Asia Pacific Journal of Educators and Education, Vol. 28, 33–51, 2013

ENHANCING PRIMARY PUPILS' GEOMETRIC THINKING THROUGH PHASE-BASED INSTRUCTION USING THE GEOMETER'S SKETCHPAD

Chew Cheng Meng* and Lim Chap Sam
School of Educational Studies, Universiti Sains Malaysia,
11800 Pulau Pinang
*Corresponding author: cmchew@usm.my

Abstract: The purpose of this study was to enhance primary pupils' geometric thinking through phase-based instruction using The Geometer's Sketchpad (GSP) based on the van Hiele theory of geometric thinking. Specifically, it sought to examine Year Four pupils' van Hiele levels of geometric thinking about equilateral triangle, square, regular pentagon and regular hexagon before and after phase-based instruction using GSP, and whether there was any significant difference in the pupils' van Hiele levels of geometric thinking about the regular polygons after the intervention. The researchers employed an exploratory case study research design and purposeful sampling to select a class of 26 mixed-ability Year Four pupils from a primary school in Selangor. A van Hiele level test based on Mayberry's (1981) test and scoring criteria was devised and administered to the pupils before and after the intervention to assess their van Hiele levels of geometric thinking about the regular polygons. The results of the pre-test showed that the pupils' initial van Hiele levels were predominantly at Level 0 (Pre-recognition) for regular pentagon and regular hexagon but at Level 1 (Recognition) for equilateral triangle and square. However, the results of the post-test revealed that the pupils' van Hiele levels after the intervention were predominantly at Level 2 (Analysis) for all the regular polygons. In addition, the results of the Wilcoxon test showed that there was a significant difference in the pupils' van Hiele levels of geometric thinking for all the regular polygons after phase-based instruction using GSP. The median van Hiele level in the post-test was higher than the median van Hiele level in the pre-test for all the regular polygons, indicating that the intervention had significantly enhanced the pupils' geometric thinking about the regular polygons.

Keywords: van Hiele theory of geometric thinking, phase-based instruction, The Geometer's Sketchpad, van Hiele levels of geometric thinking, regular polygons

Abstrak: Tujuan kajian ini adalah untuk mempertingkatkan pemikiran geometri murid-murid sekolah rendah melalui pengajaran berasaskan fasa dengan menggunakan *The Geometer's Sketchpad* (GSP) berdasarkan teori pemikiran geometri van Hiele. Khususnya, kajian ini mengkaji tahap pemikiran geometri van Hiele murid Tahun Empat tentang segi tiga sama sisi, segi empat sama, pentagon sekata dan heksagon sekata sebelum dan selepas pengajaran berasaskan fasa dengan menggunakan GSP, dan sama ada terdapat perubahan yang signifikan dalam tahap pemikiran geometri van Hiele murid tentang poligon sekata selepas intervensi. Pengkaji menggunakan reka bentuk kajian kes eksploratori dan persampelan bertujuan untuk memilih sebuah kelas Tahun Empat yang terdiri daripada 26 orang murid yang berbeza kebolehan dari sebuah sekolah rendah di

Selangor. Ujian tahap van Hiele berdasarkan ujian dan kriteria penskoran Mayberry (1981) dibina dan ditadbirkan kepada murid sebelum dan selepas intervensi untuk menilai tahap pemikiran geometri van Hiele mereka tentang poligon sekata. Dapatan ujian pra menunjukkan bahawa tahap van Hiele awal bagi kebanyakan murid adalah pada Tahap 0 (Pra-pengenalan) bagi pentagon sekata dan heksagon sekata tetapi pada Tahap 1 (Pengenalan) bagi segi tiga sama sisi dan segi empat sama. Namun, dapatan ujian pasca menunjukkan bahawa tahap van Hiele selepas intervensi bagi kebanyakan murid adalah pada Tahap 2 (Analisis) bagi semua poligon sekata itu. Tambahan pula, keputusan ujian Wilcoxon menunjukkan bahawa terdapat perbezaan yang signifikan dalam tahap pemikiran geometri van Hiele murid tentang semua poligon sekata selepas pengajaran berasaskan fasa dengan menggunakan GSP. Median tahap pemikiran van Hiele dalam ujian pasca adalah lebih tinggi daripada median tahap pemikiran van Hiele dalam ujian pra bagi semua poligon sekata itu. Hal ini menunjukkan bahawa intervensi tersebut dapat mempertingkatkan tahap pemikiran geometri murid tentang semua poligon sekata itu.

Kata kunci: Teori pemikiran geometri van Hiele, pengajaran berasaskan fasa, *The Geometer's Sketchpad*, tahap pemikiran geometri van Hiele, poligon sekata

BACKGROUND OF THE STUDY

The study of geometry is important as it has been recognised as a basic skill in mathematics (National Council of Supervisors of Mathematics, 1977; National Council of Teachers of Mathematics, 2000) and also it has important applications to topics in basic mathematics (Sherard, 1981). For example, geometric regions and shapes are essential for teaching fractions, decimals and percents (Hatfield, Edwards, Bitter, & Morrow, 2000). Geometry is also essential for learning functions and calculus. The derivative of a function, for instance, can be visualised as the slope of the tangent line to the graph of the function or the definite integral as the area under a curve (Usiskin, 1980). In addition, geometry is a foundation for study in such fields as science, engineering, architecture, geology and astronomy (van de Walle, 2001). Further, geometry has important applications to real-life problems (Sherard, 1981) such as arranging a living room, making frames, planning a garden, as well as in various aspects of construction work (Hatfield et al., 2000; van de Walle, 2001). Moreover, there are cultural and aesthetic values to be derived from the study of geometry (O'Daffer & Clemens, 1992), which enable pupils to "appreciate the importance and beauty of mathematics" (Malaysian Ministry of Education, 2003, p. 2).

