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First-order trigonometric network in the former Yugoslavia

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Abstract State surveys in a number of countries still use networks developed according to the principles that were applicable over 100 years ago or more. During that period, the rule was that the networks were developed in the classical geodetic datum, using the principle of relative positioning. Basic measured data were the angles and basis networks for scale definition, and measured data was not rectified to reflect the influence of physical parameters of the Earth (deflection of vertical). Given the types of measured values, reduction and positioning, these networks are called trigonometric and are divided into orders. The main network is 1st order and was used to develop networks of lower ranks—up to the point for details surveying. In the former Socialist Federal Republic of Yugoslavia trigonometric networks of the 1st order were developed in accordance with the recommendations of international organizations. We deliberately say "networks" because the 1st order trigonometric network of the former Yugoslavia was not developed as a single entity, but consisted of several parts. It is useful to know the history of the development and the accuracy of 1st order trigonometric networks because all surveys rely on this network, as the data obtained with new technology (GPS) is expected to fit into the existing system (Völgyesi in Transformation of Hungarian Unified National and Gauss-Krüger Projection System into WGS-84, Reports on Geodesy, 1997). The paper will present the historical facts relating to the development of 1st order trigonometric networks in the former Yugoslavia, without any analysis of the results of measurements and adjustment of individual parts of the network.

Keywords History of trigonometric network · Survey · Base networks · Accuracy

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1 Background

The 1st order trigonometric network (TN1), as the base for the state survey (data of which is still in use) in the territory of former Yugoslavia, fully in accordance with the International Association of Geodesy (IAG) requirements of the time, was first established in 1948. The initial works on creating such a network had started in 1872 with the relevant works of Militär-Geographisches Institut (MGI) in Vienna on creating the 1st order triangulation of the Österreich-Ungarischen Monarchie. For that purpose in the territory of former Yugoslavia two chains of triangles were firstly developed—from Slovenia to Vojvodina and from Slovenia through Dalmatia to Montenegro, which were later linked by the third covering the territory of Bosnia and Herzegovina (Fig. 1). This triangulation was defined in the classical geodetic Datum: latitude and longitude the initial point of the 1st order Hermannskogel near Vienna, azimuth of the 1st order trigonometric side Hermannskogel–Hundsheimberg and parameters of the Bessel ellipsoid (RGA 1976; FGA 1953a, b). The results of this works were published in 1902 in the "*Ergebnisse der Triangulierung…*", and served as the base for all subsequent works on the creation of triangulation for former Yugoslavia.

However, measurements for determination of trigonometric points' coordinates were done in this territory earlier, from 1855–1857 to 1871–1875 by the surveyors of the Habsburg Monarchy (Kovács and Timár 2009). The measurement data produced then were lost in the meantime as well as the points of measurement, resulting in giving up on their use in subsequent works.

In the territory of former Serbian Kingdom the works on establishment of TN1 started under the supervision of professor Milan J. Andonović, engineer, in 1887 with the establishment of the Geodetic Institute at the Royal College at the time. Preliminary measuring tests were done on the selected and erected base near Paraćin using the *Maeyer-Bessen* base device (Bošković 1939). The works were interrupted in 1894 and continued in 1899 when the Geo-

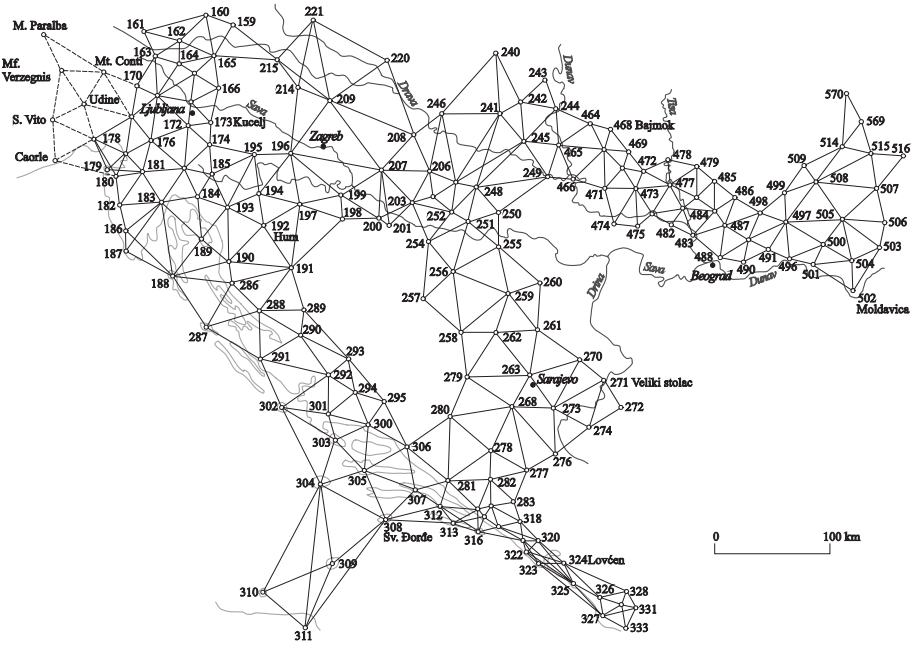

Fig. 1 MGI 1st order network

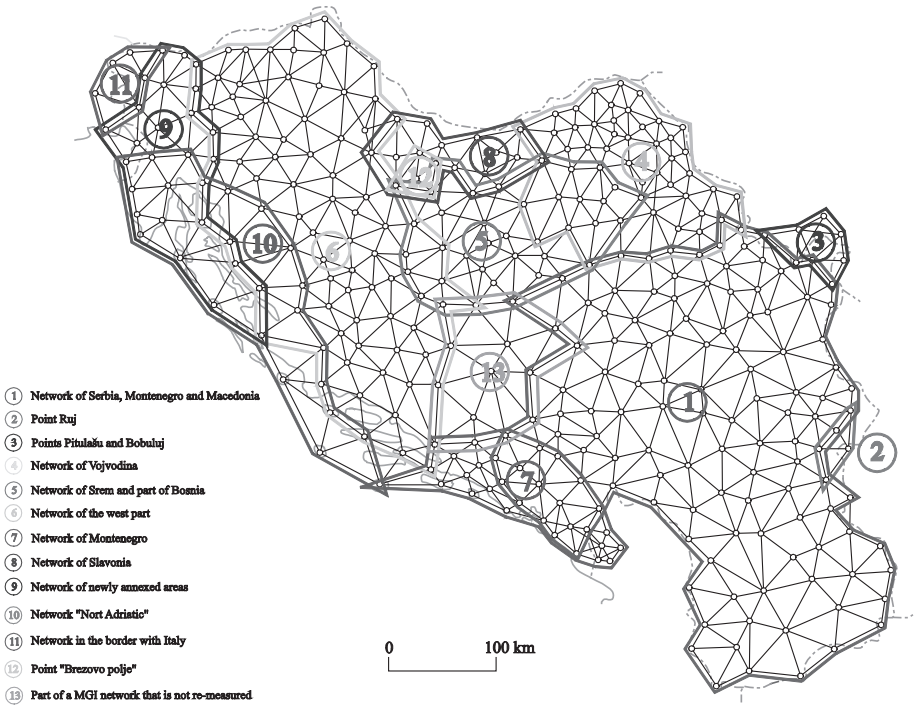

Fig. 2 TN1—epoch 1900–1948 (Bratuljević et al. 1995)

graphic Department of the Military General Staff (later Military Geographic Institute—VGI) became in charge of the 1st order triangulation in Serbia. Apart from measuring the horizontal angles and baselines, the astronomical latitude and azimuth definitions were also done in approximately 30 triangulation points under the supervision of General Stevan Bošković.

