Dual Solutions of Stagnation-Point Flow of a Fluid on a Shrinking Surface of another Quiescent Fluid

Azizah Mohd Rohni[1], Zurni Omar[2] and Noraziah Hj Man[3]

Abstract— A problem of an orthogonal stagnation point flow for an incompressible two-dimensional impingement of a lighter fluid on the surface of a heavier fluid is considered. The governing equations in partial differential equations are first transformed into ordinary differential equations using similarity transformation. The resulting equations are then solved numerically using shooting technique. It is found that dual solutions exist for certain parameters considered.

Keywords— Two fluid; stagnation point; dual solutions

I. Introduction

THE flow induced by shrinking surface is compressed and attracted towards a fixed point and makes it different from the stretching case. As mentioned by Lok et al in [1], flow over the shrinking sheet is not confined within a boundary layer, and it is unlikely to exist. From a physical point of view, there are two situations for which the similarity solutions likely to exist; either an adequate suction is imposed on the boundary or a stagnation flow is added to confine the vorticity within the boundary layer [2].

In recent years considerable amount of interest has been given to the stagnation point flows due to their great importance in both theoretical and practical points of view [3] and the two-dimensional stagnation flow on a solid surface has been considered by many researchers ([1], [2], [3], [4]), to mention just a few. To our present knowledge, the study of stagnation flow of shrinking surface over another quiescent fluid with distortion (suction) has not been considered in literature. Therefore, we make an attempt to study the problem theoretically.

II. Problem Formulation

Consider an incompressible viscous fluid of density ρ_1 and kinematic viscosity ν_1 impinging orthogonally on a permeable shrinking surface of another quiescent, heavier incompressible viscous fluid of density ρ_2 and kinematic viscosity ν_2. Let (x, y_1) denote the Cartesian coordinates for the upper fluid with $x = 0$ as the symmetry plane, and x - axis is taken along the interface between the two fluids. It is assumed that the constant mass velocity is v_0, where $v_0 < 0$ for suction and $v_0 > 0$ for injection or withdraw of the fluid, respectively. It is also assumed that the sheet is shrinking with the velocity $u_w(x) = cx$, where c is a constant. The coordinate system for the lower fluid is (x, y_2). Under the boundary layer approximations, the governing equations of continuity and momentum are [5]:

For the upper fluid, we have the following governing equations:

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y_1} = 0 \tag{1}$$

$$u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y_1} = U_1 \frac{dU_1}{dx} + \nu_1 \frac{\partial^2 u_1}{\partial y_1^2} \tag{2}$$

The irrotational stagnation-point flow in the upper fluid towards the shrinking interface is described by

$$u_1(x) = cx, \quad v_1(y_1) = v_0 \tag{3}$$

where c is a constant. Thus, the boundary conditions for u_1 and v_1 at infinity are

$$\begin{aligned} &u_1(x) \to U_1(x) = ax, \\ &v_1(y_1) \to V_1(y_1) = -ay_1 \quad \text{as} \quad y_1 \to \infty \end{aligned} \tag{4}$$

For the lower fluid, we have the following governing equations:

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y_2} = 0 \tag{5}$$

$$u_2 \frac{\partial u_2}{\partial x} + v_2 \frac{\partial u_2}{\partial y_2} = U_2 \frac{dU_2}{dx} + \nu_2 \frac{\partial^2 u_2}{\partial y_2^2} \tag{6}$$

with the boundary condition at the interface

$$u_2(x) = cx, \quad v_2(y_2) = v_0 \tag{7}$$

Azizah Mohd Rohni[1] is with the School of Quantitative Sciences, Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia.

Zurni Omar[2], is with the School of Quantitative Sciences, Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia (e-mail: zurni@uum.edu.my).

Noraziah Hj Man[3] is with the School of Quantitative Sciences, Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia.

Since the lower fluid is at rest at infinity, the boundary conditions for u_2 and v_2 are

$$u_2(x) = U_2(x) \to 0, \quad v_2(y_2) = V_2(y_2) \to 0 \quad \text{as} \quad y_2 \to \infty \tag{8}$$

For similarity transformation, we take:

$$u_1 = a\,x\,f'(\eta), \quad v_1 = -\sqrt{a\,\nu_1}\,f(\eta), \quad \eta = \frac{y_1}{\sqrt{\nu_1 / a}} \tag{9}$$

for the upper fluid, where a prime denotes differentiation with respect to η. Clearly with u_1 and v_1 given above, the Eq. (1) is satisfied. Similarly for the lower fluid, we take:

$$u_2 = a\;x\,h'(\xi), \quad v_2 = -\sqrt{\nu_2\,a}\;h(\xi), \quad \xi = \frac{y_2}{\sqrt{\nu_2 / a}} \tag{10}$$

with u_2 and v_2 given above, it is readily seen that Eq. (5) is identically satisfied. Using (9), Eq. (2) gives for the upper fluid flow

$$f''' + ff'' - (f')^2 + 1 = 0 \tag{11}$$

with the boundary conditions

$$f(0) = \alpha, \quad f'(0) = c/a \quad \text{(at interface)} \qquad f'(\eta) \to 1 \quad \text{as} \quad \eta \to \infty \quad \text{(at infinity)} \tag{12}$$