In spite of its importance, the performance of Malaysian secondary students in geometry was still unsatisfactory as highlighted in a number of reports on international assessment studies. Specifically, in the Third International Mathematics and Science Study (TIMSS) 1999 Report, the average geometry achievement of Malaysian Form Two students (497) was far behind that of their counterparts in the top five Asia-Pacific countries of Japan (575), Republic of Korea (573), Singapore (560), Chinese Taipei (557), and Hong Kong (556). In addition, it was not significantly higher than the international average (487). As a result, Malaysia was ranked 16th in average geometry achievement out of 38 participating countries (Mullis, Martin, Gonzalez, Gregory, Garden, et al., 2000).

Subsequently, although the average geometry achievement of Malaysian Form Two students (495) was significantly higher than the international average (467) in the Trends in International Mathematics and Science Study (TIMSS) 2003 Report, it was still far below that of their counterparts in the top five Asia-Pacific countries of Republic of Korea (598), Chinese Taipei (588), Hong Kong (588), Japan (587), and Singapore (580). In fact, Malaysia was ranked sixteenth in average geometry achievement out of 46 participating countries and 4 benchmarking participants (Mullis, Martin, Gonzalez, & Chrostowski, 2004).

But, in the Trends in International Mathematics and Science Study (TIMSS) 2007 Report, the average geometry achievement of Malaysian Form Two students (477) was not only significantly lower than the TIMSS scale average (500) but also it was even further behind that of their counterparts in the top five Asia-Pacific countries of Chinese Taipei (592), Republic of Korea (587), Singapore (578), Japan (573), and Hong Kong (570). Consequently, Malaysia was ranked twenty-fourth in average geometry achievement out of 49 participating countries and 7 benchmarking participants (Mullis, Martin, & Foy, 2008).

The low rankings of Malaysian Form Two students in the TIMSS 1999, 2003 and 2007 Reports indirectly reflected that their levels of geometric thinking were still far from satisfactory. In addressing this concern, it is imperative that primary pupils are provided with a firm foundation of geometry in order to develop their geometric thinking.

Research has shown that instruction using GSP could enhance students' learning of plane geometry (Choi, 1996; Choi-Koh, 1999; Driskell, 2004; Elchuck, 1992; Frerking, 1995; Thompson, 2006) as well as solid geometry (Chew, 2007; July, 2001; McClintock, Jiang, & July, 2002). More specifically, research has shown that phase-based instruction using GSP based on the van Hiele theory of geometric thinking could enhance secondary students' geometric thinking. Choi (1996) and Choi-Koh (1999) showed that phase-based instruction using GSP could enhance secondary students' van Hiele levels of geometric thinking about special triangles, namely equilateral triangle, isosceles triangle and right-angled triangle. Chew (2007) found that phase-based instruction using GSP could enhance Form One students' van Hiele levels of geometric thinking about cubes and cuboids. However, there has been a lack of research that specifically examines if primary pupils' geometric thinking could be enhanced through phase-based instruction using GSP based on the van Hiele theory of geometric thinking.

PURPOSE OF THE STUDY

The purpose of this study was to enhance primary pupils' geometric thinking through phase-based instruction using GSP based on the van Hiele theory of geometric thinking. Specifically, this study aimed to answer the following research questions:

1. What were the pupils' van Hiele levels of geometric thinking about equilateral triangle, square, regular pentagon, and regular hexagon before and after phase-based instruction using GSP?
2. Was there a significant difference in the pupils' van Hiele levels of geometric thinking about the regular polygons after the intervention?

THEORETICAL FRAMEWORK

According to the van Hiele theory of geometric thinking postulated by Pierre and Dina van Hiele, pupils progress sequentially through five hierarchical levels of thinking in the process of learning geometry. But, for this study, only the descriptors of the first two levels of geometric thinking as described by Mayberry (1981) were quoted below because the geometry content of the Year 4 mathematics syllabus is only up to Level 2 (Malaysian Ministry of Education, 2003):

Level 1 (Recognition)

"Figures are recognised by appearance alone. A figure is perceived as a whole, recognisable by its visible form, but properties of figures are not distinguished. At this level a student should (1) recognise and name figures; (2) discriminate a given figure from others which look somewhat the same" (Mayberry, 1981, p. 47).

Level 2 (Analysis)

"Properties are perceived. Properties are isolated and unrelated. Since each property is seen separately, no relationship between properties is perceived. Relations between different figures are not perceived. A student on this level should recognise and name properties of geometric figures" (Mayberry, 1981, p. 48).

Besides these levels, Clements and Battista (1992) proposed the existence of Level 0 (Pre-recognition) in order to characterise pupils who are unable to distinguish between geometric figures because they only notice a subset of the visual characteristics of the geometric figures (Mason, 1997).