In the period between the two world wars, the VGI worked on improvement and completion of 1st order triangulation in Vojvodina, parts of Bosnia and Herzegovina and Croatia, as well as on the astronomical and gravimetric works needed for converting the 1st order triangulation into astro-geodetic network (Adamik 1949). However, astronomic and gravimetric works were not used for establishing the 1st order triangulation. From 1919, the newly founded General Cadastre Directorate (subsequently Department of Cadastre and National Goods) started participating both in the field and computing works on the establishment of the 1st order network.

After WWII up to 1948 the VGI and the Federal Geodetic Directorate (SGU) worked jointly on 1st order triangulation on Slovenia, Istria, coastal areas and Slavonia. That way the entire territory of the former Yugoslavia was covered by 1st order triangulation comprised of 595 triangles. However, this network was never adjusted as a whole and points coordinates were obtained by adjustment of individual parts of the network, linking to MGI points without Laplace azimuth, vertical deflection and ellipsoid heights. In its base, the 1st order trigonometric network of the former Yugoslavia is a collection of 13 individually developed networks merged into one unit (Fig. 2; Table 1). This 1st order triangulation was linked to 1st order triangulations of the neighbouring countries—Albania, Bulgaria, Greece, Italy and Romania. At this point, with all of its shortcomings, this 1st order triangulation represents the base for the state survey, which since 1924 is done in Gauss-Krüger projection.

Table 1 Overview of developing a 1st order trigonometric network (TN1)

Nr.	Network name	Year of realization
1	Network of Serbia Montenegro and Macedonia	1900–1928
2	Individual determined point Ruj	1921
3	In particular determined point Pitulašu and Bobuluj	1934
4	Network of Vojvodina	1928–1938
5	Expansion of the network 4 in Srem and part of Bosnia	1935–1946
6	Network of Western countries	1937–1940
7	Network of Montenegro	1946
8	Network of Slavonia	1946
9	Network of newly annexed areas	1947
10	Network "North Adriatic"	1948
11	Network in the border with Italy	1948
12	Individual determined point Brezovo polje	1951
13	MGI part of a network that is not remeasured	1872–1902

2 Trigonometric network in Serbia, Montenegro and Macedonia (Network 1)

2.1 Network datum

Network datum for Serbia, Montenegro and Macedonia is defined by a group of selected points from MGI network, total of 7 points (although the original planned number was 11), marked in Fig. 3, as follows:

- From the west, to points 1/261 Konjuh, 2/270 Džep brdo, 3/271 Veliki Stolac, 47/273 Borovac, 72/276 Maglić, 73/277 Bjelasica and 74/283 Tisac—points of definition of the network datum,
- From north-east, over Danube, to points 19/491 Dumača, 20/496 Antina livada, 26/501 Moldavica and 27/502 Kukujova—points that were supposed to be used for defining the network datum that were disregarded in the calculation process.

2.2 Network characteristics

The network has 130 points that form 209 triangles. The lengths in the network range from 8.9 to 60.6 km, with average value of 34.7 km. The network has three angles less than 25°, with the smallest angle being 19°42′. Network lengths, for defining the scale, (total of 8) were calculated from basis network, as indicated in Fig. 3.

Majority of points are marked by an underground mark digged in at the depth 1.2–1.3 m and a massive concrete pillar with dimensions 0.45 × 0.45 × 1.40 m, which is placed onto a concrete plate, with height of 1.10 m.

Erection of points was done using four-side pyramids covered by multiple layers of wood planks. Sighting was done on the pyramids, and with certain angles with greater distances between the points the sighting was done in unfavourable conditions using heliotropes.

Angle measurements were done exclusively according to repetition method (in 12 sets) in the period 1903–1914 (Northern part) and 1920–1923 (Southern part) using the theodolites with a microscope-micrometers (data $1'' - 2''$) type Starcke-Kammerer (Fig. 4), Bamberg

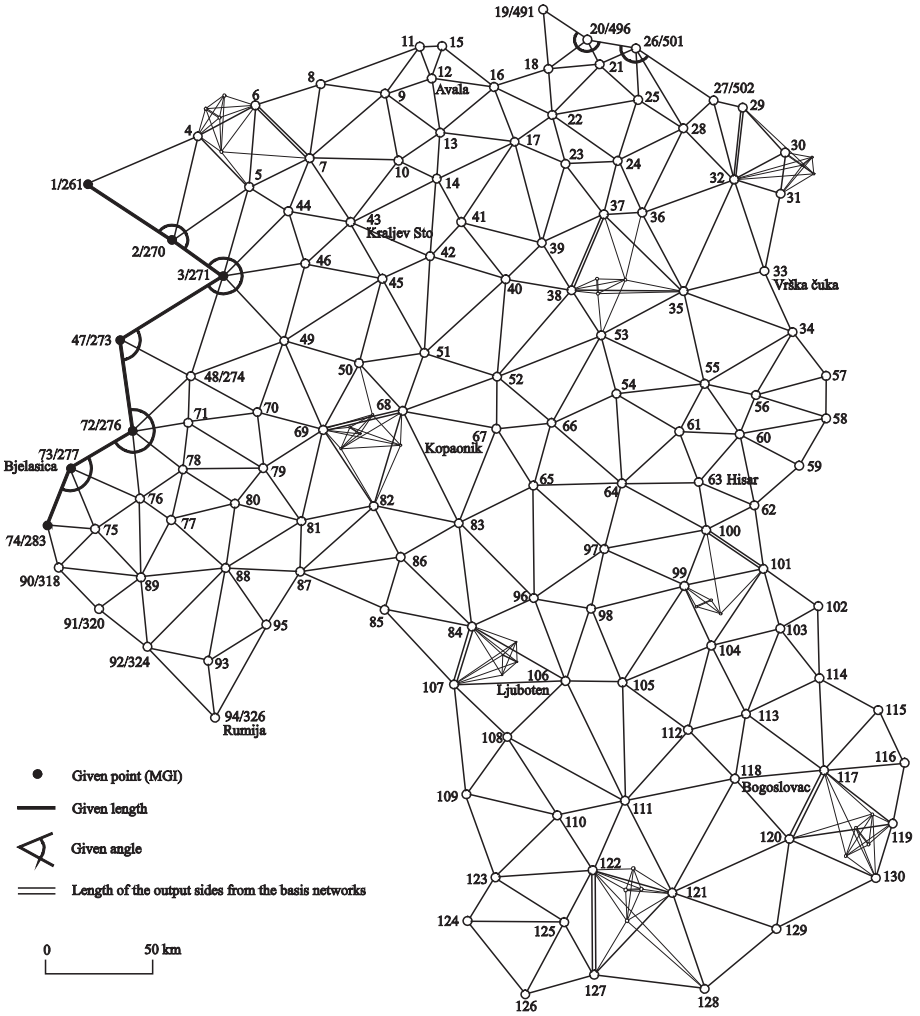

Fig. 3 Trigonometric network in Serbia, Montenegro and Macedonia (1900–1928) (Delčev 2001)

and Hildebrand. With measurements in 1904 the network was linked to MGI triangulation through given points (FGA 1953a).

Using the existing data, the angle average error was calculated from non-closed triangles using the Ferrero formula—$m_a = \pm\sqrt{\frac{[f^2]}{3n}} = \pm 0''.90$, where f is the misclosure error of a triangle and n is the number of the triangles within the network. The network has ten triangles with misclosure errors above $3''$ (ranging from $3''.08$ to $4''.98$) (Jovanović 1971).

