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Accelerating Cognitive Development Using a Mathematical Thinking Skills Course to Target Metacognitive Processes

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ABSTRAK *Seramai 300 orang pelajar telah mengikuti satu kursus untuk meningkatkan lagi kemahiran pemikiran matematik mereka. Kumpulan ini telah dibandingkan dengan satu kumpulan kawalan melalui ujian awal (pre-test), ujian akhir (post-test) dan temuduga berstruktur. Alat-alat untuk menilai tahap perkembangan kognitif and kebolehan menggunakan kemahiran metakognitif dan strategik telah direka oleh pelapor. Pelajar-pelajar yang mengikuti kursus ini telah mendapat keputusan yang lebih tinggi dan signifikan di dalam ujian akhir metakognitif dan kognitif apabila dibanding dengan kumpulan kawalan. Walaupun kemahiran kognitif tidak diajar secara langsung dalam kursus ini, boleh diandaikan bahawa pengajaran kemahiran metakognitif boleh membantu meningkatkan lagi kemahiran kognitif para pelajar.*

Introduction

"The Practical Applications of Mathematics Project" was an action research project funded by the Welsh Office during 1991/2. The project aimed to develop approaches and materials to teach and assess thinking skills involved in using and applying mathematics in practical, modelling situations, with students aged between 11 and 16. (Tanner & Jones, 1993a, 1993b)

Phase two of the project (funded by the Welsh Office and the University of Wales 1993/4) aimed to develop and evaluate a thinking skills course to accelerate students' cognitive development in mathematics. The practical activities and teaching approaches developed in phase one formed the basis of the course.

Mathematical Thinking Skills

Coles (1993) has identified three dimensions: skills, dispositions and attitudes; which are generic to any discussion of the teaching of thinking. In terms of mathematics, a student would know how to perform a procedure, when and why it should be used, and gain a certain satisfaction from using these skills. Any course purporting to teach thinking would, therefore, have to develop the necessary conceptual knowledge, the metacognitive ability to select the appropriate knowledge and strategies, and also the motivation to succeed at the task.

From a Piagetian viewpoint, adolescence marks the onset of formal thought - the ability to reason from a hypothesis and to see reality as a reflection of theoretical possibilities (Halford, 1978). Formal thought has been described (Sutherland 1992) as a systematic way of thinking; a generalized orientation towards problem-solving with an improvement in the student's ability to organize and structure the elements of a problem. However, these key aspects of problem-solving are metacognitive rather than conceptual in nature. It can be argued, therefore, that formal thought is underpinned by the development of metacognitive skills.

Accelerating Cognitive Development

The mathematics curriculum for adolescents in England and Wales requires students to hypothesize and test, to generalize, and to justify and prove their conclusions. That is, students are required to think on a formal level.

Recent research has suggested that cognitive development can be accelerated (eg: Shayer and Adey, 1992; Novak, 1990; Elawar, 1992). A key feature of these studies has been their deliberate enhancement of metacognitive abilities. Indeed, metacognition has been identified by McGuinness (1993) as a primary tool for conceptual development.

The conception of thinking used in this paper is that of thinking as sense-making (McGuinness, 1993), which is embedded in a socio-constructivist epistemology. Learning is a social activity with both cognitive and affective aspects. The culture of the classroom determines the quality and the nature of the learning that occurs. Successful teaching programmes must take into account the social context of the classroom.

The Thinking Skills Course

There are two strands to the course:

- a structured series of cognitive challenges to stimulate the progressive evolution of key skills in the areas of strategy, logic and communication;
- the use and development of teaching techniques which will encourage the maturation of the metacognitive skills of planning, monitoring and evaluation.

Underpinning both strands is a continual emphasis on the need to explain rather than describe, to hypothesize and test, and to justify and prove.

Activities are structured to encourage the development of a small number of general strategic or cognitive tools. Each activity is targeted on at least one of the schema of formal operations, eg: controlling variables, proportionality, correlation, probability, manipulation of symbols.