Furthermore, the van Hieles propose a sequence of five phases of learning, also called phase-based instruction to help pupils progress from one level of geometric thinking to the next (van Hiele, 1959/1984, 1986, 1999; van Hiele-Geldof, 1959/1984). Each of these phases of learning as employed in this study is discussed below:

Phase 1 (Inquiry)

In the first phase, the pupils examined examples and non-examples of equilateral triangles (Fuys & Geddes, 1984) using the pre-constructed GSP sketch (see Figure 1) so that the teacher (the first author) learned what prior knowledge the pupils had about equilateral triangles, and the pupils learned what direction further study would take (Crowley, 1987). Then, they examined examples and non-examples of squares, regular pentagons and regular hexagons (Fuys & Geddes, 1984) using the pre-constructed GSP sketches (the GSP sketches were obtained by clicking tabs 2, 3 and 4 respectively as shown in Figure 1) so that the teacher learned what prior knowledge the pupils had about the regular polygons, and the pupils learned what direction further study would take (Crowley, 1987).

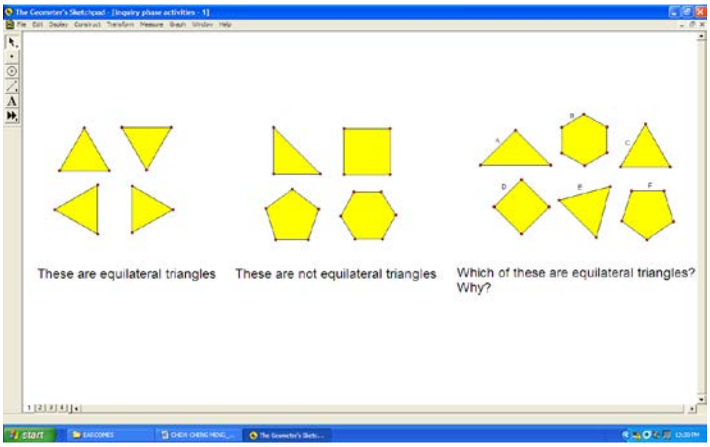


Figure 1. Examples and non-examples of equilateral triangles

Phase 2 (Guided orientation)

Next, the pupils dragged a vertex of the equilateral triangle in the pre-constructed GSP sketch (see Figure 2) and observed the side lengths, angle measures and number of axes of symmetry in order to identify the properties of equilateral triangle. Then, they dragged a vertex of the square, regular pentagon and regular hexagon in the pre-constructed GSP sketches respectively (the GSP sketches were obtained by clicking tabs 2, 3 and 4 respectively as shown in Figure 2) and observed the side lengths, angle measures and number of axes of symmetry in order to identify the properties of the regular polygons (Clements & Battista, 1992; Crowley, 1987; Hoffer & Hoffer, 1992).

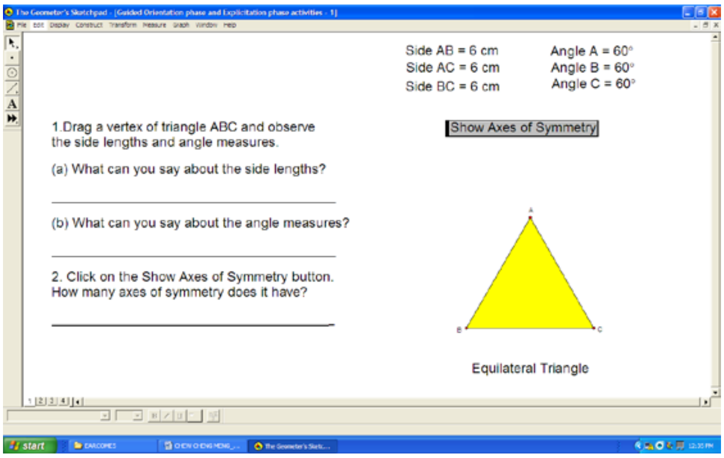


Figure 2. Properties of equilateral triangles

Phase 3 (Explicitation)

The teacher led the pupils' discussion of the properties of the regular polygons using their own words, based on the pre-constructed GSP sketches in Phase 2, and then introduced the relevant geometric terminology when appropriate (Clements & Battista, 1992; van Hiele, 1999).

Phase 4 (Free orientation)

The pupils were challenged with the task of finding the relationship between the number of axes of symmetry and the number of sides of regular polygons using the pre-constructed GSP sketch as shown in Figure 3 (Crowley, 1987).

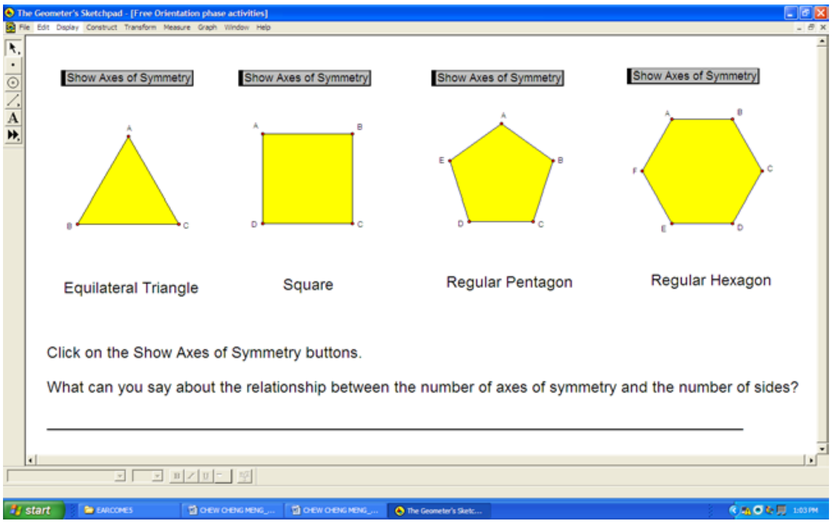


Figure 3. Relationship between number of axes of symmetry and number of sides

Phase 5 (Integration)

The pupils reviewed and summarised what they had learned about the properties of the regular polygons using the pre-constructed GSP sketch as well as the relationship between the number of axes of symmetry and the number of sides of regular polygons as shown in Figure 4 (van Hiele, 1999).