2.2.1 Base networks

Lengths in the network were determined from base networks. The bases were stabilized using natural stone pillars situated on the concrete plate. The stone pillar has a bolt with a fine cross installed.

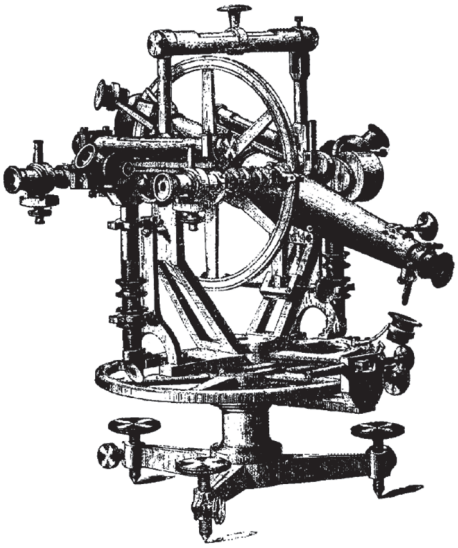

Fig. 4 Theodolite Starcke-Kammerer

During 1904 four bases were measured—Paraćin, Negotin, Loznica and Vranje, followed by Prilep, Strumica and Prizren in 1922 and Sjenica in 1924. All these bases were measured using Jäderin apparatus (invar wires). Measurements of the bases were done using one wire which was called operational, while others were used merely for control of the length of the measuring wire. One of the characteristics was that the length of bases, as well as the lengths of individual sections, were not dividable by 24 m, and therefore the remainder was measured using the invar tape of 12 m length. The lengths of the bases were corrected for the influence of temperature variations and non-symmetry of a catenary and then reduced to ellipsoid.

The bases were linked to 1st order triangulation through base networks in which directions were measured according to repetition method using theodolites of Fennel type, while they were sighted using wooden pyramids. The measurement records for some of these networks were missing during WWII. For the measurement accuracy is known the angle average error calculated from non-closed triangles using the Ferrero-formula: base networks from 1904 $m_a = \pm 0''.65$, and from 1922 to 1924 $m_a = \pm 0''.83$. All the base networks were adjusted according to the conditional measurements. The accuracy data for base networks from measurements and adjustment, as well as some characteristics, are given in Table 2.

2.3 Network adjustment

The network was adjusted according to the conditional measurements method, using approximation method. The measurement results were the directions and lengths of known sides of base networks. The directions were reduced from physical areas of the Earth onto an ellipsoid by adding corrections for:

- influence of the absolute height of the sighting points:

$$v_1'' = k \cdot H,$$

  where the value $k = \rho'' \cdot \frac{e^2}{2a} \cdot \cos^2 \phi_m \cdot \sin 2\alpha$ was taken from the table,
- replacement of the normal intersection with geodetic line:

Table 2 Base networks list

Title	D base km	D outp. km	Year of meas.	Nr. pts.	Nr. dir.	Nr. trian.	Nr. cond. equat.	$f''_{\Delta\,\max}$	$(m_a)''_{\Delta}$	$(m_a)''_{\text{adj}}$
Paraćin	5.60	36.61	1904	8	40	22	20	1.84	0.60	0.98
Negotin	4.66	33.99	1904	7	38	23	19	2.87	0.78	1.46
Vranje	4.97	32.03	1904	6	22	8	8	0.85	0.22	0.21
Loznica	5.03	35.21	1904	8	42	26	22	2.81	0.65	0.85
Prizren	5.38	27.67	1922	6	30	16	20	2.99	0.95	0.94
Strumica	6.62	34.27	1922	8	42	24	22	2.40	0.70	0.76
Prilep	5.98	48.14	1922	8	44	28	24	2.32	0.62	1.04
Sjenica	5.57	36.62	1924	9	48	27	25	4.14	0.98	1.20

$$v''_2 = l \cdot D^2,$$

where $l = \frac{\rho''}{12} \cdot \left(\frac{e}{a}\right)^2 \cdot \cos^2 \varphi_m \cdot \sin 2\alpha$ is a table look-up value (e and a are the ellipsoid parameters).

From the angle formed by the directions reduced to the ellipsoid, the triangles are closed and corrected for the spherical excess:

$$\varepsilon'' = [4] \cdot a \cdot b \cdot \sin \gamma,$$

where $[4] = \frac{\rho''}{2r^2}$ are a table look-up value, and a, b and γ are the triangular elements.

Lengths of output sides were obtained by adjustments of base networks using the method of conditional adjustment. The measured directions were processed the same way as in the 1st order network. The lengths of baselines, reduced to ellipsoid, were considered absolutely accurate. Those way output sides of the ellipsoid were obtained and were given for the adjustments of the 1st order network. Astronomical data, primarily azimuth, are not entered in the adjustment.

Northern part of the network started adjusting the VGI according to conditional measurements in Clark ellipsoid. By 1924 this adjustment was not completed. In the meantime, all measurements in the southern part were completed and works on lower order networks initiated, and the Supervisory Board for State Survey had already adopted the suggestion by the former Cadastre General Directorate on the introduction of Gauss-Krüger projection and Bessel ellipsoid. The Special Board for studying the 1st order network adjustment was established, and in March 1925 he suggested that the entire network be adjusted at simultaneously, according to the conditional measurements method, the approximation method that was suggested by prof. Ivan Sviscev and to be linked to MGI triangulation. Network adjustment started in the autumn of 1925 and was completed in March 1927, which was extremely fast for that time.

After solving the conditional equations and obtaining definitive, adjusted direction values, the coordinates of all points in the network were calculated, on Bessel ellipsoid and in Gauss-Krüger projection. Seven MGI points were adopted as given in the west, across Drina River, marked on Fig. 3. The points across Danube and Sava were not used because discrepancies between coordinates appeared which were published by MGI and calculated from this network (by 2–3 meter) (Svečnikov 1962).

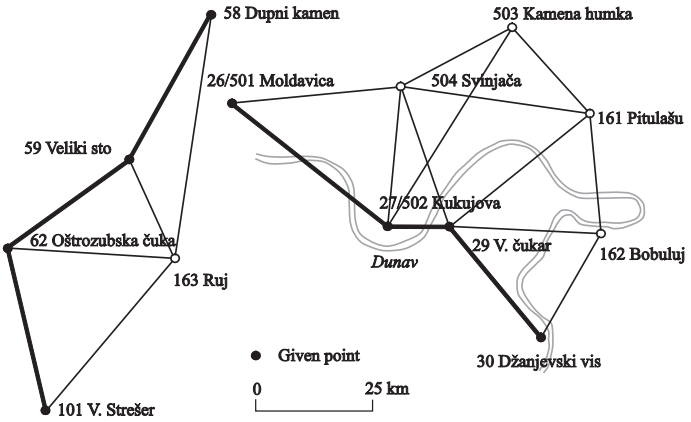

Fig. 5 Individual determined points (1921 and 1934)

3 Individually determined point Ruj (Network 2)

In the border with Bulgaria there is a trigonometric point of 1st order 163 Ruj (Fig. 5) from which observations were done in 1921, which however was not included in the network adjustment due to observations that did not meet the required accuracy. Two out of three triangles (linked to point 62) do not close by $+6''.38$ and $-4''.02$.

4 Specially determined points Pitulašu and Bobuluj (Network 3)

In 1934, the Cadastral Department, for the purpose of more accurate determination of lower order network points, and with the permission of the Romanian government, determined two points of 1st order in the Romanian territory. These points are called 161 Pitulašu and 162 Bobuluj (Fig. 5). The network linked to these points is comprised of seven triangles. From lack of closure of these triangles the angle mean error is obtained using Ferrero formula—$m_a = \pm 1''.18$.

