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LEARNING BASIC ADDITION FACTS THROUGH SENSE-MAKING AND UNDERSTANDING

Poon Cheng Yong[1]*, Yeo Kee Jiar[2] and Noor Azlan Ahmad Zanzali[3]

[1]Jabatan Ilmu Pendidikan, Institut Pendidikan Guru Kampus Tun Abdul Razak, 94300 Kota Samarahan, Sarawak
[2, 3] Faculty of Education, Universiti Teknologi Malaysia, 81310 Skudai, Johor

*Corresponding author: jasmine_mpsm@yahoo.com

Abstract. The traditional diagnostic and remedial approaches which are based on the behaviourist framework of learning might not facilitate students in acquisition of conceptual understanding and procedural knowledge. This research was intended to help students with mathematics learning difficulties improving their mathematical knowledge of basic addition facts through an instructional model for remedial intervention of mathematics. This model involves application of concrete materials, application of a mixed instructional approach, delivery of conceptual and procedural knowledge, and use of problem-solving activity. Using a case study design, a remediation class teacher and four students at a suburban elementary school were involved. Data was collected using observation, students' work, and interview. Qualitative data was analysed using a qualitative approach. Research findings indicate that concrete materials could be used as a tool for sense-making or counting by students. It depends on the individual differences of students and the instructional approach of teacher. In general, a teacher-directed learning process was carried out but because students were allowed to make their decisions in problem-solving, some students managed to construct their knowledge rather than follow procedures prescribed by their teacher. Learning of both conceptual and procedural knowledge was facilitated through problem-solving activity and incorporation of some constructivist approaches. Mathematical knowledge of students was improved in the remedial intervention which was based on the instructional model. For remedial intervention, teachers should reduce the use of behaviourist approaches gradually, and incorporate more student centred approaches, based on the student individual differences and usual practice.

Keywords: basic addition facts, conceptual understanding, instructional model, procedural knowledge

INTRODUCTION

Students in the elementary schools are expected to master basic skills of reading, writing, arithmetic, and reasoning (Ministry of Education Malaysia, 2010). Those who have not mastered the above skills in Malay Language and Mathematics during the first three years of their schooling would be pulled out from regular classroom and assigned to a teacher who is appointed as the remediation teacher

(Special Education Department, 2003). Although remediation program is officially implemented in primary schools in Malaysia, research finding indicates that it was not given emphasis in implementation by some schools (Rashida, 1996; Mathialagan, 2000). Some regular class teachers also did not offer cooperation and support to this program. Many remediation program teachers needed more knowledge and skills in carrying out mathematics remediation (Mathialagan, 2000). Researches about mathematics instruction for students who are weak in mathematics should be improved (Ketterlin-Geller, Chard, & Fien, 2008). Lack of knowledge in effective instruction becomes a limitation to mathematics learning of slow learners in mathematics.

To help students with learning difficulties acquire the basic skills in number sense, their mathematics learning is commonly supported by an instructional approach, which is based on a behaviourist framework of learning (Cawley & Parmar, 1992; Joyce, Weil & Calhoun, 2009). Typically, instruction for students with mathematics learning difficulties is provided through diagnostic and remedial approaches by using drill-and-practice or direct instruction (Tournaki, 2003; Moscardini, 2009). These approaches might involve students in learning activities that foster over-reliance on prescriptive pedagogies that prevent them from active thinking and sense-making process (Ketterlin-Geller et al., 2008). Lacking of experiences in these authentic processes might prevent the students from acquisition of mathematical knowledge which they need to progress to higher mathematics learning.

Mathematics remediation for these students is generally focused on arithmetic with emphasis on memorization of basic facts and mastery of mathematical automacy (Cawley & Parmar, 1992; Mercer & Miller, 1992; Fuchs & Fuchs, 2001; Tournaki, 2003; Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Poon, Yeo, & Noor Azlan, 2012). Consequently, instructional activities could not engage them in acquiring conceptual understanding and mathematical process skills (Kettlerlin-Geller et al., 2008; Cawley & Parmar, 1992) such as problem solving, making connection, representing, communicating, and reasoning (National Council of Teachers of Mathematics, 2000). Procedural knowledge becomes a core part of instruction in the remediation classroom. Although students are able to apply certain concepts and perform procedures during initial instruction, they might not maintain their knowledge and skills over time (Ketterlin-Geller et al., 2008). Furthermore, if students are not involved in learning activities that promote mathematical processes such as problem solving and reasoning, they will not be able to make sense of mathematics in order to gain conceptual understanding as well as procedural knowledge.

