that the strength of the perturbation in Eq. (8) depends adaptively on the local dynamics of the system. Indeed, when $A(x_i,t_n)$ naturally tends to shadow the goal pattern $g(x_i,t_n)$, this implies a temporal decreasing behavior of $\delta_i(t)$, and a consequently negative $\lambda_i(t)$, and therefore a reduction of the weight factor $K_i(t)$ in Eq. (9). Conversely, whenever the natural evolution of the dynamics tends to push the system away from the goal pattern, this is reflected by a growth of $K_i(t)$. In other words the perturbation is adapted to the local dynamics, since the further (closer) the system is to the goal pattern, the larger (smaller) is the weight given to the perturbation. It should be remarked that the limit $\sigma\to 0$ of the above algorithm recovers the Pyragas' control method of Ref. [4], implying a constant weight K_0 in Eq. (9). The positive quantity σ represents the sensitivity of the method, and it plays a crucial role in assuring the smallness of the perturbations as well as the effectiveness of the control [5].

Figure 1 reports the control of one of the unstable plane waves (2) for $\sigma=0.1$ and $K_0=1$. The control procedure implies the suppression of the defects, until the controlled amplitude relaxes to a constant value. The arrow indicates the instant at which control is switched on. The control procedure is effective for a large range of σ and K_0 values.

The control process here introduced works also in PT, with similar features as in Fig. 1. In this case, the absence of defects allows an even larger range of σ and K_0 values for the effectiveness of the control procedure.

Let us now discuss the robustness of the control method against white noise. For this purpose, in addition to the control perturbation $U(x,t)$, to the right hand side of Eq. (1) we add a Gaussian noise $\pi(x,t)$ with zero average and δ correlated in space and time $[\langle\pi(x,t)\rangle_t=0$ and $\langle\pi(x,t)\pi^*(x',t')\rangle=\gamma\delta(x-x')\delta(t-t')]$. The results are shown in Fig. 2. for a noise strength of 1% of the unperturbed dynamics A. The control process still leads to the appearance of the desired goal pattern for relatively high noise strengths (up to 4%). The lower part of the right picture shows that noise cancellation is effective only at the controller points.

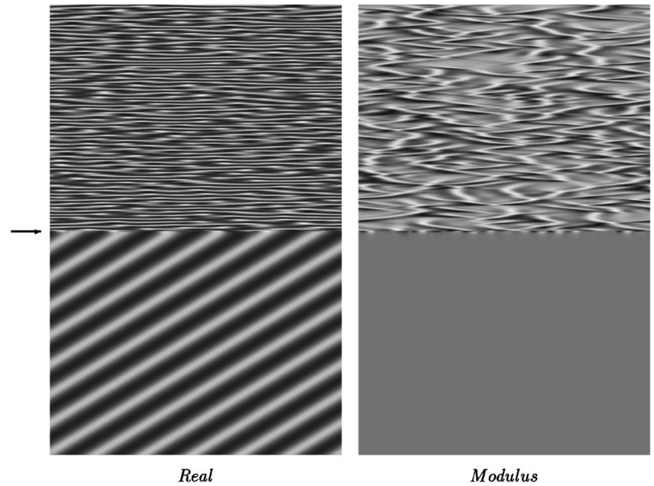

FIG. 1. Space (horizontal)–time(vertical) plots of the real part of A (left) and the modulus of A (right). Time increases downwards from 1000 to 1800 (u.t.). The first 1000 time units correspond to the transient before the system reaches the chaotic (AT) domain starting from random initial conditions. The patterns have been coded in 256 gray levels (white corresponds to maxima). The parameters are $\mu_1=2.1$, $\mu_2=-1.3$, $dt=0.001$, $L=64$, and $dx=0.125$. The control ($\sigma=0.1$, $K_0=1$) starts at $T=1400$ (indicated by an arrow). The goal dynamics is chosen to be the particular plane wave solution (2) having $q=0.589$ (corresponding to six wavelengths for this system size). The associated frequency and amplitude are $\omega=0.12$ and $A_q=0.808$. Under these conditions, the control is reached after a very fast transient and with only $M=17$ controllers.

Finally, we discuss the problem of chaos synchronization. In this case we consider two complex fields $A_1(x,t)$ and $A_2(x,t)$, each obeying Eq. (1) with the same parameters μ_1 and μ_2 as in the above case. The two fields evolve from different random initial conditions, thus producing two space

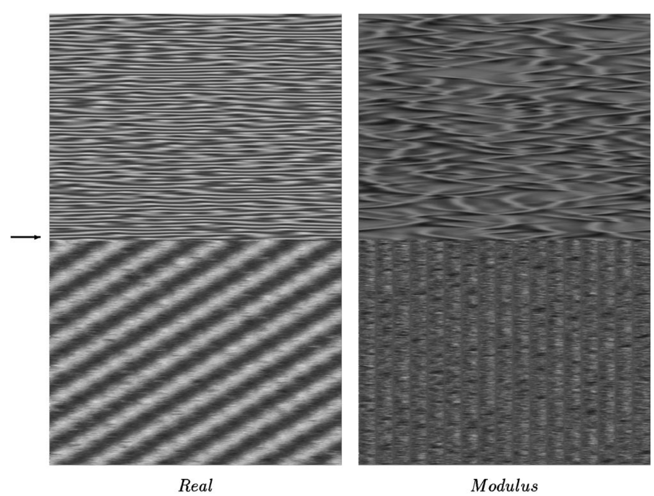

FIG. 2. Same as Fig. 1 with the addition of Gaussian noise with a standard deviation $0.01\times A_q/\sqrt{2}$ to all points of the mesh at each time step. This noise is added to both the real and imaginary parts of the field $A(x,t)$. The trace of the $M=17$ equispaced controllers is now visible on the modulus.