Activities do not attempt to develop process skills divorced from content - process skills learned in isolation are unlikely to be integrated into conceptual schema, and courses which fail to focus on content are unlikely to gain general acceptance amongst teachers.

The Structure of The Materials

The Cognitive Strand:

The pilot course ran over a five month period. During this period teachers of experimental classes were asked to select from groups of activities which were responsive to a range of strategies set in key mathematical contexts, linked to Piaget's schemata.

The activities were responsive to a variety of strategies, including:

* identification of variables or attributes;
* systematic working - simple cases first;
* coping with real data - estimating, averaging, error.

The strategies were not addressed separately in the activities - skill in comparing and selecting strategies was required. Each group of activities was responsive to a small number of target strategies and a student who had attempted an activity from each group would have encountered a wide range of strategies. Possible routes through the activities were indicated in the course documentation.

The activities in the course did not address directly the questions used in the test of cognitive ability. We were not "teaching to the test" but were hoping to establish "transfer".

The Metacognitive Strand:

Metacognitive skills were not taught through the content of the materials but through the teaching approaches used. The teaching approaches employed were considered to be more significant than the activities chosen to provide contexts for learning.

Vygotsky (1978) suggests that a child learns by interacting with more capable others who provide sufficient support for the task to be completed. The teacher acts as `a vicarious form of consciousness' (Bruner, 1985 p.24), structuring tasks and controlling the path of solutions until such time as the child achieves conscious control of a new function or conceptual system. Vygotsky viewed such internalization as a social process mediated by language, with external speech used for communication with others and inner speech for planning and self regulation.

Hirabayashi and Shigematsu (1987) argued that students develop their concepts of metacognition by copying their teacher's behaviour, and thus, their executive or control functions represent an `inner' teacher. Vygotsky (1978) suggested that all such higher order functions originate as actual relationships between individuals, thus before students can `internalize' these skills they must develop them explicitly with others. Discussion and questioning within a supportive group leads students to construct a `scaffolding' framework for each other, which enables them to solve problems collaboratively before they can solve such problems individually (Forman and Cazden, 1985).

Several researchers have argued that the use of thinking strategies improve learning and have called for the explicit teaching of such strategies (see Christensen 1991 for a review). However, Christensen found that children who had been explicitly taught learning strategies failed to use them as efficiently or as appropriately as those children who had invented strategies for themselves.

The teaching approaches utilized in the course were intended to develop students' metacognitive skills and, by so doing, to encourage them to construct and evaluate their own strategies. The teaching approaches which were found to be successful in developing metacognitive skills during phase one of the project emphasized social processes. The teaching approach which we hoped the teachers would follow was based on a socio-constructivist viewpoint. That is, mathematics is actively constructed by students rather than transmitted by teachers, and that this construction takes place in a social context. Students validate their constructions against those of others through discussion and debate.

The teaching approaches which were successful in phase one parallel in some ways the sequence of stages described by Lipman (1993) and emphasize the following key aspects:

***Questioning using organizational prompts*:** a list of organizing questions was provided and supplemented with oral questions which were asked on a regular basis, eg: "Can you explain your plan to me?", "Does that always happen?". The aim was to encourage students to develop a framework of questions to organize their thoughts. An expectation developed that such questions would be asked and students seemed able to internalize them for use in planning.

***Internalization of scientific argument*:** groups of students were required to present interim approaches and findings. Presentations were followed up by questioning and constructive criticism. Questioning was led by the teacher at first, with a gradual increase in the amount of student-initiated questioning. Students began to copy the form of question used by the teacher when framing their own. It became clear that groups were anticipating the same form of question about their own presentation and preparing a suitable response. The students were learning how to conduct a scientific argument (Wheatley, 1991).

***Start, stop, go*:** this approach combined the internalization of organizational prompts and scientific argument with an emphasis on self-monitoring and reflection. Tasks began with a few minutes of silent reading and planning. Small groups then discussed possible approaches. A whole class brainstorm followed before returning to small group planning. This ensured that all students engaged with the task and began to plan but that a variety of perceptions and plans was examined and evaluated.