Advocates of the diagnostic and remedial approaches recommended students to learn concepts and procedures of mathematics through hands-on manipulation of concrete materials and pictorial representations using the concrete-representation-abstract sequence (Mercer & Miller, 1992; Fuchs & Fuchs, 2001, Bryant et al., 2008, Flores, 2009). Through the use of systematic and explicit instruction, this strategy was found effective in improving students' basic skills in arithmetic and facilitating the students' understanding of mathematics ideas. However, these students might only benefit from their learning at the initial stage and still face difficulties in subsequent learning (Ketterlin-Geller et al., 2008). Students with learning difficulties can only benefit from their learning if they are encouraged to think and reason. Merely perform steps in solving problems by following what is demonstrated does not help children to internalise the concepts and thus might not understand those steps. This view is supported by Ma (1999) that students' misconceptions in mathematics are likely a result of being taught rules and algorithms which are demonstrated by their teacher in early mathematics. These teachers with traditional disposition tend to use materials to demonstrate procedures for their students to re-enact. Giving student specific tactics to apply in solving problems could hamper them in learning with understanding (Moscardini, 2009).

INSTRUCTIONAL MODEL FOR LEARNING MATHEMATICS THROUGH SENSE-MAKING AND UNDERSTANDING

Mathematics remediation should be aimed at helping students to master basic skills so that they could continue their study in the regular classroom (Ministry of Education Malaysia, 2010). Merely gaining arithmetic skills is not sufficient for these students. Hence, they should be engaged in instructional activities that could enhance both their conceptual and procedural knowledge.

In this research project, we proposed an instructional model as shown in Figure 1 which is focused on the content, strategy, technique, and approach of instruction. As suggested by Van de Walle (2001), learning of mathematics should consist of concepts and procedures. Based on empirical evidence of an iterative model provided by Rittle-Johnson, Siegler and Alibali (2001), and Rittle-Johnson and Koedinger (2009), there is an interactive relationship between these two types of knowledge. They develop optimally when they are both emphasised during the teaching and learning process. In terms of instructional approach, we proposed the application of both behaviourist and constructivist approaches during mathematics remedial intervention based on recommendation from Gurganus (2007) and Hallahan, Lloyd, Kauffman, Weiss, & Martinez (2005). Students with learning difficulties might encounter difficulties with this indirect approach which is emphasised by the mathematics educators. Hence, their teacher might need to use the behavioural learning approach to support them in mathematics learning. For instructional strategy, we applied the remedial approach which emphasises learning mathematics through the sequence of concrete objects, drawing or picture, and symbols (Mercer & Miller, 1992). The purpose is to help students master a mathematical skill based on their concept understanding. Problem solving was recommended as a mathematics teaching approach in official documents such as Agenda for Action (National Council of Teachers of Mathematics, 1980) and Cockcroft Report (Cockcroft, 1982). It is coherent with the reformation in mathematics education. This activity enables students to understand concept while they have discussion with peers about a problem they intend to solve (Slavin, 2009).