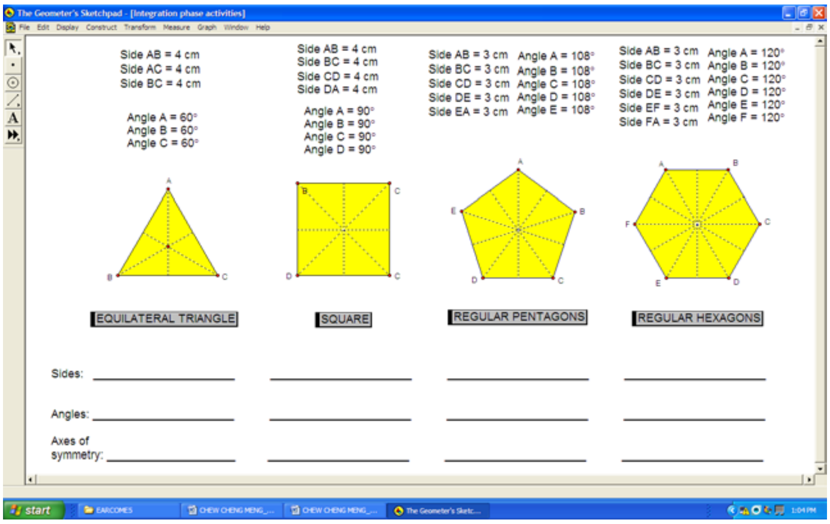


Figure 4. Summary of properties of the regular polygons

METHODOLOGY

Research Design and Sample

The researchers employed an exploratory case study research design (Berg, 2000; Yin, 1994) and purposeful sampling to select the sample since the goal of the case study was not to generalise the results of the study from the sample to the population from which it was drawn. The criteria for selecting the sample were: (a) one class of Year Four pupils studying in a public academic primary school that has a computer laboratory equipped with a teacher's laptop, an LCD projector, sufficient computer equipment and facility; (b) the class was of mixed ability in terms of mathematics achievement in the 2009 school first monthly test; and (c) the pupils were of mixed gender.

Based on the above selection criteria, a class of 26 mixed-ability Year Four pupils from a primary school in Selangor was chosen as the sample for this study. Table 1 shows the participants' achievement level in Mathematics in the 2009 school first monthly test.

Table 1. Participants' mathematics achievement level

Mathematics Achievement	Frequency
Average	12
High	6

Instrument

The 20-item multiple-choice paper-and-pencil van Hiele level test was devised by the researchers to assess the pupils' van Hiele levels of geometric thinking based on Mayberry's (1981) van Hiele level test and scoring criteria. This is because Mayberry's van Hiele level test was designed to assess students' van Hiele levels of geometric thinking about specific geometric concepts. However, only items for the first two van Hiele levels were devised (see Table 2) because the geometry content of the Year 4 mathematics syllabus is only up to Level 2 (Malaysian Ministry of Education, 2003).

Table 2. Distribution of items in the van Hiele level test

Concept	Level	Question Type	Question Number	Possible Score	Criterion for Question	Criterion for Level
Equilateral triangle	Level 1	Name	1	1		1 of 2
		Discriminate	5	2	2/2 = 1	
			9	1		
	Level 2	Properties	13	1		2 of 3
			17	1		
Square	Level 1	Name	2	1		1 of 2
		Discriminate	6	2	2/2 = 1	
			10	1		
	Level 2	Properties	14	1		2 of 3
			18	1		
Regular pentagon	Level 1	Name	3	1		1 of 2
		Discriminate	7	2	2/2 = 1	
			11	1		
	Level 2	Properties	15	1		2 of 3
			19	1		
Regular hexagon	Level 1	Name	4	1		1 of 2
		Discriminate	8	2	2/2 = 1	
			12	1		
	Level 2	Properties	16	1		2 of 3
			20	1		

Source: Adapted from Mayberry (1981)

The items for Level 1 assessed pupil's ability to: (1) recognise and name the regular polygons, namely equilateral triangle, square, regular pentagon and regular hexagon (a sample Level 1-item for equilateral triangle is shown in Figure 5); and (2) discriminate the regular polygons from other polygons (a sample Level 1-item for equilateral triangle is shown in Figure 6). The items for Level 2 assessed pupil's ability to identify properties of equilateral triangle, square, regular pentagon and regular hexagon (a sample Level 2-item for equilateral triangle is shown in Figure 7). The van Hiele level test required about 30 minutes to complete. The test was piloted with 30 Year Four pupils in another primary school which yielded a KR-20 reliability coefficient of 0.92, suggesting that the reliability of the test was high (Gay & Airasian, 2003).