Similarly, using (10), Eq. (6) for the lower fluid, we obtain

$$h''' + hh'' - (h')^2 = 0 \tag{13}$$

with the boundary conditions

$$h(0) = \beta, \quad h'(0) = c/a \quad \text{(at interface)} \qquad h'(\xi) \to 0 \quad \text{as} \quad \xi \to \infty \quad \text{(at infinity)} \tag{14}$$

where primes denote the differentiation with ξ now. c/a is the shrinking velocity, and $\alpha = -v_0 / \sqrt{\nu_1\,a}$ and $\beta = -v_0 / \sqrt{\nu_2\,a\,\lambda}$ are the constant suction ($\alpha, \beta > 0$) or injection ($\alpha, \beta < 0$) parameters.

III. Results and Discussion

To check the accuracy of the present results, we compared qualitatively with Liu [6] in Fig. 1 and the agreement is very good. Therefore we confident that our results are accurate and this is a reinforcement to further study this problem.

Hence, we plot the skin friction coefficient for the upper fluid and the lower fluid in Fig. 2 and Fig. 3 respectively. As we can see from Fig. 2 and Fig. 3, the pattern of skin friction for the lower fluid resembles the pattern of skin friction of the upper fluid. However, the region of solution for lower fluid is smaller compared to the upper fluid. Further, when suction is absence in the lower fluid, it seems that the solutions exist for stretching case only and we found no solution when the interface is shrinked. This is different from the upper fluid where the solution is still exist for shrinking case even though without the presence of suction.

In Fig. 4 and Fig. 5, we plot the velocity profiles for the upper and lower fluid respectively. As we can see in Fig. 4, the velocity increases as more suction is imposed. Suction reduces the skin friction and hence helps the flows move faster. This can be seen in both first and second solutions. Based on Fig. 4, the boundary layer thickness for the second solutions are greater than compared to the first solutions. At the beginning of the second solutions, it seems that the velocity decreases as suction increases and it can be seen that the reverse flows occurred. This means that the suction retard the flow at the beginning but it changes after a certain point. The patterns of first and second solutions are different as can be seen from the figures.

The same behaviour of the first solution can be seen in the lower fluid in Fig. 5. The velocity increases as the suction increases. However, the opposite trend can be observed in the second solutions, where the suction decreases the flow velocity. The patterns of first and second solutions are almost similar with different boundary layer thickness. There occurs reverse flow in the upper fluid while there is no reverse flow in the lower fluid.

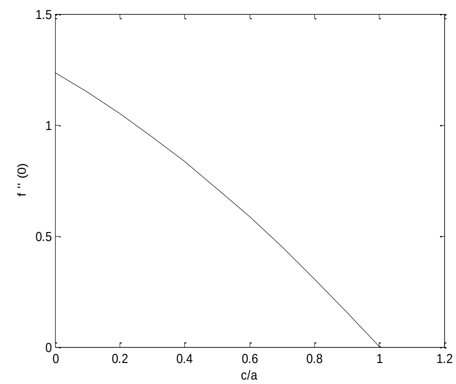

Fig. 1 Variation of skin friction $f''(0)$ for the upper fluid with shrinking variable c/a when suction is absence.

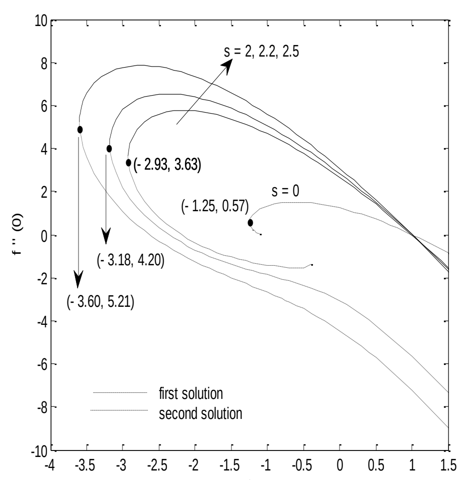

Fig. 2 Variation of skin friction coefficient $f''(0)$ with shrinking variable c/a for the upper fluid when different suction is imposed.