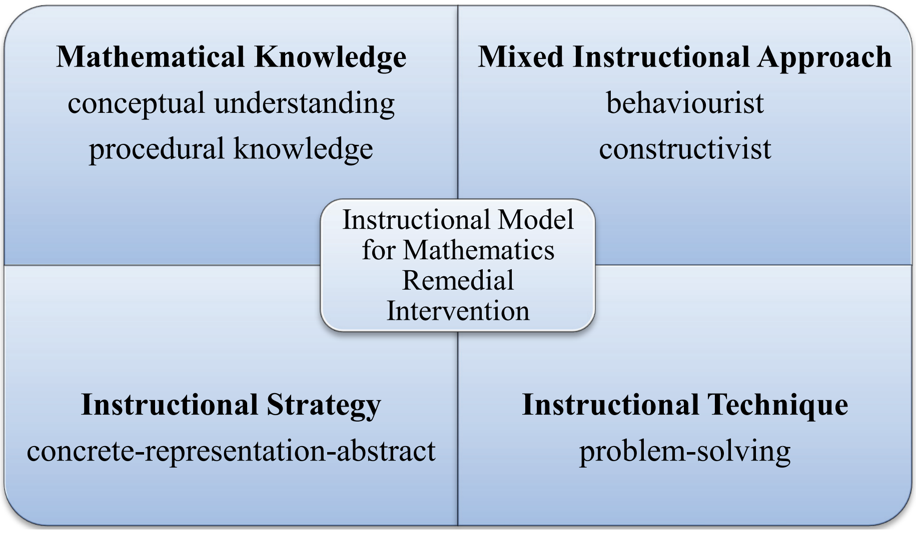

Figure 1. Instructional model for Mathematics remedial intervention

Mathematical Knowledge

Van de Walle (2001) suggested that students should be involved in learning conceptual knowledge and procedural knowledge. In gaining conceptual understanding, students construct internally the logical relationships of mathematical ideas. As part of the mental network of a learner's ideas, it can reflect the understanding of the learner's procedural knowledge. Procedural knowledge consists of rules and procedures that are used by the learner to perform mathematical tasks. As it does not help developing conceptual understanding of a learner, students might lack the ability to use efficient strategies in solving arithmetic problems if they are only taught memorisation of rules and procedures. Hence, learning of procedural knowledge should not occur without conceptual understanding but that is common in mathematics classroom (Van de Walle, 2001; Byrnes, 2008).

Children with learning difficulties in arithmetic tend to demonstrate difficulties in representing arithmetic facts or retrieving facts (Micallef & Prior, 2004). The ability of children to memorise or retrieve those facts effectively could facilitate problem solving and written or mental computation with multi-digits. However, it is a major obstacle for many school children. Baroody, Bajwa and Eiland (2009) also suggested that children learn basic facts in three phases: counting, reasoning, and mastery. According to the constructive view of learning, children need a lot of experiences to learn at the phases of counting and reasoning before meaningful memorisation occurs (Baroody et al., 2009; Byrnes, 2008; Reys, Lindquist, Lambdin, & Smith, 2007). In the process of counting objects and making sense, pupils build a part-part-whole schema for numbers and understand the key principles of additive composition by which parts are combined to form a whole (Resnick, 1989; Van de Walle, 2001). Cathcart, Pothier, Vance & Bezuk (2011) also recommended that children should develop an understanding of meaning and properties of number operations. Some children might use properties of addition to add whole numbers while others use their understanding of addition to develop quick recall of basic addition facts (Cathcart et al., 2011). Therefore, students should be engaged in learning addition based on the join model which proposes addition as the combination of two groups of objects (Cathcart et al., 2011; Reys et al., 2007; Usiskin, 2007).

Mixed Instructional Approach

Explicit instruction and drill-and-practice are found effective in teaching arithmetic skills to students with learning difficulties (Gurganus, 2007; Joyce et al., 2009). These approaches have their theoretical base on the behaviour modification of human beings. In response to the feedback of the progress of a task, human beings could use their self-correcting communication system to adjust their behaviour. This knowledge helps educators to design instruction for learning computation, developing social skills and other basic skills. Reinforcement is used widely to help students with learning difficulties to regulate their academic behaviour (Joyce et al., 2009; O'Donnell, Reeve, & Smith, 2007). Teachers usually use incentives to elicit the intended behaviours. Next, they use prompts to maintain the behaviours. They also use positive reinforcement to strengthen the behaviours and encouraged students to perform that again in future. Teachers would teach new behaviours by means of reinforcement for small steps toward the desired goal (Slavin, 2009). However, over reliance on prescriptive pedagogies as used in the diagnostic and remedial approaches might prevent students from active thinking and sense-making (Ketterlin-Geller et al., 2008; Moscardini, 2009). Hence, teachers should reduce the use of approaches which are based on the behaviourist framework of learning and incorporate more constructivist approaches to instruction.