1. What is the name of this figure?

A. Scalene triangle
B. Equilateral triangle
C. Isosceles triangle
D. Right-angled triangle
E. Obtuse-angled triangle

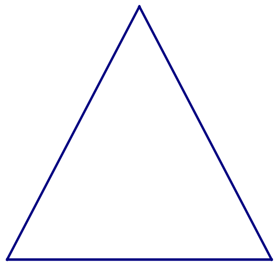
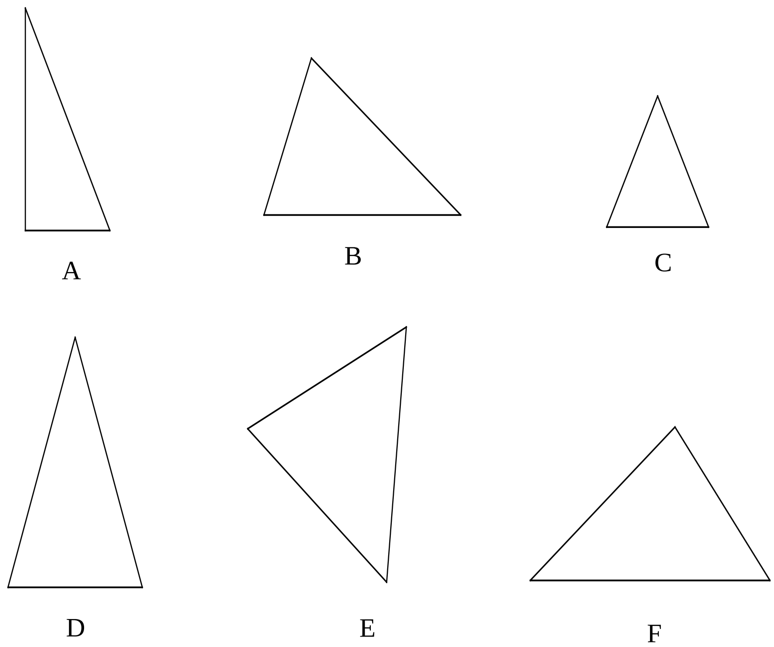


Figure 5. Sample Level 1-item for recognising equilateral triangle



4. Which of these figures are equilateral triangles?

Figure 6. Sample Level 1-item for discriminating equilateral triangle

17. Equilateral triangles have

A. three axes of symmetry
B. four axes of symmetry
C. five axes of symmetry
D. six axes of symmetry

Figure 7. Sample Level 2-item for identifying properties of equilateral triangle

Procedure

Prior to phase-based instruction using GSP, a pre-test was administered to all the pupils to determine their initial van Hiele levels of geometric thinking about equilateral triangle, square, regular pentagon and regular hexagon. Next, the first author carried out phase-based instruction using GSP which comprised two double-period 70-minute lessons. The first two phases of learning (Inquiry and Guided Orientation phases) as described above were carried out in the first lesson while the last three phases of learning (Explicitation, Free Orientation and Integration phases) as described above were carried out in the second lesson in the school computer laboratory. After phase-based instruction using GSP, a post-test was administered to all the pupils to determine their van Hiele levels of geometric thinking about the regular polygons.

RESULTS

Pupils' van Hiele Levels of Geometric Thinking Before and After the Intervention

The results of the pre-test and post-test for the van Hiele levels of geometric thinking about equilateral triangle, square, regular pentagon and regular hexagon are presented in Table 3. The results of the pre-test showed that the pupils' initial van Hiele levels of geometric thinking about the regular polygons ranged from Level 0 to Level 2 in different frequencies and percentages. In particular, the pupils' initial van Hiele levels were predominantly at Level 0 for regular pentagon (69.2%) and regular hexagon (69.2%) but at Level 1 for equilateral triangle (57.7%) and square (57.7%). In contrast, the results of the post-test revealed that the pupils' van Hiele levels of geometric thinking after the intervention were predominantly at Level 2 for all the regular polygons (84.6%) indicating that they had progressed from either Level 0 to Level 2 or from Level 1 to Level 2.

Table 3. Frequency and percentage of pupils at each van Hiele level

	Equilateral triangle		Square		Regular pentagon		Regular hexagon	
	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
Level 0	6 (23.1%)	0 (0.0%)	5 (19.2%)	0 (0.0%)	18 (69.2%)	0 (0.0%)	18 (69.2%)	0 (0.0%)
Level 1	15 (57.7%)	4 (15.4%)	15 (57.7%)	4 (15.4%)	7 (26.9%)	4 (15.4%)	7 (26.9%)	4 (15.4%)
Level 2	5 (19.2%)	22 (84.6%)	6 (23.1%)	22 (84.6%)	1 (3.8%)	22 (84.6%)	1 (3.8%)	22 (84.6%)
Total	26 (100.0%)	26 (100.0%)	26 (100.0%)	26 (100.0%)	26 (100.0%)	26 (100.0%)	26 (100.0%)	26 (100.0%)

Difference in van Hiele Levels of Geometric Thinking After the Intervention

To answer the second research question, the data were analysed using SPSS version 17.0 for Windows. The results of the Wilcoxon signed ranks tests are presented in Table 4. For equilateral triangle, 21 pupils had a higher post-test score than their pre-test score, 5 pupils had similar score in their pre-test and post-test, but none had a lower post-test score than the pre-test score. The Wilcoxon signed ranks test indicated that there was a significant difference in the pupils' van Hiele levels of geometric thinking about equilateral triangle after the intervention, $Z = -4.41$, $p < 0.01$. The median van Hiele level of 2 in the post-test was higher than the median van Hiele level of 1 in the pre-test, indicating that the intervention had significantly enhanced the pupils' geometric thinking about equilateral triangle.