At intervals the class was stopped for reporting back. Students began to anticipate not only the form of questioning which would be used, but also that reporting back would occur. Groups began to monitor their progress in anticipation, which restrained impulsive planning and encouraged self-monitoring.

***Using peer and self assessment to encourage reflection*:** Students were required to write up their work individually, but selected groups also presented their final report to the class for peer assessment. Reflecting on the work of others led students inevitably to reflect back on their own work. Through assessing the work of others, students learned to evaluate and regulate their own thinking.

Students were encouraged to assess their own work against a self-assessment framework for each activity. This formed the basis for a dialogue between the student(s) and the teacher which helped them to understand the criteria against which they were being assessed.

Experimental Design

An action research network of six comprehensive schools, drawing students from a variety of social and ethnic backgrounds, was established. The sample was not random due to the degree of commitment demanded from the teachers involved and consequent difficulties of self selection. It may best be described as an opportunity sample approximating to a stratified sample of English-medium schools in Wales.

The action research paradigm was chosen due to the novelty of some of the activities proposed. Both qualitative and quantitative approaches were used. Two teachers from each school, who were to be involved in teaching intervention lessons, attended an initial one day induction course to familiarise them with the theoretical underpinning to the project and the outcomes of previous work, in particular, effective teaching strategies. Teachers involved in the project attempted to integrate these approaches into their own teaching styles.

Intervention lessons were led by normal class teachers rather than outside "experts". The advantages of this approach in terms of realism, pupil-teacher relationships and teacher development are clear. The approach carries the disadvantage, however, that the experiences of the intervention classes were not standardised. Regular participant observation by the university research team was necessary to record the nature of the interventions made. These observations revealed that the extent to which teachers were able to integrate the approach with their styles was very variable. In one case at least, the attempt to marry contrasting styles resulted in confusion.

Two matched pairs of classes were identified in each school to act as control and intervention groups. One pair was in year seven and one pair in year eight (ages 11-plus and 12-plus). Matched classes were either of mixed ability or parallel sets in every case. Before intervention lessons began, a written assessment paper and an attitude questionnaire was given to each student in the control and intervention classes to act as a pre-test.

In addition to the written paper, the metacognitive skills of a sample of 48 students were assessed through one to one structured interviews while attempting a practical investigation, (a pendulum experiment in the pre-test and a toppling experiment in the post-test). The sample of 48 students was generated by asking teachers to identify one high ability and one low ability student in each of the control and intervention classes. The interview based assessments of metacognitive skills were compared with those obtained in the written papers.

Regular network meetings were held at which experiences were exchanged, strategies discussed and new activities devised and refined.

The pilot course and intervention teaching lasted for approximately five months.

Assessing Cognitive Ability

The written assessment papers are based loosely on a neo-Piagetian framework, in that they assume that children's development progresses through stages and that each of these stages has characteristic forms and limitations of cognitive operation, but that although development may be seen as the formation of increasingly complex cognitive structures, it is limited by the capacity of working memory, (Pascual-Leone, 1976; Halford, 1978; Case, 1985; Boulton-Lewis & Halford, 1991).

Thus the facility of an item is affected by its structural complexity and associated demands on the capacity of working memory as much as its level of cognitive sophistication.

A pragmatic approach was taken to the design of the written paper. The study is set in the context of school mathematics and not the laboratory, so due regard was paid to the National Curriculum for England and Wales. Items were placed in the context of the four content domains: Number, Algebra, Shape and Space, and Probability and Statistics. Items were aimed at testing comprehension rather than simple knowledge.

Items were classified as identifying one of four stages of development, which we referred to as: early concrete, late concrete, early formal and late formal, in line with the Piagetian framework, but account was also taken of the anticipated memory requirements, the assessment structure of the National Curriculum, and the results of large scale studies such as the Concepts in Secondary Mathematics and Science Project (CSMS) and its sequels (Hart, 1981).