The constructivist approach plays an important role in the current mathematics education. Its ideas about teaching and learning of mathematics are greatly influenced by Piaget's work about cognitive developmental stages and theory on cognitive development (Slavin, 2009). Vygotsky's idea on learning involves the acquisition of signs through instruction and information from others (Slavin, 2009). He believed that cognitive development occurs if children are allowed to work within their zone of proximal development with the help of peers and teachers. Slavin (2009) suggested that students work together in small groups to solve problems. The problem context should be compelling to students so that it has motivational values of connecting problem-solving to students' real life. In the process, teachers facilitate the discussion of strategies in finding the solution. They also need to provide feedback on the process by which the students arrive at the solutions.

Instructional Strategy

Mercer and Miller (1992) suggested the use of concrete-representation-abstract sequence of teaching as an instructional strategy in the mathematics remedial classrooms. At the first phase, teachers carry out demonstration using manipulative to help students understand a concept. Later, the students are guided to manipulate concrete materials until they understand the concept. At the representation phase, steps in the concrete phase are repeated but manipulative are replaced by use of pictures or drawings. The purpose is to help students master a mathematical skill based on their concept understanding. At the abstract phase, students solve problems by merely using symbols and numbers. The application of this instructional strategy was teacher-centred.

In the view of constructivist approach, the concrete-representation-abstract sequence should be based on Bruner's principle of structure. Bruner suggested three modes of representation. Teaching and learning of knowledge progress through the modes of enactive, iconic and symbolic (Sprinthall, Sprinthall & Oja, 1994). In learning mathematical concepts using the enactive mode, students manipulate concrete objects. During the learning experiences in the enactive mode, students should change their knowledge and skills to a suitable language. They are expected to solve problems using the knowledge and skills that they learn in the enactive modes.

Instructional Technique

Problem-solving is recommended for mathematics learning by National Council of Teachers of Mathematics (2000) and Ministry of Education Malaysia (2010). However, the actual implementation of this activity depends on teachers who carry out the curriculum in the daily mathematics classrooms. The problem-solving activities might be focused on solving word problems using the examination-oriented approach and explicit instruction. Slavin (2009) suggested the use of real-life problem context and cooperative learning method in order to engage students in mathematical processes such as making connections, representations, and reasoning. Through these processes, students could be helped in constructing their knowledge and skills in mathematics.

RESEARCH PURPOSE

The challenge in this research is to help students with learning difficulties engage in mathematical activities that encourage them to make sense of mathematics and thus facilitate mastering of basic addition facts with understanding. This research investigated the teaching and learning process during instructional activities that involved direct modelling of basic addition facts using concrete materials. Students were expected to acquire conceptual understanding and procedural knowledge of mathematics.

The research questions of this study include:

1. What is the teaching process during the implementation of an instructional model in a mathematics remedial intervention?
2. What is the learning process during the implementation of an instructional model in a mathematics remedial intervention?

METHODOLOGY

Research Design

As this research was intended to investigate the process of teaching and learning in a mathematics remediation classroom, a case study research design was used to gain an in-depth understanding and to reflect on that process (Creswell, 2008; Merriam, 1998).

Setting and Participants

To understand the effect of using concrete materials in helping students with mathematics learning difficulties in the remediation classroom, a remediation class teacher and his four students were selected through purposeful sampling (Creswell, 2008). The teacher, Mr. Harris, was officially assigned to the Special Remediation Program. He was selected to participate in this research because he was currently practicing behaviourist approach and intended to try constructivist approaches for mathematics remediation. The participating students, Nasrah, Farib, Fatimah and Najib, were selected after administration of a screening test and followed by a diagnostic test. These nine-year-old students were from a nearby village and had not mastered basic skills in addition including basic addition facts.

Data Collection and Analysis

As the purpose of this research is to gain a holistic understanding of the teaching and learning process in the mathematics remediation classroom, data collection instruments included observation protocol, interview, and work of students. Data were collected before actual implementation of our instructional activities to understand the usual practice and behaviour of the participating teacher and students in the mathematics remediation classroom. Based on the information obtained, we developed instructional activities which would be implemented by the teacher and his students. After that, a post-activity interview was carried out with the teacher and also the students to understand their behaviours during the remedial intervention. We checked the work of students to understand their responses to the knowledge delivered, the use of concrete-representational-abstract sequence, and the application of instructional approach, in the remedial intervention.