The pre-test and post-test scores for square also showed an almost similar trend. 20 pupils had a higher post-test score than their pre-test score, 6 pupils had similar score in their pre-test and post-test, but none had a lower post-test score than the pre-test score. The Wilcoxon signed ranks test indicated that there was a significant difference in the pupils' van Hiele levels of geometric thinking about square after the intervention, $Z = -4.38$, $p < 0.01$. The median van Hiele level of 2 in the post-test was higher than the median van Hiele level of 1 in the pre-test, indicating that the intervention had significantly enhanced the pupils' geometric thinking about square.

For regular pentagon, 25 pupils had a higher post-test score than their pre-test score, 1 pupil had similar score in their pre-test and post-test, but none had a lower post-test score than the pre-test score. The Wilcoxon signed ranks test indicated that there was a significant difference in the pupils' van Hiele levels of geometric thinking about regular pentagon after the intervention, $Z = -4.51$, $p < 0.01$. The median van Hiele level of 2 in the post-test was much higher than the median van Hiele level of 0 in the pre-test, indicating that the intervention had significantly enhanced the pupils' geometric thinking about regular pentagon.

Table 4. Wilcoxon signed ranks test for difference in van Hiele levels of geometric thinking

		N	Mean Rank	Sum of Ranks	*Z*	Asymp. Sig. (2-tailed)
Post-test for Equilateral Triangle – Pre-test for Equilateral Triangle	Negative Ranks	0[a]	0.00	0.00	–4.413[d]	0.000
	Positive Ranks	21[b]	11.00	231.00		
	Ties	5[c]				
	Total	26				
Post-test for Square – Pre-test for Square	Negative Ranks	0[a]	0.00	0.00	–4.379[d]	0.000
	Positive Ranks	20[b]	10.50	210.00		
	Ties	6[c]				
	Total	26				
Post-test for Regular Pentagon – Pre-test for Regular Pentagon	Negative Ranks	0[a]	0.00	0.00	–4.512[d]	0.000
	Positive Ranks	25[b]	13.00	325.00		
	Ties	1[c]				
	Total	26				
Post-test for Regular Hexagon – Pre-test for Regular Hexagon	Negative Ranks	0[a]	0.00	0.00	–4.512[d]	0.000
	Positive Ranks	25[b]	13.00	325.00		
	Ties	1[c]				
	Total	26				

Notes: [a] Post-test < Pre-test; [b] Post-test > Pre-test; [c] Post-test = Pre-test; [d] Based on negative ranks

Similarly, for regular hexagon, 25 pupils had a higher post-test score than their pre-test score, 1 pupil had similar score in their pre-test and post-test, but none had a lower post-test score than the pre-test score. The Wilcoxon signed ranks test indicated that there was a significant difference in the pupils' van Hiele levels of geometric thinking about regular hexagon after the intervention, $Z = –4.51$, $p < 0.01$. The median van Hiele level of 2 in the post-test was much higher than the median van Hiele level of 0 in the pre-test, indicating that the intervention had significantly enhanced the pupils' geometric thinking about regular hexagon.

The boxplots showing the distributions of the median van Hiele levels in the pre-test and post-test for equilateral triangle, square, regular pentagon and regular hexagon are presented in Figure 8.

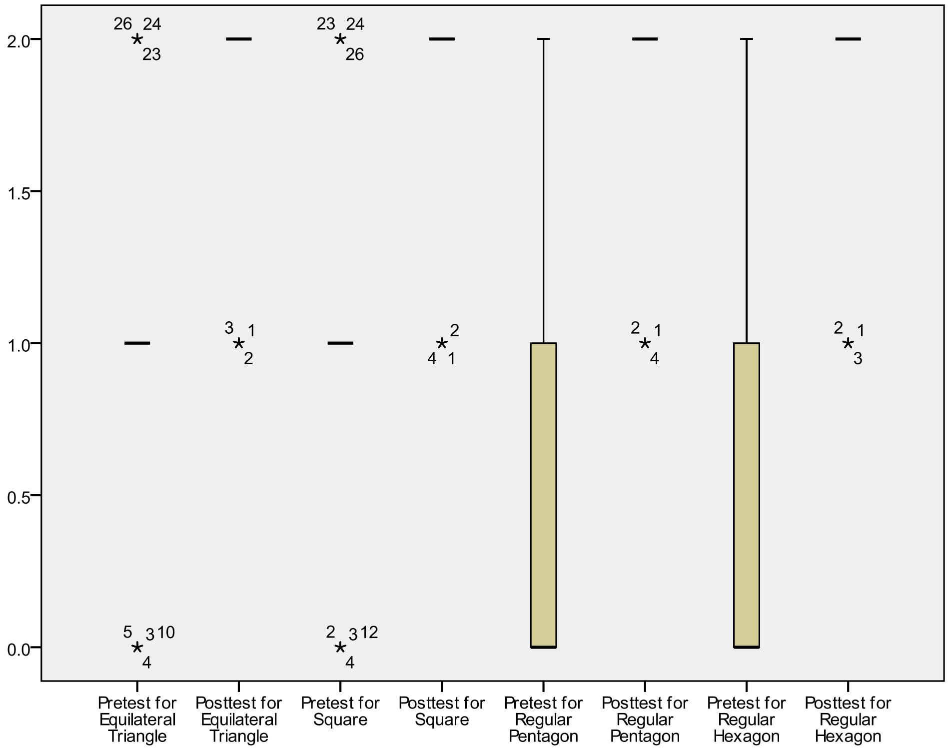


Figure 8. Distributions of the median van Hiele levels in the pre-test and post-test for each regular polygon

DISCUSSION AND CONCLUSION

The results of the pre-test showed that the pupils' initial van Hiele levels of geometric thinking about the regular polygons ranged from Level 0 to Level 2. After phase-based instruction using GSP the results of the post-test revealed that the pupils' van Hiele levels of geometric thinking were predominantly at Level 2 for all the regular polygons indicating that they had progressed from either Level 0 to Level 2 or from Level 1 to Level 2. In addition, the results of the Wilcoxon signed ranks test showed that there was a significant difference in the pupils' van Hiele levels of geometric thinking for all the regular polygons after phase-based instruction using GSP. The median van Hiele level in the post-test was higher than the median van Hiele level in the pre-test for all the regular polygons, indicating that the intervention had significantly enhanced the pupils' geometric thinking about the regular polygons.