The mean direction error was calculated from residuals determined in adjustment and its value is $\pm 1''.83$, while the largest residual of direction is $2''.35$

5 Network of Vojvodina, Srem and part of Bosnia (Networks 4 and 5)

5.1 Network datum

The network is dependant on the following given points (Fig. 6):

- MGI triangulation—to the points Kučerina 203, 247 Papuk, 248 Kapovac, 465 Trojnaš, 466 Čvorksovo brdo, 254 Šabića han, 1/261 Konjuh, 258 Vlašić and 262 Tvrtkovac;
- Triangulation of the western part of the country (Network 6)—to the points Kabajkovac 243, 257 Tisovac stari and 369 Ranča;
- Network of Serbia, Montenegro and Macedonia (Network 1)—to the points 4 Gučevo, 6 Cer, 8 Podgorica, 9 Bukovik, 12 Avala, 16 Udovice, 18 Leštar, 19/491 Dumača and 20/496 Antina livada.

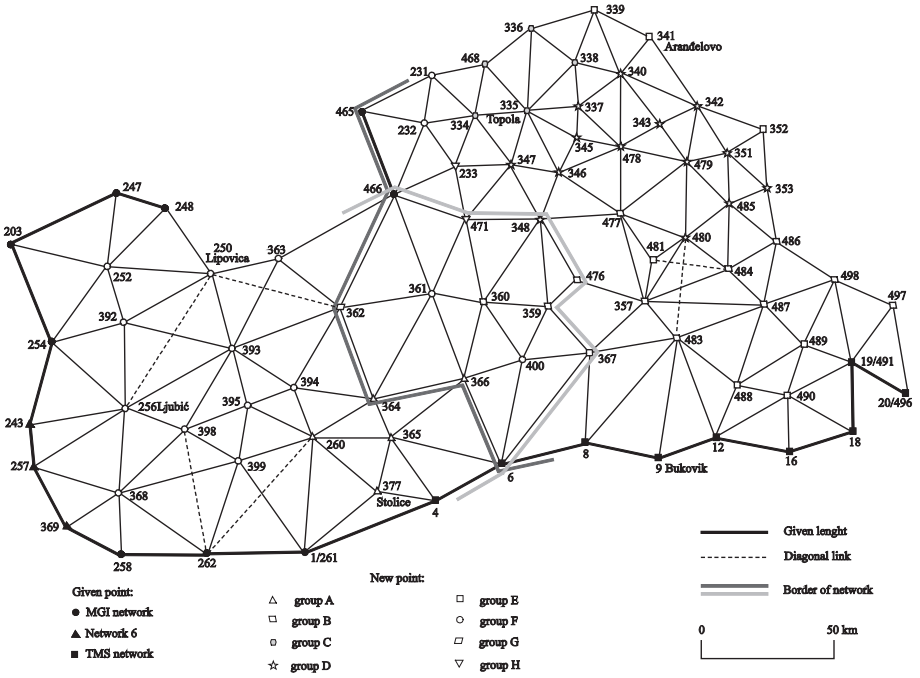

Fig. 6 Trigonometric Network of Vojvodina, Srem and part of Bosnia (1928–1946)

5.2 Network characteristics

The network has 82 points which form 124 triangles with six diagonal links (as shown in Fig. 6) and two tetragons with no diagonal links.

Lengths in network range from 11.9 to 61.7 km, with average value of 28.8 km. There are total of 32 sides in the network the length of which is shorter than 20 km. The smallest angle is 16°03′, at point 357 Koševac.

The points are stabilized by concrete pillars of the same dimensions as in Network 1, with points in the eastern, particularly north-eastern, part of the network mostly being church towers. In the western part of the network high pyramids were erected in multiple points, e.g. point 366 Sremska Raca the height of which was 46 m.

Directions were measured in the network according the repetition method in 12 sets. Considering that the network has a large number of church towers from which eccentric observations were made, as well as a large number of wooden pyramids the pillars of which are susceptive to folding, the obtained accuracy of the measurement is reduced. In comparison to other networks, this one has the lowest accuracy, and the mean angle error was calculated according to Ferrero-formula from non-closing triangles: $m_a = \pm 1''.06$. The network has ten triangles with non-closures higher than 3″, ranging from 3″.19 to 4″.92.

In the network territory the base network Vrsac is situated, the base of which, length 4,022.655 m, was measured in 1895 by MGI using the measuring equipment with bars. The base network has 7 points with 32 directions. The accuracy of the network is characterized by angle mean error calculated from non-closures of triangles according to Ferrero-formula: $m_a = \pm 0''.76$. Three known sides belong to the base network, but during the adjustment none of them were taken as given.

5.3 Network adjustment

The network was adjusted using the conditional method. The results of the measurement were the directions processed in the same way as in the previous networks. Coordinates of given points from three different epochs were used (Fig. 6). Due to different epochs of determination, the analysis of coordinates of given points was conducted, and depending on the results, only those coordinates of which were coherent were used for adjustment. The network was adjusted in parts, "piece by piece", and each was marked as a group of adjustments.

5.3.1 Group A

In 1935 Petrovaradin Municipality asked for a triangulation and survey of forestry complex surrounding Sremska Raca is conducted. For that purpose, the VGI adjusted the group of 5 points, connecting them to given points, i.e. sides: 1/261 Konjuh, 4 Gučevo and 6 Cer. Lower order points were calculated using the adjusted coordinates. In Fig. 6 points determined from this part of the network are marked by triangles.

5.3.2 Group B

After the completed measurements in Vojvodina, Network 4, calculations and adjustments were taken in 1936, according to conditional method. As given were taken only points from the Network 1 and point 364 Sandić and 366 Sremska Rača from *Group A*, but not the point of the MGI network. The adjustment needed to be repeated and from it were taken only some of points.

5.3.3 Group C

By 1938 the adjustment of *Group B* was not completed and the Cadastral Department decided to conduct the survey of Subotica and the area of Beljske forests. In order to complete these works, it was necessary to obtain the 1st order point in a very short period and it was therefore decided to adjust only one part of the network that covers the subject area. In order to obtain the coordinates of points maximally adjusted to those published by MGI, it was decided that points 465 Trojnas and 466 Cvorkovo brdo be included as given. As points 364 Sandic and 366 Sremska Raca are located relatively close to them, and were already calculated from *Group A*, the adjustment of the triangle chain between these points was done (the chain had the total of 9 points). The results of the adjustment of this chain were unspecific—chain adjustments were big, yet acceptable if absolutely necessary (results of these adjustments were lost during the war). Coordinates of point 362 V. Gradiste were calculated from the adjusted angles and sides (***Group G***) and were taken as given for this part of the network. In order to adjust the coordinates to the eastern part of network, points 359 Crveni Čot, 360 Liske and 477 Čurug were taken as given and they were obtained based on the results of the first adjustment with insufficient number of decimals.

After calculating the coordinates of all given points, group of points was calculated using the method of conditional measurements. Coordinates of all points were calculated from adjusted directions and lengths, but only those used in triangulation is Subotica were kept as definite (total of 5 points). Based on two of these coordinates, 336 Subotica and 338 Komorci, by triangle adjustments, coordinates of point 339 Horgos were calculated, which also was needed for surveying this area.