Data collected using the qualitative approach from classroom observations and interviews were recorded in the form of video clips and analysed using a qualitative approach recommended by Creswell (2008) which involved transcribing, segmenting, coding, creating themes, and inter-relating themes. Qualitative data from interview with the participating teacher and students helped us to understand their perception and behaviour. Work of students was compared with the video clips of classroom observation to help us understand the behaviour of the students and the conditions under which the work was produced.

Instructional Activities

The activities were planned for the students to learn basic addition facts which include understanding the meaning of whole numbers addition based on the join model. The focus of this article was on remedial intervention for basic addition facts of doubles and 'a single digit plus 6, 7, 8, and 9'. During implementation of the activity, Mr. Harris was expected to apply the instructional model to help the students improve their mathematical knowledge on basic addition facts. The teaching and learning process took approximately one hour.

RESULTS

Instruction of Mr. Harris

Two egg trays, each with ten holders, and twenty balls were used for the activity. Mr. Harris added two balls to the tray to represent all the cases in the doubles systematically. As shown in Figure 2, for '2 + 2 = 4', he arranged four balls on the tray where two of the balls were put on the left column, and the other two balls were put on the right column. After each case was shown, he asked the students to write a math sentence to represent it. The final product would be a list of doubles.

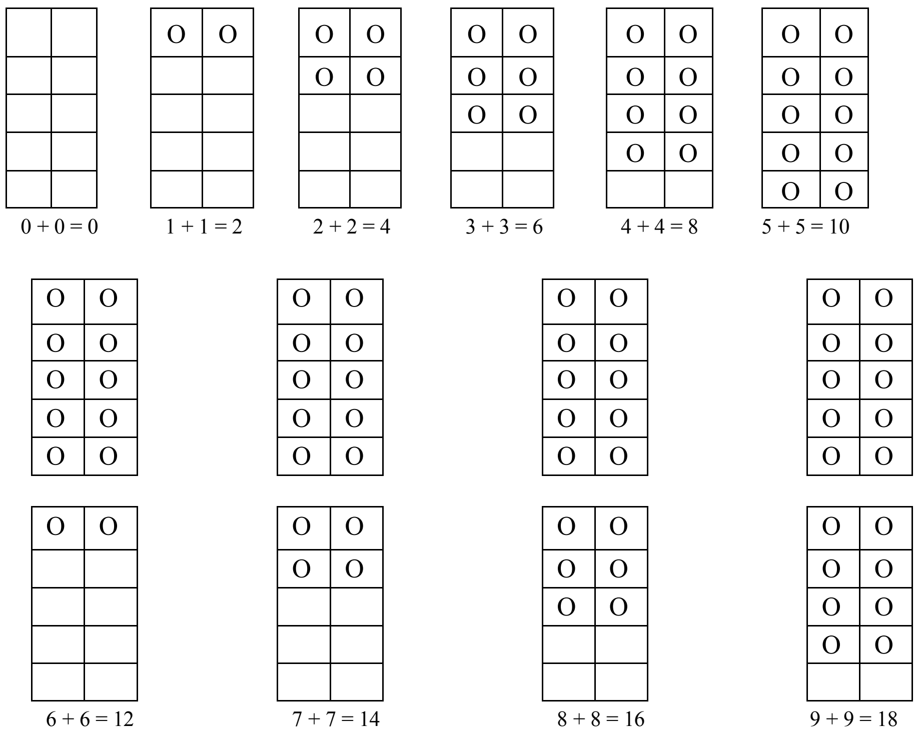

Figure 2. Arrangement of balls on egg trays to show doubles

In helping the students learn basic addition facts from the family of doubles and '9 + □', Mr. Harris explained a context where the balls represented the green bean *kuih* that were given by him to the students. Apart from the balls and egg trays, he also asked the students to use their fingers as 'concrete objects' to represent the red bean *kuih*. He put nine balls on an egg tray to represent the first addend, '9', which was 'in the students' head'. The students were required to use their fingers to represent the second addend. Using these concrete objects, Mr. Harris encouraged the students to use 'finger count-on' technique in order to master those basic addition facts. He told them that there were nine pieces of *kuih* on the tray already, to find the total pieces of *kuih*, they needed to use their fingers and count on from the second addend. The students were required to "keep nine in your head, raise your fingers according to the second number (addend), and count on".