The assessment paper was trialled with 60 students from years seven and eight. Items which did not discriminate well within a hierarchy were rejected. Discrimination means that items classified as late formal, for example, should only be successfully completed by children who were generally successful at lower rated items. The final version of the assessment paper included two items at each level in each of the four domains.

An attitude questionnaire involving 45 statements using a Likert type scale which had been trialled and developed for an earlier project (Hendley, Stables, Parkinson and Tanner, 1995) was given to all students at each assessment point.

Assessing Metacognitive Ability

Metacognitive skills are associated with awareness and control of one's own learning, (Brown, 1987). They include an awareness of what one knows and does not know, the ability to predict the success of one's efforts (Royer, Cicero & Carlo, 1993), planning, monitoring and evaluating one's work (Gray, 1991), and an ability to reflect on the learning process and know what one has learned.

Observing essentially hidden metacognitive processes is far from easy, not least because people are adept at using small verbal or non-verbal cues to attempt to provide the responses which they think are expected. Several methods for eliciting information about thinking processes have been identified (Rowe, 1991) and a variety of direct and indirect approaches were used in the project to study students' metacognitive skills.

Assessing Self-Knowledge

One aspect of metacognitive ability assessed in the written paper was awareness of one's own knowledge through the ability to predict one's own accuracy. Each of the four cognitive assessment sections began with the question:

> "There are 8 questions in this section. Read them through now. How many do you think you will get right? __/8."

Each section ended with:

> "Read through your answers. Put a tick in the box next to the question if you think your answer is right. How many do you think you right? __/8."

Planning, Monitoring and Evaluating

The metacognitive skills of question posing, planning, evaluation of results and reflection were assessed through a section in the written paper entitled "Planning and doing an experiment". In this section the students were required to apply their mathematical knowledge to solve a practical problem set in a scientific context. We were testing their ability to use their mathematics in a novel situation.

Students were told that some string and a place to hang it from, a weight holder and some 20g weights, a tape measure and a stop watch were available. They were then asked to think of one interesting question to investigate using the equipment and to write down their plan under the four headings:

My question
My plan
I would take these measurements
How I would present my results.

Answers were assessed according to a set of criteria which focused on factors such as:

- the number of variables investigated eg: "How long does it swing?" or "I would compare swing with weight",
- whether variables were controlled,
- whether a relationship was sought and the quality of that relationship, eg: binary - "long string versus time and short string versus time" or continuous - "time measured for 20cm, 30cm, 40cm, 50cm, etc."
- the presentation of results eg: bar chart, ordered table, graph of ... against ..., seeking an equation or relationship.

The results of an experiment were presented and the students were invited to plot them on a graph, make a prediction, test it against a formula and suggest how the results could have been made more accurate. Different problems allowing similar lines of development were used in the post-tests.

Interviews

Interviews were conducted on a one to one basis between the university researchers and students whilst attempting to organize and conduct a mathematical investigation into a practical task.

Students were assessed through a form of dynamic assessment, (cf: Feuerstein, 1979; Brown & Ferrara, 1985; Newman, Griffin & Cole, 1991). The researchers aimed to provide the minimum level of structure necessary for students to progress. The intention was to work in the student's "zone of proximal development" (Vygotsky, 1978 p.86). Rather than observing students either succeed or fail in a task without intervention, we recorded how much help students required to make progress in a task.

Interviews followed a strict script which included settling down questions, instruction on how to use the equipment and a series of prompts to be used if students failed to progress. The researcher had to make a judgement as to whether a prompt was needed to ensure progress. Interviews were tape recorded and transcribed. Assessments were made against specific criteria for levels of ability in planning, monitoring, evaluating and reflecting during the experiment. These assessments were then checked against transcripts.

Students were encouraged to think aloud during the task by such devices as

> "Pretend that I'm your partner, but I'm not as clever as you. You have to explain things clearly so that I can understand what we are doing."