The results of this study are consistent with the results of previous studies on phase-based instruction using GSP (Chew, 2007; Choi, 1996; Choi-Koh, 1999). Further, not all the pupils were on the same van Hiele level of geometric thinking across the regular polygons after phase-based instruction using GSP. This result concurs with the findings of Chew's (2007) and Choi's (1996) studies that not all the students were on the same van Hiele level of geometric thinking across cubes and cuboids, and equilateral triangle, isosceles triangle and right-angled triangle, respectively after phase-based instruction using GSP.

The pupils' progress into higher van Hiele levels of geometric thinking about the regular polygons after phase-based instruction using GSP might be attributed to the appropriate use of pre-constructed GSP sketches based on the van Hiele theory (Chew, 2007). The pre-constructed GSP sketches for the regular polygons were employed in phase-based instruction because construction activities which require Level 2 thinking are inappropriate for pupils who are at Level 0 or Level 1. That is pupils at Level 0 or Level 1 have difficulty constructing sketches of the regular polygons in GSP because they do not yet know the properties of the regular polygons (de Villiers, 1999). Besides, the pre-constructed GSP sketches enabled the Level 0 or Level 1 pupils to first investigate visually the shapes of the regular polygons and then analyse their properties instead of getting bogged down in constructing the GSP sketches themselves. In this way, the pupils were better able to recognise the shapes of the regular polygons as well as identify their properties by directly manipulating the pre-constructed GSP sketches to generate many examples of the regular polygons and followed by observations of those actions.

Apart from the appropriate use of pre-constructed GSP sketches, the teacher's guidance and facilitation based on the van Hiele theory might also contribute to the pupils' progress into higher van Hiele levels of geometric thinking about the regular polygons after phase-based instruction using GSP (Chew, 2007). More specifically, during "Inquiry phase" the teacher engaged the pupils in examining examples and non-examples of the regular polygons using GSP. During "Guided Orientation phase", he carefully sequenced the GSP instructional activities for the pupils to examine the properties of the regular polygons. During "Explicitation phase", he encouraged the pupils to share their findings using their own words and then introduced the relevant geometric vocabulary when appropriate. During "Free Orientation phase", he carefully chose appropriate geometric problem for the pupils to solve. Finally, he guided the pupils in reviewing and summarising the properties of the regular polygons in Integration phase.

However, after the intervention four pupils only managed to progress from Level 0 to Level 1 for all the regular polygons. The lack of progress into Level 2 thinking for the four pupils might be attributed to their low achievement in mathematics and English language (Chew, 2007). They seemed to have difficulty remembering geometric terms such as equal sides, equal angles and axes of symmetry. This finding concurs with the results of Fuys and Geddes (1984) as well as (Chew, 2007) that progress into higher levels of geometric thinking was also influenced by instruction and ability, especially language ability. Another possible explanation is that the short period of this study and whole class instruction did not permit sufficient time needed for the four pupils to revisit the relevant phases of learning so as to understand better the properties of the regular polygons.

In conclusion, we acknowledge our limitations in making any generalisations from the results of this study. Nonetheless, the findings of this study suggest for this sample that phase-based instruction using GSP had significantly enhanced the pupils' geometric thinking about the regular polygons. This implies that with well-designed phase-based GSP instructional activities as well as teacher's guidance and facilitation, primary pupils can learn the regular polygons with increasing understanding (Chew, 2007).

REFERENCES

Berg, B. L. (2000). *Qualitative research methods for the social sciences*. Boston: Allyn and Bacon.

Chew, C. M. (2007). *Form one students' learning of solid geometry in a phase-based instructional environment using The Geometer's Sketchpad.* Unpublished doctoral dissertation, Universiti Malaya, Kuala Lumpur.

Choi, S. S. (1996). *Students' learning of geometry using computer software as a tool: Three case studies* (Doctoral dissertation, University of Georgia, 1996). Ann Arbor, MI: UMI Publications.

Choi-Koh, S. S. (1999). A student's learning of geometry using the computer. *Journal of Educational Research, 92*(5), 301–311.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420–464). New York: MacMillan Publishing Co.

Crowley, M. L. (1987). The van Hiele model of the development of geometric thought. In M. M. Lindquist & A. P. Shulte (Eds.), *Learning and teaching geometry, K-12, 1987 Yearbook* (pp. 1–16). Reston, VA: National Council of Teachers of Mathematics.

de Villiers, M. D. (1999). *Rethinking proof with the Geometer's Sketchpad.* Emeryville, CA: Key Curriculum Press.

Driskell, S. O. S. (2004). *Fourth-grade students' reasoning about properties of two-dimensional shapes* (Doctoral dissertation, University of Virginia, 2004). Ann Arbor, MI: UMI Publications.

