

Order Parameter Fragmentation after a Symmetry-Breaking Transition

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As a nonlinear optical system consisting of a Kerr medium inserted in a feedback loop is exposed to a light intensity growing linearly from below to above the threshold for pattern formation, the critical slowing down around threshold freezes the defect population. The measured number of defects immediately after the transition scales with the quench time as predicted by Zurek for a two-dimensional Ginzburg-Landau model. The further temporal evolution of the defect number is in agreement with a simple annihilation model, once the drift of defects specific for our system is taken into account.

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Symmetry-breaking transitions are a fundamental subject of research in many fields of physics from condensed matter in thermodynamical equilibrium [1] to nonequilibrium systems that undergo bifurcations [2], possibly leading to patterned states in spatially extended systems [3]. With reference to cosmological models, Kibble [4] stressed that topological defects may have played a fundamental role in the evolution of the early Universe. Zurek [5] captured the general feature of the rapid crossing of the threshold region, pointing out the formal similarity between the cosmological phase transition and some accessible laboratory situations, thus opening the field to a series of theoretical and experimental considerations [6–9]. All these phenomena can be described by a complex field in a quartic potential, as the coefficient of the quadratic term (control parameter) is driven from positive (below threshold) to negative (above threshold) over a quench time τ_q . A central issue is the evaluation of the correlation length of the field as a function of τ_q which is the time scale of the control parameter variations. This correlation length has been shown to be given by the average separation of nearest-neighbor topological defects of the field under consideration [10,11]. The transient behavior has been modeled and verified in a recent series of papers [12–14] in which the defect number N at the end of the quench time τ_q and its successive evolution have been reported. From an experimental point of view, phase transitions in liquid crystals [15] and in superfluid helium [6–9] have revealed the occurrence of transient phenomena related to defects, but the key prediction of the power-law dependence of the defect number with τ_q has not yet been verified.

In this Letter we report the statistics of defects that form in a two-dimensional (2D) nonlinear optical system as the incident light intensity (control parameter) ramps linearly from below to above the threshold for pattern formation, within a quench time τ_q . The initial number N at the end of the quench follows the scaling law $\tau_q^{-1/2}$ and its successive time evolution is ruled by the combined action of mutual defect annihilation and escape due to drift.

The experimental setup consists of a liquid crystal light valve (LCLV) illuminated by a spatially uniform laser beam and inserted in a feedback loop (see Fig. 1). Within a suitable parameter range, the LCLV acts as an optical Kerr medium. As a result, the beam reflected from the front face of the LCLV undergoes a phase retardation proportional to the light intensity applied to the rear face of the valve. Optical pattern formation in a Kerr-like medium with various types of feedback has been studied both theoretically [16] and experimentally [17]. When the feedback loop includes a free propagation length L introducing diffraction in the optical wave, patterns are due to the interplay of the Kerr effect with the diffusion of the refractive index perturbation and the diffraction. Introduction of a nonlocal interaction by means of a transverse displacement of the optical wave front within the feedback loop induces a new class of pattern-forming instabilities [18,19]. The equation for the evolution of the phase of the beam reflected by the LCLV front face is

$$\tau \frac{\partial u}{\partial t} = -u(x, y, t) + l_d^2 \nabla_{\perp}^2 u(x, y, t) + \alpha I_{fb}(x + \Delta x, y, t), \quad (1)$$

where τ and l_d are, respectively, the response time and the diffusion length of the liquid crystals, and (x, y) are the coordinates in the plane transverse to the propagation direction z . The cell thickness is less than a diffusion length, hence u has no z dependence and ∇_{\perp}^2 is the Laplacian operator in the (x, y) plane. $I_{fb}(x + \Delta x, y, t)$ is the feedback intensity displaced by the amount Δx along x , and α gives the strength and sign of the Kerr nonlinearity. We use α negative (defocusing case). I_{fb} is proportional to the input intensity I_0 , which we take as the control parameter, and it includes the effects of dephasing through the cell, diffraction along the propagation length L , and lateral displacement Δx . Calling I_{0th} the intensity at the threshold for pattern formation, we rescale I_0 as $\epsilon \equiv \frac{I_0 - I_{0th}}{I_{0th}}$. Close to threshold, the normal form equation for the slowly varying amplitude A of u corresponding to

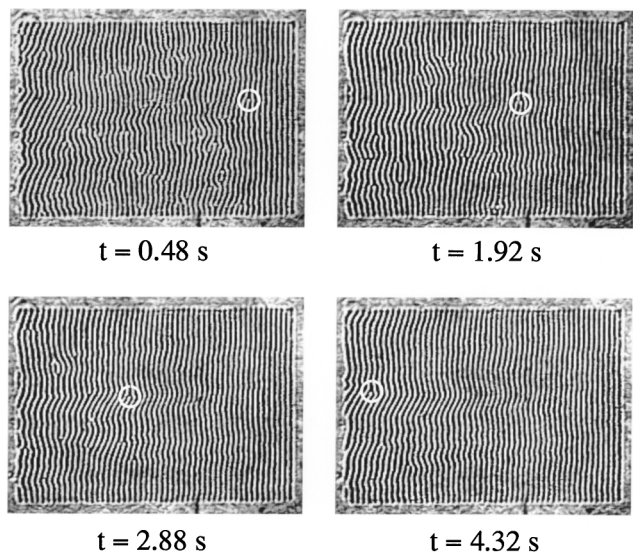


FIG. 3. Intensity patterns showing the evolution of the number of defects with time for a phase transition with $\tau_q = 0.41$ s. An open circle traces the drift of a defect far enough from other ones, to avoid annihilation within the observation.

no sense to apply a ramp on a time scale faster than, or comparable to, the inertial time scale τ_0 that is of the order of 100 ms. On the side of long τ_q , it is meaningless to let the system evolve over a time longer than the time l/v over which any perturbation will be swept away by coherent transport. In our experiment the system size is $l_0 = 10.35$ mm and the group velocity $v \approx 2$ mm/s for $\epsilon = 0.64$, leading to a maximum value of τ_q at about 5 s.