For the pre-test interviews a simple pendulum was set up by the researcher in front of the student and then dismantled. Students were then asked to set up a similar arrangement for themselves. They were encouraged to keep talking throughout the experiment.

> "Talk to me as much as you can. I'm interested in all your ideas"

Students were then encouraged to identify variables.

> "Your pendulum didn't have to be exactly the same as mine. What things can you think of which you might have changed?"

A series of prompts followed until sufficient variables were identified. They were then asked to hypothesize about which might affect time, using further prompts. They were then asked to set up an experiment to investigate the pendulum.

Marks were awarded for each level achieved in planning, monitoring, evaluating and reflecting. Marks were deducted for prompts given in each section. If prompts exceeded marks achieved, zero was awarded for that section. An example of a criterion statement:

> 3 marks: Shows evidence of planning to control variables and work systematically using binary logic, eg: times for long string and short string.

An example of a prompt:

Prompt 3 : You said we could change ... How could we test to see if it made a difference?

The script was trialled and developed through several different versions before arriving at its final form.

The Reliability of The Assessment Instruments

Analysis of the results of the pretest revealed acceptable internal consistency for the written assessment of cognitive ability (table 1). The internal reliability of the metacognitive interview scale is acceptable (table 2). The shorter scale used to assess reflection produced lower correlations with the other skills.

The levels achieved by year eight students were higher than those achieved by year seven students, as might be expected if the test is assessing a developmental level (Table 3).

The results show a later development of formal thought for this sample than suggested by Piaget and more in line with Sutherland (1992), or Shayer, Kuchemann and Wylam (1976) who found concrete operations attained at an average age of 12 or the first year of secondary school. The median student in the pre-test sample was judged to be late concrete.

The levels achieved in the separate sections on number, algebra, shape and space, and statistics and probability varied. Although there is a significant correlation between the scores gained in these sections (one tailed significance = .001), they lend support to the view that children do not become "formal" in all areas simultaneously. There is horizontal "decalage" or lag.

TABLE I. Reliability of The Cognitive Assessment (pre-test)

	Number	Algebra	Shape	Prob & stats	Metacog
Number	1.0000				
Algebra	.5350	1.0000			
Shape & space	.5304	.4612	1.0000		
Prob & stats	.5574	.5012	.4313	1.0000	
Metacog	.5615	.4824	.4472	.5524	1.0000

(1 tailed significance = .001 for all correlations)

Number of cases = 604
Cronbach's alpha = .8553

TABLE 2. Reliability of Metacognitive Assessment by Interview (pre-test)

	Plan	Monitor	Evaluate	Reflect
Plan	1.0000			
Monitor	.7965	1.0000		
Evaluate	.6882	.6377	1.0000	
Reflect	.4187	.3580	.5078	1.0000

Number of cases = 48
Cronbach's alpha = .8418

TABLE 3. Comparing Cognitive Development for Years 7 and 8

Variable	Number of Cases	Mean	SD	SE of Mean
YEAR 7	307	11.036	4.253	.243
YEAR 8	298	3.307	4.873	.282

Mean Difference = -2.2712 2-tail sig for t-test = .000

Comparing The 3 Different Metacognitive Assessments

The metacognitive skill of self-knowledge was assessed by asking students to predict their score before and after attempting a section of work. Values for this skill were calculated by the following formulae:

FORECAST = [abs(predict1-number)+abs(predict2-algebra) +abs(predict3-shape)+abs(predict4-prob)]/4.

POSTCAST = [abs(right1-number)+abs(right2-algebra) +abs(right3-shape)+abs(right4-prob)]/4.

Similar formulae without the use of absolute values were used to examine the nature of predictive errors.

The great majority of children overestimated rather than underestimated their performance. There was a significant negative correlation between predictive and cognitive ability, and a small but significant negative correlation between predictive and metacognitive ability as measured in the test, (the higher the score in forecast or postcast the lower the accuracy). Students of low cognitive ability were unable to predict whether they would succeed in a question or not (table 4).