After an example was given using explicit explanation and demonstration, the students took turn to practice the skill so that Mr. Harris could guide everyone individually. After that, they were required to complete the list for family of '9 + □' on a piece of paper. They could use the balls on the egg tray to help them if needed. Next, Mr. Harris encouraged the students to complete the list for families of '8 + □', '7 + □' and '6 + □' by practicing the technique that he taught them without using the balls and egg trays. The students were required to imagine that the seven or six balls and egg trays were in their head, and represent the second addend using their fingers. After that, they needed to use the count-on technique to find the sum. However, they still could use those objects if they needed to.

Responses of Nasrah

During one-to-one guided practice for doubles and '9 + □', Nasrah made mistakes. The students were shown an empty egg tray with ten holders. It had two rows and five holders in each row. Mr. Harris did not put any ball on the egg tray. Nasrah said "zero plus five equals to five" when Mr. Harris expected '0 + 0 = 0'. She explained that her math sentence meant 'zero occupied holder plus five empty holder equals to five holders'. Obviously, she was making the connection based on a join model of addition although it had different meaning from the desired answer. After Nasrah had completed her list, she managed to identify a pattern among the doubles. She explained that the addends and sum were in ascending order.

When she was asked to complete '9 + 7', she applied the finger-counting technique taught by Mr. Harris and answered "eighteen". She explained that she put up nine fingers and kept seven, instead of nine, in her head. Later, she 'moved' one out of the nine fingers to her head and 'joined the seven' to make a ten. So she still had eight fingers which were raised, and she added the ten in her head and the eight fingers to get 'eighteen'.

Mr. Harris immediately gave her feedback and explanation regarding the finger-counting technique so that she could do the computation again. Nasrah practiced it according to Mr. Harris's instruction, and finally could use it confidently. She completed the list for family of '9 + □' on her own. For the first four, she retrieved the answers from her memory. Later, she completed the list without doing any computation. Nasrah explained that the first four facts were '9 + 0 = 9', '9 + 1 = 10', '9 + 2 = 11', and '9 + 3 = 12'. She managed to identify a number pattern from the sums, '9, 10, 11, and 12'. Thus she completed the other facts in the list based on this number pattern. Nasrah used finger-counting to find the first few facts in the list for family of '8 + □', '7 + □', and '6 + □'. She completed all the lists based on the number pattern that she found in the previous activity.

Responses of Farib

Farib could complete the math sentence for the doubles in the first tray from his memory but manage to identify a pattern for '6 + 6' until '10 + 10'. He explained that the total increased in 2s. For basic addition facts in the family of '9 + □', Farib was able to retrieve them from his memory if the second addend was between 0 and 4. He used mental strategy if the second addend was 5 or greater. Sometimes, he used finger-counting that was taught by Mr. Harris if he wanted to check his answer. He explained that he used a strategy as shown in Figure 3 when he performed mental computation. It was actually the 'making tens' strategy (Reys et al., 2007). However, Mr. Harris still encouraged him to use the finger-counting technique to minimise the risk of making mistake.

$$\begin{aligned} 9 + 5 &= 9 + 1 + 4 \\ &= 10 + 4 \\ &= 14 \end{aligned}$$

Figure 3. 'Making Tens' strategy applied by Farib

To complete the first few basic addition facts in the list for the other families, Farib would either retrieve the facts from his memory or use 'mental-counting' to find the sum. According to Farib, he counted his fingers mentally instead of doing it using his fingers. He assumed that this was a better technique if compared to finger counting. After that, he would complete the other facts based on the number pattern which he described as "*nombor tu (jumlah) menaik, dan tambah satu*" (The sums were in ascending order, and plus one).

Responses of Fatimah

Fatimah counted the balls on the egg trays one by one slowly to represent each situation shown by Mr. Harris using a math sentence. To complete the basic addition facts in the family of '9 + □', Fatimah used the technique taught by Mr. Harris. She performed it slowly but her answer was always correct. Sometimes, Fatimah looked at the ceiling and did not use her fingers while she did her computation. She explained that in completing the list for this group of facts, she could use 'mental-counting' if the second addend was "*nombor kecil*" (a small number). For example, when she was solving '9 + 4', she would imagine that there were four fingers in her head, and thus she counted 'ten, eleven, twelve, and thirteen' for the four fingers.