5.3.4 Group D

In the meantime, the adjustment of the network marked as *Group B* was coming to an end and it was therefore possible to compare the coordinates of points in the northern part of network obtained with connection to MGI network (through points Trojnaš and Čvorkovo brdo) and without that connection. From this comparison (*Group B* and *Group C*) it was concluded that discrepancies by coordinate axes have the values of 2.5–3 m, which is a significant discrepancy. There were two possibilities for solving this problem—to keep the coordinates from the adjustment of *Group B* or keep the coordinates obtained through connection to MGI network and then do the repeated adjustment of the entire network situated between these points and points from Network 1. As during the years 1937–1938 the network developed in the western part of the country relied on MGI triangulation network, and at the same time the survey was done on the part of the network marked as *Group C* that relied on the MGI triangulation points, the change of coordinates would have caused great complications.

It was therefore decided that the part of the network close to Network 1 should maintain the points' coordinates from *Group B* adjustment and that the adjustment should be repeated for part of the network situated between these and points of MGI network. That way the coordinates from the uncompleted adjustments for 10 points were kept marked in Fig 7. Part of the network that was calculated using the method of indirect measurement is comprised of 15 points relying on 12 given points from three different epochs:

1. From *Group C* at points: 334 Telečka, 335 Topola, 338 Komarci and 339 Horgoš,
2. From *Group B*, incomplete adjustment, at points: 357 Koševac, 359 Crveni Čot, 360 Liske, 476 Petrovaradin, 484 Orlovat and 486 Boka,
3. At points 233 Brestovac and 471 Parabuć (***Group H***), obtained by readjustment chain of triangles between MGI network points 465 Trojnaš and 466 Čvorkovo brdo and Network 1 points—4 Gučevo and 6 Cer (this is a chain that is described in *Group C*, but was extended and previous adjustment was discarded).

5.3.5 Group E

This group includes points that are determined as individual, using the direction intersection method. It is comprised of 7 points, which were calculated from three to five points.

5.3.6 Group F

After the attempted adjustment of the western part of the network, territories of Srem and north-eastern part of Bosnia, it turned out that this network, inserted between the MGI network points from the north, west and south and with already defined Vojvodina network points from the east, could not be adjusted. The residuals were much greater than the real accuracy of measurements. It was therefore decided that the network should be expanded east, resulting in the creation of Network 5 (Fig. 6). But regardless of this expansion, the residuals obtained from the adjustment, that was done using the method of indirect measurements, were huge (12 of 161 directions have the residuals above $3''$). The mean error of direction from the adjustment amounts to $2''.05$ while the mean error calculated from non-closure of triangles amounts to $0''.75$. These big improvements happened because a network was inserted between the two chains of MGI triangulation that were shifted by about 3 m.

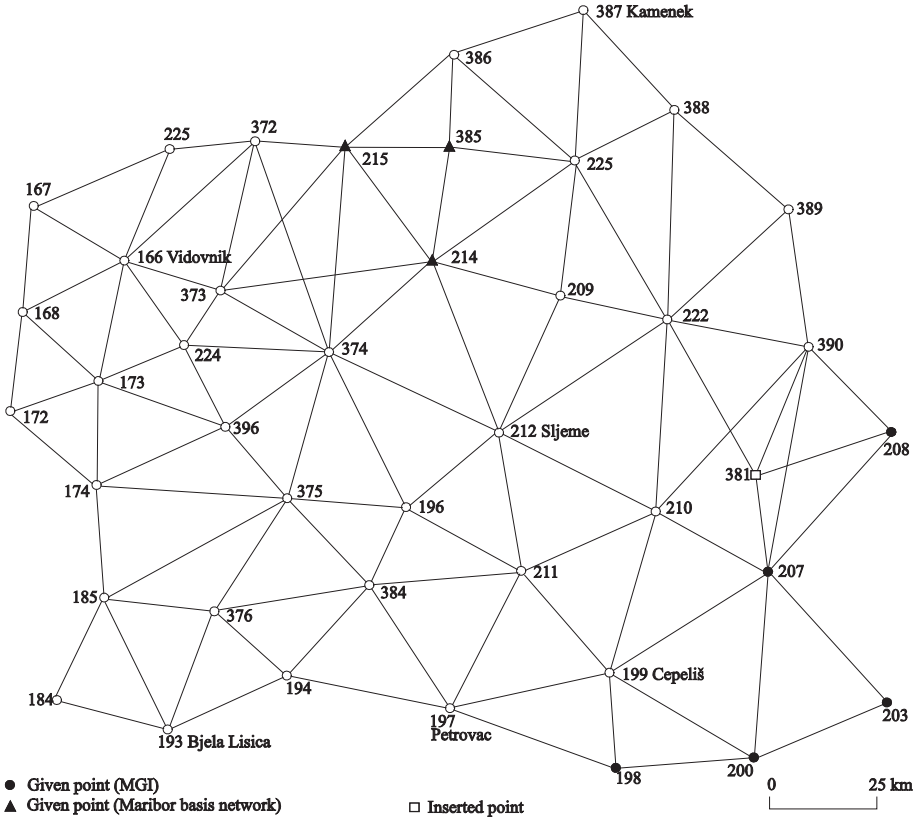

Fig. 7 The northern western part of the network state (1937–1940)

6 Network of the west part (Network 6)

Although developed in two parts—southern part first (Fig. 7) and then expanded by the northern part (Fig. 8)—this network is treated as a whole.

6.1 Network datum

The network has total of 22 given points, 19 from MGI triangulation and 3 from adjustment of Maribor basis network.

6.2 Network characteristics

Network as a whole has 93 points, 82 of which were observed by the Cadastre Department and 11 by VGI. The lengths of sides range from 14.8 to 69.6 km, with average value of 33.2 km. The smallest angle in the network is at point 191 Gola Plješevica—19°55′. The northern part of network has two diagonal connections, same as subsequently inserted point 381 Štefanje (Fig. 7).

The points have been marked according to the procedure described in "The Temporary Instruction for Triangulation Works" which was published in 1937 by the VGI and according

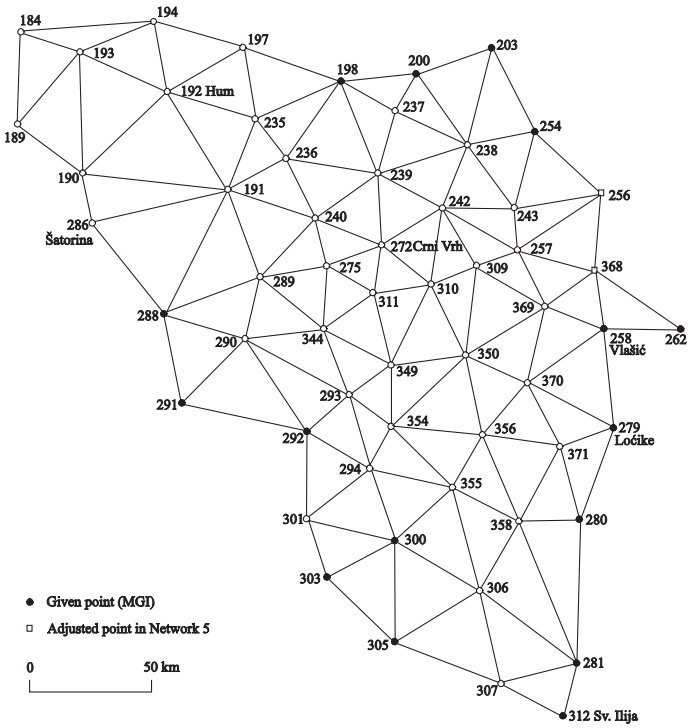

Fig. 8 South part of the western part of the network state (1937–1940)

to which all the works were performed. This instruction defined triple marking of points: underground—stone plate with carved in cross, ground—concrete pillar with dimensions of $0.45 \times 0.45 \times 1.30$ m and lateral—with four stones with carved in crosses dug in to depth of 0.5 m. Concrete pillar from which the surveying was done was placed onto the concrete plate thickness 10–15 cm and height 1.10 m.