Correlations between metacognitive assessments made in the written paper and the interviews were good (table 5). These correlations lend support to the claim that these metacognitive skills are associated. High correlations were found between levels of cognitive ability and metacognitive ability when measured by interview (0.75, table 6), the written test (0.65, table 7) or the forecast prediction (-0.45, table 4).

TABLE 4. Correlations between Forecasting, Postcasting, Cognitive and MetacognitiveAbilities (By Test)

Correlations:	Forecast	Metacog	Cognitive
Forecast	1.0000		
Metacog	-.2711	1.0000	
Cognitive	-.4513	.6353	1.0000

N of cases: 493 (1 tailed significance = .001 for all correlations)

TABLE 5. Correlations Between Metacognitive Assessments

Correlations:	Forecast	Metacog by test	Interview
Forecast	1.0000		
Metacog (test)	-.5257	1.0000	
Interview	-.5226	.5793	1.0000

N of cases: 42 (1 tailed significance = .001 for all correlations)

TABLE 6: Correlations Between Cognitive and Metacognitive Abilities by Test, Prediction and Interview

	Cognitive	Metacog	Metacognitive test
Cognitive ability	1.0000		
Metacognitive interview	.7516	1.0000	
Metacognitive test	.6195	.5908	1.0000
No of cases = 48	1-tailed significance = -.001		

TABLE 7. Correlations Between Cognitive and Metacognitive Abilities by Test

	Metacognitive ability
Cognitive ability	.6504
No of cases = 604	1-tailed significance = .001

Comparing Intervention and Control Classes After The Intervention

The post-test results of the intervention and control classes were compared using analysis of covariance, taking the pre-test score as the covariate in each case. The main hypotheses to be tested through the quasi-experiment were as follows:

H1. Pupils following the course would have their mathematical development accelerated and would improve their scores in the post-test more than the control groups.

H2. The metacognitive skills of the intervention classes, as measured by the metacognitive section of the post-test would be accelerated.

H3. Accelerated cognitive development, as measured by the cognitive sections of the post-test would be observed in classes where metacognitive skills were taught.

The null hypothesis was that there was no difference between the performances of the intervention and control classes.

As shown in table 8, both control and intervention groups improved their performance on the test paper during the study but the analysis reveals a difference in performance between them in favour of the intervention groups which is significant at the .001 (0.1%) level.

Analysis of those sections of the test which assess metacognitive skills shows improved performance by intervention classes and little change in control groups. These differences are significant at the .001 (0.1%) level. The teachers in the intervention classes succeeded in teaching metacognitive skills.

Hypothesis three contended that cognitive acceleration would take place when metacognitive skills had been learned. Qualitative data collected during school visits indicated that the extent to which teachers were able to adopt the required teaching approaches was variable. In three cases it was clear that the required approach was not employed and metacognitive skills were not taught. Data from these schools was therefore rejected.

When these three classes and their associated control groups were removed, covariate analysis of the nine remaining control and intervention pairs revealed accelerated cognitive development for the intervention groups which was significant at the 5% (0.05) level (table 9). These cognitive skills had not been taught directly in the intervention lessons and improvement here could be explained as transfer of learning.

Recent research into the teaching of thinking skills (Shayer & Adey, 1992; Perkins, 1987) suggests that immediate cognitive acceleration should not be expected after the development of metacognitive skills. It may be necessary for students to encounter fresh experiences to interpret with their new skills before advantages can be seen. In fact a deterioration in cognitive scores might have been expected due to the time spent teaching metacognitive skills. This did not occur.

The cognitive sections of the assessment paper probed deep understanding rather than simple recall of facts or algorithms. It may be that the students in the intervention classes were more able to apply the mathematics which they knew due to their enhanced metacognitive skills. Further acceleration may occur as metacognitive skills are applied to the learning of new mathematics. Additional research is needed to determine if the acceleration will be sustained in future public examinations.