For the list of other families, Fatimah explained that if the second addends are between 0 and 3, she could do the counting mentally. She would use finger-counting technique that was taught by Mr. Harris if the number was greater than 3.

Responses of Najib

When a case was demonstrated using concrete objects, Najib counted the balls on the tray one by one. After Mr. Harris showed '0 + 0 = 0' until '3 + 3 = 6', Najib completed the list without observing demonstration of Mr. Harris. He explained that he had identified the patterns in the list. He found the addends increased by one while the sum increased by two.

For the basic addition facts in the families of '9 + □', '8 + □', '7 + □', and '6 + □', Najib relied on using the finger-counting technique that was taught by Mr. Harris. He was not confident with any mental strategy as he always made mistake when he tried to do mental-counting. Mr. Harris drilled Najib to use finger-counting but Najib also encountered difficulty in counting using fingers. After the lists were prepared, Najib identified the patterns that appeared among the basic addition facts in every list. He was able to list all the facts systematically again and immediately. Najib explained that he wrote the ten facts in each list systematically as he found that the second addends and the sums were in ascending order. For example, for the family of '9 + □', the first addend was '9' while the second addend of the facts in the list was in the sequence of '0, 1, 2, ..., 9'. The sum of the facts in the list also progressed in the sequence of '9, 10, 11, ..., 18'.

DISCUSSION

Manipulation of Concrete Objects

The students were concrete-operational learners (Slavin, 2009; Woolfolk, 1995). Hence, concrete experiences helped them in making connections between each situation and the related math sentence or basic addition fact. All the students could identify the connections and thus represent each situation using written symbols. These connections would improve their knowledge of the join model of addition. Apart from that, they also identified patterns from the arrangement of the concrete objects and the list of basic addition facts. These patterns could help the students to master the procedural knowledge of basic addition facts. The performance of these students confirmed our argument that teachers should offer more chances for students with learning difficulties to participate in active thinking and sense-making as suggested by Van de Walle (2001) rather than simply following rules and procedures as practiced in the traditional remedial approach (Mercer & Miller, 1992). However, teacher should facilitate the discussion of these alternative strategies so that the more-able peers could justify their solution while the less-able peers could learn these strategies (Slavin, 2009).

In finding the sum to a math sentence, Fatimah and Najib relied on counting concrete objects or using finger count-on technique in retrieving basic addition facts. Fatimah only could use mental-counting if the second addend was a small number. Therefore, there might be over reliance on concrete objects as a tool for counting rather than developing reasoning skill.

On the other hand, Farib and Nasrah managed to retrieve many basic addition facts from their memory. When they failed to recall certain facts especially those with sum more than 10, they were found using a less mature mental computation technique. Although they did not use finger-counting, they counted their fingers in their imagination. Farib had shown his ability in retrieving some facts through application of 'making tens' strategy. Concrete objects were used by Farib and Nasrah to develop their mental strategy, not merely as a tool for making connections between real-world situations and mathematics. Our finding was also supported by research finding of Moscardini (2009) where his student participants had shown reasoning abilities in solving arithmetic problems.

A Mixed Instructional Approach

Generally, the instruction of Mr. Harris was based on the behaviourist approaches. He did not encourage Farib to practice reasoning skill in retrieving basic addition facts, as he preferred his students in using finger count-on to minimise mistakes. Thus, Mr. Harris also did not ask the students to explain their alternative strategies in retrieving basic addition facts to let other less-able peers learn. Although students should progress from counting to reasoning before they could master basic facts (Baroody et al., 2009), Mr. Harris was not confident with his students in using reasoning to learn mathematics. He assumed that sense-making was for understanding mathematical concepts, not for mastering procedural knowledge.

The teacher-directed instruction of Mr. Harris ignored the constructivist approaches suggested by Slavin (2009) and Van de Walle (2001). He emphasized the role of practice rather than learning. By teacher-directed problem-solving activity, he reduced the opportunities for students to make sense and explore mathematical ideas. Whenever his students encountered learning difficulties, he would immediately provide feedback on their answer and explicit instruction, rather than discussion on their process of arriving at the solution and using scaffolding technique.