Elchuck, L. M. (1992). *The effects of software type, mathematics achievement, spatial visualization, locus of control, independent time of investigation, and van Hiele level on geometric conjecturing ability* (Doctoral dissertation, The Pennsylvania State University, 1992). *Dissertation Abstracts International*, *53*(05), 1435A. Retrieved 10 April 2004, from http://wwwlib.umi.com/dissertations/ fullcit/9226687

Frerking, B. G. (1995). *Conjecturing and proof-writing in dynamic geometry* (Doctoral dissertation, Georgia State University, 1995) *Dissertation Abstracts International*, *55*(12), 3772A. Retrieved 10 April 2004, from http://wwwlib.umi.com/ dissertations/fullcit/9507424

Fuys, D., & Geddes, D. (1984). *An investigation of van Hiele levels of thinking in geometry among sixth and ninth grades: Research findings and implications.* Paper presented at the 68th Annual Meeting of the American Educational Research Association, New Orleans, LA, 27 April. (ERIC Document Reproduction Service No. ED245 934).

Gay, L. R., & Airasian, P. (2003). *Educational research: Competencies for analysis and applications* (7th ed.). Upper Saddle River, NJ: Merrill Prentice Hall.

Hatfield, M. M., Edwards, N. T., Bitter, G. G., & Morrow, J. (2000). *Mathematics methods for elementary and middle school teachers* (4th ed.). New York: John Wiley & Sons, Inc.

Hoffer, A. R., & Hoffer, S. A. K. (1992). Geometry and visual thinking. In T. R. Post (Ed.), *Teaching mathematics in grades K-8: Research-based mathematics* (2nd ed., pp. 249–227). Boston: Allyn and Bacon.

July, R. A. (2001). *Thinking in three dimensions: Exploring students' geometric thinking and spatial ability with The Geometer's Sketchpad* (Doctoral dissertation, Florida International University, 2001). Ann Arbor, MI: UMI Publications.

Malaysian Ministry of Education. (2003). *Integrated curriculum for primary schools: Curriculum specifications, Mathematics Year 4.* Kuala Lumpur: Curriculum Development Centre.

Mason, M. M. (1997). The van Hiele model of geometric understanding and mathematically talented students. *Journal for the Education of the Gifted, 21*(1), 39–53.

Mayberry, J. W. (1981). *An investigation of the van Hiele levels of geometric thought in undergraduate preservice teachers* (Doctoral dissertation, University of Georgia, 1981). Ann Arbor, MI: UMI Publications.

McClintock, E., Jiang, Z., & July, R. (2002). Students' development of three-dimensional visualization in the Geometer's Sketchpad Environment. *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (ERIC Document Reproduction Service No. ED 471 759).

Mullis, I. V. S., Martin, M. O., & Foy, P. (2008). *TIMSS 2007 International Mathematics Report.* Boston: TIMSS & PIRLS International Study Center. Retrieved 7 April 2009, from http://timss.bc.edu/TIMSS2007/PDF/ 07/T07_M_IR_Chapter3.pdf

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. (2004). *TIMSS 2003 International Mathematics Report.* Boston: TIMSS & PIRLS International Study Centre. Retrieved 2 February 2007, from http://timss.bc.edu/timss2003/mathD.html

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Gregory, K. D., Garden, R. A., O'Connor, K. M., Chrostowski, S. J., & Smith, T. A. (2000). *TIMSS 1999 International Mathematics Report.* Boston: International Study Centre.

National Council of Supervisors of Mathematics. (1977). Position paper on basic skills. *Arithmetic Teacher, 25*(1), 19–22.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

O'Daffer, P. G., & Clemens, S. R. (1992). *Geometry: An investigative approach* (2nd ed.). Reading, MA: Addison-Wesley Publishing Company.

Sherard, W. H. (1981). Why is geometry a basic skill? *The Mathematics Teacher, 74*(1), 19–21.

Thompson, E. (2006). Euclid, the van Hiele levels, and the Geometer's Sketchpad. (Master's thesis, Florida Atlantic University, 2006). *Masters Abstracts International, 44(*06), 2529. Retrieved 2 February 2007, from http://wwwlib.umi.com/ dissertations/fullcit/1435803

Usiskin, Z. (1980). What should *not* be in algebra and geometry curricula of average college-bound students? *Mathematics Teacher, 73*(6), 413–424.

van de Walle, J. A. (2001). *Elementary and middle school mathematics: Teaching developmentally* (4th ed.). New York: Addison Wesley Longman, Inc.

van Hiele, P. M. (1959/1984). The child's thought and geometry. In D. Fuys, D. Geddes, & R. Tischler (Eds.), *English translation of selected writings of Dina van Hiele–Geldof and Pierre M. van Hiele* (pp. 243–252). Brooklyn, NY: Brooklyn College, School of Education (ERIC Document Reproduction Service No. 287 697).

van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education.* New York: Academic Press.

van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics, 5*(6), 310–316.

van Hiele-Geldof, D. (1959/1984). Last article written by Dina van Hiele-Geldof entitled: Didactics of geometry as learning process for adults. In D. Fuys, D. Geddes, & R. Tischler (Eds.), *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele* (pp. 215–233). Brooklyn, NY: Brooklyn College, School of Education (ERIC Document Reproduction Service No. ED 287697).

Yin, R. K. (1994). *Case study research: Design and methods* (2nd ed.). Thousand Oaks, CA: Sage Publications, Inc.