Attitudes as revealed by the attitude questionnaire were remarkably stable over the period and comparable between the groups. There was no significant difference in attitude between the groups at the 5% level. The similarities in attitude score suggest that there was little Hawthorne effect at work.

Following the analysis of the post-tests, the project teachers were invited to comment on the results:

Researcher: How do you feel about the results that we have? Do you think that we are right in saying that the intervention group did better than the control group? Or is this just a statistical aberration?

Sue: I definitely think it has helped their thinking skills. I said at the beginning that if you could convince me you could convince anybody because I was completely against it but now, I definitely can see the worth of it.

In the new classes formed for the new academic year some of the teachers now had students from both intervention and control groups. They were convinced that there was a marked difference between such students:

Doreen: Well, the content that they were taught by us last term was exactly the same, both classes have done exactly the same work. But looking at the work this term, the intervention class metacognitively, planning and evaluating and that, the intervention class are, no doubt at all, far better. I have had much better work in from that half of the class - I've got the best of both classes now in the top set in year 9 from the intervention and control groups in year 8. In the investigations they have been far more adventurous in trying to use algebra but they were taught formulas in exactly the same way as the other class.

Sue: Test and homework results this year so far are better from the students from last term's intervention class. They seem to be able to think more clearly.

Joanne: They (previous intervention students) seem to be less bothered by the harder problems - they got right through one of the chapters in one lesson and it's not an easy text book.

Such comments corroborate the statistical findings.

TABLE 8. Overall Intervention/Control Covariate Analysis

OVERALL		N	Mean Pre	Mean Post	SD Pre	SD Post	Prob	I > C ?
TEST	I	265	16.7	21.7	7.4	9.1		
	C	257	16.0	18.7	7.2	7.1	.000	Yes
METACOG	I	266	4.1	6.4	3.3	4.0		
	C	259	4.0	4.2	3.3	3.3	.000	Yes
COGNITIVE	I	266	12.6	15.3	4.8	5.6		
	C	257	11.9	14.4	4.7	4.5	.282	ns
ATTITUDE	I	189	3.5	3.5	0.5	0.5		
	C	192	3.5	3.5	0.5	0.5	.417	ns

TABLE 9. Post-Test V Pretest - Covariate Analysis (Valid Classes)

VALID CLASSES		N	Mean		SD		Prob	I > C ?
			Pre	Post	Pre	Post		
TEST	I	214	17.4	22.9	7.3	8.8		
	C	203	16.8	19.3	7.0	7.1	.000	Yes
METACOG	I	214	4.4	7.0	3.4	4.1		
	C	204	4.3	4.5	3.3	3.4	.000	Yes
COGNITIVE	I	215	13.1	16.0	4.6	5.3		
	C	203	12.5	14.8	4.5	4.4	.014	Yes
ATTITUDE	I	161	3.5	3.5	0.5	0.5		
	C	161	3.5	3.5	0.5	0.5	.097	ns

Conclusion

The results of the post-tests indicate that the first aim of the project has been achieved, namely that metacognitive skills have been successfully taught. This close transfer was not unexpected but, in view of the limited time-scale of the intervention project, it was surprising that the differences between control and intervention groups were so pronounced.

In contrast to the metacognitive skills, the cognitive sections of the test had not been taught directly. Improvement in these sections may be explained by transfer of learning. Three possible mechanisms are suggested. The improvement in the students' metacognitive skills may have enabled them to apply their existing mathematical knowledge more effectively. Their improved thinking skills may have enhanced their learning of "normal school mathematics" by enabling them to organise their mathematical learning more effectively. An expectation may have developed that mathematics should be understood rather than simply memorized, resulting in deep learning.

Both the quantitative and qualitative data strongly suggest that cognitive acceleration can be achieved as a result of improved metacognition. The close association between cognitive and metacognitive skills reported here emphasizes the need for longer term studies in this area.

NOTE

All names are, of course, pseudonyms.

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