The sighting in the network was done only partially using heliotropes, and mostly using pyramids that were covered by multiple layers of plywood, and, depending on the terrain, their heights ranged from 5.5 to 10 m, with 8 m being the average height.

Measurements were considered correct when the difference between the individual sets, when measuring a single angle, did not exceed $4''$. The achieved accuracy of the measurements, expressed through mean angle error calculated according to Ferrero-formula was:

- For southern part (surveyed in 1937 and 1938): $m_a = \pm 0''.75$,
- For northern part (surveyed in 1939 and 1940): $m_a = \pm 0''.85$.

These mean errors show that the northern part of the network has slightly lower measurements accuracy. Also, in the southern part of the network the largest triangle non-closure was $2''.92$, while in the northern part has three triangles with the non-closure higher than $3''$ (highest one being $3''.46$).

For angle measurements five theodolites by Otto-Fennel Company, with data on the microscope-micrometer $2''$, were used. According to the said Instruction, which requires that the "run" of the microscope should be defined at each 1st order point, during the measurements conducted in 1937 at each station the "run" was determined by individual measurements. The angles were measured using Schreiber method, with adjusted angle weight:

$$p = \frac{n \cdot s}{2} = const = 12$$

with:

n-number of sets and
s-number of directions.

The territory covered by the network includes three baselines measured by MGI—they are: Maribor (surveyed 1860), Sinj (surveyed 1870) and Dubica (surveyed in 1878–1879). These baselines were surveyed using the equipment with bars and their real measurement accuracy is not known. The internal accuracy, from the measuring difference front–back, ranges from 1:1 600 000 to 1:4 200 000.

Directions were measured in base networks. The accuracy of measurements is characterized by mean angle error calculated according to Ferrero-formula: $m_a = \pm 0''.76$.

6.3 Network adjustment

The network was adjusted using the indirect method. The results of the measurement were the directions obtained from adjustment of angles in stations, processed as in the previous networks. Lengths of known sides of base networks were not entered directly, but indirectly—as given values (through given coordinates). Coordinates of points from MGI triangulation were used for Datum definition (or coordinates emanating from them).

Prior to the adjustment of the network in 1939 the Maribor Base Network was again surveyed and adjusted. This was done for the purpose of obtaining definite coordinates of given points in that part of the network. The base network was adjusted using the method of conditional measurements and the length of base was kept from MGI measurements. In this way coordinates of points 214 Donačka gora, 215 Žigertov vrh and 385 Grmada were obtained and kept as given.

Due to urgency of obtaining the coordinates for individual areas the network was adjusted using the indirect method in five groups. The adjustment of the network did not include points 256 Ljubić and 368 Očauš; instead their coordinates were calculated while adjusting Network 5 (expansion of network in Vojvodina). Point 381 Štefanje was determined as an individual point by intersection of directions. The adjustment resulted in the mean direction error: $m_d = \pm 1''.07$.

The network had only one direction the residual of which exceeded $2''$ (this residual was $2''.33$). In comparison to the previous networks these indicators show that the highest accuracy of measurements was achieved here.

7 Montenegro network (Network 7)

During 1940 the works on the 1st order network were not definitely completed. The network of Slavonija was not completed either, and the network in the coastal area needed to be redone because of the poor shape of triangles. Prior to the war, only the design of the network in

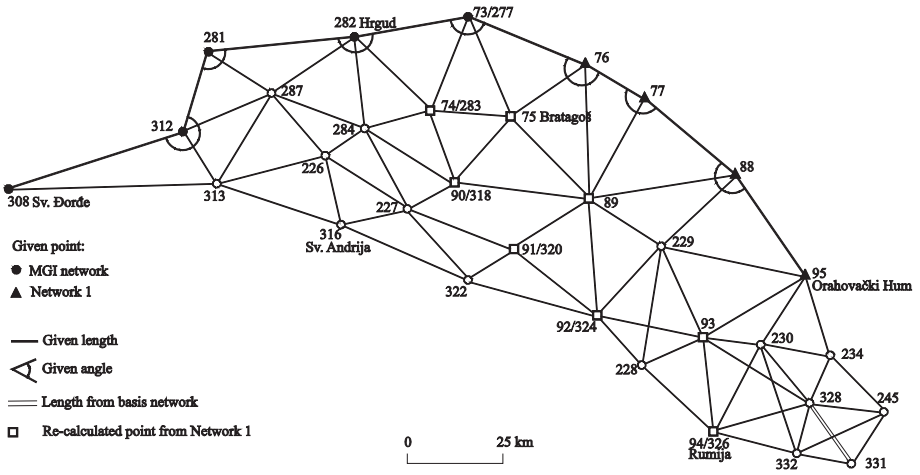

Fig. 9 Network of Montenegro (1946)

the coastal area of Montenegro was done and was supposed to be completed during 1941. However, due to the war, the works were stopped and continued in 1946.

7.1 Network datum

Network (Fig. 9) had a 9 given points 3 of which were from MGI network (73/277 Bjelasica, 281 Babina gomila and 282 Hrgud), while the other six were from Network 1 (triangulation of Serbia, Montenegro and Macedonia).

7.2 Network characteristics

The network has 32 points that form 51 triangles with four diagonal connections. Majority of points belonged to MGI and Network 1, and only five were newly established. The lengths ranged from 11.9 to 56.8 km with average value of 26.5 km. The smallest angle in the network is 16°53′.

Concrete surveying pillars were either newly erected or fixed at all points. The shape and dimensions of concrete pillars were the same as in the previously described networks.

Erection of points, except 7 far eastern ones, was done using pyramids, height 5–8 m, coated by plywood. The sighting was done partly by heliotropes, partly by pyramids.

The angles in the network were measured according to Schreiber method, with the adjusted angle weight of 12. Four theodolites of Otto-Fennel were used for measurements together with two theodolites of Starcke-Kammerrer with microscope-micrometre (data 2″). Points 75 Bratagoš, 76 Golija and 77 Vojnik were not used for new measurements, but rather observations of directions from Network 1 were used for adjustments (dating from 1928). Accuracy of measurements is characterized by the mean angle error calculated from non-closure of triangles: $m_a = \pm 0''.83$. The network has only one triangle with non-closure higher than 3″.

Lengths in the network were determined from Skadar (Albania) base network. During the occupation, the Italians re-surveyed the base and the base network, but their points were not identical with MGI base network. However, the length of the exit side 328 Gruda—331 Jubani, calculated by the Italians, well coincided with the length 88 Maganik—95 Orahovački

hum, determined in Network 1, and was therefore included in the adjustment as a given. Other indicators of the accuracy of the base and base networks are not known.

7.3 Network adjustment

The network was adjusted according to the conditional method. The results were directions obtained from angles adjustment at the station, processed the same way as in previously networks, with the scale defined trough given lengths. Points from MGI network and Network 1 were taken as given (Fig. 9).

79 conditional equations were set, as follows:

- 47 equations for triangle condition,
- 7 equations for given angle condition,
- 8 equations for given side condition,
- 17 sinus conditional equations.

In the adjustment process it turned out that three directions had residuals higher than $2''$, two of which were old (taken over from network 1). The mean direction error was calculated based on the residuals and was: $m_d = \pm 1''.15$.