Although the learning environment was teacher-directed, Mr. Harris allowed his students to use alternative strategies in retrieving basic addition facts. A relaxed and flexible environment also enabled his students to identify patterns among those facts and thus improve their conceptual understanding because conceptual knowledge also consists of logical relations (Van de Walle, 2001). The students' experiences in identifying patterns and relations among basic addition facts helped them to solve the problem by listing the facts based on the properties. Thus, the students actually had learned to apply a problem-solving strategy that was stated in the curriculum (Ministry of Education Malaysia, 2010).

The problem context used in this intervention seemed to motivate the students in participating in the instructional activity as it was suggested by Mr. Harris based on his understanding of the *kampung* (village) life. As it was closely related to students' everyday life, it also facilitated them in understanding the meaning of those basic addition facts. Moreover, Mr. Harris often tried to help the students understand the problem context by asking them to imagine they were the characters in the context.

In short, although behaviourist approach was applied in this teacher-directed problem-solving activity, some constructivist practices were incorporated and thus enabled the students to engage in mathematical processes. Their participation in these processes facilitated their learning of conceptual and procedural knowledge of basic addition facts.

Improving Conceptual Understanding And Procedural Knowledge Through Problem Solving

Problem-solving activity facilitated the students in learning basic addition facts by providing a meaningful context for them to make sense actively. The meaningful context helped them to understand the relation between the join model of addition (Reys et al., 2007; Cathcart et al., 2011) and the basic addition facts that they had written. This experience helped them to realise that these facts were meaningful because they were the product of a real-world context. In the process of counting and making sense, the students understood and mastered the part-part-whole schema (Resnick, 1989; Van de Walle, 2001) for addition. They were convinced of this schema as they identified it from all the facts. Apart from the schema that they identified in each basic addition fact, they also could identify the patterns that appeared among the facts. Understanding these patterns actually could improve their conceptual understanding as logical relations were part of the knowledge. Therefore, experiences to learn basic addition facts through counting and sense-making might help the students to understand these logical relations and thus memorise the facts meaningfully (Baroody et al., 2009; Byrnes, 2008; Reys et al., 2007). Although the cases were shown systematically by the teacher, the students prepared a list of basic addition facts that enabled them to identify the patterns that appeared. The facts that were arranged systematically also facilitated the students to memorise them efficiently.

The problem-solving activity was teacher-directed but a few constructivist approaches were applied and thus still enabled the students to make sense. The students were guided to understand the problem context through the use of concrete objects and a compelling context. Systematic presentation of the cases which were related to the basic addition facts stimulated the students to think actively about the relations and patterns. As a result, they solved the problem using their own strategy when they listed the remaining facts based on the patterns they observed. This finding supported the suggestion of Slavin (2009) that problem-solving activity could engage students in mathematical processes. Through these processes, the students organised and improved their knowledge of basic addition facts.

CONCLUSION

The instructional model used in the remedial intervention of this research involved application of concrete materials, application of a mixed instructional approach, delivery of conceptual and procedural knowledge, and use of problem-solving activity. Concrete materials could enable students to identify connections between mathematical ideas and real-world context, and hence engage them in active thinking and sense-making. However, this instructional strategy also resulted in reliance of some students in using it as a tool of counting physically in order to find the sum of math sentences. Other students proactively used the concrete materials to help them develop their 'mental-counting' and mental-reasoning skills. Generally, the teaching and learning process was teacher-directed. After demonstration and explanation were carried out by the teacher, students were required to apply the knowledge and skills in guided practices. However, the teacher allowed the students to apply alternative strategies in solving problems. Thus, flexibility in his instruction to incorporate some constructivist approaches resulted in some mathematical processes. The emphasis of learning both conceptual and procedural knowledge through problem-solving activity facilitated the students to master basic addition facts meaningfully and efficiently. The students also learned some process skills such as problem-solving strategy, reasoning skill, making connections and representations. Hence, a mixed instructional approach with the use of concrete materials and problem-solving might help students to learn conceptual and procedural knowledge concurrently. Their mathematical process skills could be improved when they are required to engage in active thinking and sense-making. However, teachers should reduce the use of behaviourist approaches gradually, and incorporate more student-centred approaches, based on the student individual differences and usual practice.

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