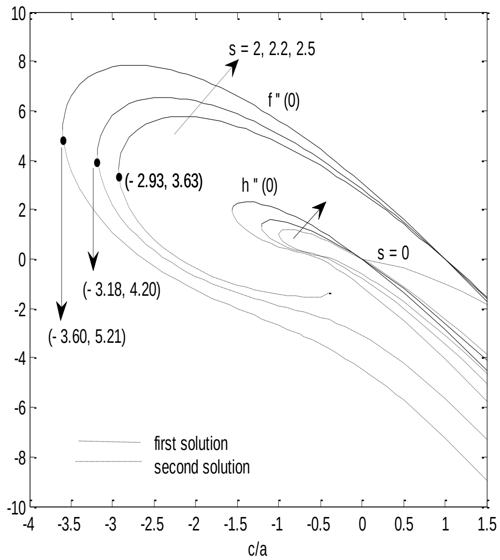

Fig. 3 Variation of skin friction $f''(0)$ for the upper fluid and for the lower fluid with shrinking variable c/a when different suction is imposed.

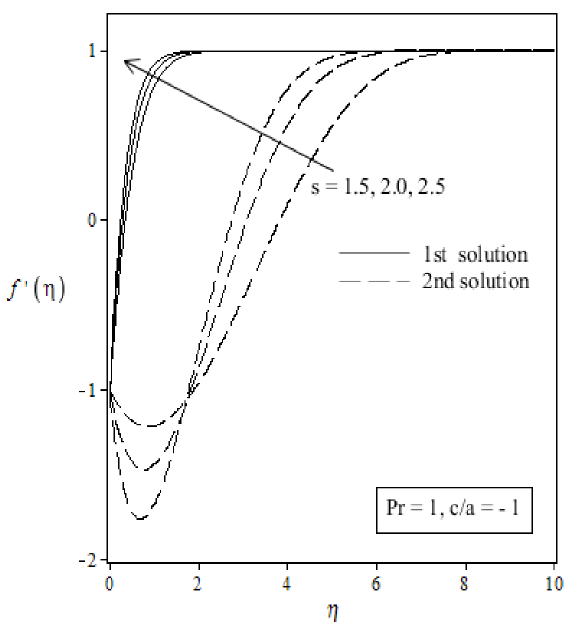

Fig. 4 Velocity profiles for the upper fluid when shrinking parameter $c/a = -1$ with different values of suction parameter

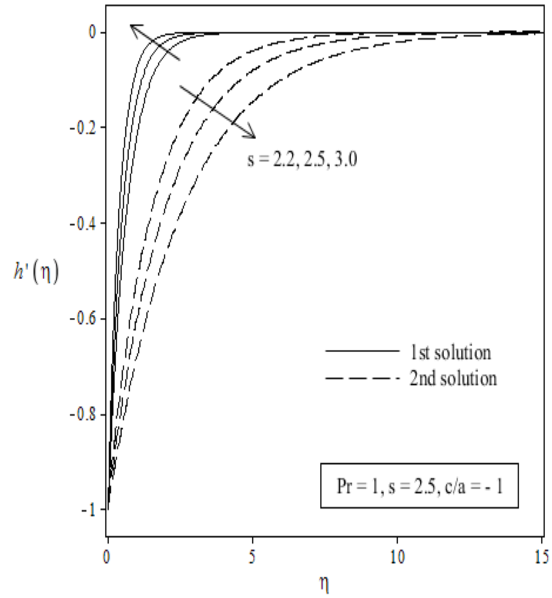

Fig. 5 Velocity profiles for the lower fluid when shrinking parameter $c/a = -1$ with different values of suction parameter.

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REFERENCES

[1] Lok Y. Y., Ishak, A. Pop, I. (2011). MHD Stagnation point flow with suction towards a shrinking sheet, Sains Malaysiana 40 (10): 1179 – 1186.

[2] Mahapatra, T.R., Nandy, S.K., (2012) Stability of dual solutions in stagnation-point flow and heat transfer over a porous shrinking sheet with thermal radiation, Meccanica, DOI 10.1007/s11012-012-9579-5 http://dx.doi.org/10.1007/s11012-012-9579-5

[3] Mehmood, A, Ali A. (2008), An explicit analytical solution of steady three dimensional stagnation point flow of a second grade fluid toward a heated plate, Journal of Applied Mechanics, 75: 061003-1 - 061003-7

[4] Rohni, A.M, Ahmad, S., Pop, I., (2014). Flow and heat transfer at a stagnation-point over an exponentially shrinking vertical sheet with suction, International Journal of Thermal Sciences, 75: 164 – 170. http://dx.doi.org/10.1016/j.ijthermalsci.2013.08.005

[5] Reza, M and Gupta, A.S (2012) MHD stagnation-point flow of an electrically conducting fluid on the surface of another quiescent fluid. Acta Mech. 223: 2303–2310. http://dx.doi.org/10.1007/s00707-012-0708-y

[6] Liu, T. S (1992), Nonorthogonal stagnation flow on the surface of a quiescent fluid – An exact solution of the Navier – Stokes equations, Quart. Appl. Math., 1 (1): 39 – 47.