During this adjustment 8 points from Network 1 were given new coordinates (Fig. 9). The deviations of these points, newly adjusted, in comparison to the old coordinates from Network 1 were from +17 to −83 cm along the x-axis and from +78 to −62 cm along the y-axis, and in relation to MGI network from −28 to −96 cm along the x-axis and from +187 to −528 cm along the y-axis.

8 Slavonija network (Network 8)

8.1 Network datum

The network has 10 given points 6 of which are from MGI network and 4 from Network 5 (Fig. 10).

8.2 Network characteristics

Network has 19 points that form 21 triangles. The lengths in the network range from 18.4 to 50.2 km, with average value 30.7 km. The smallest angle in the network is $25°04'$.

The points were stabilized by concrete pillars the same way as in the above networks.

Erection of points, except the church towers, was done by pyramids coated with wood boards. At points 250 Lipovica and 397 Podgorje, the pyramids were about 20 m high. The sighting was done partly on heliotropes, partly by pyramids.

The angle measurements were done using theodolites of Otto-Fennel and Starcke-Kammerrer with microscope-micrometre (data $1''$). The angles were measured according to Schreiber method with the weight of the adjusted angle of 12, while the mean error from non-closure of triangles was: $m_a = \pm 0''.76$.

The network has only one angle with non-closure exceeding $3''$.

8.3 Network adjustment

Network adjustment was done according to mean measurements method. The measuring results were directions extracted from the angle adjustments on the sides and processed

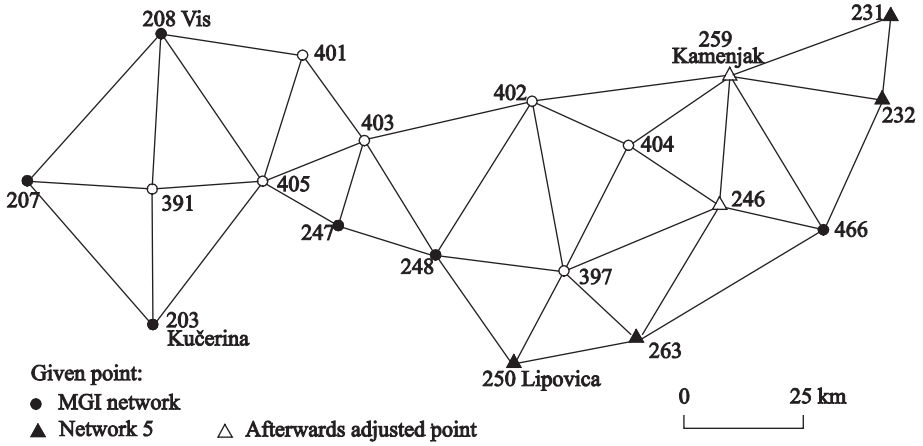

Fig. 10 Slavonija network (1946)

the same way as in the above networks. Points from MGI and Network 5 were used as given.

The mean direction error calculated based on residuals was: $m_d = \pm 1''.41$.

Coordinates of points 246 Osijek and 259 Kamenjak (Fig. 10) were adjusted separately, while the remaining 7 points were adjusted at once.

9 Network of the newly annexed area (Network 9)

9.1 Network datum

The network has 11 given points out of which two come from MGI network, while nine come from Network 6 (Fig. 11).

9.2 Network characteristics

Network has 28 points that form 33 triangles with one diagonal connection. The lengths range from 17.2 to 56.8 km with average value of 34.3 km. The smallest angle in the network is $14°54'$.

No new measurements were done in this network, but rather the data from MGI surveying was used, so the network was simply readjusted. Prior to deciding not to undertake new measurements, the testing of the network quality was done through mean angle error calculation according to Ferrero-formula: $m_a = \pm 0''.96$.

The network had only two triangles with non-closure that exceeded $3''$.

9.3 Network adjustment

The adjustment of the network was done according to the conditional method. The measuring results were the direction processed the same way as in the above networks. Coordinates of points from MGI network and Network 6 were used as given. Lengths (total of 10) and angles (9) were also considered as given among the given points (Fig. 11). The mean direction error calculated based on residuals from adjustment was: $m_d = \pm 1''.26$.

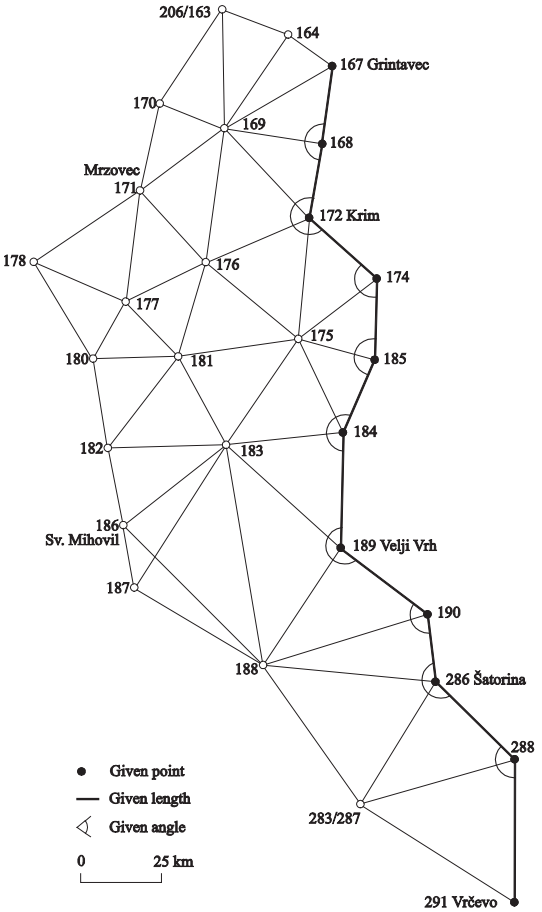

Fig. 11 Network of newly annexed area (1947)

10 "Northern Adriatic" network (Network 10)

10.1 Network datum

The network has 16 given points (Fig. 12) from three different epochs:

- 4 points from MGI network,
- 7 points from Network 6,
- 5 points from Network 9.

10.2 Network characteristics

The network has 27 points that form 31 triangles. The lengths range from 19.6 to 79.2 km, with average value of 42.4 km. The smallest angle in the network is 19°55′.

The points were stabilized using concrete pillars, same as the previously networks. All the points were erected using pyramids coated by wood. The sighting was done partly on heliotropes, partly on pyramids. Angles were measured according to Schreiber method with

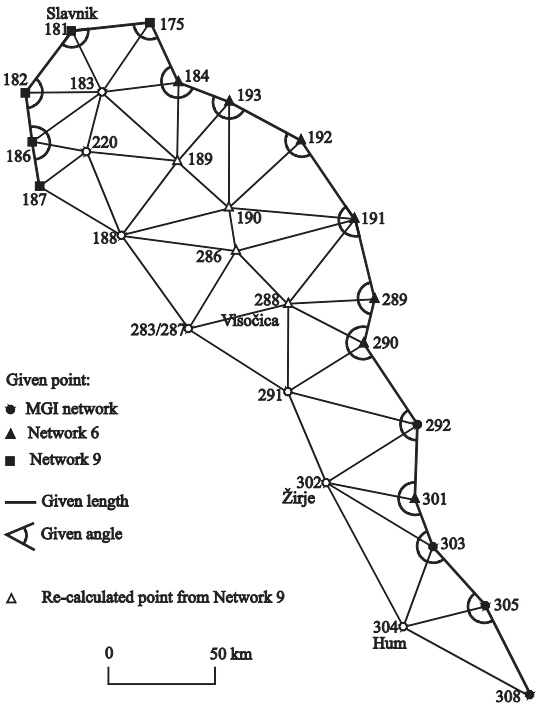

Fig. 12 "Northern Adriatic" Network (1948)

the weight of the adjusted angle 12, using theodolites of Otto-Fennel. The measurement accuracy is characterized by the angle mean error calculated according to Ferrero formula: $m_a = \pm 0''.79$.