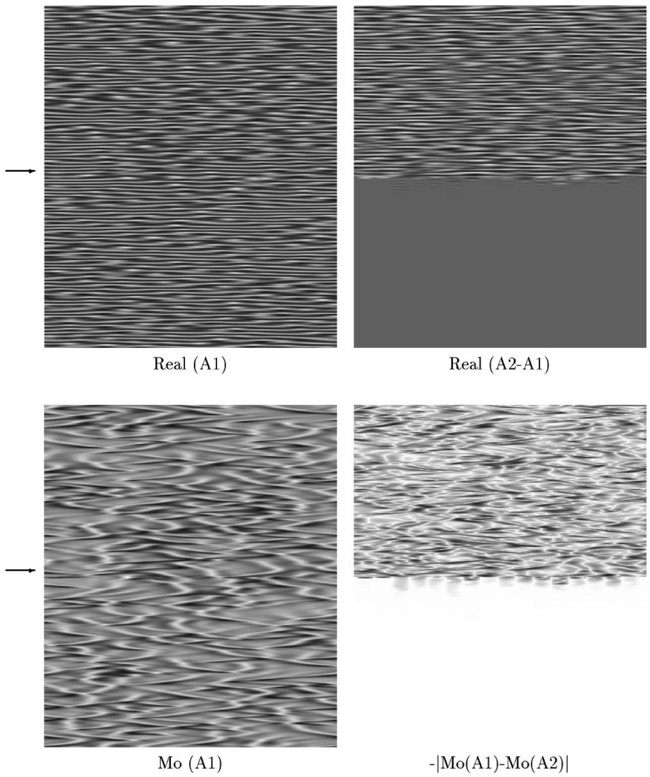

FIG. 3. Synchronization of two identical systems $A_1(x,t)$ (left column) and $A_2(x,t)$, both in the AT regime (same parameters as in Fig. 1). The right columns display the differences between the two patterns (upper, real parts; lower, moduli). The time runs from 1000 to 1600 (u.t), and the synchronization starts at $T=1300$ (indicated by an arrow).

and time unsynchronized AT dynamics. In this case, the algorithm of Eqs. (6)–(9) is used in order to select the perturbations at each controller point x_i. Now, the goal dynamics for $A_1(x,t)$ is $A_2(x,t)$, and vice versa. In other words, the local controllers symmetrically force each complex field to collapse into the other one. The results are shown in Fig. 3, for $\sigma=0.1$ and $K_0=1$. The arrow indicates the instant at which the controllers become active. Rather than suppressing the defects, here the final synchronized state $A_1(x,t) = A_2(x,t)$ remains amplitude turbulent, but the process determines the synchronization of the defects as shown by the equality of the amplitudes A_1 and A_2. Also in this case, the process is effective in PT, and it is robust against external noise up to 4% of the amplitude of both complex fields.

It is important to point out that, while the proposed control process crucially relies on a knowledge of the goal plane wave, here the synchronization procedure is independent of any previous knowledge of the system, since the local goal values for the two fields can be directly measured by the same controllers at any time and at any controller location. In the control case, one should first individuate the coefficients μ_1 and μ_2 in Eq. (1) by using a preliminary learning task on the unperturbed system. Then, the use of the dispersion relation (3), and of the expression of A_q in Eq. (2) allows one to calculate the desired plane wave at all times and at all spatial locations. Conversely, the synchronization process can be implemented without any kind of previous knowledge on the system.

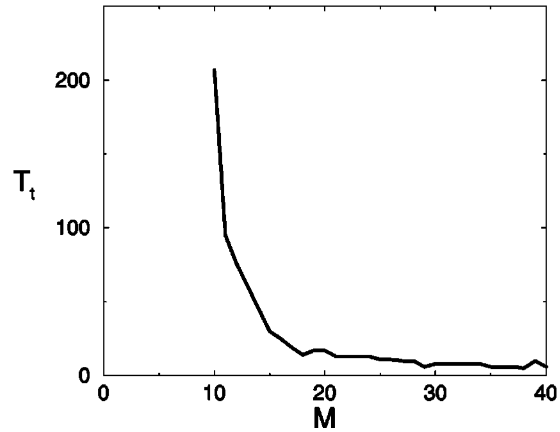

FIG. 4. Plot of the transient time T_t before achieving control as a function of the number M of equidistantly spaced controllers. Same parameters as in the caption of Fig. 1: $\mu_1=2.1$, and $\mu_2=-1.3$, AT regime. The proposed method fails for $M<8$, whereas a controller for each double correlation length is enough to achieve control.

Let us now discuss the problem of the minimum number of requested local perturbations. In Fig. 4 we report the transient time T_t for achieving control of the same plane wave and in the same parameter conditions as in Fig. 1, as a function of M. Looking at Fig. 4 one easily realizes that T_t diverges to $+\infty$ for $M<8$. Recalling that $L=64$ and $\xi=4.39$, so that $N=17$, Fig. 4 actually tells us that control is possible, unless associated with a larger transient time, even with a controller distance $\nu \simeq 2\xi$, that is, with a number of controllers about one half the number of correlation domains. This improvement suggests that our adaptive method can overcome the difficulties encountered so far for experimental implementations of control of space time chaotic states.

In conclusion, we have shown that control and synchronization of a space-extended system can be realized by means of a finite number of local controllers, affecting different points of the system, which can be mutually separated by more than a space correlation length. Therefore, the minimum controller number comes out to be smaller than the number of correlation domains. The robustness of the procedure against external noise has been proved in the special case of the amplitude and phase turbulent regimes of a one-dimensional complex Ginzburg-Landau equation.

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