We then measure the evolution of the defect number with time, by counting the defects in a series of images of the type shown in Fig. 3 for the case $\tau_q = 0.41$ s. Since here we are studying the decay of the defect number starting from the end of the quench, that occurs at $t_{\text{fin}} = \tau_q/2$, we take this value as the time origin. In the absence of drift, N decreases because of annihilation processes. The annihilation rate is expected to be of the form $\dot{N} = -\chi N^2$, being proportional to the frequency with which defects encounter one another [13]. This leads to the evolution law $N(t)^{-1} = N_{(t=0)}^{-1} + \chi t$, already confirmed in a liquid crystal experiment [15].

In our experiment, the decrease of the defect number in the course of time is due both to the annihilation mechanism with a rate $\dot{N} = -\gamma N^2/A(t)$ and to the escape from the boundary at a rate $-Nv/l(t)$. We thus have the solution

$$N(t) = \frac{N_0(1 - vt/l_0)}{1 + \gamma N_0 t/A(0)}, \quad (2)$$

where N_0 is the initial number of defects in the area $A(0)$. In Fig. 4 we report the experimental measurements and their relative fits with Eq. (2) for three different values of τ_q . The theoretical curves are in good agreement with the experimental data as we can see from the values of γ and v reported in the figure caption.

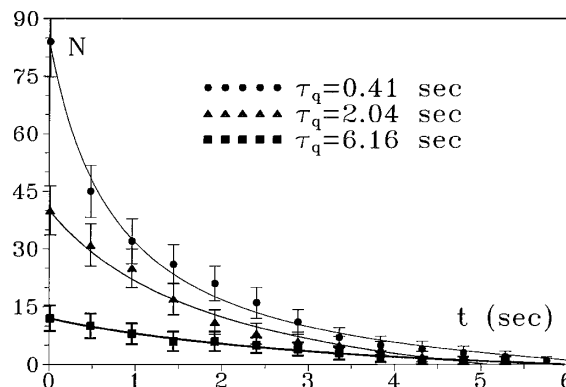


FIG. 4. Experimental measurements (symbols) and relative fits with Eq. (2) (lines) of the temporal evolution of the defect number for three different values of τ_q . The best fit values for the parameters v and γ result: $v = 1.37$ mm/s, $\gamma = 0.71$ mm²/s; $v = 1.63$ mm/s, $\gamma = 0.66$ mm²/s; $v = 1.1$ mm/s, $\gamma = 0.88$ mm²/s.

In order to have an insight to the range of times over which the process of annihilation dominates, we report also $1/N$ vs t for $\tau_q = 0.41$ s in Fig. 5. The plot includes the experimental points, the fit with Eq. (2), and the straight line obtained by putting $v = 0$ in Eq. (2). Expanding the expression of $1/N$ for $t \rightarrow 0$, the quadratic term in t becomes appreciable for $t \geq t^* = 1.93$ s. As we can clearly see in Fig. 4, for $t < t^*$ the linear fit is good, meaning that annihilation dominates the drift, thus recovering the results expected in [13] and observed in [15].

In summary, we have studied experimentally the transient statistics of the topological defects in a system with a complex order parameter swept in time across a supercritical bifurcation. Both the dependence of the early defect number on the sweep speed as well as its successive delay rate at later times confirm the theoretical predictions. In the first case, the defect rate dependence upon the sweep

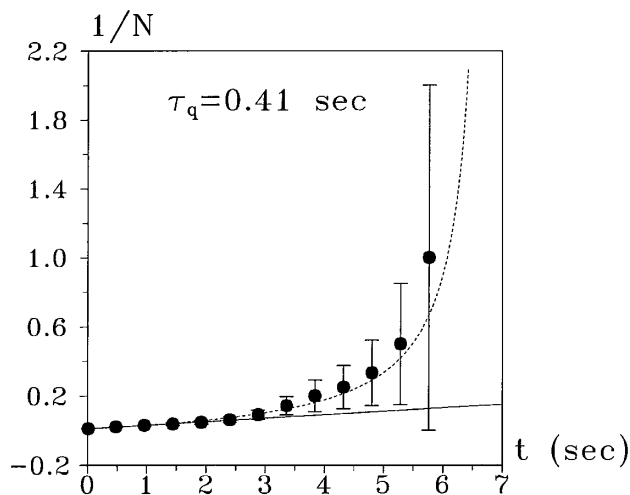


FIG. 5. $1/N$ vs t for $\tau_q = 0.41$ s: experimental data (dots), fit with Eq. (2) (dashed line), and straight line obtained by putting $v = 0$ in Eq. (2).

time is a power law with a $-1/2$ exponent. In the second case, once we account for a drift peculiar to our system, the defect annihilation model, based on a mass-action-like law, results are confirmed.

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