The network doesn't have any triangles with non-closure higher than $3''$.

10.3 Network adjustment

The network was adjusted according to conditional method. The results were directions derived from the adjustment of angles on the stations, calculated the same way as in the above listed networks. Coordinates of points from MGI network were used as given as well as from Networks 6 and 9. Lengths (15) between given points were also entered as given together with given angles (14), Fig. 12. The mean direction error calculated based on the adjustment results was: $m_d = \pm 1''.23$, with the biggest residual amounting to $-2''.31$.

During the network adjustment, four points marked in the Figure, which were already calculated in Network 9, were recalculated. It is safe to say that due to these points the new network was developed. More specifically, during the works in the lower order networks it was determined that these points were not identical to old MGI points.

11 Network on the border with Italy (Network 11)

After the demarcation of the border with Italy, 1st order trigonometric network was expanded onto the north-west part of the country by annexing of three points (Fig. 13). As some of these points fall on the Italian territory, Italian triangulators did the measurements from three measuring points.

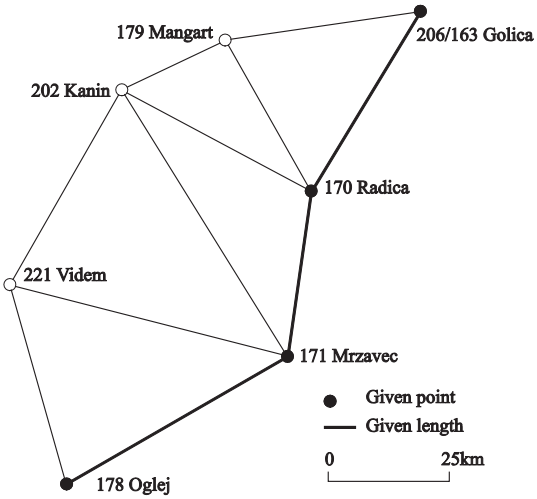

Fig. 13 Network on the border with Italy (1948)

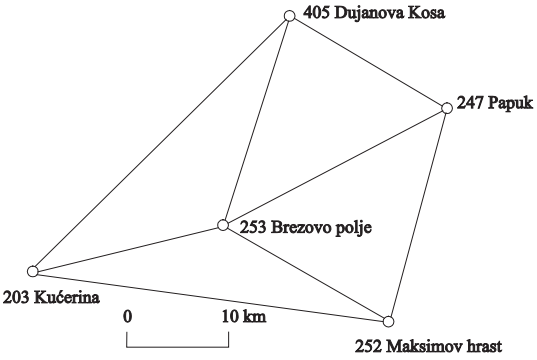

Fig. 14 Point 253 Brezovo polje (1951)

When measuring the angles according to Schreiber method, the mean angle error was calculated according to Ferrero-formula: $m_a = \pm 0''.60$.

The network was adjusted according to conditional measurements method. The results were directions extrapolated from the adjustment of angles on the stations, calculated the same way as in the above listed networks. Four points from Network 9 were taken as given. The mean direction error calculated from the adjustment was $m_d = \pm 1''.11$, while the biggest residual of direction was $+1''.61$.

12 Individually determined point "Brezovo Polje" (Network 12)

During the works conducted on the Slavonia network, it was not possible to include point 253 Brezovo Polje (Fig. 14) because a tall pyramid needed to be erected at that point. New measurements were conducted in 1950–1951 as this was one MGI network point. The biggest non-closure of the triangle was $-2''.15$.

Adjustment of the central system was done according to the conditional measurements method. The mean direction error obtained from the adjustment was $m_d = \pm 2''.69$, while three adjustments were above $3''$. These large adjustments can be explained by the fact that

Table 3 Basic characteristics of certain parts of the network TN1

Index	Year of realiz.	No of points		No	$D_{aver.}$	Meas. results	Sight to	Method of adjust.	Mean ang. errs ” from	
		Total	Given	Δ	km				Δ	Adj.
1	1900–1928	130	7	209	34.7	Directions, 8 basis	pyr. hel.	Condit. approxim.	0.90	2.36
2	1921	5	4	3	/	Directions	/	Condit.	/	/
3	1934	8	4	7	/	Directions	/	Condit.	1.18	2.59
4 and 5	1928–1946	82	21	124	28.8	Directions	pyr.	Condit., partially	1.06	/
6	1937–1940	93	22		33.2	Angles, 3 basis	pyr. hel.	Indirect, 5 groups	0.82	1.51
7	1946	32	9	51	26.5	Angles, 1 basis	pyr. hel.	Condit.	0.83	1.63
8	1946	19	10	21	30.7	Angles	pyr.	Indirect	0.76	2.00
9	1947	28	11	33	34.3	Taken from MGI	/	Condit.	0.96	1.78
10	1948	27	16	31	42.4	Angles	pyr.	Condit.	0.79	1.74
11	1948	7	4	5	/	Angles	/	Condit.	0.60	1.57
12	1951	4	3	3	/	Angles	pyr.	Condit.	/	3.80
13	1902	3	/	/	/	Directions	/	Condit.	/	/

the given points are from two different epochs, from Networks 5 and 8 and are very close to one another.

13 Part of the network that was not re-surveyed (Network 13)

During the process of development of new networks that were supposed to cover the entire territory of the Socialist Federal Republic of Yugoslavia, intentionally or accidentally, a part of the network was left that was not replaced by new surveys. More specifically, three points 263 Bukovik, 268 Bjelašnica and 278 Velež were not re-surveyed and their coordinates were therefore left out of the MGI triangle chain.

At the end of this paper, Table 3 provides and overviews of all parts of the 1st order trigonometric network, showing its basic characteristics.

14 Instead of a conclusion

All of the above said leads to the overall conclusion that there is no unified national 1st order trigonometric network for the territory of the former Yugoslavia. The network as a whole was never adjusted at one time, it is of inhomogeneous accuracy, its reliability is poor and there are measurement errors. The evidence for all these claims came with the conduct of subsequent works (FGA 1953a, b; Svečnikov 1962). Consequently in 1953 the decision was made to implement a new astro-geodetic network in accordance with the standards of the time. In this new network the measurements of lengths and angles were done, while the astronomic measurements were done for the purpose of defining the verticals deflections. However, the network was never put into real use due to the political situation.

Lack of homogeneity of the 1st order network is primarily reflected in the achieved measurement accuracy that is diverse in different segments of the network (Table 3). In some parts of the network through subsequent measurements errors were detected in some of the measured values. For example, in 1981 surveys done using invar wires, it was detected that Paraćin base has an error of 1 dm, which caused the exit side error of 1 m. This error could not be discovered earlier, because due to poor reliability, in the adjustment it was only possible to detect the minimum error of 1.5 m (Delčev 2001). During the adjustment of the part of the network for Serbia, Macedonia and Montenegro using the method of indirect measurements it was determined that the 1st order trigonometric network has an average positioning error of 0.6 m (Delčev 2001). It can be assumed, considering the methods of measuring and processing applied, that the situation is no different in the other parts of the network.

With the application of GPS technologies and measurements at the existing points in the entire network it was reconfirmed that the network is inhomogeneous and has distortions. Therefore, unified parameters of transformation could not be uniformly established for any part of the network and had to be defined partially, in smaller segments (Stopar and Kuhar 2003).

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